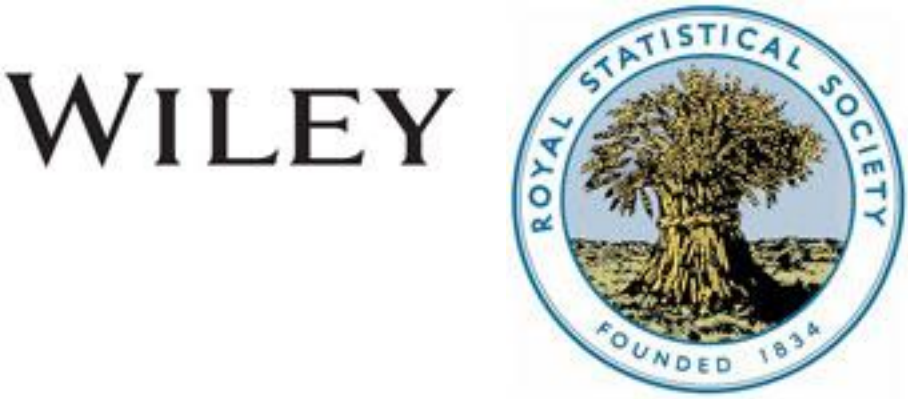

The Applications of the Method of Correlation to Social and Economic Statistics
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Source: *Journal of the Royal Statistical Society*, Vol. 72, No. 4 (Dec., 1909), pp. 721-730
Published by: Wiley for the Royal Statistical Society
Stable URL: http://www.jstor.org/stable/2340140
Accessed: 25-06-2016 04:37 UTC

confidence placed near 20*s.* + $\frac{10}{18}$ of 5*s.* = 22·8*s.* As we are making the minimum estimate we will call this 22*s.* 6*d.*

3. The most plausible estimate:—

	Number in group	Under 20*s.*	20*s.* to 25*s.*	25*s.* to 30*s.*	30*s.* and over.
A	6,500,000	1,200 000	1,800,000	1,200,000	2,300,000
B	1,000,000	800,000	200,000	0	0
C	500,000	500,000	0	0	0
Together	8,000,000	2,500,000	2,000,000	1,200,000	2,300,000

The median now is at 20*s.* + $\frac{15}{20}$ of 5*s.*, *i.e.*, 23*s.* 9*d.*, and from the two previous calculations we know this to be correct within 5 per cent.

It is probable that in the course of two years we shall have material capable of giving an estimate as close as this for Great Britain, and very likely for the whole of the United Kingdom. Actually I believe that I could make the calculation with material already existing.

The question I wish to bring earnestly before the members of this Institute is, whether it is not possible, from their knowledge of the meaning of the statistics existing in their own countries, and of the economic condition of the groups concerning whom exact information is wanting, to make similar estimates, correct at any rate within 10 per cent., for international comparison.

II.—*The Applications of the Method of Correlation to Social and Economic Statistics.*[1] By G. Udny Yule.

The method of correlation has found many applications of recent years to the study of biology, more probably than to any other branch of science in which statistical methods are of service, but the applications to the problems that specially interest the student of social and economic statistics have, it seems to me, been relatively scanty. The following brief survey of the development of the method and of the work that has been done, so far as it is known to me, has been written in the hope of directing the attention of others to this very interesting method, and of suggesting further applications.

[1] Slightly condensed from a paper read before the Twelfth Congress of the International Statistical Institute at Paris, July, 1909.

Development of the method.—The method of correlation is only an application to the purposes of statistical investigation of the well-known *method of least squares.* It is impossible, therefore, entirely to separate the special literature of the theory of correlation from that of the theory of error or of the method of least squares. Of the numerous memoirs on the theory of error the most important in the present connection is that of A. Bravais (1),[2] who as long ago as 1846 discussed the theory of error for points in space, regarding the errors as either independent or correlated, from the standpoint of the normal law of errors. He did not, however, use a single symbol for a correlation coefficient, although the product-sum formula may be regarded as due to him, in a sense—*i.e.*, the product-sum is introduced into his expression for the frequency of various combinations of errors. Sir Francis Galton was the first to devise the practical statistical method (2 and 4), and it is to him that we owe the conception of a numerical measure of the intensity of correlation—a "coefficient of correlation," as it is now termed, or "Galton's function," as it was called for some time. Sir Francis Galton also discussed the distribution of frequency exhibited by the data with which he was dealing, and in an accompanying memoir (3 and 5) Mr. J. D. H. Dickson showed that the characteristics observed were in accordance with the law of error. The general theory of two, three or more variables was investigated, again from the standpoint of the normal distribution of frequency, by Edgeworth (6) and by Pearson (7), and the latter introduced into practice, in the memoir cited, the product-sum formula for the correlation coefficient. The method used by Galton to determine his coefficient was a graphical approximation.

As the form of frequency distribution given by the law of error is not common in economic statistics, a deduction of the formulæ of correlation, and of their properties, without reference to the form of the frequency distribution is of value; this the present writer attempted in (10) and (11), the latter including worked out arithmetical examples that may be of use to the student. The author's deduction has been justified by Edgeworth (17 and 35) and by Bowley (20), on the ground that the normal law will hold for averages of samples of x_1 and x_2, even though it may not hold for the single observations, and that the correlation is the same in the two cases. The formula for partial correlation (or net correlation as I first termed it, the word partial being suggested shortly after by Professor Pearson) was first given, I believe, in (10). In the general case of n variables, the mathematics of the subject become somewhat complicated, and a special notation devised by the present writer (34) permits of considerable simplification both in the algebra and the arithmetical processes.

A formula for the probable error of the coefficient of correlation was given by Pearson in (7), but is slightly erroneous; the correct result was given two years later in the memoir by Pearson and Filon (12). The result applies in strictness to the case of the normal distribution of frequency only.

[2] The references are to the Bibliography on p. 729.

I have already referred to (11) as containing worked-out illustrations which may be of service to those unfamiliar with the necessary work. The reader will also find proofs and illustrations in Mr. Bowley's *Elements of Statistics* (14), and in his small book on *The Measurement of Groups and Series* (20), as well as in Mr. Palin Elderton's work (27), in which the illustrations are mainly taken from cases of interest to actuaries, in the memoir by M. Lucien March (24), and in the recent communication by Professor Bresciani in the *Giornale degli Economisti* (39). For those who are not well endowed with mathematical knowledge Mr. Hooker's "elementary explanation" (36) will be of advantage, and Mr. Darbishire's tables (32) afford an extraordinarily pretty graphical illustration, based on actual records of the throws of dice, of the significance of values of the correlation coefficient ranging from 0 to 1 by steps of $\frac{1}{12}$.

The principal direction in which the theory of correlation has been recently, and probably will be further, extended is the consideration of cases in which the use of regression equations of non-linear forms is essential. Professor Pearson has considered such cases in (25), apart from any theory of frequency; and, indeed, the second part of the memoir (19) may also be regarded as a contribution to the subject. Mr. Blakeman's discussion of tests for linearity of regression may also be referred to in this connection (22). From the standpoint of the theory of frequency, Professor Edgeworth's work on the generalised law of error (26) is perhaps the most important of recent years.

Illustrative applications.—The first case of economic interest to which the theory of correlation was applied was, I believe, that discussed in two notes published in the *Economic Journal* for December, 1895, and December, 1896 (8). Mr. Charles Booth had argued that there was little evidence that the giving of out-door relief—*i.e.*, relief in the home of the recipient—under the English poor law, in lieu of relief in the workhouse, had any marked tendency to increase pauperism, the unions (districts) in which out-relief was given freely not being appreciably worse than the unions which had adopted a strong policy against the giving of out-relief. In the first note I worked out the correlation between *total pauperism* (percentage of the population in receipt of relief of any kind on one day in the year) and the *out-relief ratio*, or ratio of the number in receipt of out-relief, to the number in receipt of relief in the workhouse, for all the unions of England in 1871 (593 unions) and 1891 (580 unions). The coefficients of correlation were $+0{\cdot}26$ and $+0{\cdot}39$, respectively. In the second note the pauperism of males over 65 years of age was considered on the basis of two returns issued in 1890, and, dividing the unions into groups according to their rural or urban character, correlations were found ranging from $+0{\cdot}08$ to $+0{\cdot}43$. In a footnote the results were given of an interesting little investigation into the relations subsisting between (1) pauperism, (2) out-relief and (3) the earnings of agricultural labourers in rural unions. Taking the estimated average earnings of such labourers in 38 rural unions investigated by the Labour

Commission, the *total* and *partial* correlation coefficients found were:—

	Correlation.	
	Total.	Partial.
Earnings and pauperism	− 0·66	− 0·74
Out-relief and pauperism	+ 0·60	+ 0·70
,, earnings	− 0·12	+ 0·46

These figures were of considerable interest. In the first place it seemed that administration in such unions, as indicated by the proportion of out-relief, was nearly as important a factor in determining the amount of pauperism as poverty itself (earnings). In the second place the value of the partial coefficient between pauperism and out-relief (+ 0·70) eliminated the possible argument that pauperism and out-relief were merely found together because both were natural sequents of poverty. Finally, the positive partial correlation between earnings and out-relief suggested that the giving of out-relief at the time the paper was written could have no sensible effect on the normal wage-rate, whatever may have been the case earlier in the century when such relief was given much more freely. It remained possible, of course, that it might influence the age up to which normal wages were given, or the semi-charitable wages given to the old.

These investigations had interested me in the problem of pauperism, and were followed by a much more elaborate investigation, published in the *Journal of the Royal Statistical Society* of London three years later (13); the memoir was termed part i, as I hoped to return to the subject and clear up some points that were left obscure, but part ii has never been written. In this investigation the method was changed, in so far as the quantities correlated were changes in different districts during a given interval, instead of the values at a given time; the method seemed better adapted to the facts, for even though a reduction in the proportion of out-relief were always followed by a reduction in pauperism, it would not be a necessary consequence that at any one time pauperism and out-relief should be positively correlated. Further, the relation of changes in pauperism in every district to three other variables were now considered, viz., changes in (1) the ratio of out-relief to indoor relief, (2) the proportion of the aged (over 65 years of age) in the population, (3) the population itself—the growth or decrease of the population of each district being the best index I could find to its industrial prosperity. The changes in each of these variables were tabulated for all the unions of England for the two inter-censal decades 1871-81 and 1881-91, and the correlations worked out for four separate groups of unions, classified by density of population. The results were discussed by means of the regression equations in four variables, the partial correlations not being used. The full conclusions are too lengthy to reproduce here, but it may be stated that changes in the total pauperism were markedly

correlated with changes in the out-relief ratio and very little with changes either in the total population or in the proportion of old. The issue of the report of the Poor Law Commission in England, and the reconstruction of the Poor Law to which it will probably lead, render any further examination of the British pauperism statistics of recent decades of little more than historic interest, but I cannot help thinking that the statistics of other countries should offer analogous data the discussion of which would serve a useful purpose.

Following the historical order, the next subject of economic interest to which we find the method of correlation applied is the relation between the marriage-rate and the price of wheat in England and Wales, over a period of years, taken as an illustration by Mr. Bowley in his *Elements of Statistics* (14). But such cases offer special difficulty, for the changes in each of the variables are of two distinct kinds, (*a*) slow secular movements, (*b*) more or less rapid changes, of sensible magnitude from year to year, and often of a quasi-periodic character; the slow movements may be quite unrelated while the short-period changes are in extraordinarily close correspondence. Mr. Hooker (15) suggested a very simple means of obviating this difficulty, namely, correlating the deviations of the two variables from the *instantaneous* average of each, instead of from the average of the whole period; for practical purposes the "instantaneous average" may be defined as the mean of the values in the seven, nine, or eleven years surrounding each year. Using this method, Mr. Hooker found a high correlation between the wave-like movements in the marriage-rate, and those in imports, in exports and in the clearing-house returns, and a somewhat smaller correlation with the movements in the price of wheat. Further, he not only correlated the deviation of the marriage-rate with the deviation of the measure of trade in the same year, but also with that in the years following and preceding. Thus the correlation of the deviation in marriage-rate with that of the amount of clearing in the following year is $-0{\cdot}19$, in the same year $+0{\cdot}47$, in the year before $+0{\cdot}92$, in the year but one before $+0{\cdot}76$; interpolating between these figures, the maximum correlation would appear to subsist between the marriage-rate and the clearing of about a year and a quarter before. It would appear then that the movement of the marriage-rate lags by about a year and a quarter behind the clearing-house returns. In his memoir of 1905 (24), M. March gives several illustrations drawn from vital statistics, using both a method practically identical with that just described (but, apparently, obtaining the instantaneous average by graphical interpolation), and also the method of correlating the changes from year to year, a method that was likewise suggested by Mr. Hooker (16) and applied to problems in another field. Amongst other examples, M. March takes the relation between fluctuations in the marriage-rate and unemployment, and in the marriage-rate and the birth-rate, in England. For the correlation between annual movements in the marriage-rate and in unemployment he finds the value $-0{\cdot}73$. In the following year, not knowing of M. March's work, I applied

Mr. Hooker's moving-average method to similar data, and found a correlation of −0·87 (31, p. 96). For the correlation between fluctuations in the marriage-rate and the birth-rate in the same and the immediately following years, M. March finds the successive (positive) values 0·076, 0·278, 0·341, 0·329, 0·041; interpolation between these figures would seem to give a maximum correlation between the marriage-rate and the birth-rate of about 2·24 years after. Working again at precisely the same problem (31, p. 123), but not using data for quite the same period, I found for the correlations of the marriage-rate with the birth-rate of one, two, and three years after (deviations from instantaneous means) the values 0·352, 0·749, 0·418. These are slightly higher values than M. March's, the deviations in the earlier years that he had included being somewhat discordant, but they give almost the same value for the lag of the birth-rate with respect to the marriage-rate, viz., 2·17 years. In the memoir just cited, considerable space was devoted to the discussion of the actual nature of the relation between these fluctuations in the marriage-rate and trade, and it was concluded that the marriage-rate was more strictly related not to the actual value of the trade in the same or in any preceding year, but to the difference between the trade in the same year and the trade in the fifth or sixth preceding year; for the full argument reference must be made to the original. Mr. Heron, to elucidate the relation between fertility and social status (28), has discussed the correlation between the birth-rates (per 1,000 married women of fertile ages) and various measures of position in the social scale for different districts of London, at two different times. As measures of the social standing of the district were used such data as the proportion of servants to the population, the proportion of professional men, the proportion of general labourers, and the proportion of pawnbrokers. The correlations are in 1901 all high, and negative for any measure positively correlated with high social standing, *i.e.*, the birth-rates, as usual, are highest in the lowest strata of the population. But the correlations have undergone a great change between 1851 and 1901. In the former year they are much smaller, and may possibly be accounted for solely by the younger average age of wives in the lower classes; the necessary data as to age are lacking in the 1851 census. In 1901 the correlations are much larger, and cannot be accounted for by the different age-distributions of wives. Further, one very remarkable change has supervened in the last decade only. In 1851 the correlation between birth-rate (for wives) and infantile mortality was negative; it remained negative, though becoming gradually insensible till 1891, but in 1901 it was large and positive.

In such problems of vital statistics as those dealt with in the preceding memoirs there would seem to be still a very wide field for which the present method is especially adapted. Not only have the memoirs cited referred almost solely to English statistics, but more light has yet to be thrown, I feel certain, on the decrease of the birth-rate, and on the nature of the relation between the fluctuations of trade and the oscillations of the marriage-rate. Further, very little work has been done in connection with the

statistics of mortality, and I feel sure that the method could elucidate the influence of statistically measurable conditions on the death-rate. Newsholme (30 and 38) has used the correlation coefficient in a discussion of the causes of the decrease of phthisis, but this is the only work that I can cite. The relation between the occupation of married women for gain, in factories or otherwise, and infantile mortality, is, for example, one problem still awaiting fuller treatment.

Passing to another subject, the first application to statistics of prices we find in the paper by Mr. Hooker on the suspension of the Berlin produce exchange, published in 1901 (16). Mr. Hooker worked out the correlations between the daily corn prices at Berlin, Liverpool and Chicago for the years between 1892 and 1900, in order to see whether the Berlin market was as intimately connected with the rest of the world during the period 1897-99, while the exchange was suspended, as previously. The results were irregular and unsatisfactory, and Mr. Hooker concluded that it would be better to correlate the daily price movements instead of the prices themselves. The results of the altered procedure were not published till 1905 (23). They were more consistent than with the former method, though the coefficients were smaller, and on the whole it seemed that Berlin prices were not less dependent upon the quotations at other markets while the produce exchange was suspended than they were before. Mr. Hooker gives, in the same note, an excellent illustration of the necessity for eliminating the secular movement when correlation between short period changes only is to be expected. For the years 1870-99 the correlation between the total production of maize in the United States and the farm price of maize in Iowa is $-0{\cdot}28$ only; the correlation between the annual changes is $-0{\cdot}84$. An illustration used in a short note on a point of theory by Mr. Hooker and myself (29) possesses also some interest of its own. It was shown that the price of wheat in England appears to exert almost as much influence on the amount of wheat exported from India as does the amount of the Indian crop, the partial regressions of the exports on the amount of the crop and on the price in England being almost the same for the period considered (1889-1904). I know of no other papers in which the theory of correlation is applied to the investigation of the relation between the prices of commodities in different markets, or between prices and supply, unless the terms may be applied to the "price" and supply of money. Dr. J. P. Norton published in 1901 (18) a small volume of most interesting studies in the New York money market, in which the method is freely used in a very able manner. Tables are given showing the correlation between the ratio of reserves to deposits in American banks and the rate of discount, and also between the reserve deviations and the rate of discount. Apart from their immediate application, the former tables are of some special interest, as the regression is very far from linear; it may, in fact, be closely represented by a rectangular hyperbola. Dr. Norton also discusses the correlations between some periodic movements by an "instantaneous average" method, but uses an

interpolated logarithmic curve, instead of the mean of a small number of observations, like Mr. Hooker, to obtain his instantaneous average. Thus, for the correlation between the movement in the reserves and that in the loans in the same and the immediately following weeks, he finds the values 0·489, 0·615, 0·872, 0·958, 0·914. It is clear that the maximum in the annual oscillation of the loans occurs a little more than three weeks after the maximum in the reserves. The only other application of correlation to financial statistics that I can cite is one of the illustrations in M. March's memoir (24), in which he utilises the annual returns of the Bank of France respecting reserves, discount, deposits and payments, &c.

The influence of the weather on the crops is of great economic importance, and no excuse is necessary for introducing a notice of an investigation thereon into this brief survey. The memoir by Mr. Hooker (33) refers only to the crops in one large district of England, but it is to be hoped that it will be followed by similar investigations in other countries for which the necessary data are available. Indeed some work appears to have been done already for crops in India, but I have not been able to include the memoir in my bibliography since I have only seen a brief abstract (Asiatic Society of Bengal, Calcutta, Mr. S. M. Jacob, "On the correlation of areas of matured crop and the rainfall, and certain allied problems in agriculture and meteorology": abstract in *Nature*, London, 18th March, 1909, p. 89). Mr. Hooker considers ten different crops, for which annual estimates of yield are available for twenty-one years, and, as factors of the weather, rainfall and accumulated temperature above 42° Fahrenheit. The weather factors are averaged for periods of eight weeks overlapping by four weeks, and the crop is correlated with the weather in each eight-weekly interval from a period near, or rather later than, the usual harvest, back to a time preceding the previous harvest. The inclusion of a period so long before the crop is even sown was found to be necessary, as the weather of one year influences the character of the seed-crop, and hence indirectly the yield of the crop in the next following year; speaking generally, moreover, the results indicate a certain opposition between the conditions necessary for good seed and those necessary for a bulky crop. For the detailed conclusions reference must be made to the original.

The short note by Miss Alice Lee in the *Economic Journal* of last year (37) breaks new ground. Miss Lee correlates the percentage of not-unemployed in certain trades-unions, as shewn by the Board of Trade Returns, with the rate of increase, from year to year, in the value of imports of articles wholly or mainly manufactured, and finds a positive correlation of 0·31. Taking the rate of increase of the not-unemployed with the rate of increase of manufactured imports, the correlation is + 0·47. Miss Lee concludes that manufactured imports do not check but favour the tendency to greater employment. It appears to me, however, that the matter requires further investigation; the result given is mainly dependent on the short period movement of trade; in a period of booming trade imports of all kinds rise and employment increases. The method

does not seem adequate for elucidating the existence or otherwise of a secular relation.

It is unlikely that, in the list of memoirs attached to this review of recent work, I have included all those which should have found a place, and I must apologise in advance for any omissions. In certain cases, of course, memoirs have been deliberately excluded; thus, with the exception of Sir Francis Galton's work, I have purposely omitted all memoirs on heredity and allied subjects, since, though they are often of high importance from the sociological standpoint, I hardly consider that they fell within the intended scope of this note.

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