Review
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law. To suppose all science to be dynamics would not upset this position; for we should still give a preference to repetition arrangements if a conflict arose. In defining uniform time by the repetition method, we appeal, not to any particular type of experiment, but to the general concurrence of the results given by all physical operations capable of approximate repetition. It may be noted that Newton calls the standard uniform time, which he postulates, " absolute " time; and that he mentions the recently invented pendulum clock as affording one piece of evidence of the need for the adoption of a measure of time superior to that defined by any particular example of motion.

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REVIEWS.

The Universal Solution for Numerical and Literal Equations, by which the Roots of Equations can be expressed in Terms of their Coefficients. By M. A. M'GINNIS. Pp. x., 196 (Swan, Sonnenschein & Co.), 5/-

It is amusing to find a thoroughly paradoxical work like this, which has escaped the eye of a publisher's reader. It has all the usual features; the laboured proof of the obvious, the misunderstanding of the question at issue, and the insertion, as it were in passing, of a *petitio principii* which ruins the whole argument. It would be easy to make fun of poor Mr. M'Ginnis, with his childish conceit and amazing incompetence; but it will be more useful to say a few words on the problem which he has attacked, because it is so frequently misunderstood, and the facts that are known about it are so often incorrectly stated.

The well-known formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

for the solution of the quadratic $ax^2 + bx + c = 0$, gives us directions for performing certain arithmetical operations upon the coefficient; besides the four ordinary operations we have to extract a square root. A similar formula can be given for solving a cubic equation: in this case, however, the chain of operations includes a square root, and at least one cube root. These formulae, moreover, apply to the equations, *whatever their coefficients may be*, provided, of course, that they represent ordinary arithmetical or algebraic quantities. Now, what Abel proved is, that a root of a general equation of a degree higher than the fourth cannot be specified by any rule which starts with the coefficients and combines them by a finite chain of operations, restricted to the four rules of arithmetic and the extraction of roots; more briefly, there are no general formulae for equations of the fifth and higher orders which are analogous to those which have been formed for orders lower than five. This has been proved, and no mathematician, acquainted with the subject, wastes his time upon a problem which has been shown to be impossible. But algebraic formulae can be given for certain special equations of every degree; for instance, the solution of $x^7 - a = 0$ is given by $x = \sqrt[7]{a}$: thus the question arises,—What are the equations which can be solved algebraically, *i.e.* by the four elementary operations and the extraction of roots? This problem was put by Abel, partly answered by him, and practically solved in its general form by Kronecker. There are, however, many interesting points which remain for discussion.

It may be added that formulae for the solution of the *general* equation of the fifth order have been constructed, which involve certain transcendental functions, such as elliptic functions, elliptic modular functions, or the icosahedral function. These rules are analogous to that for the extraction of roots by logarithms, or the solution of a cubic by trigonometrical tables.

It is not superfluous to point out that all this is quite independent of processes for the approximate calculation of the numerical values of the roots of an equation with given numerical coefficients. Except for the labour involved, this can always be done to any prescribed degree of accuracy for an equation of any order. A great deal of Mr. M'Ginnis's book is quite irrelevant to the problem he professes to solve, and consists of numerical approximations by trial and error, which are often ingenious enough, but quite beside the point.

It may be worth while to point out the weak point in Mr. M'Ginnis's "General Solution of the Sixth Degree." He assumes

$$x^6 + mx^5 + nx^4 + ox^3 + px^2 + tx + q = \left(x^2 + \frac{m}{a}x + y\right)\left(x^2 + \frac{m}{b}x + z\right)\left(x^2 + \frac{m}{c}x + w\right), \quad \text{.............(1)}$$

and then puts

$$n - \frac{m^2}{A} = \frac{A_0}{2m} - \frac{m^2}{2A^2} = y + z + w, \quad \text{.......................(2)}$$

$$p - \left(\frac{m^2 n}{B^2} - \frac{m^4}{B^3}\right) = \frac{Bt}{m} = yz + zw + wy. \quad \text{....................(3)}$$

Thus

$$oA^3 - 2mnA^2 + 2m^3A - m^3 = 0,$$

$$tB^4 - mpB^3 + m^3nB - m^5 = 0,$$

whence A, B are to be found: then (2), (3) with $yzw = q$ gives y, z, w by a cubic equation. Finally, Mr. M'Ginnis says "it is evident" that, by comparing coefficients in (1), $\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{c}$ can be obtained. Quite apart from this, there is the fatal flaw that we have 8 unknowns, y, z, w, a, b, c, A, B, with 9 equations to be satisfied: namely, the four included in (2), (3), and the five obtained by equating coefficients in (1). Thus there is a necessary condition to be satisfied by the equation, and I leave it to others to express this in its simplest form.

Mr. W. M. H. Woodward professes to demolish the proof of the impossibility of solving the general quintic by radicals given (after Wantzel) in Serret's *Algèbre Supérieure.* He entirely fails to appreciate the argument, and asserts that "The conclusion of Wantzel, that the roots cannot be indicated in algebraic language, is equivalent to saying that there are no roots"!

At the beginning of his preface Mr. M'Ginnis tells us that his book appears "at the request of many able mathematicians, teachers, and scholars throughout the United States," and follows this by a list of "a few" of them, which includes, with others, two professors of mathematics (one of whom professes "Languages" as well), a President of a College, a Principal of a High School, and a State Superintendent. Assuming that they have not been unkind enough to play a practical joke, it is difficult to form a high opinion of their intellectual capacity.

G. B. MATHEWS.

Proportion; a Substitute for the Fifth Book of Euclid. By Professor G. A. GIBSON, M.A., F.R.S.E. (John Lindsay, Edinburgh; 8vo., pp. 27.)

Prof. Gibson's pamphlet has received the formal approval of the Edinburgh Mathematical Society, and is printed in the *Proceedings* of the Society for the current year. We welcome it as a genuine and not unsuccessful attempt to provide a satisfactory substitute for the Fifth Book of Euclid. By a fortunate coincidence, it appears at the same time as a more elaborate attempt of the same nature by Prof. M. J. M. Hill, F.R.S. ("Euclid, Books V. and VI.," Cambridge University Press.) The coincidence will have happy results if it leads to a general discussion and to some practical improvements in the teaching of proportion in elementary geometry. That such improvements are much needed is very clearly shown by Prof. Hill, both in the book referred to and in the *School World* for September and October, 1899. We hope that Prof. Hill's work will be reviewed later in these columns, and we refer to it at present solely for the sake of comparison.

In criticising Prof. Gibson's pamphlet we are not questioning its present opportuneness and value, but merely giving expression to personal views and predilections on a debateable question of method and procedure. Prof. Gibson advocates an entire departure from Euclid's method by recommending two fundamental alterations: first, that ratio should be defined as a number from the outset, and second, that the consideration of the ratio of like commensurable magnitudes should be separated from and precede the consideration of the ratio of like incommensurable magnitudes. In § 3 it is apparently implied, though not formally stated, that the symbol $\frac{m}{n}A$ represents m times the n^{th} part of the magnitude A, when m and n are positive integers. In § 6 the following definition is given:—"If A and B be two like magnitudes having a common measure M, so that $A = m\,M$, $B = n\,M$, and therefore $A = \frac{m}{n}B$, the ratio of A to B is defined to be the fraction $\frac{m}{n}$." In § 16 the definition is extended to incommensurable magnitudes as follows:—"If A, B are two like incommensurable magnitudes, and if B be divided into any number n of equal parts of which A contains