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Generalized Q-Neutrosophic Soft Expert Set for Decision under Uncertainty

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Abstract: Neutrosophic triplet structure yields a symmetric property of truth membership on the left, indeterminacy membership in the centre and false membership on the right, as do points of object, centre and image of reflection. As an extension of a neutrosophic set, the Q-neutrosophic set was introduced to handle two-dimensional uncertain and inconsistent situations. We extend the soft expert set to generalized Q-neutrosophic soft expert set by incorporating the idea of soft expert set to the concept of Q-neutrosophic set and attaching the parameter of fuzzy set while defining a Q-neutrosophic soft expert set. This pattern carries the benefits of Q-neutrosophic sets and soft sets, enabling decision makers to recognize the views of specialists with no requirement for extra lumbering tasks, thus making it exceedingly reasonable for use in decision-making issues that include imprecise, indeterminate and inconsistent two-dimensional data. Some essential operations namely subset, equal, complement, union, intersection, AND and OR operations and additionally several properties relating to the notion of generalized Q-neutrosophic soft expert set are characterized. Finally, an algorithm on generalized Q-neutrosophic soft expert set is proposed and applied to a real-life example to show the efficiency of this notion in handling such problems.

Keywords: algorithm; decision making; expert set; generalized neutrosophic set; neutrosophic sets; Q-neutrosophic; soft sets.

1. Introduction

Zadeh established the concept of fuzzy set [1] as a way to handle uncertain information, by assigning a number to each element that shows the degree of membership of the element. Intuitionistic fuzzy set [2] is another way to handle uncertainty that assigns two numbers to each element. These numbers show the the degree of membership and the degree of nonmembership of the element. However, these theories fail to handle an indeterminate environment, hence Smarandache established the idea of neutrosophy [3] as an extension of fuzzy set and intuitionistic fuzzy set to mitigate such situations. Neutrosophic set (NS) [4] is recognized via three independent membership functions that depict the degrees of truth (T), indeterminacy (I), and falsity (F). Soft set [5] is another commonly used method in handling uncertainties. It has been extended extensively to fuzzy soft set [6], vague soft set [7–9] and neutrosophic soft set [10]. Although these concepts are widely applicable to different life branches, they lack the ability to handle two-dimensional problems. This motivates the definition of Q-fuzzy soft set [11,12] that served the uncertainty and two-dimensionality simultaneously. Recently, this was extended to the theory of Q-neutrosophic soft set (Q-NSS) [13] by extending the theory of Q-fuzzy soft set to a neutrosophic set. Q-NSS is a tri-component two-dimensional set, enabling it to address inconsistent, indeterminate and imprecise data in which the indeterminacy is measured unequivocally and truth, indeterminacy and falsity memberships are independent. Relations between Q-NSSs were studied in [13] while their measures of distance, similarity and entropy were discussed in [14].

Decision-making is an exploration point of research especially in uncertain environments. Recently, many researchers applied neutrosophic sets to real-life decision making problems containing uncertain, indeterminate and incompatible information [15–25].

Recently, the need for models incorporating opinions of experts and validating information supplied by observers has been recognized. In such cases, it is pertinent to have the conclusions of a specialist to approve the information obtained from observers before these data can be utilized to make a decision. The lack of this feature was one of the major problems that was inherent in fuzzy and soft sets, and their hybrid models, including Q-NSSs. The concept of soft expert sets (SESs) [26] is the first model in literature to deal with this issue, by presenting the opinions of the experts, with no extra task. Although SES was considered a novel idea at the time of its initiation, it does not have the capacity to represent the uncertainty that appears in most real issues. Many generalizations of the SES model were introduced to overcome this issue such as fuzzy soft expert sets [27], neutrosophic soft expert sets [28], neutrosophic vague soft expert set [17], complex neutrosophic soft expert set [18], vague soft expert sets [29–31], generalized neutrosophic soft expert set [32] and Q-neutrosophic soft expert set (Q-NSES) [33]. Q-NSES has the capacity to handle indeterminacy and two-dimensionality simultaneously, since it incorporates the elements of both soft expert set and Q-neutrosophic set. The structure of this concept enables it to provide the opinions of experts to activate the data obtained from individuals and able to present the ideas within a two-dimensional indeterminate environment which makes it suitable to describe many real problems.

In this study, we redefine the operations of Q-NSES [33] and introduce the conception of generalized Q-neutrosophic soft expert set (GQ-NSES) as an extension of Q-NSES by attaching the parameterization of fuzzy sets while defining a Q-NSES. The proposed concept is more practical as it includes uncertainty in the selection of a fuzzy set corresponding to each value of the parameter. We will introduce some concepts related to this model along with basic operations relevant to GQ-NSESs, namely the union, intersection and complement operations. The commutative and associative laws of these operations will be proposed and an application of GQ-NSES in decision-making will be illustrated.

2. Preliminaries

We review some basic ideas of soft set, neutrosophic set and Q-neutrosophic soft expert set that are related to the study in this work.

2.1. Neutrosophic Set

In the following, we recall the notion of neutrosophic set [4] with the operations of subset, complement, intersection and union [3].

Definition 1 (see [4]). *A neutrosophic set Γ on the universe X is defined as*

$$\Gamma = \{\langle x, (T_\Gamma(x), I_\Gamma(x), F_\Gamma(x))\rangle : x \in X\}, \textit{where } T, I, F : X \to]^-0, 1^+[$$

and

$$^-0 \leq T_\Gamma(x) + I_\Gamma(x) + F_\Gamma(x) \leq 3^+.$$

Definition 2 (see [3]). *Let Γ and Ψ be two neutrosophic sets. Then, we say that Γ is a subset of Ψ denoted by $\Gamma \subseteq \Psi$ if and only if $T_\Gamma(x) \leq T_\Psi(x)$, $I_\Gamma(x) \geq I_\Psi(x)$ and $F_\Gamma(x) \geq F_\Psi(x)$ for all $x \in X$.*

Definition 3 (see [3]). *The union of two neutrosophic sets Γ and Ψ in the universe X is denoted by $\Gamma \cup \Psi = \Lambda$, where*

$$\Lambda = \{\langle x, (\max\{T_\Gamma(x), T_\Psi(x)\}, \min\{I_\Gamma(x), I_\Psi(x)\}, \min\{F_\Gamma(x), F_\Psi(x)\})\rangle : x \in X\}.$$

Definition 4 (see [3]). *The intersection of two neutrosophic sets* Γ *and* Ψ *in the universe* X *is denoted by* $\Gamma \cap \Psi = \Lambda$, *where*

$$\Lambda = \{\langle x, (\min\{T_\Gamma(x), T_\Psi(x)\}, \max\{I_\Gamma(x), I_\Psi(x)\}, \max\{F_\Gamma(x), F_\Psi(x)\})\rangle : x \in X\}.$$

Definition 5 (see [3]). *The complement of a neutrosophic set* Γ *in the universe* X *is denoted by* Γ^c, *where*

$$\Gamma^c = \{\langle x, (1 - T_\Gamma(x), 1 - I_\Gamma(x), 1 - F_\Gamma(x))\rangle : x \in X\}.$$

The neutrosophic empty set Γ_0 *in the universe* X *is* $\Gamma_0 = \{\langle x, (0,1,1)\rangle : x \in X\}$.

2.2. Q-Neutrosophic Soft Expert Set

Abu Qamar and Hassan [13] proposed Q-neutrosophic set (Q-NS) for dealing with two-dimensional inconsistent, indeterminate and uncertain information.

Definition 6 (see [13]). *Let* X *be a universal set and* Q *be a nonempty set. A Q-neutrosophic set* Γ_Q *in* X *and* Q *is an object of the form*

$$\Gamma_Q = \{\Big\langle (x,q), T_{\Gamma_Q}(x,q), I_{\Gamma_Q}(x,q), F_{\Gamma_Q}(x,q) \Big\rangle : x \in X, q \in Q\},$$

where $T_{\Gamma_Q}, I_{\Gamma_Q}, F_{\Gamma_Q} : X \times Q \to]^-0, 1^+[$ *are the true membership function, indeterminacy membership function and false membership function, respectively, with* $^-0 \leq T_{\Gamma_Q} + I_{\Gamma_Q} + F_{\Gamma_Q} \leq 3^+$.

Hassan et al. raised the notion of Q-neutrosophic soft expert [33].

Let X be a universe, Q be a nonempty set, E a set of parameters, U a set of experts (agents), and $O = \{1 = agree, 0 = disagree\}$ a set of opinions. Let $Z = E \times U \times O$ and $A \subseteq Z$.

Definition 7 (see [33]). *A pair* $(\widehat{\Gamma}_Q, A)$ *is called a Q-NSES over X, where* $\widehat{\Gamma}_Q$ *is a mapping given by* $\widehat{\Gamma}_Q : A \to QNSES$ *such that QNSES is the set of all QNSES over U.*

In the following, we refined the basic operations of Q-NSES introduced in [33].

Definition 8. *Let* $(\widehat{\Gamma}_Q, A)$ *and* $(\widehat{\Psi}_Q, B)$ *be two Q-NSESs over X. Then,* $(\widehat{\Gamma}_Q, A)$ *is said to be Q-NSE subset of* $(\widehat{\Psi}_Q, B)$, *denoted by* $(\widehat{\Gamma}_Q, A)\widehat{\subseteq}(\widehat{\Psi}_Q, B)$ *if* $A \subseteq B$ *and* $T_{\widehat{\Gamma}_{Q(a)}}(x,q) \leq T_{\widehat{\Psi}_{Q(a)}}(x,q)$, $I_{\widehat{\Gamma}_{Q(a)}}(x,q) \geq I_{\widehat{\Psi}_{Q(a)}}(x,q)$, $F_{\widehat{\Gamma}_{Q(a)}}(x,q) \geq F_{\widehat{\Psi}_{Q(a)}}(x,q)$ $\forall a \in A, (x,q) \in X \times Q$.

Definition 9. *The complement of* $(\widehat{\Gamma}_Q, A)$ *is defined as* $(\widehat{\Gamma}_Q, A)^c = (\widehat{\Gamma}^c_Q, A)$, *where* $\widehat{\Gamma}^c_Q = A \to P(X \times Q)$ *and*

$$\widehat{\Gamma}^c_Q = \Big\{\Big\langle a, T_{\widehat{\Gamma}^c_Q(a)}(x,q), I_{\widehat{\Gamma}^c_Q(a)}(x,q), F_{\widehat{\Gamma}^c_Q(a)}(x,q) \Big\rangle : a \in A, (x,q) \in X \times Q\Big\},$$

such that $\forall a \in A, (x,q) \in X \times Q$

$$T_{\widehat{\Gamma}^c_Q(a)}(x,q) = 1 - T_{\widehat{\Gamma}_Q}(x,q),$$
$$I_{\widehat{\Gamma}^c_Q(a)}(x,q) = 1 - I_{\widehat{\Gamma}_Q}(x,q),$$
$$F_{\widehat{\Gamma}^c_Q(a)}(x,q) = 1 - F_{\widehat{\Gamma}_Q}(x,q).$$

Now, we propose the union and intersection of two Q-NSESs.

Definition 10. *The union of* $(\widehat{\Gamma}_Q, A)$ *and* $(\widehat{\Psi}_Q, B)$ *is a Q-NSES* $(\widehat{\Upsilon}_Q, C)$*, defined as* $(\widehat{\Gamma}_Q, A)\widehat{\cup}(\widehat{\Psi}_Q, B) = (\widehat{\Upsilon}_Q, C)$*, where* $C = A \cup B$ *and for all* $c \in C$ *and* $(x,q) \in X \times Q$ *the truth, indeterminacy and falsity memberships of* $(\widehat{\Upsilon}_Q, C)$ *are as follows:*

$$T_{\widehat{\Upsilon}_{Q(c)}}(x,q) = \begin{cases} T_{\widehat{\Gamma}_{Q(c)}}(x,q) & \text{if } c \in A - B, \\ T_{\widehat{\Psi}_{Q(c)}}(x,q) & \text{if } c \in B - A, \\ \max\{T_{\widehat{\Gamma}_{Q(c)}}(x,q), T_{\widehat{\Psi}_{Q(c)}}(x,q)\} & \text{if } c \in A \cap B, \end{cases}$$

$$I_{\widehat{\Upsilon}_{Q(c)}}(x,q) = \begin{cases} I_{\widehat{\Gamma}_{Q(c)}}(x,q) & \text{if } c \in A - B, \\ I_{\widehat{\Psi}_{Q(c)}}(x,q) & \text{if } c \in B - A, \\ \min\{I_{\widehat{\Gamma}_{Q(c)}}(x,q), I_{\widehat{\Psi}_{Q(c)}}(x,q)\} & \text{if } c \in A \cap B, \end{cases}$$

and

$$F_{\widehat{\Upsilon}_{Q(c)}}(x,q) = \begin{cases} F_{\widehat{\Gamma}_{Q(c)}}(x,q) & \text{if } c \in A - B, \\ F_{\widehat{\Psi}_{Q(c)}}(x,q) & \text{if } c \in B - A, \\ \min\{F_{\widehat{\Gamma}_{Q(c)}}(x,q), F_{\widehat{\Psi}_{Q(c)}}(x,q)\} & \text{if } c \in A \cap B. \end{cases}$$

Definition 11. *The intersection of* $(\widehat{\Gamma}_Q, A)$ *and* $(\widehat{\Psi}_Q, B)$ *is a Q-NSES* $(\widehat{\Upsilon}_Q, C)$*, defined as* $(\widehat{\Gamma}_Q, A)\widehat{\cap}(\widehat{\Psi}_Q, B) = (\widehat{\Upsilon}_Q, C)$*, where* $C = A \cap B$ *and for all* $c \in C$ *and* $(x,q) \in X \times Q$ *the truth, indeterminacy and falsity memberships of* $(\widehat{\Upsilon}_Q, C)$ *are as follows:*

$$\begin{aligned} T_{\widehat{\Upsilon}_{Q(c)}}(x,q) &= \min\{T_{\widehat{\Gamma}_{Q(c)}}(x,q), T_{\widehat{\Psi}_{Q(c)}}(x,q)\}, \\ I_{\widehat{\Upsilon}_{Q(c)}}(x,q) &= \max\{I_{\widehat{\Gamma}_{Q(c)}}(x,q), I_{\widehat{\Psi}_{Q(c)}}(x,q)\}, \\ F_{\widehat{\Upsilon}_{Q(c)}}(x,q) &= \max\{F_{\widehat{\Gamma}_{Q(c)}}(x,q), F_{\widehat{\Psi}_{Q(c)}}(x,q)\}. \end{aligned}$$

Definition 12. *If* $(\widehat{\Gamma}_Q, A)$ *and* $(\widehat{\Psi}_Q, B)$ *are two Q-neutrosophic soft expert sets on X, then* $(\widehat{\Gamma}_Q, A)$ *AND* $(\widehat{\Psi}_Q, B)$ *is the Q-neutrosophic soft expert set denoted by* $(\widehat{\Gamma}_Q, A)\widehat{\wedge}(\widehat{\Psi}_Q, B)$ *and defined by* $(\widehat{\Gamma}_Q, A)\widehat{\wedge}(\widehat{\Psi}_Q, B) = (\widehat{\Lambda}_Q, A \times B)$*, where* $\widehat{\Lambda}_Q(a,b) = \widehat{\Gamma}_Q(a)\widehat{\cap}\widehat{\Psi}_Q(b)$ *for all* $(a,b) \in A \times B$ *is the operation of intersection of two Q-neutrosophic sets on X.*

Definition 13. *If* $(\widehat{\Gamma}_Q, A)$ *and* $(\widehat{\Psi}_Q, B)$ *are two Q-neutrosophic soft expert sets on X, then* $(\widehat{\Gamma}_Q, A)$ *OR* $(\widehat{\Psi}_Q, B)$ *is the Q-neutrosophic soft expert set denoted by* $(\widehat{\Gamma}_Q, A)\widehat{\vee}(\widehat{\Psi}_Q, B)$ *and defined by* $(\widehat{\Gamma}_Q, A)\widehat{\vee}(\widehat{\Psi}_Q, B) = (\widehat{\Lambda}_Q, A \times B)$*, where* $\widehat{\Lambda}_Q(a,b) = \widehat{\Gamma}_Q(a)\widehat{\cup}\widehat{\Psi}_Q(b)$ *for all* $(a,b) \in A \times B$ *is the operation of union of two Q-neutrosophic sets on X.*

3. Generalized Q-Neutrosophic Soft Expert Set

In this section, we propose the generalized Q-neutrosophic soft expert set (GQ-NSES) and proceed to introduce several concepts related to this model. We will put forward the operations of union, intersection and complement of GQ-NSESs, and proceed with the properties of the commutative and associative laws of these operations.

We begin by proposing the definition of GQ-NSES, followed by an illustrative example.

Let X be a universe, Q be a nonempty set, E a set of parameters, U a set of experts (agents), and $O = \{1 = agree, 0 = disagree\}$ a set of opinions. Let $Z = E \times U \times O$, $A \subseteq Z$ and f be a fuzzy set; that is, $f : A \to [0,1]$.

Definition 14. *A pair* $(\widehat{\Gamma}_Q^f, A)$ *is called a GQ-NSES over X, where* $\widehat{\Gamma}_Q^f$ *is a mapping given by*

$$\widehat{\Gamma}_Q^f : A \to P(X \times Q) \times I,$$

where $P(X \times Q)$ *denotes the power Q-neutrosophic soft expert set. For all* $a \in A$, $\widehat{\Gamma}_Q$ *is referred to as the Q-neutrosophic expert value set of the parameter a, i.e,*

$$\widehat{\Gamma}_Q(a) = \left\{ \left\langle (x,q), T_{\widehat{\Gamma}_Q}(x,q), I_{\widehat{\Gamma}_Q}(x,q), F_{\widehat{\Gamma}_Q}(x,q) \right\rangle \right\}$$

presents the degree of belongingness, indeterminacy belongingness and non-belongingness of elements of X in $\widehat{\Gamma}_Q$, *where* $\forall (x,q) \in X \times Q, \forall a \in A, T_{\widehat{\Gamma}_Q}, I_{\widehat{\Gamma}_Q}, F_{\widehat{\Gamma}_Q}$ *representing the membership functions of truth, indeterminacy and falsity, respectively. The values* $T_{\widehat{\Gamma}_Q}, I_{\widehat{\Gamma}_Q}, F_{\widehat{\Gamma}_Q} \in [0,1]$ *and*

$$0 \leq T_{\widehat{\Gamma}_Q}(x,q) + I_{\widehat{\Gamma}_Q}(x,q) + F_{\widehat{\Gamma}_Q}(x,q) \leq 3.$$

The GQ-NSES $(\widehat{\Gamma}_Q^f, A)$ *is a parametrized family of Q-neutrosophic soft expert sets on X, which has the degree of preference of the approximate value set which represented by* $f(a)$ *for each paremeter a. The GQ-NSES can be written as:*

$$(\widehat{\Gamma}_Q^f, A) = \left\{ \left\langle a, (\widehat{\Gamma}_Q(a), f(a)) \right\rangle : a \in A, \widehat{\Gamma}_Q(a) \in P(X \times Q), f(a) \in [0,1] \right\}.$$

In short, for each parameter a, $\widehat{\Gamma}_Q^f(a)$ *gives not only the extent to which each element in X belongs, indeterminacy belong or not belong to* $\widehat{\Gamma}_Q$ *but also indicates how much such belonging is preferred.*

Example 1. *Suppose a company wants to fill a position to be chosen by an expert committee. There are three candidates* $X = \{x_1, x_2, x_3\}$ *with two types of qualifications* $Q = \{q_1 = master, q_2 = doctorate\}$ *and the hiring committee takes into consideration a set of parameters* $E = \{e_1 = computer\ knowledge, e_2 = experience\}$. *Let* $U = \{u_1, u_2\}$ *be the set of two committee members. Then, we can view the GQ-NSES* $(\widehat{\Gamma}_Q^f, A)$ *as consisting of the following collection of approximation:*

$$\begin{aligned}
(\widehat{\Gamma}_Q^f, A) = \Big\{ & \Big\langle (e_1,u_1,1), \Big([(x_1,q_1),0.1,0.2,0.6], [(x_1,q_2),0.4,0.3,0.7], [(x_2,q_1),0.5,0.2,0.1], \\
& \qquad [(x_2,q_2),0.7,0.2,0.3], [(x_3,q_1),0.8,0.3,0.1], [(x_3,q_2),0.2,0.3,0.6], 0.7 \Big) \Big\rangle, \\
& \Big\langle (e_1,u_2,1), \Big([(x_1,q_1),0.6,0.4,0.2], [(x_1,q_2),0.5,0.3,0.2], [(x_2,q_1),0.5,0.4,0.3], \\
& \qquad [(x_2,q_2),0.9,0.4,0.2], [(x_3,q_1),0.6,0.8,0.4], [(x_3,q_2),0.4,0.1,0.5], 0.5 \Big) \Big\rangle, \\
& \Big\langle (e_2,u_1,1), \Big([(x_1,q_1),0.7,0.2,0.3], [(x_1,q_2),0.8,0.4,0.6], [(x_2,q_1),0.3,0.6,0.9], \\
& \qquad [(x_2,q_2),0.1,0.3,0.3], [(x_3,q_1),0.4,0.7,0.5], [(x_3,q_2),0.8,0.7,0.4], 0.4 \Big) \Big\rangle \\
& \Big\langle (e_2,u_2,1), \Big([(x_1,q_1),0.7,0.6,0.5], [(x_1,q_2),0.7,0.4,0.1], [(x_2,q_1),0.8,0.6,0.3], \\
& \qquad [(x_2,q_2),0.7,0.6,0.2], [(x_3,q_1),0.3,0.4,0.2], [(x_3,q_2),0.5,0.3,0.7], 0.6 \Big) \Big\rangle, \\
& \Big\langle (e_1,u_1,0), \Big([(x_1,q_1),0.5,0.5,0.3], [(x_1,q_2),0.8,0.8,0.4], [(x_2,q_1),0.7,0.1,0.3], \\
& \qquad [(x_2,q_2),0.9,0.7,0.5], [(x_3,q_1),0.9,0.6,0.5], [(x_3,q_2),0.6,0.3,0.3], 0.8 \Big) \Big\rangle, \\
& \Big\langle (e_1,u_2,0), \Big([(x_1,q_1),0.2,0.2,0.5], [(x_1,q_2),0.7,0.4,0.9], [(x_2,q_1),0.8,0.7,0.5], \\
& \qquad [(x_2,q_2),0.8,0.3,0.3], [(x_3,q_1),0.3,0.2,0.8], [(x_3,q_2),0.5,0.5,0.5], 0.2 \Big) \Big\rangle,
\end{aligned}$$

$$\Big\langle (e_2,u_1,0),\Big([(x_1,q_1),0.4,0.8,0.8],[(x_1,q_2),0.9,0.6,0.6],[(x_2,q_1),0.9,0.7,0.5],$$
$$[(x_2,q_2),0.2,0.1,0.6],[(x_3,q_1),0.4,0.7,0.9],[(x_3,q_2),0.8,0.2,0.1],0.3\Big)\Big\rangle,$$
$$\Big\langle (e_2,u_2,0),\Big([(x_1,q_1),0.9,0.4,0.5],[(x_1,q_2),0.7,0.1,0.6],[(x_2,q_1),0.1,0.8,0.4],$$
$$[(x_2,q_2),0.5,0.7,0.8],[(x_3,q_1),0.4,0.4,0.4],[(x_3,q_2),0,6,0.8,0.2],0.3\Big)\Big\rangle\Big\}.$$

Each element of the GQ-NSES implies the opinion of each expert based on each parameter about the candidates with their qualifications and the degree of preference of the approximate value set. For example, $[(x_1,q_1),0.1,0.2,0.6]$ *under the parameter* $(e_1,u_1,1)$ *shows the degree to which expert* u_1 *agree that the candidate* x_1 *with a master qualification* q_1 *has a computer knowledge, whereas* $[(x_1,q_1),0.5,0.5,0.3]$ *under the parameter* $(e_1,u_1,0)$ *shows the degree to which expert* u_1 *disagree that the candidate* x_1 *with a master qualification* q_1 *has a computer knowledge.*

Now, we present the ideas of the subset of two GQ-NSESs and the equality of two GQ-NSESs.

Definition 15. *Let* $(\widehat{\Gamma}^f_Q,A)$ *and* $(\widehat{\Psi}^g_Q,B)$ *be two GQ-NSESs over X. Then,* $(\widehat{\Gamma}^f_Q,A)$ *is said to be GQ-NSE subset of* $(\widehat{\Psi}^g_Q,B)$*, denoted by* $(\widehat{\Gamma}^f_Q,A)\sqsubseteq(\widehat{\Psi}^g_Q,B)$ *if* $A\subseteq B$ *and for* $a\in A$*, the following conditions are satisfied:*

1. $f(a)$ *is a fuzzy subset of* $g(a)$*, that is* $f(a)\le g(a)$*,*
2. $\widehat{\Gamma}_Q(a)$ *is a Q-neutrosophic soft expert subset of* $\widehat{\Psi}_Q(a)$*, that is* $T_{\widehat{\Gamma}_Q(a)}(x,q)\le T_{\widehat{\Psi}_Q(a)}(x,q)$, $I_{\widehat{\Gamma}_Q(a)}(x,q)\ge I_{\widehat{\Psi}_Q(a)}(x,q)$, $F_{\widehat{\Gamma}_Q(a)}(x,q)\ge F_{\widehat{\Psi}_Q(a)}(x,q)\forall(x,q)\in X\times Q$.

Definition 16. *Let* $(\widehat{\Gamma}^f_Q,A)$ *and* $(\widehat{\Psi}^g_Q,B)$ *be two GQ-NSESs over X. Then,* $(\widehat{\Gamma}^f_Q,A)$ *is said to be equal to* $(\widehat{\Psi}^g_Q,B)$*, denoted by* $(\widehat{\Gamma}^f_Q,A)\widehat{=}(\widehat{\Psi}^g_Q,B)$ *if* $(\widehat{\Gamma}^f_Q,A)$ *is a GQ-NSE subset of* $(\widehat{\Psi}^g_Q,B)$ *and* $(\widehat{\Psi}^g_Q,B)$ *is a GQ-NSE subset of* $(\widehat{\Gamma}_Q,A)$*.*

Next, we give the definitions of an agree- GQ-NSES and a disagree- GQ-NSES.

Definition 17. *An agree- GQ-NSES* $(\widehat{\Gamma}^f_Q,A)_1$ *over X is a GQ-NSES subset of* $(\widehat{\Gamma}_Q,A)$ *defined as*

$$(\widehat{\Gamma}^f_Q,A)_1=\Big\{\widehat{\Gamma}^f_{Q_1}(a):a\in E\times U\times\{1\}\Big\}.$$

Definition 18. *A disagree- GQ-NSES* $(\widehat{\Gamma}^f_Q,A)_0$ *over X is a GQ-NSES subset of* $(\widehat{\Gamma}^f_Q,A)$ *defined as*

$$(\widehat{\Gamma}^f_Q,A)_0=\Big\{\widehat{\Gamma}^f_{Q_0}(a):a\in E\times U\times\{1\}\Big\}.$$

In the following, we discuss the operations of complement, union and intersection of GQ-NSESs.

Definition 19. *The complement of* $(\widehat{\Gamma}^f_Q,A)$ *is defined as*

$$(\widehat{\Gamma}^f_Q,A)^c=((\widehat{\Gamma}^f_Q)^c,A)$$
$$=\Big\{\Big\langle a,(\widehat{\Gamma}_Q{}^c(a),f^c(a))\Big\rangle:a\in A,\widehat{\Gamma}_Q(a)\in P(X\times Q),f(a)\in[0,1]\Big\},$$

where, $f^c(a)=1-f(a)$ *and* $\widehat{\Gamma}^c_Q=\Big\{\Big\langle a,T_{\widehat{\Gamma}^c_Q(a)}(x,q),I_{\widehat{\Gamma}^c_Q(a)}(x,q),F_{\widehat{\Gamma}^c_Q(a)}(x,q)\Big\rangle:a\in A,(x,q)\in X\times Q\Big\}$*, such that* $\forall a\in A,(x,q)\in X\times Q$

$$T_{\widehat{\Gamma}^c_Q(a)}(x,q) = 1 - T_{\widehat{\Gamma}_Q}(x,q),$$
$$I_{\widehat{\Gamma}^c_Q(a)}(x,q) = 1 - I_{\widehat{\Gamma}_Q}(x,q),$$
$$F_{\widehat{\Gamma}^c_Q(a)}(x,q) = 1 - F_{\widehat{\Gamma}_Q}(x,q).$$

Example 2. *Consider the approximation given in Example 1, where*

$$\widehat{\Gamma}^f_Q(e_1,u_1,1) = \Big\{\Big([(x_1,p),0.1,0.2,0.6],[(x_1,q),0.4,0.3,0.7],[(x_2,p),0.5,0.2,0.1], [(x_2,q),0.7,0.2,0.3],[(x_3,p),0.8,0.3,0.1],[(x_3,q),0.2,0.3,0.6],0.7\Big)\Big\}.$$

By using the GQ-NSES complement, we obtain the complement of the approximation given by

$$(\widehat{\Gamma}^f_Q)^c(e_1,u_1,1) = \Big\{\Big([(x_1,p),0.9,0.8,0.4],[(x_1,q),0.6,0.7,0.3],[(x_2,p),0.5,0.8,0.9], [(x_2,q),0.3,0.8,0.7],[(x_3,p),0.2,0.7,0.9],[(x_3,q),0.8,0.7,0.4],0.3\Big)\Big\}.$$

Proposition 1. *If $(\widehat{\Gamma}^f_Q, A)$ is a GQ-NSES over X, then $((\widehat{\Gamma}^f_Q, A)^c)^c = (\widehat{\Gamma}^f_Q, A)$.*

Proof. Suppose that $(\widehat{\Gamma}^f_Q, A)$ is a GQ-NSES over X defined as $(\widehat{\Gamma}^f_Q, A) = \Big\{\Big\langle a, \Big((T_{\widehat{\Gamma}^f_Q(a)}(x,q), I_{\widehat{\Gamma}^f_Q(a)}(x,q), F_{\widehat{\Gamma}^f_Q(a)}(x,q)), f(a)\Big)\Big\rangle : a \in A, (x,q) \in X \times Q\Big\}$. The complement of $(\widehat{\Gamma}^f_Q, A)$ denoted by $(\widehat{\Gamma}^f_Q, A)^c = ((\widehat{\Gamma}^f_Q)^c, A)$ is as defined below:

$$\begin{aligned}(\widehat{\Gamma}^f_Q)^c &= \Big\{\Big\langle a, \Big((T_{(\widehat{\Gamma}^f_Q)^c(a)}(x,q), I_{(\widehat{\Gamma}^f_Q)^c(a)}(x,q), F_{(\widehat{\Gamma}^f_Q)^c(a)}(x,q)), f^c(a)\Big)\Big\rangle : a \in A, (x,q) \in X \times Q\Big\} \\ &= \Big\{\Big\langle a, \Big((1 - T_{\widehat{\Gamma}^f_Q(a)}(x,q), 1 - I_{\widehat{\Gamma}^f_Q(a)}(x,q), 1 - F_{\widehat{\Gamma}^f_Q(a)}(x,q)), 1 - f(a)\Big)\Big\rangle : a \in A, (x,q) \in X \times Q\Big\}.\end{aligned}$$

Thus,

$$\begin{aligned}((\widehat{\Gamma}^f_Q, A)^c)^c &= \Big\{\Big\langle a, \Big(((1 - T_{\widehat{\Gamma}^f_Q(a)}(x,q))^c, (1 - I_{\widehat{\Gamma}^f_Q(a)}(x,q))^c, (1 - F_{\widehat{\Gamma}^f_Q(a)}(x,q))^c), (1 - f(a)))^c\Big)\Big\rangle \\ &\quad : a \in A, (x,q) \in X \times Q\Big\} \\ &= \Big\{\Big\langle a, \Big((1 - (1 - T_{\widehat{\Gamma}^f_Q(a)}(x,q)), 1 - (1 - I_{\widehat{\Gamma}^f_Q(a)}(x,q)), 1 - (1 - F_{\widehat{\Gamma}^f_Q(a)}(x,q))), 1 - (1 - f(a))\Big)\Big\rangle \\ &\quad : a \in A, (x,q) \in X \times Q\Big\} \\ &= \Big\{\Big\langle a, \Big((T_{\widehat{\Gamma}^f_Q(a)}(x,q), I_{\widehat{\Gamma}^f_Q(a)}(x,q), F_{\widehat{\Gamma}^f_Q(a)}(x,q)), f(a)\Big)\Big\rangle : a \in A, (x,q) \in X \times Q\Big\} \\ &= (\widehat{\Gamma}^f_Q, A).\end{aligned}$$

This completes the proof. □

Now, we define the union and intersection of two GQ-NSESs.

Definition 20. *The union of* $(\widehat{\Gamma}_Q^f, A)$ *and* $(\widehat{\Psi}_Q^g, B)$ *is a GQ-NSES* $(\widehat{\Upsilon}_Q^h, C)$*, defined as* $(\widehat{\Gamma}_Q^f, A)\widehat{\bigcup}(\widehat{\Psi}_Q^g, B) = (\widehat{\Upsilon}_Q^h, C)$*, where* $C = A \cup B$ *and for all* $c \in C$ *and* $(x, q) \in X \times Q$ *the truth, indeterminacy and falsity memberships of* $(\widehat{\Upsilon}_Q^h, C)$ *are as follows:*

$$T_{\widehat{\Upsilon}_Q^h(c)}(x,q) = \begin{cases} T_{\widehat{\Gamma}_Q^f(c)}(x,q) & \text{if } c \in A - B, \\ T_{\widehat{\Psi}_Q^g(c)}(x,q) & \text{if } c \in B - A, \\ \max\{T_{\widehat{\Gamma}_Q^f(c)}(x,q), T_{\widehat{\Psi}_Q^g(c)}(x,q)\} & \text{if } c \in A \cap B, \end{cases}$$

$$I_{\widehat{\Upsilon}_Q^h(c)}(x,q) = \begin{cases} I_{\widehat{\Gamma}_Q^f(c)}(x,q) & \text{if } c \in A - B, \\ I_{\widehat{\Psi}_Q^g(c)}(x,q) & \text{if } c \in B - A, \\ \min\{I_{\widehat{\Gamma}_Q^f(c)}(x,q), I_{\widehat{\Psi}_Q^g(c)}(x,q)\} & \text{if } c \in A \cap B, \end{cases}$$

$$F_{\widehat{\Upsilon}_Q^h(c)}(x,q) = \begin{cases} F_{\widehat{\Gamma}_Q^f(c)}(x,q) & \text{if } c \in A - B, \\ F_{\widehat{\Psi}_Q^g(c)}(x,q) & \text{if } c \in B - A, \\ \min\{F_{\widehat{\Gamma}_Q^f(c)}(x,q), F_{\widehat{\Psi}_Q^g(c)}(x,q)\} & \text{if } c \in A \cap B, \end{cases}$$

and $h(c) = \max\{f(c), g(c) : \forall c \in C\}$.

Definition 21. *The intersection of* $(\widehat{\Gamma}_Q^f, A)$ *and* $(\widehat{\Psi}_Q^g, B)$ *is a GQ-NSES* $(\widehat{\Upsilon}_Q^h, C)$*, defined as* $(\widehat{\Gamma}_Q^f, A)\widehat{\bigcap}(\widehat{\Psi}_Q^g, B) = (\widehat{\Upsilon}_Q^h, C)$*, where* $C = A \cap B$ *and for all* $c \in C$ *and* $(x, q) \in X \times Q$ *the truth, indeterminacy and falsity memberships of* $(\widehat{\Upsilon}_Q^h, C)$ *are as follows:*

$$\begin{aligned} T_{\widehat{\Upsilon}_Q^h(c)}(x,q) &= \min\{T_{\widehat{\Gamma}_Q^f(c)}(x,q), T_{\widehat{\Psi}_Q^g(c)}(x,q)\}, \\ I_{\widehat{\Upsilon}_Q^h(c)}(x,q) &= \max\{I_{\widehat{\Gamma}_Q^f(c)}(x,q), I_{\widehat{\Psi}_Q^g(c)}(x,q)\}, \\ F_{\widehat{\Upsilon}_Q^h(c)}(x,q) &= \max\{F_{\widehat{\Gamma}_Q^f(c)}(x,q), F_{\widehat{\Psi}_Q^g(c)}(x,q)\}, \end{aligned}$$

and $h(c) = \min\{f(c), g(c) : \forall c \in C\}$.

Example 3. *Assume that two GQ-NSESs* $(\widehat{\Gamma}_Q^f, A)$ *and* $(\widehat{\Psi}_Q^g, B)$ *are defined as follows:*

$$\begin{aligned} (\widehat{\Gamma}_Q^f, A) = \Big\{ &\Big\langle (e_1, u_1, 1), \Big([(x_1, q_1), 0.2, 0.5, 0.4], [(x_1, q_2), 0.1, 0.3, 0.3], [(x_2, q_1), 0.8, 0.5, 0.5], \\ &\qquad [(x_2, q_2), 0.8, 0.8, 0.2], [(x_3, q_1), 0.6, 0.3, 0.5], [(x_3, q_2), 0.4, 0.3, 0.1], 0.4 \Big) \Big\rangle, \\ &\Big\langle (e_1, u_2, 1), \Big([(x_1, q_1), 0.2, 0.4, 0.7], [(x_1, q_2), 0.5, 0.1, 0.2], [(x_2, q_1), 0.1, 0.2, 0.6], \\ &\qquad [(x_2, q_2), 0.6, 0.5, 0.2], [(x_3, q_1), 0.6, 0.4, 0.1], [(x_3, q_2), 0.7, 0.2, 0.4], 0.2 \Big) \Big\rangle, \\ &\Big\langle (e_1, u_1, 0), \Big([(x_1, q_1), 0.3, 0.3, 0.3], [(x_1, q_2), 0.7, 0.3, 0.5], [(x_2, q_1), 0.5, 0.6, 0.2], \\ &\qquad [(x_2, q_2), 0.7, 0.3, 0.1], [(x_3, q_1), 0.3, 0.5, 0.2], [(x_3, q_2), 0.7, 0.5, 0.6], 0.5 \Big) \Big\rangle, \\ &\Big\langle (e_1, u_2, 0), \Big([(x_1, q_1), 0.4, 0.4, 0.3], [(x_1, q_2), 0.2, 0.3, 0.6], [(x_2, q_1), 0.1, 0.4, 0.5], \\ &\qquad [(x_2, q_2), 0.6, 0.4, 0.2], [(x_3, q_1), 0.2, 0.3, 0.6], [(x_3, q_2), 0.2, 0.1, 0.1], 0.4 \Big) \Big\rangle \Big\}, \end{aligned}$$

$$(\widehat{\Psi}^g_Q, A) = \Big\{ \Big\langle (e_1, u_1, 1), \Big([(x_1, q_1), 0.7, 0.3, 0.2], [(x_1, q_2), 0.4, 0.3, 0.1], [(x_2, q_1), 0.4, 0.4, 0.5], [(x_2, q_2), 0.6, 0.5, 0.5], [(x_3, q_1), 0.6, 0.6, 0.2], [(x_3, q_2), 0.3, 0.5, 0.5], 0.4 \Big) \Big\rangle,$$
$$\Big\langle (e_1, u_2, 1), \Big([(x_1, q_1), 0.7, 0.6, 0.3], [(x_1, q_2), 0.6, 0.7, 0.2], [(x_2, q_1), 0.1, 0.5, 0.7], [(x_2, q_2), 0.7, 0.5, 0.1], [(x_3, q_1), 0.3, 0.2, 0.3], [(x_3, q_2), 0.1, 0.1, 0.4], 0.3 \Big) \Big\rangle,$$
$$\Big\langle (e_1, u_1, 0), \Big([(x_1, q_1), 0.5, 0.7, 0.4], [(x_1, q_2), 0.3, 0.4, 0.1], [(x_2, q_1), 0.5, 0.1, 0.6], [(x_2, q_2), 0.8, 0.6, 0.2], [(x_3, q_1), 0.7, 0.8, 0.6], [(x_3, q_2), 0.5, 0.8, 0.6], 0.6 \Big) \Big\rangle,$$
$$\Big\langle (e_1, u_2, 0), \Big([(x_1, q_1), 0.3, 0.2, 0.7], [(x_1, q_2), 0.4, 0.8, 0.1], [(x_2, q_1), 0.1, 0.1, 0.4], [(x_2, q_2), 0.8, 0.3, 0.2], [(x_3, q_1), 0.4, 0.1, 0.2], [(x_3, q_2), 0.2, 0.5, 0.3], 0.6 \Big) \Big\rangle \Big\}.$$

Then,

$$(\widehat{\Gamma}^f_Q, A) \widehat{\bigcup} (\widehat{\Psi}^g_Q, A) = \Big\{ \Big\langle (e_1, u_1, 1), \Big([(x_1, q_1), 0.7, 0.3, 0.2], [(x_1, q_2), 0.4, 0.3, 0.1], [(x_2, q_1), 0.8, 0.4, 0.5], [(x_2, q_2), 0.8, 0.5, 0.2], [(x_3, q_1), 0.6, 0.3, 0.2], [(x_3, q_2), 0.4, 0.3, 0.1], 0.4 \Big) \Big\rangle,$$
$$\Big\langle (e_1, u_2, 1), \Big([(x_1, q_1), 0.7, 0.4, 0.3], [(x_1, q_2), 0.6, 0.1, 0.2], [(x_2, q_1), 0.1, 0.2, 0.6], [(x_2, q_2), 0.7, 0.5, 0.1], [(x_3, q_1), 0.6, 0.2, 0.1], [(x_3, q_2), 0.7, 0.1, 0.4], 0.3 \Big) \Big\rangle,$$
$$\Big\langle (e_1, u_1, 0), \Big([(x_1, q_1), 0.5, 0.3, 0.3], [(x_1, q_2), 0.7, 0.3, 0.1], [(x_2, q_1), 0.5, 0.1, 0.2], [(x_2, q_2), 0.8, 0.3, 0.1], [(x_3, q_1), 0.7, 0.5, 0.2], [(x_3, q_2), 0.7, 0.5, 0.6], 0.6 \Big) \Big\rangle,$$
$$\Big\langle (e_1, u_2, 0), \Big([(x_1, q_1), 0.4, 0.2, 0.3], [(x_1, q_2), 0.4, 0.3, 0.1], [(x_2, q_1), 0.1, 0.1, 0.4], [(x_2, q_2), 0.8, 0.3, 0.2], [(x_3, q_1), 0.4, 0.1, 0.2], [(x_3, q_2), 0.2, 0.1, 0.1], 0.6 \Big) \Big\rangle \Big\}.$$

The standard commutative and associative laws relevant to the operations of union and intersection are satisfied and stated below.

Proposition 2. *Let* $(\widehat{\Gamma}^f_Q, A)$, $(\widehat{\Psi}^g_Q, B)$ *and* $(\widehat{\Upsilon}^h_Q, C)$ *be GQ-NSESs over a universe X. Then, the following properties hold true:*

(1) $(\widehat{\Gamma}^f_Q, A) \widehat{\bigcup} (\widehat{\Psi}^g_Q, B) = (\widehat{\Psi}^g_Q, B) \widehat{\bigcup} (\widehat{\Gamma}^f_Q, A)$,
(2) $(\widehat{\Gamma}^f_Q, A) \widehat{\bigcap} (\widehat{\Psi}^g_Q, B) = (\widehat{\Psi}^g_Q, B) \widehat{\bigcap} (\widehat{\Gamma}^f_Q, A)$,
(3) $((\widehat{\Gamma}^f_Q, A) \widehat{\bigcup} (\widehat{\Psi}^g_Q, B)) \widehat{\bigcup} (\widehat{\Upsilon}^h_Q, C) = (\widehat{\Gamma}^f_Q, A) \widehat{\bigcup} ((\widehat{\Psi}^g_Q, B) \widehat{\bigcup} (\widehat{\Upsilon}^h_Q, C))$,
(4) $((\widehat{\Gamma}^f_Q, A) \widehat{\bigcap} (\widehat{\Psi}^g_Q, B)) \widehat{\bigcap} (\widehat{\Upsilon}^h_Q, C) = (\widehat{\Gamma}^f_Q, A) \widehat{\bigcap} ((\widehat{\Psi}^g_Q, B) \widehat{\bigcap} (\widehat{\Upsilon}^h_Q, C))$.

Proof. (1) We will prove that $(\widehat{\Gamma}^f_Q, A) \widehat{\bigcup} (\widehat{\Psi}^g_Q, B) = (\widehat{\Psi}^g_Q, B) \widehat{\bigcup} (\widehat{\Gamma}_Q, A)$ by using Definition 20 and we consider the case when $c \in A \cap B$ as the other cases are trivial:

$$(\widehat{\Gamma}^f_Q, A) \widehat{\bigcup} (\widehat{\Psi}^g_Q, B) = \Big\{ \Big\langle c, \Big(\big(\max\{T_{\widehat{\Gamma}^f_Q(c)}(x,q), T_{\widehat{\Psi}^g_Q(c)}(x,q)\}, \min\{I_{\widehat{\Gamma}^f_Q(c)}(x,q), I_{\widehat{\Psi}^g_Q(c)}(x,q)\}, \min\{F_{\widehat{\Gamma}^f_Q(c)}(x,q), F_{\widehat{\Psi}^g_Q(c)}(x,q)\} \big), \max\{f(c), g(c)\} \Big) \Big\rangle : (x,q) \in X \times Q \Big\}$$
$$= \Big\{ \Big\langle c, \Big(\big(\max\{T_{\widehat{\Psi}^g_Q(c)}(x,q), T_{\widehat{\Gamma}^f_Q(c)}(x,q)\}, \min\{I_{\widehat{\Psi}^g_Q(c)}(x,q), I_{\widehat{\Gamma}^f_Q(c)}(x,q)\}, \min\{F_{\widehat{\Psi}^g_Q(c)}(x,q), F_{\widehat{\Gamma}^f_Q(c)}(x,q)\} \big), \max\{g(c), f(c)\} \Big) \Big\rangle : (x,q) \in X \times Q \Big\}$$
$$= (\widehat{\Psi}^g_Q, B) \widehat{\bigcup} (\widehat{\Gamma}^f_Q, A).$$

(2) The proof is similar to that of part (1).

(3) We want to prove that $((\widehat{\Gamma}^f_Q, A)\widehat{\bigcup}(\widehat{\Psi}^g_Q, B))\widehat{\bigcup}(\widehat{\Upsilon}^h_Q, C) = (\widehat{\Gamma}^f_Q, A)\widehat{\bigcup}((\widehat{\Psi}^g_Q, B)\widehat{\bigcup}(\widehat{\Upsilon}^h_Q, C))$ by using Definition 20 and we consider the case when $c \in A \cap B$ as the other cases are trivial

$$(\widehat{\Gamma}^f_Q, A)\widehat{\bigcup}(\widehat{\Psi}^g_Q, B) = \Big\{\Big\langle c, \Big(\big(\max\{T_{\widehat{\Gamma}^f_Q(c)}(x,q), T_{\widehat{\Psi}^g_Q(c)}(x,q)\}, \min\{I_{\widehat{\Gamma}^f_Q(c)}(x,q), I_{\widehat{\Psi}^g_Q(c)}(x,q)\}, \min\{F_{\widehat{\Gamma}^f_Q(c)}(x,q), F_{\widehat{\Psi}^g_Q(c)}(x,q)\}\big), \max\{f(c), g(c)\}\Big)\Big\rangle : (x,q) \in X \times Q\Big\}.$$

Considering the case when $c \in C$, then we have

$$\begin{aligned}
&((\widehat{\Gamma}^f_Q, A)\widehat{\bigcup}(\widehat{\Psi}^g_Q, B))\widehat{\bigcup}(\widehat{\Upsilon}^h_Q, C) \\
&= \Big\{\Big\langle c, \Big(\big(\max\big\{\max\{T_{\widehat{\Gamma}^f_Q(c)}(x,q), T_{\widehat{\Psi}^g_Q(c)}(x,q)\}, T_{\widehat{\Upsilon}^h_Q(c)}(x,q)\big\}, \\
&\quad \min\big\{\min\{I_{\widehat{\Gamma}^f_Q(c)}(x,q), I_{\widehat{\Psi}^g_Q(c)}(x,q)\}, I_{\widehat{\Upsilon}^h_Q(c)}(x,q)\big\}, \\
&\quad \min\big\{\min\{F_{\widehat{\Gamma}^f_Q(c)}(x,q), F_{\widehat{\Psi}^g_Q(c)}(x,q)\}, F_{\widehat{\Upsilon}^h_Q(c)}(x,q)\big\}\big), \max\big\{\max\{f(c), g(c)\}, h(c)\big\}\Big)\Big\rangle : \\
&\quad (x,q) \in X \times Q\Big\} \\
&= \Big\{\Big\langle c, \Big(\big(\max\big\{T_{\widehat{\Gamma}^f_Q(c)}(x,q), T_{\widehat{\Psi}^g_Q(c)}(x,q), T_{\widehat{\Upsilon}^h_Q(c)}(x,q)\big\}, \\
&\quad \min\big\{I_{\widehat{\Gamma}^f_Q(c)}(x,q), I_{\widehat{\Psi}^g_Q(c)}(x,q), I_{\widehat{\Upsilon}^h_Q(c)}(x,q)\big\}, \\
&\quad \min\big\{F_{\widehat{\Gamma}^f_Q(c)}(x,q), F_{\widehat{\Psi}^g_Q(c)}(x,q), F_{\widehat{\Upsilon}^h_Q(c)}(x,q)\big\}\big), \max\big\{f(c), g(c), h(c)\big\}\Big)\Big\rangle : \\
&\quad (x,q) \in X \times Q\Big\} \\
&= \Big\{\Big\langle c, \Big(\big(\max\big\{T_{\widehat{\Gamma}^f_Q(c)}(x,q), \max\{T_{\widehat{\Psi}^g_Q(c)}(x,q), T_{\widehat{\Upsilon}^h_Q(c)}(x,q)\}\big\}, \\
&\quad \min\big\{I_{\widehat{\Gamma}^f_Q(c)}(x,q), \min\{I_{\widehat{\Psi}^g_Q(c)}(x,q), I_{\widehat{\Upsilon}^h_Q(c)}(x,q)\}\big\}, \\
&\quad \min\big\{F_{\widehat{\Gamma}^f_Q(c)}(x,q), \min\{F_{\widehat{\Psi}^g_Q(c)}(x,q), F_{\widehat{\Upsilon}^h_Q(c)}(x,q)\}\big\}\big), \max\big\{f(c), \max\{g(c), h(c)\}\big\}\Big)\Big\rangle : \\
&\quad (x,q) \in X \times Q\Big\} \\
&= (\widehat{\Gamma}^f_Q, A)\widehat{\bigcup}((\widehat{\Psi}^g_Q, B)\widehat{\bigcup}(\widehat{\Upsilon}^h_Q, C)).
\end{aligned}$$

(4) The proof is similar to that of part (3).
□

Next, we define AND and OR operations of GQ-NSESs.

Definition 22. *If $(\widehat{\Gamma}^f_Q, A)$ and $(\widehat{\Psi}^g_Q, B)$ are two generalized Q-neutrosophic soft expert sets on X, then $(\widehat{\Gamma}^f_Q, A)$ AND $(\widehat{\Psi}^g_Q, B)$ is the generalized Q-neutrosophic soft expert set denoted by $(\widehat{\Gamma}^f_Q, A)\widehat{\bigwedge}(\widehat{\Psi}^g_Q, B)$ and defined by $(\widehat{\Gamma}^f_Q, A)\widehat{\bigwedge}(\widehat{\Psi}^g_Q, B) = (\widehat{\Lambda}^h_Q, A \times B)$, where $\widehat{\Lambda}^h_Q(a,b) = \widehat{\Gamma}^f_Q(a)\widehat{\bigcap}\widehat{\Psi}^f_Q(b)$ and the truth, indeterminacy and falsity memberships of $(\widehat{\Lambda}^h_Q, A \times B)$ are as follows:*

$$\begin{aligned}
T_{\widehat{\Upsilon}^h_Q(a,b)}(x,q) &= \min\{T_{\widehat{\Gamma}^f_Q(a)}(x,q), T_{\widehat{\Psi}^g_Q(b)}(x,q)\}, \\
I_{\widehat{\Upsilon}^h_Q(a,b)}(x,q) &= \max\{I_{\widehat{\Gamma}^f_Q(a)}(x,q), I_{\widehat{\Psi}^g_Q(b)}(x,q)\}, \\
F_{\widehat{\Upsilon}^h_Q(a,b)}(x,q) &= \max\{F_{\widehat{\Gamma}^f_Q(a)}(x,q), F_{\widehat{\Psi}^g_Q(b)}(x,q)\},
\end{aligned}$$

and $h(a,b) = \min\{f(a), g(b) : \forall a \in A$ and $b \in B\}$.

Definition 23. *If $(\widehat{\Gamma}^f_Q, A)$ and $(\widehat{\Psi}^g_Q, B)$ are two generalized Q-neutrosophic soft expert sets on X, then $(\widehat{\Gamma}^f_Q, A)$ OR $(\widehat{\Psi}^g_Q, B)$ is the generalized Q-neutrosophic soft expert set denoted by $(\widehat{\Gamma}^f_Q, A)\widehat{\bigvee}(\widehat{\Psi}^g_Q, B)$ and defined by $(\widehat{\Gamma}^f_Q, A)\widehat{\bigvee}\ (\widehat{\Psi}^g_Q, B) = (\widehat{\Lambda}^h_Q, A \times B)$, where $\widehat{\Lambda}^h_Q(a,b) = \widehat{\Gamma}^f_Q(a)\widehat{\bigcup}\widehat{\Psi}^g_Q(b)$ and the truth, indeterminacy and falsity memberships of $(\widehat{\Lambda}^h_Q, A \times B)$ are as follows*

$$\begin{aligned}
T_{\widehat{\Upsilon}^h_Q(a,b)}(x,q) &= \max\{T_{\widehat{\Gamma}^f_Q(a)}(x,q), T_{\widehat{\Psi}^g_Q(b)}(x,q)\},\\
I_{\widehat{\Upsilon}^h_Q(a,b)}(x,q) &= \min\{I_{\widehat{\Gamma}^f_Q(a)}(x,q), I_{\widehat{\Psi}^g_Q(b)}(x,q)\},\\
F_{\widehat{\Upsilon}^h_Q(a,b)}(x,q) &= \min\{F_{\widehat{\Gamma}^f_Q(a)}(x,q), F_{\widehat{\Psi}^g_Q(b)}(x,q)\},
\end{aligned}$$

and $h(a,b) = \max\{f(a), g(b) : \forall a \in A \text{ and } b \in B\}$.

Example 4. *Assume that two GQ-NSESs $(\widehat{\Gamma}^f_Q, A)$ and $(\widehat{\Psi}^g_Q, B)$ are defined as follows:*

$$\begin{aligned}
(\widehat{\Gamma}^f_Q, A) = \Big\{ &\Big\langle (e_1,u_1,1), \Big([(x_1,q_1),0.2,0.5,0.4], [(x_1,q_2),0.1,0.3,0.3], [(x_2,q_1),0.8,0.5,0.5],\\
&[(x_2,q_2),0.8,0.8,0.2], [(x_3,q_1),0.6,0.3,0.5], [(x_3,q_2),0.4,0.3,0.1], 0.4\Big)\Big\rangle,\\
&\Big\langle (e_1,u_1,0), \Big([(x_1,q_1),0.3,0.3,0.3], [(x_1,q_2),0.7,0.3,0.5], [(x_2,q_1),0.5,0.6,0.2],\\
&[(x_2,q_2),0.7,0.3,0.1], [(x_3,q_1),0.3,0.5,0.2], [(x_3,q_2),0.7,0.5,0.6], 0.5\Big)\Big\rangle,
\end{aligned}$$

$$\begin{aligned}
(\widehat{\Psi}^g_Q, A) = \Big\{ &\Big\langle (e_1,u_1,1), \Big([(x_1,q_1),0.7,0.3,0.2], [(x_1,q_2),0.4,0.3,0.1], [(x_2,q_1),0.4,0.4,0.5],\\
&[(x_2,q_2),0.6,0.5,0.5], [(x_3,q_1),0.6,0.6,0.2], [(x_3,q_2),0.3,0.5,0.5], 0.4\Big)\Big\rangle,\\
&\Big\langle (e_1,u_1,0), \Big([(x_1,q_1),0.5,0.7,0.4], [(x_1,q_2),0.3,0.4,0.1], [(x_2,q_1),0.5,0.1,0.6],\\
&[(x_2,q_2),0.8,0.6,0.2], [(x_3,q_1),0.7,0.8,0.6], [(x_3,q_2),0.5,0.8,0.6], 0.6\Big)\Big\rangle.
\end{aligned}$$

Then,

$$\begin{aligned}
(\widehat{\Gamma}^f_Q, A)\widehat{\bigvee}(\widehat{\Psi}^g_Q, A) = \Big\{ &\Big\langle ((e_1,u_1,1),(e_1,u_1,1)), \Big([(x_1,q_1),0.7,0.3,0.2], [(x_1,q_2),0.4,0.3,0.1], [(x_2,q_1),0.8,0.4,0.5],\\
&[(x_2,q_2),0.8,0.5,0.2], [(x_3,q_1),0.6,0.3,0.2], [(x_3,q_2),0.4,0.3,0.1], 0.4\Big)\Big\rangle,\\
&\Big\langle ((e_1,u_1,1),(e_1,u_1,0)), \Big([(x_1,q_1),0.5,0.5,0.4], [(x_1,q_2),0.3,0.3,0.1], [(x_2,q_1),0.8,0.1,0.5],\\
&[(x_2,q_2),0.8,0.6,0.2], [(x_3,q_1),0.7,0.3,0.5], [(x_3,q_2),0.5,0.3,0.1], 0.4\Big)\Big\rangle,\\
&\Big\langle ((e_1,u_1,0),(e_1,u_1,1)), \Big([(x_1,q_1),0.7,0.3,0.2], [(x_1,q_2),0.7,0.3,0.1], [(x_2,q_1),0.5,0.4,0.2],\\
&[(x_2,q_2),0.7,0.3,0.1], [(x_3,q_1),0.6,0.5,0.2], [(x_3,q_2),0.7,0.5,0.5], 0.5\Big)\Big\rangle,\\
&\Big\langle ((e_1,u_1,0),(e_1,u_1,0)), \Big([(x_1,q_1),0.5,0.3,0.3], [(x_1,q_2),0.7,0.3,0.1], [(x_2,q_1),0.5,0.1,0.2],\\
&[(x_2,q_2),0.8,0.3,0.1], [(x_3,q_1),0.7,0.5,0.2], [(x_3,q_2),0.7,0.5,0.6], 0.6\Big)\Big\rangle\Big\}.
\end{aligned}$$

Proposition 3. *Let $(\widehat{\Gamma}^f_Q, A), (\widehat{\Psi}^g_Q, B)$ and $(\widehat{\Upsilon}^h_Q, C)$ be GQ-NSESs over a universe X. Then, the following properties hold true:*

(1) $((\widehat{\Gamma}^f_Q, A)\widehat{\bigwedge}(\widehat{\Psi}^g_Q, B))\widehat{\bigwedge}(\widehat{\Upsilon}^h_Q, C) = (\widehat{\Gamma}^f_Q, A)\widehat{\bigwedge}((\widehat{\Psi}^g_Q, B)\widehat{\bigwedge}(\widehat{\Upsilon}^h_Q, C))$,

(2) $((\widehat{\Gamma}^f_Q, A)\widehat{\bigvee}(\widehat{\Psi}^g_Q, B))\widehat{\bigvee}(\widehat{\Upsilon}^h_Q, C) = (\widehat{\Gamma}^f_Q, A)\widehat{\bigvee}((\widehat{\Psi}^g_Q, B)\widehat{\bigvee}(\widehat{\Upsilon}^h_Q, C))$.

Proof. The proof is similar to the proof of part 3 of Proposition 2. □

4. Application of Generalized Q-Neutrosophic Soft Expert Set

In this section, an application of GQ-NSES in decision making is discussed. The problem we consider is as follows:

An investment company considers several business options to increase its portfolio. The set of possible alternatives is $X = \{x_1 = \text{car company}, x_2 = \text{food company}, x_3 = \text{computer company}\}$, $Q = \{q_1 = local, q_2 = international\}$. The company will choose the best option according to the following three criteria $E = \{e_1 = \text{risk}, e_2 = \text{growth}, e_3 = \text{environmental impact}\}$. Let $U = \{u_1, u_2\}$ be a set of experts. After deliberation, the experts construct the following GQ-NSES:

$$
\begin{aligned}
(\widehat{\Gamma}_Q, A) = \Big\{ &\Big\langle (e_1, u_1, 1), \Big([(x_1, q_1), 0.4, 0.3, 0.7], [(x_1, q_2), 0.2, 0.1, 0.4], [(x_2, q_1), 0.3, 0.3, 0.2], \\
&\qquad [(x_2, q_2), 0.5, 0.1, 0.5], [(x_3, q_1), 0.5, 0.6, 0.3], [(x_3, q_2), 0.8, 0.4, 0.2], 0.2 \Big) \Big\rangle, \\
&\Big\langle (e_1, u_2, 1), \Big([(x_1, q_1), 0.4, 0.6, 0.3], [(x_1, q_2), 0.6, 0.1, 0.1], [(x_2, q_1), 0.5, 0.6, 0.6], \\
&\qquad [(x_2, q_2), 0.7, 0.5, 0.6], [(x_3, q_1), 0.6, 0.7, 0.8], [(x_3, q_2), 0.2, 0.7, 0.5], 0.6 \Big) \Big\rangle, \\
&\Big\langle (e_2, u_1, 1), \Big([(x_1, q_1), 0.3, 0.3, 0.3], [(x_1, q_2), 0.4, 0.9, 0.2], [(x_2, q_1), 0.8, 0.2, 0.1], \\
&\qquad [(x_2, q_2), 0.4, 0.5, 0.5], [(x_3, q_1), 0.6, 0.7, 0.7], [(x_3, q_2), 0.5, 0.5, 0.5], 0.9 \Big) \Big\rangle, \\
&\Big\langle (e_2, u_2, 1), \Big([(x_1, q_1), 0.9, 0.6, 0.2], [(x_1, q_2), 0.2, 0.1, 0.8], [(x_2, q_1), 0.1, 0.8, 0.6], \\
&\qquad [(x_2, q_2), 0.6, 0.4, 0.8], [(x_3, q_1), 0.2, 0.2, 0.2], [(x_3, q_2), 0.9, 0.6, 0.4], 0.7 \Big) \Big\rangle, \\
&\Big\langle (e_3, u_1, 1), \Big([(x_1, q_1), 0.5, 0.4, 0.3], [(x_1, q_2), 0.3, 0.8, 0.9], [(x_2, q_1), 0.3, 0.9, 0.1], \\
&\qquad [(x_2, q_2), 0.1, 0.1, 0.4], [(x_3, q_1), 0.7, 0.1, 0.3], [(x_3, q_2), 0.4, 0.5, 0.7], 0.5 \Big) \Big\rangle, \\
&\Big\langle (e_3, u_2, 1), \Big([(x_1, q_1), 0.6, 0.6, 0.2], [(x_1, q_2), 0.1, 0.2, 0.2], [(x_2, q_1), 0.9, 0.4, 0.2], \\
&\qquad [(x_2, q_2), 0.6, 0.7, 0.4], [(x_3, q_1), 0.5, 0.5, 0.3], [(x_3, q_2), 0.4, 0.5, 0.1], 0, 3 \Big) \Big\rangle, \\
&\Big\langle (e_1, u_1, 0), \Big([(x_1, q_1), 0.5, 0.7, 0.3], [(x_1, q_2), 0.4, 0.7, 0.9], [(x_2, q_1), 0.2, 0.8, 0.9], \\
&\qquad [(x_2, q_2), 0.3, 0.2, 0.8], [(x_3, q_1), 0.8, 0.1, 0.4], [(x_3, q_2), 0.7, 0.9, 0.6], 0.8 \Big) \Big\rangle, \\
&\Big\langle (e_1, u_2, 0), \Big([(x_1, q_1), 0.7, 0.4, 0.2], [(x_1, q_2), 0.6, 0.5, 0.8], [(x_2, q_1), 0.3, 0.4, 0.8], \\
&\qquad [(x_2, q_2), 0.6, 0.7, 0.9], [(x_3, q_1), 0.4, 0.3, 0.2], [(x_3, q_2), 0.6, 0.5, 0.9], 0.6 \Big) \Big\rangle, \\
&\Big\langle (e_2, u_1, 0), \Big([(x_1, q_1), 0.2, 0.1, 0.2], [(x_1, q_2), 0.4, 0.5, 0.8], [(x_2, q_1), 0.6, 0.6, 0.6], \\
&\qquad [(x_2, q_2), 0.4, 0.4, 0.4], [(x_3, q_1), 0.8, 0.2, 0.5], [(x_3, q_2), 0.3, 0.7, 0.8], 0.5 \Big) \Big\rangle, \\
&\Big\langle (e_2, u_2, 0), \Big([(x_1, q_1), 0.1, 0.5, 0.6], [(x_1, q_2), 0.9, 0.4, 0.4], [(x_2, q_1), 0.7, 0.6, 0.5], \\
&\qquad [(x_2, q_2), 0.5, 0.1, 0.2], [(x_3, q_1), 0.3, 0.2, 0.2], [(x_3, q_2), 0.7, 0.3, 0.8], 0.7 \Big) \Big\rangle, \\
&\Big\langle (e_3, u_1, 0), \Big([(x_1, q_1), 0.1, 0.7, 0.9], [(x_1, q_2), 0.4, 0.3, 0.8], [(x_2, q_1), 0.9, 0.9, 0.3], \\
&\qquad [(x_2, q_2), 0.8, 0.8, 0.1], [(x_3, q_1), 0.7, 0.4, 0.6], [(x_3, q_2), 0.4, 0.8, 0.2], 0.8 \Big) \Big\rangle, \\
&\Big\langle (e_3, u_2, 0), \Big([(x_1, q_1), 0.7, 0.3, 0.2], [(x_1, q_2), 0.5, 0.3, 0.3], [(x_2, q_1), 0.7, 0.4, 0.1], \\
&\qquad [(x_2, q_2), 0.3, 0.1, 0.9], [(x_3, q_1), 0.4, 0.6, 0.5], [(x_3, q_2), 0.6, 0.1, 0.1], 0.4 \Big) \Big\rangle \Big\}.
\end{aligned}
$$

The experts may use the following algorithm to choose the best option for the investment:

1. Input the GQ-NSES $(\widehat{\Gamma}^f_Q, A)$.
2. Find the values of $|T_{\widehat{\Gamma}^f_Q} + I_{\widehat{\Gamma}^f_Q} - F_{\widehat{\Gamma}^f_Q}|$ for each element $(x, q) \in X \times Q$, where $T_{\widehat{\Gamma}^f_Q}, I_{\widehat{\Gamma}^f_Q}, F_{\widehat{\Gamma}^f_Q}$ representing the truth, indeterminacy and falsity membership functions.
3. Compute the score of each element $(x, q) \in X \times Q$ by taking the sum of the products of the numerical grade of each element with the corresponding values of $f(a)$ (the degree of preference) for the agree-GQ-NSES and disagree-GQ-NSES, denoted by ν_i and η_i, respectively.
4. Find the values of the score $\delta_j = \nu_i - \eta_i$.
5. Find m for which $\delta_m = \max \delta_i$.

Table 1 presents the values of $|T_{\widehat{\Gamma}_Q} + I_{\widehat{\Gamma}_Q} - F_{\widehat{\Gamma}_Q}|$ for each element $(x, q) \in X \times Q$.

Table 1. Values of $|T_{\widehat{\Gamma}^f_Q} + I_{\widehat{\Gamma}^f_Q} - F_{\widehat{\Gamma}^f_Q}|$ for each element $(x, q) \in X \times Q$.

$E \times U \times O$	(x_1, q_1)	(x_1, q_2)	(x_2, q_1)	(x_2, q_2)	(x_3, q_1)	(x_3, q_2)	f(a)
$(e_1, u_1, 1)$	0	−0.1	0.4	0.1	0.8	1	0.2
$(e_1, u_2, 1)$	0.7	0.6	0.5	0.6	0.5	0.4	0.6
$(e_2, u_1, 1)$	0.3	1.1	0.9	0.4	0.6	0.5	0.9
$(e_2, u_2, 1)$	1.3	−0.5	0.3	0.2	0.2	1.1	0.7
$(e_3, u_1, 1)$	0.6	0.2	1.1	−0.2	0.5	0.2	0.5
$(e_3, u_2, 1)$	1	0.1	1.1	0.9	0.7	0.1	0.3
$(e_1, u_1, 0)$	0.9	0.2	0.1	−0.3	0.5	1	0.8
$(e_1, u_2, 0)$	0.9	0.3	−0.1	0.4	0.5	0.2	0.6
$(e_2, u_1, 0)$	0.1	0.1	0.6	0.4	0.5	0.2	0.5
$(e_2, u_2, 0)$	0	0.9	0.8	0.4	0.3	0.2	0.7
$(e_3, u_1, 0)$	−0.1	−0.1	1.5	1.5	0.5	1	0.8
$(e_3, u_2, 0)$	0.8	0.5	1	−0.5	0.5	0.6	0.4

Tables 2 and 3 present the grades for the agree-GQ-NSES and disagree-GQ-NSES, taken from the first six rows and the last six rows of Table 1, respectively.

Table 2. Numerical grades for agree-GQ-NSES.

$E \times U \times \{1\}$	(x_1, q_1)	(x_1, q_2)	(x_2, q_1)	(x_2, q_2)	(x_3, q_1)	(x_3, q_2)	f(a)
$(e_1, u_1, 1)$	0	−0.1	0.4	0.1	0.8	1	0.2
$(e_1, u_2, 1)$	0.7	0.6	0.5	0.6	0.5	0.4	0.6
$(e_2, u_1, 1)$	0.3	1.1	0.9	0.4	0.6	0.5	0.9
$(e_2, u_2, 1)$	1.3	−0.5	0.3	0.2	0.2	1.1	0.7
$(e_3, u_1, 1)$	0.6	0.2	1.1	−0.2	0.5	0.2	0.5
$(e_3, u_2, 1)$	1	0.1	1.1	0.9	0.7	0.1	0.3

Table 3. Numerical grades for disagree-GQ-NSES.

$E \times U \times \{0\}$	(x_1, q_1)	(x_1, q_2)	(x_2, q_1)	(x_2, q_2)	(x_3, q_1)	(x_3, q_2)	f(a)
$(e_1, u_1, 0)$	0.9	0.2	0.1	−0.3	0.5	1	0.8
$(e_1, u_2, 0)$	0.9	0.3	−0.1	0.4	0.5	0.2	0.6
$(e_2, u_1, 0)$	0.1	0.1	0.6	0.4	0.5	0.2	0.5
$(e_2, u_2, 0)$	0	0.9	0.8	0.4	0.3	0.2	0.7
$(e_3, u_1, 0)$	−0.1	−0.1	1.5	1.5	0.5	1	0.8
$(e_3, u_2, 0)$	0.8	0.5	1	−0.5	0.5	0.6	0.4

Let ν_i and η_i represent the score of each numerical grade for the agree-GQ-NSES and disagree-GQ-NSES, respectively. These values are given in Table 4.

Table 4. The score $\delta_j = \nu_i - \eta_i$.

ν_i	η_i	δ_j
$\nu(x_1, q_1) = 2.2$	$\eta(x_1, q_1) = 1.55$	$\delta(x_1, q_1) = 0.65$
$\nu(x_1, q_2) = 2.01$	$\eta(x_1, q_2) = 1.14$	$\delta(x_1, q_2) = 0.87$
$\nu(x_2, q_1) = 2.28$	$\eta(x_2, q_1) = 2.48$	$\delta(x_2, q_1) = -0.2$
$\nu(x_2, q_2) = 1.05$	$\eta(x_2, q_2) = 1.48$	$\delta(x_2, q_2) = -0.43$
$\nu(x_3, q_1) = 1.6$	$\eta(x_3, q_1) = 1.76$	$\delta(x_3, q_1) = -0.16$
$\nu(x_3, q_2) = 1.79$	$\eta(x_3, q_2) = 2.2$	$\delta(x_3, q_2) = -0.41$

As can be seen in Table 4, $\max \delta_i = \delta(x_1, q_2)$. Therefore, the best option is to invest in an international car company.

5. Comparative Analysis

A generalized Q-neutrosophic soft expert model offers better compatibility, accuracy and flexibility than existing models. This can be confirmed by a comparison utilizing generalized Q-neutrosophic soft expert with the strategy utilized in [33] as seen in Table 5. We can note that the proposed method include a degree of preference in the decision process, thus make it highly effective in decision-making. The comparison is similarly conducted as the illustration in Section 4, whereby the ranking order is found to be consistent.

Table 5. Comparison of GQ-NSES to Q-NSES.

Method	GQ-NSES	Q-NSES
True	Yes	Yes
Falsity	Yes	Yes
Indeterminacy	Yes	Yes
Expert	Yes	Yes
Q	Yes	Yes
Degree of preference	Yes	No
Ranking	$(x_1, q_2) > (x_1, q_1) > (x_3, q_1) > (x_2, q_1) > (x_3, q_2) > (x_2, q_2)$	$(x_3, q_1) > (x_1, q_2) > (x_2, q_1) > (x_2, q_2) > (x_3, q_2) > (x_1, q_1)$

From Table 5, it can be seen that the results of the ranking on the six companies obtained by the proposed GQ-NSES method in this paper is different from the ranking obtained by the method introduced in [33]. The main reason is that the proposed method has some crucial advantages over Q-NSES, which can take the preference of decision-makers into consideration.

6. Conclusions

The concept of GQ-NSES was initiated by incorporating the idea of SESs to Q-NSs and attaching a degree of preference corresponding to each parameter. The proposed concept is significantly superior and improved generalization of Q-NSES, which delivers better results, particularly for decision-making problems. The basic operations on GQ-NSES were defined and subsequently the basic properties were proven. An algorithm incorporating GQ-NSES is introduced and applied to a real-life example. The notion of GQ-NSES extended current neutrosophic theories for dealing with indeterminacy and will stimulate further studies on extensions and applied usage. In the future, one could study the measures of distance, similarity and entropy of GQ-NSES by generalizing the results in [14]. Furthermore, the algebraic structures such as group, ring and field of the Q-NSS and Q-NSES and their generalizations may be studied.

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