III—It is shown that sodium nitroprusside can be used as an internal indicator to show the end-point of the disappearance of the sulfide in the titration with iodine.

IV—This work affords further evidence of the interesting fact that when an iodine solution or a dilute acid is carefully added to a solution containing calcium polysulfide and thiosulfate, the polysulfide can be quantitatively decomposed before the thiosulfate is attacked.

V—A rapid accurate method of weighing the precipitated sulfur in the iodine and hydrochloric acid titrations of a lime sulfur solution is proposed.

VI—Two methods (B and C), not heretofore used for the determination of thiosulfate in lime-sulfur solutions, are described, both being theoretically and practically sound and accurate.

VII—The accuracy of the iodine titration method (or Harris method) for the analysis of such solutions is confirmed.

Agricultural Experiment Station
Lexington, Kentucky

LABORATORY AND PLANT

THE FLOW OF VISCOUS LIQUIDS THROUGH PIPES

By W. K. Lewis

Received May 2, 1916

The carrying capacity of pipes for water under various pressure drops has been experimentally studied by many engineers and while the results are not very concordant on account of the extreme sensitiveness to varying conditions, none the less our knowledge of the resistance to flow of water through pipe lines is relatively satisfactory and complete. On the other hand, practically no work has been published on the resistance to flow through pipes of liquids other than water despite the fact that information of this sort is of vital importance to the chemical engineer. Davis, in his "Handbook of Chemical Engineering," points out the extremely small carrying capacity of pipes for viscous liquids such as sulfuric acid and glycerin as compared with their capacity for water, but he gives no suggestions as to methods of estimating the size of pipes required for specific cases. There are in this country thousands of miles of pipe lines transmitting mineral oils as well as the piping systems of chemical plants handling acids, solvents, and all sorts of liquids in large quantities. While a wealth of information along this line is undoubtedly available to the engineers of individual corporations, the lack of any published figures and the importance of the whole problem led this laboratory to undertake an investigation along these lines which has extended over the last four years. After certain preliminary work, a series of tests was made studying the flow of mineral oils of varying viscosity at relatively low velocities. The results of this work are reported as Series A. Later, in order to supplement the results of these first experiments by study of flow at higher velocities, another separate investigation was carried out which is reported as Series B. While the results are limited to relatively small pipes and low viscosities, certain generalizations can be drawn which are so well confirmed that their use, even beyond the scope of the present experimental range, seems justified, especially as a working basis upon which to develop further investigation.

NOMENCLATURE

P = pressure drop in grams/cm². or lbs./sq. ft.

μ = coefficient of absolute viscosity in sec. dynes/cm². or sec. poundals/sq. ft.

l = length of the pipe in cm. or ft.

r = radius of pipe in cm. or ft.

ρ = density of the fluid.

V_c = mean velocity at point where sinuous flow changes to parallel flow, cm./sec. or ft./sec.

V_m = mean velocity in cm./sec. or ft./sec.

f = hydraulic frictional coefficient.

GENERAL DISCUSSION

It has long been known that, for the flow of fluids through capillary tubes up to 4 or 5 mm. in diameter, the formula of Poiseuille, $P = \frac{8\mu l V_m}{gr^2}$, holds quantitatively. Though for the flow of liquids other than water through large tubes or pipes, quantitative data have not been published, the paths along which the particles of liquid travel have been studied qualitatively by introducing air or dyes into the fluids and photographing the effects produced by forcing them through glass containers of various shapes and sizes.[1] These observations show that at low velocities liquids move in straight lines parallel to the axis of the tube, but when the velocity is sufficiently increased, the lines of flow become distorted, the filament forming violent eddies of constantly changing form and position. At the walls of the container there is always a film of liquid which is retarded by the friction of the solid surface, so that it continues to move in straight lines. The mean velocity of the fluid, at the point where the change in type of motion takes place, is commonly known as the critical velocity and all flow below this point is called parallel, direct, or viscous motion, while that above is known as turbulent, indirect, or sinuous flow.

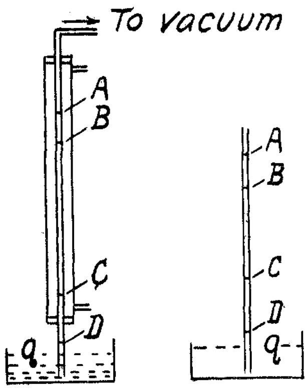
From this it is obvious that at least two different laws must govern the flow of fluids, the one above and the other below the critical velocity, with the possibility of a third for an intermediate state. Poiseuille's law would be the one to use below the critical point, since his formula applies entirely to straight line motion. For flow above the critical point, we have but one suggestion as to the manner in which flow may take place, from observation of the formulae for the flow of steam, air, and water above the critical rate. These equations may all be written $P = \frac{f\rho l V_n}{gr}$,

[1] H. S. Hele-Shaw, *Engineering*, **65** (1898), 420, 444, 477, 510.

and it is surprising to find that they all have approximately the same constant, f, and also nearly the same value of n. It would seem possible, then, that this formula may be general and hence apply to any fluid, but this is an hypothesis which can be verified only by experiment.

SERIES A[1]

Two horizontal lines of pipe were built, one consisting of standard 1-in. pipe 136 ft. long, and the other 1/2-in. pipe 63.8 ft. long. In both lines a return bend was placed in the middle. At the end of these lines several bends and drops were necessary to accommodate the pipe to the receiving tank. As it was desired, however, to study flow in straight pipe only, a differential manometer was placed at the return bend in each line by means of a T separated from the bend by a close nipple, and an ordinary manometer was attached at a point near the end of the system, before the bends or drops were introduced. The pressure recorded by these manometers was subtracted from the pressure registered by a gauge at the beginning of the line and thus the pressure drop throughout the *straight pipe* was obtained.



FIG·I FIG·II

TABLE I—RESULTS OF TYPICAL RUNS (SERIES A)

Run No. (c)	Coeff. Abs. Visc. μ	Dens. of Fluid ρ	Delivery through Pipe: Lbs. per Min.	Delivery through Pipe: Cc. per Sec. (b) Calc. (d)	Cc. per Sec. (b) Obs.	Per cent Deviation Obs. from Calc.	Pressure Drop (a): Lbs. per sq. in.	Pressure Drop (a): Dynes per sq. cm.
11	0.938	0.95	31.25	256.0	248.0	3.1	10.8	74.3
24	0.917	0.95	39.80	318.0	316.0	0.6	13.12	90.3
39	0.920	0.95	65.50	520.0	520.0	0.0	21.7	156.0
44	0.920	0.95	50.00	397.0	397.0	0.0	16.45	113.0
54	0.920	0.95	11.20	88.5	89.0	0.6	3.67	25.2
64	0.790	0.91	8.33	66.8	66.2	0.6	12.0	82.5
69	0.718	0.91	11.91	91.4	94.9	—3.8	14.9	102.2
74	0.670	0.91	16.75	131.3	133.0	—1.3	18.58	127.5
79	0.620	0.91	22.42	179.0	178.0	0.6	22.45	154.1
98	0.308	0.89	4.28	32.1	33.9	—4.6	4.89	33.6
107	0.304	0.89	6.95	57.0	55.3	3.0	8.49	58.3
133	0.310	0.89	14.1	120.0	112.0	6.7	18.28	125.5
143	0.261	0.88	50.0	415.0	397.0	4.4	5.32	36.5
148	0.246	0.88	82.5	676.0	656.0	3.1	8.17	56.0
154	0.235	0.88	97.0	810.0	770.0	5.2	9.37	64.3
158	0.227	0.88	112.5	1000.0	894.0	11.8	11.2	77.9
160	0.224	0.88	115.8	1125.0	919.0	22.4	12.25	84.2
187	0.081	0.85	156.2	5480.0	1243.0	293.0	22.0	151.0
167	0.218	0.88	123.5	1519.0	983.0	53.5	16.23	111.5
171	0.217	0.88	133.2	1810.0	1060.0	69.8	19.25	132.1
175	0.215	0.88	144.3	2130.0	1148.0	85.5	22.35	153.5
181	0.086	0.86	96.8	1978.0	770.0	156.0	8.52	58.5
182	0.084	0.85	111.5	3090.0	888.0	250.0	8.40	57.6
185	0.082	0.85	139.5	4450.0	1108.0	331.0	17.48	120.0

(*a*) The conversion factor for changing lbs. per sq. in. to dynes per sq. in. is 6.87×10^4.

(*b*) The conversion factor for changing lbs. per min. to cc. per sec. is 7.95.

(*c*) *In Runs 1–60 and 95–187, the 1-in. standard steam pipe was used.* The 1/2-in. standard steam pipe was used in the remaining runs.

(*d*) Calculated by Poiseuille's equation.

Five lubricating oils of different viscosities were used, the oil being pumped into a constant head tank by means of a Gould rotary pump and thence into the pipe line. After passing through the line, the oil was discharged into a tank supported by platform scales, where the time and quantity of discharge were recorded. The density of the oil was determined by a Westphal balance and the viscosity by a modified Gurney Viscosimeter.[1]

In Table I are given representative laboratory data. It is remarkable that Poiseuille's equation, which was derived for capillary tubes, should hold (within the limits of experimental error) for fluids flowing through 136 ft. of 1-in. pipe with a pressure drop of 22.8 lbs./sq. in, and a velocity of 1.1 cu. ft./min.

The runs above the critical point were necessarily made at very high velocities and the pump, rated at only 10 gals. per minute, was consequently running at approximately its theoretical capacity. Moreover, the driving motor was running at from 50 to 60 per cent overload, thus causing great difficulty in properly regulating the pressure. At the highest velocities attained, if constant conditions were to be maintained in the pipe line, no oil could pass through the by-pass, used in previous runs to shunt part of the oil back to the reservoir. With this apparatus

[1] The work in this series of runs was carried out by Messrs. Greenough and Dinsmore in 1914 and submitted as a thesis in partial fulfilment of the requirements for the S.B. degree at the Massachusetts Institute of Technology.

[1] Fig. I shows a Gurney viscosimeter as described in *J. Am. Chem. Soc.*, **34** (1912), 24. The apparatus consists simply of an upright capillary tube, provided with a jacket, through which may be passed water, or the vapor of some constant boiling liquid, in order that a constant temperature may be maintained. There is also an arrangement for applying a vacuum to draw the liquid into the tube.

The formula which Gurney published is useless, mainly because he did not take into account the non-uniformity of the cross-section of glass tubes. No capillary tube is of constant cross-section; the effect of the irregularity is a complicated function of the distance from the bottom of the tube, and to correct Gurney's formula for this point would be out of the question. If a tube is smaller at one end than at the other, as from the method of manufacture is usually the case, the liquid will flow much more rapidly when the large end is at the bottom than when the tube is reversed. Furthermore, Gurney's correction terms, although small and perhaps negligible, are of doubtful validity. For example, the term $\frac{0.005\rho l V^2}{r}$, which is used for frictional resistance, holds for non-viscous fluids flowing in turbulent motion and is incorrect for parallel flow.

We propose a new method of calculation suggested by Lang (Thesis, Mass. Inst. Tech., **1914**), which can be obtained directly from the fundamental equation of Poiseuille. Let the tube (Fig. II) be so adjusted that the capillary head, Dq, is maintained constant. Let the fluid be raised to A and allowed to flow by gravity from A to C. For capillary tubes, as previously stated, Poiseuille's formula, $P = \frac{8\mu l V}{gr^2}$, holds *quantitatively*. Therefore, as pressure is proportional to the density of the fluid, $h\rho = \frac{8\mu l V}{gr^2}$.

For two different fluids, 1 and 2,

$$h\rho_1 = \frac{8\mu_1 l V_1}{gr} \quad (1), \quad \text{and} \quad h\rho_2 = \frac{8\mu_2 l V_2}{gr^2} \quad (2).$$

Dividing (1) by (2),

$$\frac{\rho_1}{\rho_2} = \frac{\mu_1 V_1}{\mu_2 V_2}.$$

Solving,

$$\mu_1 = \frac{\rho_1 \mu_2 V_2}{\rho_2 V_1}, \text{ or } \mu_1 = \frac{\rho_1 T_1 \mu_2}{\rho_2 T_2}.$$

Therefore, if the tube be standardized by measuring the time of efflux of a liquid between two fixed points, B and C, the viscosity of any other fluid may be obtained by simply measuring its density and time of flow between the same points, providing ρ and μ are known for the first liquid. This automatically corrects for unevenness of bore. We would suggest, as did Gurney, that the fluid be raised each time to A as described above, but that the time be taken between B and C for greater accuracy in time of starting.

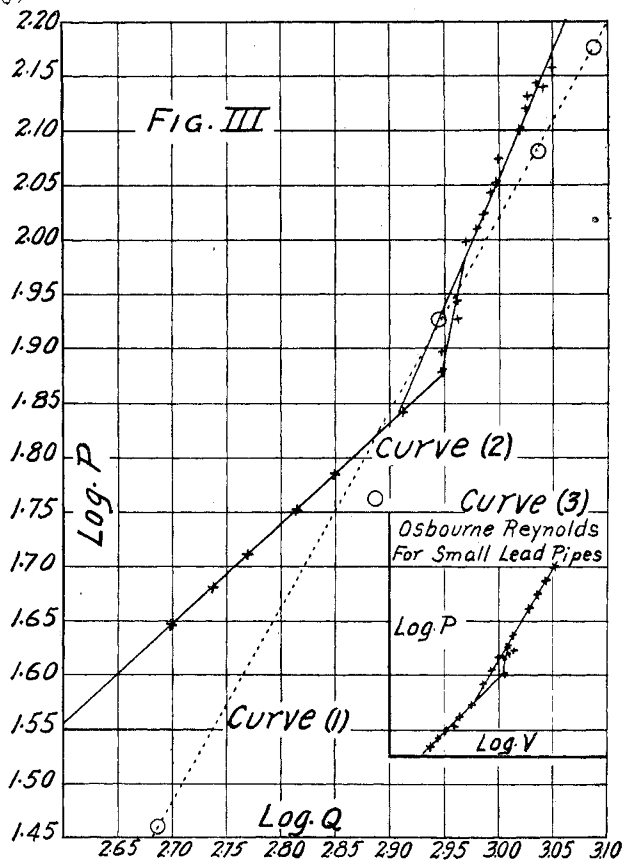
For high precision over wide ranges of temperature, the standardization and the viscosity determination should be carried out at nearly the same temperature, or the following correction may be made for expansion of the glass:

$$T_2 = \frac{T_1(1 + a_1 t)}{(1 + 2a_1 t)},$$

where a = linear coefficient of expansion of glass, and t = temperature difference between that employed and the temperature of standardization.

the experimental error for runs at high velocities was therefore large.

In order to determine the critical velocity of the oils and also to ascertain whether the equation $P = (f)V^n$ would hold above the critical velocity, we plotted



the logarithm of the pressure drop against the logarithm of the velocity.[1]

In Fig. III, *Curve 1* represents the least viscous oil ($\mu = 0.089$) flowing above the critical velocity. *Curve 2* represents the oil ($\mu = 0.23$) flowing above and below the critical velocity. *Curve 3*, plotted from data collected by Osbourne Reynolds on the critical velocity of water, gives a means of comparison between water and a fluid twenty times as viscous.

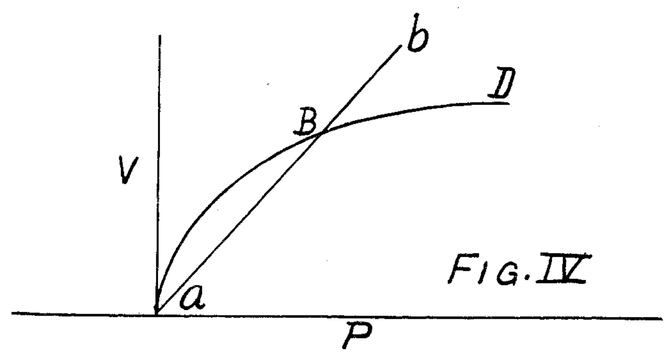
Even a casual glance at the two latter curves will reveal their extraordinary similarity. Each liquid has two critical velocities: the first represents the higher one, where nonsinuous motion becomes sinuous; the other shows the point where turbulent flow changes to direct motion. Usually, in industrial practice, a fluid enters a pipe with a turbulent motion produced by a rotary or centrifugal pump, compressed air or by some other means whereby violent eddies are set up in the field. We, however, were able to realize both conditions because of the introduction of a constant-head tank between our pump and the pipe line. This tank was provided with an air dome and insured fairly uniform and quiet conditions at the mouth of the pipe line. Nevertheless, this higher critical velocity is a point of unstable equilibrium and seldom reached in practice; hence when critical velocity[1] is referred to in this article, the lower point—the change from turbulent to nonsinuous motion—will be understood. It is to be noticed that the curves do not deviate from straight lines up to the point of intersection, or, in other words, there is no intermediate flow between straight and turbulent motion.

The average value of n from the two curves is approximately 2 and we therefore make the same assumption made in hydraulics, namely that $P = \frac{pflv^2}{gr}$, realizing that neither oil nor water follows this law without a varying constant. The frictional coefficients for those runs above the critical velocity were found to be nearly the same as those for water under the same conditions of flow. All exact work on flow above the critical velocity is, however, taken from Series B.

In this first series of runs it was impossible to carry the velocities of flow far above the critical point and in all runs at these high velocities, the experimental error was great, owing to inadequate equipment. In order to study the flow of liquids at varying velocities at rates well above the critical point, a second entirely separate series of runs was undertaken, using

[1] For if $P = kV^n$, $\log P = \log k + n \log V$, the equation of a straight line having the form $x = a + ny$, the slope being $\frac{\log P}{\log V}$ which is unaffected by $\log k$ and is equal to n.

[1] In order to derive an equation for the critical velocity of any fluid, let us consider the pressure-velocity plots for Poiseuille's and the sinuous formula. If a fluid is moving through a pipe below the critical velocity Poiseuille's formula, as above stated, holds quantitatively or, in other words, $P = kV$. The pressure-velocity plot is therefore a straight line (Fig. IV) passing through the origin and determined experimentally by



locating points between a and b. Let the same liquid flow above the critical velocity and accordingly follow the sinuous formula $P = kV^2$ This curve is a parabola passing through the origin with all experimentally determined points lying between B and D. If $P = kV^{(2 + n)}$, where n was plus or minus a small decimal, the curve BD would cease to be a parabola but would remain a curve which nevertheless has the same general shape and in the following discussion would make no difference.

Between points a and B, the velocity is greater for a given pressure drop in turbulent motion than in straight line flow, while above B the reverse is true. It is clear that for a liquid moving with a greater velocity per unit pressure, less power is necessary to transmit a given amount of this fluid through a pipe. Below the point B therefore more energy is consumed by viscous than by sinuous flow. Above point B the reverse is true. This experimental work has, however, proven that below B the flow is viscous and sinuous above. In both cases the flow takes place in that way involving the greater energy consumption. This may be looked upon as a specific instance of the general rule that if a change can take place in a variety of ways the change tends to follow that path which corresponds to the greatest decrease in free energy.

At the critical velocity the two types of motion give identical pressure drops, *i. e.*,

$$P_c = \frac{f\rho l v_c^{\,n}}{gr} = \frac{8\mu l V_c}{gr^2}.$$

If the value of 2 be assigned to n, $V_c = \frac{8\mu}{f\rho r}$.

The value to use for f will be developed under Series B.

not only higher velocities, but also a 2-in. pipe in addition to the smaller pipes of the previous series.

SERIES B[1]

From a supply tank of 400 gals. capacity liquids were pumped by a Duplex Steam Pump into an air dome and thence through four pipe lines one of 2-in. wrought iron, one of 1-in. wrought iron, one of 1/2-in. wrought iron, and one of 1/2-in. steel pipe. In each case the liquid passed through a length of straight pipe around a return bend and back to the starting point, where it flowed into a slightly larger pipe which delivered it through two quick opening valves to two tanks set on platform scales.

A by-pass was used to shunt part of the liquid from the pump directly back to the supply tank in order to regulate the pressure as desired. The initial pressure was taken by calibrated gauges, the drop in pressure around the bend by a differential manometer and the back pressure by a U-tube manometer. As in all hydraulic experimentation, this work was subject to more or less error. The reciprocating pump would not give absolutely constant pressure even with the help of an air dome with 10 cu. ft. capacity. The best average values were obtained by throttling the gauge and the readings were again averaged by the graphical consideration of the data, but the values of p cannot be relied up to better than 5 per cent. The 1/2-in. wrought iron pipe, although new, was very rough and gave in all runs about 25 per cent greater friction loss than the steel. In small pipes the condition of the surface plays such an important part in the friction loss and the roughness varies so greatly with different pipes that we have assumed the 1/2-in. steel pipe as a standard for this size. The liquids used were water and gas oil. The viscosity of the former was obtained from the data of Thorpe and Rodger as found in Landolt-Börnstein. The viscosity of the latter was determined as in the previous series. The temperature and therefore the viscosity of the oil and water was changed by means of a steam coil in the supply tank. Series of runs were made with hot and cold water and with hot, warm and cold oil, the viscosity being determined in each case from the average temperature of the series. The viscosities are as follows:

	Temp.	Dynes/cm².	Poundals/In.²
Cold oil	24.5° C.	0.169	0.0000789
Warm oil	40.0° C.	0.1045	0.0000488
Hot oil	60.0° C.	0.0596	0.0000278
Cold water	20.0° C.	0.0100	0.00000466
Hot water	48.5° C.	0.0056	0.00000261

It is to be noted that a wide viscosity range was covered as the cold oil was over 30 times as viscous as the hot water.

Runs were made at velocities up to 20 ft. per sec. in the 1/2- and 1-in. pipes and as high as 12 ft. per sec. in the 2-in pipe. The gauge pressure of each run was kept constant by the regulation of the by-pass valve. The results appear in Table II.

[1] The work in this series of runs was carried out by Messrs. Haylett and Lucey and submitted as a thesis in partial fulfilment of the requirements for the S.B. degree at the Massachusetts Institute of Technology in 1915.

Friction-velocity curves were constructed from the experimental data by plotting the pressure drop against the velocity. It was found that for each pipe the pressure drop p varied with a power of the velocity v, or, $p = av^n$.

The value of n was determined by plotting log p against log v and determining the average slope of the curves of the four pipes. The lines were nearly parallel with an average slope of 1.85. To determine the variation of the pressure drop with the diameter, values of $\log (p/v^{1.85})$ were plotted against the logarithm of the diameter for oils of the same

TABLE II—PRESSURE DROP: LBS. PER SQ. IN. PER 100-FT. LENGTH OF PIPE (SERIES B)

SIZE OF PIPE	Velocity in feet per second							
1/2-in. Steel	4	6	8	10	12	14	16	18
COLD OIL:								
Observed	..	20.8	32.5	49.0	71.2	98.8	...	...
Calculated	..	20.1	34.0	51.5	71.8	71.8	...	...
Deviation	..	—3.3	+4.6	+5.1	+0.8	—2.8	...	...
WARM OIL:								
Observed	..	21.5	33.6	47.7	64.1	83.0	104.2	...
Calculated	..	17.8	30.2	45.7	64.8	85.2	109.0	...
Deviation	..	..	—10	—4.2	+1.1	+2.6	+4.6	...
HOT OIL:								
Observed	..	19.3	29.9	43.3	59.9	79.2	101.0	...
Calculated	..	16.3	27.6	41.8	58.4	78.0	99.8	...
Deviation	..	..	—7.2	—5.8	—2.5	—1.5	—1.2	...
COLD WATER:								
Observed	..	13.2	22.2	33.1	46.3	79.2	61.3	99.8
Calculated	..	14.3	24.2	36.6	51.2	73.2	93.7	109.0
Deviation	..	+8.3	+9.0	+9.5	+10	+16	+16	+19
HOT WATER:								
Observed	..	12.0	20.0	30.5	43.0	58.2	76:5	97.3
Calculated	..	14.2	24.1	36.5	51.0	73.0	93.5	108.0
Deviation	..	+18	+20	+20	+18	+25	+22	+11
1-in. Wrought Iron								
COLD OIL:								
Observed	4.5	10.3	19.0	29.3	41.5	55.0	68.8	...
Calculated	5.5	11.7	19.8	30.0	41.9	56.0	71.7	...
Deviation	+22	+11	+4.2	+2.4	+0.9	+1.8	+4.2	...
WARM OIL:								
Observed	5.1	10.7	17.8	26.7	36.7	48.0	60.4	...
Calculated	4.8	10.3	17.4	26.3	36.7	49.0	62.8	...
Deviation	—5.9	—3.7	—2.2	—1.5	0.0	+2.1	+3.8	...
HOT OIL:								
Observed	5.8	10.4	16.2	23.7	32.6	43.2	55.2	68.0
Calculated	4.4	9.2	15.7	23.7	33.0	44.2	56.6	70.2
Deviation	—2.70	—10	—3.1	0.0	+1.2	+2.3	+2.5	+2.9
2-in. Wrought Iron								
COLD OIL:								
Observed	2.64	5.50	9.08	13.67	..	...	...	...
Calculated	2.46	5.22	8.85	13.40	..	...	...	...
Deviation	—6.8	—5.1	—2.5	—1.5	..	...	...	...
WARM OIL:								
Observed	1.64	4.29	8.05	12.79	...	...	...	...
Calculated	2.15	4.56	7.78	11.70	...	...	...	...
Deviation	31.0	+6.3	—3.8	—8.3	..	...	...	...
HOT OIL:								
Observed	1.41	3.82	6.90	10.37	14.19	...	...	...
Calculated	1.92	4.09	6.92	10.50	14.60	...	...	...
Deviation	26.7	+7.0	+0.3	+0.9	+2.6	...	...	...

viscosity and again nearly parallel lines were obtained. The average slope of these was —1.22, *i. e.*, p varies inversely as $d^{1.22}$, or $P = \frac{fv^{1.85}}{d^{1.22}}$ is the equation obtained where f varies with the viscosity of the liquid. This last relation was determined by plotting f against the viscosity, by which a straight line was obtained for the 1- and 2-in. pipes and another parallel to it for the 1/2-in. steel pipe. The equation of these lines was of the form $f = k\mu + c$. The slope k was found to be 115 and the intercept c, 0.0114—for the 1- and 2-in. pipes (0.0140 for the 1/2 in.). Thus, the completed equation becomes $p = (115\mu + 0.0114) \frac{v^{1.85}}{d^{1.22}}$, where p is the pressure drop due to friction expressed in lbs. per sq. in. per 100 ft. of pipe, μ the absolute viscosity in poundals per sq. in., v the velocity in ft. per sec. and d the diameter of the pipe in sq. ft.

Table II contains illustrations of the results of these runs giving the velocity in ft. per sec., the observed pressure drop in lbs. per sq. in. per 100 ft. of pipe, the pressure drop calculated by the use of the above formula, and the percentage deviation between the two. The equation

$$P = \frac{(115\mu' + 0.0114)v^{1.85}}{d^{1.22}}$$

may be the form of a general law,

$$P = \frac{(k\mu + c)y^n}{d^m},$$

for all sizes of pipes, with varying velocities and different degrees of internal roughness. The constants, however, doubtless vary, as do all hydraulic constants, and only experimental data can possibly give the ones to use for any particular case. However, until further research has been carried out in this field, these laws, with the derived constants, will serve as a first approximation.

FRICTIONAL COEFFICIENTS FOR VISCOUS LIQUIDS

We have shown that flowing liquids of all viscosities when in sinuous motion follow substantially the same equation, differing only in the coefficient of that equation: *i. e.*, the flow of any liquid in sinuous motion may be expressed as a constant times some function of velocity, length, and radius. The constant f depends upon the viscosity of the liquid flowing, but the function of the velocity, length and radius is independent of the viscosity. In all ordinary hydraulic calculations it is assumed that this function of velocity, length, and radius is $\frac{v^2 l\rho}{gr}$, and the fact that the flow does not exactly follow this formula is provided for by the use of tables for the coefficient f, these tables showing the value of f as determined by radius and velocity, the two factors which influence it. Inasmuch as we have shown that the flow of viscous liquids has the same function of velocity, length and radius as the flow of water, it follows that the ratio of the constant to be employed for the flow of viscous liquids to that of water itself is the same as the ratio of the constant employed in the more exact exponential formula which we have derived above. If f_w be the hydraulic coefficient for the flow of water through a pipe and f that of any liquid, both to be employed on the assumption that the lost head or pressure drop is directly proportional to the length, to the square of the velocity and inversely to the radius, it follows that

$$\frac{f}{f_w} = \frac{0.0114 + 115\mu}{0.0114 + 115(0.00000467)^*}$$

$$= 0.955 + 0.045\frac{\mu}{0.00000467},$$

or, $f = f_w(0.955 + 0.045z)$,

where z is the *relative viscosity* of the liquid in question to water. The best way to determine z is to measure the relative time of efflux of the liquid and water through the same capillary tube. The tube should not be constricted at the ends and the use of the Gurney viscosimeter as outlined above for this purpose is recommended.

* Viscosity of water at ordinary temperature (20° C.).

We believe that the use of this formula

$$f = f_w(0.955 + 0.045z)$$

is at present the best means of estimating the carrying capacity of a pipe for viscous liquids. From the formula it is evident that for liquids of low viscosity the capacity is exactly the same as that for water, but for high viscosities the capacity rapidly decreases. We have experimentally confirmed the accuracy of this formula by the data given above up to viscosities of 20-fold that of water. Up to this point, the correction term for viscosity in the formula above is small, but for very high viscosities, such as are encountered in heavy mineral oils, glycerin, etc., this term becomes very great. We personally doubt the validity of this formula for high viscosities, believing that the pressure drops will be decidedly less than this formula indicates. On the other hand, the use of this formula should be safe, inasmuch as the actual carrying capacity of a pipe designed by its use will exceed the calculated rather than otherwise. We have hitherto not been in a position to confirm the use of this formula for more viscous liquids.

CALCULATION OF CARRYING CAPACITY

To estimate the carrying capacity of a pipe line for any liquid, proceed as follows: Determine the density of the liquid and its viscosity relative to water, doing the latter either with the Gurney viscosimeter or by measuring the time of efflux through a capillary tube for which the time of efflux of water or of any other liquid of known absolute viscosity has been measured. The relative viscosity times 0.0671[1] gives the absolute viscosity in poundals/sec./sq. ft. Now employ the two formulae,

$$p = \frac{8\mu l v}{gr^2}, \text{ and } p = \frac{fl\rho v^2}{gr},$$

where $f = f_w(0.955 + 0.045z)$. Choose that result which indicates the greatest resistance to flow.

A single illustration will make the procedure clear.

It is required to find the velocity of an oil of sp. gr. 0.91 through 600 ft. of standard 1-in. pipe (inside diameter, 1.07 in.) under a head of 30 ft. The time of efflux of the oil at 20° C. from a pipette which is not constricted at the tip is 108 sec., water flowing from the same pipette in 4.90 sec.

$$p = h\rho = 30(0.91)(62.3) = 1701 \text{ lbs./sq. ft.}$$

$$z = \frac{108}{4.90} = 22.05 \qquad \mu = 0.0671z = 1.478$$

$$r = \frac{1.07}{(12)(2)} = 0.0446 \qquad g = 32.2$$

Assume that f_w, from hydraulic tables for the pipe in question, is 0.0075.

For viscous flow, $v = \frac{(1701)(32.2)(0.0446)^2}{(8)(1.478)(600)} = 0.0153$ ft./sec.

For sinuous flow,

$$v = \sqrt{\frac{(30)(32.2)(0.0446)}{(600)[0.995 + 0.045(22.05)](0.0075)}} = 2.22 \text{ ft./sec.}$$

The flows in viscous motion being but a small fraction of that required for sinuous flow, viscous motion

[1] The viscosity of water at 20° C.

will result and the low discharge is the one which must be expected. Had the sinuous formula given the lower value of v, the result by the formula for viscous flow would have been rejected.

SIZE OF PIPE FOR VISCOUS LIQUIDS

Liquids of even moderate viscosity flowing under low heads follow viscous motion unless the pipes be very large. It is very important to keep in mind the fact that, so long as the motion is viscous, doubling the size of the pipe increases the velocity 4-fold and the discharge 16-fold for the same pressure drop. For the same discharge a pipe twice the size requires only one-sixteenth the pressure drop and therefore but one-sixteenth the power. If a pipe is carrying liquid in viscous motion, increase in size of the pipe is always well worth consideration, owing to this very great effect on carrying capacity and power consumption. Decrease in size will ultimately result in converting the flow into sinuous motion, after which the effect of size is greatly lessened, being inversely proportional to only the first power of the diameter.

SUMMARY

Liquids flowing through pipes flow either in straight line motion in which case they follow Poiseuille's formula, $p = \frac{8\mu lv}{gr^2}$, or in sinuous motion, the pressure drop being represented by $p = \frac{fl\rho v^2}{gr}$. The flow will follow that formula which requires the higher pressure drop, the higher radius, or gives the lower velocity, as the case may be. Both formulae must therefore be employed and the result chosen according to the above rule. To obtain the coefficient f of the formula for sinuous motion: look up, in suitable hydraulic tables, the value of the coefficient for water flowing in the same size pipe at the same velocity and multiply this coefficient by the expression $(0.955 + 0.045z)$ wherein z is the viscosity relative to water of the liquid flowing.

These formulae have been experimentally substantiated only for use in pipes up to 2 in. in diameter and for the flow of liquids of viscosity (relative to water at 20°) of 20. They are probably safe for use in larger pipes and at higher viscosities, but more exact expressions for these conditions must be determined by further experimentation.

Research Laboratory of Applied Chemistry
Massachusetts Institute of Technology
Boston

THE DESIGN AND OPERATION OF OZONE WATER PURIFICATION SYSTEMS

By Sheppard T. Powell[1]

Received February 7, 1916

Although the first attempt to purify water on a practical scale by means of ozone was made less than thirty years ago, still this gas was generated and its chemical and physical properties have been studied by many investigators for more than a century. In all probability ozone has been recognized by scientists since the earliest ages, if not by name at least by its characteristic properties. The first authentic record that we possess of the manufacture of this gas was in 1783 when Van Marum, a Dutch scientist, termed it "a smell of electricity," as a result of its production by this means.

It was not, however, until the exhaustive studies of Schoenbein in 1840 that the active properties of ozone were well understood or any analytical methods were devised for measuring this gas. Schoenbein recognized this active oxidizing agent as a distinct gas to which he gave the name of ozone.

For more than fifty years after Schoenbein's researches nothing was accomplished in placing ozone within the scope of a commercial possibility, although its active and oxidizing power was well known. With the development of the alternating current generators and transformers, which so materially reduced the cost of production, and the increasing knowledge of the bacteriology and chemistry of water, ozone was recognized as a water purification agent of great value. Berthelot, a French chemist, in 1890 undertook, with some degree of success, to apply this method of water purification, and from then on Europe, as well as America, has been practically flooded with numerous designs of generators and patented appliances for water treatment.

All the ozonizers that have been devised are based upon the same general principle, *viz.*, the production of the allotropic form of oxygen, O_3, from the oxygen of the atmosphere. This is accomplished by passing a current of air over a brush discharge which takes place between electrodes connected to a high voltage alternating current circuit; these usually have a solid dielectric interposed between them. There are, of course, many other ways of generating this gas, but none of these processes other than the one described has proven a commercial success.

Numerous theories have been advanced to account for the production of ozone in this manner but the one generally accepted is based on the theory of molecular motion.[1] This theory, as stated by Dr. C. P. Steinmetz, regards the chemical effect of all ether radiations recognized by light, heat and electrical waves as more or less specific for various compounds to definite frequencies of their movement setting up resonance effects upon the natural molecular or atomic motion.

Pure ozone is colorless, has a distinct and peculiar odor and instantly decomposes at 260° C. It can be liquefied by a pressure of 1840 lbs. per sq. in. and at a temperature of —103° C. In this condition it is highly magnetic but is not so powerful an oxidizing agent as the gas.

The great affinity of ozone for organic matter renders it peculiarly suited for water purification, in that it not only removes the bacteria by direct oxidation but will eliminate to a considerable degree other organic substances contained therein.

All ozonation plants for the purification of water consist of two distinct parts—the ozone generator and

[1] Chemist and bacteriologist of the Baltimore County Water and Electric Company, 100–102 West Fayette Street, Baltimore, Md.

[1] *Engineering News*, **63** (1910), 488.