THE CEPHALIC INDEX

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I

The top views of the skulls of the races of man show great differences in form. For this reason the shape of the skull has come to be one of the best-studied racial characters. The more or less elongated form of the skull has been proved to be a good means of characterizing varieties of man. The degree of elongation is concisely expressed by the proportion between the antero-posterior diameter or length and the transversal diameter or breadth of the skull. Generally the latter diameter is expressed in percents of the former, and this value is called the cephalic index. The object of the following investigation is a study of the biological significance of this index.

The statement that in a certain race the breadth of the skull is, on the average, a certain percentage of its length, would seem to imply that there exists a certain characteristic relation between length and breadth, so that individuals of a certain length of head would, on the average, have a breadth of head corresponding to the length multiplied by the cephalic index. It is well known that this is not the case, but that the heads which have absolutely the greatest lengths have the lowest indices. I have obtained the following results from a study of 239 Sioux Indians which were measured for me by Mr G. H. Kaven in 1892:

Number of Individuals.	*Length of Head.*	*Correlated Cephalic Index.*
11	180–184 mm.	84.9
35	185–189	81.5
86	190–194	79.9
70	195–199	78.3
25	200–204	77.6
12	205–210	75.9

The cephalic index is also greatly influenced by causes other than the length and breadth of head. A comparison of stature, height of face, and breadth of face with the cephalic index of the same series of Sioux Indians, is given in the following table:

Number of Individuals.	*Stature.* (mm.)	*Correlated Cephalic Index.* (%)	*Number of Individuals.*	*Height of Face.* (mm.)	*Correlated Cephalic Index.* (%)	*Number of Individuals.*	*Breadth of Face.* (mm.)	*Correlated Cephalic Index.* (%)
16	1600–1649	81.6	27	115–119	78.8	18	135–139	78.3
49	1650–1699	78.9	71	120–124	80.0	44	140–144	78.5
86	1700–1749	79.4	60	125–129	80.2	85	145–149	78.6
68	1750–1799	79.6	55	130–134	79.1	85	150–154	80.5
19	1800–1849	78.4	16	135–139	78.1	17	155–159	81.1

Although these values do not change quite regularly, they clearly show correlations between the three measurements which I selected and the cephalic index.

The index—like all other indices—is a complex value depending, as it does, on two measurements. In order to gain an insight into its significance, it will be best to investigate the correlations of its constituent elements. The correlation of length and breadth of head, determined from a series of 923 male adult Indians of the Sioux, Ojibwa, and Crow tribes, is as follows:

Group.	I	II	III	IV	V	VI
Length of Head (mm.)	180–184	185–189	190–194	195–199	200–204	205–209
Average Breadth of Head.	153.8	153.8	154.8	156.2	157.8	159.4

This table shows that the average breadth of head of individuals whose length of head is very great or very small, differs little from the average breadth.

When we investigate the correlation of length and breadth of head with stature, it is found that the length of head is more influenced by stature than the breadth of head. Correlation between breadth of face and horizontal diameters of the head shows the two transversal diameters to be very closely correlated, while the length of head is more closely correlated with height of face. The following table illustrates these observations:

243 Sioux Indians. (Adult males, 20–59 years.)

Number of Individuals.	*Stature.* (mm.)	*Correlated Length of Head.* (mm.)	*Correlated Breadth of Head.* (mm.)
16	1600–1649	187.5	153.1
49	1650–1699	193.3	152.6
86	1700–1749	194.1	154.3
68	1750–1799	194.5	155.3
19	1800–1849	196.5	154.3

Number of Individuals.	*Height of Face.* (mm.)	*Correlated Length of Head.* (mm.)	*Correlated Breadth of Head.* (mm.)
27	115–119	191.3	150.9
71	120–124	192.1	154.8
60	125–129	193.3	154.6
55	130–134	196.0	154.7
16	135–139	198.7	155.5

Number of Individuals.	*Breadth of Face.* (mm.)	*Correlated Length of Head.* (mm.)	*Correlated Breadth of Head.* (mm.)
8	135–139	187.8	147.3
44	140–144	190.9	150.1
85	145–149	194.4	152.9
86	150–154	194.5	156.5
17	155–159	197.1	160.0

When we wish to consider the joint influences of all these causes upon either length or breadth of head, we must subdivide each group according to the varying values of the new cause and calculate the correlated averages of the breadth of head. It will be seen at once that, in order to carry on the investigation in this manner, practically unlimited material must be available, otherwise the number of individuals in each class will be very small, owing to the great number of classes. Problems of this kind were first discussed by Francis Galton,[1] but the method of treatment has been fully developed by Karl Pearson.[2]

It may be well, on account of the importance of this method, to give an elementary deduction. The correlations of several organs are to be investigated, the first of which has the average value l_1. The various observed values of this organ may be expressed by $l_1 + x_1$, where x_1 designates the difference between an observed value and the average value. The second, third, and fourth organs have the average values, respectively, l_2, l_3, l_4, . . .

[1] *Natural Inheritance.*

[2] *Mathematical Contributions to the Theory of Evolution*, III; Philosophical Transactions of the Royal Society of London, vol. 187, pp. 253 ff.

and the observed values may be expressed by $l_2 + x_2$, $l_3 + x_3$, $l_4 + x_4$, . . . The values x_1, x_2, . . . are determined by a great many very small causes which we will call ε. Then we may say—

$$\begin{aligned} x_1 &= \alpha_{11}\,\varepsilon_1 + \alpha_{12}\,\varepsilon_2 + \alpha_{13}\,\varepsilon_3 + \ldots \\ x_2 &= \alpha_{21}\,\varepsilon_1 + \alpha_{22}\,\varepsilon_2 + \alpha_{23}\,\varepsilon_3 + \ldots \\ &\cdot \\ &\cdot \\ x_n &= \alpha_{n1}\,\varepsilon_1 + \alpha_{n2}\,\varepsilon_2 + \alpha_{n3}\,\varepsilon_3 + \ldots \end{aligned}$$

where the α are constants. When we wish to investigate the correlations of the series of organs with the first organ, we may substitute the value of ε_1 from the first equation and we find :

$$(1) \qquad \left\{ \begin{aligned} x_2 &= q_{21}\,x_1 + \beta_{12}\,\varepsilon_2 + \beta_{13}\,\varepsilon_3 + \ldots \\ &\cdot \\ &\cdot \\ x_n &= q_{n1}\,x_1 + \beta_{n2}\,\varepsilon_2 + \beta_{n3}\,\varepsilon_3 + \ldots \end{aligned} \right.$$

Here the q and β are new constants which result from the substitution. Since the causes ε are subject to chance only, their averages will disappear. If, then, we assume x_1 to be constant, we have

$$(2) \qquad \text{Average } x_n = q_{n1}\,x_1.$$

Now we will form the products $x_1\,x_n$. These may be arranged in such order that all the x_1 that have equal values shall be grouped together. Then each of these values for x_1 will be multiplied by the sum of all the correlated values of x_n. Supposing there are p individuals in which the first organ has the deviation from its average x_1, we may substitute for the sum of all the correlated values x_n p times the average of x_n, or according to (2), $p\,q_{n1}\,x_1$. Therefore the sum total

$$\Sigma\,x_1\,x_n = \Sigma\,x_1\,q_{n1}\,x_1 = \Sigma\,q_{n1}\,x_1^2.$$

Now, the sum of all the x_1^2 is equal to the mean square variation (or standard variation) of l_1, which we will call μ_1. Therefore

$$\Sigma\,x_1\,x_n = \Sigma\,q_{n1}\,\mu_1^2,$$

or when n the total number of individuals measured

$$= n q_{n1}\,\mu_1^2.$$

We can show, in the same manner, that

$$\Sigma x_1 x_n = nq_{1n} \mu_n^2.$$

Therefore

$$q_{n1} \mu_1^2 = q_{1n} \mu_n^2.$$

We will call

$$q_{n1} = r \frac{\mu_n}{\mu_1};$$

Then

$$q_{1n} = r \frac{\mu_1}{\mu_n}, \qquad \text{and}$$

$$\Sigma x_1 x_n = n r \mu_1 \mu_n$$

(3)
$$r = \frac{\Sigma x_1 x_n}{n \mu_1 \mu_n}$$

We call r the coefficient of correlation, q the coefficient of regression, because it measures the regression of the correlated value toward the average.[1]

It remains to determine the variability of the array of values x_n which are correlated to a given value x_1. According to (1) this variability does not depend upon x_1, since the ε are not functions of x_1, but are entirely independent. The variability of each array is equal to the mean square of the differences between the members of the array and their average. The latter is $l_n + q_{n1} x_1$, while the value for each member of the array is $l_n + x_n$. Their difference is

$$x_n - q_{n1} x_1$$

and the mean square variability of the array

$$\begin{aligned} \Sigma (x_n - q_{n1} x_1)^2 &= \Sigma x_n^2 + \Sigma q_{n1}^2 x_1^2 - 2 x_n q_{n1} x_1. \\ &= \Sigma x_n^2 - \Sigma q_{n1}^2 x_1^2 \ . \end{aligned}$$

Since the value on the left-hand side of this equation remains the same for all values of x_1, we may substitute for it ρ, the variability of the array. If we form the sum of these equations for all the values of x_1, we have

$$\Sigma x_n^2 = n \mu_n^2,$$
$$\Sigma x_1^2 = n \mu_1^2.$$

Therefore

$$n \rho^2 = n \mu_n^2 - n q_{n1}^2 \mu_1^2$$
$$\rho = \mu_n \sqrt{1 - r^2}$$

[1] See Galton, *Natural Inheritance*, pp. 95 ff.

II

We will apply this result to a discussion of the correlation between length and breadth of head. Pearson[1] has calculated the following data:

Adult Male Crania:	r
Ancient Egyptians (Flinders-Petrie)........	0.2705
Bavarian peasants (Ranke)................	0.2849
Modern Parisians (Broca).................	0.0474

I have calculated the coefficients of correlation for the following series:

	r
Sioux, living, 243 adult males..................	+ 0.24
Skulls of 57 adult male Sioux Indians[2]..........	+ 0.24
Skulls of 47 adult male Eskimo, Smith sound[3]...	+ 0.47
Indians of the northern coast of British Columbia; adult males..............................	+ 0.08
Shuswap Indians; adult males.................	+ 0.04
Skulls of adult males, Baden[4]..................	+ 0.09
Bágdi caste of Bengal; adult males[5]...........	+ 0.13

It appears from these data that the degree of correlation between length and breadth of the head is very slight, and that its values differ considerably among various races. The latter fact is rather surprising, since we might expect that in the human species the same organs are subject to the same laws. The coefficient of correlation in the Parisians, for instance, is exceedingly low, in fact so low that we may say that the breadth of head in the Parisians is entirely independent of the length of head.

The most plausible explanation of this phenomenon lies in the effect of mixture of types upon the coefficient of correlation.

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[1] *Loc. cit.*, pp. 279, 280.

[2] George L. Otis, *List of Specimens in the Anatomical Section of the U. S. A. Medical Museum*, 1880, pp. 104 ff.

[3] Ibid., pp. 9 ff.

[4] *Die Schädel in der Grossherzoglichen Anatomischen Anstalt zu Heidelberg; Archiv für Anthropologie*, vol. 24, pp. 24 ff.

[5] H. H. Risley, *The Tribes and Castes of Bengal*, vol. I, pp. 16 ff.

Supposing two types which have the measurements $l_1, l_2, \ldots l_n$, and $\lambda_1, \lambda_2, \ldots \lambda_n$, respectively, inhabit the same area, so that it is impossible to determine to what type each individual belongs. If the component types are not known, we should find for the whole series the averages $L_1, L_2, \ldots L_n$ with the variabilities $M_1, M_2 \ldots M_2$, and the coefficient of correlation calculated according to the method given above would be

$$R = \frac{\Sigma X_1 X_n}{n M_1 M_n}$$

If we divide this sum into two groups, the one embracing all the individuals of one type, the other those of the other type, we may substitute for the former

$$\begin{aligned} L_1 + X_1 &= l_1 + x_1, & X_1 &= l_1 - L_1 + x_1 \\ L_n + X_n &= l_n + x_n; & X_n &= l_n - L_n + x_n \end{aligned}$$

for the latter

$$\begin{aligned} L_1 + X_1 &= \lambda_1 + \xi_1 & X_1 &= \lambda_1 - L_1 + \xi_1 \\ L_n + X_n &= \lambda_n + \xi_n & X_n &= \lambda_n - L_n + \xi_n \end{aligned}$$

$$R = \frac{\Sigma (x_1 + l_1 - L_1)(x_n + l_n - L_n)}{n M_1 M_n} + \frac{\Sigma (\xi_1 + \lambda_1 - L_1)(\xi_n + \lambda_n - L_n)}{n M_1 M_n}$$

$$= \frac{\Sigma x_1 x_n}{n M_1 M_n} + \frac{\Sigma \xi_1 \xi_n}{n M_1 M_n} + \frac{n_1 (l_1 - L_1)(l_n - L_n) + n_2 (\lambda_1 - L_1)(\lambda_n - L_n)}{n M_1 M_n}$$

and by substituting the values of r_1, r_n and of

$$L = \frac{n_1 l + n_2 \lambda}{n}$$

$$R = r_1 \frac{n_1 \mu_1 \mu_n}{n M_1 M_n} + r_n \frac{n_2 \mu_1' \mu_n'}{n M_1 M_n} + \frac{n_1 n_2 (l_1 - \lambda_1)(l_n - \lambda_n)}{n^2 M_1 M_n}$$

Supposing that in the component elements of the series r_1 and r_n, μ_1 and μ_1', μ_n and μ_n' are equal, we have

$$R = r \frac{\mu_1 \mu_n}{M_1 M_n} + \frac{n_1 n_2 (l_1 - \lambda_1)(l_n - \lambda_n)}{n_1^2 M_1 M_n}$$

It will be noticed that whenever the second term of this sum is negative, R will be less than r. This will be the case when l_1 is larger than λ_1 and l_n smaller than λ_n, or vice versa.

I think this effect of mixture is a sufficient explanation of the low value of the coefficient of correlation for Paris, where we find a very heterogeneous population embracing narrow- and long-headed subjects from northern France and very broad- and short-headed ones from southern France. The same explanation seems plausible for the Shuswap and the tribes of northern British Columbia. The high value for Eskimo skulls may be due to the fact that a number of female skulls were counted as male skulls. This would have the effect of raising the coefficient of correlation between the diameters of the head.

When the type is dis-homogeneous owing to mixed descent, the coefficient of correlation will depend upon the law of heredity. If the mixed race should range in a probability curve around a type intermediate between the parental types, we should expect to find the coefficient of correlation slightly influenced by mixture, if any. If the mixture should result in a tendency of reversion to either parental type, the effect would be similar to that observed in the case of mechanical mixture which was discussed before. Evidently in such cases the coefficient of correlation has no direct biological significance.

III

It is quite evident that both breadth and length of head are influenced by a great number of causes, some of which act upon both measurements in the same manner, while others may influence each in a peculiar way. Such causes may be investigated by determining their correlations with length and breadth of head separately. Thus, we may determine the correlation between capacity of skull, stature, dimensions of face, and the two diameters of the head. I have calculated a number of such correlations for adult males of a few Indian tribes with the following results :

Coefficient of Correlation.

	Indians of Northern Coast of British Columbia.	*Shuswap Indians.*	*Sioux Indians.*
Stature and Length of Head..........	....	....	+ 0.26
Stature and Breadth of Head.........	....	....	+ 0.09
Breadth of Face and Length of Head..	+ 0.30	+ 0.28	+ 0.27
Breadth of Face and Breadth of Head.	+ 0.53	+ 0.62	+ 0.52
Height of Face and Length of Head..	+ 0.24	+ 0.27	+ 0.30
Height of Face and Breadth of Head..	— 0.10[1]	+ 0.22	+ 0.22

If we desire to eliminate the influences of these causes from the *apparent* correlation between length and breadth of head, we must consider each of these a function not only of the other, but also of all the measurements the effect of which we desire to exclude. In doing so, we may pursue the same method that we used in the beginning (p. 450) and by a series of substitutions in (1) we find

(4) Average $x_1 = q_{123\ldots p}\, x_2 + q_{132\ldots p}\, x_3 + \ldots\, q_{1p2\ldots(p-1)}\, x_p$

The values q may be determined in the following manner. We will call

$$q_{abc\ldots} = r_{abc\ldots} \cdot \frac{\mu_a}{\mu_b} \tag{5}$$

By multiplying (4) successively with $x_2, x_3 \ldots x_p$, we have

$$(6)\left\{\begin{array}{l} r_{12} = r_{123\ldots p} + r_{132\ldots p}\, r_{32} + \ldots + r_{1p2\ldots(p-1)}\, r_{p2} \\ r_{13} = r_{123\ldots p}\, r_{23} + r_{132\ldots p} + \ldots + r_{1p2\ldots(p-1)}\, r_{p3} \\ \cdot \\ \cdot \\ \cdot \\ r_{1p} = r_{123\ldots p}\, r_{2p} + r_{132\ldots p}\, r_{3p} + \ldots + r_{1p2\ldots(p-1)} \end{array}\right.$$

By multiplying the last of these equations successively by $r_{p2}, r_{p3} \ldots$ and subtracting from the first, second ... equation, we find

$$\begin{array}{l} r_{12} - r_{1p} r_{p2} = r_{123\ldots p}(1 - r_{2p} r_{p2}) + r_{132\ldots p}(r_{32} - r_{3p} r_{p2}) \\ \cdot \qquad + \ldots\ldots + r_{1(p-1)2\ldots p}(r_{(p-1)2} - r_{(p-1)p} r_{p2}) \\ \cdot \\ \cdot \end{array}$$

[1] This low value may be due to the cause that on the coast of British Columbia two types come into contact, one with a very high face, the other with a low face. Their mixture would probably result in a decrease of the correlations for height of face in the mixed race.

We call

$$\frac{r_{12} - r_{1p} r_{p2}}{1 - r_{2p} r_{p2}} = r_{12p};$$

$$\frac{r_{13} - r_{1p} r_{p3}}{1 - r_{3p} r_{p3}} = r_{13p} \quad \text{etc.}$$

$$(7) \begin{cases} r_{12p} = r_{123\ldots\ldots p} + r_{132\ldots p}\, r_{32p} + \ldots + r_{1(p-1)2\ldots p}\, r_{(p-1)2p} \\ r_{13p} = r_{123\ldots\ldots p}\, r_{23p} + r_{132\ldots p} + \ldots + r_{1(p-1)2\ldots p}\, r_{(p-1)3p} \\ \vdots \\ r_{1(p-1)p} = r_{123\ldots p}\, r_{2(p-1)p} + r_{132\ldots p}\, r_{3(p-1p)} + \ldots + r_{(1p-1)2\ldots p} \end{cases}$$

It will be seen that (7) is identical with (6) except insofar as to all the r representing correlations between two measurements, the element p has been added, and insofar as the number of equations and of unknown quantities has been decreased by one. We may, therefore, continue a series of successive substitutions which will always result in equations of the same form. The next substitution would be

$$r_{12(p-1)p} = \frac{r_{12p} - r_{1(p-1)p}\, r_{(p-1)2p}}{1 - r_{2(p-1)p}\, r_{(p-1)2p}}$$

The last substitution will give us

$$r_{123\ldots p} = \frac{r_{12(45\ldots p)} - r_{12(45\ldots p)}\, r_{32(45\ldots p)}}{1 - r_{23(45\ldots p)}\, r_{32(45\ldots p)}}$$

Thus the coefficients of correlation between p variables may be reduced to those between $(p - 1)$ variables.

The variability ρ of the array of x_1 which is correlated to a series of values for $x_2, x_3 \ \ldots$

$$\begin{aligned}\rho^2 &= \text{Average of } (x_1 - q_{123\ldots}\, x_2 - q_{132\ldots}\, x_3 - \ldots)^2 \\ &= \text{Average of } (x_1^2 + q_{123}^2\ldots x_2^2 + q_{132}^2\ldots x_3^2 + \ldots \\ &\qquad - 2q_{123\ldots}\, x_1 x_2 - 2q_{132\ldots}\, x_1 x_3 - \ldots \\ &\qquad + 2q_{123\ldots}\, q_{132\ldots}\, x_2 x_3 + \ldots)\end{aligned}$$

By substituting the values of the types r and μ^2 for the averages of the types xx and x^2 and for q its related r, according to (5), we find

$$\begin{aligned}\rho^2 = \mu_1^2 (1 &+ r_{123}^2\ldots + r_{132}^2\ldots + \ldots \\ &- 2r_{123\ldots}\, r_{12} - 2r_{132\ldots}\, r_{13} - \ldots \\ &+ 2r_{123\ldots}\, r_{132\ldots}\, r_{23} + \ldots)\end{aligned}$$

By introducing in the negative elements of this sum the values for $r_{12}, r_{13} \quad . \quad . \quad .$ from (6), we obtain

(8) $\rho^2 = \mu_1^2 (1 - r_{123\,..}^2 - r_{132\,..}^2 - 2 r_{132\,..}\, r_{132\,..}\, r_{32} - \,..),$

or in a shorter form

$$\rho^2 = \mu_1^2 \left[1 - \Sigma \left(r_{1a\,..\,p} \cdot r_{1b\,..\,p} \cdot r_{ab}\right)\right],$$

in which sum a and b must be made to assume all the values from 2 to p.

IV

I have treated, according to this method, the measurements of the 57 skulls of adult male Sioux Indians to which I referred above (page 453). I have compared length (l) and breadth (b) of skull with its height (h), the bizygomatic diameter of the face (z), and with the capacity of the skull. For the last purpose I have made a reduction which seemed necessary, because the capacity of the skull is a cubic measure, while all the others are linear measures. For this reason I have compared the latter with the cubic root of the capacity of the skulls (c). The averages and variabilities of these measurements are as follows:

	c	*l*	*b*	*h*	*z*
Average (mm.)........	112.9	181.7	143.5	133 0	141.6
Standard variability....	± 2.6	± 6.3	± 5.6	± 4.7	± 6.3

The following coefficients of correlation were found:

I. *Coefficients of Single Correlation.*

Determined by:

	c	*l*	*b*	*h*	*z*
Average c......	....	+0.54	+ 0.67	+ 0 44	+ 0.49
Average l.......	+ 0.54	....	+ 0.24	+ 0.36	+ 0.39
Average b......	+ 0.67	+ 0.24	....	0.00	+ 0.61
Average h......	+ 0.44	+ 0.36	0.00	....	+ 0.19
Average z......	+ 0.49	+ 0.39	+ 0.61	+ c.19	....

It appears from this table that the correlations of the diameters with the capacity are strongest, while that between length and breadth is one of the lowest values in the table. This condition is still more strongly brought out when double, triple, and quadruple correlations are considered.

II. *Coefficients of Double Correlation.*

Average c Determined by:				*Average l Determined by:*				*Average b Determined by:*			
l	b	h	z	c	b	h	z	c	l	h	z
+0.40	+0.57	....	....	+0.69	−0.22	....	....	+0.76	−0.17	....	....
+0.44	....	+0.29	....	+0.47	....	+0.15	....	+0.83	....	−0.36	....
+0.41	....	....	+0.33	+1.46	....	....	+0.17	+0.49	....	....	+0.37
....	+0.67	+0.44	....	....	+0.24	+0.36	....	....	+0.28	−0.10	....
....	+0.59	....	+0.13	....	0.00	....	+0.38	....	0.00	....	+0.61
....	....	+0.36	+0.43	....	....	+0.30	+0.33	....	....	−0.13	+0.64

Average h Determined by:				*Average z Determined by:*			
c	l	b	z	c	l	b	h
+0.35	+0.17	....	....	+0.39	+0.18	....	....
+0.80	....	−0.53	....	+0.15	....	+0.51	....
+0.46	....	....	−0.04	+0.51	....	....	−0.04
....	+0.38	−0.10	....	....	+0.26	+0.55	....
....	+0.34	....	+0.06	....	+0.37	....	+0.06
...	....	−0.19	+0.30	....	....	+0.61	+0.19

III. *Coefficients of Triple Correlation.*

Average c Determined by:				*Average l Determined by:*				*Average b Determined by:*			
l	b	h	z	c	b	h	z	c	l	h	z
+0.28	+0.54	+0.35	....	+0.63	−0.17	+0.09		+0.89	−0.11	−0.36	....
+0.41	+0.59	....	−0.03	+0.65	−0.38	....	+0.29	+0.60	−0.25	....	+0.41
+0.32	....	+0.27	+0.32	+0.39	....	+0.16	+0.17	+0.66	....	−0.37	+0.36
....	+0.67	+0.45	0.00	....	+0.06	+0.31	+0.29	....	+0.04	−0.14	+0.63

Average h Determined by:				*Average z Determined by:*			
c	l	b	z	c	l	b	h
+0.75	+0.07	−0.53	....	−0.04	+0.28	+0.58	....
+0.37	+0.19	....	−0.07	+0.43	+0.19	....	−0.06
+0.78	....	−0.65	+0.25	0.00	....	+0.61	+0.18
....	+0.34	−0.19	+0.18	....	+0.22	+0.56	+0.12

IV. *Coefficients of Quadruple Correlation.*

Determined by	*c*	*l*	*b*	*h*	*z*	*Variability of Array.*
Average c..	....	+ 0.32	+ 0.67	+ 0.36	− 0.08	of c = 0.47 μ_c
Average l..	+ 0.63	....	− 0.36	+ 0.03	+ 0.29	of l = 0.79 μ_l
Average b..	+ 0.69	− 0.16	....	− 0.52	+ 0.41	of b = 0.54 μ_b
Average h..	+ 0.77	+ 0.03	− 0.65	...	+ 0.18	of h = 0.78 μ_h
Average z..	− 0.17	+ 0.27	+ 0.70	+ 0.19	....	of z = 0.69 μ_z

These data show that the diameters of the skull are primarily determined by its capacity. The height of the skull appears to be most closely associated with its capacity, the length seems to be least closely related to it. I presume this is due to the fact that the development of the frontal sinuses and of the occipital protuberances does not depend upon the form of the inner cavity of the skull, but upon the general development of the skeleton. Since this is partly expressed by stature, we might expect that this influence would partly be eliminated by the introduction of stature in the series of correlations. Unfortunately this element cannot be introduced on account of lack of data. Furthermore, the errors of the values of multiple correlations are so great that it is not advisable to carry on the investigation of a series of no more than 57 skulls beyond quadruple correlations which may be considered approximately correct in their first decimal.

It is of great interest to note that when capacity is introduced in our consideration a compensatory growth is found to exist between breadth of head on the one hand, and height and length of head on the other. We find, therefore, as a result of our investigation, that the law of compensation which Virchow formulated after an analysis of the forms of skulls with premature synostosis of sutures, holds good also in normal skulls. *Among skulls belonging to the same type a breadth above the average is compensated by a height and a length below the average.* The correlation between length and breadth is not an expression of a biological relation between the two measurements, but an effect of the changes which both undergo when the capacity of the skull increases or decreases. The cephalic index, therefore, is not the expression of a law of direct relation between length and breadth of the skull. The proportion between the diameters of the skull and its capacity, on the other hand, expresses an intimate biological relation between these measurements. It appears that the diameters of the head must be considered as due to the tendency of the inner cavity of the skull, or more probably of the brain, to assume a certain size and form in a given type of man, this form being expressed by the proportion of the diameters of the brain and its size. If one of the diameters differs from the norm in being excessively large, the others will tend to be too small. This is definitely shown to be the case when the transversal diameter differs from the norm.

The variabilities of the arrays show that the reduction of variability of capacity is greatest. This proves that the four linear measurements which we have treated largely determine the capacity. The variability of the length and height of skull is very slightly reduced. This shows that these measurements are largely influenced by causes which we have not included in our considerations. The same is true of the bizygomatic diameter of the face. When we compare the reduction in variability of the linear measurements as determined by *c*, and as determined by the quadruple correlation, we find the following :

Variability of Array.

	Determined by c.	*Determined by Quadruple Correlation.*
l	0.84	0.79
b	0.74	0.54
h	0.90	0 78
z	0.87	0.69

This comparison proves that breadth, height, and bizygomatic diameter have an insignificant effect upon length as compared with the effect of capacity. The great reduction of variability for breadth of head and bizygomatic diameter is due to the intimate correlation between these two values.

It follows from these considerations that while the cephalic index is a convenient practical expression of the form of the head, it does not express any important anatomic relation. On the other hand, the relation between capacity and head diameters is found to be of fundamental importance, and among these the relation between transversal diameter and capacity is most significant. Since in measurements on the living we are unable to measure capacity of the head, it is necessary to find a substitute. It would seem that circumferences are the most available means for judging cranial size. Therefore such circumferences should be included in all anthropometrical schedules designed to investigate racial characters.