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# Flow Shop Scheduling Problem in Neutrosophic Environment Using Trapezoidal Fuzzy Numbers

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**Abstract:** In this paper, we propose to solve the flow shop scheduling problem using Branch and Bound method. This paper presents an algorithm with the help of branch and bound approach for a flow shop scheduling problems consisting of 4 jobs and 4 machines in which the processing time as Neutrosophic trapezoidal fuzzy numbers which are further converted in to crisp value using graded mean ranking method. Numerical example is given to illustrate the study of solving flow shop scheduling under Neutrosophic trapezoidal fuzzy number using branch and bound technique.

**Keywords:** Neutrosophic Fuzzy Set, Trapezoidal Fuzzy Number, Neutrosophic Trapezoidal Fuzzy Number, Flow shop scheduling Problem.

## INTRODUCTION

Scheduling problems are common Occurrence in our real life. e.g. Programs to be run is a sequence at a computer centre, ordering of jobs for processing in a manufacturing plant. etc. The scheduling problem practically depends upon the important factors namely, transportation time, break down effect, etc. Flow shop scheduling is one of the most important decision making concept to arrange the task that is to be performed (or) processed in a machine in that particular order.

A geometrical model and a graphical algorithm for a sequencing problem were advanced by William. W. Hardgrave and George L.Nemhauser in 1963. These include a geometric approach a "Branch and Bound" approach, a combinatorial analysis approach, and an approximation approach.

This technique was developed by Little, et.al. All possible jobs are placed into the first sequence position and lower bounds on total time are calculated. In 1965, Zadeh [9] introduced the idea of fuzzy sets which gives best solution for impreciseness or vagueness. In 1986, K. Atanassov developed the concept of Intuitionistic Fuzzy Set (IFS) which is characterized by the membership degree and the non-membership degree.

In 1995, The new tool which is an Neutrosophic set theory was defined by Smarandache[7],to handle problems involving incomplete, indeterminate and inconsistent information, that cannot be solved with fuzzyset and Intuitionistic fuzzyset, Neutrosophic fuzzy set plays animportant role in handling imprecise condition.

In this paper, flow shop scheduling problems has been solved under Neutrosophic fuzzy environment. Here processing time is being taken in Neutrosophic trapezoidal fuzzy numbers and with the idea of Branch and bound technique we get the optimal solution to calculate total elapsed time.Numerical example is given to illustrate the study of solving flow shop scheduling under Neutrosophic trapezoidal fuzzy number using branch and bound technique

## PRELIMINARIES

### Definition

A **fuzzy set**$\widetilde{A}$ is defined by $\widetilde{A}$ = {(x, $\mu_{\widetilde{A}}$(x)): x∈A}. In the pair (x, $\mu_{\widetilde{A}}$(x)), the first element belong to the classical set A, the second element $\mu_{\widetilde{A}}$(x), belong to the interval [0, 1] is called the membership function.

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### Definition

An **Intuitionist fuzzy set** $\widetilde{A}^{I}$ is defined by $\widetilde{A}^{I}$ = {<{(x, $\mu_{\widetilde{A}^I}$(x), $\upsilon_{\widetilde{A}^I}$(x))>/$x \in X$}, where $\mu_{\widetilde{A}^I}$(x), $\upsilon_{\widetilde{A}^I}$(x): X→[0,1] are function such that 0≤ $\mu_{\widetilde{A}^I}$(x)+$\upsilon_{\widetilde{A}^I}$(x)≤ 1 for all x∈X. Here $\mu_{\widetilde{A}^I}$(x) represented as the degree of membership and $\upsilon_{\widetilde{A}^I}$(x) represented as the degree of non-membership of the element.

### Definition

A **Neutrosophic fuzzy set** $\widetilde{A}^{N}$ is defined as $\widetilde{A}^{N}$ **=**{x,$T_A$(x), $I_A$(x),$F_A$(x): $x \in$ X} where $T_A$(x), $I_A$(x),$F_A$(x): X→[0,1] and 0≤$T_A$(x)+ $I_A$(x)+$F_A$(x) ≤3 where $T_A$(x) is membership, $I_A$(x) is inderministic function and $F_A$(x) is non-deterministic function.

### Definition

**Fuzzy number** $\widetilde{A}$ is a fuzzy set on the real line $\Re$, must satisfy the following conditions.

(i) $\mu_{\widetilde{A}}(x_0)$ is piecewise continuous

(ii) There exist atleast one $x_0 \in \Re$ with $\mu_{\widetilde{A}}(x_0)$ =1

(iii) $\widetilde{A}$ must be normal & convex.

### Triangular Fuzzy Number

A fuzzy number $\tilde{\bar{A}} = (a_1, a_2, a_3)$ is said to be triangular fuzzy number if its membership function is given by,

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & for\, x < a_1 \\ \dfrac{(x-a_1)}{(a_2-a_1)} & for\, a_1 \le x \le a_2 \\ \dfrac{a_3-x}{(a_3-a_2)} & for\, a_2 \le x \le a_3 \\ 0 & for\, x > a_3 \end{cases}$$

where $a_1 \le a_2 \le a_3$ are real numbers.

### Trapezoidal Fuzzy Number

A fuzzy number $\widetilde{A} = (a_1, a_2, a_3, a_4)$ is said to be trapezoidal fuzzy number if its membership function is given by,

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4 \\ 0, & x > a_4 \end{cases}$$

where $a_1 \le a_2 \le a_3 \le a_4$ are real numbers.

### Definition

A **Neutrosophic trapezoidal fuzzy set** $\widetilde{A}^{N}$ in U is represented by

$$\widetilde{A}^{N} = \{< x: \overline{T}_{\widetilde{A}^N}(x), \overline{I}_{\widetilde{A}^N}(x), \overline{F}_{\widetilde{A}^N}(x) >; x \in U\}$$

$\overline{T}_{\widetilde{A}^N}(x), \overline{I}_{\widetilde{A}^N}(x), \overline{F}_{\widetilde{A}^N}(x): U \to M[0,1]$, where

$\overline{T}_{\widetilde{A}^N}(x)$ denote the truth membership value,

$\overline{I}_{\widetilde{A}^N}(x)$ denote the indeterminacy value and

$\overline{F}_{\widetilde{A}^N}(x)$ denote the falsity-membership value of $x \text{ in } \tilde{A}^N$ and for every $x \in U$.

Here $\overline{T}_{\widetilde{A}^N}(x) = (T^1_{\widetilde{A}^N}(x), T^2_{\widetilde{A}^N}(x), T^3_{\widetilde{A}^N}(x), T^4_{\widetilde{A}^N}(x))$ ; $\overline{I}_{\widetilde{A}^N}(x) = (I^1_{\widetilde{A}^N}(x), I^2_{\widetilde{A}^N}(x), I^3_{\widetilde{A}^N}(x), I^4_{\widetilde{A}^N}(x))$

$$\overline{F}_{\widetilde{A}^N}(x) = (F^1_{\widetilde{A}^N}(x), F^2_{\widetilde{A}^N}(x), F^3_{\widetilde{A}^N}(x), F^4_{\widetilde{A}^N}(x))$$

## RANKING FUNCTION OF TRAPEZOIDAL FUZZY NUMBER

An efficient approach for comparing the fuzzy number is by the use of a ranking function based on their graded means.

That is for every $\widetilde{A}$ = $(a_1, a_2, a_3, a_4)$ ∈ F(R), the ranking function R: F(R) →R by graded means is defined a function.

$$R(\tilde{A}) = \frac{(a_1+2a_{2+}2a_3 + a_4)}{6}$$

## BRANCH AND BOUND ALGORITHM

Branch and Bound is an algorithm design paradigm which is generally used for combinatorial optimization problems. It is a solution approach that can be applied to a number of different types of problems. This method uses a tree diagram of nodes and branches to organize the solution partitioning, it represent subsets of the solution set ,the branch is checked against lower estimated bounds of the optimal solution. It maintain provable lower bounds in global objective value.

Here let k be the level number in the branch tree and ∈ be the assignment in the current node of a branching tree. Assume root node be 0.

Let$p_{\in}^{k}$ be the assignment at level k of the branching tree and $v_{\in}$ be the lower bound of the partial assignment up to$p_{\in}^{k}$ such that

$$\boldsymbol{v}_{\in} = \sum_{\cancel{i,j \in x}} \boldsymbol{a}_{i,j} + \sum_{\cancel{i \in x}} \sum_{\cancel{j \in y}} \boldsymbol{min a}_{i,j}$$

Where $a_{i,j}$ is the processing time of the machine

## NUMERICAL EXAMPLE

We Consider 4 jobs, 4machine flow shop scheduling problem where processing time of the jobs are represented as Neutrosophic trapezoidal fuzzy numbers. Our objective is to obtain an optimal schedule and the total elapsed time. Using branch and bound technique.

| Machines/ jobs | Machine A | Machine B | Machine C | Machine D |
|---|---|---|---|---|
| $J_1$ | ((9,12,15,21); (10,13,16,22); (15,18,21,27)) | ((0,1,2,6); (1,4,7,7); (9,12,15,21)) | ((7,8,9,13); (5,1015,17); (13,16,19,25)) | ((16,19,22,28); (14,20,26,32); (18,21,24,30)) |
| $J_2$ | ((1,4,7,7); (12,15,18,24); (14,17,20,32) | ((2,5,8,14); (3,6,9,15); (8,11,14,14)) | ((0,1,2,6); (1,4,7,7); (9,12,15,21)) | ((10,13,16,22); (16,19,22,22); (19,22,25,31)) |
| $J_3$ | (7,10,13,19); (9,12,15,21); (13,16,19,19)) | ((2,3,4,8); (4,6,8,10); (8,11,14,20)) | ((1,2,3,7); (5,6,11,15); (14,20,26,32)) | ((10,13,16,22); (20,23,26,32); (21,24,27,33)) |
| $J_4$ | ((3,6,9,19); (4,7,10,16); (24,27,30,36) | ((3,5,7,9); (5,10,15,17); (11,13,19,21)) | ((7,8,9,13); (8,11,14,14); (11,13,19,21)) | ((22,25,28,34); (25,28,31,37); (29,32,35,41)) |

**Step (1)**converting the processing time of above Neutrosophictrapezoidal fuzzy number in to Neutrosophic number using graded mean ranking method.

| Machines/ jobs | Machine A | Machine B | Machine C | Machine D |
|---|---|---|---|---|
| $J_1$ | (14,15,20) | (2,5,14) | (9,12,18) | (21,23,23) |
| $J_2$ | (15,17,20) | (7,8,12) | (2,5,14) | (15,20,34) |
| $J_3$ | (12,14,17) | (4,7,13) | (3,9,23) | (15,25,26) |
| $J_4$ | (7,9,29) | (6,12,16) | (9,12,16) | (27,30,34) |

**Step (2)**Apply branch and bound method Compute the lower bound $p_{11}^{1}$

$$\boldsymbol{v}_{\in} = \sum_{\cancel{i,j \in x}} \boldsymbol{a}_{i,j} + \sum_{\cancel{i \in x}} \sum_{\cancel{j \in y}} \boldsymbol{min a}_{i,j}$$

Where $\in = (1,1), X = (2,3,4)\ and\ Y = (2,34)$

$$v_{11} = a_{11} + \sum_{X\in 2,3,4}\sum_{Y\in 2,3,4} mina_{i,j}$$

$$p_{11}^1=(14,15,20)+(2,5,14)+(4,7,13)+(6,12,16)=(26,39,63)$$
$$p_{21}^1=(5,17,20)+(2,5,14)+(4,7,13)+(6,12,16)=(17,41,63)$$
$$p_{31}^1=(12,14,17)+(2,5,14)+(2,5,14)+(6,12,16)=(22,36,61)$$
$$p_{41}^1=(7,9,29)+(2,5,14)+(2,5,14)+(4,7,13) = (15,26,70)$$

**Further branching:** Further branching is done from the terminal node which has the least lower bound .At this stage; the nodes $p_{11}^1, p_{21}^1, p_{31}^1, p_{41}^1$ are the terminal nodes .The node $p_{41}^1$ has the least lower bound. Hence, further branching is started from the node

Here $p_{41}$is minimum therefore eliminate fourth row and first column further branching is starting from this node .

$$v_{22} = a_{41} + a_{i2} + \sum_{X\in 3,4}\sum_{Y\in 3,4} mina_{i,j}$$

$$p_{12}^2 = (7,9,29)+(2,5,14)+(2,5,14)+(3,9,23\ ) = (14,28,80)$$
$$p_{22}^2 = (7,9,29)+(7,8,12)+(9,12,18)+(3,9,23)= (26,38,82)$$
$$p_{32}^2 = (7,9,29)+(4,7,13)+(9,12,18)+(2,5,14) = (22,33,74)$$

Here $p_{12}$ is minimum and therefore eliminate first row and second column

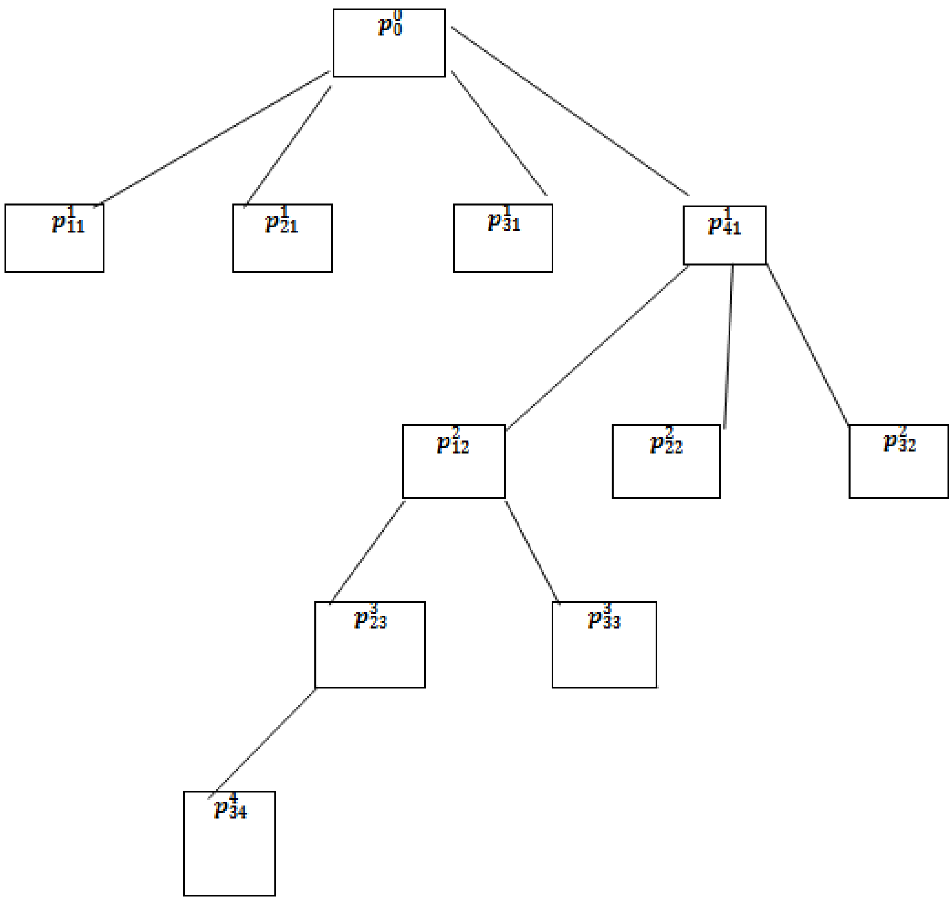
Further branching is starting from this node

$$v_{33} = a_{41} + a_{12} + a_{i3} + \sum_{i\in 4}\sum_{j\in 4} mina_{i,j}$$

$$p_{23}^3 = (7, 9,29) +(2,5,14) +(2,5,14)+(15,25,26) = (26,44,83)$$
$$p_{33}^3 = (7,9,29) +(2,5,14) +(3,9,23) +(15,20,24) = (27,43,90)$$

Here $p_{23}$ is minimum and eliminate third row and third column. Therefore remaining $a_{ij}$ is in $p_{34}$.We get the sequence as

$$p_{41} - p_{12} - p_{23} - p_{34}$$




From the lower bounds, we can determine that the optimal sequence will be taken as 4-1-2-3

**Step (3)** Total elapsed time

| Machines/ jobs | Machine A In | Out | Machine B In | Out | Machine C In | Out | Machine D In | Out |
|---|---|---|---|---|---|---|---|---|
| **$J_4$** | - | (7,9,29) | (7,9,29) | (13,21,45) | (13,21,45) | (22,33,61) | (22,33,61) | (49,63,95) |
| **$J_1$** | (7,9,29) | (21,24,49) | (21,24,49) | (23,2963) | (23,29,63) | (32,41,81) | (49,63,95) | (70,86,118) |
| **$J_2$** | (21,24,49) | (36,41,69) | (36,41,69) | (43,49,81) | (43,49,81) | (45,54,95) | (70,86,118) | (85,106,152) |
| **$J_3$** | (36,41,69) | (48,55,86) | (48,55,86) | (52,62,99) | (52,62,99) | (55,71,122) | (85,106,152) | **(100,131,178)** |

Neutrosophic fuzzy total elapsed time= **(100,131,178) Hours**

## CONCLUSION

We proposed the flow shop scheduling problem in Neutrosophic fuzzy environment is solved by branch and bound method after defuzzification, which is easy to understand. It helps to formulate uncertainty for the decision makers in real life situation.

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