GEOMETRIC CONSTRUCTIONS.

By W. W. Beman,
University of Michigan.

The solution of problems as presented in the study of Euclidean geometry is effected by the use of the classical instruments, the straight edge and the compasses in combination. In the secondary schools of this country, at least, little attention has been given to the possibility of dealing with these problems by the use of the straight edge alone, the compasses alone, the ruler (with parallel edges), the square (the carpenter's square ungraduated, i. e., a right-angled triangle without hypotenuse), and so on. It is obvious that the straight edge alone is less powerful than the straight edge and the compasses together, but it is by no means so manifest that all problems that can be solved by the use of the straight edge and the compasses together can be solved by the use of the compasses alone, by the use of the ruler alone, by the use of the square alone, by the use of the straight edge in combination with a fixed circle; nor is it manifest that by the use of two squares one can trisect an angle, duplicate a cube, construct a regular heptagon, in fact solve any equation of the third or fourth degree.

The general theory of geometric constructions by various of these methods has been discussed in Klein's Vorträge über ausgewählte Fragen der Elementargeometrie, Leipzig, 1895 (English translation with title, Famous Problems of Elementary Geometry, Ginn & Co., Boston); more fully in Enrique's Questioni riguardanti la geometria elementare, Bologna, 1900 (German translation of Part II with title, Fragen der Elementargeometrie, Leipzig, 1907); and perhaps best of all in Adler, Theorie der geometrischen Konstruktionen, Leipzig, 1906.

There seems little doubt that problems to be solved by these more modern methods may rouse interest on the part of wide-awake students of elementary geometry and accordingly a short list is appended.

Constructions with the Straight Edge Alone.

1. Through a given point to draw a line that shall pass through the inaccessible intersection of two given lines.
2. Through a given point to draw a line that shall be parallel to a given line on which two equal adjacent segments have been laid off.
3. To bisect a line segment lying on one of two given parallels.
4. Through a given point to draw a line parallel to two given parallels.
5. Having given a parallelogram, through a given point to draw a line parallel to a given line.

CONSTRUCTIONS WITH THE COMPASSES ALONE.*

1. Through a given point to draw a line parallel to a given line.
2. To double, treble a given line segment.
3. To determine the symmetric of a given point with respect to a given line.
4. To construct the fourth proportional to three given line segments.
5. To determine the center of the circle passing through three given points not collinear.
6. To bisect a given line segment.
7. To bisect a given arc.
8. To construct a square with two vertices given.

CONSTRUCTIONS WITH THE RULER ALONE.

1. Through a given point to draw a line parallel to a given line.
2. Through a given point to draw a line perpendicular to a given line.
3. To bisect an angle.
4. To double, treble, a given angle.

CONSTRUCTIONS WITH THE SQUARE ALONE.

1. To double, treble, a given line segment.
2. To bisect a given line segment.
3. To double, treble, a given angle.
4. To bisect a given angle.
5. Through a given point to draw a line making with a given line an angle equal to an angle given in magnitude and sense.

*In constructions with the compasses alone no straight lines are actually given or drawn. The line is determined by two points upon it.

A novel method of recovering a sunken cargo has been adopted by the United States Steel Company. A large magnet, 3½ feet in diameter and weighing 3,000 pounds, has been employed in raising kegs of nails from a barge that was sunk in the Mississippi River near New Orleans. The magnet raised five or six kegs at a time, or about a ton at each lift. The advantage of this method was that it avoided breaking open the kegs, as would have been the case had a dredge been used. The magnet is soon to be used for raising a sunken load of woven wire, and also for a load of steel baling strips.

The most recent report of the Commissioner of Education states that the total number of graduates of public high schools in June, 1908, was 90,232, of whom 28,895 were college preparatory students. In addition there were 11,940 graduates preparing for higher institutions.

In other words, 45.25 per cent of all our public high school graduates in 1908 were planning to go on with their education in higher institutions, and 32.02 per cent were actual college preparatory students. In private schools there were 11,279 graduates in 1908, of whom 4,877, or 43.24 per cent, had prepared for college.