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THE CORONA VOLTMETER AND THE ELECTRIC STRENGTH OF AIR

A Natural Secondary Standard of Voltage

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An improved form of the corona voltmeter is described. Precision measurements of crest values of high alternating voltage taken in the high-tension circuit are compared with the indications of the corona voltmeter.

The law of corona has been determined to a higher degree of accuracy, and a modification in the form of the law as heretofore accepted is revealed.

As based on the precision voltage measurements the corona voltmeter is proposed as a natural secondary standard of high voltages. Its advantages as a standard, and its practical operation are described.

I. INTRODUCTION

THE corona voltmeter is an instrument for measuring accurately the crest values of high alternating voltages. It makes use of the fact that corona forms on a clean round wire in air at a sharply marked definite value of voltage dependent in a simple relation on the density of the air. The range of the instrument using a single wire is extended to wide limits by enclosing the wire and varying the density of the air.

The essential elements of the instrument are a central rod or wire on which corona forms, an outer concentric cylinder forming the opposite terminal, an outer air-tight containing case in which the air pressure may be varied, and convenient means for determining accurately the first appearance of corona. The principle and method of operation, including the use of gaseous ionization and sound as corona indicators, and two earlier forms of the instrument have been described in an earlier paper.[1] An improved type of

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[1] A bilbiography of all references will be found at the end of the paper.

the instrument for voltages in the neighborhood of 150,000 volts is described below and shown in Figs. 9 and 10.

The principal object of this paper is to describe a series of experiments in which the values of corona forming crest voltages have been determined by precision measurements made in the high-voltage circuit. Also to show that the law followed is so definite, and the indications of the instrument so constant, that it constitutes not only an accurate measuring instrument, but also through the results of the present investigation, a natural secondary standard of high voltage possessing many advantages over others at present in use.

An important result of the work is the discovery of an interesting modification of the law of corona formation.

The various precautionary and check measurements taken to ensure the accuracy of the final readings constitute in themselves prime evidence of the accuracy of the corona as a measure of voltage, and also of its constancy and reliability in operation in the corona voltmeter. In addition some further notes on the operation of the voltmeter are given towards the end of the paper.

II. THE CORONA AS A STANDARD OF VOLTAGE

Two striking properties of the high-voltage corona in air have led to the suggestion of its use for the measurement of voltage and to the development of the corona voltmeter. The first is the remarkable constancy of the value of voltage at which, under fixed conditions, the corona appears on a round wire or rod; and the second is the simplicity of the law connecting the critical or corona-forming voltage with the diameter of the rod and the condition of the surrounding gas.

The former of these properties has been noted by a number of observers and in particular by one of the present authors in the first of a series of papers on the electric strength of air,[2] and again especially in a paper on the corona voltmeter.[1] Using a clean round rod and the best type of portable voltmeter in the low-tension circuit, on repeated raising and lowering of the voltage corona appears sharply at exactly the same value throughout, that is, at a value constant to within say one-tenth or one-quarter per cent. Under more refined conditions the constancy is shown to be even closer.

The empirical law connecting the critical or corona-forming voltage gradient E in kilovolts per cm., at the surface of the wire, the radius of the wire r in centimeters, and the relative density of the gas δ, is usually stated in the form

$$E = A\,\delta\left(1 + \frac{B}{\sqrt{\delta\,r}}\right) \tag{1}$$

A more convenient form for our present purposes is

$$E/\delta = A + \frac{B'}{\sqrt{\delta\,r}} \tag{2}$$

which gives a linear relation between E/δ and $\frac{1}{\sqrt{\delta\,r}}$; obviously $B' = A\,B$.

The value of δ is given by

$$\frac{3.92\,p}{273 + t} \tag{3}$$

in which p is the pressure in centimeters of mercury and t is the temperature in degrees centigrade.

The above relatively simple relations have now been corroborated by a number of observers and with quite close agreement as to the values of A and B. The influence of the diameter of the wire on corona-forming voltage was first emphasized by H. J. Ryan,[3] who was also the first to point out the possibilities of the corona as a voltage indicator. The exact nature of this influence and the presence of the two constants A and B were first shown by one of the present authors.[4] The precise influence of the density of the air was first shown by F. W. Peek, Jr.,[5] in one of the most important contributions yet made to the knowledge of the subject. Moisture in the air has no effect on the critical intensity.[2]

The form of the above law is the same for both continuous voltages and crest values of alternating voltages. With continuous voltage, however, there are appreciable differences in the values of the constants A and B, as between positive and negative corona-forming wire, the form of the law in each case remaining the same.[6] One of the most important results of the present work is the fact that this difference between positive and negative corona is reflected in the alternating corona, and that the law as given by formulas (**1**) and (**2**) must be modified. Briefly stated, the modification consists in the use of different values of the constants A and B above and below a definite value of $\frac{1}{\sqrt{\delta r}}$, the form of the law, however, remaining the same in each case, as will be seen below.

It has generally been accepted that within the commercial range frequency has no influence on the corona-forming voltage. Observations with the accurate methods used in the experiments show a slight influence of frequency within the range mentioned.

Since corona formation through the constancy of its appearance and the simplicity of its law offers a ready means for the measurement of high voltage, it is important that the constants A and B be determined accurately. When this is once done, such an instrument as the corona voltmeter has a calibration dependent only on its dimensions, and so constitutes a natural secondary standard of voltage.

Nearly all determinations of alternating corona voltages have been based on observations of voltage and crest factor taken in the low-tension circuit, and computed from transformer ratios. As is well known, this method is subject to serious errors on both accounts. If therefore advantage is to be taken of the constancy of corona voltage as a standard and as a method of measurement, it is necessary that the constants A and B be determined by direct measurement in the high-voltage circuit of the crest values of corona voltage, and to a relatively high degree of accuracy in terms of accepted standards. These determinations once made over a sufficiently wide range of values of δ, corona formation, by reason of the simplicity of the relation of formula (**1**), and its freedom from outside influence, becomes a far more reliable standard than the sphere gap, the potential transformer, or any other standard at present proposed.

III. PRECISION MEASUREMENT OF CORONA CONSTANTS

For the determination of the values of A and B we must (1) measure accurately the crest value of alternating voltage at which corona appears, (2) be able to observe to as small a difference of voltage as possible the first appearance of corona, and (3) measure δ and provide a wide range of its values.

The crest value of voltage (1) may be determined from the average value of the charging current of an air condenser in the high-voltage circuit. This method first used by Chubb and Fortescue,[7] was modified by Whitehead and Gorton,[8] and is now further improved as described below.

For (2) the accurate observation of the first appearance of corona, two methods are used,—(a) the telephone for detecting the sound of the corona, and (b) the galvanometer for detecting the conductivity of the air caused by the corona. Both methods are used in the corona voltmeter and are described in detail in an earlier paper;[1] further observations are reported below. The visibility of corona is neither convenient nor accurate as a means of determining its first appearance.

The corona voltmeter with its air-tight outer casing provides the method (3) for the observation of δ, the relative air density, and its variation over a wide range. Pressure and temperature are read and the pressure may be adjusted to any chosen value, thus permitting setting for any value of δ.

III. 1. MEASUREMENT OF VOLTAGE

If an alternating voltage of maximum value E volts and frequency f be impressed on a condenser of capacity C, the average charging current is

$$i = 4fCE; \tag{4}$$

if f and C are known and i is measured E the maximum for the maximum value of charging voltage is determined. When used for the high values of voltage pertaining to corona formation, one side of the condenser is grounded and the charging current measured in the ground connection. Since the condenser must withstand the full maximum voltage and have no dielectric or other loss, the most convenient form is that of concentric cylinders with wide radial separation, and with air as dielectric. This, however, means small capacity per unit axial length, and small total capacity if the outside dimensions are to be kept within reasonable limits. Consequently the use of this method involves the use of a large air condenser of small capacity and a determination of the value of the capacity.

Chubb and Fortescue[7] constructed a cylindrical condenser consisting of two wooden forms, each covered with sheet metal surfaces. The diameters of the two members were 60 cm. and 162.8 cm. respectively, and the outer member was provided with two flaring guard ring ends. The capacity between the inner member and the central section of the outer member was calculated as 2.65×10^{-11} farad, no attempt at measurement being made, doubtless owing to the difficulty of measuring so small a value. Chubb and Fortescue measured the charging current in the ground connection of the central section of the outer member of the condenser by means of a d'Arsonval galvanometer and a synchronous commutator connected as a shunt suppressor.

In the present experiments the same type of condenser is used, *i. e.*, the cylindrical guard ring type with voltage applied to the inside member and charging current measured in the ground connection of the central section of the outside member. The capacity, however, was measured, as described below. Further, the charging current was measured by the use of two rectifying kenotrons, thus obviating the irregularities and uncertainties of the synchronous commutator. The commutator was, however, frequently used for comparison and certain auxiliary tests.

A diagram of the principal connections is shown in Fig. 1. Voltage is applied from the transformer *A* to the corona voltmeter *B* and the air condenser *C*. The charging current of the central section of the latter passes to ground in alternate half waves through the resistances and kenotrons R_1, K_1, and R_2, K_2. The currents in R_1 and R_2 are therefore pulsating but unidirectional and so may be read by a continuous-current instrument in series or in shunt, as shown in Fig. 1, G_1 being a sensitive d'Arsonval galvanometer critically damped. A second galvanometer G_2 and a telephone T are used to detect the first appearance of corona on the central rod of the corona voltmeter as described below. A number of auxiliary circuits have been omitted from Fig. 1 and will be referred to in connection with the various measurements.

We will now describe in turn the methods of measuring the charging current, the frequency, and the capacity of the condenser, together with the precautions taken, the limits of accuracy, and all leading to the determination of the value of voltage present on the first appearance of corona in the corona voltmeter B.

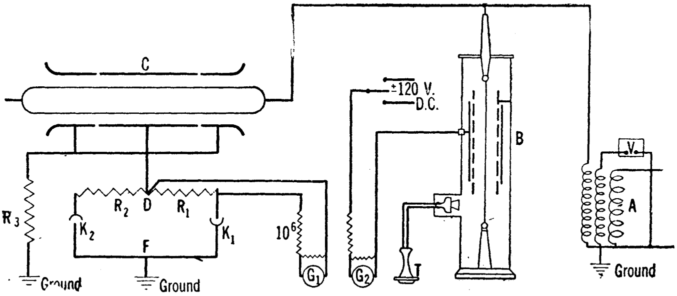

FIG. 1—PRINCIPAL CONNECTIONS

(*a*) *Charging Current.* Balance in kenotron circuit. In formula (4) i is the average value of the charging current. In Fig. 1 all positive half waves will pass through one kenotron and all negative half waves through the other. The d'Arsonval galvanometer in shunt to the resistance R_1 will therefore receive a pulsating unidirectional current and show a deflection proportional to its average value. Obviously the combination may be calibrated directly in terms of continuous current in R_1 and in terms of such a calibration the galvanometer will read one-half the average value of the charging current. In view of the foregoing it is of first importance that when no charging current is passing there be no continuous current flowing in the closed circuit $K_1 D K_2 F$. This condition was realized by the adjustment of the point of connection to the filament exciting circuits of the kenotrons as indicated at $P_1 P_2$ in Fig. 2. This in effect interposes a small adjustable e. m. f. counter to the normal direction of conductivity, at each kenotron. If this is not done the normal leak of electrons from the filament, particularly at its negative end, results in a small current in the circuit $K_1 D K_2 F$. In making these adjustments the galvanometer G_1 was connected first in series in the circuit $K_1 D K_2 F$ and then between the points D and F, repeating with adjustments at P_1 and P_2 until both readings are simultaneously zero. After balancing the kenotron circuit, as above, it was connected into the condenser ground circuit and with alternating voltage on was further balanced for equal resistance in the two branches by adjustment for zero current in the galvanometer connected across $D F$; on removal of the alternating voltage the circuit $K_1 D K_2 F$ is still balanced. Without these adjustments a small error is possible, the galvanometer G_1 in the connection of Fig. 1 showing at times a deflection of 0.5 mm.; after the adjustments mentioned no deflection can be detected.

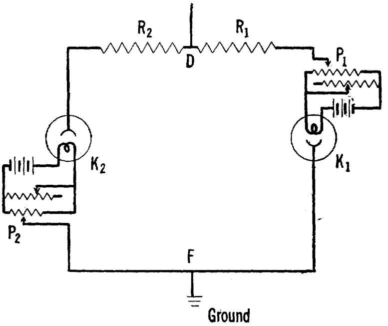

FIG. 2—KENOTRON CONTROL

(*b*) *Influence of Wave Form.* The use of the kenotrons for rectifying the charging current introduces an error if the wave form of voltage is not smooth, *i. e.*, if it has more than one maximum in each half wave. In this case there is a reversal of condenser current following every such maximum or elevation in the wave and since the kenotron passes current in only one direction, this reverse current passes through the opposite kenotron and so does not contribute to the galvanometer reading. Similarly in the next half cycle the reverse current is recorded in the galvanometer as positive. Thus due to both half waves the result is a galvanometer reading higher than that corresponding to the average charging current.

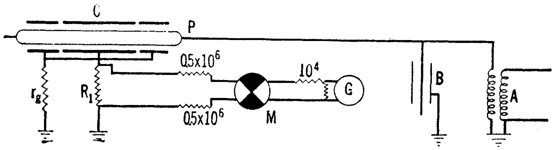

FIG. 3—MEASUREMENT OF CONDENSER CHARGING CURRENT WITH SYNCHRONOUS COMMUTATOR

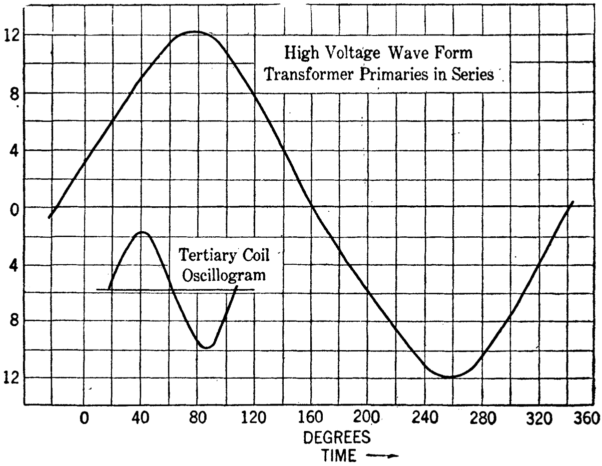

FIG. 4—HIGH-VOLTAGE WAVE FORM

The generator used in the experiments has a surface wound armature and shows a smooth wave on an oscillogram. The inserts of Figs. 4 and 5 show the voltage waves as taken from a low-tension tertiary coil on the transformer *T*, Fig. 1, for series and parallel connections respectively of the two primary coils. In order, however, to answer this question definitely, the wave form of the voltage at the high-tension terminals was taken by the method indicated in Fig. 3, in which

TABLE I.
WAVE FORM OF HIGH VOLTAGE

Brushes degrees	Galvanometer deflection cm.					
	Full wave			Half wave		
	Left	Right	Mean	Left	Right	2 X Mean
22.5	7.80	7.76	7.78	3.72	4.10	7.82
30	8.41	8.47	8.44	4.09	4.39	8.48
37.5	9.14	9.10	9.12	4.42	4.70	9.12
90	11.91	11.87	11.89	5.87	6.03	11.90
120	10.68	10.60	10.64	5.32	5.30	10.62

M is the synchronous commutator connected as full rectifier or as half-wave suppressor. In this method first pointed out by Bedell,[9] for any position of the brushes the galvanometer reading is proportional to

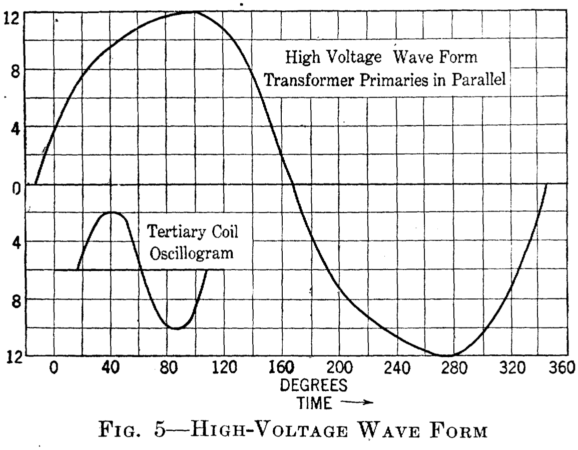

FIG. 5—HIGH-VOLTAGE WAVE FORM

the average value of the charging current for any particular half-wave interval between the brushes, and this in turn is proportional to the instantaneous value of the voltage on the condenser. Figs. 4 and 5 show the wave forms so taken for series and for parallel connections respectively of the transformer primary coils, which together with Table I, giving a section from the complete sheet of readings, indicate the conditions of accuracy. The mean values of right and left readings of the galvanometer are taken in all cases in order to eliminate a slight right and left dissymmetry probably due to electrostatic disturbance, generally noticeable in the very sensitive galvanometer, in spite of most careful screening. For obvious reasons this disturbance is more pronounced in the half-wave measurements in which the galvanometer is used as a half-wave suppressor.

The curves of Figs. 4 and 5 were each taken at the critical or corona-forming voltage using the same corona rod and equal values of air density. Although there are noticeable differences in wave form and in the values of effective voltage at the terminals of the tertiary coil (38 volts and 34.5 volts respectively) it is seen that the maxima of the two waves have very closely the same values. Further evidence that no error was present due to irregularities of wave form is found below in the comparison of corona readings taken with kenotrons and with commutator.

(*c*) *Comparison of Kenotrons with Commutator.* With the connections shown in Fig. 1, since the kenotron conducts in only one direction, the galvanometer receives a unidirectional pulsating current, the successive pulses being separated by time intervals of one-half period. The same conditions may be obtained in the galvanometer by the method shown in Fig. 3, the resistance R_1 being connected straight to ground and the synchronous commutator being connected as a shunt suppressor, *i. e.*, so that the galvanometer is short-circuited during alternate half-cycles. With fixed conditions in the high-tension circuit therefore both these methods should give the same galvanometer reading. In the experiments recorded in the following Table II, the voltage was set at the corona-forming value for a 0.955-cm. (0.376-in.) diameter rod in the corona voltmeter, for each of the two wave forms pertaining to the two methods of connection of the transformer primaries. The table gives for each method of connection first the readings leading to the commutator setting for maximum galvanometer deflection, and this is done using the galvanometer both as complete rectifier and as half-wave suppressor. It is seen that these two sets of readings are closely in the relation 2 to 1, and that the commutator setting is indicated to within one and one-half electrical degrees. The readings at maximum setting are then given and directly below them the corresponding kenotron

TABLE II.

COMPARISON OF READINGS WITH KENOTRONS AND WITH COMMUTATOR.

Brush setting degrees	Transformer primaries in parallel					
	Full wave			Half wave		
	Left	Right	Mean	Left	Right	Mean
90	11.99	12.01	12.00	6.07	5.98	6.02
93	12.00	12.07	12.03	6.09	5.99	6.04
96	12.00	12.07	12.03	6.09	5.98	6.03
99	11.93	12.02	11.97	6.01	5.99	6.00
94.5	11.99	12.07	6.01 × 2	6.08	5.98	6.03
With kenotrons				5.99	6.01	6.00
	Transformer primaries in series.					
72	11.82	11.90	11.86	6.01	5.89	5.95
75	11.92	12.02	11.97	6.04	5.93	5.98
78	11.98	12.07	12.03	6.08	5.94	6.01
81	11.97	12.03	12.20	6.07	5.97	6.02
84	11.89	12.00	11.94	6.02	5.93	5.97
79.5	11.96	12.07	6.01 × 2	6.07	5.97	6.02
With kenotrons						6.01
				5.99	6.01	6.00
				5.98	6.02	6.00

readings. It is seen that there is excellent agreement particularly when the full rectification readings are included. A further interesting observation was made in connecting the galvanometer and commutator as shunt suppressor across each of the resistances R_1 and R_2 of Fig. 1 in turn, using the maximum brush settings and other conditions of Table II. For one of the resistances the full galvanometer deflections of Table II were obtained, but for the other the mean of the right and left readings of the galvanometer was accurately zero, as is to be expected since for the half wave during which the kenotron conducts, the galvanometer is short-circuited by the commutator. This observation also indicates the absence of reverse currents due to inequalities in the voltage wave.

(*d*) *Calibration of Galvanometer.* The galvanometer used for measuring the charging current of the air condenser was a late American type d'Arsonval read by telescope and scale. Its constants were as follows: resistance 115 ohms; sensitivity 40 megohms; free period 9.5 seconds; critical damping resistance 560 ohms. Throughout all the observations the galvanometer was shunted with 560 ohms and the combination used in series with 10^6 ohms for measuring the potential drop across the resistances R_1 and R_2 in Fig. 1. Since the maximum voltage is measured through the current in R_1 or R_2, it is obviously of the first importance that the galvanometer be accurately calibrated. The calibration directly in amperes was effected by passing continuous current through R_1 or R_2 and measuring this current through the resulting potential drop over a resistance of 499 ohms always in this auxiliary circuit; the potential drop was measured on a precision potentiometer in terms of a Weston cell. The value of the resistance was determined to within 1/25 of 1 per cent by comparison with certified laboratory standards. Two certified Weston cells were used, one checking the constancy of that in use with the potentiometer; at the end of the observations their values were equal to the fourth decimal.

The galvanometer was calibrated for every series of observations and usually at approximately the scale reading pertaining to the particular charging current being measured; see Table VII.

In order to investigate a possible error due to the pulsating character of the galvanometer current when the instrument is calibrated for continuous current, an extensive series of observations was made using the commutator for breaking up continuous current and for cutting out alternate half waves of alternating current. These experiments have been described in another paper,[10] and show that when the galvanometer is connected as in Fig. 1 or Fig. 3 the calibration with continuous current is accurately the same as that with pulsating current whether rectangular or of approximately sine shape.

(*e*) *Resistance of Ground Connection.* Since the central section of the air condenser is connected to ground through the resistance and kenotron circuits of Fig. 1, two questions arise as to the effect of the resistance of these circuits on the charging current of the condenser. First, if the resistance of this circuit is sufficiently high the voltage across the terminals of the condenser may be appreciably lower than the total voltage between high-tension terminal and ground; and second, if the resistance in question is sufficiently high, suitable adjustment must be made to ensure equal potential between the central section of the condenser and the guard rings, or otherwise the current in the kenotron circuit would not be that due to the capacity between the high-voltage member of the condenser and the central section of the grounded member alone.

With reference to the first of these questions, the calculated capacity of the central member of the condenser is 8.262×10^{-11} farads. The measured value is 8.286×10^{-11} farads, (see paragraph (h) below). The corresponding reactance at 60 cycles is therefore 3.2×10^{7} ohms. The resistances R_1 and R_2, Fig. 1, were 2000 ohms each throughout the experiments. The equivalent resistance of one kenotron varies with its filament current and also with the current that the kenotron is transmitting. The values of these resistances were determined by taking the volt-ampere characteristics of the kenotrons with continuous currents. It is not thought necessary to reproduce these readings here, as the characteristics of the kenotron are well-known. The maximum value of kenotron resistance obtaining in the experiments was approximatley 4000 ohms.

The filament currents of the two kenotrons were always adjusted to the same value as indicated by a direct-reading continuous-current instrument. Slight variations in the filament current have no effect on the transmitting power of the kenotron and only produce small variations in the equivalent resistance of the kenotron.

Consequently the maximum aggregate resistance in the ground circuit of the central section at any time was approximately 6000 ohms. This non-inductive resistance is in series with the capacity reactance of 3.21×10^7 ohms of the central section of the condenser and has therefore a quite negligible effect either in elevating the potential of the outer member of the condenser above ground, or in reducing the voltage at the condenser terminal below the full applied value.

The accuracy of the above deductions was tested by connecting the central section of the condenser to ground through resistance only and similarly the two guard rings in parallel to ground through resistance only. The voltage drop over these resistances was studied by means of the commutator and galvanometer. The results showed that the voltage drop over each resistance was accurately proportional to its value. They also showed that the values so measured on either resistance were independent of the value of the other resistance within the range 0 to 10,000 ohms, the study not being carried further.

With reference to the second question raised above, namely, as to the effect of a difference of potential between the guard rings and the central section on the values of the charging current to ground from the central section, it would appear from the foregoing that, since the difference of potential between the two members is negligibly small, and furthermore since the capacity between them is also small, no influence on the

TABLE III.

INFLUENCE OF RESISTANCE IN GROUND CONNECTION OF CONDENSER.

R_3	Volts over *D-F*.			Volts over R_2			Volts over R_1			Volts over R_3		
	Left	Right	Mean	Left	Right	Mean	Left	Right	Mean	Left	Right	Mean
9999	10.7	10.73	10.71	3.3	3.35	3.32	3.3	3.35	3.32	17.9	18.1	18.0
5970	10.73	10.82	10.77	3.29	3.36	3.32	3.29	3.36	3.32	10.68	10.8	10.79
0	10.73	10.82	10.77	3.29	3.36	3.32	3.29	3.36	3.32	0.1	0.1	0.

$R_1 = R_2 = 2000$ ohms. Filament current 2.75 amperes.

charging current of the central section would be found even if the guard rings were connected directly to ground. However, it was thought best to investigate the matter experimentally and also to determine the proper value of resistance to connect between the guard rings and ground in order to ensure that the guard rings are at the same potential as the central member. The results of this study are given in Table III.

Referring to Table III and Fig. 1, the first column gives the value of the resistance R_3 between guard rings and ground; the next three columns right, left and mean readings of the galvanometer connected between D and F of Fig. 1 using the commutator for rectification; the next three columns voltage over R_2; and the next three that over R_1 both without use of commutator; and the last three columns the voltage over R_3 with the aid of the commutator. These results show that the voltage between the point D and ground and the voltages over the resistances R_1 and R_2 are all independent of the value of the resistance R_3 up to 10,000 ohms. Furthermore they show that the guard rings are brought to the same potential as the central section when connected to ground through a resistance R_3 of 5,970 ohms. R_3 was kept at this value throughout the course of the work.

(*f*) *Measurement of Frequency.* The value of frequency enters directly from Formula (**4**) into the expression for the maximum value of voltage. Constancy of frequency therefore and as accurate a determination of its value as possible are highly essential to an accurate determination of the maximum voltage.

As regards constancy, the 5-kw. single-phase generator was driven by a continuous-current motor run as the only load on a large storage battery. This constant source of supply was further supplemented by an automatic speed control illustrated in Fig. 6. In this method the ultimate source of constant speed is an electrically operated tuning fork. A small rotary converter, R, of the same frequency as the tuning fork is driven from the direct-current end and is loaded with a resistance on the alternating-current end, the load circuit being taken through a pair of contacts on the tuning fork, contact being made once during each half wave. The time interval of contact by the tuning fork is a fraction of the whole alternating-current period. If the speed rises, contact is made at an instant when there is a greater electromotive force and current, thus resulting in a greater load on the converter. If the speed goes down, the conditions are reversed, consequently the tendency of the change of load is to maintain the speed of the converter constant. This tendency can be made greater or more positive by inserting resistance in the armature circuit of the continuous-current end of the converter.

The shaft of the small converter and that of the larger machine which is to be controlled are each supplied with a crown commutator, K_1 and K_2. The number of commutator segments for each is chosen so that by their speeds the frequencies of reversal of the two commutators are the same. The commutators are connected together electrically, as indicated in Fig. 6, in such a way as to short-circuit a small resistance in the armature circuit of the main driving motor for a greater or less period, according as the commutators depart more or less from the position of exact coincidence of phase. The machines automatically find such a relative commutator phase relation that an increase in speed decreases the duration of short circuit of the armature resistance; and vice versa, so that the average voltage on the motor armature is such as to more or less exactly maintain the speed in a constant relation to that in the small converter, which in its turn is maintained constant by the tuning fork. The

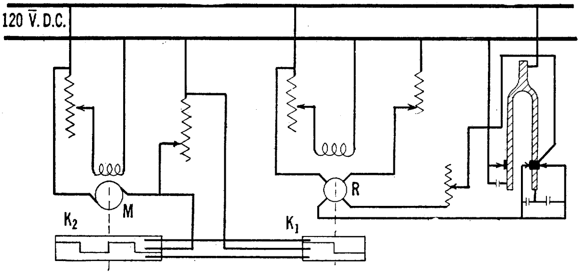

Fig. 6—Control of Frequency by Tuning Fork

introduction of the small converter is necessary, since the tuning fork contacts will not carry the large currents interrupted in the control of the considerably larger direct-current motor.

The use of the foregoing method prevented slow changes of speed due to temperature changes in the motor, or to variations of applied voltage, etc. Occasionally changes of this character were sufficiently great to overcome the regulating power of the tuning fork and the resulting upset in frequency was immediately indicated by the "beats" between the two frequencies detected by means of a pair of stroboscopic disks, one on each machine and through either of which the tuning fork might be viewed. In this way it was possible to tell at any instant by a glance whether the frequency was constant.

Observations taken with the stroboscopic disks indicate that by the above method the frequency was maintained constant at within 0.5 per cent. In this connection it is to be noted, however, that as read by the galvanometer, the charging current of the air condenser is read as an average value. Consequently, so long as the frequency is kept to an average constant value, momentary changes of frequency will not be registered in the galvanometer. This was borne out by the general character of the galvanometer reading, this reading being always absolutely stationary and not subject to any momentary variations which could be detected.

(*g*) *Value of Frequency.* Most of the measurements described in this paper were made at 60 cycles and this frequency is to be understood unless particular note is made of some other value. The frequency of the tuning fork was measured each day by bringing the small rotary converter into synchronism with the fork by the method described above and then taking the speed of the rotary by means of a "tachascope" or, combined revolution counter and stop watch. The speed of the rotary corresponding to 60 cycles for the alternating generator was 1800 rev. per min. The usual method of observation was to take the number of revolutions of the rotary within a period of three minutes, taking this observation three times and taking the mean of these for the determination of frequency. Table IV gives an example of the measurements which indicate that the average frequency was determined to within 0.04 per cent.

TABLE IV.
MEASUREMENT OF FREQUENCY.

Time	Revolutions of control rotary		
3 min.	5400 + 4.5	5400 − 4	5400 − 6
3 min.	5400 − 1	5400 − 3	5400 − 2
3 min.	5400 + 3.5	5400 + 5	5400 + 2
Average...............	5400 + 2.3	5400 − 2	5400 − 2
Average frequency.....	60.02	59.97	59.98

(*h*) *Capacity of Air Condenser.* As already stated, the air condenser was of concentric cylinder type. It had a continuous inside member and an outer member consisting of a central section protected at each end with guard rings. A photograph is shown in Fig. 7. Both members were made from standard cast iron pipe.

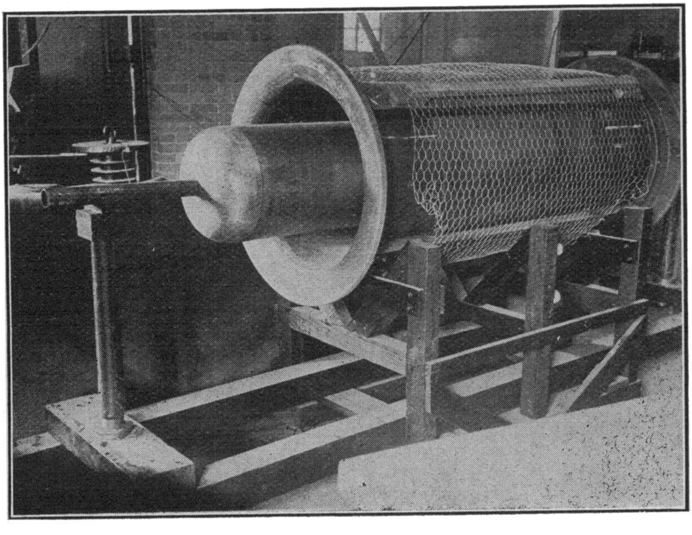

Fig. 7—Air Condenser

The surface of the inner member was turned off so as to have a uniform diameter and to be free from surface irregularities. The ends were filled with rounded wooden plugs covered with tin foil and the whole inner member was supported by a 1½ in. pipe through central holes in these wooden plugs. The guard ring ends of the outer member consisted of straight sections on the outer ends of which were mounted standard flanges screwed on, the inside surfaces being turned off so as to provide a smooth flaring end to each guard ring.

The central or inner member was supported on dry oak posts boiled in paraffin. Sliding and screw adjustments at the top and bottom of these posts permitted accurate centering in relation to the outer member. The two guard ring ends were tied together by means of four stout oak pieces maintaining them in line. The central section of the outer member was supported on eight small plate glass insulators mounted in the four oak pieces mentioned. The whole was then supported in a wooden cradle, as indicated in Fig. 7.

Following are the principal dimensions of the air condenser:

Diameter of inner member	29.50 cm.	(11.61 in.)
Average diameter of outer member	49.30 cm.	(19.42 in.)
Length of central section of outer member	76.20 cm.	(30.00 in.)
Length of guard ring ends	30.5 cm.	(12.00 in.)

The inside diameter of the outer member was not strictly uniform. It was measured in twelve places, the extreme variation being between 49.20 cm. and 49.51 cm. The calculated value of the capacity between the central section of the outer member and the inner member, based on the average diameter of the outer member given above, is 8.249×10^{-11} farads. In view of the uncertainty introduced by slight irregularities of this character, it was decided to measure the capacity.

The measurement of so small a value of capacity is a matter of considerable difficulty and requires much care. Maxwell's bridge method was used, following in general the experimental method of Rosa and

Dorsey[11]. We were fortunate in being able to borrow from the National Bureau of Standards the special commutator constructed by them.

The diagram of the connections of this method is shown in Fig. 8. The principle is well-known and consists in replacing one of the resistances in a Wheatstone bridge with a condenser, and a suitable device for rapidly charging and discharging the condenser. In the arrangement shown in Fig. 8 the charging current only is used, the condenser being short-circuited in the alternate intervals. If, as in our case, the condenser has guard rings, it is necessary that they follow the same cycle of potential as the central section to be measured. This is accomplished by the double bridge shown in Fig. 8. *R S* and *R′ S′* are contacts carried on the special form of rotating commutator constructed by Rosa and Dorsey for effecting the simultaneous charge and discharge of the central and guard ring members.

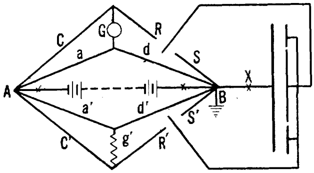

FIG. 8—MEASUREMENT OF CAPACITY OF AIR CONDENSER (8.28×10^{-11} FARADS)

The bridge is balanced by varying the arms *a* and *a′* and when in proper adjustment corresponding arms in the two bridges must have identical values. The value of the capacity between the inner member and the central section of the outer member is

$$C = \frac{a\,F}{n\,c\,d} \qquad (5)$$

a, *c* and *d* are values of resistance; *n* is the number of charges of the condenser per second; and *F* is a correction factor depending on the method of the relative values of the resistances, and differing but little from unity; in our work by about 3 parts in 10,000.

The values used by us were as follows:

$c = c' = 521{,}000$ ohms
$d = d' = 50{,}000$ ohms
$g = g' = 1{,}124$ ohms

The high resistances were of manganin made up of specially wound non-inductive units lent by the Bureau of Standards. The remaining resistances were of standard, high-grade, laboratory type, and all values as given were measured in terms of laboratory standards to 0.04 per cent.

For the determination of n, the number of charges per second, the commutator which had sixteen segments was geared directly to the small rotary converter controlled by a tuning fork by the method of Fig. 6, as already described. Two speeds differing by the factor 2 were possible with this arrangement, control of the machine speed by the fork being effective at either speed. Moreover, observation by means of the stroboscopic disk as to the constancy of the speed was also possible. The fork thus served for maintaining the frequency of charge and discharge at the average uniform value pertaining to either of two speeds. The value of this frequency, and thereby the value of n, was determined by a contact-making device on the commutator and a stop watch. The normal speed of the rotary was 1800 rev. per·min. and this corresponded to the value of $n = 480$. The accuracy of the speed determination and so of the value of n was therefore within 0.04 per cent.

In measuring the capacity it was necessary to make a correction due to the capacity of the lead wires between the condenser and the bridge. This was done by measuring the capacity first with the condenser in and then with the connection to the central member of the condenser opened at the point X, Fig. 8. Two precautions have to be taken following this method; first, as to the insulation resistance of the capacity to be measured and the insulation resistance of the central section of the outer member to ground; and second, the capacity between the bridge wiring and the central member when the connection at X is opened.

With reference to the insulation resistances mentioned, it was found that in moist weather the values were relatively low, say of the order a few hundred megohms, doubtless owing to the rather large surfaces of support of the heavy parts. However, in clear dry weather the resistances were greater than one-half million megohms, a figure which reduces the possible error on this account to quite negligible proportions. These figures include the commutator insulation and were checked at each series of observations for the measurement of the capacity.

As to the possible error due to a charging current from the open contact at X through the capacity to the central member and thence to the outer member, this error was avoided by completely enclosing the projecting ends of the central member of the condenser by means of large cones made of galvanized sheet iron, the connection to the central member being carried through a small opening in one of the cone ends. The opening X was made at this place and under these circumstances the interior central member was entirely screened during the measurement for the correction due to the capacity of all the auxiliary wiring.

The most difficult part of this measurement is the adjustment of the brushes on the commutator for simultaneous charge and discharge of the central section and the guard ring ends. This requires very careful study of the conditions of contact of the brushes on the commutator sections, so as to prevent mechanical vibration and to ensure a reasonable permanence of a proper adjustment when once it is effected. Probably the best test of these conditions is by means of an auxiliary circuit through the brush contacts, indicating by means of a galvanometer the ratio of the running to the standstill deflections. By taking the two sets of brushes in turn singly and then in series, a quite accurate idea of the conditions is obtainable. Throughout the measurements of capacity, observations of this character were taken both before and after the capacity readings, thus ensuring satisfactory conditions. As an additional precaution in this direction a double set of readings was always taken, the duplicate contacts of the commutator being exchanged for the two readings between the central section of the condenser and the guard rings. Table V gives the readings which were selected as having been taken under the most favorable conditions, that is to say, those in which there was least difference between the two values of capacity corresponding to the exchange of the two sets of brush contacts.

TABLE V.
MEASUREMENT OF CAPACITY OF AIR CONDENSER.

	Half speed $n = 240.2$ $ncd = 62.56 \times 10^{11}$ A positive		Full speed $n = 480.3$ $ncd = 125.12 \times 10^{11}$ A positive	
X	Left	Right	Left	Right
Closed..........	940			1851
Open............	419	417	805	810
Closed..........	939	934	1839	1854
Open............	419	417	805	814
Closed..........		934	1835	
Difference......	520.5	517	1032	1040.5
Capacities......	8.320	8.264	8.248	8.316
Mean..........	8.296×10^{-11}		8.282×10^{-11}	
X	A negative		A negative	
Closed..........		930		1867
Open............	420	415	826	828
Closed..........	941	929	1864	1864
Open............	420	415	826	828
Closed..........	942		1864	1858
Difference......	521.5	514.5	1038	1035
Capacities......	833.6	8.224	8.296	8.272
Mean..........	8.280×10^{-11}		8.289×10^{-11}	

Mean of means 8.286×10^{-11} farads.
Calculated value 8.262×10^{-11} farads.

The figures given in Table V are the values of the resistance "*a*" in ohms. The words "right and "left" refer to the particular set of commutator brushes used. It will be noted also that readings were taken for two speeds and for each speed a reading for a reversal of the polarity of the battery of the bridge. The value of the capacity in each case is calculated by means of Formula (**5**). The final average value of these readings is 8.286×10^{-11} farads. The calculated values based on independent measurements of the dimensions by each of the authors were 8.28×10^{-11} and 8.245×10^{-11} farads.

In computing the maximum value of voltage the measured value of the capacity, 8.286×10^{-11}, has been used.

(*i*) *Electrostatic Screening.* In view of the small value of the current to be measured, especial care had to be taken for the elimination of all electrostatic inductive effects between the high-tension circuit, the outer surface of the condenser, and the various measuring circuits in the ground connection of the condenser. With the sensitive galvanometer used it was found that the unscreened exposure of relatively small and distant portions of the wiring could introduce considerable error. It was necessary for the elimination of errors of this character to completely enclose in grounded casing all of the low-voltage measuring circuits of Fig. 1, including the various auxiliary circuits not shown; thus the kenotrons, the resistances, R_1, R_2 and R_3 and their various control circuits were enclosed in one sheet-iron box. The entire equipment of the observer's station, including the galvanometers for the measurement of the charging current and for the detection of corona and the various calibrating circuits were all enclosed in a small chamber completely surrounded with wire mesh. The condenser itself was surrounded with an outer screen of wire mesh. All connections between these various parts were likewise carried in metal conduit. All of the screening coverings described were connected together and to ground.

The test for completeness of electrostatic screening consisted in opening the high-voltage connection to the central member of the air condenser at P, Fig. 3, applying the voltage, and observing the galvanometer in the method of connection of Fig. 3. In this observation the value of R_1 was 9900 ohms and is about five times the value used in the charging current measurements. The sensitivity of the galvanometer used was 1250 megohms and no deflection could be detected.

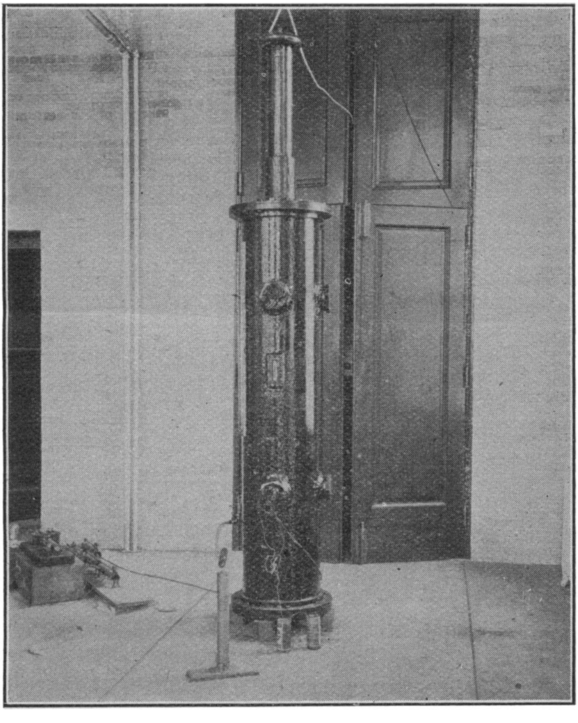

FIG. 9—CORONA VOLTMETER—200,000 VOLTS

The galvanometer could be read to 0.2 millimeter which, in accordance with the magnitude of the deflections corresponding to the charging currents measured, means that an error on account of electrostatic induction, if present, is less than 0.1 or 0.2 per cent.

III. 2. DETECTION OF CORONA

All of the corona observations on which the measurements are based were taken with the corona voltmeter shown in Fig. 9. A vertical section (not to scale) is shown in Fig. 10. The principal dimensions are,—height overall 9 ft. 10 in.; outside diameter 22 in.; diameter of grounded electrode cylinder 24.67 cm. (9.715 in.); length 60.95 cm. (24 in.). This cylinder was perforated over its whole surface with 0.952-cm. (0.375-in.) diameter holes on 1.27-cm. (0.5-in.) centers. The corona rods, 11 in number, of diameters ranging from 0.1038 to 1.2665 cm. were of tool steel polished and nickel-plated, and were all of the same length and equipped with similar threaded end fittings. A small opening *A* in the top cover permits easy insertion or removal of a rod, attachment of fittings to top and bottom insulators being made through two hand holes in the sides (see *A*, *B*, *B*, Fig. 10). These openings are closed with air-tight covers. The cover at *A* has a glass window permitting observation of visual corona. The locations of the thermometer *M*, the telephone, the connections *T* and *G* for telephone and galvanometer, and other auxiliaries are indicated in Fig. 10.

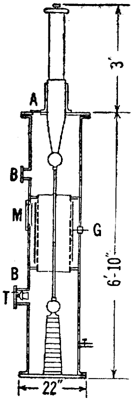

FIG. 10—CORONA VOLTMETER—200,000 VOLTS

(*a*) *Visual Corona.* Attention was first drawn to the phenomenon of the high-voltage corona by the power loss between transmission lines. This was promptly found to be accompanied by the visual corona around the conductors. All of the early studies of corona formation, of which the most notable were those of Ryan, gaged the first presence of corona by visual means. The visual method, however, is neither convenient nor accurate. It necessitates working in a dark room and is subject to error, in that accurate observation depends upon the state of the eye as regards its recent usage, fatigue, etc.

Nevertheless, under carefully controlled conditions using long time intervals for eye rests, it is possible to secure consistent observations with the visual corona. In the earlier papers of one of the authors comparisons of corona voltages as between the visual and other methods have been recorded showing that they have identical values. In the present work observations of this character have been repeated in some of the auxiliary experiments and they show the identity of corona voltages as observed by the visual method and by the far more convenient and accurate galvanometer. The visual method has, however, not been used for the measurements, the telephone and galvanometer being far more accurate and convenient.

(*b*) *The Telephone as Detector.* The presence of corona may be detected by its sound even in an open space. If the corona conductor is surrounded by an outer casing, such as a cylinder forming the opposite conductor, the sound within this enclosing space is confined and intensified. It may be conveniently used as a detector of the first presence of corona by means either of a direct tube connection between the ear and the enclosing chamber, or by means of a telephone transmitter within the enclosing chamber and a receiver at the ear. The latter method is necessary if the air pressure is to be varied and so is used in the corona voltmeter.

A number of experiments have been recorded in earlier papers, showing the identical values of corona voltages as observed visually, with the telephone, and with the galvanometer method described below. During all of the present work the telephone and galvanometer have been used simultaneously and the work throughout makes it certain that either of these methods may be used for detecting the first presence of corona.

The present work, however, has revealed that the character of the note in the telephone is different under different conditions as regards the size of the corona conductor and the density of the air surrounding it. This feature has considerable value after slight experience in the operation of the corona voltmeter, as it gives the observer information as to the conditions under which he is working. As regards its bearing on the accuracy of the telephone as a method of telling the first presence of corona, it is only necessary to note that the first uniform continuous note, whether it be faint and high or considerably louder and of lower tone, is to be taken as the signal of the presence of corona. The first type mentioned pertains to large wires at low pressure and the second to smaller wires and higher pressures.

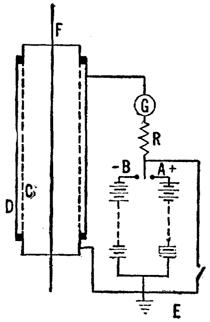

Fig. 11—The Galvanometer as Detector of Corona

Irregularities or other surface imperfections of the corona rod can usually be detected in the telephone by a characteristic crackling note quite distinct from that of corona.

(*c*) *The Galvanometer as Detector.* The essential elements of the galvanometer method of detecting corona are shown in Fig. 11, in which *C* is the perforated cylinder in the corona voltmeter which is connected to ground and to one side of the voltage to be measured; *D* is a surrounding cylinder only slightly larger in diameter than *C*, from which it is carefully insulated. The remaining elements of the circuit are obvious from Figs. 1 and 10. When corona appears on the central rod *F*, the surrounding air is copiously ionized and this ionization extends through the perforations to the space between the cylinders *C* and *D* which thus becomes highly conducting, resulting in a deflection of the galvanometer *G*.

An extensive series of observations has been made with corona rods of various size and under different conditions as to temperature and pressure on the relation between the voltage on the corona rod *F* and the resulting galvanometer deflections. A characteristic series of observations are given in Table VI, and the corresponding curves are plotted in Figs. 12, 13 and 14. In connection with these observations it may be noted that the sensitivity of the undamped galvanometer was 1280 megohms, and when critically damped with a shunt of 3400 ohms the sensitivity was 428 megohms. The resistance *R* was 50,000 ohms and the battery ± 115 volts. From the dimensions of the corona voltmeter already given, the length of the cylinder *C* was 60.95 cm., its diameter 24.67 cm., and its space separation from the cylinder *D* 0.317 cm.

TABLE VI.

GALVANOMETER AND TELEPHONE AS DETECTORS OF CORONA.

0.314 cm. diameter rod.

Positive electrode		Negative electrode		Electrode zero potential	
T. C. volts	Deflection cm.	T. C. volts	Deflection cm.	T. C. volts	Deflection cm.
0	0.0	0	0.0	0	0
31.4	0.0	31.4	0.0	31.4	0
31.5*	0.0	31.5*	11.7	31.5*	1.4
31.7	0.0	31.6	16.7	32	5.1
31.8	0.0			33	7.3
31.9	0.06			34	8.7
32	0.1			35	9.1
32.1	0.18			36	8.4
32.2	0.58			37	6.3
32.5	1.08			38	3.6
33	5.9			39	0.1
34	19.8			40	−3.9

*Telephone.

From these constants and from the observations the resistance of the space between *C* and *D* when in the initial conducting condition corresponding to the start of corona is about 1600 megohms.

Figs. 12, 13 and 14, pertaining to three different sizes of corona rod, are plotted with galvanometer deflections in centimeters as ordinates and transformer tertiary coil volts as abscissas. Each contains three sets of curves taken at different values of air density. The upper smaller curves are taken with the value of galvanometer sensitivity used throughout the voltage measurements. The larger curves are taken with galvanometer sensitivity reduced to 1/10 in order to extend the curves. Three curves were taken at each pressure, one each for the electrode *D* of Fig. 11 at 115 volts positive potential, one at 115 volts negative potential, and one at ground potential. The value of voltage at which the telephone is first heard is also indicated. It is to be noted that in all of these curves negative potential on the electrode *D* is best for the detection of the first presence of corona in that the curve rises most sharply. This is especially

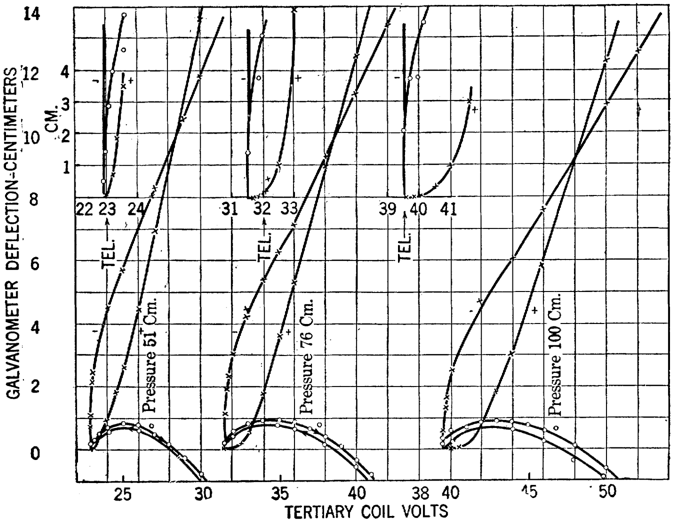

FIG. 12—GALVANOMETER AS DETECTOR OF CORONA—0.314-CM. DIAM. ROD

noticeable with small rods. With larger rods the advantage of negative over positive potential holds at low pressure, but tends to disappear at high pressures.

With larger rods at low pressures where the negative electrode should be used for first detection of corona, the telephone gives a pure high note of relatively faint volume within the interval of voltage between the curves of negative and positive electrodes, the full volume of sound appearing when the latter curve begins. At high pressures the curves come together; and with either positive or negative electrode the first corona is accompanied by a full sound in the telephone. With the smaller rods, although there is also a lag of the rise of the curve of positive electrode behind that of negative electrode, the faint initial high tone in the telephone is absent and except at very low pressures there is no marked variation in the telephone note, this note being clear and full with the first appearance of corona with negative electrode.

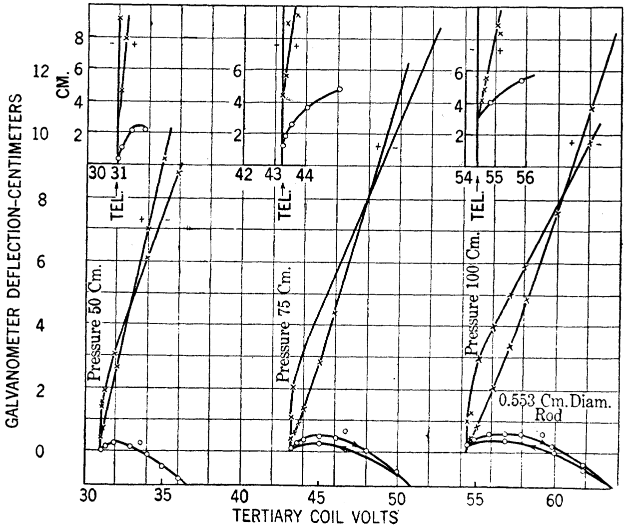

FIG. 13—GALVANOMETER AS DETECTOR OF CORONA—0.553-CM. DIAM. ROD

Considering the foregoing, therefore, from the standpoint of accuracy of determination of the first appearance of corona, negative potential should be used on the electrode *D* in all cases. Conditions of observation with both telephone and galvanometer are better at values of air density above that of normal atmosphere, rather than below, if large rods are used. Consequently, for reading low voltages better conditions are obtained by using a small rod rather than by using a large rod with reduced air density.

The difference in shape of the curves of positive and negative electrode has been noted in an earlier paper.

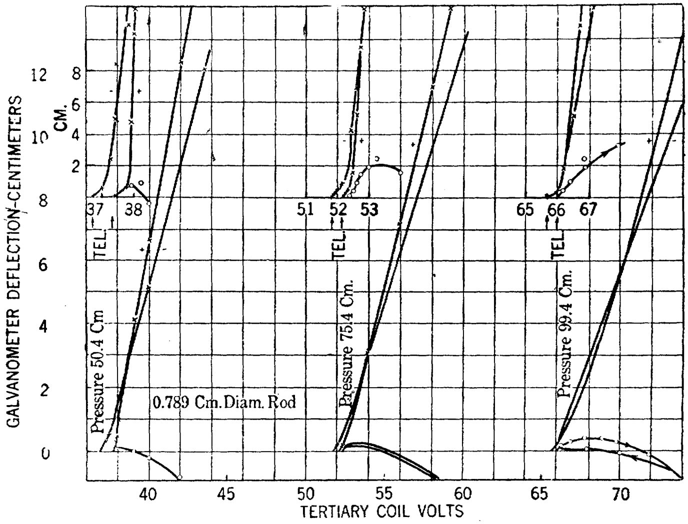

FIG. 14—GALVANOMETER AS DETECTOR OF CORONA—0.789-CM. DIAM. ROD

The greater sensitivity of the negative electrode is obviously due to the fact that corona formation or ionization of the air occurs first due to the motion of the negative electrons. The acceleration of the electron is greatest when it is moving toward the positively charged corona conductor. Under these circumstances the positive ions, as products of the process of ionization, would be repelled and would therefore give maximum current in the galvanometer circuit of Fig. 11 when the electrode *D* is negative. The exact shape of the curves probably depends on the wave form of voltage, moisture content of the air, and possibly on the frequency. It is only the initial slope of the most sensitive of these curves that is of importance in the detection of corona; their shape above the region of starting is of no present interest. It may be noted in passing, however, that it is possible to eliminate the voltage for the electrode *D* entirely and still observe the beginning of corona, although at a considerably lessened sensitivity. The curves showing the galvanometer deflection when the electrode *D* is at ground potential are seen to show reversal of galvanometer deflection at the voltage at which the curves of positive and negative electrodes intersect.

With reference to the actual degree of accuracy to which the beginning of corona could be observed, it will be noted from Fig. 12 that the galvanometer deflection begins and increases so sharply as to be practically instantaneous. Thus, for example, in Table VI and Fig. 12 the galvanometer deflection increases from 0 to 11.7 cm. within the voltage interval 31.4 to 31.5, the telephone coming in sharply at the same point. This indicates an accuracy of a very small fraction of 1 per cent. With the larger rod in Fig. 14 at high pressure we have a deflection of 1.4 cm. within the voltage interval 65.9 to 66. At low pressures the deflection is 2.4 cm. within the voltage interval 36.7 to 37.2. From this it will be seen that the sensitivity of corona detection is still quite high even under the unfavorable conditions of large rod at low pressure.

Throughout the observations the telephone and the galvanometer were read simultaneously, each checking the other. If an appreciable galvanometer deflection occurred without a corresponding clear telephone indication, or vice versa, an explanation could usually be found in a local spark or other surface impurity on the rod. We believe, in view of the foregoing that throughout the work the accuracy with which the beginning of corona has been read is better than 1/10 of 1 per cent. In this connection it may be pointed out that the initial flat portion or low rate of rise of the negative curves with large wires can only be detected by use of a very sensitive instrument. These initial portions are probably due to slight surface imperfections rather than to full corona. This is borne out by the fact that the full telephone note comes out at a point corresponding to the steep portion of the curve. With an instrument of lower sensitivity, such as would normally be used with the corona voltmeter, these initial portions of the curve cannot be detected and the instrument takes an initial sharp deflection accompanied by the simultaneous telephone note.

III. 3 MEASUREMENT OF AIR DENSITY

The relative air density δ as given in Formula (**3**) is

$$\delta = \frac{3.92 \times p}{273 + t} \qquad (\mathbf{3})$$

p being the absolute pressure in centimeters of mercury. and t the temperature in degrees centigrade. Thus δ has the value 1 at 76 cm. mercury and 25 deg. centigrade.

(*a*) *Pressure.* In the experiments the pressure was read on an open mercury manometer, (see Fig. 10), the accuracy of observation therefore being to about 0.2 millimeter. The usual correction for temperature was applied. All of the observations leading to the precision formulas (**7**) and (**8**) for the electric strength of air were taken within the range of pressures 25 cm. and 139 cm. absolute, all of which was covered by the mercury manometer. A number of observations studying the performance of the corona voltmeter at higher pressures were made with a standard direct reading gage by Schaeffer & Budenberg, calibrated within their common range in terms of the mercury manometer.

At the highest and the lowest pressures slight leaks in the outer casing of the voltmeter were detected. These leaks were, however, quite slow and pressure readings were taken at the beginning and end of each series of voltage readings. The average value of pressure within the interval was usually taken in these cases and an inspection of Table VII indicates that the error on this account was negligible. In calculating δ, p is the absolute pressure in centimeters of mercury.

The open mercury manometer reads pressure with relation to the atmosphere. Atmospheric pressure was determined from the laboratory standard barometer with the usual correction for temperature.

(*b*) *Temperature.* The temperature within the voltmeter casing was read on an ordinary laboratory centigrade thermometer hung within the casing near its wall and so that it could be viewed through a small glass window. The thermometer could be read to within about 0.2 degree. From an inspection of the value of δ this indicates that the absolute temperature was read to within less than 1/10 per cent.

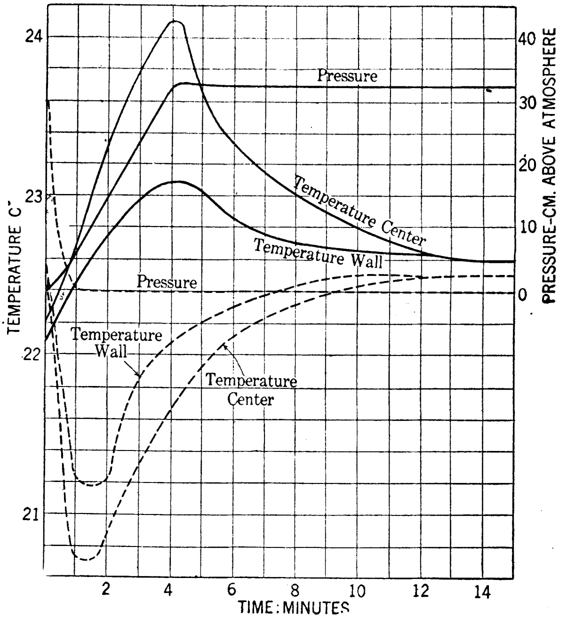

FIG. 15—TEMPERATURE AT SURFACE OF CORONA ROD

The determination of the temperature in this way assumes that the temperature of the air at the surafce of the corona rod is the same as that near the outer wall of the voltmeter. There is an obvious possibility of error here, so the matter was investigated experimentally by comparing the thermometer mentioned above with another hung immediately adjacent to the corona rod and viewed by telescope through another glass window. The curves of Fig. 15 show the differ-

TABLE VII.
CORONA VOLTAGE READINGS.

0.4765 cm. diam. rod

Freq.	Bar. Press.	Temp.	Press. Guage		Corr. Abs. Press.	Voltage Galvanometer							Ter. Coil Volts
						At start of Corona			Calibration				
			Left	Right		Left	Right	Mean	Left	Right	Volts 499 ohms	Milamp. per div.	
60.03	76.56	18.9	72.30	8.60	139.74	10.22	10.30	10.26	10.26	10.29	0.3890	0.07590	65.9
						10.23	10.30	10.26					65.95
						10.22	10.30	10.26					65.95
		19.0	72.00	8.90		10.22	10.30	10.26	10.23	10.30	0.3891		65.95
60.04	76.12	19.8			76.12	6.22	6.23	6.22	6.19	6.21	0.2348	0.07591	40.2
						6.19	6.23	6.21					40.1
						6.19	6.22	6.20					40.1
		20				6.19	6.22	6.20	6.18	6.22	0.2348		40.05

60.03	76.50	18.8	19.97	60.48	36.25	3.46	3.48	3.47	3.42	3.44	0.1308	0.07642	22.5
						3.47	3.49	3.48					22.2
						3.48	3.49	3.48					22.3
		18.7	20.07	60.33		3.48	3.50	3.49	3.42	3.44	0.1308		22.4
0.7109 cm. diam. rod													
60.03	76.02	20.8	60.30	19.85	116.13	11.01	11.09	11.05	11.01	11.10	0.4189	0.07584	70.9
						11.01	11.08	11.04					70.95
						11.01	11.10	11.05					70.95
		20.8	60.12	20.05		11.00	11.09	11.04	11.01	11.10	0.4189		70.95
60.11	75.97	21.7	30.02	50.17	55.98	6.08	6.12	6.10	6.01	6.03	0.22785	0.07585	39.05
						6.08	6.11	6.095					39.05
						6.08	6.11	6.095					39.1
		21.7	30.08	50.08		6.08	6.12	6.10	6.00	6.04	0.22785		39.1
60.03	75.07	21.7	20.35	60.10	35.60	4.17	4.29	4.23	4.18	4.30	0.1605	0.07588	27.1
						4.18	4.29	4.235					27.1
						4.19	4.3	4.245					27.3
		21.7	20.49	60.00		4.19	4.32	4.255	4.18	4.30	0.1605		27.4

ence in temperature between these two thermometers resulting from rapid expansion and compression of the air within the voltmeter. They indicate that on compression from atmosphere to 30 cm. above atmosphere there is a resulting difference of temperature of about 1 deg. and on expansion of approximately ½ deg. between the two thermometers. In the former case the two thermometers reached the same temperature within five or six minutes and within a shorter interval in the latter case. In the observations no such sharp changes of pressure occurred, the common maximum change being about 10 cm. In all cases, however, sufficient time was allowed to ensure that the observed temperature was sensibly the same as that near the corona rod.

IV. EXPERIMENTAL OBSERVATIONS

Many hundreds of observations were taken with eleven different sizes of corona rod of diameters, as follows: 1.266, 0.955, 0.790, 0.710, 0.654, 0.5536, 0.4765, 0.3142, 0.2060, 0.1197, 0.1038 cm. The figures for the diameters given in the tables and computations are the average values taken from 20 micrometer measurements on each rod. Except in the cases of the smallest rods, the maximum variations from the mean diameters as given were quite small, that is, in the neighborhood of 0.2 or 0.3 per cent. The rods were all of tool steel and nickel-plated after polishing, this material and treatment yielding the most accurate cylindrical shape and smoothest surface that we have found. The extremes of absolute pressure reached in the precision determinations were 25 cm. and 139 cm. of mercury, although not all the rods were carried through the entire range.

The usual sequence in taking observations was as follows: Adjust frequency control and read its value; set pressure in corona voltmeter to desired value; adjust filament current and kenotron circuit for zero current in the closed kenotron circuit; read temperature and pressure in voltmeter; raise voltage slowly until corona begins, as indicated by galvanometer and telephone; read value of charging current of condenser by galvanometer.

One observer took the temperature and pressure and read the charging current galvanometer. A second observer, who raised the voltage, was equipped with telephone head-piece and read the galvanometer indicating the beginning of corona. This observer also took a reading of an ordinary direct-reading voltmeter connected to the tertiary coil of the high-tension transformer. This last reading was useful as a check of the constancy of circuit conditions and for rough comparisons, but its readings were not used in any computations. At the instant that the presence of corona was detected by galvanometer and telephone the slow elevation of voltage would be stopped and the first observer would take the reading of the charging current galvanometer. Obviously the voltage elevation could be made as slow as desired and frequent check readings were taken which do not appear in the record. The degree of constancy of the corona voltage is discussed in the following paragraph. After each series of observations the charging current galvanometer was calibrated at a deflection approximately equal to that of the observation.

Table VII gives six typical sets of readings of the principal data, these sets being selected at random from the observation sheets. The first six columns give the frequency, temperature and pressure, leading to the corrected absolute pressure in column 6. Columns 7 and 8 give the right and left readings of the galvanometer measuring the condenser charging current. It will be noted that there are four pairs of such readings for each value of pressure. Between the readings of each pair and between each of the pairs the voltage was lowered below corona value and raised again. The readings of these columns therefore are a good indication as to the accuracy with which corona formation repeats itself.

Since the beginning of corona is indicated by the telephone and by a sudden sharp deflection of the galvanometer rather than by the magnitude of the deflection of the latter, this latter reading was not recorded. (See however Figs. 12, 13 and 14.) A direct relative indication of the alternating voltages at which successive coronas start is available, however, in the tertiary coil voltmeter and the readings of this instrument are given in the last column of Table VII.

Referring to the readings of the pressure, it will be noted that there is usually a difference in the net readings of the mercury manometer at the beginning and at the end of each series of four pairs of corona readings. These differences are due to slight leakage of the voltmeter casing and are therefore greatest for the highest and lowest absolute pressures. The differences are never very great and when they occur they are followed by a corresponding change in the corona voltage as indicated by the condenser charging current. Therefore in taking for each set of readings the average value of the pressure and the average value of the galvanometer reading no appreciable error is introduced.

Three sets of auxiliary observations should be recorded here bearing, respectively, on the perforation of the grounded cylinder, the influence of frequency and the permanence of the surface of the corona forming rod.

As regards the holes in the grounded cylinder, a number of observations from time to time have shown that these perforations have no effect on the value of the corona-forming voltage; for example, in the paper "The Electric Strength of Air.—III"[13] a number of experiments were conducted with an outer cylinder made of wire mesh of a quite wide opening. The values of corona voltage observed in this case were sensibly the same as those obtained with cylinders with continuous walls. Further, numerous readings taken with outer cylinders having continuous walls have shown that the voltages at which visual corona appears are in accord with those observed in the corona voltmeter. In order, however, to make a direct test of this question, two series of observations of visual corona voltage were taken, using in the two cases outer cylinders cut from the same length of brass tubing, one piece being perforated and the other not perforated.

The two tubes were each 9.5-cm. inside diameter and of length 24 cm. One was perforated with 0.27-cm. diameter holes drilled as closely as possible to each other and in a number of cases being actually tangent to each other. A clean rod 0.315 cm. in diameter was centered in each of these tubes in turn and visual corona observations taken with a rested eye in a darkened room. The corona voltages as measured on the transformer tertiary coil in the two cases were: with perforated cylinder 23.5, 23.4, 23.4; with unperforated cylinder 23.4, 23.5, 23.6, 23.5, 23.5. There appears therefore no reason to question that the perforations in the grounded cylinder have no effect on the value of corona-forming voltage.

As regards the influence of frequency on corona-forming voltage, in the paper "The Electric Strength of Air.—II",[14] experiments were described which indicated a slight lowering of corona voltage with increasing frequency between the range 10 and 100 cycles. Subsequently Whitehead and Gorton[8] have extended the range of frequency up to 3000 cycles and have shown that within the range 500 to 3000 cycles there is practically no influence of the frequency on the corona-forming voltage. They noted, however, that the value of corona voltage within this range was from 3 to 4 per cent lower than at 60 cycles. F. W. Peek records that within the range of commercial values the frequency has no influence on corona-forming voltage.

Having at hand the accurate method of measurement already described, a series of observations was taken on the influence of frequency between the values 20 and 90 cycles. The readings were taken on a rod 0.48 cm. in diameter and at atmospheric pressure. Several readings at each frequency were taken and all were reduced to the same value of $\frac{1}{\sqrt{\delta r}}$. The corresponding values of E/δ are given in Table VIII.

The above figures indicate that there is a small influence of frequency on corona-forming voltage within the range 20 to 90 cycles. For example, the corona-forming voltage at 25 cycles is shown to be about 0.8 per cent higher than at 60 cycles. All of the other observations of this paper were taken at 60 cycles and therefore the laws of corona formation, as discussed below, pertain to that frequency.

As regards the permanence of the surface of the corona-forming rod, experiments have shown that a practically indefinite number of corona observations may be taken without deterioration of the surface of the nickel-plated steel rods used. These experiments consisted in maintaining corona on a rod for a long period of time, interrupting it at regular intervals and taking the corona-forming voltage. One of several such tests consisted in maintaining a 0.476-cm. diam. rod at a voltage 2.5 per cent above the corona-forming value for one hour, and taking the corona-forming voltage six times during the interval. At start the corona-forming voltage as indicated at the tertiary coil terminals was 40.1; at successive intervals through the hour the values were 40.05, 40.05, 40.05, 40.15, 40.2, 40.1.

TABLE VIII.
INFLUENCE OF FREQUENCY.

Frequency cycles per second	Values of $\frac{E}{\delta}$ at $\frac{1}{\sqrt{\delta r}} = 2.03$	Aver.
20	50.41, 50.50	50.45
40	50.25, 50.25, 50.39, 50.5	50.35
60	49.95, 50.07, 50.13	50.05
90	49.24	49.24

If the voltage is carried higher, increasing the volume of corona discharge, the surface will ultimately develop local spark points, leading to local sparks at voltages lower than corona-forming values. Normally, however, with an initially clean rod and dust-free air inside the voltmeter, the surface of the rods in the corona voltmeter shows a most satisfactory degree of permanence.

V. Summary of Observations

From the complete data of which Table VII is a small portion, the values of the relative density δ are computed from formula (**3**). The charging current galvanometer is calibrated directly in terms of current in the resistances R_1 and R_2 of Fig. 1. As already described, the readings of this galvanometer then lead through formula (**4**) to the crest value of alternating voltage. From this value and the dimensions of the rod and the inner cylinder of the corona voltmeter, the critical or corona-forming electric intensity in kilovolts per centimeter at the surface of the conductor is readily computed. Some of these steps are given in Table VII and the more important ones are collected in Table IX.

Table IX shows about one-fourth of the total number of derived values for computing the law of corona. The summary of all these values is given in Table X. Each reading of voltage in Table IX corresponds to four pairs of right and left galvanometer readings at each pressure, as set forth in Table VII. It will be noted that on the average Table IX presents three sets of readings for each value of pressure. This means that for each pressure there are 12 observations of corona forming voltage. Table IX also includes both observed and computed values of $\frac{E}{\delta}$ and $\frac{1}{\sqrt{\delta\, r}}$, the latter based on the law of corona deduced from all the observations as set forth in the following section.

VI. THE LAW OF CORONA

As the work proceeded the values of $\frac{E}{\delta}$ and $\frac{1}{\sqrt{\delta\, r}}$ were plotted. It was found that the resulting curve was a straight line, in accordance with formula (**1**), for the larger values of $\frac{1}{\sqrt{\delta\, r}}$. However, on extending the study to larger corona rods, and especially at the higher pressures, it was found that the points departed from the straight line indicated for smaller rods and lower pressures. This fact at first was quite disturbing as it suggested either a departure from the simple law

TABLE IX.
COMPUTED VALUES FOR LAW OF CORONA.

Rad. r cm.	Temp. t deg. cent.	Press. p cm.	δ	Kv. max.	Surf. int. E kv/cm max.	$\frac{E}{\delta}$	$\frac{1}{\sqrt{\delta r}}$	$\frac{E}{\delta}$ calc.	Diff. per cent.
0.2383	18.0	46.92	0.6322	32.51	34.57	54.69	2.577	55.02	−0.60
“	18.0	47.20	0.6360	32.71	34.79	54.71	2.569	54.95	−0.44
“	18.1	47.37	0.6381	33.00	35.10	55.01	2.565	54.92	+0.16
“	18.7	35.97	0.4835	26.42	28.09	58.11	2.946	58.17	−0.10
“	18.75	36.25	0.4872	26.73	28.42	58.35	2.935	58.08	+0.47
“	20.3	26.65	0.3563	20.61	22.14	62.16	3.432	62.33	−0.27
“	20.5	27.06	0.3615	21.06	22.39	61.96	3.408	62.13	−0.27
0.1571	22.4	115.29	1.5301	52.24	76.21	49.80	2.040	50.11	−0.62
“	22.3	114.76	1.5238	52.20	76.15	49.97	2.044	50.15	−0.36
“	22.2	114.32	1.5185	52.09	75.99	50.04	2.048	50.19	−0.30
“	22.8	96.10	1.2739	45.00	65.65	51.53	2.235	52.05	−1.00
“	22.8	95.88	1.2710	45.04	65.69	51.69	2.238	52.08	−0.75
“	22.7	95.70	1.2690	45.05	65.72	51.78	2.240	52.09	−0.60
“	21.85	76.00	1.0107	37.37	54.51	53.94	2.510	54.45	−0.90
“	22.0	75.98	1.0099	37.54	54.76	54.23	2.511	54.46	−0.42
“	22.1	75.95	1.0092	37.54	54.76	54.27	2.512	54.46	−0.35
“	22.9	55.20	0.7315	28.98	42.28	57.80	2.950	58.21	−0.71
“	23.05	55.39	0.7336	29.17	42.35	58.00	2.946	58.27	−0.29
“	23.2	55.50	0.7347	29.24	42.66	58.06	2.944	58.25	−0.15
“	23.4	45.19	0.5978	24.89	36.31	60.74	3.263	60.90	−0.26
“	23.45	45.38	0.6002	25.03	36.51	60.83	3.257	60.84	−0.02
“	23.4	45.48	0.6016	25.04	36.52	60.70	3.253	60.81	−0.02
“	18.2	35.66	0.4802	21.15	30.86	64.26	3.641	64.13	+0.22
“	18.3	36.19	0.4871	21.42	31.25	64.14	3.615	63.91	+0.38
“	18.3	36.33	0.4890	21.46	31.30	64.01	3.608	63.85	+0.27
0.1030	26.0	115.48	1.5144	40.88	82.94	54.78	2.532	54.64	+0.26
“	26.0	115.01	1.5082	40.91	82.98	55.03	2.537	54.68	+0.64
“	26.0	114.68	1.5039	40.75	82.75	55.03	2.541	54.71	+0.59
“	20.6	114.80	1.5331	40.85	82.86	54.05	2.517	54.51	−0.85
“	20.75	113.97	1.5213	40.86	82.89	54.49	2.526	54.59	−0.18
“	20.8	113.70	1.5174	40.86	82.89	54.63	2.530	54.61	+0.04
“	26.7	96.05	1.2566	35.25	71.52	56.90	2.780	56.75	+0.26
“	26.6	95.77	1.2534	35.26	71.55	57.07	2.783	56.78	+0.51
“	26.6	95.62	1.2514	35.25	71.52	57.14	2.785	56.80	+0.60
“	19.9	76.05	1.0181	29.88	60.62	59.54	3.088	59.40	+0.24
“	20.0	76.03	1.0174	29.97	60.80	59.76	3.089	59.41	+0.59
“	20.0	76.00	1.0170	30.00	60.86	59.85	3.090	59.41	+0.74

TABLE X.

No.	Rad. cm.	$\frac{1}{\sqrt{\delta r}}$	$\frac{E}{\delta}$		Per cent discrep.
			obs.	calc.	
1	0.633	1.253	42.38	42.29	+0.21
5	“	1.386	43.71	43.61	+0.23
2	0.477	1.282	42.56	42.58	−0.05
4	“	1.353	43.02	43.28	−0.6
6	“	1.43	43.92	44.05	−0.3
7	“	1.431	44.20	44.06	+0.32
10	“	1.530	44.99	45.04	−0.11
12	“	1.661	46.28	46.34	−0.13
15	“	1.852	48.22	48.24	−0.04
16	“	1.870	48.57	48.42	+0.31
	“	2.117	50.79	50.88	−0.18
20	“	2.493	54.1	54.29	−0.35
3	0.355	1.349	43.24	43.24	0.0
8	“	1.480	44.60	44.55	+0.11
13	“	1.667	46.58	46.40	+0.39
	“	1.944	49.41	49.15	+0.53
19	“	2.418	54.01	53.66	+0.65
18	“	2.379	53.49	53.32	+0.32
9	0.238	1.503	44.55	44.78	−0.51
11	“	1.653	46.28	46.26	+0.04
14	“	1.798	47.70	47.70	0.0
	“	2.035	49.94	50.05	−0.22
	“	2.045	49.91	50.15	−0.48
	“	2.194	51.31	50.64	−0.65
17	“	2.375	53.07	53.29	−0.41
24	“	2.570	54.79	54.95	−0.29
26	“	2.941	58.22	58.11	+0.19
30	“	3.413	62.06	62.16	−0.16
	0.157	2.044	49.94	50.15	−0.42
	“	2.238	51.67	52.08	−0.79
21	“	2.511	54.15	54.45	−0.55
27	“	2.946	57.95	58.17	−0.38
29	“	3.257	60.76	60.84	−0.13
33	“	3.621	64.14	63.94	+0.31
23	0.103	2.537	54.95	54.68	+0.49
22	“	2.524	54.39	54.56	−0.31
25	“	2.783	57.04	56.78	+0.46
28	“	3.089	59.72	59.40	+0.54
32	“	3.610	64.25	63.87	+0.6
35	“	4.024	67.38	67.40	−0.03
38	“	4.562	72.21	72.00	+0.29
36	0.0598	4.041	67.99	67.54	+0.67
39	“	4.716	73.33	73.31	+0.03
41	“	5.22	77.48	77.63	−0.19
43	“	5.983	83.42	84.16	−0.88
31	0.0519	3.537	63.77	63.24	+0.84
34	“	3.865	66.57	66.04	+0.81
37	“	4.356	70.23	70.24	−0.01
40	“	5.097	76.46	76.58	−0.16
42	“	5.529	79.93	80.28	−0.44
44	“	6.254	85.60	86.47	−1.01

of formula (**1**) or the presence of some error in method or observation. Many readings were repeated but resulted only in confirming the earlier ones.

Further study and investigation of the foregoing interesting results led to the work of Whitehead and Brown[6] on "The Corona at Continuous Voltages," which shows that while both the positive and negative corona obey a law of the form of formula (**1**), yet the constants A and B are different in the two cases. This means that if the law for each case is put into the form of the linear relation between $\frac{E}{\delta}$ and $\frac{1}{\sqrt{\delta\, r}}$ the two lines will intersect and that below the point of intersection negative corona appears first, while above the point of intersection positive corona appears first.

Extending the foregoing to corona formation at alternating voltage, we should find, if we plot between $\frac{E}{\delta}$ and $\frac{1}{\sqrt{\delta\, r}}$ that below the value $\frac{1}{\sqrt{\delta\, r}} = 1.895$, representing the intersection of the positive and negative corona curves, the alternating corona should obey the same law as the negative continuous corona and that above that value it should obey the law found for positive corona. This is exactly the result that we have found and it therefore constitutes a necessary and in fact important, modification of the law of corona.

The foregoing conclusions are immediately obvious if all of the observations are plotted, and as the results are very consistent throughout, a quite close approximation to the exact values of the constants of formula (**1**) is possible by this graphical method. However, it is obviously better to derive the values of these constants from the figures themselves, and for this purpose the "Sigma-Delta" method[12] for evaluating the constants, has been used.

If we attempt to apply the Sigma-Delta method to the entire set of observations, a part of which are given in Table IX, it becomes very laborious indeed. It appeared to us, therefore, that this could be avoided by a still further averaging of the results for one pressure corresponding to each of the groups of readings in Table IX, this averaging being done on the values of $\frac{E}{\delta}$ and $\frac{1}{\sqrt{\delta r}}$. There being a linear relation between these two quantities, no error is thereby introduced. In this way the values of Table X are reached. The first column of Table X gives the sequence numbers as used in the Sigma Delta method; the third and fourth columns the mean values of $\frac{1}{\sqrt{\delta r}}$ and $\frac{E}{\delta}$; the fifth column the calculated value of $\frac{E}{\delta}$ as derived from the Sigma-Delta method; and the sixth column the error as between observed and calculated values of $\frac{E}{\delta}$ expressed in per cent.

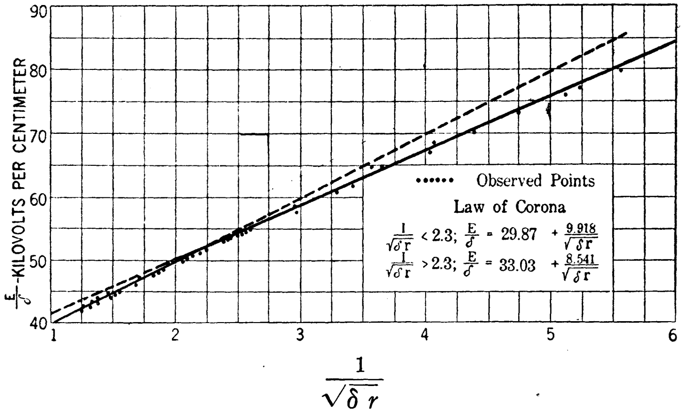

FIG. 16—THE LAW OF CORONA

In applying the Sigma-Delta method to the figures of Table X it was thought best, in view of the uncertainty as to the exact point of intersection of the two straight lines referred to above, to omit the points in the immediate neighborhood of this point of intersection. Consequently the points were plotted, as shown in Fig. 16, and the approximate position of the point of intersection of the two lines thus determined roughly. This being done, the readings corresponding to values of $\frac{1}{\sqrt{\delta r}}$ between 1.9 and 2.3 were omitted from the computations for the reason mentioned. In this way the equation of the line below the point of intersection was determined from the first sixteen readings of Table X, comprising the interval 1.253 to 1.870, for $\frac{1}{\sqrt{\delta r}}$. There are twenty-eight readings above the point of intersection comprising values from 2.375 to 6.254 for $\frac{1}{\sqrt{\delta r}}$. These were used for determining the equation of the line above the point of intersection. The results of the computation give the following formulas:

For values of $\frac{1}{\sqrt{\delta r}}$ below 2.295 and in this range negative corona appears first:

$$\frac{E}{\delta} = 29.84 + \frac{9.938}{\sqrt{\delta r}} \tag{7}$$

For values of $\frac{1}{\sqrt{\delta r}}$ above 2.295 and in this range positive corona appears first:

$$\frac{E}{\delta} = 32.96 + \frac{8.559}{\sqrt{\delta r}}. \tag{8}$$

The point of intersection of the two lines is $\frac{1}{\sqrt{\delta r}} = 2.26$ corresponding to a value of $\frac{E}{\delta} = 52.39$.

It is interesting to note in connection with Fig. 16 that observations on two of the rods used (0.238-cm. and 0.157-cm. radii) give several points each on both sides of the point of intersection of the two lines.

The extension of the observations in the direction of larger values of $\frac{1}{\sqrt{\delta r}}$ was readily accomplished by using smaller rods and lower pressures. The extreme values in this direction were reached in using a 0.0519-cm. radius rod and an absolute pressure of 36 cm. of mercury giving the value 6.28 for $\frac{1}{\sqrt{\delta r}}$.

In the opposite direction, *i. e.*, smaller values of $\frac{1}{\sqrt{\delta r}}$ there is also a wide range possible in increasing the pressure and using larger rods. Our largest rod was 0.633-cm. radius and our greatest pressure about 135-cm. mercury, giving 1.25 for $\frac{1}{\sqrt{\delta r}}$. As indicated elsewhere, larger rods are not desirable, but obviously much higher pressures can be used. Our limit in this direction was found in the break-down voltage of the air condenser, precluding precision measurements below 1.25 for $\frac{1}{\sqrt{\delta r}}$. However, we have made a number of observations of the performance of the corona voltmeter at higher pressures, with the precision voltage measurement omitted, and have found nothing to suggest a departure from the simple linear relation indicated in formula (7) and Fig. 16.

VII. ADVANTAGES OF THE CORONA VOLTMETER AS A STANDARD

The law expressed in formulas (7) and (8) constitutes a definite standard over a wide range of voltage. The range has been tested by precision measurements up to the neighborhood of 150,000 volts and with every evidence that the law continues beyond that value. The only quantities which enter are the radius of the corona rod, the radius of the outer cylinder, and the density of atmospheric air. This is equivalent to saying that the calibration of the corona voltmeter depends only on its physical dimensions.

As regards its availability in practical measurements, a workable corona voltmeter may be set up in practically any laboratory with very little trouble. A straight clean wire stretched on the axis of a surrounding metal cylinder will give very reliable indications in a darkened room with a visual observation of corona formation. With little additional trouble a galvanometer may be used as corona indicator. A considerable although not a continuous, range may be had by using corona wires of different diameters.

The construction of the complete corona voltmeter itself, moreover, is a relatively simple and inexpensive matter. Up to 100,000 volts it may be readily constructed in any well-equipped laboratory and for higher ranges offers no serious difficulties. With the complete instrument a wide and continuous voltage setting is available using a single rod and observations sharply marked may be taken with an ordinary laboratory galvanometer or with a telephone receiver.

The usual method of setting for a definite voltage is to read the temperature inside the instrument, set the pressure at a particular value based on the value of δ, corresponding to the desired voltage setting, and which may be read from a table based on formulas (7) and (8) and then slowly raise the voltage until corona appears, as indicated by telephone or galvanometer. While this would probably be the more common usage in connection with insulation testing and other similar service, the instrument may also be used for measuring an unknown voltage. In this case the pressure would be set for a voltage known to be higher than that to be measured and the pressure then allowed to fall slowly, its value being read at the instant corona appears. Formulas (7) and (8), or tables computed from them, would then give the value of the voltage.

The corona voltmeter would appear to have several advantages over the needle and sphere gaps; among them may be mentioned the following:

(a) Freedom from disturbance by proximity of neighboring conductors or extraneous electrostatic fields.

(b) A 2 per cent inaccuracy is the minimum claimed for the sphere gap. With careful manipulation and good circuit conditions it is certain that a corresponding figure of better than 0.5 per cent is possible with the corona voltmeter.

(c) No manipulation of high-voltage circuit. Each change of setting of the sphere gap requires altering the distance between the discharge spheres. The setting of the corona voltmeter for different voltages requires the change of air pressure only.

(d) No discharge of high-voltage circuit and no series resistance necessary. The reading of the corona voltmeter is continuous and stationary and draws no current from the high-voltage circuit.

(e) Measurement of an unknown voltage. This cannot be done with the sphere gap except through repeated opening of circuit and successive approximation.

(f) All parts of the corona voltmeter are grounded except the leading-in wire of the high-tension terminal. All dimensions remained fixed. All auxiliary circuits are at low values of continuous voltage.

(g) Permanence. The surface of a corona-forming rod remains unaffected under the continuous application of initial corona over long periods.

As regards outside dimensions, the earlier paper[1] on the corona voltmeter described a corona voltmeter for voltages under 50,000 having dimensions of 76 cm. in length and 24 cm. in diameter. The instrument shown in Fig. 9 is 9 ft. 10 in. high (of which 3 ft. is in the insulating bushing) and 1 ft. 10 in. in diameter. The inside cylinder forming the grounded terminal is 24.6 cm. in diameter and 61 cm. long. The most convenient diameters of corona rod are between 0.3 cm. and 0.9 cm. This instrument has been used for voltages up to 150,000 volts without sign of distress, this being the maximum voltage obtainable under the conditions of test. It was used for this voltage at an internal air pressure of about 135 cm. of mercury absolute, *i. e.,* not quite 15 lb. per sq. in. above atmosphere. Pressures three or four times this value could readily be used. The limiting voltage would probably be found in the flash-over voltage of the insulating terminal bushing. This bushing has a normal rating of

150,000 volts effective. It is probable therefore that the instrument shown in Fig. 9 may safely be operated without trouble to 200,000 volts maximum value.

An instrument rated normally at 300,000 volts, and which it is expected may reach 400,000 volts, is now in process of construction.

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DISCUSSION ON "THE CORONA VOLTMETER AND THE ELECTRIC STRENGTH OF AIR" (WHITEHEAD AND ISSHIKI), WHITE SULPHUR, W. VA., JUNE 30, 1920.

C. L. Fortescue: In regard to the use of corona voltmeter as a secondary standard, there are a number of things to be said. One is that for practical work it is very hard to use sensitive galvanometers, on account of vibration and other disturbances. It is difficult to get men with sufficient judgment to note such a thing as a corona point, and even to use a device like the telephone. It is much easier to have something that they can see, like a voltmeter.

Now, in using the sphere gap, the fact that you cannot measure varying voltages is not important. In nearly all practical work we have to test apparatus for a certain maximum voltage, and it is only necessary in making a test, to have the apparatus connected in, set the sphere gap to the value you want to use, spark over the gap, and note the reading of the voltmeter on the secondary winding of transformer or tertiary winding, and after resetting the sphere gap for a slightly higher spark-over voltage test the apparatus by holding the voltage by means of the voltmeter; in other words, note the correspondence, and keep the apparatus at that voltage. It is evident therefore that the sphere gap does not offer any difficulties in that respect.

As regards the accuracy of the sphere gap, which is in the neighborhood of two per cent in practical work, that is a sufficient degree of accuracy. While I think Dr. Whitehead's corona voltmeter would have other uses; than for practical work I still think that the sphere gap is a better secondary standard for that purpose.

Now possibly the method that Dr. Whitehead used for calibrating his voltmeter might be used for a secondary standard as well as for a primary standard—in fact, we have been using that method for a crest voltmeter measuring device, in conjunction with the kenotron, and found it quite reliable within certain limits. Dr. Whitehead defined those limits; they depend on the following characteristics of the vacuum valve, so long as the voltage wave does not have more than one maximum, that is to say, so long as the slope of the voltage wave does not become zero at more than one point in a half wave—the kenotron device will give accurate results. There is a prospect of using the commutator device instead of the kenotron. We encountered the same difficulties in regard to that, that I have also mentioned in regard to observations—the ordinary tester is not a very good observer, and you cannot depend on his judgment a great deal. The main difficulty with the commutator device is the angle setting, the setting of the angle of the commutator so that you get the maximum value. It requires good judgment in order to do that properly. I still have hopes that we may be able to apply the commutator absolute measurement scheme also for secondary standards.

A. E. Kennelly: I would like to ask Dr. Whitehead whether he does not look on his galvanometric method as the primary method, and merely checks it up with his telephone to make sure that it is functioning properly?

Joseph Slepian: I would like to ask Dr. Whitehead, now that he has found that the corona makes a good peak voltmeter, whether he concludes that, like the sphere gap, it has a unity impulse ratio, or whether under periodic voltage, which exceeds a definite peak, the corona begins to develop through several cycles?

This would be of importance in considering the application of impulse discharges, such as are made in tests on insulators for lightning impulses.

L. W. Chubb: The point that Dr. Slepian just brought up is a very pertinent one; in some cases we do need a big voltmeter that has an impulse ratio of 1, in other words, it has a great discharge, and that is one of the disadvantages of the sphere gap in ordinary testing. If there is a slight surge it sparks over and ruins your test in a great many cases. The calibration curves on an ordinary circuit are regular gunshot diagrams. and it seems as though Dr. Whitehead's method, which checks so very close should be given consideration as a standard for the Institute. I think the other method of integrating the current in an air condenser is probably more practical as a working standard, but this is a very accurate method of detecting corona, and its constancy makes it a rather good, you might say, sub-primary standard.

F. W. Peek, Jr.: It is interesting to point out that high voltages anywhere may be readily determined by means of a piece of wire and a formula which involves a simple arithmetical calculation.

Dr. Whitehead has developed the corona voltmeter to a high state. There will undoubtedly be certain fields in which it will prove useful. It rarely happens that any method replaces all others. The accuracy of the corona voltmeter should be of the same order as that of the sphere gap. The phenomena is the same.

Corona is spark-over to space. In the sphere the spark occurs from metal to metal.

In my work I have found that in making investigations at operating frequencies a voltmeter coil in the transformer is by far the most convenient method and gives the best results. If a sine wave is not available a crest voltmeter should be used on the voltmeter coil.

When transient or lightning voltages are studied the sphere gap is practically the only available method. It also has the additional advantage that it measures to a high degree of accuracy voltages varying from direct current to transients of a fraction of a microsecond duration. Our knowledge of transients has been greatly extended during the last few years because of these characteristics of the sphere gap.

Transient voltages can also be measured by means of the corona voltmeter but to a less degree of accuracy than with the sphere. There is a difference in the appearance between the corona on a wire when it is (+) or (—). In my investigations I found that corona produced by transient voltages lasting less than a millionth of a second could not only be readily seen but that the eye could distinguish between a (+) and (—) wire.[1] For the smaller wires there is lag and the starting voltage is higher than at the lower frequencies

J. B. Whitehead: As regards the objection to the use of the galvanometer, the question raised by Mr. Fortescue, I may say that since the experiments described in this paper were completed, we have used an ordinary indicating instrument, a milammeter or voltmeter, as corona detector in place of the galvanometer. We do this with vacuum tube amplifiers. The additional equipment is not serious for a standard instrument. As to the sphere gap and the standard air condenser used directly as working methods, I think that the advantages that can be offered for the corona voltmeter are its greater accuracy and the facts, that it is compact in shape, that there are no exposed high-tension terminals or working parts except the top end of the insulating bushing, and the fact that it is entirely free from disturbances due to external fields.

I do not desire to make the suggestion that the corona voltmeter is any more suitable for shop testing than existing methods, but it does appear to me, that in view of the advantages I have mentioned, and especially the fact that as regards accuracy, it certainly

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[1] Peek, Effect of Transient Voltages on Dielectrics, TRANS. A. I. E. E., 1915, Vol. XXXIV, p. 1857.

far exceeds any figures which have been offered for the sphere gap, there is a great deal to be said for it as an ultimate secondary standard.

As regards Prof. Kennelly's question as to the relative value of the telephone and galvanometer, as indicators, certainly in the hands of the man in the machine shop, the telephone appeals to me as being the better. It is very positive indeed in its indications, and does not involve the use of any instrument. On the other hand, the galvanometer is certainly the better instrument for laboratory work.

Mr. Peek has kindly answered for me the question raised by Mr. Slepian, as to the value of the corona voltmeter for measuring transients, and their impulse ratio. As the observations by telephone and galvanometer are based on the continuous presence of corona, the voltmeter would probably not respond to transients of short duration. Possibly the telephone might.

It is reassuring to know, as stated by Mr. Peek, that the corona due to the transient voltage may be seen. If that is the case, there is no reason why the peekhole in the top of the case of the corona voltmeter should not be used for detecting the corona due to the transient voltage. I am interested in the statement of Mr. Peek, in terms merely of the crest values. I do not recall the work which led to that conclusion, but I would be interested, if he would give me, the reference.