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XVI. On some relations between the optical and the electrical qualities of metals

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and would then become inappreciable, or at any rate so small that it could not be discharged from the probable effects of selective absorption. The effect of these greater masses is not denied, but their existence would not sensibly affect our present results, which apply only to the electron of small mass carrying a negative charge. Drude, in his discussion of optical constants in connexion with his electron theory of metals, assumed the existence of two kinds of conducting electrons, and could by means of them account separately for the observed values for ν and for κ .

My purpose is a more limited one, but the results obtained seem to me to be more definite on account of the small number of assumptions which are made.

Looking at the results of these calculations, the simplest explanation of metallic conduction would appear to be that each atom has one, or possibly two or three, negative electrons which are easily detached, and follow freely the electric force, even for such rapid oscillations as those of light. Drude assumes that they are always detached from the atom and behave very much as the ions in electrolytes are supposed to behave. He has worked out his idea in two important papers*, and given it substantial support in a variety of directions.

XVI. *On some Relations between the Optical and the Electrical Qualities of Metals.* By Prof. E. HAGEN and Prof. H. RUBENS †.

MAXWELL'S electromagnetic theory of light—which in its original form does not consider the molecules and their vibratory periods, but simply expresses the optical property of a single wave-length—demands the existence of analogous relations between the transparencies of metals and their electric conductivities. These relations have often been examined, without being confirmed in any way ‡. The theory did not seem to hold good, either with regard to the absolute amount of transparency, or with regard to the order in which the metals can be arranged according to their transparency, and which ought to be the order of their electrical resistances.

* Wied. *Ann.* xxxix. p. 537, xlii. p. 189.

† Cf. *Sitzungsberichte der K. Akademie der Wissensch. in Berlin* p. 269 & p. 410 (1903), and *Ann. d. Physik*, xi. p. 873 (1903). Authors translation communicated by Prof. C. Vernon Boys, F.R.S.

‡ Cf. W. Wien, Wied. *Ann.* xxxv. p. 48 (1888); and E. Cohn, Wied. *Ann.* xlv. p. 55 (1892), and others.

In a former paper* we have shown that some of these contradictions begin to disappear if, instead of examining the ultra-violet or the visible spectrum, we advance to longer wave-lengths. We found that platinum, which, in the visible and ultra-violet spectrum, is much more opaque than gold and silver, becomes more transparent than these metals in the infra-red. We have lately stated that the same is true in a higher degree with respect to bismuth. A thin layer of bismuth of about $90 \mu\mu$ thickness, which scarcely transmits 1/1000 part in the red, possesses a transparency of 10 per cent. at a wave-length not longer than $\lambda = 4 \mu$. Consequently it did not seem improbable that, passing to still greater wave-lengths, values in accordance with Maxwell's theory would be obtained for the diathermancy of metals. Supposing Maxwell's theory to be correct, this would mean that the influence of the molecular periods of the different metals vanishes gradually in the infra-red region, with increasing wave-lengths.

Relations quite similar to those that Maxwell's theory demands for the transparency of metals, can be foreseen for the intensity of the radiation penetrating into the metals, and for the power of emission. But these values are much easier to determine than the transparencies. In the first place, investigations of the transparency require considerably greater intensities of radiation. Secondly, no substances exist which—in that part of the spectrum—are sufficiently transparent for heat-rays, and can at the same time serve as supporters of such thin layers of metal †. Lastly, the number of metals suited to the examination of transparencies is much more limited than that suited to measurements of the emission or reflecting-power. This is owing to the fact that the construction of good reflecting-mirrors is much easier than that of metallic layers of equal thickness and perceptible transparency.

The intensity of radiation entering into the metals can be measured in different ways. The simplest method is by determining the reflecting-power, or by measuring the power of emission.

If R represents the reflecting-power of a metal for normal incidence expressed in percents of the incident radiation, the intensity of the radiation entering the metal is $I = (100 - R)$. In those parts of the spectrum in which the

* E. Hagen & H. Rubens, *Ann. d. Phys.* viii. p. 432 (1902).

† Rocksalt, sylvine, and silver chloride, which possess a sufficing transparency, cannot be used in consequence of their unfavourable chemical qualities.

energy suffices for exact measurements, and R is sufficiently different from 100, the determination of I can be successfully performed by measuring the reflecting-power. But it is just for the greatest attainable wave-lengths that these conditions are not sufficiently fulfilled. Therefore, in these regions it is preferable to determine I by measurements of the power of emission. If E be the emission of an opaque polished metal surface, and e that of "a black body" of the same temperature—both for the same wave-length—we obtain $I = E/e$ according to Kirchhoff's law.

Consequently our experimental research consists of two parts. In the first I is determined with the aid of the reflecting-power, in the second by measurement of the power of emission.

I.

The Reflecting-Power of Metals in the Infra-Red.

The reflecting-power of metals has often been the object of experimental research. In a paper published in 1889 by one of us*, the reflecting-power of a series of metals has been examined. From the results the conclusion was drawn that the better conductors of heat and electricity (silver, copper, gold) show a higher reflecting-power for infra-red rays than do the other metals. In addition to this paper, and that of A. Trowbridge†, our own recent research must be mentioned. But owing to the imperfection of the material and the surface of the mirrors employed, the results of the older researches do not permit any definite comparison with the electromagnetic theory. On the other hand, our own measurements‡ did not pass beyond the limit of 1.5μ , a wave-length not sufficing for this purpose.

Method of Observation.

The arrangement of our apparatus is given in fig. 1. H designates the slit of a spectrometer, furnished with silver mirrors (K and L) instead of lenses, in such way that $H K$ represents the collimator, and $L T$ the observing telescope. T is a linear thermopile, described in an earlier paper by one of us§. The prism P was of fluorite for the wave-lengths

* H. Rubens, Wied. *Ann.* xxxvii. p. 249 (1889).

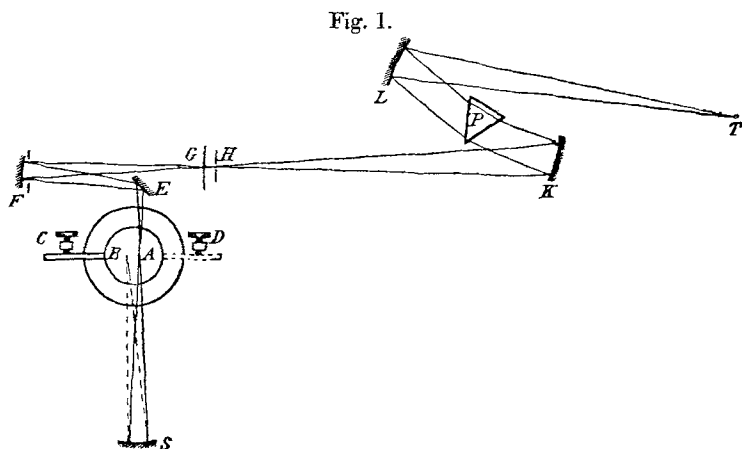
† A. Trowbridge, Wied. *Ann.* lxxv. p. 595 (1898). The papers of S. P. Langley, Phil. Mag. xxvii. p. 10 (1889), E. F. Nichols, Wied. *Ann.* lx. p. 401 (1897), and F. Paschen, *Ann. d. Phys.* iv. p. 304 (1901) contain only measurements of the reflecting-power of silver in the infra-red.

‡ E. Hagen & H. Rubens, *Ann. d. Phys.* viii. p. 1 (1902).

§ H. Rubens, *Zeitschrift für Instrumentenkunde*, xviii. p. 65 (1898).

between 1 and $8\ \mu$, and of sylvine for the interval between 8 and $14\ \mu$. The galvanometer used with the thermopile was an iron-clad instrument of du Bois-Rubens' construction*, well protected against magnetic disturbances. The observer at the galvanometer could, by aid of a special arrangement, lift or drop the screen G, placed in front of the slit H.

In addition to this, the figure shows two supplementary mirrors, E and F, the source of light A, and the concave mirror S the reflecting-power of which was to be examined.



Our source of light was a Nernst incandescent lamp, the fibre of which (1 mm. thick and 2 mms. wide) was attached to a small turning-table in such a way as to permit a kind of excentric rotation. This table, as shown in the figure, was placed between the mirrors S and E. It was furnished with an arm which could be brought into contact with either of the two fixed screws C and D, thus permitting the transference of the Nernst fibre from position A into position B (dotted in fig. 1), and *vice versa*. Immediately in front of the fibre was adjusted a small slit (2 mms. wide), containing two crosses of very thin platinum wire 8 mms. above each other. The excentricity of the Nernst fibre and the position of the slit in front of it were adjusted by two different micrometric screws.

After undergoing reflexions from the plane mirror E and the concave mirror F, the rays coming from A were united on the slit H to an image of exactly the same dimensions as that produced directly by the source of light. The mirror S

* H. du Bois & H. Rubens, *Zeitschrift für Instrumentenkunde*, xx. p. 65 (1900).

was so adjusted that its centre of curvature lay on the axis of the turning-table. By this means an image of equal size also appears on the slit H when the lamp is at B. A proof for the correctness of our adjustment is given by the fact that the images of the two wire crosses on H are equally exact in both positions of the lamp.

We need simply to add that the slit H was 6 mms. high and 1 mm. wide, and that a diaphragm of 24×24 mms. was placed in front of the mirror F, in order to prevent any but the central part of the cone of rays from entering the slit. Thus it is evident that the rays follow the same path in both cases, the only difference being the additional distance B S A when the lamp is at B. On dividing the deflexion obtained in this position by that observed in the first case, the reflecting-power of the mirror S is at once obtained.

We undertook measurements for about 15 wave-lengths of the spectrum. While the Nernst fibre was in the position A, the total distance traversed by the rays was 210 cms., in the position B it amounted to 270 cms. Thus it became necessary to limit the observations to wave-lengths for which the absorption produced by the water-vapour and the carbon dioxide* of the atmosphere is not considerable. For our purpose the absorption is small enough, in most parts of the spectrum, to prevent perceptible errors. An additional distance, small compared with the whole path of the rays, scarcely increases the absorption already produced. Of all the wave-lengths concerning which measurements were made, $\lambda = 7 \mu$ is the only one forming an exception to this rule. Here a correction of about 3.1 per cent. was necessary, owing to the absorption produced by the water-vapour.

The Mirrors.

All the metals were used as concave mirrors of 30 cms. radius and 4 cms. aperture. They were made of very pure material. Three different modifications of silver, gold, and platinum, and two of nickel were examined. All these mirrors, except that of cast bismuth, possessed perfect surfaces and gave excellent images.

Results of the Observations.

Table I. (p. 162) contains the results of our measurements for seven pure metals and for six alloys. The curves of fig. 2 (p. 163) show the reflecting-power of ten of our mirrors for different wave-lengths. In the red, as well as in the neighbouring regions of the infra-red, our observations agree

* Cf. F. Paschen, *Wied. Ann.* li. p. 51, lii. p. 209, and liii. p. 335 (1894); also H. Rubens & E. Aschkinass, *Wied. Ann.* lxiv. p. 598 (1898).

Phil. Mag. S. 6. Vol. 7. No. 38. Feb. 1904. M

TABLE I.—Reflecting-power R of Metals.

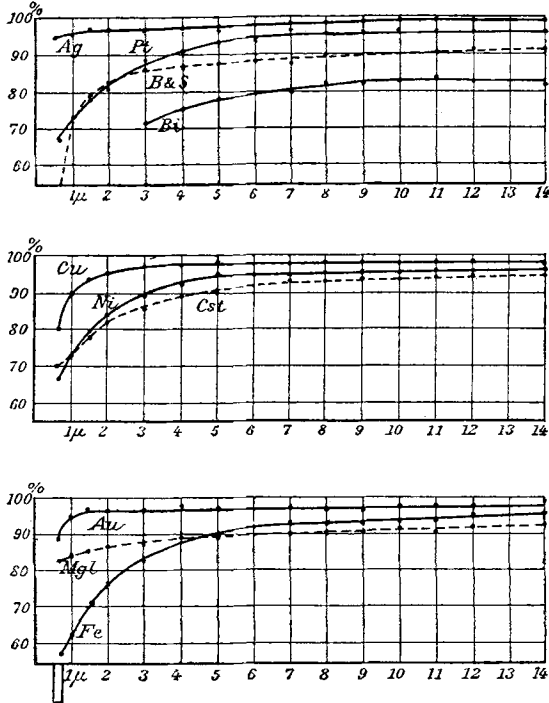
Wave-length λ in μ .	Silver.			Copper.		Gold.			Platinum.		Nickel.		Steel.	Bismuth.		Constantan,		Patent Nickel.		Brades and Schunemann's Alloy 98 [32 Cu + 34 Sn + 29 Ni + 5 Fe.]	Rose's Alloy, 68.2 Cu + 31.8 Sn.	Magnesium, 69 Al + 31 Mg.
	Massive, pure.	Pre-precipitated.	Dispersed.	Massive, pure.	Electrolytic.	Pre-precipitated.	Dispersed.	Electrolytic.	Dispersed.	Massive, pure.	Electrolytic.	Unchilled.	Cast.	60 Cu + 40 Ni.	M.	P.	73.0	51.2	65.3			
0.65*	96.6	95.9	94.6	...	88.6	89.6	86.1	69.8	63.8	67.2	67.3	59.8	...	70.0	71.0	73.0	51.2	65.3	88.2			
0.70	96.1	96.2	...	82.9	92.3	91.3	71.2	71.4	73.8	...	70.5	84.1			
1.0	96.4	97.5	95.5	90.1	...	93.6	72.9	70.4	73.0	72.0	...	63.1	...	72.4	72.6	75.0	...	70.5	85.5			
1.5	97.3	97.9	...	93.8	...	96.7	94.8	77.7	75.3	79.9	78.6	70.8	...	77.8	79.3	79.9	79.1	80.4	86.7			
2.0	97.3	97.8	96.8	95.6	96.8	96.5	94.9	80.6	79.8	83.5	83.5	76.7	...	82.3	83.5	84.1	82.3	86.7				
3.0	97.3	98.1	97.4	97.1	...	95.6	88.8	88.5	88.5	88.1	88.7	83.0	71.7	82.6	89.1	88.8	85.8	86.2	87.4			
4.0	97.7	98.5	97.6	97.3	96.0	97.2	96.0	91.5	91.6	92.5	91.1	87.8	75.2	88.3	91.4	92.1	87.1	88.5	88.7			
5.0	97.3	98.1	97.3	97.9	97.0	96.9	95.7	93.5	90.8	94.7	94.4	89.0	77.2	89.5	92.2	92.2	87.3	89.1	89.0			
7.0	98.5	98.5	98.3	98.3	98.3	97.3	95.7	95.5	93.0	94.8	94.3	92.9	79.5	92.7	92.3	92.1	88.6	90.1	90.0			
8.0	99.0	98.5	98.1	98.6	97.8	96.8	96.1	95.1	92.5	95.0	95.7	93.0	81.5	92.7	93.5	92.5	89.8	91.0	90.9			
9.0	98.9	98.7	98.1	98.4	98.0	96.7	96.1	95.4	92.5	95.6	95.6	92.9	81.4	93.0	93.7	92.2	90.3	92.2	90.6			
10.0	99.0	98.7	98.5	98.5	97.8	97.7	96.4	95.9	93.1	95.4	95.6	93.5	82.3	94.0	93.0	92.6	90.6	92.9	91.8			
11.0	99.0	98.8	98.8	98.4	98.3	97.7	96.5	95.6	92.7	95.9	95.9	94.0	83.2	93.4	93.0	93.8	90.2	92.9	90.7			
12.0	98.9	98.8	98.1	98.4	97.8	97.9	97.2	96.5	94.9	95.7	96.1	95.1	82.2	94.0	93.0	94.3	90.9	92.9	92.5			
14.0	98.8	98.3	...	97.9	97.9	98.7	96.7	96.4	94.7	95.6	97.2	96.0	81.6	94.2	92.6	93.4	90.3	93.6	92.2			

* As $\lambda = 0.65$ and $\lambda = 0.7 \mu$, a water-cell was introduced into the path of rays in order to exclude the influence of diffused infra-red radiation.

very satisfactorily with the results obtained in our previous researches.

It is well known that the reflecting-power of metals is subject to very rapid changes in the ultra-violet and visible spectrum. That is not the case in the infra-red; for this spectral region the curves of fig. 2 show a very regular form

Fig. 2.



Cst. stands for Constantan, *Mgl.* for Magnalium, *B. & S.* for Brandes and Schünemann's alloy. The curves for Patent Nickel M and P lie very close to that of Constantan. The visible spectrum (from $\lambda=0.45 \mu$ to $\lambda=0.7 \mu$) is marked by the two small vertical lines at the left-hand corner of the curves.

for all the metals and alloys. In every case the curve rises asymptotically to $R=100$ per cent. On comparing the reflecting-powers of different metals with each other in the region of greater wave-lengths, we find that—relative to their reflecting-power—the metals always take the same order of succession. This law is still more obvious when we consider the intensity entering the metals ($100-R$) instead of the reflecting-power (R). Although $(100-R)$ diminishes continually with the increasing wave-length, it does so in the same

proportion for all the metals, so that it is of no consequence at what part of this spectral region the comparison is made.

Table II. contains the values of $100 - R$ at $\lambda = 4 \mu$, 8μ , and 12μ for all the pure metals examined and for five alloys, the electric conductivity of which was accurately known. The numbers given for $(100 - R)$ are always the average values obtained for the different modifications of the same material. The only results we have herefrom excluded are those obtained by aid of the mirrors produced by cathodic dissipation. Their reflecting-power was found to be a little smaller than that of the other mirrors. For silver, this difference is scarcely perceptible; for gold it amounts to 1 per cent., for platinum to 2-3 per cent. We cannot decide whether these differences are due to a deviating molecular structure of the dissipated layers, or whether they are caused by a very slight oxidation of the metals.

In addition to the value $(100 - R)$, Table II.* contains the electric conductivity κ †, its square root, and finally the product $(100 - R) \cdot \sqrt{\kappa}$ for $\lambda = 4, 8$, and 12μ . It is evident that this product has approximately the same value with all the metals for $\lambda = 12 \mu$; *i. e.* the intensities entering the metals are—in the region of long waves—in inverse proportion to the square root of the electrical conductivity,

$$(100 - R)\sqrt{\kappa} = \text{const.} \quad (1)$$

But also for $\lambda = 8 \mu$, and even for $\lambda = 4 \mu$, this equation is approximately verified. At $\lambda = 4 \mu$ the products $(100 - R) \cdot \sqrt{\kappa}$ vary with an average deviation of 21 per cent. from the number $C_4 = 19.4$. At $\lambda = 8 \mu$ the average value of the products is $C_8 = 13.0$; the average deviation is 14.5 per cent. At $\lambda = 12 \mu$, lastly, the mean value is $C_{12} = 11.0$, and the mean deviation is not more than 9.6 per cent.

The values registered in Table II. for bismuth are bracketed, and not considered in the calculation of the average values of C . The first reason is, because they do not possess the exactness of the other numbers. This is accounted for by the deficiency of the curvature and polish of our bismuth mirrors. The cutting and polishing of cast bismuth is very difficult, because of the crystalline structure of this material and the irregular consistency of its surface. The mirror of cast bismuth could therefore only serve for the investigation of the change of the reflecting-power with increasing wavelength; the absolute values had to be determined with the aid of mirrors made by cathodic dissipation. Although these

* R is given in per cents. of the incident radiation.

† W. Jäger and H. Diesselhorst, *Wissenschaftliche Abhandlungen der Physikal. Techn. Reichsanstalt*, iii. p. 269 (1900).

TABLE II.

Metals.	Conductivity, κ_{18} .	$\sqrt{\kappa_{12}}$.	$\lambda = 4\mu.$		$\lambda = 8\mu.$		$\lambda = 12\mu.$	
			(100-R).	$C_4 = \frac{100-R}{\sqrt{\kappa_{12}}}$	(100-R).	$C_8 = \frac{100-R}{\sqrt{\kappa_{18}}}$	(100-R).	$C_{12} = \frac{100-R}{\sqrt{\kappa_{18}}}$
Silver.....	61.4	7.85	1.9	14.9	1.25	9.8	1.15	9.0
Copper.....	57.2	7.36	2.7	20.6	1.4	10.6	1.6	12.1
Gold.....	41.3	6.43	3.4	21.9	2.7	17.4	2.15	13.8
Platinum.....	9.24	3.04	8.5	25.8	4.6	14.0	3.5	10.6
Nickel.....	8.5	2.92	8.2	23.9	4.65	13.6	4.1	12.0
Steel.....	5.02	2.24	12.2	27.3	7.0	15.7	4.9	11.0
Bismuth.....	0.84	0.916	(24.8)	(22.7)	(18.5)	(16.9)	(17.8)	(16.3)
Patent Nickel P.....	3.81	1.95	7.9	15.4	7.5	14.6	5.7	11.1
Patent Nickel M.....	2.94	1.71	8.6	14.8	6.5	11.1	7.0	12.0
Constantan.....	2.04	1.43	11.7	16.7	7.3	10.6	6.0	8.6
Rosse's Alloy.....	2.07	1.44	11.5	16.6	9.0	13.0	7.1	10.2
Brandes and Schünnemann's Alloy.....	1.48	1.22	12.9	15.7	10.2	12.3	9.1	11.1
Mean value of C_4 for $\lambda=4, 8$, and 12μ , obs.	—	—	—	$C_4 = 19.4$	—	$C_8 = 13.0$	—	$C_{12} = 11.0$
Mean deviation δ of the single products from C_4	—	—	—	$\delta_4 = 21.0\%$	—	$\delta_8 = 14.5\%$	—	$\delta_{12} = 9.6\%$
Theoretical value of C_4 , $C_4' = \frac{36.5}{\sqrt{\lambda}}$	—	—	—	$C_4' = 18.25$	—	$C_8' = 12.90$	—	$C_{12}' = 10.54$

had a good spherical shape, they were neither quite free from oxidation nor were they thick enough, so that they necessarily gave too small a reflecting-power. But the want of exactness of the values obtained for bismuth is not the only reason why we excluded them from our calculations. Bismuth follows the law given above in no respect, as will be proven by our further experiments.

Comparison with the Theory.

Maxwell's original theory, which (as has been mentioned) does not consider the molecules and their vibration, leads to this simple expression for the reflecting-power *

$$R = 100 - \frac{200}{\sqrt{\Lambda\tau}}$$

* Cf. P. Drude, 'Physik des Aethers,' p. 574, Formula (66), 1894; and E. Cohn, 'Das electromagnetische Feld,' p. 444 (1900); also M. Planck, *Sitzungsber. d. k. Akad. d. Wissensch. zu Berlin*, p. 278 (1903). Prof. Planck arrives at the equation (2) in this way:—

If a plane linear polarized light-wave propagates in a metal which does not possess any dielectric qualities and absorbs the vibrations only by ordinary galvanic conduction, this process is represented by the Maxwell-Hertz equations

$$\frac{\partial E}{\partial t} = c \frac{\partial H}{\partial x} - 4\pi\Lambda E,$$

$$\frac{\partial H}{\partial t} = c \frac{\partial E}{\partial x}.$$

Herein E and H mean the intensity of the electric and magnetic field, *c* the velocity of light in the vacuum, and Λ the galvanic conductivity of the metal in absolute electrostatic measure. From these equations we obtain

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2} - 4\pi\Lambda \frac{\partial E}{\partial t}.$$

This equation is satisfied by the expression

$$E = A \cdot e^{n(it - \frac{p}{c}x)},$$

if the further condition is fulfilled

$$ni(1 + p^2) + 4\pi\Lambda = 0;$$

wherein *n* means the number of vibrations in 2π seconds and

$$p = g + i\nu,$$

\nu is the ratio between the wave-length in vacuum and that in the metal; *g* means the coefficient of extinction, defined by the law, that the intensity of a ray proceeding in the metal is reduced to $e^{-4\pi g}$ of its initial intensity after having passed over a distance as long as one wave-length in vacuum.

By substituting the value of *p*, and by separating the real and imaginary terms, we obtain

$$-2g\nu n + 4\pi\Lambda = 0$$

and $1 + g^2 - \nu^2 = 0.$

or for the entering intensity

$$(100 - R) = \frac{200}{\sqrt{\Lambda\tau}} \dots \dots \dots (2)$$

Herein Λ means the electric conductivity of the metal in absolute electrostatic measure and τ the period of oscillation in seconds. The formulæ are only valid for sufficiently large Λ , which make the product $\Lambda\tau$ very large compared with unity, a condition always fulfilled with sufficient approximation here. When introducing into equation (2) the wavelength λ measured in μ , in place of the period of vibration,

Besides, if we introduce τ , the period of the vibration in seconds

$$\tau = \frac{2\pi}{n},$$

we have

$$\nu = \frac{1}{2}(\sqrt{4\Lambda^2\tau^2 + 1} + 1)$$

and

$$g = \frac{1}{2}(\sqrt{4\Lambda^2\tau^2 + 1} - 1).$$

These two equations characterize the whole optical behaviour of the metal.

For normal incidence the reflecting-power R_1 of the metal surface bordering on the vacuum is expressed—as is well known—by the formula

$$R_1 = \frac{(\nu - 1)^2 + g^2}{(\nu + 1)^2 + g^2},$$

the intensity of the incident radiation being equal to unity; and, after substituting our values of ν and g ,

$$R_1 = \frac{\sqrt{4\Lambda^2\tau^2 + 1} + 1 - \sqrt{2(\sqrt{4\Lambda^2\tau^2 + 1} + 1)}}{\sqrt{4\Lambda^2\tau^2 + 1} + 1 + \sqrt{2(\sqrt{4\Lambda^2\tau^2 + 1} + 1)}}.$$

For sufficiently long waves, *i. e.* for sufficiently large values of τ , the unit may be neglected as compared with $4\Lambda^2\tau^2$, and we obtain, in first approximation,

$$g = \nu = \sqrt{\Lambda\tau}$$

and
$$R_1 = 1 - \frac{2}{\sqrt{\Lambda\tau}}.$$

Finally, if we express the reflecting-power in percent. of the incident radiation, as has been done in this paper, we arrive at

$$R = 100 - \frac{200}{\sqrt{\Lambda\tau}},$$

which is identical with our equation (2).

and the value κ^* of Table II. in place of the conductivity Λ given in electrostatic measure, we obtain

$$(100 - R) = \frac{36.5}{\sqrt{\kappa\lambda}} \dots \dots \dots (3)$$

or

$$(100 - R) \cdot \sqrt{\kappa} = \frac{36.5}{\sqrt{\lambda}} = C_\lambda \dots \dots \dots (4)$$

Therefore the relation between the reflecting-power and the conductivity of metals, experimentally given in the region of long waves, corresponds perfectly with the demands of Maxwell's theory. The value of the constant $C_\lambda = (100 - R) \sqrt{\kappa}$ is, according to Table II.,

- 19.4 at $\lambda = 4\mu$
- 13.0 „ $\lambda = 8\mu$
- 11.0 „ $\lambda = 12\mu$

and the corresponding theoretical values of the constant C_λ , computed from the Maxwell equation (4), are

- 18.25 for 4μ
- 12.90 „ 8μ
- 10.54 „ 12μ

They show a better agreement with the values given above than might have been expected or even hoped for.

TABLE III.

Metals.	(100 - R) for $\lambda = 12\mu$.	
	Observed.	Computed.
Silver.....	1.15	1.3
Copper	1.6	1.4
Gold	2.1	1.6
Platinum	3.5	3.5
Nickel	4.1	3.6
Steel	4.9	4.7
Bismuth	(17.8)	11.5
Patent Nickel P	5.7	5.4
Patent Nickel M	7.0	6.2
Constantan	6.0	7.4
Rosse's Alloy.....	7.1	7.3
Brandes and Schünemann's } Alloy	9.1	8.6

* κ is the reciprocal value of the resistance of a conductor one metre long and of one square millimetre cross-section in ohms.

A similar proof for the agreement of experiment and theory results from Table III. Here the values of $(100-R)$ determined experimentally are placed beside those computed from formula (4), and solely dependent on the electric conductivity and the wave-length $\lambda=12\mu$.

Excepting the values given for bismuth, the agreement is good, particularly when we consider that the numbers calculated from formula (4) are absolute values and do not contain any arbitrary coefficient.

II.

The Emissive Power of the Metals for $\lambda=25\cdot5\mu$.

The formula (4), found by our experiments relating to reflexion and deduced from Maxwell's theory, was now to be examined for rays of much greater wave-length; for instance, for the "residual rays"* of fluorite. For the accomplishment of this purpose, the investigation of $(100-R)$ by aid of the reflecting-power is not an advantageous method, as was mentioned above. For all metals the reflecting-power R approaches 100 per cent. asymptotically with increasing wave-lengths, and the difficulty of experimentally determining $(100-R)$ increases accordingly †.

But if—instead of the reflecting-power—the emission-power is made the object of research, the course of investigation becomes much easier. In that case the metal surfaces require merely to possess the same temperature for the comparison of their radiating-power with that of an absolutely

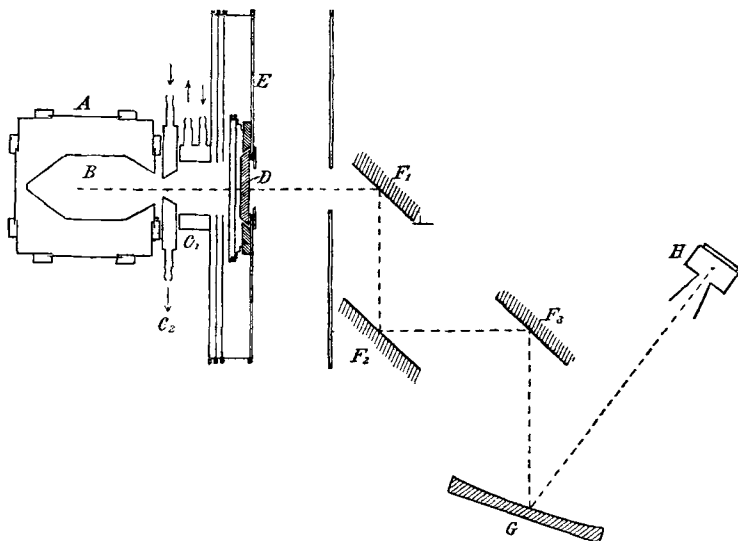
* By "residual rays" we mean the still surviving part of the whole radiation of a source of heat after having undergone several reflexions on surfaces of a certain substance; for instance, of rock-salt, sylvine, or fluorite. In the infra-red these substances possess rather sharply-limited bands of metallic absorption, at which the reflecting-power attains very high values (sometimes more than 90 per cent.), whilst for other wave-lengths it is small. The residual rays of sylvine attain their maximum energy at a wave-length of about 61μ , those of rock-salt at 51μ . The residual rays of fluorite are less homogeneous than those of rock-salt and sylvine. They begin at 22μ , attain a sharply-defined maximum at $\lambda=24\mu$, and decrease first rapidly, then slower towards the longer waves. On account of the unsymmetrical form of the energy-curve the mean wave-length of the assemblage of rays—after three reflexions from fluorite—amounts to $25\cdot5\mu$ (H. Rubens, *Wied. Ann.* lxi. p. 576, 1899). For further particulars on residual rays, *vide* H. Rubens & E. F. Nichols, *Wied. Ann.* lx. p. 418 (1897); and H. Rubens & E. Aschkinass, *Wied. Ann.* lxxv. p. 241 (1898).

† *Cf.* H. Rubens & E. F. Nichols, *Wied. Ann.* lx. p. 418 (1897).

black body for long waves, a process causing no difficulty when sufficient energy is at hand*.

Fig. 3 shows the arrangement of apparatus which served

Fig. 3.



for these experiments. A is a copper box that could be turned at will : its four sides contained round openings provided with screws, into which the metal plates † (radiating

* A similar method has been tried by O. Wiedeburg (*Wied. Ann.* lxxvi. p. 92, 1898) to compare the total emission of different metals with that of silver. The results which he obtained support those already published by H. Rubens (*Wied. Ann.* xxxvii. p. 249, 1889), that the good conductors for heat and electricity show a higher reflecting-power than the other metals. But, in view of our measurements of the reflecting-power, Wiedeburg's numerical data of the emission-power seemed so improbable, that we repeated his experiments for several metals. Doubtless his experiments are influenced in a very high degree by diffused foreign radiation, a fact which is proved by the following table. Here we give the emission of four metals at 100°, silver being the unit, in the first place according to O. Wiedeburg's, and secondly according to our own observations. The latter cannot possibly be influenced by any foreign radiation.

Total emission at 100°.

Observers.	Silver.	Platinum.	Steel.	Manganin.
O. Wiedeburg	1·00	1·23	1·31	1·32
Hagen and Rubens..	1·00	4·65	6·66	8·00

† It is a special advantage of this method that the metal plates used need only to be pure and well polished, but not plane.

surface 50 mms. diameter) or the "black body" could be introduced. The "black body" was a copper cylinder B of 47 mms. internal diameter, furnished with conic ends, and coated with lampblack on the inside. Its aperture at the outer end was 22 mms. in diameter. Boiling aniline served as heating liquid, which was in continual contact with the metal sheets to be heated. The heating was done electrically by means of a spiral of constantan ribbon inside the copper case. The temperature was generally regulated to 170°. The heating-box was put in front of a metal diaphragm C₁ or C₂, cooled by flowing water. Behind this diaphragm stood a screen D, kept at the temperature of the room. When this screen was lifted, the radiation to be measured underwent successive reflexions from three or four suitably set-up surfaces of fluorite F₁ F₂ F₃, and by means of a large concave silver mirror G were finally brought to focus on a Rubens thermopile H. By this process the "residual rays" of fluorite, corresponding to a wave-length of about 26 μ , were separated from the total emission.

A direct comparison between the radiations of the metal surfaces and that of the "black body" being impossible on account of the different sizes of the respective radiating surfaces, we had recourse to the following method. A thin copper disk, covered on one side with a very thin glass plate (5 cms. diameter) was introduced into one of the openings of the heating case. We then determined the ratio between the radiation of the "black body" and that of the glass plate by aid of the smaller diaphragm C₁ (14 mms. diameter). The value we found was 1.29. In our further experiments, the radiation of the different metal surfaces was compared with that of the glass plate. In all these cases we could make use of the large diaphragm C₁ (diameter 32 mms.). The numbers thus obtained had only to be divided by 1.29 in order to give the emission-power of the different metals, compared with that of the black body. These results are given in Table IV.

Mercury was the only metal that required another treatment. We therefore changed our arrangement in the following manner. Instead of the copper cube, a cup filled with mercury, heated to 100°, was placed opposite and a little below the diaphragm C₁. In front of the diaphragm and above the mercury surface, there was adjusted a polished plate of fluorite. By means of this fluorite mirror the heat-rays emanating from the mercury were horizontally reflected through the diaphragm. In order to compare this radiation with that of the black body, we covered the surface of the

TABLE IV.

1	2	3	4	5	6	7	8
Metals.	Conductivity at 18°. κ_{18} .	Temperature-coefficient of resistance $10^3 \alpha$.	Conductivity at 170°. κ_{170} .	$\sqrt{\kappa_{170}}$.	Emission-power for $\lambda = 25.5\mu$ and 170°. Computed by Formula (5).	Emission-power $I = (100 - E)$ Observed.	Product C $(100 - E) \sqrt{\kappa_{170}}$ taken from columns 5 and 7.
Silver.....	61.4	4.0	39.2	6.26	1.15	1.13	7.07
Copper.....	51.8	4.2	32.5	5.70	1.27	1.17	6.67
Gold.....	41.3	3.68	27.2	5.21	1.30	1.56	8.10
Aluminium.....	31.6	3.90	20.4	4.52	1.60	1.97	8.91
Zinc.....	15.83	3.94	10.2	3.19	2.27	2.27	7.24
Cadmium.....	13.13	4.25	8.40	2.86	2.53	2.55	7.29
Platinum.....	9.24	3.84	5.98	2.44	2.96	2.82	6.88
Nickel.....	8.50	4.38	5.26	2.29	3.16	3.20	7.33
Th.....	8.28	4.65	5.01	2.24	3.23	3.27	7.32
Steel.....	5.02	3.69	3.30	1.81	3.99	3.66	6.62
Mercury.....	1.044	0.92	0.916†	0.957	7.35	7.66	7.33
Bismuth.....	0.84	4.54	0.513	0.716	10.69	(25.6)	(18.3)
"Rohguss" *.....	7.89	0.8	7.05	2.65	2.73	2.70	7.16
Manganu.....	2.38	0.03	2.37	1.54	4.69	4.63	7.16
Constantan.....	2.04	0.01	2.04	1.43	5.05	5.20	7.43
Patent Nickel P.....	3.81	0.25	3.69	1.92	3.77	4.05	7.77
Patent Nickel M.....	2.94	0.20	2.86	1.69	4.28	4.45	7.41
							7.53

* "Rohguss" contains 85.7 Cu + 7.2 Zn + 6.4 Sn.

† At 100°.

mercury with a very thin sheet of the same glass as had always served in our previous comparisons.

No diffuse radiation of small wave-lengths could possibly have influenced our results. This was proved by the insertion of a plate of fluorite or rock-salt, 1 cm. thick, that perfectly absorbed the whole radiation*.

Column 6 of Table IV. contains the values of $(100-R)$ computed by means of formula 3, for $\lambda=25\cdot5\mu$, whereas column 7 shows the values of $(100-R)$ obtained by observation. The agreement between the two columns is so good as to form a sufficient proof for the correctness of the law

$$(100-R)\sqrt{\kappa}=\text{const.},$$

found by our previous experiments on shorter waves. The same is shown by column 8, containing the product $(100-R)\sqrt{\kappa}$, taken from our observations †. The average value of this constant amounts to 7·33 for the pure metals, to 7·41 for the alloys; the theoretical value, calculated by means of formula 4, is $36\cdot5/\sqrt{25\cdot5}=7\cdot23$. Aluminium alone gives a rather considerable deviation, and bismuth furnishes a complete exception to our law ‡. But we can scarcely wonder at this, since bismuth forms an exception in various other ways §.

Dependence of the Emission-Power of Metals on Temperature.

The good agreement of the emission-power $(100-R)$ obtained from our experiments with that computed from Maxwell's theory, justifies the conclusion that the variation of the conductivity of metals with temperature requires a

* Cf. H. Rubens and A. Trowbridge, *Wied. Ann.* lx. p. 724 (1897).

† Of the numbers in column 8 those for Zn, Cd, Ni, Sn, Hg, manganin, and constantan agree particularly well with the theoretical value 7·23. This is most probably owing to the fact that for these metals the conductivity was very accurately known. It was certainly the case with the above-named three alloys and with mercury, which, in comparison with the solid metals, can easily be obtained very pure. As to gold, we must remark that the conductivity of this metal at 18° is only 41·3 if the gold is *absolutely* pure. If it contains even $\frac{1}{1000}$ of iron or copper, its conductivity is reduced to 24·7, and the temperature-coefficient sinks from 3·68 to 2·03.

‡ Both the aluminium and the bismuth surfaces could not be kept free from oxide for a considerable length of time. With aluminium we are nearly sure that the observed deviation is due to this fact. But with bismuth the greater part of the deviation is apparently due to other causes.

§ Cf. P. Lenard, *Wied. Ann.* xxxix. p. 626 (1890).

corresponding change of the emission-power for long waves*. Otherwise the constant $C = (100 - R)\sqrt{\kappa_{170}}$ for pure metals would have been found 25 per cent. smaller than that for the alloys with exceedingly small temperature-coefficients, and could agree only by a mere chance with the theoretical value 7.23.

Our experiments on emission therefore enable us to calculate the change of resistance with the temperature, since the emission increases with the square root of the resistance. The following experiments have been undertaken in order to obtain a still stronger proof of this relation. A hollow case † of platinum-foil, heated by an electric current, was placed in front of the diaphragm C_1 . The temperature of the case could be measured by aid of a thermo-element LeChatelier, the welded end of which was placed inside. For higher temperatures (800° – 1560°) the hollow case was replaced by a single platinum strip, cut from the same piece of platinum-foil of which the case had been made. The temperature of the incandescent platinum strip was determined by means of the optical pyrometer, lately described by Holborn and Kurlbaum ‡. The “black temperature” observed herewith was transformed into Celsius degrees, according to the table of correction given by these authors. For the experiments with higher temperatures (above 800°) we were obliged to increase the number of the reflecting fluorite surfaces from 3 to 4; otherwise the “Reststrahlen” would not have been sufficiently pure. The still remaining small impurities, caused by radiation of short wave-length, were determined by the interposition of a plate of rock-salt, and the amount deducted. For the high temperatures we used the diaphragm C_2 , only 14 mms. wide, thus permitting of direct comparison between the radiation of the hot platinum-foil and that of our “black body” at 170° Celsius. But when the intensity of radiation of the black body for “residual rays” of fluorite is known for one temperature, it is easily determined for every other §; since the intensity of the observed “residual rays”—for

* In the visible spectrum the change of the optical constants of the metals with the temperature is exceedingly small, as is shown by the observations of Messrs. R. Sissingh (*Arch. Néerland.* xx. p. 172, 1886), P. Drude (*Wied. Ann.* xxxix. p. 538, 1890), B. Zeemann (*Commun. of the Lab. of Physics at the Univers. of Leyden*, No. 20, 1895), and A. Pflüger (*Wied. Ann.* lviii. p. 493, 1896).

† A hollow case of this kind was first employed by Messrs. O. Lummer and F. Kurlbaum, *Verhandl. d. Phys. Gesellsch. zu Berlin*, xvii. p. 106 (1898).

‡ L. Holborn & F. Kurlbaum, *Ann. d. Phys.* x. p. 225 (1903).

§ H. Rubens & F. Kurlbaum, *Ann. d. Phys.* iv. p. 649 (1901).

temperatures above 20° C.—is proportional to the difference of temperature between the black body and the thermopile. (This fact has also been expressed by M. Planck's law of radiation*.) Consequently the radiation of the hot platinum-foil could always be referred to the radiation of an equally heated black body; that is tantamount to a direct determination of the value (100-R). The continuity of these experiments at *high* temperature with those at *lower* was secured by observing the galvanometer-deflexions with both arrangements of apparatus between 700° and 800° Celsius. By forming the ratio of two deflexions, corresponding to the two arrangements, the temperature of the source of radiation being the same, one obtains a reduction factor by which the deflexions observed at higher temperatures must be multiplied so that the results may become at once comparable with those observed at low temperatures. Table V. shows the results of our observations, after this calculation.

TABLE V.

Temperature of the Platinum-foil <i>t</i> in Celsius degrees.	Observed Deflexion.	Remarks.	
1556	200	Strip of platinum in front of the narrow diaphragm C ₂ . 4 surfaces of fluorite. Measurement of temperature with the optical pyrometer. The black body gave at 170° a (reduced) deflexion of 196 mms.	
1438	177		
1320	153		
1214	130		
1100	109.5		
976	89.7		
858	74.6		
762	63.6		
804	68.0		
695	53.2		
614	45.4		Hollow case of platinum in front of the diaphragm C ₁ . 3 surfaces of fluorite. Measurement of temperature with the thermoelement.
493	32.2		
404	24.4		
323	18.2		
261	12.4		
169	6.5		

By the ordinary electrical method of measurement, the specific conductivity of our platinum-foil was determined, $\kappa_0 = 6.5$ (specific resistance = 0.154), the temperature-coefficient amounting to $\alpha = 0.0024$, between 18° and 65°. These numbers show that the platinum employed was not pure; but for our purpose this was of no importance.

A very good agreement between the computed and the

* M. Planck, *Ann. d. Phys.* iv. p. 553 (1901).

observed emission of platinum-foil is obtained by means of the equation

$$w_t = w_0(1 + \alpha t + \beta t^2), \quad \dots \quad (5)$$

in which

$$\begin{aligned} w_0 &= 0.154 \\ \alpha &= 0.0024 \\ \beta &= 0.0000033. \end{aligned}$$

This is proved in Table VI., the first column of which contains a few temperatures in Celsius degrees. The second

TABLE VI.

1	2	3	4	5	6	7
Temp. t in Celsius degrees.	Specific resist- ance $w_t = w_0(1 + \alpha t + \beta t^2)$	Conduc- tivity κ_t .	Emission-power computed $(100 - R) = \frac{7.23}{\sqrt{\kappa_t}}$	Emission of the black body γ_t .	Observed deflexion α .	Emission power $(100 - R)$ observed.
170	0.233	4.31	3.49	196	6.6	3.36
220	0.260	3.84	3.68	261	9.6	3.68
300	0.312	3.22	4.04	366	15.7	4.29
600	0.559	1.79	5.40	758	42.8	5.65
900	0.900	1.11	6.86	1150	79.6	6.93
1200	1.33	0.751	8.34	1540	128.0	8.32
1500	1.85	0.540	9.84	1940	189.5	9.78

shows the respective specific resistances, computed from formula 5; the third, the corresponding conductivity κ_t , and the fourth the emission-power of the employed platinum, computed from the equation

$$100 - R = \frac{7.23}{\sqrt{\kappa_t}}.$$

The fifth column gives the radiation γ_t of the "black body," as derived, for the respective temperature, from the radiation of the black body at 170° by linear extrapolation. The sixth column contains the observed galvanometer-deflexions α for the emission of the heated platinum-foil. These numbers were derived from Table V. by interpolation. By forming the ratio of the corresponding numbers of columns 6 and 5, and by multiplying it by 100, one obtains the "observed" emission-powers, given in column 7. The agreement of these numbers with the "computed" ones of column 4 is the more

conclusive because they are absolute values, not containing any arbitrary factor.

The coefficients w_0 and α of equation (5) are directly obtained by ordinary electrical measurement. In the region of lower temperatures (down to about 250°) where the influence of the square term βt^2 is only small, the *observed* emission values agree perfectly with those *computed* from the electrical conductivity. But in order to obtain a similar agreement between observed and computed emission values in the region of high temperatures, the adoption of a square term with the coefficient β is absolutely necessary.

The coefficient $\beta = 0.0000033$ has a positive sign in equation (5), thereby indicating a more rapid increase of the resistance at high temperatures. This is in contradiction with measurements of Messrs. Benoît*, L. Callendar†, and Holborn & Wien‡, who have all observed a slower change of resistance at high temperatures, which corresponds to a small negative value of β .

Accordingly, we must suppose that the increase of resistance at higher temperatures—as computed from our observations on emission—is only apparent, and that other facts influence the change of the emission of platinum in that region. Particularly a change (roughening) of the surfaces at high temperatures is not improbable, and that would account for a remarkable rising of the emissive power. Lastly, it is not improbable that the observed deviations are connected with perhaps the insufficient homogeneity of the “residual rays.”

Summary of the obtained Results.

1. The reflecting-power of the investigated metals from $\lambda = 0.65\mu$ to $\lambda = 14\mu$ is given in Table I.; the emission-power for $\lambda = 25.5\mu$ and 170° C. is to be found in Table IV.

2. For long waves the intensity entering into the metals ($100 - R$) is in inverse proportion to the square root of the electrical conductivity κ , and to the square root of λ , the wave-length of the incident radiation. This law, derivable from Maxwell’s theory, holds good the more strictly, the longer the waves are. This is proved in Table VII., which gives the observed and computed values of the constant

$$C_\lambda = (100 - R) \sqrt{\kappa} \quad \text{and} \quad C_\lambda = \frac{36.5}{\sqrt{\lambda}}$$

for four different wave-lengths of the infra-red spectrum

* R. Benoît, *Compt. Rend.* lxxvi. p. 342 (1873).

† L. Callendar, *Phil. Mag.* [5] xlvii. p. 191 (1899).

‡ L. Holborn & W. Wien, *Wied. Ann.* lvi. p. 360 (1895).

TABLE VII.

λ .	$C_\lambda = (100 - R) \sqrt{\kappa}$ observed.	Mean deviation of the single pro- ducts from the average value.	$C_\lambda = \frac{36.5}{\sqrt{\lambda}}$ computed.
4 μ	19.4	21.0 per cent.	18.25
8	13.0	14.5	12.90
12	11.0	9.6	10.54
25.5	7.36	4.9	7.23

Moreover, this table contains the average deviation (expressed in percents) which the single products $(100 - R) \sqrt{\kappa}$ of the different metals show, as compared with the average value given in the second column,

3. According to Maxwell's theory the quantity $100 - R$ undergoes a change corresponding to the change of resistance which the metals show with increasing temperature. This has been verified by our experiments.

These two facts (2 and 3) form a new important proof of Maxwell's theory, and that in a sphere, in which hitherto no connexion between the observed facts and the theoretically computed laws had been recognized.

4. From our observations the conclusion must be drawn, that the periods of vibration of the molecules scarcely influence the optical behaviour of metals in the region of long waves.

5. No influence of the magnetic qualities of iron and nickel* on their behaviour towards these rays could be detected. Had such influence made itself manifest relative to these metals, the value $(100 - R)$ ought to have been considerably larger than it appears when computed from the conductivity for constant electric current† by aid of formula (4).

6. In the region of long waves we are, therefore, justified in assuming the agreement of the other optical constants with the values computed from Maxwell's theory. This

* This fact can be explained by Prof. Drude's theory of magnetism (Cf. *Verhandl. d. Deutschen Physikal. Gesellsch.* v. pp. 143 & 148, 1903).

† In the experiments of Mr. V. Bjerknes (*Wied. Ann.* xlvii. p. 69, 1892) the influence of magnetism very strongly manifests itself for vibrations of about 10^8 per second. This is partly due to his method of observation, which greatly differs from ours.

theory leads—as previously shown by Mr. Drude*—to the equation †

$$g = \nu = 5.48 \sqrt{\kappa\lambda}, \quad (6)$$

where g is the coefficient of extinction, ν the index of refraction of the metals for normal incidence. This equation, which is only approximately correct, shows that the index of refraction and the extinction coefficient are numerically equal for long waves.

Besides

$$R = 100\left(1 - \frac{2}{\nu}\right) = 100\left(1 - \frac{2}{g}\right),$$

therefore

$$g = \nu = \frac{200}{100 - R}.$$

Consequently both values are definable from the emission-power alone.

7. A further consequence, resulting from the agreement of our researches with the electromagnetic theory of light, deserves special mention. Besides abstract numbers, the theoretical computation of the constant C contains only the velocity of light and the wave-length, both of which can be determined by experiments on radiation. By dividing the emission-power of a metal for the wave-length λ (the emission of the black body being rated at 100) by the constant C , and by squaring the ratio, we obtain the electrical resistance in ohms of a wire of the respective metal (1 m. length and 1 mm.² cross-section). So it is now possible to undertake absolute determinations of electrical resistances solely by the aid of measurements on radiation.

* P. Drude, *Physik des Aethers*, p. 575 formula (68), 1894; and M. Planck, *l. c.* In the footnote p. 166 of this paper we have given Planck's enunciation of formula (6).

† It follows from formula (6) that the extinction-coefficient (g) increases, for long waves, with the square-root of the wave-length. The absorption-coefficient $a_0 = \frac{4\pi g}{\lambda}$, which characterizes the real absorption of the metals, consequently diminishes proportionally to the square-root of the wave-length. Nevertheless the absorption of the metals remains very considerable, even for waves longer than one metre. A metal layer of about $\frac{1}{150}$ mm. thickness must necessarily absorb the whole infra-red spectrum. It is therefore impossible that infra-red rays of great wave-length pass through layers of aluminium half a millimetre thick, and it follows that M. Blondlot's so-called "Rayons N" cannot possibly be infra-red rays.