28.

Demonstratio duarum celeberrimi Gaussii propositionum.

(Disqu. arithm. p. 17.)

(Auct. *Th. Clausen.*)

Denotent (α), (α,β), (α,β,γ), etc. quantitates, quae ita a se invicem pendent:

$$(\alpha) = \alpha,$$
$$(\alpha,\beta) = \beta(\alpha)+1,$$
$$(\alpha,\beta,\gamma) = \gamma(\alpha,\beta)+(\alpha),$$
$$(\alpha,\beta,\gamma,\delta) = \delta(\alpha,\beta,\gamma)+(\alpha,\beta), \text{ etc.}$$

ubi lex progressionis obvia est. Nunc dico, si fuerit:

$$(1.)\quad (\alpha,\beta\ldots\lambda,\mu)(\beta,\gamma\ldots\varkappa,\lambda)-(\alpha,\beta\ldots\varkappa,\lambda)(\beta,\gamma\ldots\lambda,\mu) = \pm 1,$$

fore etiam

$$(\alpha,\beta\ldots\mu,\nu)(\beta,\gamma\ldots\lambda,\mu)-(\alpha,\beta\ldots\lambda,\mu)(\beta,\gamma\ldots\mu,\nu) = \mp 1;$$

in qua serie numerus quantitatum $\alpha,\beta\ldots$ unitate auctus est. Habemus enim:

$$(\alpha,\beta\ldots\mu,\nu) = \nu(\alpha,\beta\ldots\lambda,\mu)+(\alpha,\beta\ldots\varkappa,\lambda),$$
$$(\beta,\gamma\ldots\mu,\nu) = \nu(\beta,\gamma\ldots\lambda,\mu)+(\beta,\gamma\ldots\iota,\varkappa),$$

quibus valoribus in aequationem posteriorem substitutis, aequatio (1.) statim sequitur. Nunc vero, cum sit:

$$(\alpha,\beta,\gamma)\beta-(\alpha,\beta)(\beta,\gamma) = (\alpha\beta\gamma+\alpha+\gamma)\beta-(\alpha\beta+1)(\beta\gamma+1) = -1;$$

vel cum aequatio (1.) valeat pro numero quantitatum $\alpha,\beta\ldots 3$; sequitur illam etiam pro numero 4, omnibusque ulterioribus valere, sumto signo superiore, quando ille est par, inferiore vero quando est impar.

Cum sit: $(\alpha,\beta\ldots\varkappa,\lambda) = \lambda(\alpha,\beta\ldots\iota,\varkappa)+(\alpha,\beta\ldots\theta,\iota)$,

$$(\tau,\sigma\ldots\mu,\lambda) = \lambda(\tau,\sigma\ldots\nu,\mu)+(\tau,\sigma\ldots\xi,\nu),$$

erit:

$$(\tau,\sigma\ldots\nu,\mu)(\alpha,\beta\ldots\varkappa,\lambda)+(\tau,\sigma\ldots\xi,\nu)(\alpha,\beta\ldots\iota,\varkappa)$$
$$= (\tau,\sigma\ldots\mu,\lambda)(\alpha,\beta\ldots\iota,\varkappa)+(\tau,\sigma\ldots\nu,\mu)(\alpha,\beta\ldots\theta,\iota);$$

si enim aequatio prior in partem aequationis primam, posterior in secundam substituantur, aequatio fit identica. Eodem modo habetur: pars posterior aequationis

$$= (\tau,\sigma\ldots\lambda,\varkappa)(\alpha,\beta\ldots\theta,\iota)+(\tau,\sigma\ldots\mu,\lambda)(\alpha,\beta\ldots\eta,\theta);$$

unde concludere licet:

$(\tau,\sigma)(\alpha,\beta\ldots\pi,\varrho)+(\tau)(\alpha,\beta\ldots o,\pi) = (\tau,\sigma\ldots\delta,\gamma)(\alpha,\beta)+(\tau,\sigma\ldots\varepsilon,\delta)(\alpha)$, vel
$\tau\sigma(\alpha,\beta\ldots\pi,\varrho)+\tau(\alpha,\beta\ldots o,\pi)+\alpha,\beta\ldots\pi,\varrho) = (\alpha\beta(\tau,\sigma\ldots\delta,\gamma)+\alpha(\tau,\sigma\ldots\varepsilon,\delta)+(\tau,\sigma\ldots\delta,\gamma)$;
$\tau(\alpha,\beta\ldots\varrho,\sigma)+(\alpha,\beta\ldots\pi,\varrho) = \alpha(\tau,\sigma\ldots\gamma,\beta)+(\tau,\sigma\ldots\delta,\gamma)$; vel denique

$$(2.)\quad (\alpha,\beta\ldots\sigma,\tau) = (\tau,\sigma\ldots\beta,\alpha).$$