

Constraints on IRS Control: An Alternative Approach to Tax Gap Analysis

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Abstract—A tax authority wants to take actions it knows will foster the greatest degree of voluntary taxpayer compliance to reduce the “tax gap.” This paper suggests that even if a tax authority could attain a state of complete knowledge, there are constraints on whether and to what extent such actions would result in reducing the macro-level tax gap. These limits are not merely a consequence of finite agency resources. They are inherent in the system itself. To show that this is one possible interpretation of the tax gap data, the paper formulates known results in a different way by analyzing tax compliance as a population with a single covariate. This leads to a standard use of the logistic map to analyze the dynamics of non-compliance growth or decay over a sequence of periods. This formulation gives the same results as the tax gap studies performed over the past fifty years in the U.S. given the published margins of error. Limitations and recommendations for future work are discussed, along with some implications for tax policy.

Keywords—Tax law, tax compliance, tax gap, income tax.

I. INTRODUCTION

THE stoic philosopher Epictetus wrote that human suffering stems from our confusion over the things we can control and the things we cannot [1]. To date, tax authorities have viewed the landscape of the things they can control related to what is known as “voluntary tax compliance” as all-inclusive [2]. The only question has been, “How do tax authorities *better* control this landscape?” At no time have tax authorities asked the more restrictive (and introspective) question, “Are there limits to a tax authority’s control over voluntary compliance?” In asking this question, this paper does not consider the limitations imposed by constrained resources. It argues that the limits to a tax authority’s control over voluntary compliance is, instead, inherent in the voluntary tax compliance system itself.

The U.S. tax authority, the Internal Revenue Service (IRS), has commissioned studies going back half a century to estimate what has become known as the “tax gap,” which is the difference between taxes legally due the government for a specific period and those voluntarily and timely paid [3]. The methodological formulation in U.S. tax gap studies has produced estimates of the voluntary compliance rate and its complement, the non-compliance rate. Although the tax gap estimation method changed considerably in 2001 and is constantly being refined, the methodology behind its formulation has remained fundamentally unchanged. In short, the formulation attempts to estimate the tax gap by taking a sample of tax returns and, when necessary, auditing them [4]. This has produced estimates of compliance rates with a ± 2 percent margin of error [3]. If one takes this margin of error

into account, the total U.S. non-compliance rate has remained statistically unchanged since the first tax gap study [5]. This suggests that the tax gap estimation methods used prior to and since 2001 are equivalent.

Over the past two decades, the tax gap has gone from a mere statistic to a major political focus of congressional demands made on the IRS [6]. This focus has led to a redeployment of IRS resources to raise additional tax revenues through increased domestic enforcement and to lobby for legislation such as the Foreign Account Tax Compliance Act (FATCA), which gives the IRS unprecedented access to the non-U.S. activities of taxpayers [7]- [9]. In 2018, the IRS announced a strategic initiative to inform its administrative operations by examining drivers of tax compliance from behavioral economics [10]. Each of these actions were taken in whole or in part as a response to increased political pressure to reduce the tax gap. Thus, the strategic focus of the IRS has been to decrease the overall tax gap and the non-compliance rate.

Merely knowing the size of the overall tax gap does little else but alert tax policymakers to the issue of non-compliance. To address the tax gap itself, or to determine if it even can be addressed, one must dig deeper to understand the dynamics behind the numbers [3]. Others have attempted this in various ways. Allingham and Sandmo famously imposed an “economics-of-crime” model to describe a taxpayer’s choice whether to voluntarily comply with the tax laws as an expected utility function [11]. Some have criticized this deterrence theory holding that, if it is correct, a tax compliance puzzle exists since the tax gap should be significantly greater than it actually is [12].

Others disagree, arguing that third-party information matching provides increased incentives for certain subgroups of taxpayers, such as wage earners, to voluntarily comply [13], [14]. Other subgroups who know that the IRS does not receive third-party information, such as self-employed individuals, are less likely to voluntarily comply since there is a lower probability of non-compliance detection [15]. Disaggregated (micro-level) tax gap data support this observation, showing that groups for which third-party information reporting exists exhibit significantly higher levels of voluntary compliance than groups for which there is little to no information reporting [16]. This reinforces an assumption that the government has the ability to control voluntary compliance levels if its efforts are properly focused on encouraging structural systems that foster compliance [17].

Yet, a tax compliance puzzle remains, just a different kind. There exists an assumption of scalable linearity in the argument that the right kinds of third-party reporting (e.g., brokers reporting the basis in taxpayer investments)

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will have a significant impact on the tax gap [18]. Historical data, however, do not support this assumption. For example, aggregate (macro-level) voluntary compliance rates appear unaffected by government increases in the matching of third-party information or any other increases in the probability of non-compliance detection for the past half century.

Until 1974, matching programs that check tax returns against third-party information did not exist [19]. Since 1974, the government's ability to match third-party information has significantly improved, due in part to structural systems such as enabling legislation [20]. Still, macro-level compliance rates remain statistically identical to what they were prior to the start of third-party information matching programs. Some data even suggest compliance remained unchanged for the decade between 1963 and 1973 [19]. If matching produces a higher probability of detection and taxpayers act in response to this probability increase, one would expect the non-compliance rate for years prior to 1974 to be significantly higher than for years after the matching program's initiation and later maturation. The fact that the non-compliance rate is statistically the same for years prior to the matching program as it is for all subsequent years suggests that there exist other determinants of macro-level compliance measures.

These results are consistent with agent-based models (ABMs) of tax compliance, which show that higher probabilities of detection do affect individual taxpayers' compliance decisions, but significant changes in the probability of detection do not affect macro-level compliance [5]. Micro-level compliance appears directly affected by statutory and administrative structural systems such as information matching, but the macro-level voluntary compliance rate has remained statistically unchanged since the government's initial studies.

If statutory and other structural systems positively influence voluntary compliance at micro-levels, why don't these same structural systems also improve the macro-level tax gap? In this way, deterrence theory might not be complete even if it is correct. This is an important distinction because the macro-level tax gap measure is the one congressional authorities are pressuring the IRS to improve. In other words, this is the measure that seems to politically matter the most. Yet, attempting to solve the macro-level problem with a micro-level solution is proving futile. Therefore, inquiring as to the limits of the tax authority's ability to control macro-level compliance rates seems like a logical first step in discussing any government effort related to reducing the tax gap.

This paper describes what is essentially a second formulation of compliance rates published in U.S. tax gap studies. The formulation is statistically equivalent to the usual formulation. Therefore, there are no fundamentally new results in these compliance rate measures. Still, there is value in recognizing old things from a new point of view [21]. Additionally, there is always the hope that the new point of view will inspire an idea for the modification of present theories on tax non-compliance generally; a modification necessary to better align theory with observed compliance phenomena.

For those wondering why one would introduce a mathematical model into the tax gap discussion, consider the following in light of *lex parsimoniae*: Is it more likely that (1) structural

systems such as legislation affect micro-level compliance and, coincidentally, undulating compliance responding to these "speed bumps" just happen to cancel each other out leaving a statistically identical macro-level non-compliance rate for the past fifty years; or (2) a property characteristic of the macro-level tax compliance system is simply letting voluntary compliance play out according to that characteristic? If it is the latter, a mathematical model is the best way to examine this characteristic of the system. According to Fry, "Mathematics is about abstracting away from reality, not replicating it. And it offers real value in the process. By allowing yourself to view the world from an abstract perspective, you create a language that is uniquely able to capture and describe the patterns and mechanisms that would otherwise remain hidden" [22].

It is also important to define terms. A *model*, as the term is used here, refers to the relations which speculatively describe a certain phenomenon, in this case the phenomenon of tax non-compliance. Quantities that are measurable by independent observation are *variables*. Here, the variables are the non-compliance rates observed as estimates from the tax gap studies. To formulate these relations, this approach also introduces a *parameter* to the model that represents some inherent property of the system [23].

The paper seeks first and foremost to present a new approach to thinking about the macro-level tax gap other than assuming that government authorities need to simply figure out the next best steps to reduce it. It does not attempt to answer the question, "What causes the tax gap?" It only provides a mathematically-viable alternative explanation that potentially has nothing to do with direct government intervention. In this way, it aspires to begin a new dialogue on the tax gap instead of simply adding additional conclusions to the literature based on the existing assumption that government policies have a direct effect on this macro-level measure. It attempts this by analyzing tax compliance as a population.

The paper first discusses the characteristics of a wicked system, which prior work suggests might be an appropriate domain of any voluntary tax compliance system [24]. It then establishes a framework for analysis that considers a population of monetary units that are legally due to the tax authority for a specified period, which is the same basis of tax gap analyses. This framework allows one to analyze tax compliance as a population and graph the progression of tax non-compliance given a parameter that is a property of a specific jurisdiction's tax system. Through this analysis, the paper concludes that the parameter defining the system is itself defined by a fixed point around which the non-compliance rates for various periods orbit.

The paper also offers some limitations of this approach; specifically, verification limits given the lack of adequate time series data on tax non-compliance. It concludes that the tax non-compliance observed at the macro level is a result of the system feeding back on itself. Effects of tax authority actions end up being negligible in analyzing the system *qua* system even though these actions certainly affect individual taxpayers' compliance decisions and those of disaggregated groups of taxpayers. The paper also offers future research ideas based on results from this new approach and some implications for

tax policy.

II. ASSUMPTIONS AND FORMALIZATION

The formulation presented here contains as its essential idea the concept of tax compliance measured as a population of monetary units in what has been previously dubbed a “wicked system.” It is, therefore, worthwhile to review the concept of a wicked system and its essential characteristics.

A wicked system has both structural complicatedness and self-organizing complexity. Legislatures and tax authorities impose structural constraints on taxpayers in an effort to increase voluntary compliance. At the same time, taxpayers react to and operate within these constraints and interact with individuals such as other taxpayers, tax advisors, etc. Hundreds of millions of these interactions for any one tax period generate some form of self-organization. Therefore, any model of a wicked system must account for both aspects.

Previous work on tax compliance as a wicked system suggests such a system is difficult to model [24]. This is because problems that tax policymakers and administrators must deal with tend to defy efforts to delineate their boundaries and identify their causes. These tendencies also hide the problematic nature of attempts to legislate and regulate tax compliance behavior [25]. One way to attempt an imperfect model is to “chunk off” elements of the system, or analyze a collection of “snapshots” much like an atlas is a collection of snapshots that amounts to an imperfect model of a globe [26].

Prior work with ABMs suggest that increasing or decreasing the local probability of non-compliance detection has little to no global effect on the voluntary compliance rate. Yet, a system-wide parameter does produce a positive non-linear effect on compliance [5]. An attempt at even an imperfect model, then, must assume the self-organizing complexity of a large number of taxpayers represented by their actions (measured in monetary units that are legally due to the tax authority for a given period), and impose a global parameter that is a characteristic of a specific jurisdiction’s tax system.

A. Establishing a Tax Compliance Function

It is simpler to work with the same units (money) and states (compliance) as extant tax gap analyses. Consider the population of n monetary units that are legally due to the tax authority for the period t , which can be any taxable period. Time is not considered continuous here since tax returns and payments are typically filed and paid in discrete taxable periods. All references to a monetary unit henceforth should be understood as a monetary unit legally due to the tax authority for period t .

Notate the micro-state of monetary unit i with the covariate p^i , where $p^i \in \{|V\rangle, |U\rangle\}$; that is, a monetary unit can exist in either the micro-state $|V\rangle$ or the micro-state $|U\rangle$. Here, $|V\rangle$ is the “voluntary compliance” micro-state of the system, while $|U\rangle$ is the “non-compliance” micro-state. States $|V\rangle$ and $|U\rangle$ are the only two possible micro-states. The complete macro-state $|N\rangle$ of the population n for period t is, then, the sum of the micro-states at t . Thus, any monetary unit in the

population and defined only by the two micro-states is in the set $\{p^1, p^2, \dots, p^i, \dots, p^{n-1}, p^n\}$.

This is just one snapshot of the wicked system. As complicated as it might first appear, this formalization is still an oversimplification. Whenever $p^i = p^j$ (that is, whenever two monetary units look the same in terms of the observed covariate only) they have the same probability of non-compliance. This is not to say the monetary units i and j are the same in all other respects. In fact, they might differ in many very important ways and with regard to unmeasured covariates, e.g., $q^i \neq q^j$. Yet, for the purposes of this analysis these differences do not affect the probability of non-compliance. This approach is equivalent to the “chunking” discussed previously as a way to analyze a wicked system.

Define x_t as the non-compliance rate at period t . The system has a non-compliance rate configuration space $S(x) = \{x \in \mathbb{R} : 0 \leq x \leq 1\}$. The system also has a set of possible non-compliance rate configurations as functions of discrete (non-continuous) time $\{x(t) \in \mathbb{R} : 0 \leq x(t) \leq 1, t \geq 0, t \in \mathbb{N}\} \subseteq S(x)$, where $x(t) \equiv x_t$.

The goal is to find a function f that represents x at the next period $t + 1$ given some state of the population of monetary units with respect to x at the current period t ; that is, one wants to find a function f that maps $x_t \mapsto x_{t+1}$. Assume f is a function of (1) the non-compliance rate x at t ; and (2) some other function g that maps to the scalar parameter λ , which is a property characteristic of the system. In its curried form, this becomes

$$g(x_t \mapsto \lambda \mapsto f(x_t, \lambda)). \quad (1)$$

Thus, $f(x_t, \lambda)$ is a function of x_t and a parameter λ . Neither are dynamically dependent on time.

B. Analyzing Tax Compliance as a Population

Considered as a population, the compliance state of monetary units can grow or decay as with any other population [27]- [29]. Beginning with first principles one might ask, “Why does voluntary compliance exist?” [30]. There are three rather simple answers that are sufficient for the purposes of this paper. Voluntary compliance exists at t because:

- 1) p^i first appears in state $|V\rangle$ at t ;
- 2) p^i transitions from $|U\rangle$ to $|V\rangle$ at t ;
- 3) p^i does not transition from $|V\rangle$ to $|U\rangle$ at t .

Additionally, there are both qualitative and quantitative states of the total population of monetary units for a given period. One can regard the qualitative state abstractly as

$$|N\rangle = |V\rangle + |U\rangle. \quad (2)$$

If one includes the number of monetary units in each state at t this becomes

$$n_t |N\rangle = v_t |V\rangle + u_t |U\rangle, \quad (3)$$

where v_t is the total number of monetary units in the voluntary compliance micro-state and u_t is the total number in the non-compliance micro-state.

Since the raw number of monetary units in each state changes for each period, one must normalize the quantities

to examine the dynamics of the population at sequential periods. One can do this by thinking of compliance and non-compliance as rates instead of raw numbers associated with abstract states. Dividing (3) through by n_t yields

$$|N\rangle = \left(\frac{v_t}{n_t}\right)|V\rangle + \left(\frac{u_t}{n_t}\right)|U\rangle. \quad (4)$$

Note that this gives the quantitative state of $|N\rangle$ in terms of the non-compliance rate $x_t := (u_t/n_t)$ and the voluntary compliance rate $(1 - x_t) := (v_t/n_t)$. Equation (4) allows one to consider x as a normalized population of monetary units in the non-compliance micro-state for any period.

Because the transition of a qualitative state $|U\rangle$ at t to $|V\rangle$ at $t + 1$ depends on the quantitative state $u|U\rangle$ at t , x_{t+1} will always depend at a minimum on x_t . In other words, $x_t \mapsto x_{t+1}$, which meets the condition from §II-A.

Table I data suggest that tax non-compliance seems to stabilize around a certain measure. If thought of as a population, one would expect compliance rates to stay the same from one period to the next if and only if $(1 - x) = x$. This is only an expectation (average) since it is possible x might fluctuate wildly for each period even though there ends up being an equilibrational balance over many periods that cancel the positive and negative fluctuations leaving only the expectation value.

TABLE I
 NON-COMPLIANCE RATES BASED ON U. S. TAX GAP STUDIES

t	x (w/o $\pm 2\%$ error)
1973	17.4%
1976	19.3%
1979	20.2%
1981	18.4%
1982	18.2%
1984	20.0%
1986	20.8%
1987	18.2%
1988	17.7%
1992	16.9%
2001	16.3%
2006	16.9%
2012	18.3%

Regarded as the function $f(x_t)$, one sees that an expected change in the function with respect to x_t is equilibrational if $\langle \partial f(x_t)/\partial x_t \rangle = 0$. Because the maximum change is unity when $x_t = 0$ and negative unity when $x_t = 1$, this becomes $\langle \partial f(x_t)/\partial x_t \rangle = (1 - x_t) - (x_t) = 0$. From this, one can state more generally the expected change in the function even if not in equilibrium as

$$\left\langle \frac{\partial f(x_t)}{\partial x_t} \right\rangle = (1 - x_t) - (x_t) = 1 - 2x_t. \quad (5)$$

Yet, it is clear from the data in Table I that $(1 - x_t) \neq x_t$. Therefore, to achieve an expected stabilization point there must also exist a constant parameter that represents a characteristic of the system itself. Recall from (1) that the function mapping x_t to x_{t+1} is dependent on both x_t and the parameter λ . Thus, the expected change in the complete function is

$$\left\langle \frac{df(x_t, \lambda)}{dx_t} \right\rangle = \lambda(1 - 2x_t). \quad (6)$$

Integrating (6) with respect to x_t yields

$$f(x_t, \lambda) = x_{t+1} = \lambda \int (1 - 2x_t) dx_t = \lambda(x_t - x_t^2). \quad (7)$$

This function is the *logistic map* often employed to study various populations. It maps the normalized non-compliance population value at any time step to its value at the next step [31]. Thus, the normalized non-compliance population level x_{t+1} is a function of the previous time step's population level x_t and the parameter λ . This iterative mapping provides a simple, one-dimensional, discrete equation to use as a snapshot of the tax compliance system.

If $\lambda < 1$, non-compliance will always decay to zero over time. Higher values of λ might settle toward a fixed point $\xi : x_{t+1} = x_t$, or fluctuate across different values, just as any other population might fluctuate across a series of booms and busts [32].

C. The System Parameter

The preceding implies that non-compliance is somehow dependent on the parameter λ . But what is λ ? Given that tax compliance is an open system, one would expect the parameter λ to produce some fixed point ξ about which observations of the phenomenon of tax non-compliance for different periods not only fluctuate but orbit. If tax compliance was a closed system, ξ would serve as an attractor and eventually all expected non-compliance rates would settle on this fixed point as long as it was stable. Yet, tax compliance is not a closed system. It is an open system. For every period, the number of taxpayers and the number of monetary units change. Moreover, period $t + 1$ might see individual taxpayers act differently with regard to the covariate p^i than they did at t . These variations perturb the non-compliance rate so as to create an orbiting dynamic about ξ .

In this way, the tax compliance system of a jurisdiction is characterized by its fixed point ξ . As long as the system remains only perturbed and not fundamentally altered, then ξ (and thereby λ) should remain the same for all periods. On the other hand, if the system is not merely perturbed but modulated in such a way that it becomes fundamentally altered, then ξ and λ would increase or decrease accordingly, thereby changing the various non-compliance rates' locus of orbit over multiple periods. Accordingly, the parameter λ appears to be some function of ξ .

D. Graphing Compliance Dynamics

What would this fundamental change in the system look like if the parameter λ were to vary? One can examine this as a non-continuous time series that generates the sequence

$$x_0, x_1, x_2, \dots, x_\tau \quad (8)$$

where $\tau = t + k : \tau \in \mathbb{N}, \tau \gg 0$, and $k \in \mathbb{N}$. Note that this sequence is both uniform and independent.

One can represent the dynamics of this process in two dimensions on a state space where x_{t+1} is the ordinate and x_t is the abscissa. In other words, x_{t+1} is the next value in the sequence following x_t , x_{t+2} is the next value following x_{t+1} ,

and so on. This reflects the sequence in (8) if $t = 0$. The graph must include both $x_{t+1} = f(x_t, \lambda)$ (equation of iterated values) and the identity function $x_{t+1} = x_t$ (fixed points at specific iterated values) where the range of x for all t is limited to the interval $[0, 1]$ since x is a rate and not a quantity.

Start with some initial value of x_t at $t = 0$ and plot the point from $(x_0, 0)$ to (x_0, x_1) . Next, locate (x_1, x_1) since it is the new starting point for period $t + 1$ and move to (x_1, x_2) . The point representing the beginning of the next iteration will be (x_2, x_2) , etc. If one continues with this progression there will be one of two possible results. Either the iteration will converge to a fixed point or it will diverge around one.

The system always has at least one fixed point ξ . The question is not whether ξ exists but whether it is stable or unstable. Expected tax non-compliance rates converge when this fixed point is stable and diverge when it is unstable.

Equation (7) shows that $x_{t+1} = \lambda x - \lambda x^2$, which means that ξ can be defined as

$$\xi = \lambda \xi - \lambda \xi^2. \quad (9)$$

A little bit of algebraic manipulation yields

$$\xi(\xi \lambda - \lambda + 1) = 0. \quad (10)$$

This produces two solutions:

$$\xi^* = 0 \quad (11)$$

and

$$\xi = 1 - \frac{1}{\lambda}. \quad (12)$$

Equation (11) shows that there is always a fixed point at $(x_t, x_{t+1}) = (0, 0)$, but this is trivial (hence, the asterisk). What is more interesting is the solution in (12). Since the parameter can only be non-negative and to avoid triviality, one is interested in solutions for (12) where $\lambda > 1$.

This is the lower bound of the stable, non-trivial interval for λ , but one can also solve for the upper bound. A stable ξ must have a derivative on the interval $(-1, 1)$. In other words, $\|f'(x, \lambda)\| < 1$. If $\|f'(x, \lambda)\| \geq 1$ then ξ is unstable.

The stability dynamic is due to $f'(x, \lambda)$ giving the rate of change of $f(x, \lambda)$. If the magnitude of the rate of change is less than unity then after multiple iterations the function will get closer and closer to a fixed point (it will be "pulled in" to that point). Something is considered stable if it is perturbed just a little and it tends to come back to that point rather than tending to move away from it. If the magnitude of the rate of change is greater than unity then after multiple iterations the function will get farther and farther away from a fixed point (it will be "flung out" from that point). Therefore, what is salient for this paper's purpose is the magnitude of the change, not its sign. Whether it is positive or negative is just the direction in which it is flung. The direction is unimportant for purposes of this analysis.

Equation (6) shows that $f'(x, \lambda) = \lambda(1 - 2x)$. For ξ^* in (11), $f'(0, \lambda) = \lambda$. Thus, the trivial fixed point is stable for $0 \leq \lambda < 1$. For the non-trivial solution ξ in (12), $f'(1 - \frac{1}{\lambda}, \lambda)$ is equal to

$$\lambda \left[1 - 2 \left(1 - \frac{1}{\lambda} \right) \right] = 2 - \lambda. \quad (13)$$

This means ξ is stable for $\|2 - \lambda\| < 1$. Consequently, the non-trivial solution produces stable fixed points for parameters on the interval $(1, 3)$. If λ is on this interval, $\langle x_t \rangle$ will converge to a stable fixed point at ξ in a closed system or will orbit around ξ in an open system. Given (12), ξ can take any value on the open interval $(0, \frac{2}{3})$. However, if $\lambda \geq 3$ then $\|2 - \lambda\| \geq 1$ and $\langle x_t \rangle$ will neither converge to a stable fixed point nor orbit about it. Rather, it will diverge around ξ .

III. RESULTS

The published U.S. tax gap studies in Table I show observed non-compliance rates, each with a $\pm 2\%$ margin of error. From (12), one sees that

$$\lambda = \frac{1}{1 - \xi}. \quad (14)$$

Consequently, (7) becomes

$$x_{t+1}(1 - \xi) = x_t(1 - x_t), \quad (15)$$

which means that $\xi = x_{t+1} = x_t$. Thus, given the error in the results displayed in Table I, it is not inconsistent to assume that the U.S. tax system is characterized by a parameter that yields an idealized non-compliance rate of ξ . Again, since tax compliance is an open system $\langle x \rangle$ is always slightly perturbed at $t + 1$ from its value at t . This would cause $\langle x \rangle$ to orbit about ξ .

TABLE II
 NON-COMPLIANCE RATES FROM ITERATED $f(x_t, \lambda)$

t	$f(x, \lambda)$	Change
1973	17.4%	0.0%
1976	17.6%	-1.7%
1979	19.0%	-1.2%
1981	19.7%	1.3%
1982	18.4%	0.2%
1984	18.2%	-1.8%
1986	19.6%	-1.2%
1987	20.1%	1.9%
1988	18.2%	0.5%
1992	17.8%	0.9%
2001	17.2%	0.9%
2006	16.7%	0.2%
2012	17.2%	-1.1%

Treating the tax compliance system as a population of monetary units characterized by the parameter λ with $f(x, \lambda)$ iterated over each period yields identical results to those observed in the tax compliance studies in Table I given the stated error. Beginning with the 1973 report as x_0 in the graphing method described in §II-D, one gets the results displayed in Table II with the difference from the non-compliance rate reported in each study listed under the heading "Change." Notice that each difference from the respective x -value in Table I is within that study's assumed margin of error, thereby making the results of iterated functions $f(x_t, \lambda)$ statistically identical to the published non-compliance rate for each period. This implies that the tax compliance system feeds back on itself to regulate the system-level of compliance based on ξ . It also means that one can define an empirically valid vector field in terms of x and t where the trajectory of x is proportional to

$$\xi + \left[\cos \left(1 + \frac{\tau}{2} \right) \times \{\text{error}\} \right].$$

Additionally, in the phase space defined by λ and ξ one would expect a modified S -curve traced out by points of negative divergence around ξ for $0 \leq \lambda \leq 3$ and points of positive divergence around ξ for $\lambda > 3$. One would also expect a non-zero measure of curl since tax compliance is an open system; although after $\lambda = 4$, parts of the region of the curl around each point ξ would escape the boundary interval $[0, 1]$. For this reason, and to avoid triviality, it makes sense to work only with the (ξ, λ) coordinates between $(0, 1)$ and $(\frac{3}{4}, 4)$ on this phase space, although ξ does asymptote to unity as $\lambda \rightarrow \infty$.

This interpretation of the tax gap data does not require an assumption that tax authority action is an essential characteristic of the system *qua* system (i.e., at the macro level), even if government policies are shown to have significant effects on micro-level compliance trends within the same system. Although this interpretation might at first appear counterintuitive, it is important to remember that such differences occur throughout observable experience in other disciplines. For example, the number of quarks in a proton have a significant effect on quantum dynamics, but they are irrelevant to one studying cellular composition in a biological system [33]. The same might be true for expected macro-level compliance in a voluntary system. A tax authority's control over the discriminants of macro-level compliance becomes negligible in a voluntary compliance system. This might be true even with maximum third-party information matching.

IV. LIMITATIONS AND FUTURE RESEARCH

A. Dimensionality of the Tax Compliance Rate

This rudimentary snapshot has multiple limitations. For example, what if x is actually a component of some unknown vector, i.e., $x \in \mathbf{x}$, or $x = x^k$? In other words, available data might not provide information on the variables of the underlying system, but rather on some *function* of those variables. Further, the formal model as constructed here is univariate in that it is only quadratic in x with a parameter λ , but what if it is bivariate and quadratic in both x and λ ? These would constitute unknown (and perhaps unknowable) unknowns in formalizing a voluntary compliance system, although the result would be some kind of parabolic manifold.

If tax compliance is only one dimension of larger social, environmental, and even biological interactions, then tax policymakers should manage their expectations about what studying tax compliance can explain about other aspects of society, the environment, and even life itself. In other words, this mathematical relation might be powerful, but it might also be incomplete because it is local only to the issue of tax compliance and does not consider the global context of all human interaction, including the complex interactions of the tax authority and of other regulatory bodies in the overall socio-political ecosystem. The system would not be reducible to a single difference equation since each value of x would be compatible with the multiple components of the abstract vector \mathbf{x} [34].

One can think of this system as a multi-dimensional snapshot. Formal tools for modeling this are currently beyond the

norm in the field of tax law and administration. Therefore, the next step in this research is to explore the unrestricted rational mappings of both the x and the λ hyperplanes in \mathbb{R}^k or even \mathbb{C}^k . This next step has already been explored abstractly by Mandelbrot [35]. The focus of future research is to explore how these abstract bounds translate to real social interactions, especially with respect to legislation and regulations.

If the model presented here is just one of many snapshots of the system, a proper arrangement of those snapshots might require something akin to a metric tensor, which one can denote as $g_{\alpha\beta}$, instead of the one-dimensional scalar $g(x) \mapsto \lambda$. A metric tensor is a function which takes as input a pair of tangent vectors at a point on a Riemannian manifold and produces a scalar. The metric tensor would allow an analysis of an entire assortment of components that collectively make up property characteristics of the tax compliance system. Such a metric tensor in k -dimensional general curvilinear coordinates could take the form of an $a \times b$ matrix of abstract vectors

$$g_{\alpha\beta} = \begin{pmatrix} g_{11} & g_{12} & g_{13} & \cdots & g_{1b} \\ g_{21} & g_{22} & g_{23} & \cdots & g_{2b} \\ g_{31} & g_{32} & g_{33} & \cdots & g_{3b} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{a1} & g_{a2} & g_{a3} & \cdots & g_{ab} \end{pmatrix}$$

with the appropriate constraints, such as $a = b = k$. The indices α and β take values $1, 2, 3, \dots, k$.

The metric tensor gives the square of the magnitude of the infinitesimal displacement in metric space of a curve on a differentiable manifold with respect to the non-compliance rate:

$$\|ds(x)\|^2 = \sum_{\alpha\beta} \left(g_{\alpha\beta} dx^\alpha dx^\beta \right). \quad (16)$$

By parameterizing a curve such that $x(t)$, the arc length of the curve between x_t and x_{t+1} becomes

$$\|s(x)\| = \int_t^{t+1} dt \sqrt{\sum_{\alpha\beta} \left(g_{\alpha\beta} \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt} \right)}, \quad (17)$$

which for purposes of this paper would be the magnitude of the displacement in state space between states $|N\rangle$ at t and $t + 1$. The only trade-off to using this method is that one must assume temporal continuity. As previously discussed, this is not an exact reflection of observable tax compliance phenomena since taxpayers typically file returns and pay taxes at discrete times.

A related area for future research would include whether this state transition is subject to the principle of least action, which appears to be fundamental in the state transitions of organized systems and many other fields. If the state transitions along the geodesic are subject to this principle then the transition from state $|N\rangle_t$ to state $|N\rangle_{t+1}$ would be the one with the least action and, hence, the highest organization [36]. If this is the case, the assumption of temporal continuity should not present a problem since the "path" taken in state space between t and $t + 1$ would be constrained by the action principle.

B. Lack of Adequate Data to Validate Theory

There is also a very practical limitation with this snapshot; that is, the small sample of time series data currently available. The formal theory presented in this paper relies heavily on multiple iterations to discover both the stable and unstable points ξ of a voluntary taxpayer compliance system. Yet, it remains difficult, if not impossible at present, to validate this theory without significantly robust time series data. Not only is such time series data necessarily finite, it is extremely limited and incredibly difficult to granularize since it is only reported in the aggregate by way of annual statistical publications from a tax authority, such as the IRS's annual *Data Book*.

Publicly-available tax enforcement data from which one can derive a certain expected measure of compliance are reported, at best, on an annual basis. Such a limitation can create a kind of false positive in that a linear random system, which this paper argues against, can generate effects that mimic a non-linear deterministic yet dynamic system, which this paper argues for, if the time series data is relatively small [37].

Currently, the limited time series data makes it difficult to discern whether the time series is generated by a linear stochastic system or a non-linear wicked system. Further, it is also possible to have a mixed variant where a wicked system contains some random stochastic components [38]. These will remain difficult hurdles to overcome as long as the available time series data remain small and aggregated.

V. TAX POLICY IMPLICATIONS

If the results in §III are correct, the effects on tax policy are considerable. It has been famously said that “tax administration *is* tax policy” [39]. What the tax authority can administer effectively becomes a jurisdiction's policy on what and how much to tax. But *how* a jurisdiction increases revenue is just as important as the increase itself. Good tax administration is almost never simply about getting more revenue [40]. As Christians notes, issues of equity and justice are inescapable [41]. Consequently, the three traditional criteria of tax policy analysis are equity, efficiency, and administrability. This section touches on just three tax policy issues: tax expenditures, third-party information matching, and the voluntary compliance system itself.

A. Tax Expenditures

The Congressional Budget and Impoundment Control Act of 1974 defines tax expenditures as “revenue losses attributable to provisions of the Federal tax laws which allow a special exclusion, exemption, or deduction from gross income or which provide a special credit, a preferential rate of tax, or a deferral of tax liability” [43]. The Joint Committee on Taxation provides official revenue estimates for all tax legislation considered by Congress. The Treasury Department also provides estimates, although the two do not always coincide. United States government officials have long hinted at, if not officially recommended, that the elimination of tax expenditures would significantly decrease the tax gap [44]. The usual reason given is that tax expenditures create complexity, which creates increased opportunities for both taxpayer mistake and evasion

[44]. Eliminating certain tax expenditures, so the argument goes, would eliminate these opportunities and thereby decrease the tax gap.

Others go further. In her 2012 Annual Report to Congress, the IRS's National Taxpayer Advocate opined in support of tax expenditure elimination that “if Congress were to eliminate all tax expenditures, straight math indicates it could cut individual income tax rates by 44 percent and still generate the same amount of revenue it collects under current rules” [45].

This is a potentially dangerous leap. Although tax expenditures are defined as revenue reductions, they do not represent a one-to-one mapping to potential tax revenue increases. The Treasury Department's Office of Tax Policy (OTP) recognizes this and regularly warns Congress about the fallacious assumption. Repealed tax expenditures do not necessarily equal an increase in tax revenues because (1) eliminating a tax expenditure might alter taxpayer behavior, which in turn might affect the resulting voluntary compliance measured in monetary units; and (2) repeal of a tax expenditure might increase or decrease tax revenues independent of changes in taxpayer behaviors. For example:

[R]epeal of an itemized deduction could increase the revenue costs from other deductions because some taxpayers would be moved into higher tax brackets. Alternatively, repeal of an itemized deduction could lower the revenue cost from other deductions if taxpayers are led to claim the standard deduction instead of itemizing.

[46]

Additionally, provisions in the tax code are not completely modular. They are not like fuses on a circuit board that can be added or removed independent of other fuses. According to OTP, “If two provisions were repealed simultaneously, the increase in tax liability could be greater or less than the sum of the two separate tax expenditures, because each is estimated assuming that the other remains in force” [46].

If the alternative formalization of the tax gap explained in this paper is correct, then the effect of tax expenditures on the tax gap might be negligible as with enforcement magnitudes. There is some evidence supporting this conjecture. First, Congress radically modified the number and type of tax expenditures since the 1974 Act, especially leading up to the Tax Reform Act of 1986 [47]. The most recent report from OTP enumerates 167 items, compared to the 1974 report that enumerated 67 items [44]. Yet, the normalized tax gap has remained statistically unchanged. Since Congress has significantly changed tax expenditures over the past 50 years creating undulations in the monetary value of tax expenditures compared to gross tax revenues and the Gross Domestic Product, and these changes do not map one-to-one to tax revenue increases, it would be an improbable coincidence that the normalized tax gap would remain unchanged by the “expert” congressional planning of adding, removing, and modifying tax expenditures in just the right way so as to leave the normalized tax gap unchanged. A more probable explanation is the one given here relating to the U.S. voluntary compliance system itself.

Second, there is evidence in the tax gap data that even if there is a significant change in taxpayer behavior relating to tax expenditures such as credits, the macro-level tax gap remains unaffected. Comparing tax gap reports for 2001, 2006, and 2012, one finds significant undulations (greater than the 2 percent error) in certain disaggregated groups of taxpayers (micro-level), but no significant change in the macro-level tax gap.

For example, between 2001 and 2006, the change in the Individual Business Income Tax underreporting gap was -4.5 percent. This decrease was offset by a change in Corporation Income Tax underreporting gap of 6.2 percent, for a net change of 1.7 percent. This period also witnessed an insignificant net change of -1.3 percent in Individual Income Tax underreporting due to Credits. Yet, the macro-level tax gap only saw a net change of -0.6 percent. Between 2006 and 2012, the change in Individual Business Income Tax underreporting gap was not significant at only 0.2 percent, although the change in Corporation Income Tax underreporting gap remained significant at -5.9 percent. This period also witnessed a significant change of -2.5 percent in underreporting due to Credits. Still, the macro-level tax gap experienced a non-significant change of -1.4 percent, half of which officials attribute to updated estimation methods and not an actual decrease in voluntary compliance [48].

Thus, the number of public resources marshaled to collect and analyze tax expenditure data, compile these analyses into congressional reports, and then translate the reports into legislative action might very well represent poor stewardship. If the normalized tax gap is unaffected by significant changes in tax expenditures, perhaps those resources are better focused elsewhere. Certain mainstay tax expenditures, such as the individual deduction for state and local taxes, are now at least temporarily reduced by the 2017 Tax Cuts and Jobs Act [49]. It will be interesting to see if the next tax gap study is significantly changed given these tax expenditure exclusions. If not, it will serve as further evidence that changes in tax expenditures have little to no effect of the macro-level tax gap.

Auerbach justifies tax expenditures differently. Instead of claiming that they increase voluntary compliance, he claims that tax expenditures reduce individual taxpayer burden [50]. For example, the exclusion of capital gains on owner-occupied housing eliminates the need for homeowners to maintain detailed records of all home improvements necessary to establish the basis for the home at the time of sale [51], [52]. While reduction of taxpayer burden might remain a valid policy end that justifies spending significant resources examining tax expenditures, a policy argument in favor of such analysis for purposes of reducing the tax gap will have very dull teeth if the alternative formalization in this paper is correct [53]. Therefore, this approach fundamentally changes the tax policy discussion regarding expenditures to the extent expenditures are believed to affect the macro-level tax gap.

B. Third-Party Information Matching

As discussed in §I, there is notable extant scholarship on the topic of reducing the tax gap by increasing third-party

information reporting to the IRS so the tax authority can match this information against self-reports. The hypothesis is that such reporting and matching increases a taxpayer's perceived probability of getting caught underreporting. This focus shifts the emphasis from detection probability by audit to detection probability by third-party information that a tax authority then uses to verify the original taxpayer reports. Third-party information has become central to U.S. tax collection since 1974 and exponential improvements in information technology have made reporting and matching a preferred method for tax compliance and enforcement [54].

There are data that support this hypothesis at the micro-level. Lederman shows that taxpayer groups typically subject to third-party information matching tend to have significantly higher rates of voluntary compliance than those not subject to the same centralized standard [14], [15], [18]. Viswanathan warns that congressional failure to regulate information reporting for blockchain, the "gig" economy, and any other income derived from "sources that are difficult to regulate" will lead to a sharp increase in the tax gap [55].

But is this necessarily so? This paper already addressed the fact that the macro-level tax gap shows no change for periods prior to third-party information matching and after its initiation in 1974 and its later development. While there is evidence to support the hypothesis at micro-levels, there is no evidence that changes in third-party information matching have any effect on the macro-level tax gap. Again, this is supported by previous work with ABMs showing that higher probabilities of detection increase compliance at the micro-level, but have no noticeable effect at the macro-level [5].

For tax policy purposes, this suggests that regardless of an increase in blockchain, ride sharing, or other radical changes in the informal economy, and regardless of congressional action or inaction regulating third-party information from those income sources there is little chance of any statistically significant change in the macro-level tax gap. The data confirm that certain administrative and statutory enforcement structures such as information matching do increase voluntary compliance [56], but only when such is measured at the micro-level; what Andersson and Törnberg call "sub-wicked systems" [57]. Thus, the conclusion that more deterrence leads to greater compliance is not incorrect; it is just incomplete when referring to the macro-level ("wicked system") tax gap.

At first, this appears paradoxical. Upon further reflection, however, perhaps the paradox is only in a Quinean sense [58]. An intuitive result at the micro-level but not at the macro-level creates a sense of surprise if not complete dissonance. Yet, this sense dissipates once one resolves the connections yielding strange-but-true results. One is left with the conclusion that, possibly, different forms of measurement (micro vs. macro) imply that different things are being measured. Since the causal explanation of information matching does not comport with observed data, one must look for answers elsewhere.

C. The Voluntary Compliance System

If Congress continues to demand a policy goal of reducing the tax gap, then given the results in this paper it might

not be enough to create additional structures that require more third-party information reporting. Instead, the essential characteristics of a voluntary tax compliance system itself might have to change. One such change might be that the government must control distribution of the actual income and not just information about the income distribution. Of course, such a system would by definition no longer be a voluntary compliance system. As a result, terms like “voluntary compliance rate” and “tax gap” would have no meaning in this revised system. The system itself would be fundamentally modulated, not merely perturbed.

This observation might first appear over the top, but it is indicative of the kinds of discussions tax policymakers and administrators must have if a tax authority’s influence on reducing the tax gap is as it appears: negligible. Such discussions are necessary because tax policy unintended consequences are an illusion. There are only consequences [59]. Any change in tax policy must consider the desired effects (e.g., greater revenue, less administrative costs, etc.) and also the relevant consequences that such a change might produce, including increased taxpayer discontent and an inadministrable system [60]. To paraphrase Rittel and Webber, the aim of tax policy is not to find the truth, but to improve the mechanisms of tax administration to meet a desired end *in the real world*.

It is quite possible that current government actions relating to the tax gap amount to nothing more than theater. It does not matter that this consequence is unintentional. It does not matter that this consequence is not malicious. It does not matter that it is due to a belief that attempting to do something is better than doing nothing even if doing something yields no better results and creates collateral effects. The end might still amount to nothing more than theater, and if so it is important for policymakers to recognize this.

Why? Because tax policy is not like philosophy, mathematics, or the theoretical sciences where the contributions of practitioners are just as important even if others later prove them wrong. When it comes to tax policy and administration, such immunity cannot be tolerated. Policymakers are responsible for the public actions they inspire and any resulting consequences. Rarely are these actions meaningless as they usually matter a great deal to those who are touched directly by them. Therefore, tax policymakers have “no right to be wrong” [42]. In this way, the three criteria of tax policy analysis — equity, efficiency, and administrability — find new faces in a wicked tax system.

Significantly reducing the tax gap (i.e., raising more revenue) while still maintaining the low administrative costs of a voluntary compliance system might not be an option. More revenue might require an unacceptably high investment in a fully-centralized “income control system” that ends up being both impractical and, to many, an affront to democratic ideals. It is very possible that if tax policy favors a voluntary compliance system, then the current voluntary compliance rate is the best that it gets. For this reason, difficult conversations and new ideas are here needed.

VI. CONCLUSION

Policymakers, practitioners, and researchers have tried for at least half a century to develop a better understanding of the channels through which laws, regulations, and tax authority actions can improve voluntary taxpayer compliance. Yet, there is clearly still much to learn. The one unchanging assumption in this ongoing endeavor has been that tax authority intervention of some kind can control, and thereby improve, voluntary compliance. It has been assumed that the goal, therefore, is to find the most efficient government intervention to raise revenue by reducing the tax gap without questioning if such intervention actually affects voluntary compliance.

This paper has attempted an alternative explanation for observed measures of voluntary tax compliance in the United States assuming this wicked system. It has done so by presenting a second formulation of non-compliance rates that achieves results mathematically equivalent to those observed in the tax gap studies. Instead of looking for causal explanations in the context of local taxpayer compliance and attempting to scale them to explain macro-level data, it examined the tax system *qua* system by starting with the population of monetary units legally due to the tax authority for a specific period. From this population, it derived a function that describes the non-compliance rate at the next period based on the same rate at the previous period and a system-level parameter. The paper explained that this parameter itself is a function of an expectation value of non-compliance for the system. Predictions based on this explanation were shown to be statistically identical to the tax gap estimates published in studies over the past fifty years.

The formal model here illumines only slightly the theoretical effects of tax authority actions on the tax gap. If the effect is negligible, tax policymakers and administrators might have to face the tough decision between an increase in tax revenue and maintaining a voluntary compliance system.

This paper shows that, over time, normalized non-compliance can orbit a stable expected value in a voluntary system. There remain a multitude of other “snapshots” that might provide a better understanding of the system as a whole. The one explored here hopefully starts a conversation on alternate ways of looking at voluntary tax compliance; even if through a lens, darkly. Such conversations are critical, for if the tax gap ratio in the United States is not only inveterate but incorrigible, creative alternatives to achieving tax policy ends must be explored.

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