




Article

Vector Similarity Measures of Q-Linguistic Neutrosophic Variable Sets and Their Multi-Attribute Decision Making Method

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Abstract: Since language is used for thinking and expressing habits of humans in real life, the linguistic evaluation for an objective thing is expressed easily in linguistic terms/values. However, existing linguistic concepts cannot describe linguistic arguments regarding an evaluated object in two-dimensional universal sets (TDUSs). To describe linguistic neutrosophic arguments in decision making problems regarding TDUSs, this study proposes a Q-linguistic neutrosophic variable set (Q-LNVS) for the first time, which depicts its truth, indeterminacy, and falsity linguistic values independently corresponding to TDUSs, and vector similarity measures of Q-LNVs. Thereafter, a linguistic neutrosophic multi-attribute decision-making (MADM) approach by using the presented similarity measures, including the cosine, Dice, and Jaccard measures, is developed under Q-linguistic neutrosophic setting. Lastly, the applicability and effectiveness of the presented MADM approach is presented by an illustrative example under Q-linguistic neutrosophic setting.

Keywords: Q-linguistic neutrosophic variable set; vector similarity measure; cosine measure; Dice measure; Jaccard measure; decision making

1. Introduction

Since language is used for thinking and expressing habits of humans in real life, the linguistic evaluation for an objective thing is expressed easily in linguistic terms/values [1]. Hence, they were applied to linguistic fuzzy reason [1] and linguistic decision-making (DM) problems [2–9]. Because of linguistic uncertainty and hesitancy in the linguistic evaluation for an objective thing, there exist the representations of interval/uncertain linguistic numbers or hesitant linguistic numbers. Hence, on the one hand, interval/uncertain linguistic numbers were proposed and applied to (group) DM problems in uncertain linguistic setting [10–14]. On the other hand, hesitant linguistic variables (LVs) and hesitant uncertain LVs were presented and applied in (group) DM problems in hesitant (uncertain) linguistic setting [15–19]. In addition, a linguistic cubic variable was put forward based on combining an interval LV with a unique LV and used for DM problems in linguistic cubic setting [20,21]. Further, a linguistic cubic hesitant fuzzy number/variable was presented to depict the hybrid information of its uncertain linguistic argument and its hesitant linguistic argument and utilized for DM problems in linguistic cubic hesitant fuzzy setting [22]. By combining a neutrosophic number with language, a neutrosophic linguistic number and some weighted aggregation operators of neutrosophic linguistic numbers [23] were introduced for neutrosophic linguistic number DM problems, and then the similarity measure and expected value of hesitant neutrosophic linguistic numbers [24] were further presented for DM problems with hesitant neutrosophic linguistic numbers.

In real life environments, the truth, indeterminacy, and falsity linguistic arguments regarding an objective thing are presented in a human's thinking and expressing process and linguistic neutrosophic variables/numbers (LNVs) were presented to depict truth, falsity, and indeterminacy linguistic degrees independently [25]. Then, some aggregation operators of LNVs [25,26], cosine measures of LNVs [27], and correlation coefficients of LNVs [28] were proposed, respectively, for DM problems in LNV setting. Regarding the combination of a neutrosophic linguistic number and an LNV, linguistic neutrosophic uncertain numbers and their weighted aggregation operators were presented for DM in uncertain linguistic setting [29]. By the hybrid form of an interval LV (an uncertain linguistic argument) and a single-valued LNV (an argument of confident degree), single-valued linguistic neutrosophic interval LVs, and their weighted aggregation operators were proposed for DM along with uncertain/interval linguistic arguments and their linguistic neutrosophic confident degrees [30]. Regarding hesitant LNV environment, similarity measures between hesitant LNVs were presented by the least common multiple cardinality and applied to hesitant linguistic neutrosophic DM problems [31]. By the hybrid form of LNV [25] and linguistic cubic numbers [20], linguistic neutrosophic cubic numbers and their aggregation operators were introduced for linguistic neutrosophic cubic DM problems [32,33].

However, the various linguistic concepts are all described in a unique universal set, and then in some decision situations there exist the assessment problems of alternatives over two-dimensional universal sets (TDUSs). For example, suppose a person would like to purchase a house in a group of four houses (a set of four alternatives $H = \{H_1, H_2, H_3, H_4\}$). In his/her attractive evaluation of houses, the price (x_1), environment (x_2), and traffic (x_3) of the four houses are considered as a universal set $X = \{x_1, x_2, x_3\}$, and selecting two cities c_1 and c_2 are considered as another universal set $C = \{c_1, c_2\}$. Obviously, the above various linguistic arguments cannot represent such an assessment problem for each alternative H_j ($j = 1, 2, 3, 4$) over the TDUSs $X = \{x_1, x_2, x_3\}$ and $C = \{c_1, c_2\}$ in linguistic DM setting. Then, a Q-neutrosophic set and a Q-neutrosophic soft set were put forward regarding TDUSs and applied to Q-neutrosophic soft DM problems [34]. Although they can express and handle the assessment problems with TDUSs in neutrosophic DM environments, they cannot carry out linguistic neutrosophic DM problems over TDUSs. To solve this problem, this study presents a Q-linguistic neutrosophic variable set (Q-LNVS) for the first time to express the linguistic evaluation problems of the truth, falsity, and indeterminacy over TDUSs from the predefined linguistic term set (LTS). It then puts forward the vector similarity measures of Q-LNVSs, including the cosine, Dice, and Jaccard measures of Q-LNVSs, and then establishes a multi-attribute DM approach of Q-LNVSs by the vector similarity measures of Q-LNVs to solve linguistic neutrosophic DM problems along with TDUSs. It is obvious that the proposed DM approach shows the advantage of carrying out the linguistic neutrosophic DM problems regarding TDUSs, which existing linguistic neutrosophic DM approaches [25–28] and Q-linguistic neutrosophic soft DM approaches [34] cannot solve.

The framework of this study is organized below. The second section proposes Q-LNVSs and vector similarity measures between Q-LNVSs, including the cosine, Dice, and Jaccard measures of Q-LNVSs. The third section develops a multi-attribute DM approach of Q-LNVSs by using the vector similarity measures in Q-linguistic neutrosophic setting. An illustrative example and its sensitivity analysis to weights are presented in the fourth section. The last section contains conclusions and future study.

2. Vector Similarity Measures of Q-LNVSs

First, we present the concept of Q-LNVS to depict a linguistic neutrosophic evaluation problem by the truth, falsity, and indeterminacy linguistic arguments over TDUSs in linguistic setting.

Definition 1. Set $X = \{x_1, x_2, \dots, x_n\}$ and $Q = \{q_1, q_2, \dots, q_m\}$ as TDUSs and let LTS be $S = \{s_l \mid l \in [0, g]\}$, where $g + 1$ is an odd cardinality. Then a Q-LNVS L in X and Q is defined by the following form:

$$L = \left\{ \left\langle (x_i, q_j), s_t(x_i, q_j), s_u(x_i, q_j), s_v(x_i, q_j) \right\rangle \mid x_i \in X, q_j \in Q, \right. \\ \left. s_t(x_i, q_j), s_i(x_i, q_j), s_f(x_i, q_j) \in S, j = 1, 2, \dots, m; i = 1, 2, \dots, n \right\}$$

where $s_t(x_i, q_j), s_u(x_i, q_j), s_v(x_i, q_j)$ are the truth, indeterminacy, and falsity LVs, respectively, in TDUSs for $t, u, v \in [0, g]$.

Then, the basic linguistic element $\langle (x_i, q_j), s_t(x_i, q_j), s_u(x_i, q_j), s_v(x_i, q_j) \rangle$ in L is simply denoted by $l_{ij} = \langle (x_i, q_j), s_{t_{ij}}, s_{u_{ij}}, s_{v_{ij}} \rangle$, which is called a Q-linguistic neutrosophic element (Q-LNE).

Example 1. Suppose a person would like to buy a house from a city. There is a set of two potential houses $H = \{L_1, L_2\}$ in two cities. Then, set their price (x_1), environment (x_2), and traffic (x_3) as a universal set $X = \{x_1, x_2, x_3\}$ and set the two cities as another universal set $Q = \{q_1, q_2\}$. Based on the predefined LTS $S = \{s_0 = \text{extremely low}, s_1 = \text{very low}, s_2 = \text{low}, s_3 = \text{slightly low}, s_4 = \text{medium}, s_5 = \text{slightly high}, s_6 = \text{high}, s_7 = \text{very high}, s_8 = \text{extremely high}\}$, the two Q-LNE sets obtained from S are given as follows:

$$L_1 = \{ \langle (x_1, q_1), s_6, s_1, s_2 \rangle, \langle (x_1, q_2), s_5, s_2, s_3 \rangle, \langle (x_2, q_1), s_4, s_3, s_2 \rangle, \langle (x_2, q_2), s_7, s_1, s_3 \rangle, \\ \langle (x_3, q_1), s_6, s_2, s_1 \rangle, \langle (x_3, q_2), s_6, s_1, s_1 \rangle \}, \\ L_2 = \{ \langle (x_1, q_1), s_3, s_4, s_5 \rangle, \langle (x_1, q_2), s_4, s_2, s_1 \rangle, \langle (x_2, q_1), s_4, s_2, s_2 \rangle, \langle (x_2, q_2), s_5, s_1, s_2 \rangle, \\ \langle (x_3, q_1), s_5, s_2, s_1 \rangle, \langle (x_3, q_2), s_6, s_3, s_2 \rangle \}.$$

In the following, we give the vector similarity measures between Q-LNVSs, including the cosine, Dice, and Jaccard measures of Q-LNVSs.

Definition 2. Let TDUSs be $X = \{x_1, x_2, \dots, x_n\}$ and $Q = \{q_1, q_2, \dots, q_p\}$ and let $l_{ij}^1 = \langle (x_i, q_j), s_{t_{ij}^1}, s_{u_{ij}^1}, s_{v_{ij}^1} \rangle$ and $l_{ij}^2 = \langle (x_i, q_j), s_{t_{ij}^2}, s_{u_{ij}^2}, s_{v_{ij}^2} \rangle$ ($j = 1, 2, \dots, p; i = 1, 2, \dots, n$) be two groups of Q-LNEs in two Q-LNVSs L_1 and L_2 regarding the LTS $S = \{s_l \mid l \in [0, g]\}$. Then, the cosine, Dice, and Jaccard measures of the Q-LNVSs L_1 and L_2 are defined, respectively, as follows:

$$C(L_1, L_2) = s \frac{g \sum_{j=1}^p \sum_{i=1}^n (t_{ij}^1 t_{ij}^2 + u_{ij}^1 v_{ij}^2 + v_{ij}^1 v_{ij}^2)}{\sqrt{\sum_{j=1}^p \sum_{i=1}^n [(t_{ij}^1)^2 + (u_{ij}^1)^2 + (v_{ij}^1)^2]} \times \sqrt{\sum_{j=1}^p \sum_{i=1}^n [(t_{ij}^2)^2 + (u_{ij}^2)^2 + (v_{ij}^2)^2]}}, \tag{1}$$

$$D(L_1, L_2) = s \frac{2g \sum_{j=1}^p \sum_{i=1}^n (t_{ij}^1 t_{ij}^2 + u_{ij}^1 v_{ij}^2 + v_{ij}^1 v_{ij}^2)}{\sum_{j=1}^p \sum_{i=1}^n [(t_{ij}^1)^2 + (u_{ij}^1)^2 + (v_{ij}^1)^2] + \sum_{j=1}^p \sum_{i=1}^n [(t_{ij}^2)^2 + (u_{ij}^2)^2 + (v_{ij}^2)^2]}}, \tag{2}$$

$$J(L_1, L_2) = s \frac{g \sum_{j=1}^p \sum_{i=1}^n (t_{ij}^1 t_{ij}^2 + u_{ij}^1 v_{ij}^2 + v_{ij}^1 v_{ij}^2)}{\sum_{j=1}^p \sum_{i=1}^n [(t_{ij}^1)^2 + (u_{ij}^1)^2 + (v_{ij}^1)^2] + \sum_{j=1}^p \sum_{i=1}^n [(t_{ij}^2)^2 + (u_{ij}^2)^2 + (v_{ij}^2)^2] - \sum_{j=1}^p \sum_{i=1}^n (t_{ij}^1 t_{ij}^2 + u_{ij}^1 v_{ij}^2 + v_{ij}^1 v_{ij}^2)}}. \tag{3}$$

Obviously, the above cosine, Dice, and Jaccard measures satisfy the following properties:

- (1) $s_0 \leq C(L_1, L_2), D(L_1, L_2), J(L_1, L_2) \leq s_g$;
- (2) $C(L_1, L_2) = C(L_2, L_1), D(L_1, L_2) = D(L_2, L_1), J(L_1, L_2) = J(L_2, L_1)$;
- (3) $C(L_1, L_2) = D(L_1, L_2) = J(L_1, L_2) = s_g$ if and only if $L_1 = L_2$.

When the importance of elements x_i ($i = 1, 2, \dots, n$) and q_j ($j = 1, 2, \dots, p$) is taken into account, the weight vectors corresponding to $X = \{x_1, x_2, \dots, x_n\}$ and $Q = \{q_1, q_2, \dots, q_p\}$ are given

as $w = \{w_1, w_2, \dots, w_n\}$ and $w = \{w_1, w_2, \dots, w_p\}$, respectively. Thus, the weighted cosine, Dice, and Jaccard measures of L_1 and L_2 can be presented, respectively, as follows:

$$C_w(L_1, L_2) = s \frac{8 \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i (t_{ij}^1 t_{ij}^2 + u_{ij}^1 v_{ij}^2 + v_{ij}^1 v_{ij}^2)}{\sqrt{\sum_{j=1}^p \omega_j \sum_{i=1}^n w_i [(t_{ij}^1)^2 + (u_{ij}^1)^2 + (v_{ij}^1)^2]} \times \sqrt{\sum_{j=1}^p \omega_j \sum_{i=1}^n w_i [(t_{ij}^2)^2 + (u_{ij}^2)^2 + (v_{ij}^2)^2]}} \tag{4}$$

$$D_w(L_1, L_2) = s \frac{28 \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i (t_{ij}^1 t_{ij}^2 + u_{ij}^1 v_{ij}^2 + v_{ij}^1 v_{ij}^2)}{\sum_{j=1}^p \omega_j \sum_{i=1}^n w_i [(t_{ij}^1)^2 + (u_{ij}^1)^2 + (v_{ij}^1)^2] + \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i [(t_{ij}^2)^2 + (u_{ij}^2)^2 + (v_{ij}^2)^2]} \tag{5}$$

$$J_w(L_1, L_2) = s \frac{8 \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i (t_{ij}^1 t_{ij}^2 + u_{ij}^1 v_{ij}^2 + v_{ij}^1 v_{ij}^2)}{\sum_{j=1}^p \omega_j \sum_{i=1}^n w_i [(t_{ij}^1)^2 + (u_{ij}^1)^2 + (v_{ij}^1)^2] + \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i [(t_{ij}^2)^2 + (u_{ij}^2)^2 + (v_{ij}^2)^2] - \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i (t_{ij}^1 t_{ij}^2 + u_{ij}^1 v_{ij}^2 + v_{ij}^1 v_{ij}^2)} \tag{6}$$

Example 2. Let us consider two Q-LNE sets $L_1 = \{ \langle (x_1, q_1), s_6, s_1, s_2 \rangle, \langle (x_1, q_2), s_5, s_2, s_3 \rangle, \langle (x_2, q_1), s_4, s_3, s_2 \rangle, \langle (x_2, q_2), s_7, s_1, s_3 \rangle, \langle (x_3, q_1), s_6, s_2, s_1 \rangle, \langle (x_3, q_2), s_6, s_1, s_1 \rangle \}$ and $L_2 = \{ \langle (x_1, q_1), s_3, s_4, s_5 \rangle, \langle (x_1, q_2), s_4, s_2, s_1 \rangle, \langle (x_2, q_1), s_4, s_2, s_2 \rangle, \langle (x_2, q_2), s_5, s_1, s_2 \rangle, \langle (x_3, q_1), s_5, s_2, s_1 \rangle, \langle (x_3, q_2), s_6, s_3, s_2 \rangle \}$ in the LTS $S = \{s_0, s_1, s_2, \dots, s_8\}$ with $g = 8$ and the TDUSs $X = \{x_1, x_2, x_3\}$ and $Q = \{q_1, q_2\}$. Suppose the weight vectors for $X = \{x_1, x_2, x_3\}$ and $Q = \{q_1, q_2\}$ are given as $w = (0.4, 0.25, 0.35)$ and $\omega = (0.4, 0.6)$, respectively. Then, we compute the measure values of $C_w(L_1, L_2)$, $D_w(L_1, L_2)$, $J_w(L_1, L_2)$.

By using Equations (4)–(6), their calculational processes are shown as follows:

$$C_w(L_1, L_2) = s \frac{8 \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i (t_{ij}^1 t_{ij}^2 + u_{ij}^1 v_{ij}^2 + v_{ij}^1 v_{ij}^2)}{\sqrt{\sum_{j=1}^p \omega_j \sum_{i=1}^n w_i [(t_{ij}^1)^2 + (u_{ij}^1)^2 + (v_{ij}^1)^2]} \times \sqrt{\sum_{j=1}^p \omega_j \sum_{i=1}^n w_i [(t_{ij}^2)^2 + (u_{ij}^2)^2 + (v_{ij}^2)^2]}}$$

$$= s \frac{8 \left\{ \begin{aligned} &0.4[0.4(6 \times 3 + 1 \times 4 + 2 \times 5)] + 0.25(4 \times 4 + 3 \times 2 + 2 \times 2) + 0.35(6 \times 5 + 2 \times 2 + 1 \times 1) \end{aligned} \right\} + \left\{ \begin{aligned} &0.6[0.4(5 \times 4 + 2 \times 2 + 3 \times 1)] + 0.25(7 \times 5 + 1 \times 1 + 3 \times 2) + 0.35(6 \times 6 + 1 \times 3 + 1 \times 2) \end{aligned} \right\}}{\sqrt{\left\{ \begin{aligned} &0.4[0.4(6^2 + 1^2 + 2^2)] + 0.25(4^2 + 3^2 + 2^2) + 0.35(6^2 + 2^2 + 1^2) \end{aligned} \right\} + \left\{ \begin{aligned} &0.6[0.4(5^2 + 2^2 + 3^2)] + 0.25(7^2 + 1^2 + 3^2) + 0.35(6^2 + 1^2 + 1^2) \end{aligned} \right\}} \times \sqrt{\left\{ \begin{aligned} &0.4[0.4^2(3^2 + 4^2 + 5^2)] + 0.25(4^2 + 2^2 + 2^2) + 0.35(5^2 + 2^2 + 1^2) \end{aligned} \right\} + \left\{ \begin{aligned} &0.6[0.4^2(4^2 + 2^2 + 1^2)] + 0.25(5^2 + 1^2 + 2^2) + 0.35(6^2 + 3^2 + 2^2) \end{aligned} \right\}}}$$

$$= s_{7.0913}$$

$$D_w(L_1, L_2) = s \frac{28 \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i (t_{ij}^1 t_{ij}^2 + u_{ij}^1 v_{ij}^2 + v_{ij}^1 v_{ij}^2)}{\sum_{j=1}^p \omega_j \sum_{i=1}^n w_i [(t_{ij}^1)^2 + (u_{ij}^1)^2 + (v_{ij}^1)^2] + \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i [(t_{ij}^2)^2 + (u_{ij}^2)^2 + (v_{ij}^2)^2]}$$

$$= s \frac{2 \times 8 \left\{ \begin{aligned} &0.4[0.4(6 \times 3 + 1 \times 4 + 2 \times 5)] + 0.25(4 \times 4 + 3 \times 2 + 2 \times 2) + 0.35(6 \times 5 + 2 \times 2 + 1 \times 1) \end{aligned} \right\} + \left\{ \begin{aligned} &0.6[0.4(5 \times 4 + 2 \times 2 + 3 \times 1)] + 0.25(7 \times 5 + 1 \times 1 + 3 \times 2) + 0.35(6 \times 6 + 1 \times 3 + 1 \times 2) \end{aligned} \right\}}{\left\{ \begin{aligned} &0.4[0.4(6^2 + 1^2 + 2^2)] + 0.25(4^2 + 3^2 + 2^2) + 0.35(6^2 + 2^2 + 1^2) \end{aligned} \right\} + \left\{ \begin{aligned} &0.6[0.4^2(3^2 + 4^2 + 5^2)] + 0.25(4^2 + 2^2 + 2^2) + 0.35(5^2 + 2^2 + 1^2) \end{aligned} \right\} + \left\{ \begin{aligned} &0.6[0.4(5^2 + 2^2 + 3^2)] + 0.25(7^2 + 1^2 + 3^2) + 0.35(6^2 + 1^2 + 1^2) \end{aligned} \right\} + \left\{ \begin{aligned} &0.6[0.4^2(4^2 + 2^2 + 1^2)] + 0.25(5^2 + 1^2 + 2^2) + 0.35(6^2 + 3^2 + 2^2) \end{aligned} \right\}}$$

$$= s_{7.0777}$$

$$J_w(L_1, L_2) = s \frac{8 \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i (t_{ij}^1 t_{ij}^2 + u_{ij}^1 v_{ij}^2 + v_{ij}^1 v_{ij}^2)}{\sum_{j=1}^p \omega_j \sum_{i=1}^n w_i [(t_{ij}^1)^2 + (u_{ij}^1)^2 + (v_{ij}^1)^2] + \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i [(t_{ij}^2)^2 + (u_{ij}^2)^2 + (v_{ij}^2)^2] - \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i (t_{ij}^1 t_{ij}^2 + u_{ij}^1 v_{ij}^2 + v_{ij}^1 v_{ij}^2)}$$

$$= s \frac{8 \left\{ \begin{aligned} &0.4[0.4(6 \times 3 + 1 \times 4 + 2 \times 5)] + 0.25(4 \times 4 + 3 \times 2 + 2 \times 2) + 0.35(6 \times 5 + 2 \times 2 + 1 \times 1) \end{aligned} \right\} + \left\{ \begin{aligned} &0.6[0.4(5 \times 4 + 2 \times 2 + 3 \times 1)] + 0.25(7 \times 5 + 1 \times 1 + 3 \times 2) + 0.35(6 \times 6 + 1 \times 3 + 1 \times 2) \end{aligned} \right\}}{\left\{ \begin{aligned} &0.4[0.4(6^2 + 1^2 + 2^2)] + 0.25(4^2 + 3^2 + 2^2) + 0.35(6^2 + 2^2 + 1^2) \end{aligned} \right\} + \left\{ \begin{aligned} &0.6[0.4(5^2 + 2^2 + 3^2)] + 0.25(7^2 + 1^2 + 3^2) + 0.35(6^2 + 1^2 + 1^2) \end{aligned} \right\} - \left\{ \begin{aligned} &0.4[0.4^2(3^2 + 4^2 + 5^2)] + 0.25(4^2 + 2^2 + 2^2) + 0.35(5^2 + 2^2 + 1^2) \end{aligned} \right\} + \left\{ \begin{aligned} &0.6[0.4^2(4^2 + 2^2 + 1^2)] + 0.25(5^2 + 1^2 + 2^2) + 0.35(6^2 + 3^2 + 2^2) \end{aligned} \right\} - \left\{ \begin{aligned} &0.4[0.4(6 \times 3 + 1 \times 4 + 2 \times 5)] + 0.25(4 \times 4 + 3 \times 2 + 2 \times 2) + 0.35(6 \times 5 + 2 \times 2 + 1 \times 1) \end{aligned} \right\} + \left\{ \begin{aligned} &0.6[0.4(5 \times 4 + 2 \times 2 + 3 \times 1)] + 0.25(7 \times 5 + 1 \times 1 + 3 \times 2) + 0.35(6 \times 6 + 1 \times 3 + 1 \times 2) \end{aligned} \right\}}$$

$$= s_{6.3460}$$

Obviously, the above vector measure values still belong to the LTS S.

3. DM Approach Based on the Vector Similarity Measures

This section proposes a Q-linguistic neutrosophic multi-attribute DM approach based on the Dice, cosine, and Jaccard measures (the three vector measures) of Q-LNVs in Q-LNVS setting.

Suppose there is a multi-attribute DM problem, in which $L = \{L_1, L_2, \dots, L_m\}$ is denoted by a set of m alternatives. Then, TDUSs (two kinds of attribute sets) are specified as $X = \{x_1, x_2, \dots, x_n\}$ and $Q = \{q_1, q_2, \dots, q_p\}$, respectively, and then their corresponding weigh vectors are given as $w = (w_1, w_2, \dots, w_n)$ and $w = (w_1, w_2, \dots, w_p)$. Whereas, a decision maker is required to assess the alternatives L_k ($k = 1, 2, \dots, m$) on the attributes x_i ($i = 1, 2, \dots, n$) and q_j ($j = 1, 2, \dots, p$) by Q-LNEs regarding the given LTS $S = \{s_l | l \in [0, g]\}$ with the odd cardinality $g + 1$. In the assessment process, the decision maker gives the truth, falsity, and indeterminacy linguistic values for x_i and q_j on an alternative L_k by corresponding linguistic terms in S , which are constructed as a Q-LNE $l_{ij}^k = \left\langle (x_i, q_j), s_{t_{ij}^k}, s_{u_{ij}^k}, s_{v_{ij}^k} \right\rangle$ ($j = 1, 2, \dots, p; i = 1, 2, \dots, n; k = 1, 2, \dots, m$). Hence, all the Q-LNEs provided by the decision maker can be composed of a decision matrix of Q-LNEs $L = \left(l_{ij}^k \right)_{m \times nq}$.

Thus, the proposed DM method using the vector similarity measures of Q-LNVSs is applied to the multi-attribute DM problem with Q-LNVS information. Whereas, the DM steps are depicted in detail below:

Step 1: Since $l_{ij}^* = \left\langle (x_i, q_j), s_{t_{ij}^*}, s_{u_{ij}^*}, s_{v_{ij}^*} \right\rangle = \left\langle \left\langle (x_i, q_j), \max_k \left(s_{t_{ij}^k} \right), \min_k \left(s_{u_{ij}^k} \right), \min_k \left(s_{v_{ij}^k} \right) \right\rangle \right\rangle$ is an ideal Q-LNE as the best Q-LNE, we can establish the following ideal Q-LNVS:

$$L^* = \left\{ \left\langle (x_i, q_j), s_{t_{ij}^*}, s_{u_{ij}^*}, s_{v_{ij}^*} \right\rangle \mid x_i \in X, q_j \in Q, k = 1, 2, \dots, m, j = 1, 2, \dots, p, i = 1, 2, \dots, n \right\}. \quad (7)$$

Step 2: By applying Equations (4)–(6), the cosine/Dice/Jaccard measure between L_k ($k = 1, 2, \dots, m$) and L^* is given by using the following formula:

$$C_w(L_k, L^*) = s \frac{g \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i (t_{ij}^k t_{ij}^* + u_{ij}^k v_{ij}^* + v_{ij}^k v_{ij}^*)}{\sqrt{\sum_{j=1}^p \omega_j \sum_{i=1}^n w_i [(t_{ij}^k)^2 + (u_{ij}^k)^2 + (v_{ij}^k)^2]} \times \sqrt{\sum_{j=1}^p \omega_j \sum_{i=1}^n w_i [(t_{ij}^*)^2 + (u_{ij}^*)^2 + (v_{ij}^*)^2]}}, \quad (8)$$

or

$$D_w(L_k, L^*) = s \frac{2 \times g \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i^2 (t_{ij}^k t_{ij}^* + u_{ij}^k v_{ij}^* + v_{ij}^k v_{ij}^*)}{\sum_{j=1}^p \omega_j \sum_{i=1}^n w_i^2 [(t_{ij}^k)^2 + (u_{ij}^k)^2 + (v_{ij}^k)^2] + \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i^2 [(t_{ij}^*)^2 + (u_{ij}^*)^2 + (v_{ij}^*)^2]}}, \quad (9)$$

or

$$J_w(L_k, L^*) = s \frac{g \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i (t_{ij}^k t_{ij}^* + u_{ij}^k v_{ij}^* + v_{ij}^k v_{ij}^*)}{\sum_{j=1}^p \omega_j \sum_{i=1}^n w_i [(t_{ij}^k)^2 + (u_{ij}^k)^2 + (v_{ij}^k)^2] + \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i [(t_{ij}^*)^2 + (u_{ij}^*)^2 + (v_{ij}^*)^2] - \sum_{j=1}^p \omega_j \sum_{i=1}^n w_i (t_{ij}^k t_{ij}^* + u_{ij}^k v_{ij}^* + v_{ij}^k v_{ij}^*)}} \quad (10)$$

Step 3: According to the linguistic values of the vector similarity measures, the alternatives are ranked and the best alternative L_{k^*} is chosen regarding the biggest linguistic value for X and Q .

Step 4: Based on $x_j \in X$ ($j = 1, 2, \dots, p$) or $q_i \in Q$ ($i = 1, 2, \dots, n$), we need to calculate the measure values between $L_{k^*}(x_i, q_j)$ and $L^*(x_i, q_j)$:

$$C(L_{k^*}(x_i, q_j), L^*(x_i, q_j)) = s \frac{g \sum_{i=1}^n (t_{ij}^k t_{ij}^* + u_{ij}^k v_{ij}^* + v_{ij}^k v_{ij}^*)}{\sqrt{\sum_{i=1}^n [(t_{ij}^k)^2 + (u_{ij}^k)^2 + (v_{ij}^k)^2]} \times \sqrt{\sum_{i=1}^n [(t_{ij}^*)^2 + (u_{ij}^*)^2 + (v_{ij}^*)^2]}} \quad \text{for } j = 1, 2, \dots, p, \quad (11)$$

or

$$C(L_{k^*}(x_i, q_j), L^*(x_i, q_j)) = s \frac{g \sum_{j=1}^p (t_{ij}^k t_{ij}^* + u_{ij}^k v_{ij}^* + v_{ij}^k v_{ij}^*)}{\sqrt{\sum_{j=1}^p [(t_{ij}^k)^2 + (u_{ij}^k)^2 + (v_{ij}^k)^2]} \times \sqrt{\sum_{j=1}^p [(t_{ij}^*)^2 + (u_{ij}^*)^2 + (v_{ij}^*)^2]}} \quad \text{for } i = 1, 2, \dots, n; \quad (12)$$

$$D(L_{k^*}(x_i, q_j), L^*(x_i, q_j)) = s \frac{2g \sum_{i=1}^n (t_{ij}^k t_{ij}^* + u_{ij}^k v_{ij}^* + v_{ij}^k v_{ij}^*)}{\sum_{i=1}^n [(t_{ij}^k)^2 + (u_{ij}^k)^2 + (v_{ij}^k)^2] + \sum_{i=1}^n [(t_{ij}^*)^2 + (u_{ij}^*)^2 + (v_{ij}^*)^2]} \quad \text{for } j = 1, 2, \dots, p, \quad (13)$$

or

$$D(L_{k^*}(x_i, q_j), L^*(x_i, q_j)) = s \frac{2g \sum_{j=1}^p (t_{ij}^k + u_{ij}^k + v_{ij}^k)}{\sum_{j=1}^p [(t_{ij}^k)^2 + (u_{ij}^k)^2 + (v_{ij}^k)^2] + \sum_{j=1}^p [(t_{ij}^*)^2 + (u_{ij}^*)^2 + (v_{ij}^*)^2]} \quad \text{for } i = 1, 2, \dots, n; \quad (14)$$

$$J(L_{k^*}(x_i, q_j), L^*(x_i, q_j)) = s \frac{g \sum_{i=1}^n (t_{ij}^k + u_{ij}^k + v_{ij}^k)}{\sum_{i=1}^n [(t_{ij}^k)^2 + (u_{ij}^k)^2 + (v_{ij}^k)^2] + \sum_{i=1}^n [(t_{ij}^*)^2 + (u_{ij}^*)^2 + (v_{ij}^*)^2] - \sum_{i=1}^n (t_{ij}^k + u_{ij}^k + v_{ij}^k)} \quad \text{for } j = 1, 2, \dots, p, \quad (15)$$

or

$$J(L_{k^*}(x_i, q_j), L^*(x_i, q_j)) = s \frac{g \sum_{j=1}^p (t_{ij}^k + u_{ij}^k + v_{ij}^k)}{\sum_{j=1}^p [(t_{ij}^k)^2 + (u_{ij}^k)^2 + (v_{ij}^k)^2] + \sum_{j=1}^p [(t_{ij}^*)^2 + (u_{ij}^*)^2 + (v_{ij}^*)^2] - \sum_{j=1}^p (t_{ij}^k + u_{ij}^k + v_{ij}^k)} \quad \text{for } i = 1, 2, \dots, n. \quad (16)$$

Step 5: According to the linguistic values of $C(L_{k^*}(x_i, q_j), L^*(x_i, q_j))$ or $D(L_{k^*}(x_i, q_j), L^*(x_i, q_j))$ or $J(L_{k^*}(x_i, q_j), L^*(x_i, q_j))$ for X or Q (depending on some actual situation), we can determine the best one x_{i^*} or q_{j^*} corresponding to the biggest linguistic value.

Step 6: End.

4. Illustrative Example and Sensitivity Analysis to Weights

4.1. Illustrative Example

Suppose a person would like to buy a house in one of two cities. There are four potential houses (alternatives) of L_k ($k = 1, 2, 3, 4$) in two cities. Then, set their price (x_1), environment (x_2), and traffic (x_3) as a universal set $X = \{x_1, x_2, x_3\}$ and set the two cities as another universal set $Q = \{q_1, q_2\}$. Thus, the Q-LNEs can indicate the influence of both the three attributes of houses and the two cities on his/her buying attractive degree of a house. Herewith, the two weigh vectors of X and Q are given as $w = (0.4, 0.25, 0.35)$ and $w = (0.4, 0.6)$, respectively. Whereas, the alternative L_k ($k = 1, 2, 3, 4$) are assessed over the TDUSs $X = \{x_1, x_2, x_3\}$ and $Q = \{q_1, q_2\}$ from the given LTS $S = \{s_0 = \text{extremely low}, s_1 = \text{very low}, s_2 = \text{low}, s_3 = \text{slightly low}, s_4 = \text{medium}, s_5 = \text{slightly high}, s_6 = \text{high}, s_7 = \text{very high}, s_8 = \text{extremely high}\}$ with $g = 8$. In the assessment process, the decision maker/buyer can give the truth, indeterminacy, and falsity values for x_i and q_j on an alternative L_k by corresponding linguistic terms in S , and then establish Q-LNEs $l_{ij}^k = \langle (x_i, q_j), s_{t_{ij}^k}, s_{u_{ij}^k}, s_{v_{ij}^k} \rangle$ ($j = 1, 2; i = 1, 2, 3; k = 1, 2, 3, 4$), which are constructed as the DM matrix of Q-LNEs:

$$L = \begin{matrix} L_1 \\ L_2 \\ L_3 \\ L_4 \end{matrix} \begin{bmatrix} \langle (x_1, q_1), s_6, s_2, s_1 \rangle & \langle (x_1, q_2), s_7, s_2, s_3 \rangle & \langle (x_2, q_1), s_5, s_2, s_1 \rangle & \langle (x_2, q_2), s_4, s_1, s_1 \rangle & \langle (x_3, q_1), s_5, s_2, s_2 \rangle & \langle (x_3, q_2), s_7, s_2, s_2 \rangle \\ \langle (x_1, q_1), s_6, s_1, s_2 \rangle & \langle (x_1, q_2), s_7, s_1, s_1 \rangle & \langle (x_2, q_1), s_6, s_1, s_1 \rangle & \langle (x_2, q_2), s_5, s_1, s_2 \rangle & \langle (x_3, q_1), s_6, s_1, s_3 \rangle & \langle (x_3, q_2), s_7, s_2, s_1 \rangle \\ \langle (x_1, q_1), s_5, s_2, s_3 \rangle & \langle (x_1, q_2), s_6, s_2, s_4 \rangle & \langle (x_2, q_1), s_4, s_1, s_1 \rangle & \langle (x_2, q_2), s_5, s_2, s_2 \rangle & \langle (x_3, q_1), s_4, s_2, s_2 \rangle & \langle (x_3, q_2), s_5, s_2, s_3 \rangle \\ \langle (x_1, q_1), s_5, s_1, s_1 \rangle & \langle (x_1, q_2), s_6, s_3, s_5 \rangle & \langle (x_2, q_1), s_5, s_3, s_3 \rangle & \langle (x_2, q_2), s_6, s_2, s_4 \rangle & \langle (x_3, q_1), s_5, s_1, s_1 \rangle & \langle (x_3, q_2), s_6, s_2, s_4 \rangle \end{bmatrix}.$$

Thus, the proposed multi-attribute DM approach can be used for this Q-linguistic neutrosophic DM problem. The DM steps are depicted below:

Firstly, we establish the ideal alternative from the DM matrix L by the ideal Q-LNE set:

$$L^* = \{ \langle (x_1, q_1), s_6, s_1, s_1 \rangle, \langle (x_1, q_2), s_7, s_1, s_1 \rangle, \langle (x_2, q_1), s_6, s_1, s_1 \rangle, \langle (x_2, q_2), s_6, s_1, s_1 \rangle, \langle (x_3, q_1), s_6, s_1, s_1 \rangle, \langle (x_3, q_2), s_7, s_2, s_1 \rangle \}.$$

Then, by Equations (8)–(10), the measure results and ranking of the four alternatives are given in Table 1.

Table 1. Measure results and ranking of the four alternatives.

Measure Method	Measure Value between L_k ($k = 1, 2, 3, 4$) and L^*	Ranking
$C_w(L_k, L^*)$	$S7.7472, S7.9088, S7.3465, S7.2437$	$L_2 > L_1 > L_3 > L_4$
$D_w(L_k, L^*)$	$S7.7470, S7.9087, S7.3207, S7.2387$	$L_2 > L_1 > L_3 > L_4$
$J_w(L_k, L^*)$	$S7.5095, S7.8194, S6.7478, S6.6097$	$L_2 > L_1 > L_3 > L_4$

Based on Table 1, all the ranking orders are identical regarding the cosine, Dice, and Jaccard measures. Then, the best alternative is L_2 .

Next, the measure values of Equations (11), (13), and (15) regarding Q , and the best city regarding L_2 are given in Table 2.

Table 2. Measure results regarding Q and the best city.

Measure Method	Measure Result	The Best City
$C(L_2(x_i, q_1), L^*(x_i, q_1)), C(L_2(x_i, q_2), L^*(x_i, q_2))$	$S7.8409, S7.9457$	q_2
$D(L_2(x_i, q_1), L^*(x_i, q_1)), D(L_2(x_i, q_2), L^*(x_i, q_2))$	$S7.8326, S7.9424$	q_2
$J(L_2(x_i, q_1), L^*(x_i, q_1)), J(L_2(x_i, q_2), L^*(x_i, q_2))$	$S7.6721, S7.8857$	q_2

Lastly, the results based on Table 2 indicate that the buyer should buy the house L_2 in the best city q_2 .

4.2. Sensitivity Analysis to Weights

To indicate the influence of the weights on ranking orders in the illustrative example, we consider that the two weigh vectors of X and Q are given as $w = (1/3, 1/3, 1/3)$ and $w = (1/2, 1/2)$, respectively, to analyze the sensitivity of the weights with respect to the ranking orders of the four alternatives. In this case, by Equations (8)–(10) the measure results and ranking of the four alternatives are indicated in Table 3.

Table 3. Measure results and ranking of the four alternatives with $w = (1/3, 1/3, 1/3)$ and $w = (1/2, 1/2)$.

Measure Method	Measure Value between L_k ($k = 1, 2, 3, 4$) and L^*	Ranking
$C_w(L_k, L^*)$	$S7.7470, S7.8918, S7.3878, S7.2922$	$L_2 > L_1 > L_3 > L_4$
$D_w(L_k, L^*)$	$S7.7430, S7.8917, S7.3448, S7.2892$	$L_2 > L_1 > L_3 > L_4$
$J_w(L_k, L^*)$	$S7.5019, S7.7863, S6.7888, S6.6944$	$L_2 > L_1 > L_3 > L_4$

In this case, there exists the same ranking order in Tables 1 and 3 regarding the cosine, Dice, and Jaccard measures. Then, the best alternative is still L_2 , which means the buyer should also buy house L_2 in the best city q_2 based on Table 2. It is obvious that all the ranking orders imply the decision robustness based on the cosine, Dice, and Jaccard measures regarding the change of weights in this illustrative example, which also show no sensitivity of all the ranking orders with respect to the change of the weights. In the actual DM applications, however, one of three vector measures can be selected by decision makers' preference or actual requirements.

However, existing various linguistic neutrosophic DM approaches [25–28] cannot handle the DM problems in Q-LNVS setting; while our proposed DM method can carry out both the existing DM problems with LNV information [25–28] and the DM problems with Q-LNVS information, which shows its advantage in Q-LNVS setting because the LNV set is a special case of Q-LNVS under a universal set. Furthermore, the existing Q-neutrosophic soft DM approach [34] cannot deal with the DM problems with Q-LNVS information because the Q-neutrosophic soft set [34] cannot express Q-linguistic neutrosophic information. Hence, our proposed Q-linguistic neutrosophic DM method provides a new way for linguistic neutrosophic DM with TDUSs.

5. Conclusions

This study presented the concept of Q-LNVS for the first time to describe the truth, falsity, and indeterminacy linguistic arguments in TDUSs, and then the cosine, Dice, and Jaccard measures of Q-LNVSs in vector space. Next, a Q-linguistic neutrosophic multi-attribute DM approach in Q-LNVS setting was established by using the cosine, Dice, and Jaccard measures of Q-LNVSs to solve linguistic neutrosophic DM problems regarding TDUSs. Lastly, the application of the developed DM approach was given by an illustrative example in Q-LNVS setting. The decision results show that the established multi-attribute DM approach of Q-LNVSs can solve linguistic neutrosophic DM problems regarding TDUSs (two-dimensional attribute sets) in Q-LNVS setting, which indicates its main advantage and contribution. Based on the first study, the three vector measures of Q-LNVSs will be further used for medical diagnosis, data mining, and clustering analysis for future research in Q-LNVS setting.

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