Review
Author(s): J. Milne
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troublesome in the complex case. As a matter of fact, when Mr. Bromwich discusses this point in connection with complex series, he gives an ambiguous definition, while his practice is at times certainly inconsistent with his earlier definitions. If the distinction between oscillatory and other non-convergent series is wanted at all, I believe that it would be better to use qualifying adjectives, as some German writers do, such as improperly divergent and properly divergent, and to restrict the former name to non-convergent series, such as $1-1+1-1...$, in which the sum of any number of terms always lies between two fixed limits.

But I should be sorry to end this review on a note of criticism. I began by complaining of the poverty of English writing on series; let me end by expressing the opinion that Mr. Bromwich's book is decidedly more complete, and in other ways more satisfactory, than any of the continental treatises with which it is at all comparable. ARTHUR BERRY.

King's College, Cambridge.

Modern Geometry. by C. GODFREY, MA., and A. W. SIDDONS, M.A. 162 pp. Cambridge University Press.

The authors state in their preface that the treatise covers the schedule of Modern Plane Geometry required for the Special Examination in Mathematics for the Ordinary B.A. degree at Cambridge, and that they have also had regard to the requirements of students in Physics and Engineering. The time seems almost to have arrived for entering a protest against mathematical text-books being framed too much to suit the special needs of this or any other favoured class of students, and it is a pity that a book bearing the comprehensive title *Modern Geometry* should be subject to such narrow limitations.

The work consists of 13 chapters bearing the headings: the sense of a line, infinity, the centroid, the triangle, the theorems of Ceva and Menelaus, harmonic section, pole and polar, similitude, miscellaneous properties of the circle, the radical axis and coaxal circles, inversion, orthogonal projection, cross-ratio, and the principle of duality. The first three chapters are brief, but clear, although we hardly think that a student's objection to the convention that "a point at infinity may be infinitely distant from itself" would be satisfied by the explanation that "points at infinity do not enjoy all the properties of ordinary points."

In the chapter on the triangle, the authors do not hesitate to employ trigonometrical notation, and they give us a large and well selected number of the properties which exist between the elements of a triangle and the radii of the associated circles, but although they deal pretty fully with the N.P.C. they omit to give us a demonstration of Feuerbach's Theorem. For the past 30 years this has been an object of investigation to many geometricians with more or less success, and the recent proof by Ramaswami Ayar, as revised by R. F. Davis, is as near the ideal proof as can be expected, and is of a sufficiently elementary character to be admitted into a chapter dealing with the properties of the N.P.C. In chapter vii the authors do not seem quite happy in their treatment of pole and polar, owing to their exercising the self-denial of making no reference to imaginary points, due no doubt to their respect for the feelings of the practical engineer. We would suggest that they would have done better to have taken as their fundamental proposition Ex. 262, viz. "if H be the harmonic conjugate of a fixed point T with regard to the points in which a line through T cuts a fixed circle, the locus of H is a straight line," and then taken T and the locus of H as their definition of pole and polar. The various properties would then have followed as the natural consequences of harmonic section.

In the chapter giving miscellaneous properties of the circle we have four sections dealing with orthogonal circles, the circle of Apollonius, Ptolemy's Theorem, and contact problems. In §§ 1, 4 the proofs are, as in many other cases, left to the reader. This withholding of the proof, when judiciously exercised, is a distinct advantage, but in the case of describing a circle touching three given circles it would have been advisable to give references to some of the numerous text-books which have treated this classical problem fully.

The chapter on coaxal circles is carefully done and well illustrated. Here the student's attention might have been drawn to the existence of the second common chord, so as to prepare him to find these lines occurring in pairs in conics.

The chapters on inversion and orthogonal projection are full of interest, the proofs and figures of the former being especially neat. The chapter on cross-ratio is somewhat disappointing. Of course in a subject which occupies the greater part of Chasles's *Géométrie Supérieure* it is difficult to give very much in a dozen pages, but considering its fascinating character and its supreme importance we expected, at least, an introduction to homographic ranges and their double points when they are on the same straight line, and the fundamental properties of involution, but we are not even given a method of finding the fourth point of a range in which the other three points and the cross-ratio are known.

The work concludes with a very suggestive chapter on the principle of duality, and deals with the complete quadrilateral and quadrangle, self-polar triangle, and Desargues' Theorem on triangles in perspective, followed by a useful index. In addition to the text there are 679 exercises which appear to be well chosen, and will afford the reader abundant scope for his ingenuity. But we note that Nos. 617, 650, and 654 are respectively the same as 601, 608, and 606.

Taken as a whole we can give a hearty welcome to the book, which is well arranged with good figures, and we especially welcome the few notes of human interest which we should like to see more generally introduced into mathematical text-books. They might with advantage be expanded in a second edition, for which we anticipate an early demand, *e.g.* no student can help being interested when he is told that Pascal's Theorem, as far as relates to a pair of lines, was known to Euclid (B.C. 300), and employed by him without proof in his books on Porisms, and that a proof was given by Pappus 600 years afterwards. In 1640 the theorem was given by Pascal without proof as a property of the circle, and 166 years later its correlative was published by Briauchon, in 1806. An enquiring pupil, however, would hardly be satisfied with the note on p. 20 that Apollonius (260-200 B.C.) studied and probably lectured at Alexandria, and was nick-named ϵ.

J. MILNE.

Magic Squares and Cubes. By W. S. ANDREWS and others. Chicago: The Open Publishing Company.

The construction of magic squares has a curious fascination which appeals to the mystic and the ignorant, as well as to the mathematician. The theory of the formation of simple squares has long been worked out, save for two problems. One of these, probably beyond our powers, is the number of squares of the fifth (or any higher) order. The other is a rule to enable us from any given square of the nth order to produce another square of order $n+2$ by adding $2(n+1)$ to each number in the original square, and surrounding it by a border of the remaining $4(n+1)$ numbers. It is not difficult to border a given square empirically, but a definite rule to enable us to do it in general terms is still wanting. Of late attention has been mainly directed to the construction of squares where additional conditions are imposed, and recently French mathematicians have turned their attention to the formation of double and triple magic squares.

In the earlier chapters of the book mentioned above, the condition is imposed that the square must be such that the sum of any two numbers geometrically equidistant from its centre shall be constant. A large number of the usual constructions are not affected by this restriction. The subject is treated mathematically, and bordered squares are discussed, but the writer does not seem to be aware that the essential part of his diagonal rule for odd squares was given by De Laloubère in the seventeenth century, and that the construction of compound magic squares was mentioned by Montucla in his edition of Ozanam's *Recreations*.

The interest of the book lies, however, rather in its philosophical and quasi-paradoxical parts. Dr. Carus chats about this aspect of the subject, and gives examples of magic squares in China and India. In the Introduction he says: "Magic squares are a visible instance of the intrinsic harmony of the laws of number, and we are thrilled with joy at beholding this evidence which reflects the glorious symmetry of the cosmic order." And again, they "are like a magic mirror which reflects a ray of the symmetry of the divine norm immanent in all things, in the immeasurable immensity of the cosmos not less than the mysterious depths of the human mind." This will appeal to the mystic philosopher more than to the prosaic mathematician.

Mr. Browne also contributes to the work an attempt to explain the Platonic