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Neutrosophic Triplet Non-Associative Semihypergroups with Application

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Abstract: In this paper, we extended the idea of a neutrosophic triplet set to non-associative semihypergroups and define neutrosophic triplet $\mathcal{LA}$-semihypergroup. We discuss some basic results and properties. At the end, we provide an application of the proposed structure in Football.

Keywords: LA-semihypergroups; neutrosophic triplet set; neutro-homomorphism

1. Introduction

The study of origin and features of neutralities lies in the scope of a new branch of philosophy known as Neutrosophy. In 1995, Smarandache (for the first time) used the idea of Neutrosophy and developed neutrosophic logic which is a more practical and realistic approach, to handle imprecise and vague information. He introduced the concept of (T-truth, I-indeterminacy, F-falsity) memberships. According to Smarandache Neutrosophic, logics generalizes the all previous logics such as fuzzy logic [1], intuitionistic fuzzy logic [2] and interval valued fuzzy logic [3]. Kandasamy and Smarandache [4] developed many neutrosophic algebraic structures, neutrosophic bigroup, neutrosophic vector space, neutrosophic groups and so on, based on neutrosophic logic. For practical applications, we refer the readers to [5–9]. For neutrosophic triplet sets, we refer the readers [10–13]. In 2016, Smarandache and Ali [14] gave the concept of Neutrosophic triplet groups which is a very useful addition in the theory of groups.

Hyperstucture theory was brought-out by Marty [15] in 1934, when he defined hypergroup, set about analyzing their properties and exerted them to a group. Several papers and books have been compiled in this direction, see references [16–18]. In 1990, in Greece, a congress was organized by Thomas Vougiouklis on hyperstructure, which was first named algebraic hyper structures and its applications algebraic hyper structures(AHA); however actually was the fourth, because there had been three more congresses in Italy by Corsini, on the same topic but random names. During this congress, Vougiouklis [19] presented the concept of weak structure, presently known as Hv-structure. A number of writers have gone through various aspects of Hv-structure. For instance, references [20–27]. Another book by Davvaz and Fotea in 2007 has been devoted especially to the study of hyperring theory [28].

Kazim and Naseeruddin [29] in 1970, presented the concept of left almost semigroups (LA-semigroups) and shifted the discussion toward non-associative structures. According to them, a groupoid S is called $\mathcal{LA}$-semigroups, if it is satisfies the left invertive law: $(w_1w_2)w_3 = (w_3w_2)w_1$ for all $w_1, w_2, w_3 \in S$. After that, researchers started working in this direction such as, references [30–32] and Yusuf gave the idea of left almost rings [33]. Hila and Dine [34] in 2011, shifted the non-associative structures to non-associative hyperstructures and furnished the idea of $\mathcal{LA}$-semihypergroup, which is generalization of semigroup, semihypergroup, $\mathcal{LA}$-semigroup by using left invertive law with the help of Marty's hyperoperation.Yaqoob et al. [35] expanded the work of Hila and Dine. Yousafzai et al. in [36] and Amjad et al. [37] tried to generalize different aspects of left almost semihypergroups. The concept of Hv-$\mathcal{LA}$-semigroup was laid by Gulistan et al. [38] in 2015. The idea of partially ordered left almost semihypergroups was developed by Naveed et al. [39] in 2015. Rehman et al. [40], initiated the study of $\mathcal{LA}$-hyperrings and discussed its hyperideals and hypersystems in 2017. Nawaz et al. introduced the concept of left almost semihyperrings [41]. Yaqoob et al. [42] gave the idea of left almost polygroups in 2018.

In this paper, we extended the idea of neutrosophic triplet set to non-associative semihypergroups. We define neutrosophic triplet $\mathcal{LA}$-semihypergroup. In neutrosophic triplet $\mathcal{LA}$-semihypergroup every element "w" has left neut(w) and left anti(w). In neutrosophic triplet $\mathcal{LA}$-semihypergroup left neut(w) of an element "w" may or may not be equal to left identity. We also defined the neutro-homomorphism on $\mathcal{LA}$-semihypergroups. At the end, we present an application of the proposed structure in football.

2. Preliminaries

This section of paper consists of some basic definitions, which are directly used in our work.

Definition 1 ([34])**.** *Let $\mathcal{H}$ be a non void set and $\circ : \mathcal{H} * \mathcal{H} \longrightarrow P^{\bullet}(\mathcal{H})$ be a hyperoperation, where $P^{\bullet}(\mathcal{H})$ is the family non-void subset of $\mathcal{H}$. The pair $(\mathcal{H}, *)$ is called hypergroupoid.*

For any two non-void subsets W_1 and W_2 of $\mathcal{H}$, then

$$W_1 * W_2 = \bigcup_{w_1 \in W_1, w_2 \in W_2} w_1 * w_2.$$

Definition 2 ([34])**.** *An $\mathcal{LA}$-semihypergroup is the hypergroupoid $(\mathcal{H}, *)$ with*

$$(w_1 * w_2) * w_3 = (w_3 * w_2) * w_1 \tag{1}$$

for all, $w_1, w_2, w_3 \in \mathcal{H}$. The equation (1) is called left invertive law.

Definition 3 ([35])**.** *An element e of an LA-semihypergroup $\mathcal{H}$ is called left identity (resp., pure left identity) if for all $w_1 \in \mathcal{H}$, $w_1 \in e * w_1$ (resp., $w_1 = e * w_1$). An element e of an LA-semihypergroup $\mathcal{H}$ is called right identity (resp., pure right identity) if for all $w_1 \in \mathcal{H}$, $w_1 \in w_1 * e$ (resp., $w_1 = e * w_1$). An element e of an LA-semihypergroup $\mathcal{H}$ is called identity (resp., pure right identity) if for all $w_1 \in \mathcal{H}$, $w_1 \in w_1 * e \cap e * w_1$ (resp., $w_1 = w_1 * e \cap e * w_1$).*

Definition 4 ([35])**.** *An $\mathcal{LA}$-smihypergroup with pure left identity satisfies the following property*

$$w_1 * (w_2 * w_3) = w_2 * (w_1 * w_3).$$

Definition 5 ([14])**.** *Let N be a non-void set with a binary operation $*$ and $w_1 \in N$. Then w_1 is said to be neutrosophic triplet if there exist an element $neut\,(w_1) \in N$ such that*

$$w_1 * neut\,(w_1) = neut\,(w_1) * w_1 = w_1,$$

where $neut\,(w_1)$ is different from unity element. Also there exist $anti\,(w_1) \in N$ such that

$$w_1 * anti\,(w_1) = anti\,(w_1) * w_1 = neut\,(w_1)\,.$$

If there are more $anti\,(w_1)'\,s$ for a given w_1, one takes that $anti(w_1) = w_2$ that $anti\,(w_1)$ in its turn forms a neutrosophic triplet, i.e., there exists $neut\,(w_2)$ and $anti\,(w_2)$. We denote the neutrosophic triplet w_1 by $(w_1, neut\,(w_1)\,, anti\,(w_1))$. By $neut\,(w_1)$, we means neutral of w_1.

Example 1 ([14])**.** *Consider Z_6 under multiplication modulo 6. Then 2 is a neutrosophic triplet, because $neut\,(2) = 4$, as $2 \times 4 = 8$. Similarly $anti\,(2) = 2$ because $2 \times 2 = 4$. Thus 2 is a neutrosophic triplet, which is denoted by $(2,4,2)$. Similarly 4 is a neutrosophic triplet because $neut\,(a) = anti\,(4) = 4$. So 4 is represented by as $(4,4,4)$. 3 is not a neutrosophic triplet as $neut(3) = 5$ but $anti(3)$ does not exist in Z_6 and 0 is a trivial neutrosophic triplet as $neut\,(0) = anti\,(0) = 0$. This is denoted by $(0,0,0)$.*

3. Neutrosophic Triplet $\mathcal{LA}$-Semihypergroups

In this section, we defined the neutrosophic triplet $\mathcal{LA}$-semihypergroup and some results on neutrosophic triplet $\mathcal{LA}$-semihypergroup are provided.

Definition 6. *Let $\mathcal{H}$ be a non void set with a binary hyperoperation $*$ and $w_1 \in \mathcal{H}$. Then $\mathcal{H}$ is called*

1. left neutrosophic triplet set if for every $w_1 \in \mathcal{H}$, there exist $neut\,(w_1)$ and $anti(w_1)$ such that

$$w_1 \in neut\,(w_1) * w_1,$$
$$neut(w_1) \in anti\,(w_1) * w_1.$$

2. right neutrosophic triplet set if for every $w_1 \in \mathcal{H}$, there exist $neut\,(w_1)$ and $anti(w_1)$ such that

$$w_1 \in w_1 * neut\,(w_1)\,,$$
$$neut(w_1) \in w_1 * anti\,(w_1)\,.$$

3. neutrosophic triplet set if for every $w_1 \in \mathcal{H}$, there exist $neut\,(w_1)$ and $anti(w_1)$ such that

$$w_1 \in (neut\,(w_1) * w_1) \cap (w_1 * neut\,(w_1)),$$
$$neut(w_1) \in (anti\,(w_1) * w_1) \cap (w_1 * anti\,(w_1)).$$

Definition 7. *Let $\mathcal{H}$ be a set with a binary hyperoperation $*$ and $w_1 \in \mathcal{H}$. Then $\mathcal{H}$ is called*

1. pure left neutrosophic triplet set if for every $w_1 \in \mathcal{H}$, there exist $neut\,(w_1)$ and $anti\,(w_1)$ such that

$$w_1 = neut\,(w_1) * w_1,$$
$$neut(w_1) = anti\,(w_1) * w_1.$$

2. pure right neutrosophic triplet set if for every $w_1 \in \mathcal{H}$, there exist $neut\,(w_1)$ and $anti(w_1)$ such that

$$w_1 = w_1 * neut\,(w_1)\,,$$
$$neut(w_1) = w_1 * anti\,(w_1)\,.$$

3. pure neutrosophic triplet set if for every $w_1 \in \mathcal{H}$, there exist $neut\,(w_1)$ and $anti(w_1)$ such that

$$w_1 = (neut\,(w_1) * w_1) \cap (w_1 * neut\,(w_1)),$$
$$neut\,(w_1) = (anti\,(w_1) * w_1) \cap (w_1 * anti\,(w_1))\,.$$

Example 2. *Let $\mathcal{H} = \{w_1, w_2, w_3\}$ be a se with hyperoperation defined as follows:*

$*$	w_1	w_2	w_3
w_1	w_3	$\{w_1, w_2\}$	w_1
w_2	$\{w_1, w_2\}$	$\{w_1, w_2\}$	$\{w_1, w_3\}$
w_3	w_2	$\{w_1, w_3\}$	w_2

A Cayley table 1

Here (w_1, w_2, w_2), (w_2, w_2, w_2) and (w_3, w_2, w_3) are neutrosophic triplets.

Definition 8. *Let $(\mathcal{H}, *)$ be a left (resp., right, left pure, right pure) neutrosophic triplet set. Then $\mathcal{H}$ is called left (resp., right, left pure, right pure) neutrosophic triplet $\mathcal{LA}$-semihypergroup, if the following conditions are satisfied:*

1. $(\mathcal{H}, *)$ is well defined.
2. $(\mathcal{H}, *)$ satisfies the left invertive law.

Example 3. *Let $\mathcal{H} = \{w_1, w_2, w_3, w_4, w_5\}$ be a set with the hyperoperation defined as follows:*

$*$	w_1	w_2	w_3	w_4	w_5
w_1	w_1	w_1	w_1	w_1	w_1
w_2	w_1	$\{w_3, w_5\}$	w_3	$\{w_1, w_4\}$	$\{w_3, w_5\}$
w_3	w_1	w_3	w_3	$\{w_1, w_4\}$	w_3
w_4	w_1	$\{w_1, w_4\}$	$\{w_1, w_4\}$	w_4	$\{w_1, w_4\}$
w_5	w_1	$\{w_2, w_5\}$	w_3	$\{w_1, w_4\}$	$\{w_2, w_5\}$

A Cayley table 2

Here $(\mathcal{H}, *)$ is an $\mathcal{LA}$-semihypergroup, as the element of $\mathcal{H}$ satisfies the left invertive law. Here(w_1, w_1, w_1), (w_2, w_5, w_5), (w_3, w_3, w_3), (w_4, w_4, w_4) and (w_5, w_2, w_5) are left neutrosophic triplets. Hence $(\mathcal{H}, *)$ is a left neutrosophic triplet $\mathcal{LA}$-semihypergroup.

Definition 9. *Let $(\mathcal{H}, *)$ be neutrosophic (resp., pure neutrosophic) triplet set. Then $\mathcal{H}$ is said to be neutrosophic (resp., pure neutrosophic) triplet $\mathcal{LA}$-semihypergroup, if the following condition are satisfied:*

1. $(\mathcal{H}, *)$ is a well defined.
2. $(\mathcal{H}, *)$ satisfies the left invertive law.

Example 4. *Let $\mathcal{H} = \{w_1, w_2, w_3\}$ and the hyperoperation defined in the table as follows:*

$*$	w_1	w_2	w_3
w_1	$\{w_1, w_2\}$	$\{w_1, w_2\}$	w_3
w_2	$\mathcal{H}$	$\mathcal{H}$	w_3
w_3	w_3	w_3	w_3

A Cayley table 3

Here $(\mathcal{H}, *)$ is an $\mathcal{LA}$-semihypergroup, as the element of $\mathcal{H}$ satisfies the left invertive law. Here (w_1, w_2, w_1), (w_2, w_1, w_2) and (w_3, w_3, w_3) are neutrosophic triplets. Hence $(\mathcal{H}, *)$ is a neutrosophic triplet $\mathcal{LA}$-semihypergroup.

Remark 1. *Neut(w_2) of an element "w_2" is not unique under the hyperoperation $*$ in $\mathcal{H}$ and depend on elements and hyperoperation. By the Example 4 $neut(w_2) = w_1, w_2$. Similarly $anti(w_2) = w_1, w_2$ of an element "w_2" is not unique and depends on the element and the hyperoperation $*$.*

Remark 2. *Left neut of an element is could be different from left identity.*

Definition 10. *Let $(\mathcal{H}, *)$ be a neutrosophic $\mathcal{LA}$-semihypergroup. An element $w_1 \in \mathcal{H}$, then there exist pure left $neut(w_1)$ such that $w_1 = neut\,(w_1) * w_1$ and pure left $anti(w_1)$ such that $neut(w_1) = anti\,(w_1) * w_1$.*

Proposition 1. *Let $(\mathcal{H}, *)$ be a pure left neutrosophic triplet $\mathcal{LA}$-semihypergroup with pure left identity. Then $w_2 * w_1 = w_3 * w_1$ if and only if*

$$neut\,(w_1) * w_2 = neut\,(w_1) * w_3.$$

Proof. Suppose that $w_2 * w_3 = w_3 * w_1$ for $w_1, w_2, w_3 \in \mathcal{H}$. Since $(\mathcal{H}, *)$ is a pure left neutrosophic $\mathcal{LA}$ semihypergroup, so $anti\,(w_1) \in \mathcal{H}$. Multiply $anti\,(w_1)$ to the right side of $w_2 * w_1 = w_3 * w_1$

$$\begin{aligned}(w_2 * w_1) * anti\,(w_1) &= (w_3 * w_1) * anti\,(w_1)\\ (anti\,(a) * w_1) * w_2 &= (anti\,(w_1) * w_1) * w_3\\ neut\,(w_1) * w_2 &= neut\,(w_1) * w_3.\end{aligned}$$

Conversely, let $neut\,(w_1) * w_2 = neut\,(w_1) * w_3$. Multiply to both right sides by w_1

$$\begin{aligned}neut\,(w_1) * (w_2 * w_1) &= neut\,(w_1) * (w_3 * w_1)\\ w_2 * (neut\,(w_1) * w_1) &= w_3 * (neut\,(w_1) * w_1)\\ w_2 * w_1 &= w_3 * w_1.\end{aligned}$$

This completes the proof. □

Proposition 2. *Let $(\mathcal{H}, *)$ be a pure right neutrosophic triplet $\mathcal{LA}$-semihypergroup with pure left identity. Then $w_2 * neut\,(w_1) = w_3 * neut\,(w_1)$ if $w_2 * anti\,(w_1) = w_3 * anti(w_1)$ for all $w_1, w_2, w_3 \in \mathcal{H}$.*

Proof. Suppose $(\mathcal{H}, *)$ is a pure right neutrosophic triplet $\mathcal{LA}$-semihypergroup with pure left identity and $w_2 * anti\,(w_1) = w_3 * anti(w_1)$ for $w_1, w_2, w_3 \in \mathcal{H}$. Multiply w_1 to the left side of $w_2 * anti\,(w_1) = w_3 * anti(w_1)$,

$$\begin{aligned}w_1 * (w_2 * anti\,(w_1)) &= w_1 * (w_3 * anti(w_1))\\ w_2 * (w_1 * anti\,(w_1)) &= w_3 * (w_1 * anti(w_1))\\ w_2 * neut(w_1) &= w_3 * neut\,(w_1) \quad (\text{because } neut(w_1) = w_1 * anti\,(w_1)).\end{aligned}$$

Therefore,

$$w_2 * neut\,(w_1) = w_3 * neut(w_1).$$

□

Proposition 3. *Let $(\mathcal{H}, *)$ be a pure right neutrosophic triplet $\mathcal{LA}$-semihypergroup. Then $neut(w_1) * w_2 = neut\,(w_1) * w_3$ if $w_2 * anti\,(w_1) = w_3 * anti(w_1)$ for all $w_1, w_2, w_3 \in \mathcal{H}$.*

Proof. Suppose $(\mathcal{H}, *)$ is a pure right neutrosophic triplet $\mathcal{LA}$-semihypergroup and $w_2 * anti\,(w_1) = w_3 * anti(w_1)$ for $w_1, w_2, w_3 \in \mathcal{H}$. Multiply w_1 to the right side of $w_2 * anti\,(w_1) = w_3 * anti(w_1)$,

$$
\begin{aligned}
(w_2 * anti\,(w_1)) * w_1 &= (w_3 * anti(w_1)) * w_1 \\
(w_1 * anti\,(w_1)) * w_2 &= (w_1 * anti(w_1)) * w_3 \\
neut(w_1) * w_2 &= neut\,(w_1) * w_3 \ \ \text{(because } neut(w_1) = w_1 * anti\,(w_1)\,\text{)}.
\end{aligned}
$$

Therefore,

$$neut(w_1) * w_2 = neut\,(w_1) * w_3.$$

□

Theorem 1. *Let $(\mathcal{H}, *)$ be a pure right neutrosophic triplet idempotent $\mathcal{LA}$-semihypergroup. Then $neut(w_1) * neut\,(w_1) = neut\,(w_1)$.*

Proof. Consider $neut(w_1) * neut\,(w_1) = neut\,(w_1)$. Multiply first with w_1 to the right and then again multiply with w_1 to the right, i.e.,

$$
\begin{aligned}
((neut(w_1) * neut\,(w_1)) * w_1) * w_1 &= (neut\,(w_1) * w_1) * w_1 \\
((w_1 * neut\,(w_1)) * neut(w_1)) * w_1 &= (w_1 * w_1) * neut\,(w_1) \\
(w_1 * neut(w_1)) * (w_1 * neut\,(w_1)) &= w_1 * neut\,(w_1) \\
w_1 * w_1 &= w_1 \\
w_1 &= w_1.
\end{aligned}
$$

This shows that

$$neut(w_1) * neut\,(w_1) = neut\,(w_1)\,.$$

□

Theorem 2. *Let $(\mathcal{H}, *)$ be a pure right neutrosophic triplet idempotent $\mathcal{LA}$-semihypergroup with pure left identity. Then $neut(w_1) * anti\,(w_1) = anti\,(w_1)$.*

Proof. Let $(\mathcal{H}, *)$ be a pure right neutrosophic triplet $\mathcal{LA}$-semihypergroup with pure left identity. Multiply w_1 to the left of both side $neut(w_1) * anti\,(w_1) = anti\,(w_1)$, i.e.,

$$
\begin{aligned}
w_1 * ((neut(w_1) * anti\,(w_1)) &= w_1 * anti\,(w_1) \\
neut(w_1) * (w_1 * anti\,(w_1)) &= neut(w_1) \\
neut(w_1) * neut(w_1) &= neut\,(w_1) \\
neut\,(w_1) &= neut\,(w_1) \ \ \text{(By Theorem 1)}
\end{aligned}
$$

This shows that

$$neut(w_1) * anti\,(w_1) = anti\,(w_1)\,.$$

□

Theorem 3. *Let $(\mathcal{H}, *)$ be a pure right neutrosophic triplet $\mathcal{LA}$-semihypergroup. Then*

1. $neut(w_1) * neut\,(w_2) = neut(w_1 * w_2)$ for all $w_1, w_2 \in \mathcal{H}$.
2. $ant\,(w_1) * anti\,(w_2) = anti\,(w_1 * w_2)$ for all $w_1, w_2 \in \mathcal{H}$.

Proof. 1. Consider the left hand side $neut(w_1) * neut\ (w_2)$. Multiply first with w_2 to the right and then again multiply with w_1 to the right, i.e.,

$$\begin{aligned}((neut(w_1) * neut\ (w_2)) * w_2) * w_1 &= ((w_2 * neut\ (w_2)) * neut(w_1)) * w_1 \\ &= (w_2 * neut(w_1)) * w_1 \\ &= (w_1 * neut(w_1)) * w_2 \\ &= w_1 * w_2.\end{aligned}$$

So

$$((neut(w_1) * neut\ (w_2)) * w_2) * w_1 = w_1 * w_2 \tag{2}$$

Now consider the right side $neut(w_1 * w_2)$. Multiply first with w_2 to the right and then again multiply with w_1 to the right, i.e.,

$$\begin{aligned}(neut(w_1 * w_2) * w_2) * w_1 &= (w_1 * w_2) * neut(w_1 * w_2) \\ &= w_1 * w_2.\end{aligned}$$

So

$$(neut(w_1 * w_2) * w_2) * w_1 = w_1 * w_2 \tag{3}$$

From the Equations (2) and (3) it is clear that $neut(w_1) * neut\ (w_2) = neut(w_1 * w_2)$.

2. Consider the left hand side $anti(w_1) * anti\ (w_2)$. Multiply first with w_2 to the right and then again multiply with w_1 to the right, i.e.,

$$\begin{aligned}((anti(w_1) * anti\ (w_2)) * w_2) * w_1 &= ((w_2 * anti\ (w_2)) * anti(w_1)) * w_1 \\ &= (neut(w_2) * anti(w_1)) * w_1 \\ &= (w_1 * anti(w_1)) * neut(w_2) \\ &= neut(w_1) * neut(w_2) \\ &= neut(w_1 * w_2).\end{aligned}$$

So

$$((anti(w_1) * anti\ (w_2)) * w_2) * w_1 = neut(w_1 * w_2) \tag{4}$$

Now consider the right side $anti(w_1 * w_2)$. Multiply first with w_2 to the right and then again multiply with w_1 to the right, i.e.,

$$\begin{aligned}(anti(w_1 * w_2) * w_2) * w_1 &= (w_1 * w_2) * anti(w_1 * w_2) \\ &= neut(w_1 * w_2)\end{aligned}$$

So

$$(anti(w_1 * w_2) * w_2) * w_1 = neut(w_1 * w_2) \tag{5}$$

From the Equations (4) and (5) it is clear that

$$anti(w_1) * anti\ (w_2) = anti(w_1 * w_2).$$

□

In the following example, we show that in a left neutrosophic triplet $\mathcal{LA}$-semihypergroup

$$neut(w_1) * neut\ (w_2) \neq neut(w_1 * w_2) \tag{6}$$

$$\text{and} \ \ anti(w_1) * anti\ (w_2) \neq anti(w_1 * w_2). \tag{7}$$

Example 5. *Let $\mathcal{H} = \{w_1, w_2, w_3, w_4\}$ be a set with the hyperoperation defined as follow*

$*$	w_1	w_2	w_3	w_4
w_1	$\{w_1, w_3, w_4\}$	$\{w_2, w_4\}$	$\mathcal{H}$	$\mathcal{H}$
w_2	$\{w_1, w_2, w_4\}$	$\{w_1, w_2\}$	$\{w_2, w_4\}$	$\{w_2, w_3, w_4\}$
w_3	$\{w_1, w_2, w_3\}$	$\{w_1, w_3, w_4\}$	$\{w_2, w_4\}$	$\{w_2, w_3, w_4\}$
w_4	$\mathcal{H}$	$\{w_2, w_3, w_4\}$	$\mathcal{H}$	$\{w_1, w_3\}$

A Cayley table 4

All the elements of $\mathcal{H}$ satisfies the left invertive law. Here (w_1, w_3, w_4), (w_2, w_4, w_1), (w_3, w_1, w_4) and (w_4, w_2, w_3) are left neutrosophic triplets. Hence $(\mathcal{H}, *)$ is a left neutrosophic triplet $\mathcal{LA}$-semihypergroup. Now

$$\begin{aligned} neut(w_1) * neut\,(w_2) &\neq neut(w_1 * w_2) \\ w_3 * w_4 &\neq neut(\{w_2, w_4\}) \\ \{w_2, w_3, w_4\} &\neq neut(w_2) \cup neut(w_4) \\ \{w_2, w_3, w_4\} &\neq \{w_2, w_4\}. \end{aligned}$$

Also

$$\begin{aligned} anti(w_1) * anti\,(w_2) &\neq anti(w_1 * w_2) \\ w_4 * w_1 &\neq anti(\{w_2, w_4\}) \\ \mathcal{H} &\neq anti(w_2) \cup anti(w_4) \\ \mathcal{H} &\neq \{w_1, w_3\}. \end{aligned}$$

Hence this shows that $neut(w_1) * neut\,(w_2) \neq neut(w_1 * w_2)$ and $anti(w_1) * anti\,(w_2) \neq anti(w_1 * w_2)$.

Theorem 4. *Let $(\mathcal{H}, *)$ be a pure left neutrosophic $\mathcal{LA}$-semihypergroup. Then $neut\,(anti\,(w_1)) = neut\,(w_1)$.*

Proof. Let $neut\,(anti\,(w_1)) = neut\,(w_1)$. If we put $anti\,(w_1) = w_2$, then

$$\begin{aligned} neut\,(w_2) &= neut\,(w_1)\,. \text{ Post multiply by } w_2 \\ neut\,(w_2) * w_2 &= neut\,(w_1) * w_2 \\ w_2 &= neut\,(w_1) * w_2 \\ anti(w_1) &= neut\,(w_1) * anti(w_1), \text{ as } w_2 = anti\,(w_1) \\ anti\,(w_1) &= anti\,(w_1)\,. \text{ By Theorem 1 } neut\,(w_1) * anti(w_1) = anti(w_1) \end{aligned}$$

Hence $neut\,(anti\,(w_1)) = neut\,(w_1)$. □

Theorem 5. *Let $(\mathcal{H}, *)$ be a pure left neutrosophic $\mathcal{LA}$-semihypergroup. Then $anti\,(anti\,(w_1)) = w_1$.*

Proof. Consider $anti\,(anti\,(w_1)) = w_1$. Post multiplying both sides $anti\,(w_1)$

$$\begin{aligned} anti\,(anti\,(w_1)) * anti\,(w_1) &= w_1 * anti\,(w_1) \\ neut\,(anti\,(w_1)) &= (neut\,(w_1) * w_1) * anti\,(w_1) \\ neut\,(anti\,(w_1)) &= (anti\,(w_1) * w_1) * neut\,(w_1) \text{ by left invertive law} \\ neut\,(anti\,(w_1)) &= neut\,(w_1) * neut\,(w_1) \\ neut\,(anti\,(w_1)) &= neut\,(w_1) \text{ by Theorem 1 } neut\,(w_1) * neut\,(w_1) = neut\,(w_1) \\ neut\,(w_1) &= neut\,(w_1) \text{ by Theorem 4} \end{aligned}$$

Hence $anti\,(anti\,(w_1)) = w_1$. □

Definition 11. *Let $(\mathcal{H}, *)$ be a neutrosophic $\mathcal{LA}$-semihypergroup and let K be a subset of $\mathcal{H}$. Then, K is called neutrosophic triplet $\mathcal{LA}$-subsemihypergroup, if K itself is a neutrosophic triplet $\mathcal{LA}$-semihypergroup.*

Example 6. *Let $\mathcal{H} = \{w_1, w_2, w_3, w_4, w_5\}$ be a set with the hyperoperation defined in the table as follow*

$*$	w_1	w_2	w_3	w_4	w_5
w_1	$\{w_1, w_3\}$	w_2	$\{w_2, w_3\}$	$\{w_4, w_5\}$	w_5
w_2	$\{w_2, w_3\}$	$\{w_2, w_3\}$	$\{w_2, w_3\}$	$\{w_4, w_5\}$	w_5
w_3	$\{w_2, w_3\}$	$\{w_2, w_3\}$	$\{w_2, w_3\}$	$\{w_4, w_5\}$	w_5
w_4	$\{w_4, w_5\}$	$\{w_4, w_5\}$	$\{w_4, w_5\}$	w_4	w_5
w_5	w_5	w_5	w_5	w_5	w_5

A Cayley table 5

Here $(\mathcal{H}, *)$ is an $\mathcal{LA}$-semihypergroup, because the element of $\mathcal{H}$ satisfies the left invertive law. Here (w_1, w_1, w_1), (w_2, w_3, w_3), (w_3, w_2, w_3), (w_4, w_4, w_4) and (w_5, w_4, w_4) are neutrosophic triplet. Hence $(\mathcal{H}, *)$ is a neutrosophic triplet $\mathcal{LA}$-semihypergroup. Let $K = \{w_1, w_2, w_3\}$ be subset of $\mathcal{H}$. As K is a neutrosophic $\mathcal{LA}$-semihypergroup under the $*$. Then K is called neutrosophic triplet $\mathcal{LA}$-subsemihypergroup of $\mathcal{H}$.

Lemma 1. *Let K be a non-empty subset of a neutrosophic triplet $\mathcal{LA}$-semihypergroup $\mathcal{H}$. The following are equivalent.*

1. K is a neutrosophic triplet $\mathcal{LA}$-semihypergroup.
2. For all $w_1, w_2 \in K$, $w_1 * w_2 \in K$.

Proof. The proof is straightforward. □

Definition 12. *Let $(\mathcal{H}_1, *_1)$ and $(\mathcal{H}_2, *_2)$ are two neutrosophic triplet $\mathcal{LA}$-semihypergroups. Let $f : \mathcal{H}_1 \longrightarrow \mathcal{H}_2$ be a mapping. Then f is called neutro-homomorphism if for all $w_1, w_2 \in \mathcal{H}_{1,}$ we have*

1. $f(w_1 *_1 w_2) = f\,(w_2) *_2 f\,(w_2)\,,$
2. $f\,(net\,(w_1)) = neut\,(f\,(w_1))\,,$
3. $f\,(anti\,(w_1)) = anti\,(f\,(w_1))\,.$

Theorem 6. *Let $f : \mathcal{H}_1 \longrightarrow \mathcal{H}_2$ be a neutro-homomorphism. Where $\mathcal{H}_1$and $\mathcal{H}_2$ are two neutrosophic triplet $\mathcal{LA}$-semihypergroup. Let*

1. The image of f is a neutrosophic triplet $\mathcal{LA}$-subsemihypergroup of $\mathcal{H}_2$.

2. The inverse image of f is a neutrosophic $\mathcal{LA}$-subsemihypergroup of $\mathcal{H}_1$.

Proof. The proof is straightforward. □

Remark 3. *We have the following key points;*

1. Every neutrosophic triplet $\mathcal{LA}$-semihypergroup is an $\mathcal{LA}$-semihypergroup, but the reverse may or may not true.
2. In neutrosophic triplet $\mathcal{LA}$-semihypergroup, every element must have a left $neut\,(.)$, but in an $\mathcal{LA}$-semihypergroup the left $neut\,(.)$ of an element may or may not exist.
3. In neutrosophic $\mathcal{LA}$-semihypergroup, every element must have left $anti\,(.)$, but in an $\mathcal{LA}$-semihypergroup the element may or may not have semihypergroup.
4. In neutrosophic $\mathcal{LA}$-semihypergroup pure left $neut\,(.)$ is not equal to pure left Identity.

4. Application

Neutrosophic triplet $\mathcal{LA}$-semihypergroups has many applications in different areas. Here, we present an application of neutrosophic triplet $\mathcal{LA}$-semihypergroup in football. We can use different versions of neut and anti elements like left, right, pure left and pure right that we may see in different situations. The interesting prospect of this newly defined structure is that it is not comutative, so any change from the left and same types of change from the rigth of a certain element may affect the final results with respect to neut and anti.

Consider a Football team; the centre midfield player "C_m" having a degree of performance d_1. The players "C_{ml2}" and "C_{mr1}" are the midfield player having degree of performance d_1. Thus using Definition 6, the $neut\,(C_m) \in \{C_{ml2}, C_{mr1}\}$. The players "C_{ml1}" and "C_{mr2}" are having better degree of performance d_2, thus using Definition 6, the $neut\,(C_m) \in anti\,(C_m) * Cm \cap C_m * anti\,(C_m) = C_{ml1} * C_m \cap C_m * C_{mr2} = \{d_1, d_2\}$ Neutrosophic triplet $\mathcal{LA}$-semihypergroup can help the coach to select the players for filling the position in the playground, when a player gets injured. The major advantage of neutrosophic triplet $\mathcal{LA}$-semihypergroup is that if we have a centre mid player and this player has the other players having the same performance on the right side as neut of it and it has one player on the left having better performance than it as shown in the following Figure 1.

If the performance of a player playing on the left side and right side of a centre mid player is equal to performance of a centre mid player then the structure reduces to a duplet structure. Similarly, we can find many applications in different directions.

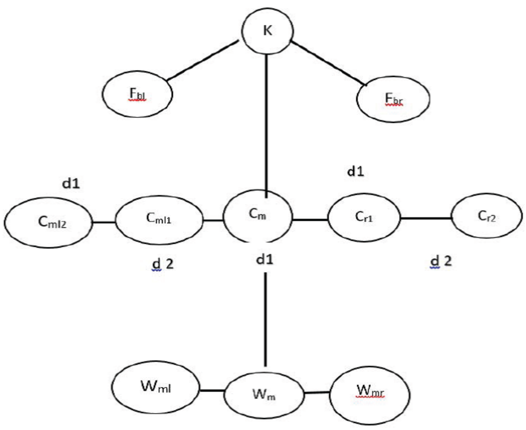

Figure 1. A view of football match.

5. Conclusions

In this paper, we apply the idea of neutrosophic triplet sets at the very useful non-associative hyperstructures, namely $\mathcal{LA}$-semihypergroups. We define neutrosophic triplet set (left, right, pure left, pure right). We discuss some basic results and an application of the proposed structure at the end. In future, we are aiming to extend this idea and give more interesting results.

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