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INTERMODAL ENTANGLEMENT IN THREE-MODE OPTOMECHANICS

Kousik Mukherjee*, ** & Paresh Chandra Jana*

* Department of Physics and Technophysics, Vidyasagar University, Midnapore, West Bengal

** Department of Physics, Government General Degree College, Gopiballavpur, West Bengal

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Abstract:

We explore the possibility of intermodal entanglement between two optical modes and a mechanical mode for three-mode optomechanics. The Hamiltonian of the system is solved analytically, using Heisenberg-Langevin equation and also Taylor's series expansion of an operator. Temporal variation of entanglement parameter corresponds to different field modes are studied under different weight factor of initial state and dependence on interaction strength is also reported. Entangled state at macroscopic level is observed.

Key Words: Optomechanics & Entanglement

Introduction:

The field of optomechanics has been emerged over last decade. Optomechanics is explored the nonlinear interaction between optical cavity field and mechanical resonator, via radiation pressure. It also lies in the interface of macroscopic and quantum level. Different interesting achievements have been realized such as weak classical force detection [1], optomechanically induced transparency [2], squeezed light generation [3], normal mode splitting [4], state conversion [5], optomechanical microwave sensor [6]. There are different types of optomechanical system (OMS) which have fundamental insights and these have also opened a new window to study different nonclassical effects. Different OMS are such as double-cavity optomechanical system [7], hybrid spin-optomechanical system [8], graphene based optomechanics [9], superconducting microwave cavity [10], cavity with micromechanical membrane [11]. Motivated by these, we are interested to study nonclassical dynamics of a three-mode optomechanics.

Entanglement is a useful and key resource for quantum information processing and quantum computing. This nonclassical effect has potential application in quantum telecloning, quantum metrology, quantum dense coding [12-14]. Study of entangled properties in various aspects has already been reported in different optical and optomechanical systems [15-18]. Keeping these facts in mind we have investigated the possibility of intermodal entanglement in three-mode optomechanical system.

The enhancement of nonlinearity may enhance the degree of nonclassicality, has been proposed in previous studies in context of quantum memory [19], mechanically induced photon antibunching [20]. The three-mode system with two optical modes may enhance nonlinearity, already discussed in references [21-22]. The previous three-mode studies were based on sub-Poissonian phonon lasing [23]. In this article we pay our attention to study intermodal entanglement in present system. In next section of the article we present model Hamiltonian of the system and operator based analytical solutions with results. At last, we give conclusions.

Theoretical Model:

We consider an optomechanical system consists with two optical mode coupled via a mechanical mode. One of the cavities is driven by a laser field [23]. The system Hamiltonian (in a rotating frame) contains four parts. First part describe Hamiltonian of mechanical system with mechanical frequency ω_m. Second part corresponds to the cavity field mode a, which is detuned w.r.t. another cavity field mode b by frequency Δ. Third one represents interaction between two cavity field modes and mechanical mode having interaction strength g. Last part corresponds to driving term with driving strength Ω.

$$H = \omega_m c^\dagger c - \Delta\, a^\dagger a + g(a\, b^\dagger c + a^\dagger\, bc^\dagger) + i\Omega(b^\dagger - b) \tag{1}$$

$a(a^\dagger)$, $b(b^\dagger)$ and $c(c^\dagger)$ are the lowering (raising) operators for two cavity field modes and mechanical modes, respectively. Such type of system and interaction has been discussed in previous studies [23-26].

Solutions:

In this section we present operator based analytical solution of the model Hamiltonian. We include cavity decay rate (k) and mechanical damping rate (γ) correspond to the respective cavity systems and mechanical system. In absence of driving strength and excluding noise term, the Heisenberg-Langevin equations for different field modes are

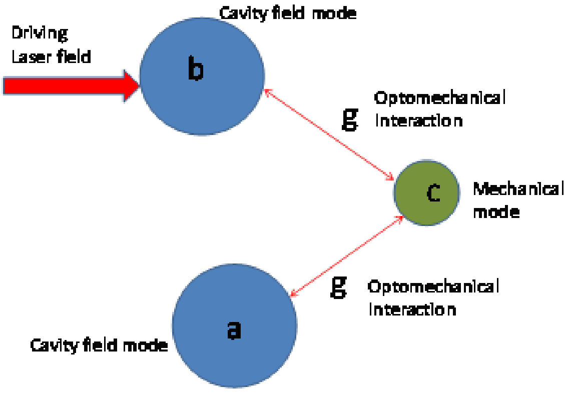


Figure 1: Schematic Diagram of the system

$$\dot{a} = -i\left[-\left(\Delta + \frac{ik}{2}\right)a + gbc^{\dagger}\right]$$

$$\dot{b} = -i\left[-\frac{ik}{2}b + gac\right]$$

$$\dot{c} = -i\left[-\left(\omega_m - \frac{i\gamma}{2}\right)c + ga^{\dagger}b\right] \qquad (2)$$

To solve above equations we use Taylor's series expansion of an operator. We assume solutions for different field modes are

$$a(t) = a(0)f_1 + b(0)c^{\dagger}(0)f_2 + a(0)c^{\dagger}(0)c(0)f_3 + a(0)b^{\dagger}(0)b(0)f_4$$

$$b(t) = b(0)h_1 + a(0)c(0)h_2 + b(0)c^{\dagger}(0)c(0)h_3 + a^{\dagger}(0)a(0)b(0)h_4$$

$$c(t) = c(0)l_1 + a^{\dagger}(0)b(0)l_2 + a^{\dagger}(0)a(0)c(0)l_3 + b^{\dagger}(0)b(0)c(0)l_4 \qquad (3)$$

Where f_i , h_i and l_i $(i = 1, \ldots, 4)$ are time dependent coefficient and also depend on detuning, coupling strength, cavity decay rate, mechanical frequency, and mechanical damping rate. These are found out from initial boundary condition $f_1 = h_1 = l_1 = 1$ and $f_i = h_i = l_i = 0$ for $i = 2,3,4$. These are as follows:

$$f_1 = \exp\left(i\Delta - \frac{k}{2}\right)t$$

$$f_2 = \frac{igf_1}{\left(i\omega_m - i\Delta - \frac{\gamma}{2}\right)}\left[1 - \exp\left(i\omega_m - i\Delta - \frac{\gamma}{2}\right)t\right]$$

$$f_3 = -\frac{g^2 f_1}{\left(i\omega_m + \frac{\gamma}{2}\right)}\left[\frac{\exp\left(i\omega_m - i\Delta - \frac{\gamma}{2}\right)t}{\left(i\omega_m - i\Delta - \frac{\gamma}{2}\right)} + \frac{\exp(-i\Delta - \gamma)t}{(i\Delta + \gamma)}\right] + \frac{g^2 f_1}{\left(i\omega_m - i\Delta - \frac{\gamma}{2}\right)(i\Delta + \gamma)}$$

$$f_4 = \frac{g^2 f_1}{\left(i\omega_m + k - i\Delta - \frac{\gamma}{2}\right)}\left[\frac{\exp(-kt)}{k} - \frac{\exp\left(i\omega_m - i\Delta - \frac{\gamma}{2}\right)t}{\left(i\omega_m - i\Delta - \frac{\gamma}{2}\right)}\right] - \frac{g^2 f_1}{k\left(i\omega_m - i\Delta - \frac{\gamma}{2}\right)}$$

$$h_1 = \exp\left(-\frac{k}{2}t\right)$$

$$h_2 = -\frac{igh_1}{\left(i\omega_m + \frac{\gamma}{2}\right)}\left[1 - \exp\left(-i\omega_m - \frac{\gamma}{2}\right)t\right]$$

$$h_3 = \frac{g^2 h_1}{\left(i\omega_m - i\Delta - \frac{\gamma}{2}\right)}\left[\frac{\exp\left(i\Delta - i\omega_m - \frac{\gamma}{2}\right)t}{\left(i\Delta - i\omega_m - \frac{\gamma}{2}\right)} + \frac{\exp(-\gamma t)}{\gamma}\right] + \frac{g^2 h_1}{\gamma\left(i\Delta - i\omega_m - \frac{\gamma}{2}\right)}$$

$$h_4 = \frac{g^2 h_1}{\left(i\omega_m - k - i\Delta + \frac{\gamma}{2}\right)}\left[\frac{\exp(-kt)}{k} + \frac{\exp\left(i\Delta - i\omega_m - \frac{\gamma}{2}\right)t}{\left(i\Delta - i\omega_m - \frac{\gamma}{2}\right)}\right] + \frac{g^2 h_1}{k\left(i\Delta - i\omega_m - \frac{\gamma}{2}\right)}$$

$$l_1 = \exp\left(-i\omega_m - \frac{\gamma}{2}\right)t$$

$$l_2 = \frac{igl_1}{\left(i\omega_m - i\Delta - k + \frac{\gamma}{2}\right)}\left[1 - \exp\left(i\omega_m - i\Delta - k + \frac{\gamma}{2}\right)t\right]$$

$$l_3 = \frac{g^2 l_1}{\left(i\omega_m + \frac{\gamma}{2}\right)}\left[\frac{\exp\left(i\omega_m - i\Delta - k + \frac{\gamma}{2}\right)t}{\left(i\omega_m - i\Delta - k + \frac{\gamma}{2}\right)} + \frac{\exp(-i\Delta - k)t}{(i\Delta + k)}\right] - \frac{g^2 l_1}{\left(i\omega_m - i\Delta - k + \frac{\gamma}{2}\right)(i\Delta + k)}$$

$$l_4 = \frac{g^2 l_1}{\left(i\omega_m - i\Delta + \frac{\gamma}{2}\right)}\left[\frac{\exp\left(i\omega_m - i\Delta - k + \frac{\gamma}{2}\right)t}{\left(i\omega_m - i\Delta - k + \frac{\gamma}{2}\right)} + \frac{\exp(-kt)}{k}\right] - \frac{g^2 l_1}{k\left(i\omega_m - i\Delta - k + \frac{\gamma}{2}\right)} \quad (4)$$

In order to calculate entanglement parameters, we assume that both cavity modes and mechanical modes are initially at coherent state. So, initial state is the tensor products of three states $|\alpha\rangle \otimes |\beta\rangle \otimes |\delta\rangle$ where $|\alpha\rangle$, $|\beta\rangle$ and $|\delta\rangle$ are the eigenket of field operators a, b and c respectively. The operator $a(t)$ operates on the product state, giving complex eigenvalue α. We used this method in our previous studies [15-16, 27-28].

Entanglement Dynamics:

To find out possibility of existence of intermodal entanglement in three-mode optomechanical system we use Hillery-Zubairy criteria [29], which is sufficient for characterization of inseparable state. The criteria expressed in-terms of expectation values of moment of field operators as $|\langle ab\rangle|^2 > [\langle N_a\rangle\langle N_b\rangle]$. We define this criterion as

$$\varepsilon_{ab} = \langle N_a\rangle\langle N_b\rangle - |\langle ab\rangle|^2 \quad (5)$$

For entangled state $\varepsilon_{ab} < 0$.

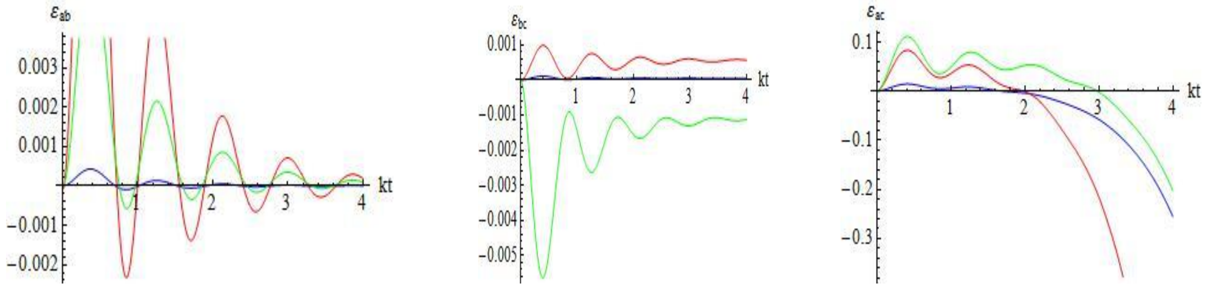
Using above criterion and solutions in equation (3) we obtain ε_{ab}, ε_{ac} and ε_{bc} for different inter-modes.

$$\varepsilon_{ab} = |\beta^2|^2|f_2|^2 - (|\alpha|^2|\beta|^2 h_4 h_1^* + c.c.)$$

$$\varepsilon_{bc} = |\beta^2|^2|l_2|^2 - (|\beta|^2|\delta|^2 l_4 l_1^* + |\beta|^2|\delta|^2 h_2^* h_1 l_2 l_1^* + c.c.)$$

$$\varepsilon_{ac} = |\beta|^2|\delta|^2|f_2|^2 - |\beta|^2|h_2|^2 - |\alpha|^2|\beta|^2|h_2|^2 - (|\alpha|^2|\delta|^2 l_3 l_1^* + \alpha^*\beta\delta^* l_2 l_1^* + |\beta|^2|\delta|^2 f_2^* f_1 l_2 l_1^* + c.c.) \quad (6)$$

As the expressions of equation (6) are not so simple, we plot these in figure 2(a-c), respectively. The negative portion of the graph gives the signature of entangled state between two cavity field modes or any cavity field mode and mechanical mode. Figure (a) represents variation of ε_{ab} with normalised time for different weight factor of the initial state and it is observed that ε_{ab} oscillates between classical and non-classical regions. But the envelope of the graph gradually decreases due to truncation of the higher order terms in calculation. Figure (b) represents variation of ε_{bc}, it is clearly evident from the study that the negativity of the entangled parameter depends on weight factor of mechanical mode i.e. δ. Variation ε_{ac} is shown in figure (c) and it is observed that its negativity arises at larger value of normalised time. It is clear from the variation that degree of entanglement for ac inter-mode is more pronounced as compared to other two inter-modes. It is also observed that degree of entanglement is also increases with coupling strength.



(a) (b) (c)

Figure 2: (Color online) Variation of entanglement parameter with rescaled time kt for $\Delta/k = 0.1$, $\omega_m/k = 7.36$, $g/k = 0.25$ with $|\alpha| = 2, |\beta| = 0.5, |\delta| = 0.2$ (Blue line) ; $|\alpha| = 2, |\beta| = 1, |\delta| = 0.2$ (Red line) ; $|\alpha| = 1$, $|\beta| = 1, |\delta| = 1$ (Green line) (a) ab mode (b) bc mode and (c) ac mode.

Here we use a set of experimental parameters with $k = 2\pi \times (20 - 500)$ MHz and $\omega_m = 2\pi \times 3.68$ GHz [30, 31].

Conclusions:

We have discussed about the possibility of intermodal entanglement between two cavity field mode and mechanical mode. The degree of entanglement depends on weight factor of the coherent states and also increases with optomechanical interaction strength between the cavity field and mechanical resonator. Degree of entanglement between un-driven cavity field mode and mechanical mode is more pronounced as compared to driven cavity field and mechanical mode. This study can be used for generation of entangled state at macroscopic scale.

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