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Neutrosophic state feedback design method for SISO neutrosophic linear systems

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Abstract

The indeterminacy of parameters in actual control systems is inherent property because some parameters in actual control systems are changeable rather than constants in some cases, such as manufacturing tolerances, aging of main components, and environmental changes, which present an uncertain threat to actual control systems. Therefore, these indeterminate parameters can affect the control behavior and performance. Then, a neutrosophic number (NN) $z = d + eI$ consists of its determinate term d and its indeterminate term eI for $d, e \in R$ (R is all real numbers and I denotes indeterminacy). In fact, NN implies a changeable interval depending on the indeterminate range of $I \in [I^L, I^U]$ and easily expresses determinate and/or indeterminate information. Unfortunately, NNs are not introduced into the modeling, analysis, and design of uncertain control systems with interval/determinate parameters in existing literature so far. To develop a new neutrosophic design method, this study firstly introduces neutrosophic state space models and the neutrosophic controllability and observability in indeterminate linear systems. Then, a neutrosophic state feedback design method is established for achieving a desired closed-loop state equation or a desired control ratio for single-input single-output (SISO) neutrosophic linear systems. Finally, the proposed control design method is used for a numerical example with NN parameters, and the simulation results demonstrate that the designed state feedback control system can reach the desired system tracking performance requirements. Meanwhile, the obtained state feedback design result demonstrates its effectiveness and robustness.

Keywords: Neutrosophic state space model; Neutrosophic controllability; Neutrosophic observability; Neutrosophic state feedback design; Neutrosophic linear system

1. Introduction

The uncertainty of parameters in actual control systems is inherent property because the parameters of the control system are changeable rather than constants in many cases, such as manufacturing tolerances, aging of main components, and environmental changes. Therefore, the indeterminacy of system parameters can affect the control behavior and performance to some extent. In fact, the parameters of the plant in conventional control problems are always treated as determinate or nominal values. However, such variations or indeterminacies of system parameters need special modeling and analysis methods of an indeterminate control system to hold the desired control performance. In existing uncertain system studies (Hussein, 2005, 2010, 2011, 2015; Kolev, 1988), many modeling methods of uncertain systems and robust stability analysis methods were proposed in interval linear time invariant systems. Kharitonov's theorem (Kharitonov,

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1979) have been introduced into the field of robust stability of systems with parametric uncertainty/interval parameters, which indicated that the strict Hurwitz property of the entire family is equivalent to the strict Hurwitz property of four specifically constructed vertex polynomials, and control system applications. For example, Meressi et al. (1993) applied Kharitonov's theorem to mechanical systems. Czarkowski et al. (1995) presented a robust stability analysis of state feedback control of the pulse width modulation (PWM) DC-DC (direct current to direct current) push-pull converter. Hote et al. (2009) introduced a robust stability analysis of the PWM push-pull DC-DC converter. Then, Precup and Preitl (2006) proposed proportional-integral (PI) and proportional-integral-derivative (PID) controllers tuning for integral-type servo systems to ensure the robust stability and controller robustness. Elkaranshawy et al. (2009) further presented a robust control of a flexible-arm robot by using Kharitonov theorem for the PID controller design. Hote et al. (2010) used the Kharitonov's theorem and Routh criterion for the stability margin of interval systems. However, existing modeling, analysis, and design methods for systems with parametric uncertainty/interval parameters are relatively complex or difficult based on the Kharitonov's theorem, which needs to satisfy the independent condition of the system parameters.

However, neutrosophic theory has been successfully applied to many areas (Abdel-Basset et al., 2017, 2018; Abdel-Basset and Mohamed, 2018; Abdel-Basset, Gunasekaran, Mohamed, & Chilamkurti, 2018; Abdel-Basset, Gunasekaran, Mohamed, & Smarandache, 2018; Abdel-Basset, Manogaran, Gamal, & Smarandache, 2018; Abdel-Basset, Mohamed, & Smarandache, 2018; Abdel-Basset, Zhou, Mohamed, & Chang, 2018; Broumi, Bakali, Talea, & Smarandache, 2016; Broumi, Bakali, Talea, Smarandache, & Vladareanu, 2016a, 2016b; Broumi, Talea, Smarandache, & Bakali, 2016) in indeterminate setting. Then, a neutrosophic number (NN) $z = d + eI$ (Kong et al., 2015; Smarandache, 1998, 2013, 2014; Ye, 2015, 2016a) consists of both its determinate term d and its indeterminate term eI for d; $e \in R$ (R is all real numbers and I denotes indeterminacy). Hence, NN easily express determinate and/or indeterminate information in indeterminate problems. Therefore, NNs have been applied to fault diagnosis (Kong et al., 2015; Ye, 2016a); multiple attribute group decision-making (Ye, 2015, 2016b); linear and nonlinear optimization/programming (Jiang and Ye, 2016; Ye, 2017a, 2018; Ye et al., 2018); traffic flow linear equations (Ye, 2017b), and the expression and analysis of rock joint roughness coefficient (Chen, Ye, & Du, 2017; Chen, Ye, Du, & Yong, 2017; Ye et al., 2016, 2017) under indeterminate environments. However, NNs are not applied to the uncertain/interval control system modeling, analysis, and design in existing literature so far. Since NN has the convenient and flexible advantage in the expression and analysis of indeterminate problems, we need to propose neutrosophic modeling and control design methods for single-input single-output (SISO) linear systems so as to satisfy the desired system performance specifications. The main contribution of this study is that the neutrosophic state space modeling, neutrosophic controllability and observability, and neutrosophic state feedback design are proposed for the first time to provide the necessary preliminary basis for the modeling, analysis, and design of neutrosophic control systems with incomplete and indeterminate information.

The arrangement of this article is given as follows. Section 2 introduces neutrosophic state space models in indeterminate systems. Section 3 presents the controllability and observability of neutrosophic linear systems. In Section 4, a state feedback design method is proposed in neutrosophic systems and used for a numerical example with NN parameters. Section 5 gives conclusions and future research.

2. Neutrosophic state space model

2.1. NN concept

In indeterminate environments, Smarandache (1998, 2013, 2014) defined the NN $z = d + eI$ for d; $e \in R$ and $I \in [I^L, I^U]$ to represent its determinate term d and its indeterminate term eI simultaneously. Obviously, it easily expresses the determinate and/or indeterminate information in real world. For instance, a capacitor C in a circuit may contain its uncertainty and deviation from the nominal value $C = 200$ μF owing to ageing, temperature, manufacturing tolerance or other disturbances. Then, the capacitor C can be expressed as the NN $z = 200 + 2I$ μF, which indicates that its determinate term (nominal value) is 200 μF and its indeterminate term is $2I$ for $I \in [I^L, I^U]$. In actual applications, however, the indeterminacy I can be specified as a possible interval range $[I^L, I^U]$ to suit actual requirements. If $I \in [-5, 5]$, it is $z \in [190, 210]$ μF; if $I \in [-10, 10]$, then it is $z \in [180, 220]$ μF. So a NN $z = d + eI$ can be also represented as a possible interval number $z = [d + eI^L, d + eI^U]$ for $z \in Z$ (Z is all NNs) and $I \in [I^L, I^U]$, which implies a changeable interval number with respect to different indeterminate ranges of $[I^L, I^U]$. Especially, there exist $z = d$ for the best case ($eI = 0$), $z = eI$ for the worst case ($d = 0$), and then z reduces to a real number when $I^L = I^U$. Obviously, NN is more suitable and more flexible than a conventional interval number in the expression of determinate and/or indeterminate information. Hence, NN indicates its expression and analysis convenience and flexibility in indeterminate problems.

Supposed that two NNs are $z_1 = d_1 + e_1 I$ and $z_2 = d_2 + e_2 I$ for d_1, e_1, d_2, $e_2 \in R$, z_1, $z_2 \in Z$, and

$I \in [I^L, I^U]$, then they contain the following operational laws (Jiang and Ye, 2016; Ye, 2017a; Ye et al., 2018):

$$z_1 + z_2 = (d_1 + e_1 I) + (d_2 + e_2 I) = d_1 + d_2 + (e_1 + e_2)I \quad (1)$$

$$z_1 - z_2 = (d_1 + e_1 I) - (d_2 + e_2 I) = d_1 - d_2 + (e_1 - e_2)I \quad (2)$$

$$z_1 \times z_2 = (d_1 + e_1 I) \times (d_2 + e_2 I) = d_1 d_2 + (d_1 e_2 + d_2 e_1)I + e_1 e_2 I^2 \quad (3)$$

$$\frac{z_1}{z_2} = \frac{d_1 + e_1 I}{d_2 + e_2 I} \quad (4)$$

2.2. Neutrosophic state space model

Control system analysis and design need mathematical models. In indeterminate systems, the differential or integral-differential equations can describe the behavior of an indeterminate system, process or component. A state space model is a description in terms of a set of first-order differential equations which are written compactly in a matrix form. This standard form has permitted the development of general computer programs, which can be used for the analysis and design for even very large systems. To establish neutrosophic state space models of indeterminate systems, the following examples are presented to show the modeling method.

Example 1. A series RLC (resistor, inductor, and capacitor) circuit composed of a resistor R, an inductor L, and a capacitor C is shown in Fig. 1. The output voltage u_o of the circuit indicated in Fig. 1 is excited by the input voltage u_i. Then, the parameters R, L, and C of the series RLC circuit imply variations or indeterminacies from their nominal values owing to ageing, temperature, manufacturing tolerances or other disturbances.

Based on the Kirchhoff's laws, the equation of the RLC circuit is given as

$$u_i = Ri + L\frac{di}{dt} + \frac{1}{C}\int idt \quad (5)$$

Let $x_1 = i$, $x_2 = \int idt$, thus there is the following form:

$$\dot{x}_2 = x_1, \quad \dot{x}_1 = -\frac{R}{L}x_1 - \frac{1}{LC}x_2 + \frac{1}{L}u_i \quad (6)$$

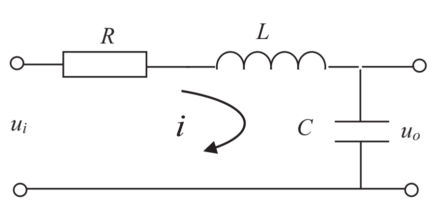


Fig. 1. Series RLC circuit.

Then, the state space model is expressed as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{LC} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u_i$$
$$y = \begin{bmatrix} 0 & \frac{1}{C} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (7)$$

where $y = u_o = x_2/C$.

Under the indeterminate environment, since R, L, and C imply some variations or indeterminacies, they are composed of determinate terms (nominal values) and indeterminate terms (changeable values). Thus R/L, 1/LC, 1/L, and 1/C can be expressed as four NNs $z_1 = d_1 + e_1 I$, $z_2 = d_2 + e_2 I$, $z_3 = d_3 + e_3 I$, and $z_4 = d_4 + e_4 I$, respectively, for $I \in [I^L, I^U]$. Then, the state space model of the RLC circuit with NNs can be represented as the following neutrosophic state space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -z_1 & -z_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} z_3 \\ 0 \end{bmatrix} u_i \quad \text{for} \quad z_1, z_2, z_3, z_4 \in Z$$
$$y = \begin{bmatrix} 0 & z_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (8)$$

$$or \quad \begin{matrix} \dot{\boldsymbol{x}} = \boldsymbol{A}(I)\boldsymbol{x} + \boldsymbol{b}(I)u_i \\ \boldsymbol{y} = \boldsymbol{c}(I)\boldsymbol{x} \end{matrix} \quad \text{for} \quad \boldsymbol{A}(I) \in Z^{2\times 2}, \boldsymbol{b}(I) \in Z^{2\times 1}, \ \boldsymbol{c}(I) \in Z^{1\times 2}, \ I \in [I^L, I^U] \quad (9)$$

In Fig. 1, it is assumed that the tolerance in all components of the circuit is to be 10%, such that $R = 500 + 500I\,\Omega$, $C = 0.01 + 0.01I$ F, $L = 0.2 + 0.2I$ H, and then $LC = 0.002 + 0.004I$ for $I \in [-0.1, 0.1]$. Thus, $z_1 = R/L = (500 + 500I)/(0.2 + 0.2I) = 2500$, $z_2 = 1/LC = 1/(0.002 + 0.004I) = 520.8334 + 1041.667I$, $z_3 = 1/L = 1/(0.2 + 0.2I) = 5.0505 + 5.051I$, and $z_4 = 1/C = 1/(0.01 + 0.01I) = 101.0101 + 101.01I$, respectively, for $I \in [I^L, I^U] = [-0.1, 0.1]$. Hence, the neutrosophic state space model can be expressed as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2500 & -(520.8334 + 1041.667I) \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 5.0505 + 5.051I \\ 0 \end{bmatrix} u_i$$
$$y = \begin{bmatrix} 0 & 101.0101 + 101.01I \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (10)$$

Example 2. Assume that a neutrosophic transfer function without zeros can be expressed as follows:

$$\frac{Y(s)}{R(s)} = \frac{z}{s^3 + z_3 s^2 + z_2 s + z_1} \quad \text{or} \quad \dddot{y} + z_3\ddot{y} + z_2\dot{y} + z_1 y = zr \quad \text{for} \quad z_1, z_2, z_3, z \in Z \quad (11)$$

A neutrosophic state space model for the indeterminate system described by this neutrosophic transfer function or equivalent neutrosophic differential equation is not unique but depends on the choice of a set of state variables. For example, let $x_1 = y$, $x_2 = \dot{y}$, $x_3 = \ddot{y}$, thus $\dot{x}_1 = x_2$,

$\dot{x}_2 = x_3$, and $\dot{x}_3 = -z_1x_1 - z_2x_2 - z_3x_3 + zr$. Then, the neutrosophic state space model is represented as follows:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -z_1 & -z_2 & -z_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix} r \quad \text{for} \quad z_1, z_2, z_3, z \in Z,$$
$$\boldsymbol{y} = [1 \ \ 0 \ \ 0]\boldsymbol{x} \tag{12}$$

or
$$\begin{matrix} \dot{\boldsymbol{x}} = \boldsymbol{A}(I)\boldsymbol{x} + \boldsymbol{b}(I)r \\ \boldsymbol{y} = \boldsymbol{c}(I)\boldsymbol{x} \end{matrix} \quad \text{for } \boldsymbol{A}(I) \in Z^{3\times3}, \quad \boldsymbol{b}(I) \in Z^{3\times1}, \ \boldsymbol{c}(I) \in Z^{1\times3}, \ I \in [I^L, I^U] \tag{13}$$

3. Controllability and observability of neutrosophic linear systems

Before introducing the state feedback design methods, we need to introduce the necessary conditions for the neutrosophic controllability and observability of indeterminate systems since optimal linear control systems are governed by the neutrosophic controllability and observability properties of indeterminate systems. An important objective of state variable control is the design of neutrosophic systems which reaches an optimum control performance. For example, in order to be able to relocate or reassign the open-loop plant poses to more desirable closed-loop locations in the *s* plane (complex plane), it is necessary that the plant satisfies the controllability property. Then, the indeterminate system analysis and design need mathematical models. To represent a common mathematical model of the indeterminate system, a neutrosophic state space model for the indeterminate linear system is described by the following neutrosophic state and output equations:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(I)\boldsymbol{x} + \boldsymbol{B}(I)\boldsymbol{u} \quad \text{(Neutrosophic state equation)} \quad \text{for} \quad \boldsymbol{A}(I) \in Z^{n\times n}, \ \boldsymbol{B}(I) \in Z^{n\times p}, \ I \in [I^L, I^U], \tag{14}$$

$$\boldsymbol{y} = \boldsymbol{C}(I)\boldsymbol{x} + \boldsymbol{D}(I)\boldsymbol{u} \quad \text{(Neutrosophic output equation) for} \quad \boldsymbol{C}(I) \in Z^{q\times n}, \ \boldsymbol{D}(I) \in Z^{q\times p}, \ I \in [I^L, I^U], \tag{15}$$

where $\boldsymbol{x}$ is the n dimensional state variable vector, $\boldsymbol{u}$ is the p dimensional input vector, and $\boldsymbol{y}$ is the q dimensional output vector; then $\boldsymbol{A}(I)$ is called the plant neutrosophic matrix or system neutrosophic matrix and $\boldsymbol{B}(I)$ as the control neutrosophic matrix; the unnamed neutrosophic matrices $\boldsymbol{C}(I)$ and $\boldsymbol{D}(I)$ relate the output variables to the state and control variables.

Based on conventional controllability and observability definitions in linear systems, we can extend them to neutrosophic linear systems and give the following neutrosophic controllability and observability definitions in neutrosophic linear systems.

Definition 1. A neutrosophic linear system is said to be completely state-controllable if for any initial time t_0 each initial state $x(t_0)$ can be transferred to any final state $x(t_f)$ in a finite time $t_f > t_0$ by means of an unconstrained control input vector $u(t)$. An unconstrained control vector has no limit on the amplitudes of $u(t)$.

The definition of controllability implies that $\boldsymbol{u}(t)$ can affect each state variable in the state equation (14).

Definition 2. A neutrosophic linear system is said to be completely observable if every initial state $x(t_0)$ can be exactly determined from the measurements of the output $y(t)$ over the finite interval of time $t_0 \le t \le t_f$.

The definition of observability implies that every state of $\boldsymbol{x}(t)$ can affect the output of $\boldsymbol{y}(t)$ in the output equation (15).

For a SISO neutrosophic linear system, the neutrosophic state space model can be expressed as the following form:

$$\begin{matrix} \dot{\boldsymbol{x}} = \boldsymbol{A}(I)\boldsymbol{x} + \boldsymbol{b}(I)u \\ \boldsymbol{y} = \boldsymbol{c}(I)\boldsymbol{x} \end{matrix} \quad \text{for} \quad \boldsymbol{A}(I) \in Z^{n\times n}, \ \boldsymbol{b}(I) \in Z^{n\times1}, \ \boldsymbol{c}(I) \in Z^{1\times n}, \ I \in [I^L, I^U] \tag{16}$$

Then, the SISO neutrosophic linear system is completely controllable if the following neutrosophic controllability has the property:

$$\text{Rank}\left[\boldsymbol{b}(I)\boldsymbol{A}(I)\boldsymbol{b}(I)\ldots A(I)^{n-1}\boldsymbol{b}(I)\right] = n \quad \text{or}$$
$$\text{Det}\left[\boldsymbol{b}(I)\boldsymbol{A}(I)\boldsymbol{b}(I)\ldots \boldsymbol{A}(I)^{n-1}\boldsymbol{b}(I)\right] \ne 0 \quad \text{for} \quad I \in [I^L, I^U] \tag{17}$$

The SISO neutrosophic linear system is completely observable if the following neutrosophic observability has the property:

$$\text{Rank}[\boldsymbol{c}(I)\boldsymbol{c}(I)\boldsymbol{A}(I)\ldots \boldsymbol{c}(I)\boldsymbol{A}(I)^{n-1}]^T = n \quad \text{or}$$
$$\text{Det}[\boldsymbol{c}(I)\boldsymbol{c}(I)\boldsymbol{A}(I)\ldots \boldsymbol{c}(I)\boldsymbol{A}(I)^{n-1}]^T \ne 0 \quad \text{for} \quad I \in [I^L, I^U] \tag{18}$$

Example 3. Assume that the neutrosophic state and output equations are as follows:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -2-I & 0 \\ -1-I & -1-2I \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \boldsymbol{u} \quad \text{for} \quad I \in [1, \ 2]$$
$$\boldsymbol{y} = [0 \quad 1]\boldsymbol{x}$$

Then, there are the following results:

$$\boldsymbol{M}_c(I) = [\boldsymbol{b}(I) \ \boldsymbol{A}(I)\boldsymbol{b}(I)] = \begin{bmatrix} 1 & -2-I \\ 1 & -2-3I \end{bmatrix} \quad \text{and}$$
$$\text{Det}(\boldsymbol{M}_c(I)) = -2I \ne 0 \quad \text{for} \quad I \in [1, 2]$$

Thus, the neutrosophic system is completely controllable if $I \in [1, 2]$.

$$\boldsymbol{M}_o(I) = \begin{bmatrix} \boldsymbol{c}(I) \\ \boldsymbol{c}(I)\boldsymbol{A}(I) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1-I & -1-2I \end{bmatrix} \quad \text{and}$$
$$\text{Det}(\boldsymbol{M}_o(I)) = 1 + I \neq 0 \quad \text{for} \quad I \in [1,2].$$

Then, the neutrosophic system is completely observable if $I \in [1, 2]$.

4. Neutrosophic state feedback design for SISO neutrosophic systems

This section covers the design method of SISO neutrosophic linear systems where the set of closed-loop eigenvalues is assigned by some state feedback. The plant state and output equations have the following form:

$$\begin{array}{l} \dot{\boldsymbol{x}} = \boldsymbol{A}(I)\boldsymbol{x} + \boldsymbol{b}(I)u \\ y = \boldsymbol{c}(I)\boldsymbol{x} \end{array} \quad \text{for} \quad \boldsymbol{A}(I) \in Z^{n\times n},\ \boldsymbol{b}(I) \in Z^{n\times 1},\ \boldsymbol{c}(I) \in Z^{1\times n},\ I \in [I^L, I^U]. \tag{19}$$

The state feedback control law is given by

$$u = r - \boldsymbol{K}\boldsymbol{x} \tag{20}$$

where r is the input and $\boldsymbol{K}$ is the state feedback vector for $\boldsymbol{K} \in Z^{1\times n}$. By combining the state equation with the state feedback control law, the closed-loop state equation is yielded as

$$\dot{\boldsymbol{x}} = (\boldsymbol{A}(I) - \boldsymbol{b}(I)\boldsymbol{K})\boldsymbol{x} + \boldsymbol{b}(I)r \tag{21}$$

Thus, the closed-loop system neutrosophic matrix produced by the state feedback is

$$\boldsymbol{A}_c(I) = \boldsymbol{A}(I) - \boldsymbol{b}(I)\boldsymbol{K} \tag{22}$$

Then, the closed-loop neutrosophic characteristic equation is obtained by

$$D(\boldsymbol{\lambda}, I) = |\boldsymbol{\lambda}\boldsymbol{U} - (\boldsymbol{A}(I) - \boldsymbol{b}(I)\boldsymbol{K})| = 0 \tag{23}$$

where $\boldsymbol{U}$ is a unit matrix and $\boldsymbol{\lambda}$ is the eigenvalue vector. Thus, Eq. (23) reveals that the closed-loop eigenvalues can be assigned by the proper selection of the state feedback vector $\boldsymbol{K}$. A necessary and sufficient condition for the selection $\boldsymbol{K}$ is that the plant is completely controllable.

Example 4. Let us consider the following neutrosophic SISO system:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2+I & -3+2I \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\boldsymbol{u} \quad \text{for} \quad I \in [0,\ 1].$$
$$y = [10 \quad 0 \quad 0]\boldsymbol{x}$$

where $\boldsymbol{x}$ is the state vector $\boldsymbol{x} = (x_1, x_2, x_3)^{\mathrm{T}}$, y is one output variable, and u is the input value. The neutrosophic system is completely controllable since from Eq. (17) there exists the following result:

$$\boldsymbol{M}_c(I) = [\boldsymbol{b}(I)\ \boldsymbol{A}(I)\boldsymbol{b}(I)\ \boldsymbol{A}(I)^2\boldsymbol{b}(I)] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2I-3 \\ 1 & -3+2I & (2I-3)^2+I-2 \end{bmatrix} \quad \text{and}$$
$$\text{Det}(\boldsymbol{M}_c(I)) = -1 \neq 0 \quad \text{for} \quad I \in [0, 1]$$

From Eq. (23), the closed-loop eigenvalues with $\boldsymbol{K} = [k_1, k_2, k_3]$ are determined as

$$D(\lambda, I) = |\lambda\boldsymbol{U} - (\boldsymbol{A}(I) - \boldsymbol{b}(I)\boldsymbol{K})| = \lambda^3 + (k_3 + 3 - 2I)\lambda^2 + (2 - I + k_2)\lambda + k_1 = 0 \tag{24}$$

Assigning the closed-loop eigenvalues $\lambda_1 = -2$, $\lambda_2 = -1 + i$, and $\lambda_3 = -1 - i$ requires the characteristic polynomial to be

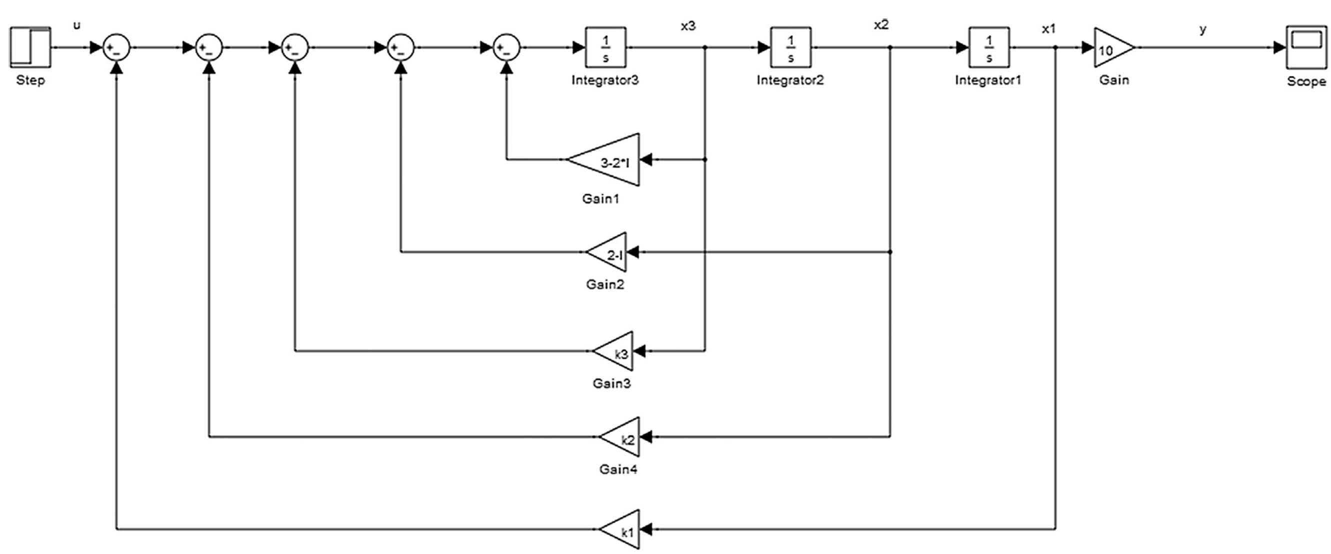
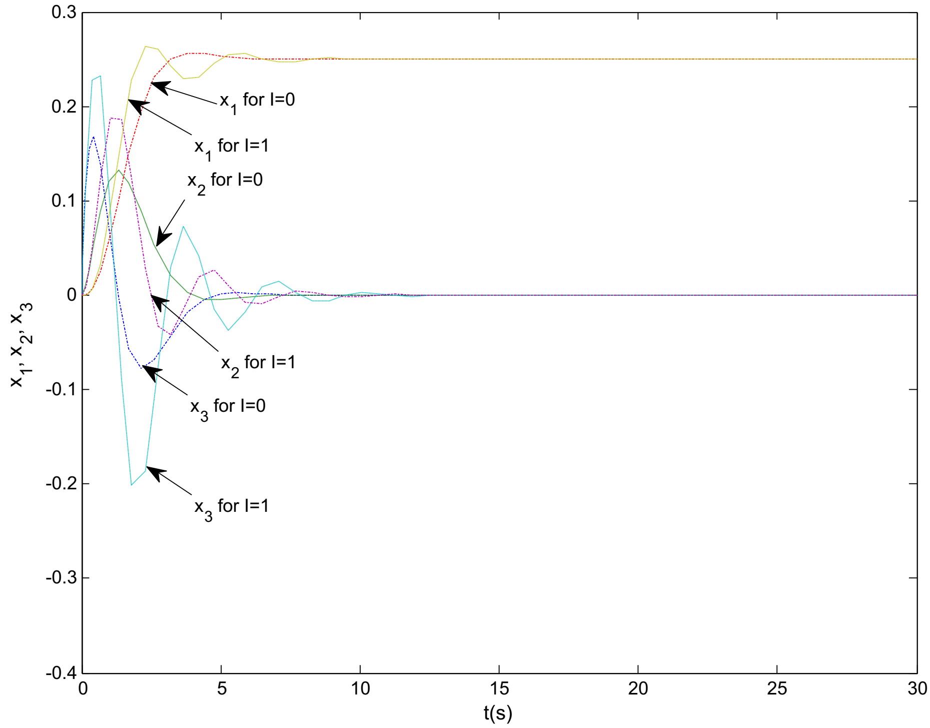
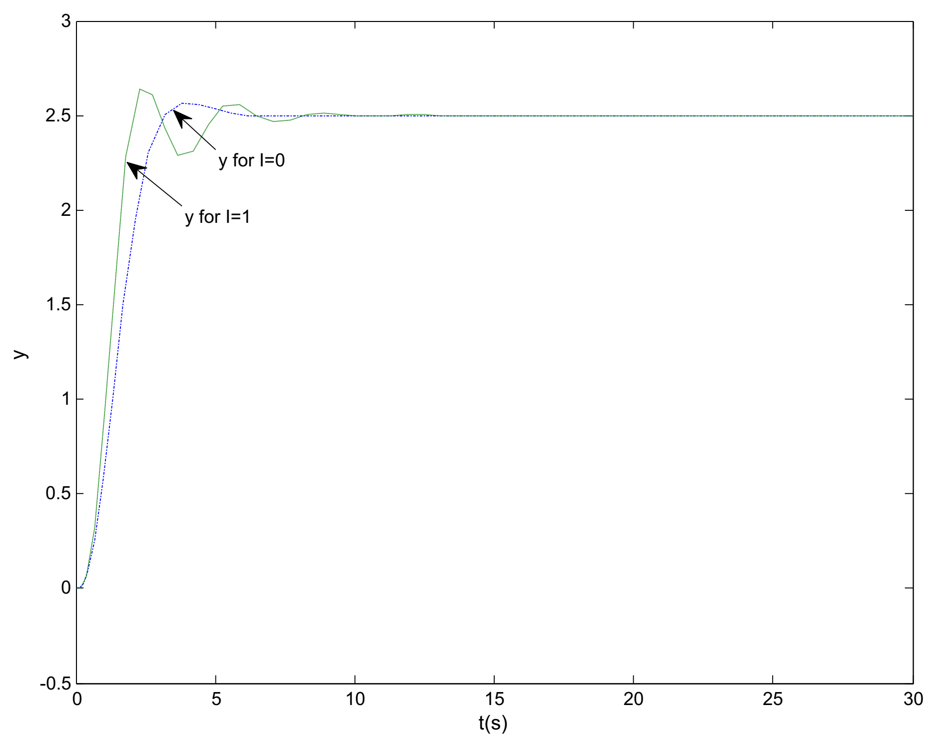


Fig. 2. Neutrosophic state feedback system with $I \in [0, 1]$.



(a)



(b)

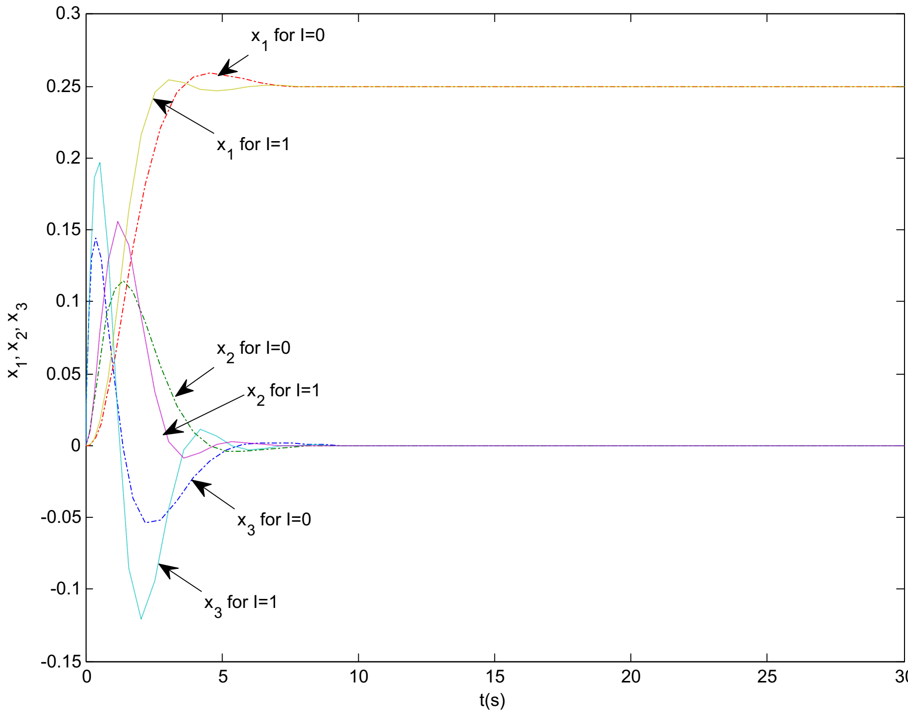
Fig. 3. Step responses of the traditional state feedback system (a special case of the neutrosophic state feedback system) corresponding to $\boldsymbol{K} = [4, 4, 1]$ and the indeterminate plant with $I = 0, 1$. (a) State variable responses; (b) output responses.

$$D_r(\lambda) = \lambda^3 + 4\lambda^2 + 6\lambda + 4 = 0. \quad (25)$$

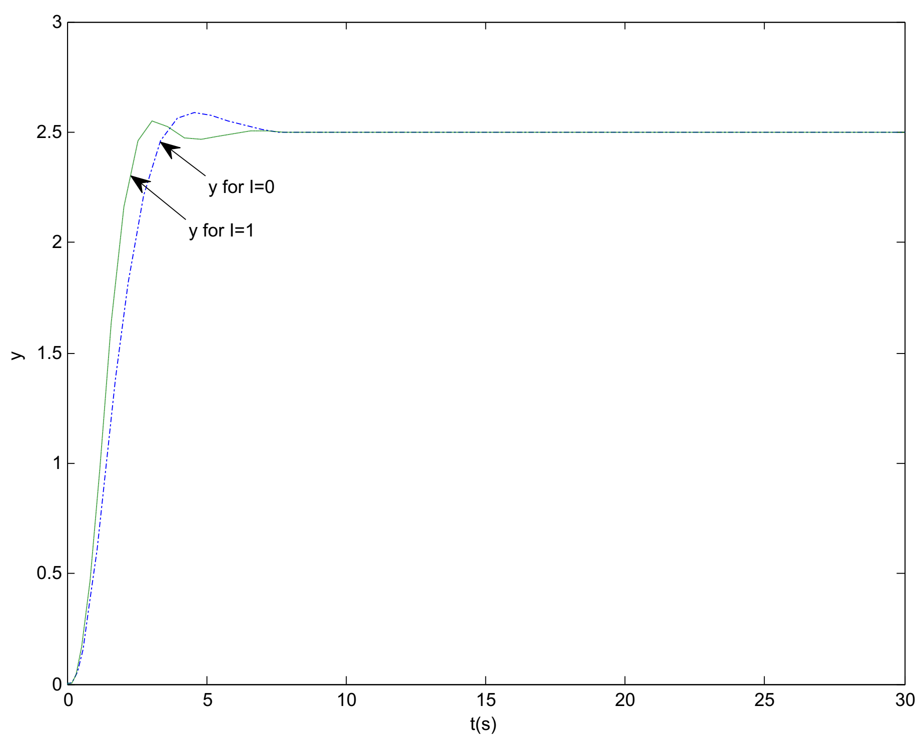
Equating the coefficients of this desired characteristic equation Eq. (25) with those of Eq. (24) results in the following values:

$k_1 = 4,\ k_2 = 4 + I \in [4, 5],$ and $k_3 = 1 + 2I \in [1, 3]$ for $I \in [0, 1]$.

Obviously, the state feedback vector $\boldsymbol{K} = [k_1, k_2, k_3]$ is an NN vector. This neutrosophic state feedback system is shown in Fig. 2.



(a)



(b)

Fig. 4. Step responses of the neutrosophic state feedback system corresponding to $\boldsymbol{K} = [4, 4.5, 2]$ and the indeterminate plant with $I = 0$, 1. (a) State variable responses; (b) output responses.

When the system has the certain/crisp parameters for $I = 0$, the state feedback vector obtained by the traditional state feedback design method is $\boldsymbol{K} = [k_1, k_2, k_3] = [4, 4, 1]$, which is the special case of the indeterminate/neutrosophic system. Since the neutrosophic state feedback values $k_1 = 4$, $k_2 \in [4, 5]$, and $k_3 \in [1, 3]$ for $I \in [0, 1]$, the neutrosophic state feedback design values usually are in the interval ranges, and then the neutrosophic state feedback systems usually indicate their response ranges/areas (indeterminate areas) in indeterminate systems. If different state feedback values are specified in the state feedback interval values, then we can get the best curves from the

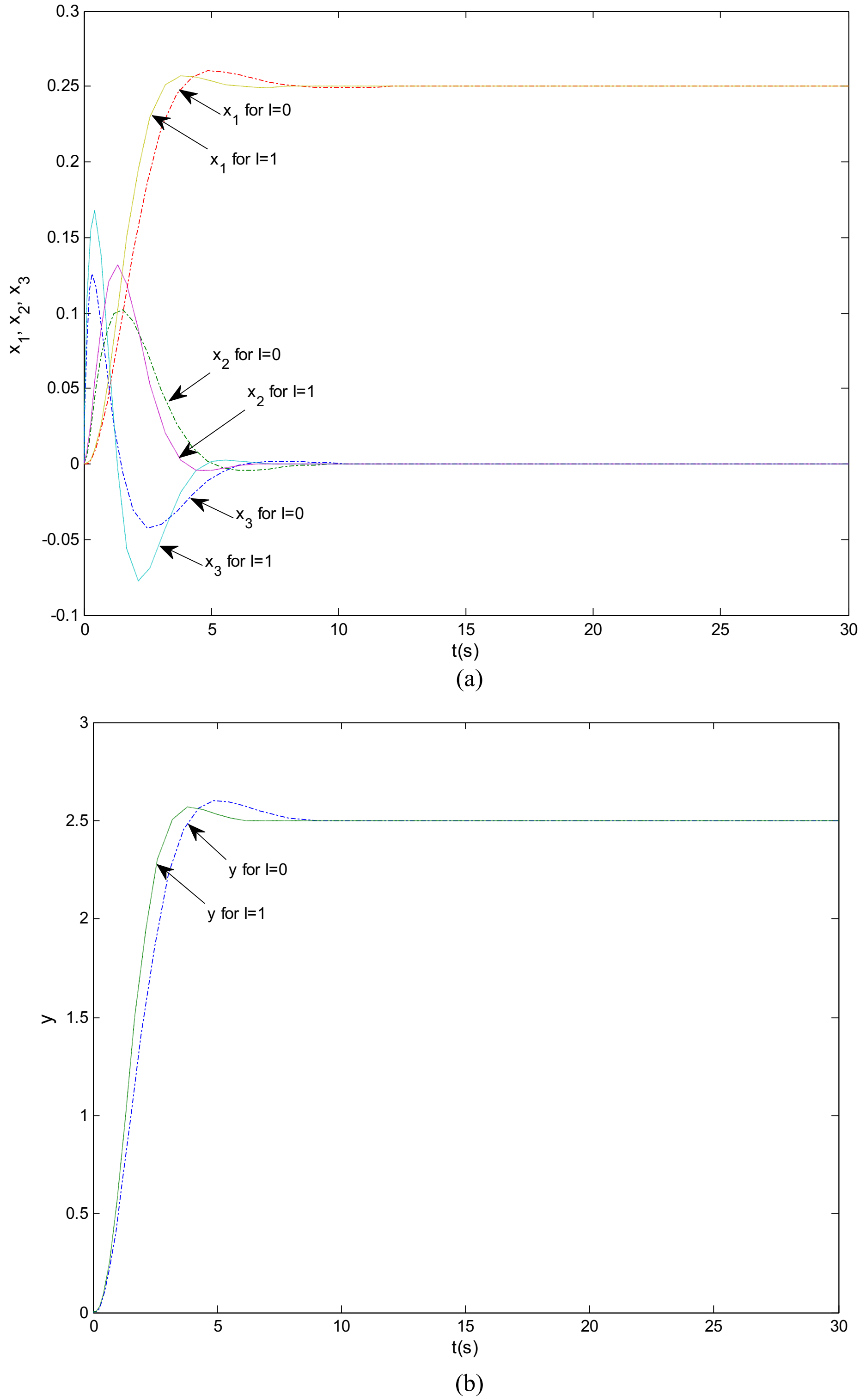


Fig. 5. Step responses of the neutrosophic state feedback system corresponding to $\boldsymbol{K} = [4, 5, 3]$ and the indeterminate plant with $I = 0, 1$. (a) State variable responses; (b) output responses.

response curves to find out the optimal state feedback values. To show the control performance and optimal state feedback values of the neutrosophic state feedback system, we only consider three specified neutrosophic state feedback vectors $\boldsymbol{K} = [4, 4, 1]$; $[4; 4.5; 2]$, $[4, 5, 3]$ for taking $I = 0, 0.5, 1$, respectively, so as to find out the optimal state feedback values. Herewith, Fig. 3 indicates the step response curves of the neutrosophic state feedback system corresponding to $\boldsymbol{K} = [4, 4, 1]$ and the indeterminate plant with $I = 0, 1$, which actually is the responses of the traditional state feedback design for $\boldsymbol{K} = [4, 4, 1]$ (taking $I = 0$) as a special case of the neutrosophic state feedback design. In Fig. 3(a), the three state variables x_1, x_2, x_3 can also show the areas/ranges of their response curves with respect to the indeterminate plant for $I = 0$, 1 in the dynamical process, and then they are tending to the corresponding determine state values in approaching to the steady state regarding the step responses of the neutrosophic state feedback system for $\boldsymbol{K} = [4, 4, 1]$. In Fig. 3 (b), the output responses of y indicate their response curve areas/ranges with respect to the indeterminate plant with $I = 0$, 1 in the dynamical process, and then their output response curves are tending to the corresponding steady output value in approaching to the steady state regarding the step responses of the neutrosophic state feedback system for $\boldsymbol{K} = [4, 4, 1]$.

Then, Figs. 4 and 5 indicate the step responses of the neutrosophic state feedback system corresponding to $\boldsymbol{K} = [4, 4.5, 2]$, $[4, 5, 3]$ and the indeterminate plant with $I = 0$, 1. In Figs. 4(a) and 5(a), the three state variables x_1, x_2, x_3 also similarly show the areas/ranges of their response curves with respect to the indeterminate plant with $I = 0$, 1 in the dynamical process, and then they are tending to the responding determine state values in approaching to the steady state regarding the step responses of the neutrosophic state feedback system; while Figs. 4(b) and 5(b) also similarly indicate that the output response curves of y show their response areas/ranges with respect to the indeterminate plant with $I = 0$, 1 in the dynamical process, and then their output response curves are tending to the responding steady output value in approaching to the steady state regarding the step responses of the neutrosophic state feedback system.

From Figs. 3–5, we see that the response curves based on the neutrosophic state feedback design are superior to ones of the traditional state feedback design for $\boldsymbol{K} = [4, 4, 1]$ and demonstrate the better response performance and robustness of the control system corresponding to the state feedback vector $\boldsymbol{K} = [4, 5, 3]$ for $I \in [0, 1]$ than ones of the traditional control system with $\boldsymbol{K} = [4, 4, 1]$. Obviously, the neutrosophic state feedback system can get better control performance in indeterminate/neutrosophic control systems, while the traditional state feedback design result is only the special case of the neutrosophic state feedback design results, but difficult to reach better control performance in indeterminate/neutrosophic systems.

5. Conclusion

This article first introduced the neutrosophic state space model of SISO linear systems in indeterminate environment and presented the controllability and observability properties of a neutrosophic system, which are important in the application of many indeterminate control system designs. Then, we proposed the neutrosophic state feedback design method for SISO neutrosophic systems, where the desired system tracking performance specifications are used to realize a state variable feedback control system. The simulation results demonstrated the effectiveness and rationality of the proposed design method for indeterminate control systems.

The main advantages of the proposed neutrosophic state feedback design method are summarized as follows:

(1) Existing state feedback design methods like the state space modeling, controllability and observability properties, and state feedback design can be extended to neutrosophic/indeterminate systems, which show the convenience of the neutrosophic state feedback design.
(2) The neutrosophic state feedback design can obtain the state feedback NNs/interval values (usually NNs but not always), which can indicate possible interval ranges of the neutrosophic state feedback values when indeterminacy $I \in [I^L, I^U]$ is specified as a possible interval range in real situations and actual requirements, so as to select a desired/optimal state feedback vector $\boldsymbol{K}$. Therefore, the proposed design method shows its flexibility and rationality for choosing the optimal state feedback values in the designed vector $\boldsymbol{K}$.
(3) The neutrosophic state feedback design is the generalization of the traditional state feedback design and more general, simpler, and more feasible in the modeling, analysis, and design than existing unconcern design methods under indeterminate environments.
(4) The neutrosophic state feedback design method was proposed for the first time to solve the neutrosophic control system problems with NNs which existing uncertain control system design methods cannot do.

However, this study of the neutrosophic control theory was proposed for the first time. Therefore, it is believed that this promising research opens the door for using the very powerful tool of the neutrosophic state feedback system modeling, analysis, and control design and provides a new effective way for neutrosophic/indeterminate control systems. In the future, we shall further propose neutrosophic state feedback design methods based on state observers and the modeling, analysis, and design methods of neutrosophic transfer functions in neutrosophic/indeterminate systems.

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