A Form of Knudsen's Vacuum Manometer

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XIX. *A Form of Knudsen's Vacuum Manometer.* *By* LEWIS F. RICHARDSON.

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IN 1911 the author was in need of a vacuum gauge for measuring pressures of the order of 1 dyne cm.$^{-2}$, or less, in electric lamp bulbs for the Sunbeam Lamp Co. The McLeod gauge, ordinarily in use in the factory, was unsatisfactory for research purposes, because it does not measure the pressure of condensible gases such as the vapours of water, oil or mercury. Sir Joseph Thomson very kindly considered the question and suggested that attention should be directed to the Knudsen Manometer, which is free from this defect.

From the Kinetic Theory of Gases, Knudsen ("Ann. der Physik," XXXII., p. 812) deduced that the pressure of gas in equilibrium in a closed apparatus varies from point to point as the square root of the absolute temperature of the gas, when the dimensions of the apparatus are very small compared with the mean free path of the molecules. This proposition will be referred to as . . . . (1)

Smoluchowski ("Ann. der Physik.," XXXIV., 182) shows that Knudsen's formula requires that the moving plate should be close to fixed plates on *both* sides.

Without going into the general theory, we may notice the peculiarities of some typical cases. Consider a gas confined between two indefinitely large plane parallel plates at uniform but different temperatures, and suppose the gas so rarefied that collisions occur with the plates only. The motion of each molecule is a series of journeys in straight lines between the plates. On each journey the molecule gives out on stopping exactly the translatory momentum in a direction normal to the plates which it received on starting. So the pressures on the two plates must be equal, however widely their temperatures may differ. At first sight this might appear to contradict Knudsen's statement (1), but it does really not do so, for it will be shown that the temperature of the gas is perfectly uniform. The kinetic energy of a particle which last collided with the hot solid is presumably greater on the average than that of one coming from the cold solid. But the temperature, measured by the mean kinetic energy, depends on the relative numbers of these two classes of molecules in the element of volume. Denote this ratio by G. From the symmetry it follows that G is uniform throughout the gas, and so the temperature is also uniform. The only way in which we can set up an inequality of temperature in the gas is by disturbing the symmetry, as, for instance, by confining the hotter area on the plates to a small patch on one of them. In the neighbourhood of the patch, G will differ from its value in the remote parts of the region.

In the present instrument the temperature is nearly uniform over each plate, but the symmetry is disturbed by the insertion between them of a small disc, which may be said to separate off a pair of compartments one on either side of it, communicating round its edge. Suppose, for simplicity, that the disc is midway between the fixed plates and parallel to them. The rates at which molecules are shot into the two compartments from the distant parts of the apparatus must be approximately equal. To the same approximation in the steady state the particles must be shot out of the two compartments at equal rates. But the rate of loss from either compartment must be proportional to the density and to the mean velocity in that compartment, as in the effusion experiments of Graham and Osborne Reynolds (Jeans, "Dynamical Theory of Gases," 2nd edition, § § 168, 170). Hence, when we compare the compartments with one another, the square roots of the absolute temperatures vary directly as the densities, and by Boyle's law inversely as the pressures, as in (1) above.

Knudsen constructed an instrument operated by these local differences of pressure. However, the instrument, as originally described by him, could only be kept in operation for a few minutes at a time, as it depended on an inequality of temperature which obliterated itself soon after it had been set up.

The following instrument was designed in consultation with Mr. R. M. Abraham, of Messrs. C. F. Casella & Co. It was made by that firm in 1912 and 1913.

The hot and cold plates were of glass and about 10 cm. in diameter. They were separated by 0·54 cm. by a glass ring. One of the plates had a hole drilled in it for the insertion of the connection to the vacuum pump. The joints were made tight by the special soft sealing wax in use at Owen's College, Manchester.

The flat glass box, so formed, was placed between two massive copper slabs each 1·15 cm. thick.

A difference of temperature of about 10°C. could be maintained indefinitely between the glass plates by warming one of the copper slabs by means of a small gas flame, or by cooling it with ice. To eliminate electrostatic forces the inner faces of the glass plates were platinized and were put in electric connection by a wire spring which pressed against both.

The moving system is shown in Fig. 1. Inside the glass box a disc of mica of 3·5 cm. diameter was free to approach or recede from the hotter glass plate by rotation about an axis in the plane of the mica, but some distance beyond its edge. This axis was formed from two pieces of tungsten wire, one above and one below in the same straight line.

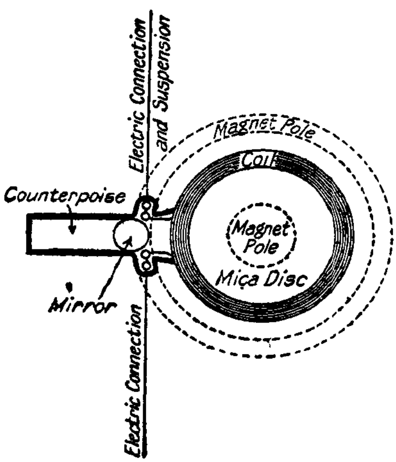


FIG. 1.

The force of the molecular bombardment tending to drive the moving disc away from the hotter glass plate, was balanced by an electromagnetic force. This force was produced by an electric current which flowed in a coil round the edge of the moving disc, and which was acted upon by a magnetic field directed along the radii of the circular coil. The poles of the magnet are shown in Fig. 1; they lay one on either side of the glass box. The object of this device was that the electromagnetic effect on the suspended system should be, not the usual couple, but a single force acting at the centre of the moving disc and at right angles to it.

The angular position of the moving system was observed by the light reflected from a mirror which it carried. The angle was brought exactly to zero by the current. The mica vane was kept approximately midway between the glass plates, but its exact position in this translatory sense could not be observed, and, on Knudsen's theory, did not matter.

A pair of thermo-junctions of copper-eureka were nearly in contact with the outer sides of the glass box opposite the centre of area of the moving system. The E.M.F. given by this circuit was balanced on a potentiometer wire, through which flowed the current in the moving coil.

A defect of the instrument as constructed was a considerable twist in the suspension wire, which required a current J_0 to balance it, when both plates were at the same temperature. When a temperature-difference had been established a different current J_r was required. Thus, $J_r - J_0$ was proportional to the mechanical force produced by the gas. If we denote by r the resistance of the potentiometer wire required to make a balance with the thermo-junctions, then $r \,.\, J_r$ was proportional to the difference of temperature $(T_1 - T_2)$ between the affixed plates. So that the mechanical force per temperature difference was proportional to

$$\frac{1}{r}\left(1 - \frac{J_0}{J_r}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Next, Knudsen's theory can be adapted to present circumstances as follows: The mechanical force per area is the difference of the gas pressures P_1 and P_2 on the two sides of the moving disc. The temperatures of the gas on the two sides may be taken to be $M + \frac{1}{4}\Delta$ and $M - \frac{1}{4}\Delta$, where M is the mean of T_1 and T_2 and Δ is their difference $T_1 - T_2$.

Now, if the gas in the instrument is in communication with a McLeod gauge which has a temperature T_3 and which registers a pressure P_3, then from Knudsen's formula (1) we shall have for the pressures P_1 and P_2 on the two sides of the moving disc:

$$\frac{P_1}{P_3} = \left\{\frac{M + \frac{1}{4}\Delta}{T_3}\right\}^{\frac{1}{2}}; \quad \frac{P_2}{P_3} = \left\{\frac{M - \frac{1}{4}\Delta}{T_3}\right\}^{\frac{1}{2}}, \quad . \quad . \quad . \quad (3)$$

whence, on expanding by the Binomial theorem,

$$\frac{P_1 - P_2}{P_3} = \frac{\Delta}{4\sqrt{MT_3}} \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

if terms in Δ^3 and higher odd powers be neglected.

So $$\frac{4\sqrt{MT_3} \,.\, (P_1 - P_2)}{\Delta} = P_3. \quad . \quad . \quad . \quad . \quad (4)$$

Now, $(P_1-P_2)/\Delta$ is the mechanical force per area per difference of temperature, and hence by (2)—

$$C\frac{\sqrt{MT_3}}{r}\left(1-\frac{J_0}{J_r}\right)=P_3, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where C is a permanent instrumental constant to be determined by experiment. Note that this formula appears to assume that the McLeod gauge and the connecting tube were both small compared with the mean free path. That condition was not satisfied. If the initial twist were zero, and if the temperature T_3 of the McLeod gauge were equal to the mean temperature M of the Knudsen apparatus, then this formula would take the very simple form,

$$\frac{P}{T}=\frac{C}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where T is this temperature; that is to say, the *number of molecules per unit volume would be inversely proportional to the resistance of the potentiometer wire.*

Facilities for testing the apparatus were kindly afforded by Sir Ernest Rutherford at Owen's College.

The apparatus was exhausted by an oil pump and then by charcoal in liquid air, and was left in that state overnight. It was then again exhausted by liquid air twice; thus, it is probable that oil and water vapour were thoroughly removed. On the second occasion the connection to the charcoal tube was left open for half an hour after the McLeod gauge had begun to register a pressure less than 0·1 dyne/cm.^{2}. At the end of this time a reading of both gauges was taken. Next, the charcoal tube was shut off, so that the pressure gradually rose owing to the small leak in the apparatus. During this stage the other readings were taken. The contents were presumably mercury-vapour and air.

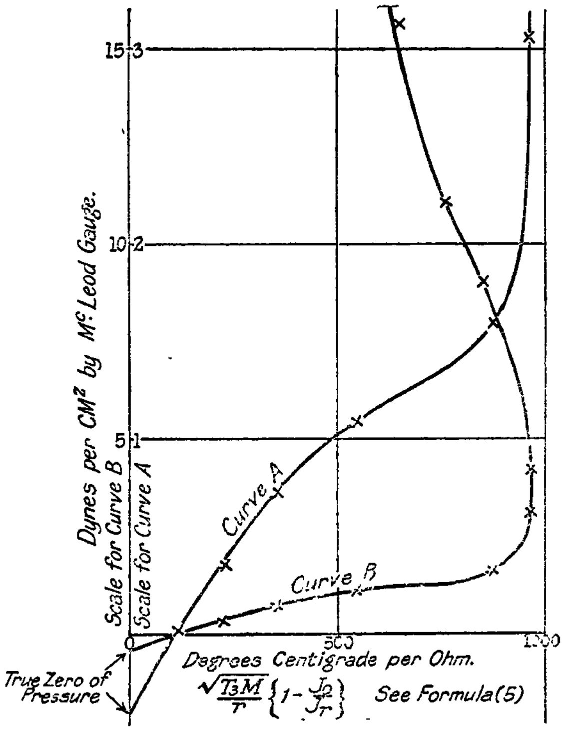
Fig. 2 shows the relation obtained in this way between the pressure P_3 by the McLeod gauge and

$$\sqrt{T_3M} \,.\, \frac{1}{r}\left(1-\frac{J_0}{J_r}\right)$$

from equation (5). The two curves represent the same function on different scales.

By extrapolation of curve A it is seen that the zero error of the McLeod gauge appears on this occasion to be about 0·4 dyne cm.$^{-2}$, which would correspond to half the maximum pressure of mercury vapour at the temperature of the McLeod gauge, namely, 0·8 dyne cm.$^{-2}$ at, say, 13°C.

If we then take the true pressure to be given by the addition of 0·4 dyne cm.$^{-2}$ to the pressure obtained from the McLeod gauge, we may say that the useful range of this instrument of Knudsen's type extended up to 2·0 dynes cm.$^{-2}$. The mean temperature of the glass box was about 300°A. On this basis mean free paths have been computed from the data given for



PRESSURE BY KNUDSEN GAUGE.

FIG. 2.

air by Jeans (" Dyn. Theory of Gases," 2nd ed., p. 341). The top of the useful range was reached at 2·0 dynes cm.$^{-2}$ when the mean free path was six times the distance of 0·54 cm. between the inner faces of the glass box.

Curve *B*, drawn on a reduced scale, shows that the mechanical force per temperature-difference attained a maximum when the corrected pressure was 4·0 dynes cm.$^{-2}$, so that the mean free path was 3·0 times the distance between the fixed plates. The maximum is, of course, outside the range of the formulæ (1) to (6), which only apply to the initial linear part.

The neglected terms in (3a) do not amount to 1/5,000 of the term which is retained, when $\Delta = 17°$C., as it was during the test, so that they cannot account for the curvature of the graph.

The chief difficulty experienced in use was that the electro magnetic force affected the period of oscillation of the coil, and in some circumstances rendered it unstable. Instability could probably be got rid of by new pole-pieces for the magnet, symmetrically designed to give as uniform a field as possible.

The vacuum-tightness of the apparatus may be measured inversely as a leak expressed as the rate of rise of pressure multiplied by the volume of all the connected vacuous cavities. It was found to be 0·2 dyne cm.$^{-2}$, sec.$^{-1}$, cm.^{3}. This would comprise also any evolution of occluded gases.

The resistance of the moving coil and its leading-in wires was 50 to 60 ohms. The current through it to balance the maximum force of the gas was about 1/700 ampere-hour per 10°C. difference between the plates, so that the electric heating of the coil was negligible.

Work upon apparatus had to be abandoned in 1913 owing to the pressure of other operations. Since then a vacuum-manometer depending on viscosity has been brought out by Dr. P. E. Shaw, and investigated theoretically by Mr. F. J. W. Whipple. A Paper on the Knudsen manometer, by J. W. Woodrow, has appeared in the "Physical Review" for December, 1914.

Summary.

The McLeod gauge has a false zero of pressure if condensible vapours are present. The Knudsen instrument is free from this defect, but has a very limited range. It operates on the principle that when the molecules collide only with the solid parts of the apparatus, then they knock a free vane away from a hotter towards a colder surface. The instrument as originally described by Knudsen could only be kept in operation for a few minutes at a time. The action of the present instrument could be maintained indefinitely. Its range extends up to 2·0 dynes cm.$^{-2}$. The force of the molecular bombardment is balanced by the effect of a magnetic field of special form, acting upon an electric current attached to the vane, and

the temperature difference is measured by a thermo-junction, the E.M.F. of which is balanced against the same current in a potentiometer. The instrument was constructed by Messrs. C. F. Casella & Co.

DISCUSSION.

Mr. R. S. WHIPPLE said he was not quite clear as to how the plate moved.

Mr. C. C. PATERSON asked if the size of the tube entering the manometer had much effect on the results.

Dr. D. OWEN asked if it would not be better to measure the temperature difference between the glass plates instead of the copper plates outside. As it was, the instrument had to be calibrated empirically, and could not be used to give a check on Knudsen's formulæ. With regard to curve 2, the author said that the mechanical force per unit temperature difference was a maximum at 4 dynes per square centimetre, and then stated that this lies outside the range of the formulæ. What, then, was the meaning of this part of the curve? What was the lowest pressure which the author was able to measure? If J_0' was nearly equal to J_r, the method evidently became inaccurate. Could not J_0 be eliminated?

The AUTHOR, in reply, said that the plate moved at right angles to its own plane. The length of leading-in tube was about a metre, and its diameter about 1 cm. He did not remember the precise figures. The difference of temperature of the glass plates could easily be measured directly; but it was easy to show that, under steady conditions, the temperature of the copper plate was a very fair measure of that of the glass. As regards a check on Knudsen's theory, there was at least a partial check, inasmuch as the first parts of the curve were straight. The linear law was not assumed, otherwise the diagram would consist of two straight lines. The lowest pressure he had reached was 0·4 dyne per square centimetre, owing to the vapour pressure of mercury in the apparatus. It would be useful to eliminate J_0 by taking extra trouble with the suspension wire; but it did not introduce inaccuracy, as it simply had to be measured.