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THE HUMANIZATION OF THE TEACHING OF MATHEMATICS[1]

WHEN the distinguished chairman of your mathematical conference did me the honor to request me to speak to you, he was generous enough, whether wisely or unwisely, to leave the choice of a subject to my discretion, merely stipulating that, whatever the title might be, the address itself should bear upon the professional function of those men and women who are engaged in teaching mathematics in secondary schools. Inexpertness, it has been said, is the curse of the world; and one may, not unnaturally, feel some hesitance in undertaking a task that might seem to resemble the rôle of a physician when, as sometimes happens, he is called upon to treat a patient whose health and medical competence surpass his own. I trust I am not wanting in that natural feeling. In the present instance two considerations have enabled me to overcome it. One of them is that, having had some experience in teaching mathematics in secondary schools, I might, it seemed to me, regard that experience, though it was gained more than a score of years ago, as giving something like a title to be heard in your counsels. The other consideration is that, in regard to the teaching of mathematics, whether in secondary schools or in colleges, I have acquired a certain conviction, a pretty firm conviction, which, were it properly presented, you would doubtless be generous enough and perhaps ingenious

[1] Address given at the meeting of the Michigan School Masters' Club, at Ann Arbor, March 28, 1912.

enough to regard as having some sort of likeness to a message.

My conviction is, that hope of improvement in mathematics teaching, whether in secondary schools or in colleges, lies mainly in the possibility of humanizing it. It is worth while to remember that our pupils are human beings. What it means to be a human being we all of us presumably know pretty well; indeed we know it so well that we are unable to tell it to one another adequately; and, just because we do so well know what it means to be a human being, we are prone to forget it as we forget, except when the wind is blowing, that we are constantly immersed in the earth's atmosphere. To humanize the teaching of mathematics means so to present the subject, so to interpret its ideas and doctrines, that they shall appeal, not merely to the computatory faculty or to the logical faculty but to all the great powers and interests of the human mind. That mathematical ideas and doctrines, whether they be more elementary or more advanced, admit of such a manifold, liberal and stimulating interpretation, and that therefore the teaching of mathematics, whether in secondary schools or in colleges, may become, in the largest and best sense, human, I have no doubt. That mathematical ideas and doctrines do but seldom receive such interpretation and that accordingly the teaching of mathematics is but seldom, in the largest and best sense, human, I believe to be equally certain. That the indicated humanization of mathematical teaching, the bringing of the matter and the spirit of mathematics to bear, not merely upon certain fragmentary faculties of the mind, but upon the whole mind, that this is a great desideration is, I assume, beyond dispute.

How can such humanization be brought about? The answer, I believe, is not far to seek. I do not mean that the answer is easy to discover or easy to communicate. I mean that the game is near at hand and that it is not difficult to locate it, though it may not be easy to capture it. The difficulty inheres, I believe, in our conception of mathematics itself; not so much in our conception of what mathematics, in a definitional sense, is, for that sense of what mathematics is has become pretty clear in our day, but in our sense or want of sense of what mathematics, whatever it may be, humanly signifies. In order to humanize mathematical teaching it is necessary, and I believe it is sufficient, to come under the control of a right conception of the human significance of mathematics. It is sufficient, I mean to say, and it is necessary, greatly to enlarge, to enrich and to vitalize our sense of what mathematics, regarded as human enterprise, signifies.

What does mathematics, regarded as an enterprise of the human spirit, signify? What is a just and worthy sense of the human significance of mathematics?

To the extent in which any of us really succeeds in answering that question worthily, his teaching will have the human quality, in so far as his teaching is, in point of external circumstance, free to be what it would. I believe it is important to put the question, and it is with the putting of it rather than with the proposing of an answer to it that I am here at the outset mainly concerned. For any one who is really to acquire possession of an answer that is worthy must win the answer for himself. I need not say to you that such an acquisition as a worthy answer to this kind of question does not belong to the category of things that may be lent or borrowed, sold or bought, donated or acquired by gift. No doubt the answers we may severally win will differ as our temperaments differ. Yet the matter is not solely

a matter of temperament. It is much more a matter first of knowledge and then of the evaluation of the knowledge and of its subject. To the winning of a worthy sense of the human significance of mathematics two things are indispensable, knowledge and reflection: knowledge of mathematics and reflection upon it. To the winning of such a sense it is essential to have the kind of knowledge that none but serious students of mathematics can gain. Equally essential is another thing and this thing students of mathematics in our day do not, or do but seldom, gain. I mean the kind of insight and the liberality of view that are to be acquired only by prolonged contemplation of the nature of mathematics and by prolonged reflection upon its relations of contrast and similitude to the other great forms of spiritual activity.

The question, though it is a question about mathematics, is not a mathematical question, it is a philosophical question. And just because it is a philosophical question, mathematicians, despite the fact that one of the indispensable qualifications for considering it is possessed by them alone, have in general ignored it. They have, in general, ignored it, and their ignoring of it may help to explain the curious paradox that whilst the world, whose mathematical knowledge varies from little to less, has always as if instinctively held mathematical science in high esteem, it has at the same time usually regarded mathematicians as eccentric and abnormal, as constituting a class apart, as being something more or something less than human. It may explain, too, I venture to believe it does partly explain, both why it is that in the universities the number of students attracted to advanced lectures in mathematics compared with the numbers drawn to advanced courses in some other great subjects not inherently more attractive, is so small; and why it is that, among the multitudes who pursue mathematics in the secondary schools, only a few find in the subject anything like delight. For I do not accept the traditional and still current explanation, that the phenomenon is due to a well-nigh universal lack of mathematical faculty. I maintain, on the contrary, that a vast majority of mankind possess mathematical faculty in a very considerable degree. That the average pupil's interest in mathematics is but slight, is a matter of common knowledge. His lack of interest is, in my opinion, due, not to a lack of the appropriate faculty in him, but to the circumstance that he is a human being, whilst mathematics, though it teems with human interest, is not presented to him in its human guise.

If you ask the world—represented, let us say, by the man in the street or in the market place or the field—to tell you its estimate of the human significance of mathematics, the answer of the world will be, that mathematics has given mankind a metrical and computatory art essential to the effective conduct of daily life, that mathematics admits of countless applications in engineering and the natural sciences, and finally that mathematics is a most excellent instrumentality for giving mental discipline. Such will be the answer of the world. The answer is intelligible, it is important, and it is good so far as it goes; but it is far from going far enough and it is not intelligent. That it is far from going far enough will become evident as we proceed. That the answer is not intelligent is evident at once, for the first part of it seems to imply that the rudimentary mathematics of the carpenter and the counting-house is scientific, which it is not; the second part of the answer is but an echo by the many of the voice of the few; and, as to the final part, the world's

conception of intellectual discipline is neither profound nor well informed but is itself in sorry need of discipline.

If, turning from the world to a normal mathematician, you ask him to explain to you the human significance of mathematics, he will repeat to you the answer of the world, of course with far more appreciation than the world has of what the answer means, and he will supplement the world's response by an important addition. He will add, that is, that mathematics is the exact science, the science of exact thought or of rigorous thinking. By this he will not mean what the world would mean if the world employed, as sometimes it does employ, the same form of words. He will mean something very different. Especially if he be, as I suppose him to be, a normal mathematician of the modern critical type, he will mean that mathematics is, in the oft-cited language of Benjamin Peirce, "the science that draws necessary conclusions;" he will mean that, in the felicitious words of William Benjamin Smith, "mathematics is the universal art apodictic;" he will mean that mathematics is, in the nicely technical phrase of Pieri, "a hypothetico-deductive system." If you ask him whether mathematics is the science of rigorous thinking about *all* the things that engage the thought of mankind or only about a few of them, such as numbers, figures, certain operations, and the like, the answer he will give you depends. If he be a normal mathematician of the elder school, he will say that mathematics is the science of rigorous thinking about only a relatively few things and that these are such as you have exemplified. And if now, with a little Socratic persistence, you press him to indicate the human significance of a science of rigorous thinking about only a few of the countless things that engage human thought, his answer will give you but little beyond a repetition of the above-mentioned answer of the world. But if he be a normal mathematician of the modern critical type, he will say that mathematics is the science of rigorous thinking about all the things that engage human thought, about *all* of them, he will mean, in the sense that thinking, as it approaches perfection, tends to assume certain definite forms, that these forms are the same whatever the subject matter of the thinking may be, and that mathematics is the science of these forms *as forms*. If you respond, as you well may respond, that, in accordance with this ontological conception of mathematics, this science, instead of thinking about *all*, thinks about *none*, of the concrete things of interest to human thought, and that accordingly Mr. Bertrand Russell was right in saying that "mathematics is the science in which one never knows what one is talking about nor whether what one says is true"—if you respond that, from the point of view above assumed, that delicious *mot* of Mr. Russell's must be solemnly held as true, and then if, in accordance with your original purpose, you once more press for an estimation of the human significance of such a science, I fear that the reply, if your interlocutor is a mathematician of the normal type, will contain little that is new beyond the assertion that the science in question is very interesting, where, by interesting, he means, of course, interesting to mathematicians. It is true that Professor Klein has said: "Apart from the fact that pure mathematics can not be supplanted by anything else as a means for developing the purely logical faculties of the mind, there must be considered here as elsewhere the necessity of the presence of a few individuals in each country developed in a far higher degree than the rest, for the purpose of keeping up and gradually raising the *general*

standard. Even a slight raising of the general level can be accomplished only when some few minds have progressed far ahead of the average." Here indeed we have, in these words of Professor Klein, a hint, if only a hint, of something better. But Professor Klein is not a mathematician of the normal type, he is hypernormal. If, in order to indicate the human significance of mathematics regarded as the science of the forms of thought as forms, your normal mathematician were to say that these forms constitute, of themselves, an infinite and everlasting world whose beauty, though it is austere and cold, is pure, and in which is the secret and citadel of whatever order and harmony our concrete universe contains, it would yet be your right and your duty to ask, as the brilliant author of "East London Visions" once asked me, namely, what is the human significance of "this majestic intellectual cosmos of yours, towering up like a million-lustered iceberg into the arctic night," seeing that, among mankind, none is permitted to behold its more resplendent wonders save the mathematician himself? But the normal mathematician will not say what I have just now supposed him to say; he will not say it, because he is, by hypothesis, a normal mathematician, and because, being a normal mathematician, he is exclusively engaged in exploring the iceberg. A farmer was once asked why he raised so many hogs. "In order," he said, "to buy more land." Asked why he desired more land, his answer was, "in order to raise more corn." Being asked to say why he would raise more corn, he replied that he wished to raise more hogs. If you ask the normal mathematician why he explores the iceberg so much, his answer will be, in effect at least, "in order to explore it more." In this exquisite circularity of motive, the farmer and the normal mathematician are well within their rights. They are within their rights just as a musician would be within his rights if he chanced to be so exclusively interested in the work of composition as never to be concerned with having his creations rendered before the public and never to attempt a philosophic estimate of the human worth of music. The distinction involved is not the distinction between human and inhuman, between social and anti-social; it is the distinction between what is human *or* inhuman, social or anti-social, and what is neither the one nor the other. No one, I believe, may contest the normal mathematician's right as a mathematical student or investigator to be quite indifferent as to the social value or the human worth of his activity. Such activity is to be prized just as we prize any other natural agency or force that, however undesignedly, yet contributes, sooner or later, directly or indirectly, to the weal of mankind. The fact is that, among motives in research, scientific curiosity, which is neither moral nor immoral, is far more common and far more potent than charity or philanthropy or benevolence. But when the mathematician passes from the rôle of student or investigator to the rôle of teacher, that right of indifference ceases, for he has passed to an office whose functions are social and whose obligations are human. It is not his privilege to chill and depress with the encasing fogs of the iceberg. It is his privilege and his duty, in so far as he may, to disclose its "million-lustered" splendors in all their power to quicken and illuminate, to charm and edify, the whole mind.

The conception of mathematics as the science of the forms of thought as forms, the conception of it as the refinement, prolongation and elaboration of pure logic, is, as you are doubtless aware, one of the great outcomes, perhaps I should say it is

the culminating philosophical outcome, of a century's effort to ascertain what mathematics, in its intimate structure, is. This conception of what mathematics is comes to its fullest expression and best defense, as you doubtless know, in such works as Schroeder's "Algebra der Logik," Whitehead's "Universal Algebra," Russell's "Principles of Mathematics," Peano's "Formulario Mathematico," and especially in Whitehead and Russell's monumental "Principia Mathematica." I cite this literature because it tells us what, in a definitional sense, the science in which the normal mathematician is exclusively engaged, is. If we wish to be told what that science humanly signifies, we must look elsewhere; we must look to a mathematician like Plato, for example, or to a philosopher like Poincaré, but especially must we look to our own faculty for discerning those fine connective things—community of aim, interformal analogies, structural similitudes —that bind all the great forms of human activity and aspiration—natural science, theology, philosophy, jurisprudence, religion, art and mathematics—into one grand enterprise of the human spirit.

In the autumn of 1906 there was published in *Poet Lore* a short poem which, though it says nothing explicitly of mathematics, yet admits of an interpretation throwing much light upon the human significance of the science and indicating well, I think, the normal mathematician's place in the world of spiritual interests. The author of the poem is my excellent friend and teacher, Professor William Benjamin Smith, mathematician, philosopher, poet and theologian. I have not asked his permission to interpret the poem as I shall invite you to interpret it. What its original motive was I am not informed—it may have been the exceeding beauty of the ideas expressed in it or the harmonious mingling of their light with the melody of their song. The title of the poem is "The Merman and the Seraph." As you listen to the reading of it, I shall ask you to regard the merman as representing the normal mathematician and the seraph as representing, let us say, the life of the emotions in their higher reaches and their finer susceptibilities.

I

Deep the sunless seas amid,
Far from Man, from Angel hid,
Where the soundless tides are rolled
Over Ocean's treasure-hold,
With dragon eye and heart of stone,
The ancient Merman mused alone.

II

And aye his arrowed Thought he wings
Straight at the inmost core of things—
As mirrored in his Magic glass
The lightning-footed Ages pass,—
And knows nor joy nor Earth's distress,
But broods on Everlastingness.
"Thoughts that love not, thoughts that hate not,
Thoughts that Age and Change await not,
All unfeeling,
All revealing,
Scorning height's and depth's concealing,
These be mine—and these alone!"—
Saith the Merman's heart of stone.

III

Flashed a radiance far and nigh
As from the vertex of the sky,—
Lo! a Maiden beauty-bright
And mantled with mysterious might
Of every power, below, above,
That weaves resistless spell of Love.

IV

Through the weltering waters cold
Shot the sheen of silken gold;
Quick the frozen Heart below
Kindled in the amber glow;
Trembling Heavenward Nekkan yearned
Rose to where the Glory burned.
"Deeper, bluer than the skies are,
Dreaming meres of morn thine eyes are
All that brightens
Smile or heightens
Charm is thine, all life enlightens,

Thou art all the soul's desire.''—
Sang the Merman's Heart of Fire.
''Woe thee, Nekkan! Ne'er was given
Thee to walk the ways of Heaven;
Vain the vision,
Fate's derision,
Thee that raps to realms elysian,
Fathomless profounds are thine''—
Quired the answering voice divine.

V

Came an echo from the West,
Pierced the deep celestial breast;
Summoned, far the Seraph fled,
Trailing splendors overhead;
Broad beneath her flying feet,
Laughed the silvered ocean-street.

VI

On the Merman's mortal sight
Instant fell the pall of Night;
Sunk to the sea's profoundest floor
He dreams the vanished Vision o'er,
Hears anew the starry chime,
Ponders aye Eternal Time.
''Thoughts that hope not, thoughts that fear not,
Thoughts that Man and Demon veer not
Times unending
Comprehending,
Space and worlds of worlds transcending,
These are mine—but these alone!''—
Sighs the Merman's heart of stone.

I have said that the poem, if it receive the interpretation that I have invited you to give it, throws much light on the human significance of mathematics and indicates well the place of the normal mathematician in the world of spiritual interests. No doubt the place of the merman and the place of the angel are not the same: no doubt the world of whatsoever in thought is passionless, infinite and everlasting, and the world of whatsoever in feeling is high and beauteous and good are distinct worlds, and they are sundered wide in the poem. But, though in the poem they are held widely apart, in the poet they are united. For the song is not the merman's song nor are its words the words of the seraph. It is the voice of the poet—a voice of man. The merman's world and the world of the seraph are not the same, they are very distinct; in conception they are sundered; they may be sundered in life, but in life it need not be so. The merman indeed is confined to the one world and the seraph to the other, but man, a man unless he be a merman, may inhabit them both. For the angel's denial, the derision of fate, is not spoken of man, it is spoken of the merman; and the merman's sigh is not his own, it is a human sigh—so lonely seems the merman in the depths of his abode.

No, the world of interests of the human spirit is not the merman's world alone nor the seraph's alone. It is not so simple. It is rather a cluster of worlds, of worlds that differ among themselves as differ the lights by which they are characterized. As differ the lights. The human spirit is susceptible of a variety of lights and it lives at once in a corresponding variety of worlds. There is perception's light, commonly identified with solar radiance or with the radiance of sound, for music, too, is, to the spirit, a kind of illumination: perceptional light, in which we behold the colors, forms and harmonies of external nature: a beautiful revelation—a world in which any one might be willing to spend the remainder of his days if he were but permitted to live so long. And there is imagination's light, disclosing a new world filled with wondrous things, things that may or may not resemble the things revealed in perception's light but are never identical with them: light that is not superficial nor constrained to paths that are straight but reveals the interiors of what it illuminates and phases that look away. Again, there is the light of thought, of reason, of logic, the light of analysis, far dimmer than perception's light, dimmer, too, than that of imagination, but far more penetrating and far more

ubiquitous than either of them, disclosing things that curiously match the things that they disclose and countless things besides, namely, the world of ideas and the relations that bind them: a cosmic world, in the center whereof is the home of the merman. There remains to be named a fourth kind of light. I mean the light of emotion, the radiance and glory of things that, save by gleams and intimations, are not revealed in perception or in imagination or in thought: the light of the seraph's world, the world of the good, the true and the beautiful, of the spirit of art, of aspiration and of religion.

Such, in brief, is the cluster of worlds wherein dwell the spiritual interests of the human beings to whom it is our mission to teach mathematics. My thesis is that it is our privilege to show, in the way of our teaching it, that its human significance is not confined to one of the worlds but, like a subtle and ubiquitous ether, penetrates them all. Objectively viewed, conceptually taken, these worlds, unlike the spheres of the geometrician, do not intersect—a thing in one of them is not in another; but the things in one of them and the things in another may own a fine resemblance serving for mutual recall and illustration, effecting transfer of attention—transformation as the mathematicians call it—from world to world; for whilst these worlds of interest, objectively viewed, have naught in common, yet subjectively they are united, united as differing mansions of the house of the human spirit. A relation, for example, between three independent variables exists only in the gray light of thought, only in the world of the merman; the habitation of the geometric locus of the relation is the world of imagination; if a model of the locus be made or a drawing of it, this will be a thing in the world of perception; finally, the wondrous correlation of the three things, or the spiritual qualities of them—the sensuous beauty of the model or the drawing, the unfailing validity of the given relation holding as it does throughout "the cycle of the eternal year," the immobile presence of the locus or image poised there in eternal calm like a figure of justice—these may serve, in contemplating them, to evoke the radiance of the seraph's world: and thus the circuit and interplay, ranging through the world of imagination and the world of thought from what is sensuous to what is supernal, is complete. It would not have seemed to Plato, as it may seem to us, a far cry from the prayer of a poet to the theorem of Pythagoras, for example, or to that of Archimedes respecting a sphere and its circumscribing cylinder. Yet I venture to say, that calm reflection upon the existence and nature of such a theorem—cloistral contemplation, I mean, of the fact that it is really true, of its serene beauty, of its silent omnipresence throughout the infinite universe of space, of the absolute exactitude and invariance of its truth from everlasting to everlasting—will not fail to yield a sense of reverence and awe akin to the feeling that, for example, pervades this choral prayer by Sophocles:

"Oh! that my lot may lead me in the path of holy innocence of word and deed, the path which august laws ordain, laws that in the highest empyrean had their birth, of which Heaven is the father alone, nor did the race of mortal men beget them, nor shall oblivion put them to sleep. The god is mighty in them and he groweth not old."

But why should we think it strange that interests, though they seem to cluster about opposite poles, are yet united by a common mood? Of the great world of human interests, mathematics is indeed but a part; but is a central part, and, in a profound

and precious sense, it is "the eternal type of the wondrous whole." For poetry and painting, sculpture and music—art in all its forms—philosophy, theology, religion and science, too, however passional their life and however tinged or deeply stained by local or temporal circumstance, yet have this in common: they all of them aim at values which transcend the accidents and limitations of every time and place; and so it is that the passionlessness of the merman's thought, the infiniteness of the kind of being he contemplates and the everlastingness of his achievements enter as essential qualities into the ideals that make the glory of the seraph's world. I do not forget, in saying this, that, of all theory, mathematical theory is the most abstract. I do not forget that mathematics therefore lends especial sharpness to the contrast in the Mephistophelian warning:

> Gray, my dear friend, is all theory,
> Green the golden tree of life.

Yet I know that one who loves not the gray of a naked woodland has much to learn of the esthetic resources of our northern clime. A mathematical doctrine, taken in its purity, is indeed gray. Yet such a doctrine, a world-filling theory woven of gray relationships finer than gossamer but stronger than cables of steel, leaves upon an intersecting plane a tracery surpassing in fineness and beauty the exquisite artistry of frost-work upon a windowpane. Architecture, it has been said, is frozen music. Be it so. Geometry is frozen architecture.

No, the belief that mathematics, because it is abstract, because it is static and cold and gray, is detached from life, is a mistaken belief. Mathematics, even in its purest and most abstract estate, is not detached from life. It is just the ideal handling of the problems of life, as sculpture may idealize a human figure or as poetry or painting may idealize a figure or a scene. Mathematics is precisely the ideal handling of the problems of life, and the central ideas of the science, the great concepts about which its stately doctrines have been built up, are precisely the chief ideas with which life must always deal and which, as it tumbles and rolls about them through time and space, give it its interests and problems, and its order and rationality. That such is the case a few indications will suffice to show. The mathematical concepts of constant and variable are represented familiarly in life by the notions of fixedness and change. The concept of equation or that of an equational system, imposing restriction upon variability, is matched in life by the concept of natural and spiritual law, giving order to what were else chaotic change and providing partial freedom in lieu of none at all. What is known in mathematics under the name of limit is everywhere present in life in the guise of some ideal, some excellence high-dwelling among the rocks, an "ever flying perfect" as Emerson calls it, unto which we may approximate nearer and nearer, but which we can never quite attain, save in aspiration. The supreme concept of functionality finds its correlate in life in the all-pervasive sense of interdependence and mutual determination among the elements of the world. What is known in mathematics as transformation—that is, lawful transfer of attention, serving to match in orderly fashion the things of one system with those of another—is conceived in life as a process of transmutation by which, in the flux of the world, the content of the present has come out of the past and in its turn, in ceasing to be, gives birth to its successor, as the boy is father to the man and as things, in general, be-

come what they are not. The mathematical concept of invariance and that of infinitude, especially the imposing doctrines that explain their meanings and bear their names—what are they but mathematicizations of that which has ever been the chief of life's hopes and dreams, of that which has ever been the object of its deepest passion and of its dominant enterprise, I mean the finding of worth that abides, the finding of permanence in the midst of change, and the discovery of the presence, in what has seemed to be a finite world, of being that is infinite? It is needless further to multiply examples of a correlation that is so abounding and complete as indeed to suggest a doubt whether it be juster to view mathematics as the abstract idealization of life than to regard life as the concrete realization of mathematics.

Finally, I wish to emphasize the fact that the great concepts out of which the so-called higher mathematical branches have grown—the concepts of variable and constant, of function, class and relation, of transformation, invariance, and group, of finite and infinite, of discreteness, limit, and continuity—I wish, in closing, to emphasize the fact that these great ideas of the higher mathematics, besides penetrating life, as we have seen, in all its complexity and all its dimensions, are omnipresent, from the very beginning, in the *elements* of mathematics as well. The notion of group, for example, finds easy and beautiful illustration, not only among the simpler geometric motions and configurations, but even in the ensemble of the very integers with which we count. The like is true of the distinction of finite and infinite, and of the ideas of transformation, of invariant, and nearly all the rest. Why should the presentation of them have to await the uncertain advent of graduate years of study? For life already abounds, and the great ideas that give it its interests, order and rationality, that is to say, the focal concepts of the higher mathematics, are everywhere present in the elements of the science as glistening bassets of gold. It is our privilege, in teaching the elements, to avail ourselves of the higher conceptions that are present in them; it is our privilege to have and to give a lively sense of their presence, their human significance, their beauty and their light. I do not advocate the formal presentation, in secondary schools, of the higher conceptions, in the way of printed texts, for the printed text is apt to be arid and the letter killeth. What I wish to recommend is the presentation of them, as opportunity may serve, in Greek fashion, by means of dialectic, face to face, voice answering to voice, animated with the varying moods and motions and accents of life—laughter, if you will, and the lightning of wit to cheer and speed the slower currents of sober thought. Of dialectic excellence, Plato at his best, as in "Phædo" or the "Republic," gives us the ideal model and eternal type. But Plato's ways are frequently circuitous, wearisome and long. They are ill suited to the manners of a direct and undeliberate age; and we must find, each for himself, a shorter course. Somebody imbued with the spirit of the matter, possessed of ample knowledge and having, besides, the requisite skill and verve ought to write a book showing, in so far as the printed page can be made to show, how naturally and swiftly and with what a delightful sense of emancipation and power thought may pass by dialectic paths from the traditional elements of mathematics both to its larger concepts and to a vision of their bearings on the higher interests of life. I need not say that such a handling of ideas implies

much more than a verbal knowledge of their definitions. It implies familiarity with the doctrines that unfold the meanings of the ideas defined. It is evident that, in respect of this matter, the scripture must read: Knowing the doctrine is essential to living the life.

C. J. KEYSER

COLUMBIA UNIVERSITY

BENJAMIN FRANKLIN THOMAS

No more unexpected and startling announcement ever came to the writer than that of the death of Professor Thomas last summer. Only a few weeks before he was apparently in rugged health and as much interested as ever in the various phases of his department of science and its applications. The workers die but the work goes on.

Professor Thomas was born at Palmyra, Ohio, October 14, 1850, and died near his summer home in Maine, July 4, 1911. He received his preparation for college under private tutorage in Fox Lake, Wisconsin, and took the degree of Master of Science at Ripon College in 1874. Then followed one year at the Fort Berthold Indian Reservation in Dakota, two years as instructor at Carlton College in Minnesota and three years as a graduate student at the Massachusetts Institute of Technology and research assistant at the Stevens Institute in Hoboken. At this last institution he earned the degree of Doctor of Philosophy in 1880. The next five years he spent as professor of physics at the University of Missouri. In 1885 he was elected to the same chair in succession to Dr. T. C. Mendenhall at the Ohio State University. To this institution he gave twenty-six years of undivided and efficient service in the cause of education. This term of service covered the critical formative period while the institution was evolving from a small "College of Agriculture and Mechanic Arts" into the great state university of the present. His compelling logic, clear mental grasp of a difficult situation, and his exceptional gift for orderly statement made his services of great value in faculty meetings, in hearings before legislative committees, and as an expert witness in important cases of litigation.

Professor Thomas was early in foreseeing the immense expansion likely to come in applied electricity, and in 1889 he was instrumental in obtaining from the legislature an appropriation for a building and its equipment to accommodate a course in electrical engineering. This was probably the first college building ever built solely for the purpose of teaching this branch of engineering. His foresight has received abundant justification in a number of graduates in this course who have shed luster on their chosen profession and on their alma mater.

Professor Thomas's especial interest was in the subject of electrical measurements and electrical applications. He was unusually successful in developing the courses in advanced electrical measurements for engineering students; and by careful planning and persistent effort, carried on steadily through many years, he succeeded in getting together a magnificent equipment for this work. At the same time he elaborated a strong course of instruction which matched the fine equipment for effective use. He was a remarkably skilful experimenter in the study of rapidly varying electrical phenomena. His oscillograms of electric discharges and of waves of electric pressure and current are the most beautiful the writer has ever seen.

Professor Thomas's practical work as an electrical expert early led him to a thorough study of the photometry of arc and incandescent lamps, and enabled him to give valuable aid in electric lighting to many great institutions of the state, such as the Ohio Soldiers' and Sailors' Orphans Home, the state house at Columbus and the State Hospitals at Athens, Dayton and Toledo. He was for a number of years director of the Ohio Meteorological Bureau before it was merged into the United States Weather Bureau. He was also the representative for the state in the meetings called by the Bureau of Standards in Washington to confer on the subject of correct weights and measures.