Nikfar Domination in Neutrosophic Graphs

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Abstract

Many various using of this new-born fuzzy model for solving real-world problems and urgent requirements involve introducing new concept for analyzing the situations which leads to solve them by proper, quick and efficient method based on statistical data. This gap between the model and its solution cause that we introduce nikfar domination in neutrosophic graphs as creative and effective tool for studying a few selective vertices of this model instead of all ones by using special edges. Being special selection of these edges affect to achieve quick and proper solution to these problems. Domination hasn't ever been introduced. So we don't have any comparison with another definitions. The most used graphs which have properties of being complete, empty, bipartite, tree and like stuff and they also achieve the names for themselves, are studied as fuzzy models for getting nikfar dominating set or at least becoming so close to it. We also get the relations between this special edge which plays main role in doing dominating with other special types of edges of graph like bridges. Finally, the relation between this number with other special numbers and characteristic of graph like order are discussed.

Keywords : Neutrosophic graph, bridge, tree, effective edge, nikfar domination.
AMS Subject Classification: 05C72, 05C69, 03E72, 94D05.

1 Introduction

Neutrosophy as a newly-born science is a branch of philosophy that studies the origin, nature and scope of neutralities.
In 1965, Zadeh introduced "fuzzy set" by the concept of degree of truth membership [12]. In 1986, Atanassow introduced "intuitionistic fuzzy set" by adding the concept of degree of false membership to the fuzzy set [2]. In 1995, Smarandache introduced "neutrosophic set" by adding the concept of degree of indeterminate membership to the intuitionistic fuzzy set [10]. There are three different types of definitions of a neutrosophic graph[1, 4, 8]. Broumi et al. [4] and Shah-Hussain [8] introduced two different definitions of neutrosophic graph by generalizations of intuitionistic fuzzy graph[9]. Akram and shahzadi introduced neutrosophic graph by using concept of neutrosophic set [1]. They also highlighted [1] some flaws in the definitions of Broumi et al. [4] and Shah-Hussain [8]. They introduced some counterexamples which state the complement of a neutrosophic graph isn't always a neutrosophic graph ([1], Example 3.5, pp. 22, 23) by using Shah-Hussain's definition of neutrosophic graph [8] and we even have much bad situations if we used Broumi et al.'s definition of neutrosophic graph ([4], Definition 3.1 p. 89) beacuse of not only we don't have complement of a neutrosophic graphs ([1], Example 3.2, p. 21) but also we don't have join of them([1], Example 3.3, pp. 21, 22). Moreover, they introduced binary operations cartesian product, composition, union, join, cross, lexicographic, strong product and unary operation complement along with proofs which show these operations hold neutrosophic property of graphs [1]. In other words, the new graph is produced by these operations, is also a neutrosophic graph.
Regarding these points, we use the definition of Akram and Shahzadi ([1], Definition 2.2, pp. 2, 3) as the main framework for our own study. The study behaviors of modeling is of spotlight by using few

parameters. Some parameters are so close to others one. if we defined being "so close" concept properly by adding some extra properties more than existence of edge between them, we would achieve the useful tool. This tool would cause solving real-world problems by deleting useless data and focusing on a few one. This leads to the concept of domination in modeling. Domination hasn't ever been introduced on any kind of neutrosophic graphs. Regarding these points, the aim of this paper is to introduce the notion of domination in this new-born fuzzy model. It is a normal question about effects of dominations in neutrosophic graphs. From here comes the main motivation for this and in this regard, we have considered some routine and fundamental framework for studying this concept.
Domination as a theoretical area in graph theory was formalized by Berge in 1958, in the chapter 4 with title " The fundamental Numbers of the theory of Graphs" ([3], Theorem 7, p.40) and Ore ([7], Chapter 13 , pp. 206, 207) in 1962. Since 1977, when Cockayne and Hedetniemi ([6], Section 3, p. 249-251) presented a survey of domination results, domination theory has received considerable attention. A set S of vertices of G ([5], Chap. 10, p. 302) is a *dominating set* if every vertex in $V(G)-S$ is adjacent to at least one vertex in S. The minimum cardinality among the dominating sets of G is called the *domination number* of G and is denoted by $\gamma(G)$. A dominating set of cardinality $\gamma(G)$ is then referred to as *minimum dominationg set*. Dominating sets appear to have their origins ([3], Example 2, p. 41) in the game of chess, where the goal is to cover or dominate various squares of a chessboard by certain chess pieces.

2 Preliminaries

We provide some basic background for the paper in this section.

Definition 2.1. (Fuzzy Set, [12])
Let V be a given set. The function $A : V \to [0, 1]$ is called a *fuzzy set* on V.

Definition 2.2. (Neutrosophic Set, [11])
Let V be a given set. A *neutrosophic set* A in V is characterized by a truth membership function $T_A(x)$, an indeterminate membership function $I_A(x)$ and a false membership function $F_A(x)$. The functions $T_A(x), I_A(x)$, and $F_A(x)$ are fuzzy sets on V. That is, $T_A(x) : V \to [0, 1], I_A(x) : V \to [0, 1]$ and $F_A(x) : V \to [0, 1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Notation 2.3. *Some special notations frequently appear in this paper. In what follows, we introduce them. Let V be a given set. For the sake of simplicity, we only use the notation E for the representation of the following set on V. $E \subseteq \{A|A \subseteq V, |A| = 2$ It means A has only two elements$\}$, where $|A|$ means cardinality of A. By Analogous to this points, the notation E_i is corresponded to V_i.*

Definition 2.4. (Neutrosophic Graph, [11], pp. 2, 3)
Let V be a given set. Also, assume E be a given set with respect to V. A *neutrosophic graph* is a pair $G = (A, B)$, where $A : V \to [0, 1]$ is a neutrosophic set in V and $B : E \to [0, 1]$ is a neutrosophic set in E such that

$$T_B(xy) \le min\{T_A(x), T_A(y)\},$$

$$I_B(xy) \le min\{I_A(x), I_A(y)\},$$

$$F_B(xy) \le max\{F_A(x), F_A(y)\},$$

for all $\{x, y\} \in E$. V is called *vertex set* of G and E is called *edge set* of G, respectively.

Definition 2.5. (Complete Neutrosophic Graph, [11], p. 3)
Let $G = (A, B)$ be a neutrosophic graph on a given set V. G is called *complete* if the following conditions are satisfied:

$$T_B(xy) = min\{T_A(x), T_A(y)\},$$

$$I_B(xy) = min\{I_A(x), I_A(y)\},$$

$$F_B(xy) = max\{F_A(x), F_A(y)\},$$

for all $\{x, y\} \in E$.

Definition 2.6. (Empty Neutrosophic Graph)
Let $G = (A, B)$ be a neutrosophic graph on a given set V. G is called *empty* if the following conditions are satisfied:

$$T_B(xy) = I_B(xy) = F_B(xy) = 0.$$

for all $\{x, y\} \in E$.

Definition 2.7. (Bipartite Neutrosophic Graph)
Let V be a given set. A neutrosophic graph $G = (A, B)$ on V is said *bipartite* if the set V can be partitioned into two nonempty sets V_1 and V_2 such that $T_B(xy) = I_B(xy) = F_B(xy) = 0$. for all $\{x, y\} \in E_1$. or $\{x, y\} \in E_2$. Moreover, if $T_B(xy) = min\{T_A(x), T_A(y)\}, I_B(xy) = min\{I_A(x), I_A(y)\}, F_B(xy) = max\{F_A(x), F_A(y)\}$, for all $\{x, y\} \in E$ then G is called a *complete bipartite neutrosophic graph*. In this case, If either $|V_1| = 1$ or $|V_2| = 1$ then the complete bipartite neutrosophic graph is said a *star neutrosophic graph*.

Definition 2.8. (Order)
Let $G = (A, B)$ be a neutrosophic graph on a given set V. Then the real number p is called the

a. *T-order,* if $p = \gamma_v(G)_T = \Sigma_{u\in V} T_A(u)$.

b. *I-order,* if $p = \gamma_v(G)_I = \Sigma_{u\in V} I_A(u)$.

c. *F-order,* if $p = \gamma_v(G)_F = \Sigma_{u\in V} F_A(u)$.

d. *order,* if was be either of $T-$order, $I-$order, and $F-$order.

Definition 2.9. (Bridge)
Let $G = (A, B)$ be a neutrosophic graph on a given set V. Then an edge xy in G is called the

a. *T-bridge,* if the strengths of each T-path P from x to y, not involving xy, were less than $T_B(xy)$.

b. *I-bridge,* if the strengths of each T-path P from x to y, not involving xy, were less than $T_B(xy)$.

c. *F-bridge,* if the strengths of each T-path P from x to y, not involving xy, were less than $T_B(xy)$.

d. *bridge,* if it was either of $T-$bridge, $I-$bridge, and $F-$bridge.

Definition 2.10. (Acyclic)
Let $G = (A, B)$ be a neutrosophic graph on a given set V. Then G is called the

a. *T-acyclic,* if there wasn't a T-path P from x to y, with only exception $x = y$., for all $x \in V$.

b. *I-acyclic,* if there wasn't a I-path P from x to y, with only exception $x = y$., for all $x \in V$.

c. *F-acyclic,* if there wasn't a F-path P from x to y, with only exception $x = y$., for all $x \in V$.

d. *acyclic,* if it was either of $T-$acyclic, $I-$acyclic, and $F-$acyclic.

Definition 2.11. (Spanning Neutrosophic Graph)
Let $G = (A, B), G_1 = (A_1, B_1)$ be a neutrosophic graph on a given set V. Then G_1 is called the *spanning neutrosophic graph* of G if $V = V_1$ but $E_1 \subseteq E$.

Definition 2.12. (Forest)
Let $G = (A, B)$ be a neutrosophic graph on a given set V. Then G is called the

a. *T-forest,* if G was T-acyclic and there is a spanning neutrosophic graph F such that for all edge xy out of F, there is a T-path P from x to y, how whose strength greater than $T_B(xy)$.

b. *I-forest,* if G was I-acyclic and there is a spanning neutrosophic graph F such that for all edge xy out of F, there is a I-path P from x to y, how whose strength greater than $I_B(xy)$.

c. *F-forest,* if G was F-acyclic and there is a spanning neutrosophic graph F such that for all edge xy out of F, there is a F-path P from x to y, how whose strength greater than $F_B(xy)$.

d. *forest,* if it was either of neutrosophic $T-$forest, neutrosophic $I-$forest, and neutrosophic $F-$forest.

Definition 2.13. (Tree)
Let $G = (A, B)$ be a neutrosophic graph on a given set V. Then G is called the

a. *T-tree,* if G was a T-forest such that there is a T-path P from x to y, for all $x, y \in V$.

b. *I-tree,* if G was a I-forest such that there is a I-path P from x to y, for all $x, y \in V$.

c. *F-tree,* if G was a F-forest such that there is a F-path P from x to y, for all $x, y \in V$.

d. *tree,* if it was either of $T-$tree, $I-$tree, and $F-$tree.

Notation 2.14. *Let V be a given set. For the sake of simplicity, we only use the notation F, p for the representation special spanning neutrosophic graph of a forest and the order a given neutrosophic graph. By Analogous to this points, the notation F_i, p_i are corresponded to G_i. Let us remind you consider three special notations in this paper by three letters. In other words, we have three correspondences for a given set, neutrosophic graph and a forest, we mean p, E_i and F_i are corresponded to G_i, V_i and G_i, respectively. Final remark is of about writing xy instead of $\{x, y\}$.*

3 Main Results

Definition 3.1. (Path)
Let $G = (A, B)$ be a neutrosophic graph on V and v_0, v_n be two given vertices such that $n \in \mathbb{N}$. Then

a. A distinct sequence of vertices $P : v_0, v_1, \cdots, v_n$ in G is called a *T-path* of length n from v_0 to v_n, if $T_B(v_i v_{i+1}) > 0$, for $i = 0, 1, \cdots, n-1$. The $\min_{i=0}^{n-1}\{T_B(v_i v_{i+1})\}$ is called the *strength* of this $T-$path and is denoted by $\mu_G(P)_T$.

b. A distinct sequence of vertices $P : v_0, v_1, \cdots, v_n$ in G is called a *I-path* of length n from v_0 to v_n, if $I_B(v_i v_{i+1}) > 0$, for $i = 0, 1, \cdots, n-1$. The $\min_{i=0}^{n-1}\{I_B(v_i v_{i+1})\}$ is called the *strength* of this $I-$path and is denoted by $\mu_G(P)_I$.

c. A distinct sequence of vertices $P : v_0, v_1, \cdots, v_n$ in G is called a *F-path* of length n from v_0 to v_n, if $F_B(v_i v_{i+1}) < 1$, for $i = 0, 1, \cdots, n-1$. The $\min_{i=0}^{n-1}\{F_B(v_i v_{i+1})\}$ is called the *strength* of this $F-$path and is denoted by $\mu_G(P)_F$.

d. A distinct sequence of vertices $P : v_0, v_1, \cdots, v_n$ in G is called a *path* of length n from v_0 to v_n, if it be $T-$path, $I-$path, and $F-$path, simultaneously. In this case, the $\min\{\mu_G(P)_T, \mu_G(P)_I, \mu_G(P)_F\}$ is called *strength* of path and is denoted by $\mu_G(P)$.

Definition 3.2. (Strength between Two Vertices)
Let $G = (A, B)$ be a neutrosophic graph on V and v_i, v_j be two given vertices such that $i > j$ and $i, j \in \mathbb{N}$. Then

a. The $\max\{\mu_G(P)_T\}$ in G is called the *T-strength* between v_i and v_j and is denoted by $\mu_G^\infty(v_i, v_j)_T$.

b. The $\max\{\mu_G(P)_I\}$ in G is called the *I-strength* between v_i and v_j is denoted by $\mu_G^\infty(v_i, v_j)_I$.

c. The $\max\{\mu_G(P)_F\}$ in G is called the *F-strength* between v_i and v_j is denoted by $\mu_G^\infty(v_i, v_j)_F$.

d. The $\max\{\mu_G^\infty(v_i, v_j)_T, \mu_G^\infty(v_i, v_j)_I, \mu_G^\infty(v_i, v_j)_F\}$ is called the *strength* between v_i and v_j in G and is denoted by $\mu_G^\infty(v_i, v_j)$.

Example 3.3. *Let $G = (A, B)$ be a neutrosophic graph on V as Figure 1. The various types of some paths of length 3 from v_1 to v_5 are investigated. $P_1 : v_1, v_4, v_2, v_5$, $P_2 : v_1, v_3, v_4, v_5$, and $P_3 : v_1, v_2, v_4, v_5$ are the T-path, I-path and F-path of length 3 from v_1 to v_5, respectively and not other ones. The distinct sequences of vertices $P_4 : v_5, v_4, v_1, v_2$ is not neither of them. $P_5 : v_3, v_4, v_5, v_2$ is all of them. In this graph, we determine various types of strength of some paths as follows. $\mu_G(P_1)_T = 0.4, \mu_G(P_2)_I = 0.2,$ and $\mu_G(P_3)_F = 0.91$. For P_5, we have $\mu_G(P_5)_T = 0.4, \mu_G(P_5)_I = 0.1, \mu_G(P_5)_F = 0.92$, and $\mu_G(P_5) = 0.1$. Finally, we discuss about various types of strength between two vertices v_1 and v_5. $\mu_G^\infty(v_1, v_5)_T = 0.4, \mu_G^\infty(v_1, v_5)_I = 0.1, \mu_G^\infty(v_1, v_5)_F = 0.91$, and $\mu_G^\infty(v_1, v_5) = 0.91$.*

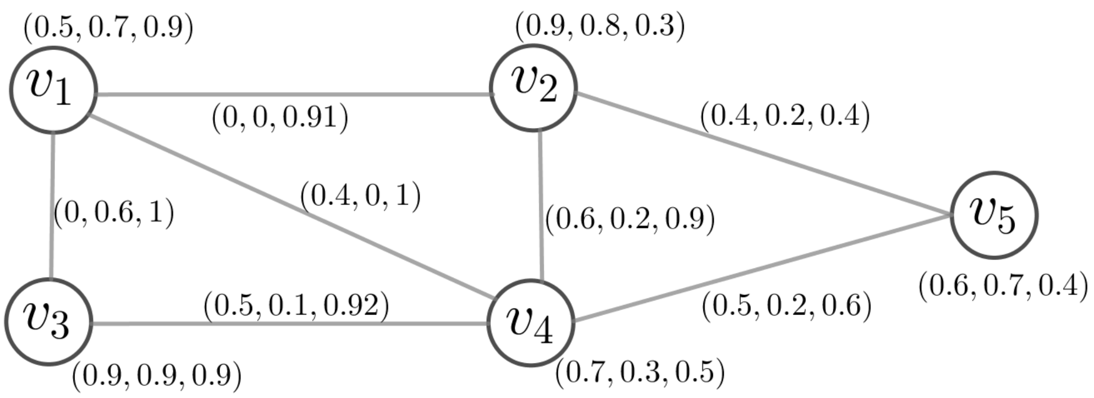

Figure 1: The strength in the neutrosophic graph $G = (A, B)$

Notation 3.4. *$\mu_{G-\{xy\}}^\infty(x, y)$ is the strength between x and y in the neutrosophic graph obtained from G by deleting the edge xy. This is as the same for the notations $\mu_{G-\{xy\}}^\infty(x, y)_T, \mu_{G-\{xy\}}^\infty(x, y)_I$, and $\mu_{G-\{xy\}}^\infty(x, y)_F$.*

In what follows, we will define four properties for edges. Based of these properties, we can construct various kindes of dominations in neutrosophic graphs.

Definition 3.5. (Effective Edges)
Let $G = (A, B)$ be a neutrosophic graph on V. Then An edge xy in G is called the

a. *T-effective,* if $T_B(xy) > \mu_{G-\{xy\}}^\infty(x, y)_T$.

b. *I-effective,* if $I_B(xy) > \mu_{G-\{xy\}}^\infty(x, y)_I$.

c. *F-effective,* if $F_B(xy) > \mu_{G-\{xy\}}^\infty(x, y)_F$.

d. *effective,* if it be either of $T-$effective, $I-$effective, and $F-$effective.

Example 3.6. *Let $G = (A, B)$ be a neutrosophic graph on V as Figure 1. In the following table, we study the properties of edges. For example, v_2v_5 has not neither of $T-$effective, $I-$effective, $F-$effective, and effective property. The edge v_3v_4 has both of $T-$effective and $I-$effective property. So it is also effective edge. The edges $\{v_1v_4, v_2v_4, v_3v_4, v_4v_5\}, \{v_1v_3, v_3v_4\}, \{v_1v_3, v_1v_4, v_2v_4\}$, and $\{v_1v_3, v_1v_4, v_2v_4, v_3v_4, v_4v_5\}$ have $T-$effective, $I-$effective, $F-$effective, and effective property, respectively. $\{v_2v_5, v_1v_2\}$ has no ones.*

Edges \ *Properties*	$T-$*effective*	$I-$*effective*	$F-$*effective*	*Effective*
v_1v_2	×	×	×	×
v_1v_3	×	√	√	√
v_1v_4	√	×	√	√
v_2v_4	√	×	√	√
v_2v_5	×	×	×	×
v_3v_4	√	√	×	√
v_4v_5	√	×	×	√

Definition 3.7. (Nikfar Domination)
Let $G = (A, B)$ be a neutrosophic graph on V and $x, y \in V$. Then

a. We say that x *dominates* y in G as *T-effective,* if the edge xy be T-effective. A subset S of V is called the *T-effective dominating set* in G, if for every $v \in V - S$, there is $u \in S$ such that u dominates v as T-effective. The *T-nikfar weight* of x is defined by $w_v(x)_T = T_A(x) + \frac{\Sigma_{xy \text{ is a T-effective edge}} T_B(xy)}{\Sigma_{xy \text{ is a edge}} T_B(xy)}$. If $\Sigma_{xy \text{ is a edge}} T_B(xy)$, for some $x \in V$. Then we consider $\frac{\Sigma_{xy \text{ is a T-effective edge}} T_B(xy)}{\Sigma_{xy \text{ is a edge}} T_B(xy)}$ equal with 0. For any $S \subseteq V$, a natural extension of this concept to a set, is as follows. We also say the *T-nikfar weight* of S, it is defined by $w_v(S)_T = \Sigma_{u \in S}(w_v(u)_T)$. Now, let U be the set of all T-effective dominating sets in G. The *T-nikfar domination number* of G is defined as $\gamma_v(G)_T = \min_{D \in U}(w_v(D)_T)$. The T-effective dominating set that is correspond to $\gamma_v(G)_T$ is called by *T-nikfar dominating set.*

b. We say that x *dominates* y in G as *I-effective,* if the edge xy be I-effective. A subset S of V is called the *I-effective dominating set* in G, if for every $v \in V - S$, there is $u \in S$ such that u dominates v as I-effective. The *I-nikfar weight* of x is defined by $w_v(x)_I = I_A(x) + \frac{\Sigma_{xy \text{ is a I-effective edge}} I_B(xy)}{\Sigma_{xy \text{ is a edge}} I_B(xy)}$ If $\Sigma_{xy \text{ is a edge}} I_B(xy)$, for some $x \in V$. Then we consider $\frac{\Sigma_{xy \text{ is a I-effective edge}} I_B(xy)}{\Sigma_{xy \text{ is a edge}} I_B(xy)}$ equal with 0. For any $S \subseteq V$, a natural extension of this concept to a set, is as follows. We also say the *I-nikfar weight* of S, it is defined by $w_v(S)_I = \Sigma_{u \in S}(w_v(u)_I)$. Now, let U be the set of all I-effective dominating sets in G. The *I-nikfar domination number* of G is defined as $\gamma_v(G)_I = \min_{D \in U}(w_v(D)_I)$. The I-effective dominating set that is correspond to $\gamma_v(G)_I$ is called by *I-nikfar dominating set.*

c. We say that x *dominates* y in G as *F-effective,* if the edge xy be F-effective. A subset S of V is called the *F-effective dominating set* in G, if for every $v \in V - S$, there is $u \in S$ such that u dominates v as F-effective. The *F-nikfar weight* of x is defined by $w_v(x)_F = F_A(x) + \frac{\Sigma_{xy \text{ is a F-effective edge}} F_B(xy)}{\Sigma_{xy \text{ is a edge}} F_B(xy)}$. If $\Sigma_{xy \text{ is a edge}} F_B(xy)$, for some $x \in V$. Then we consider $\frac{\Sigma_{xy \text{ is a F-effective edge}} F_B(xy)}{\Sigma_{xy \text{ is a edge}} F_B(xy)}$ equal with 0. For any $S \subseteq V$, a natural extension of this concept to a set, is as follows. We also say the *F-nikfar weight* of S, it is defined by $w_v(S)_F = \Sigma_{u \in S}(w_v(u)_F)$. Now, let U be the set of all F-effective dominating sets in G. The *F-nikfar domination number* of G is defined as $\gamma_v(G)_F = \min_{D \in U}(w_v(D)_F)$. The F-effective dominating set that is correspond to $\gamma_v(G)_F$ is called by *F-nikfar dominating set.*

d. We say that x *dominates* y in G as *effective,* if the edge xy be effective. A subset S of V is called the *effective dominating set* in G, if for every $v \in V - S$, there is $u \in S$ such that u dominates v as effective. We also say the *nikfar weight* of S, it is defined by $w_v(S) = \min\{w_v(S)_T, w_v(S)_I, w_v(S)_F\}$. Now, let U be the set of all effective dominating sets in G. The *nikfar domination number* of G is defined as $\gamma_v(G) = \min_{D \in U}(w_v(D))$. The effective dominating set that is correspond to $\gamma_v(G)$ is called by *nikfar dominating set.*

Proposition 3.8. *Let $G = (A, B)$ be a complete neutrosophic graph on a given set V such that there is exactly one path between two given vertices, which has*

a. *T-strength. Then* $\gamma_v(G)_T = \min_{u \in V}(T_A(u)) + 1$.

b. *I-strength. Then* $\gamma_v(G)_I = \min_{u \in V}(I_A(u)) + 1$.

c. *F-strength. Then* $\gamma_v(G)_F = \min_{u\in V}(F_A(u)) + 1$.

d. *strength. Then* $\gamma_v(G) = \min_{u\in V}(T_A(u), I_A(u), F_A(u)) + 1$.

Proof. (a). Let $G = (A, B)$ be a neutrosophic graph on a given set V. The T-strength of path P from u to v is of the form $T_A(u)\wedge\cdots\wedge T_A(v) \le T_A(u)\wedge T_A(v) = T_B(uv)$. So $\mu_G^{\infty}(u,v)_T \le T_B(uv)$. uv is a path from u to v such that $T_B(uv) = T_A(u)\wedge T_A(v)$. Therefore $\mu_G^{\infty}(u,v)_T \ge T_B(uv)$. Hence $\mu_G^{\infty}(u,v)_T = T_B(uv)$. Then $T_B(uv) > \mu^{\infty}_{G-\{xy\}}((u,v)_T$. It means that the edge uv is T-effective. All edges are T-effective and each vertex is adjacent to all other vertices. So $D = \{u\}$ is a T-effective dominating set and $\Sigma_{xy \text{ is a T-effective edge}} T_B(xy) = \Sigma_{xy \text{ is a edge}} T_B(xy)$ for each $u \in V$. The result follows.
By analogues to the proof of (a), the result is obviously hold for (b), (c), and (d). □

Proposition 3.9. *Let* $G = (A, B)$ *be an empty neutrosophic graph on a given set* V*. Then* $\gamma_v(G)_T = \gamma_v(G)_I = \gamma_v(G)_F = \gamma_v(G) = p$ *where* p *denotes the order of* G*.*

Proof. Let G be an empty neutrosophic graph on a given set V. Hence V is only T-effective dominating set in G and there is also no T-effective edge. So by Definition 3.7(a), we have $\gamma_v(G)_T = \min_{D\in S}[\Sigma_{u\in D}T_A(u)] = \Sigma_{u\in V}T_A(u) = p$. Therefore $\gamma_v(G)_T = p$.
By analogues to the proof of $\gamma_v(G)_T = p$ and Definition 3.7, the result is obviously hold for $\gamma_v(G)_I, \gamma_v(G)_F$ and $\gamma_v(G)$. □

It is interesting to note that the converse of Propositions 3.9, does not hold.

Example 3.10. *We show that the converse of Propositions 3.9, does not hold. Let* $G = (\sigma, \mu)$ *be a fuzzy graph as Figure 2. The edges* $\{v_2v_5, v_2v_4, v_3v_4, v_1v_3\}$ *are T-effective, I-effective, F-effective, and effective*

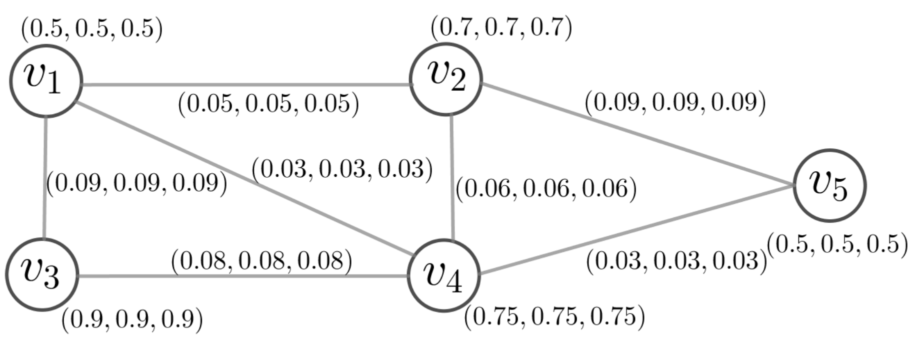

Figure 2: nikfar domination

and the edges $\{v_1v_4, v_1v_2, v_4v_5\}$ *are neither of types of being effective. So the set* $\{v_2, v_3\}$ *is all types of the effective dominating set. This set is also all types of nikfar dominating set in neutrosophic graph* G*. Hence* $\gamma_v(G) = \gamma_v(G)_T = \gamma_v(G)_I = \gamma_v(G)_F = 1.75 + 0.9 + 0.7 = 3.35 = \Sigma_{u\in V}T(u) = \Sigma_{u\in V}I(u) = \Sigma_{u\in V}F(u) = p$*. Therefore* G *isn't an empty neutrosophic graph but* $\gamma_v(G) = \gamma_v(G)_T = \gamma_v(G)_I = \gamma_v(G)_F = p$*.*

Proposition 3.11. *Let* $G = (A, B)$ *be the complete bipartite neutrosophic graph on a given set* V *such that there is exactly one path between two given vertices, which has*

a. *T-strength. Then* $\gamma_v(G)_T$ *is either* $T_A(u) + 1$, $u \in V$ *or* $\min_{u\in V_1, v\in V_2}(T_A(u) + T_A(v)) + 2$.

b. *I-strength. Then* $\gamma_v(G)_I$ *is either* $I_A(u) + 1$, $u \in V$ *or* $\min_{u\in V_1, v\in V_2}(I_A(u) + I_A(v)) + 2$.

c. *F-strength. Then* $\gamma_v(G)_F$ *is either* $F_A(u) + 1$, $u \in V$ *or* $\min_{u\in V_1, v\in V_2}(F_A(u) + F_A(v)) + 2$.

d. *strength. Then* $\gamma_v(G)_T$ *is either* $\min(T_A(u), I_A(u), F_A(u)) + 1$, $u \in V$ *or* $\min_{u\in V_1, v\in V_2}(T_A(u) + T_A(v), I_A(u) + I_A(v), F_A(u) + F_A(v)) + 2$.

Proof. (a). Let $G=(A,B)$ be the complete bipartite neutrosophic graph on a given set V such that there is exactly one path which has T-strength between two given vertices. By analogues to the proof of Theorem 3.8, all the edges are T-effective.
If G be the star neutrosophic graph with $V=\{u,v_1,v_2,\cdots,v_n\}$ such that u and v_i are the center and the leaves of G, for $1\leq i\leq n$, respectively. Then $\{u\}$ is the T-nikfar dominating set of G. Hence $\gamma_v(G)_T=T_A(u)+1$.
Otherwise, both of V_1 and V_2 include more than one vertex. Every vertex in V_1 is dominated by every vertices in V_2, as T-effective and conversely. Hence in G, the T-effective dominating sets are V_1 and V_2 and any set containing 2 vertices, one in V_1 and other in V_2. So $\gamma_v(G)_T=\min_{u\in V_1,v\in V_2}(T_A(u)+T_A(v))+2$. The result follows.
By analogues to the proof of (a) and Definition 3.7, the result is obviously hold for (b), (c), and (d). □

Proposition 3.12. *Let $G=(A,B)$ be a neutrosophic graph on a given set V and $xy\in E$. xy is a*

a. *T-effective edge if and only if xy is a T-bridge.*

b. *I-effective edge if and only if xy is a I-bridge.*

c. *F-effective edge if and only if xy is a F-bridge.*

d. *effective edge if and only if xy is a bridge.*

Proof. (a). Let $G=(A,B)$ be a neutrosophic graph on a given set V and $xy\in E$.
Suppose xy is a T-effective edge. By Definition 3.5(a), $T_B(xy)>\mu^{\infty}_{G-\{xy\}}(x,y)_T$. So $T_B(xy)=\mu^{\infty}_{G}(x,y)_T$. Therefore $\mu^{\infty}_{G}(x,y)_T>\mu^{\infty}_{G-\{xy\}}(x,y)_T$. It means xy is a bridge.
Suppose xy is a bridge. So $\mu^{\infty}_{G}(x,y)_T>\mu^{\infty}_{G-\{xy\}}(x,y)_T$. Hence $T_B(xy)=\mu^{\infty}_{G}(x,y)_T$. By $\mu^{\infty}_{G}(x,y)_T>\mu^{\infty}_{G-\{xy\}}(x,y)_T$ and $T_B(xy)=\mu^{\infty}_{G}(x,y)_T$, $T_B(xy)>\mu^{\infty}_{G-\{xy\}}(x,y)_T$. By Definition 3.5(a), it means xy is a T-effective edge.
Therefore the result follows.
By analogues to the proof of (a) and Definition 3.7, the result is obviously hold for (b), (c), and (d). □

Proposition 3.13. *Let $G=(\sigma,\mu)$ be a tree on a given set V. Then the edges of $F=(\tau,\nu)$ are just*

a. *the T-bridges, I-bridges, F-bridges, and bridges of G.*

b. *the T-effective, I-effective, F-effective, and effective edges of G.*

c. *constructed from the vertices of $T-$effective, $I-$effective, $F-$effective, and effective dominating sets in G. Hence $\gamma_v(G)_T=\gamma_v(F)_T,\gamma_v(G)_I=\gamma_v(F)_I,\gamma_v(G)_F=\gamma_v(F)_F$, and $\gamma_v(G)=\gamma_v(F)$.*

Proof. (a). Suppose that xy is an edge in F. If it were not a T-bridge, we would have a T-path P from x to y, not involving xy, of strength greater than $T_B(xy)$. By being special spanning neutrosophic graph F, P must involve edges not in F. Let u_1v_1 be an edge from P, which don't belong to F. u_1v_1 can be replaced by a T-path P_1 in F of strength than $T_B(uv)$. P_1 cannot involve xy. So by replacing each edge u_iv_i from P, which don't belong to F, by P_i, we can construct a T-path in F from x to y that does not involve xy. But G is T-acyclic. This is a contradiction. The latter of the proof is obvious. Therefore the result follows.
By Proposition 3.12(a), and (a), the result is obviously hold for (b).
By Definition 3.7(a), and (b), the result holds obviously for (c). □

Proposition 3.14. *For any neutrosophic graph $G=(A,B)$ on a given set V, we have*

a. $\gamma_v(G),\gamma_v(G)_T,\gamma_v(G)_I,\gamma_v(G)_F\leq p.$

b. $\gamma_v(G)+\gamma_v(G),\gamma_v(G)_T+\gamma_v(G)_T,\gamma_v(G)_I+\gamma_v(G)_I,\gamma_v(G)_F+\gamma_v(G)_F\leq 2p.$

Let us remind you consider p as the order of this graph.

Proof. (a). By Proposition 3.9, there is a neutrosophic graph $G = (A, B)$ such that $\gamma_v(G)_T = \gamma_v(G)_I = \gamma_v(G)_F = \gamma_v(G) = p$. So the result follows.
(b). By implementing (a) on G and $\bar{G}$, the result is obviously hold. □

4 Conclusion

The concept of neutrosophy are used as the framework in algebraic structures and fuzzy models. There are three kinds of neutrosophic graphs. As it mentioned, we chose one kind of them as the framework. In this paper, we introduce the new tool in new-born fuzzy model for analyzing its structure. In future, we would explore other elements of this fuzzy model, e. g. binary operations, unary operations and like stuff by this tool. It's extremely effective to use other tools like coloring and relations between them. It might be our future work. Also, we would like introducing neutrosophic structures along with their properties.

5 Compliance with ethical standards

Conflict of interest: The authors declare that there is no conflict of interest.

Human and animal rights: This article does not contain any studies with human participants or animals performed by any of the authors.

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