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The Arch Truss Girder again.—More upon the New System.

By S. W. Robinson, C.E., Detroit, Michigan.

My article in the March Number of your Journal, contained in its latter portion the statement, *without the proof*—of a few principles and formulæ as applied to my system of the "*Arch Truss Girder,*" where the curve of the arched chord is such,—the ties all being normal to it,—that the stress is uniform throughout the cord for a uniform horizontal loading throughout. The stress of the diagonals in the case there considered was compression.

That may be well where the girder is executed in wood; but if iron be employed it may be desirable that the diagonals resist tension. The origin was also taken at the crown.

I have found it more convenient to reckon the number of the pannel, or bay from one end.

In this article I propose to consider the case where the diagonals suffer tension; and to give rigorous analytical proof of every principle there stated, or here employed: and also, to reckon the number of pannel from the end of the truss.

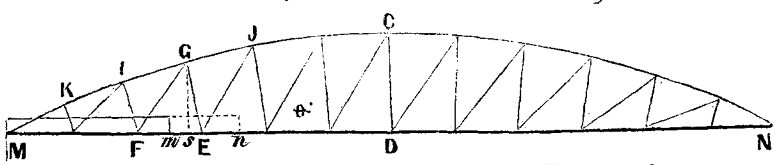
Dimensions of the Parts.

The formulæ given before would apply here; the only modification necessary being that required for reckoning the number of bay from the end.

Using the same notation as before, and taking the origin of co-ordinates at the point M in the Figure, we have

$x' = B - x$, and $y' = a - y$; or $x = B - x'$, and $y = a - y'$; which substituted in the expressions given before for the co-ordinates of K, I, G, &c., in the Figure, and dropping the accents give

$$\left.\begin{aligned} y_n &= \cos.\ i_n \left\{ a - \frac{B^2}{4a} \sin.^2 i_n \right\} \\ x_n &= B - \sin.\ i_n \left\{ \frac{B^2}{2a} + a - \frac{B^2}{4a} \sin.^2 i_n \right\} \end{aligned}\right\} \quad (1)$$



When FIGE is the n[th] pannel $n = 3$; the co-ordinates of G are x_3 and y_3; $GE = l_3$, $GF = L_3$, $GI = C_3$, $ME = b_3$,

$Mm = b'_3 = \frac{b_3 + b_2}{2} = \frac{b_n + b_{n-1}}{2} = b'_n$ and the angle at $G = i_3$ the angle of GI with $ME = i'_3$.

The angle of i_o at M is given by the formula.

$$b = \frac{B^2}{2a} \sin.\ i. \quad (2)$$

of the last article by making $b = B$.

$$\therefore \quad \sin.\ i_o = \frac{2a}{B}, \quad (3)$$

Hence for any angle i_n we have

$$i_n = i_o - \frac{n i_o}{N} = \frac{i_o}{N}(N - n). \quad (4)$$

Equation (2) gives the distance from the middle to any point.

$\therefore b_1 = B - b$ or

$$b_n = B - \frac{B^2}{2a} \sin.\ i_n \quad (5)$$

for the distances MF, ME, &c.

The expression for l as before is

$$l_n = a - \frac{B^2}{4a} \sin.^2 i_n. \quad (6)$$

The length of the diagonal FG is

$$L_n = \frac{y_n}{\sin.\ \theta_n} = \left((x_n - b_{n-1})^2 + y_n^2\right)^{\frac{1}{2}} \quad (7)$$

in which $\tan g.\ \theta_n = \frac{y_n}{x_n - b_{n-1}}$. (8)

The length of any part of the arched chord is

$$c_n = (x_n - x_{n-1}) \text{ sec. } i'_n,$$

in which $i'_n = \frac{i_n + i_{n-1}}{2}$.

Stress of the Parts.

The various parts of a truss built for any purpose, should be so proportioned as to withstand the greatest strain ever liable to come upon them. It is for this maximum stress that I have designed these formulæ for the truss.

If no other except a uniform loading throughout should ever come upon the arch truss, no diagonals would be necessary, nor would any compression ever occur upon the normals. This is the case theoretically, with the roof, but practically it is not. Snow and ice are liable to gather upon one side. This case would be represented more approximately by a load over part of it. And applied to railroad bridging the strength of each part is severely tested. Here the load may be regarded as uniform; and a long train moving over extends from the end upon which it enters, at different instances, to various parts along the roadway until we have the case of a uniform load throughout.

The stress of the diagonals being zero before the load enters, and zero after it entirely overreaches the bridge, and not being zero for a partial load, it is plain that each diagonal has a maximum as this load passes along.

I shall attempt to prove that the diagonal has its maximum as the end of the load reaches the middle of the bay to which the diagonal considered belongs: and that the chords have their maximum when the load extends entirely over.

Throughout this article in speaking of a loading, the uniform load as far as it extends is always referred to.

Now let the maximum stress of the upper or lower chord, at any points, as G or *s*, be required. Pass a plain through G, perpendicular to the straight chord. The two parts of the truss GEN and GFM being perfectly ridged may be regarded as single pieces. Then, according to principles demonstrated in "mechanics," there will exist in the upper chord a horizontal force equal and opposite to that in the lower chord, (since all the external forces are vertical) which will be the sum of the moments of the external forces taken upon either side of this section.

If the load extend, as represented in the Figure, a distance x beyond this section the sum of moments will be.

$$(\text{B}w + \text{V}) \text{ N}s - \text{W}x \frac{x}{2} = \text{H G}s.$$

Also by moments

$$\text{V} \cdot 2\text{B} = \text{W} (\text{M}s + x) \frac{\text{M}s + x}{2},$$

in which W = the weight per unit of the load,

w = the weight per unit of the truss,

and V = that portion of the vertical force at N resulting from the partial load; which substituted gives

$$H = \left(Bw + \frac{W}{4B}(Ms + x)^2\right)(2B - Ms)\frac{1}{Gs} - \frac{Wx^2}{2Gs} = \text{max.} \qquad (10)$$

Placing the first differential co-efficient equal zero the first condition for a maximum gives $x = 2B - Ms$.

The second differential co-efficient $= - Ms$, which satisfies the second condition; that is, for a maximum of either chord the load must extend entirely over the truss. Hence

For the maximum strain of the upper chord we will have the expression $W\frac{B^2}{2a}$ or,

$$T = (W + w)\frac{B^2}{2a} \quad . \quad . \quad . \quad . \qquad (11)$$

This is also equal the maximum of the straight chord at the middle, but the ties diminish it toward the ends as given in the last article.

Hence since $b'_1 = B - b'$ we will have for

Maximum strain of the straight chord

$$T = \frac{(W + w)}{2a}\left(B^4 - 4a^2(B - b^1)^2\right)^{\frac{1}{2}}. \quad . \quad . \qquad (12)$$

Tensile strain of the normals.

It is plain that the weight of the truss affects only the chords and normals, since from a uniform load there results no strain upon the diagonals.

The total tensile strain upon the normals from the middle to one end was found to be $P = \frac{wB^2}{2a}\, i_0$.

Hence the strain upon each for this case will be

$$\frac{P}{N} = (W + f\,w)\frac{B^2}{2Na}\, i_0 = p, \quad . \quad . \quad . \qquad (13)$$

in which f is a fraction expressing what part of the weight of the structure is contained in the roadway and straight chords. The value of p will be at its maximum when the load is entirely throughout, for the stress of the diagonals, being tension, tends toward diminishing that of the normals.

Stress of the diagonals.

The stress of the diagonals each have their maximum as the load in passing over, arrives at the middle of the bay considered. This strain being a function of the horizontal and vertical forces will have its maximum when the sum of the components of these forces in the diagonal have their maximum.

Let t_1 and t_2 represent these two components resulting from H and V respectively, h_1, h_2 the components in the chord.

Then, (referring to the figure,) the maximum of GF is for the load M m, upon the hypothesis assumed. If it be otherwise, let the load extend beyond m, a distance x. The expression for H is given in (10) by making $Ms = Mm = b'$, and neglecting w. This has its maximum for a load throughout, as has been shown. But the vertical force at the point considered is equal the vertical force at N, diminished by the opposite vertical force resulting from the load between m and M, or the transverse shearing at m.

Hence we will have

$$V = \frac{W}{4B}(b'+x)^2 - Wx = \frac{b'}{4B} + \frac{x}{4B}\Big(x - 2\,(2B - b)\Big). \qquad (14)$$

which has its maximum for $x = 0$, V being at its maximum for a load only to the point considered, the sum of the components from V and H may have their maximum at this point.

Resolving the forces in the diagonal GF, and the part of the chord GI which together resist H and V we get,

$$h_1 \cos. i'_n - t_1 \cos. \theta_n = H_n,\ h_1 \sin. i'_n = t_1 \sin. \theta\ ,$$
$$\text{and} - h_2 \sin. i'_n + t_2 \sin. \theta_n = V_n,\ h_2 \cos. i'_n = t_2 \cos. \theta_n,$$

from which

$$t_1 = \frac{H \text{ tang. } i'}{\sin. \theta - \cos. \theta \text{ tang. } i'}, \text{ and } t_2 = \frac{V}{\sin. \theta - \cos. \theta \text{ tang. } i'}. \qquad (15)$$

For a maximum of $t = t^1 + t_2$ we may omit the common denominator, and the expression that will give the maximum, will be $V + H \text{ tang. } i$; or, substituting H and V as given above we will have

$$\left\{\frac{W}{4By}(2B - b')(b+x)^2 - \frac{Wx^2}{2y}\right\} \text{tang. } i + \frac{W}{4B}(b'+x)^2 - Wx = \text{max.}$$

This may be transformed to

$$\frac{b'^2}{4By}\left\{(2B - b') \text{ tang. } i + y\right\} - \frac{x}{4By}(b' \text{ tang. } i - y)\,(x - 2(2B - b')) \quad (16)$$

This will have its maximum when $x = 0$. For x being the only variable in the case, the first term is constant, and in the second term b' tang. $i < y$, and $x < 2B - b'$ which makes the second term negative for any applicable positive values of x. I think no formula is necessary to show that t is no greater for a shorter load than b', since both H and V increase to the section considered;

That is, for a maximum stress of the diagonal, the loading must extend to the middle of the bay considered, and no farther.

The expression (16) does not hold for x negative, which corresponds to a shorter load than b'. For this case the moments must be taken upon the other side of the section. The conditions for a maximum from the differential calculus fail here.

The condition $\frac{dy}{d} = 0 =$ tangent, is satisfied only where an infinitessimal element of the arc constructed from $y=f(x)$ is horizontal. In the case above considered where $t=f(x)$ the ordinate t continues to increase from N to the section considered, and beyond. Consequently the tangent does not equal zero at this point. But there are points where it equals zero which would be indicated by the condition $\frac{dy}{dx}=0$.

If the curve of the arched chord have any form whatever, concave toward the straight one; or if both are straight and parallel or not, the expression (16) holds equally true. For the extraordinary case in which the curve is convex toward the straight chord, b' tangent $i>y$, and the load must extend beyond the bay considered.

If the diagonal resist compression, the expression b' tangent $i<y$, is still true: or, the conditions for a maximum of t remain unchanged.

Therefore, the maximum stress of the diagonals for every *practical* case above mentioned, will be

$$t_1+t_2=t=\frac{\mathrm{V}_n+\mathrm{H}_n \text{ tang. } i'_n}{\text{sin. }\theta_n-\text{cos. }\theta_n \text{ tang. } i'_n} \quad . \quad . \quad (17)$$

The weight of the truss having no effect upon the diagonals, the expression for H and V, to be used in this formula are those resulting from (10) and (14) by neglecting w and making $x=0$. Or

$$\mathrm{V}_n=\frac{\mathrm{W}\,b'^2_n}{4\mathrm{B}} \text{ and } \mathrm{H}_n=\frac{2\mathrm{B}-x_n}{y_n}\mathrm{V}. \quad . \quad . \quad (18)$$

Compression of the Normals.

The maximum compression of a normal occurs when the load arrives to the centre of the bay considered. Thus for the normal GE, the load extends to m: for FGJE may be regarded as a pannel, and the case comes under one already discussed above. Hence calling C the required compression and noticing that GE and GF resist H and V, the resolved forces become

$$c_n \text{ cos. } i'_n - h \text{ sin. } i'_{n+1}=\mathrm{V}_n$$
$$c_n \text{ sin. } i'_n + h \text{ cos. } i'_{n+1}=\mathrm{H}_n$$

from which

$$\mathrm{C}_n=\frac{\mathrm{V}_n+\mathrm{H}_n \text{ tang. } i'_{n+1}}{\text{cos. } i_n+\text{sin. } i_n \text{ tang. } i'_{n+1}}. \quad . \quad . \quad (19)$$

In all these formulæ the algebraic sign of the trigonometrical function of i must be observed.

These principles, for all kinds of truss bridges have been fully developed, and will probably soon appear before the public in a thorough and excellent work upon "Engineering" about to be issued by De Volson Wood,—Professor of Civil Engineering, University of Michigan.—Reference need only be had to his numerous articles published in our journals to insure all of the original, and genuine character of the work.

By means of a model constructed expressly for the purpose, he has been able to prove the results of his investigation of the truss by actual experiment, and to bring into the clearest light many errors existing in works now constantly referred to by the "*practical engineer.*"

Comparison of the new, and the ordinary parabolic systems in numerical results.

In order to show more clearly what I have obtained by these investigations upon the truss, I have computed by the preceding formulæ the value of the stresses of some of the principle parts for the two systems, which are tabulated below.

The quantities computed for the arch truss with parabolic arc, are the maximum values according to the same principles by which the new system has been treated.

The stresses of the diagonals acting as either ties or braces have been calculated for both systems. The example I have taken is for $2B = 160$, $=$ span; $a = 20 =$ depth of truss; $2N = 12 =$ whole number of bays; $w = 0$; $W = 1$; the same for both systems.

From this data I get for the compression of the arched chord throughout for this system, . .	160
The same at the middle of lower chord.	
At the end bay of lower chord, . . .	141·965
For parabolic system, arched chord at the crown, .	160
At last bay, ...	178·886
Lower chord throughout, . . .	160

THE DIAGONALS, AS TIES OR BRACES.

Bay.	New System. as ties.	New System. as braces.	Parabolic System. as ties.	Parabolic System. as braces.
2	16,480	20,220	19,524	16.500
3	20,222	21,919	20,905	18,078
4	21,111	23,786	22,686	20,488
5	23,247	24,926	23,864	22,502
6	24,615	25,183	23,811	23,766
7	25,098	24,532	23,710	24,185
8	24,650	22,986	22,322	23,673
9	23,245	20,617	20,140	22,302
10	20,893	19,227	17,402	20,125
11	17,686	14,286	14,700	17,395

It appears from these that the greatest stresses upon the diagonals are for those of the new system. The difference of the sums total of the two systems as ties is 8·183, and as braces 8·668. But this distributed among the ten diagonals are the hypothesis that an equal amount belongs to each, gives about 0·8. Or, the mean of the stresses being about 20, the increase of stress is about $\frac{4}{100}$, some more and some less. But at the same time there is a diminution in the total lengths of all the diagonals of about $\frac{1}{400}$.

But there is also an increase of stress in the normals, over the vertical ties of the parabolic form, of about $\frac{1}{20}$: but at the same time there is a gain in the length of about $\frac{1}{48}$ in favor of the new system.

Although the arguments founded upon the diagonals and normals against the new system slightly outweigh those in favor of it, still the great decrease of stress in the arched and straight chords must greatly overbalance them all. The diminution of stress for the arched chords near the abutments as may be seen above is 18·886, and in the straight chords 18·035, or a saving of about $\frac{1}{9}$ of the stress, or of area of section near the ends of the truss. These quantities vary with a or B.

If we consider the weight of the truss and make w nearly equal W, the amount of stress for the diagonals remains unchanged, *while the saving of stress to the chords is nearly doubled.*

In the "Howe's Truss" the stress of the chords diminish toward the ends. At the same time that of the diagonals is greatly augmented while the length remains the same. Probably this alone is sufficient to condemn the system for iron bridging. I have observed that iron bridges, under the arch truss system with parabolic arc, are quite common. But as iron gains the ascendency over wood as a building material for bridges,—*why is not this system with normal ties destined to become adopted as one of the most common forms?*

In the preceding part of my last article I stated, where integrating for expressions for có-ordinates to the curve, that I had not attempted the complete integration of the expression as a function of x and y only. I have since received a communication containing an algebraic solution of my final equations of $y=f(i)$ and $x=f(i)$ from Mr. E. P. Austin, Astronomer, formerly Assistant Astronomer to U.S. Lake Survey; he proceeded by transferring the origin of co-ordinates to the centre of the curve, transforming the trigonometrical functions into those of the cosine, placing the expressions $=0$, and eliminating by dividing one by the other repeatedly, until i disappeared. He gave the following for the form of the function,

$$a(x^2+y^2)^3+(b+cy^2)(x^2+y^2)^2+(d+ey^2+fy^4)(x^2+y^2)+gy^4+hy^2+i=0,$$

in which the quantities a,b,c, &c., are functions of a and ρ_0

This, he adds, is an equation of a curve which has a centre, because all the powers of the variables are of an even degree.

ERRATA TO LAST ARTICLE.

For third equation after (17) on p. 155, read $T'=\frac{WB}{2a}\left((1+\frac{4y^2}{x^2}\right)^{\frac{1}{2}}$

Equation (18) on p. 156, for sin. $\frac{-12a}{B}$ read, sin.$^{-1}\frac{2a}{B}$.

Result of Experiments on the Breaking Weight of Rolled Iron.

From the London Builder, No. 1110.

[From the paper by Mr. F. A. Paget, C. E.]

When we remember that the very best wrought iron of commerce is, to use the words of the well-known metallurgist, Saint-Claire Deville, but a metallic sponge, like platinum, the pores of which have been simply closed up by pressure or percussion; that, in one word, ordinary wrought iron has never as wrought iron, been fused, it will be seen that the uncertainties qualifying the material itself are still greater. Mr. Mallet thus found that, while the original hammered