



Semi-Compact and Semi-Lindelöf Spaces via Neutrosophic Crisp Set Theory

A.A. Salama*, I.M.Hanafy and M. S. Dabash

Department of Mathematics and Computer Science, Faculty of Sciences, Port Said University, Egypt.

Abstract

The aim of this paper is devoted to introduce and study the concepts of semi-compact (resp. semi-Lindelöf, locally semi-compact) spaces in a neutrosophic crisp topological space. Several properties, functions properties of neutrosophic crisp semi-compact spaces are studied. In addition to these, we introduce and study the definition of neutrosophic crisp semi-Lindelöf spaces and neutrosophic crisp locally semi-compact spaces. We show that neutrosophic crisp semi-compact spaces is preserved under neutrosophic crisp irresolute function and neutrosophic crisp pre-semi-closed function with neutrosophic crisp semi-compact point inverses.

Keywords: Neutrosophic crisp semi-compact spaces, Neutrosophic crisp semi-Lindelöf spaces, Neutrosophic crisp locally semi-compact spaces. Neutrosophic topological spaces

1. Introduction and preliminaries

Neutrosophic Crisp Sets were introduced by Salama & Smarandache in 2015. Neutrosophic topological spaces and many applications have been investigated by Salama et al. [5, 7, 8, 9] and [11-21]. The notions and terminologies not explained in this paper may be found in [9]. Some definitions and results which will be needed in this paper are recalled here. *In this paper, we generalize the crisp semi-compact spaces [1] and some notions in [2, 3, 4, 6] to the notion of neutrosophic crisp semi-compact spaces.*

Definition 1.1 [9] For any non-empty fixed set X , a neutrosophic crisp set (NC -set, for short) A is an object having the form $A = \langle A_1, A_2, A_3 \rangle$, where A_1, A_2 and A_3 are subsets of X satisfying $A_1 \cap A_2 = \emptyset, A_1 \cap A_3 = \emptyset$ and $A_3 \cap A_2 = \emptyset$.

Several relations and operations between NC -sets were defined in [8].

Definition 1.2 [9] A neutrosophic crisp topology (NCT , for short) on a non-empty set X is a family Γ of neutrosophic crisp subsets of X satisfying the following axioms

- i) $\emptyset_N, X_N \in \Gamma$.
- ii) $A_1 \cap A_2 \in \Gamma$ for any A_1 and $A_2 \in \Gamma$.
- iii) $\cup A_j \in \Gamma$ for any $\{A_j: j \in J\} \subseteq \Gamma$.

* Corresponding author: drsalama44@gmail.com

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In this case the pair (X, Γ) is called a neutrosophic crisp topological space (*NCTS*, for short) in X . The elements in Γ are called neutrosophic crisp open sets (*NC*-open sets for short) in X . A *NC*-set F is said to be neutrosophic crisp closed set (*NC*-closed set, for short) if and only if its complement F^c is a *NC*-open set.

Definition 1.3 [8] Let (X, Γ) be a *NCTS* and $A = \langle A_1, A_2, A_3 \rangle$ be a *NC*-set in X . Then the neutrosophic crisp closure of A ($NCcl(A)$ for short) and neutrosophic crisp interior ($NCint(A)$ for short) of A are defined by:

- (i) $NCcl(A) = \bigcap \{K: K \text{ is a } NC\text{-closed set in } X \text{ and } A \subseteq K\}$
- (ii) $NCint(A) = \bigcup \{G: G \text{ is a } NC\text{-open set in } X \text{ and } G \subseteq A\}$,

It can be also shown that $NCcl(A)$ is a *NC*-closed set, and $NCint(A)$ is a *NC*-open set in X .

Definition 1.4 [7] Let (X, Γ) be a *NCTS* and $A = \langle A_1, A_2, A_3 \rangle$ be a *NCS* in X , then A is called:

- i) Neutrosophic crisp α -open set iff $A \subseteq NCint(NCcl(NCint(A)))$.
- ii) Neutrosophic crisp semi-open set iff $A \subseteq NCcl(NCint(A))$.
- iii) Neutrosophic crisp pre-open set iff $A \subseteq NCint(NCcl(A))$.

Definition 1.5 [3,10] A subset A of space X is called semi-compact relative to X if any semi-open cover of A in X has a finite subcover of A .

Definition 1.6 [10] A subset A of a space X is called semi-Lindelöf in X if any semi-open cover of A in X has a countable subcover of A .

Definition 1.7 [5] Let (X, Γ) be a *NCTS* and $A = \langle A_1, A_2, A_3 \rangle$ be a *NCS* in X , then $f: X \rightarrow X$ is *NC*semi-continuous if the inverse image of *NC*semi-open set is *NC*semi-open.

2. Neutrosophic Crisp Semi-compact Spaces.

Definition 2.1 Let (X, Γ) be a *NCTS*.

- (i) If a family $\{\langle G_{i_1}, G_{i_2}, G_{i_3} \rangle: i \in I\}$ of *NC*-semiopen sets in X satisfies the condition $X_N = \bigcup \{\langle G_{i_1}, G_{i_2}, G_{i_3} \rangle: i \in I\}$, then it is called a *NC*-semiopen cover of X .
- (ii) A finite subfamily of a *NC*-semiopen cover $\{\langle G_{i_1}, G_{i_2}, G_{i_3} \rangle: i=1, 2, 3, \dots, n\}$ on X , which is also a *NC*-semiopen cover of X , is called a finite sub cover of *NC*-semiopen sets.

Definition 2.2 A *NCTS* (X, Γ) is called neutrosophic crisp semi-compact spaces (*NC*-semi-compact, for short) if any *NC*-semiopen cover of X has a finite subcover.

Definition 2.3

A family $\{\langle k_{i_1}, k_{i_2}, k_{i_3} \rangle: i \in I\}$ of *NC*-semiclosed sets in X satisfies the finite intersection property (*FIP* for short) iff every finite subfamily $\{\langle k_{i_1}, k_{i_2}, k_{i_3} \rangle: i=1, 2, 3, \dots, n\}$ of the family satisfies the condition $\bigcap_i \{\langle k_{i_1}, k_{i_2}, k_{i_3} \rangle: i=1, 2, 3, \dots, n\} \neq \Phi_N$.

Theorem 2.4 A *NCTS* (X, Γ) is *NC*-semi-compact iff every family $\{\langle G_{i_1}, G_{i_2}, G_{i_3} \rangle: i \in I\}$ of *NC*-semiclosed sets in X having the *FIP* has a nonempty intersection.

Proof. Let X be a NC -semi-compact space and $\mathcal{G} = \{ \langle G_{i_1}, G_{i_2}, G_{i_3} \rangle : i \in I \}$ be a cover of NC -semiopen sets of X having the FIP . Suppose that $\bigcap_i \{ \langle G_{i_1}, G_{i_2}, G_{i_3} \rangle : i \in I \} = \Phi_N$, then $\{ X \setminus \langle G_{i_1}, G_{i_2}, G_{i_3} \rangle : i \in I \}$ is a NC -semiopen cover of X and must contain a finite subcover $\{ X \setminus \langle G_{i_1}, G_{i_2}, G_{i_3} \rangle : i = 1, 2, 3, \dots, n \}$ for X . This implies that $\bigcap_i \{ \langle G_{i_1}, G_{i_2}, G_{i_3} \rangle : i = 1, 2, 3, \dots, n \} = \Phi_N$ this contradicts our assumption that \mathcal{G} has a FIP . Conversely, assume that X is not NC -semi-compact. Then there exists a NC -semiopen cover $\{ \langle G_{i_1}, G_{i_2}, G_{i_3} \rangle : i \in I \}$ for X , which contain a finite subcover for X . Thus, $\{ X \setminus \langle G_{i_1}, G_{i_2}, G_{i_3} \rangle : i \in I \}$ is a family of NC -semiclosed sets of X having the FIP . Moreover, we have $\bigcap_i \{ X \setminus \langle G_{i_1}, G_{i_2}, G_{i_3} \rangle : i \in I \} = \Phi_N$. This complete the proof.

Definition 2.5 A subset $u = \langle u_1, u_2, u_3 \rangle$ of a $NCTS (X, \Gamma)$ is called NC -semi-compact relative to X if any NC -semiopen cover of u in X has a finite subcover of u . By NC -semi-compact in X , we will mean NC -semi-compact relative to X .

Definition 2.6 A subset $u = \langle u_1, u_2, u_3 \rangle$ of a $NCTS (X, \Gamma)$ is called NC -semi-Lindelöf in X if any NC -semiopen cover of u in X has a countable subcover of u .

Remark 2.7

Since the family of all $NC\alpha$ -open subset of a $NCTS (X, \Gamma)$, denoted by Γ^α is NCT on X finer than Γ , then the family of all NC -semiopen subsets of (X, Γ^α) is equal to the family of all NC -semiopen subsets of (X, Γ) . Hence, it easily to see that a NC -set u of (X, Γ) is NC -semi-compact (resp. NC -semi-Lindelöf) in X iff it is NC -semi-compact (resp. NC -semi-Lindelöf) in (X, Γ^α) .

Theorem 2.8 The finite (resp. countable) union of NC -semi-compact (resp. NC -semi-Lindelöf) sets in a $NCTS X$ is a NC -semi-compact (resp. NC -semi-Lindelöf) in X .

Proof. obvious.

Lemma 2.9 Let $u \subseteq v \subseteq X$, where X is a $NCTS$. Then u is NC -semiopen set in v , if u is NC -semiopen set in X .

Theorem 2.10 Let v be a NC -preopen subset of a $NCTS X$ and $u \subseteq v$. If u is NC -semi-compact (resp. NC -semi-Lindelöf) in X , then u is NC -semi-compact (resp. NC -semi-Lindelöf) in v .

Proof. Suppose that $\mathcal{G} = \{ \langle G_{i_1}, G_{i_2}, G_{i_3} \rangle : i \in I \}$ is a cover of u by NC -semiopen sets in v . Using lemma 2.9, $G_{i_j} = S_{i_j} \cap v$ for each $i \in I, j = 1, 2, 3$, where S_{i_j} is NC -semiopen set in X for each $i \in I, j = 1, 2, 3$. Thus $\xi = \{ \langle S_{i_1}, S_{i_2}, S_{i_3} \rangle : i \in I \}$ is a cover of u by NC -semiopen set in X , but u is NC -semi-compact in X , so there exists $i = 1, 2, 3, \dots, n, j = 1, 2, 3$. Such that $u \subseteq \bigcup_{i=1}^n S_{i_j}$ and thus $u \subseteq \bigcup_{i=1}^n (S_{i_j} \cap v) = \bigcup_{i=1}^n G_{i_j}$. Hence u is NC -semi-compact in v .

The other case is similar.

Corollary 2.11 Let v be NC -open ($NC\alpha$ -open) set of $NCTS$ and $u \subseteq v$, if u is NC -semi-compact (resp. NC -semi-Lindelöf) in X , then u is NC -semi compact (resp. NC -semi-Lindelöf) in v .

Proof. It is obviously, since each NC -open set is $NC\alpha$ -open set and also NC -preopen set.

Lemma 2.12 Let $u \subseteq v \subseteq X$, where X is a $NCTS$ and v is a NC -preopen set in X , then u is NC -semiopen (resp. NC -semiclosed) in v iff $u = S \cap v$, where S is NC -semiopen (resp. NC -semiclosed) in X .

Proof. Obvious.

Theorem 2.13 Let v be a NC -preopen subset of $NCTS X$ and $u \subseteq v$. Then u is NC -semi compact (resp. NC -semi-Lindelöf) in X iff u is NC -semi compact (resp. NC -semi-Lindelöf) in v .

Proof. Necessity. It follows from Theorem 2.8 sufficiency. Suppose that $\xi = \{ \{S_{i_1}, S_{i_2}, S_{i_3}\} : i \in I \}$ is a cover of u by NC -semiopen sets in X . Then $\mathcal{G} = \{S_{i_j} \cap v : i \in I, j=1,2,3\}$ is a cover of u . Since S_{i_j} is NC -semiopen in X for each $i \in I$ and v is NC -preopen in X , it follows from Lemma 2.12 that $S_{i_j} \cap v$ is NC -semiopen set in v for each $i \in I, j=1,2,3$, but u is NC -semi-compact in v , so there exists $i \in I, j=1,2,3$ that $u \subseteq \bigcup_{i=1}^n S_{i_j} \cap v \subseteq \bigcup_{i=1}^n S_{i_j}$. Hence, u is NC -semi-compact in X .

The other case is similar.

Corollary 2.14 A NC -preopen subset u of X is NC -semi compact (resp. NC -semi-Lindelöf) iff u is NC -semi compact (resp. NC -semi-Lindelöf) in X .

Corollary 2.15 A NC -open ($NC\alpha$ -open) subset u of X is NC -semi compact (resp. NC -semi-Lindelöf) iff u is NC -semi compact (resp. NC -semi-Lindelöf) in X .

Theorem 2.16 Let v be a NC -semi-compact (resp. NC -semi-Lindelöf) set in a $NCTS X$ and v be is NC -semiclosed of X . Then $u \cap v$ is NC -semi-compact (resp. NC -semi-Lindelöf) in X .

Proof. Suppose that $\mathcal{G} = \{G_{i_j} : i \in I, j=1,2,3\}$ is a cover of $u \cap v$ by NC -semiopen set in X . Then $\mathcal{G} = \{G_{i_j} : i \in I, j=1,2,3\} \cup \{X \setminus v\}$ is a cover of u by NC -semiopen sets in X , but u is NC -semi-compact in X , so there exists $i=1,2,3, \dots, n, j=1,2,3$ such that $u \subseteq (\bigcup_{i=1}^n S_{i_j}) \cup \{X \setminus v\}$. Thus $u \cap v \subseteq \bigcup_{i=1}^n (S_{i_j} \cap v) \subseteq \bigcup_{i=1}^n S_{i_j}$. Hence, $u \cap v$ is NC -semi-compact in X .

The other case is similar.

Corollary 2.17 A NC -semiclosed subset u of a NC -semi-compact (resp. NC -semi-Lindelöf) space X is NC -semi-compact (resp. NC -semi-Lindelöf) in X .

Remark 2.18 From the Definition 2.1 of NC -semi-compact space, one may deduce that: NC -semi-compact space $\implies NC$ -compact space, but the inverse direction may not be true in general as show by the following example.

Example 2.19 Let (X, Γ) be a $NCTS$, where X is infinite, and $\Gamma = \{X_N, \Phi_N\} \cup \{P\}$ where $P = \{\{p_1\}, \{p_2\}, \{p_3\}\}$ be a NC -point in X . Then (X, Γ) is NC -compact but not NC -semi-compact, since $\{\{x, p_1\}, \{x, p_2\}, \{x, p_3\} : x \in X\}$ is NC -semiopen cover of X which has no finite subcover.

3. Functions and Neutrosophic Crisp Semi-compact Spaces

Definition 3.1 A function f from a $NCTSX$ into a $NCTS Y$ is called NC -irresolute if the inverse image of each NC -semiopen set in X , is a NC -semiopen set in Y .

Theorem 3.2 Let $f:(X, \Gamma_1) \rightarrow (Y, \Gamma_2)$ be a NC -irresolute function. Then

- (i) If u is NC -semi-Lindelöf in X , then $f(A)$ is NC -semi-Lindelöf in Y .
- (ii) If u is NC -semi-compact in X , then $f(A)$ is NC -semi-compact in Y .

Proof. We will proof (i) and (ii) is similar.

Suppose that $\mathcal{G} = \{ \langle G_{ij} \rangle : i \in I, j=1, 2, 3 \}$ is a cover of $f(A)$ by NC -semiopen sets in Y . Then $\mathcal{Z} = \{ \langle f^{-1}(G_{ij}) \rangle : i \in I, j=1, 2, 3 \}$ is a cover of u , but f is NC -irresolute function, so $\langle f^{-1}(G_{ij}) \rangle$ is NC -semiopen sets in X for each $i \in I, j=1, 2, 3$. Since u is NC -semi-Lindelöf in X , there exists $i_1, i_2, i_3, \dots, i_n \in I$ such that $u \subseteq \bigcup_{i=1}^n \langle f^{-1}(G_{ij}) \rangle$. Thus $f(u) \subseteq \bigcup_{i=1}^n \langle f(f^{-1}(G_{ij})) \rangle \subseteq \bigcup_{i=1}^n \langle G_{ij} \rangle$. Hence, $f(A)$ is NC -semi-Lindelöf in Y .

Corollary 3.3 If a function $f:(X, \Gamma_1) \rightarrow (Y, \Gamma_2)$ is a NC -irresolute (resp. NC -semi continuous) surjective and X is NC -semi-compact, then Y is NC -semi-compact (resp. NC -compact).

Definition 3.4 A function f from a $NCTS X$ into a $NCTS Y$ is called NC -pre-semiopen (resp. NC -pre-semiclosed) if the image of each NC -semiopen (resp. NC -semiclosed) subsets of X is NC -semiopen (resp. NC -semiclosed) subsets of Y .

Theorem 3.5 Let $f:(X, \Gamma_1) \rightarrow (Y, \Gamma_2)$ be a NC -pre-semiclosed surjection. If for each NC -point $y = \langle \{y_1\}, \{y_2\}, \{y_3\} \rangle$ in Y , $f^{-1}(y) = \langle f^{-1}\{y_1\}, f^{-1}\{y_2\}, f^{-1}\{y_3\} \rangle$ is NC -semi-compact (resp. NC -semi-Lindelöf) in X , then $f^{-1}(u)$ is NC -semi-compact (resp. NC -semi-Lindelöf) in X , where u is NC -semi-compact (resp. NC -semi-Lindelöf) in Y .

Proof. Will show the case when u is NC -semi-compact in X , the other case is similar. Let $\mathcal{G} = \{ \langle f^{-1}(G_{ij}) \rangle : i \in I, j=1, 2, 3 \}$ is a cover of $f^{-1}(u)$ by NC -semiopen sets in X . Then it follows by assumption that for each NC -point $y = \langle \{y_1\}, \{y_2\}, \{y_3\} \rangle$ in Y , there exists a finite subcollection \mathcal{G}_j^y of \mathcal{G} such that $f^{-1}(y) \subseteq \bigcup \mathcal{G}_j^y$. Let $H_{y_j} = \bigcup \mathcal{G}_j^y$. Then H_{y_j} is NC -semiopen in X where any union of NC -semiopen sets is NC -semiopen. Let $F_{y_j} = Y \setminus f(X \setminus H_{y_j})$. Then F_{y_j} is NC -semiopen in Y where f is NC -pre-semiclosed, also $y_i \in F_{y_j}$; for each $y_i \in u$, since $f^{-1}(y) \subseteq H_{y_j}$. Thus the family $\{H_{y_j} : y_j \in u\}$ is a cover of u by NC -semiclosed sets in Y , but u is NC -semi-compact in Y , so there exists $y_1, y_2, \dots, y_n \in u$ such that $u \subseteq \bigcup_{i=1}^n F_{y_{ij}}, j=1, 2, 3$. Thus $f^{-1}(u) \subseteq \bigcup_{i=1}^n f^{-1}(F_{y_{ij}}) \subseteq F_{y_{ij}}$.

Since $\mathcal{G}_j^{y_i}$ is a finite sub collection of \mathcal{G} for each $i=1, 2, \dots, n, j=1, 2, 3$, it follows that $\bigcup_{i=1}^n \mathcal{G}_j^{y_i}$ is a finite sub collection of \mathcal{G} . Hence, $f^{-1}(u)$ is NC -semi-compact in X .

Corollary 3.6 Let $f:(X, \Gamma_1) \rightarrow (Y, \Gamma_2)$ be a NC -pre-semiclosed surjection. and $f^{-1}(y)$ is NC -semi-compact in X , for each NC -point $y = \langle \{y_1\}, \{y_2\}, \{y_3\} \rangle$ in Y . If Y is NC -semi-compact, so is X .

Definition 3.7 A $NCTS (X, \Gamma)$ is called NC -Hausdorff space if for each distinct NC -points x and y of X , there exists two disjoint NC -open sets u and v of X containing x and y , respectively.

Theorem 3.8 Let $f: (X, \Gamma_1) \rightarrow (Y, \Gamma_2)$ is a NC -irresolute function from a NC -semi-compact space X into a NC -Hausdorff space Y , then

- (i) f is NC -pre-semiclosed.
- (ii) f is NC -semi-homomorphism if it is bijective.

Proof. Let u be a NC -semiclosed set of X . Then u is NC -semi-compact in X , (by Corollary 2.17). By Theorem 3.2, $f(u)$ is NC -semi-compact in Y and hence it is NC -semi-compact. Since Y is NC -Hausdorff, then $f(u)$ is NC -closed set in Y and NC -semiclosed. hence f is NC -pre-semiclosed.

- (ii) Obvious.

4. Locally Neutrosophic Crisp Semi-compact Spaces

Definition 4.1 A $NCTS X$ is said to be locally neutrosophic crisp semi-compact (LNC -semi-compact, for short) if each NC -point of X has a NC -open neighborhood which is a NC -semi-compact X .

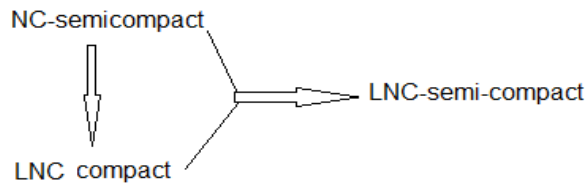
Remark 4.2 It is obvious that every NC -semi-compact space is LNC -semi-compact but the converse may not be true as show by the following example.

Example 4.3 Let (X, Γ) be an infinite discrete $NCTS$. It is obvious that (X, Γ) is LNC -semi-compact but not NC -semi-compact.

Remark 4.4 Every LNC -semi-compact space is LNC -compact, but the converse may not be true as shown by the following example.

Example 4.5 By Example 2.19 shows that a $NCTS (X, \Gamma)$ is LNC -compact but not LNC -semi-compact.

Remark 4.6 From the above discussion one can draw the following diagram:



Theorem 4.7 A $NCTS X$ is LNC -semi-compact iff for each NC -point $x \in X$, there exists a NC -open set u in X which is LNC -semi-compact containing x .

Proof. Let $u = \{ \langle u_{i_1}, u_{i_2}, u_{i_3} \rangle : i \in I \}$ be a NC -open set in X containing $x = \langle \{x_1\}, \{x_2\}, \{x_3\} \rangle$ which is LNC -semi-compact. Then there exists a NC -open neighbourhood $v = \{ \langle v_{i_1}, v_{i_2}, v_{i_3} \rangle : i \in I \}$ of x in u which is a NC -semi-compact in u . Since u is NC -open in X , so is v and by Corollary 2.11, v is NC -semi-compact in X . This shows that X is LNC -semi-compact.

The proof of the converse is obvious.

Theorem 4.8 A $NCTS X$ is LNC -semi-compact iff for each NC -point of X has a NC -open neighbourhood which is LNC -semi-compact in X .

Proof. This follows from Corollary 2.15.

Theorem 4.9 Let $f:(X, \Gamma_1) \rightarrow (Y, \Gamma_2)$ be a NC -open, NC -semi continuous surjection. and X is LNC -semi-compact space, then Y is LNC -semi-compact.

Proof. For any NC -point $y \in Y$, there exists NC -point $x \in X$ such that $f(x)=y$. Since X is LNC -semi-compact, there exists a NC -open neighborhood U_x of x which is NC -semi-compact in X . Hence $f(U_x)$ is NC -open neighborhood of y which is NC -semi-compact in Y . Therefore, by Theorem 4.8 is LNC -semi-compact.

Theorem 4.10 Let $f:(X, \Gamma_1) \rightarrow (Y, \Gamma_2)$ be a NC -pre-semiclosed, NC -continuous surjection. and $f^{-1}(y)$ is NC -semi-compact in X , for each NC -point $y \in Y$. If Y is LNC -semi-compact, so is X .

Proof. Let x is NC -point of X , by Theorem 4.8, there exists a NC -open neighborhood v of $f(x)$ such that v is NC -semi-compact in X . Then $f^{-1}(v)$ is a NC -open neighborhood of X . By Theorem 3.5, $f^{-1}(v)$ is NC -semi-compact in X . This shows that X is LNC -semi-compact.

5. Conclusion

The paper deals with the concept of semi-compactness (resp. semi-Lindelöf) in the generalized setting of a neutrosophic crisp topological space. We achieve a number of a neutrosophic crisp semi-compact (resp. neutrosophic crisp semi-Lindelöf) space. Also, we introduce and study the concept of neutrosophic crisp locally semi-compact spaces.

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