A Performance Modeling of Connectivity in Vehicular Ad hoc Networks (VANETs)

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Abstract

A Performance Modeling of Connectivity in Vehicular Ad Hoc Networks (VANETs)

Mehdi Khabazian, Ph.D.
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An emerging new type of ad hoc networks is *Vehicular Ad hoc NETworks* (VANETs) which envision Inter-Vehicle Communications. Since, nodes in VANETs are both mobile as well as carrier of information; the network may not have full communication connectivity all the time and they may form several *clusters* where the nodes in each cluster may communicate with each other directly or indirectly. Multi-clustering happens whenever the minimum distance between two adjacent nodes becomes more than the transmission range of a node. Therefore, two important performance measures which affect the functionality in VANETs are *communications connectivity* and *path availability*.

In this thesis, we study the statistical properties of these performance measures in VANETs at the steady state. First, it is assumed that the nodes travel along a multi-lane

highway which allows vehicles to overtake each other. We derive the probability distributions of the node population size and node's location in the highway segments. Then, we determine the mean population size in a cluster and probability that nodes will form a single cluster.

Then we extend the single highway model to a network of highways with arbitrary topology. We determine the joint distribution of the node populations in the highways' segments by application of the BCMP theorem. We model the number of clusters within the node population in a network path as a Markovian birth–death process. This model enables derivation of the probability distribution of the number of clusters and determination of mean durations of continuous communication path availability and unavailability times as functions of mobility and node arrival parameters. At the end, mean packet delay is presented for end to end communication in a path. We give numerical results which illustrate the effect of mobility on continuous communication path availability and communication delay. The results of this work may be helpful in studying the optimal node transmission range assignment, routing algorithms, network throughput, optimization of cross layer design schemes and MAC protocols in VANETs.

Dedication

I dedicate this thesis:

- To the Lord, who gave me this opportunity and let me to be open and learn to walk the line between cultures, deeply enjoying both sides. All glory and honor to Him.
- To Professor M.K. Mehmet Ali (my supervisor) whose detailed criticism, valuable suggestions and wise counsel have proved most invaluable to me. It has been a most rewarding learning experience to work under his guidance.
- To my wonderful wife, Maryam, who supported me throughout this entire venture. She always has been a great source of love, motivation and inspiration to my life. I love you.
- To my respectful parents, Mahmoud and Soheila, who have raised me to be the person I am today. Thank you for all the unconditional love and support that you have always given me, helping me to succeed and instilling in me the confidence that I am capable of doing anything I put my mind to.

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a	Probability that an idle node will receive a new packet for transmission during the packet transmission time.
a_c	Mathematical notation, defined in (3.14).
$a_s(m)$	Birth rate when the cluster state is $m_s = m$.
$A_{\Omega_k}(1)$	Cluster birth rate of path Ω_k when it is in single cluster state.
$B_r(z)$	Probability generating function of b_r.
b_r	Bernoulli variables that assume values of 1, 0 if two nodes in cells r and r+1 gain connectivity or remain disconnected by the time $t+\Delta t$ given that they do not have connectivity at time t.
$b_s(m)$	Death rate when the cluster state is $m_s = m$.
b_s	Bernoulli variables that assume values of 1, 0 with probabilities g_s or $1-g_s$ respectively defined in (5.48)
$B_s(z)$	Probability generating function of b_s.
$b_{x(t)}(y)$	Probability density function of $X(t)$.
$B_{x(t)}(y)$	Cumulative density function of $X(t)$.

C_1	Probability that the node population in cell one will have connectivity with the new arrival to the single highway.
C_{r+1}	Probability that the node population in cell r+1 will have connectivity with node population in cell r.
C'_{r-1}	Probability that the node population in cell r-1 will have connectivity with node population in cell r.
d	Transmission range of a node.
$d(\vec{n})$	Function of total arrivals, defined in (5.6).
$\overline{D}_{\Omega_k}$	End to end average packet delay for the path Ω_k assuming that it is in the single cluster state.
D_{ji}	Mathematical notation, defined in (5.11).
E	Number of empty cells given that there are m clusters in a segment.
e_{ji}	Total arrival rate of the nodes to the segment S_{ji}.
f_i	Probability that the random node as a cluster-head is located in cell i in the single highway.
$f_L(\ell)$	Probability density function of L.
$f_{T_s(m)}(t)$	Probability density function of $T_s(m)$.
$f_{Y_r}(y_r)$	Probability density function of Y_r.
G	Normalization constant, defined in (5.4).
g_s	Probability that node populations of the border cells of consecutive segments s and $s+1$ will have connectivity with each other.
$g_{x_{kj}(t)}(y)$	Probability density function of the distance of a node that belongs to the $\tilde{s}_{kj}$'th stream from its arrival service point.
$G_{x_{kj}(t)}(y)$	Cumulative distribution function of $g_{x_{kj}(t)}(y)$
$\tilde{g}_{x_{kj}}(y)$	Probability density function of the distance of a node from the $\tilde{s}_{kj}$'th stream at the steady-state.
$\tilde{G}_{x_{kj}}(y)$	Cumulative distribution function of $\tilde{g}_{x_{kj}}(y)$.

h_{ji}	Node population density in the segment S_{ji}.
I	Total number of cells in the single highway.
$J\tau_p$	Constant packet transmission time.
K	Number of segments in the single highway.
$\vec{k}$	A notation to refer to the segment which cell r is located on in single highway.
L	A random variable which denotes the distance between two nodes in two consecutive cells r and r+1.
ℓ_k	The length of the k'th segment in single highway.
L'	Distance between two nodes located at the consecutive cells r and r+1 at time $t+\Delta t$.
M	Number of lanes in the single highway.
$\overline{m}$	Average number of nodes in a cluster which is seen by a new arrival.
$\overline{m}'$	Average cluster size which is seen by a random node in the single highway.
M_{Ω_k}	Number of clusters in the node population of path Ω_k.
$\overline{m}_j$	Mathematical notation, defined in (4.26).
$\overline{m}'_{\ell,i,h}$	Average cluster size which is seen by a random node located in cell i and consists of all the nodes in cells r such that $\ell \le r \le h$.
m_s	Number of clusters in segment s.
n	Number of epochs during the interval $(0,t)$
N	Number of highways in the network of highways model.
$\vec{n}$	A vector that specifies the number of nodes in each highway segment of the network of highways.
n_{ji}	Number of nodes in the segment S_{ji}.
$N_{kj}(t)$	Number of nodes from the $\tilde{s}_{kj}$'th stream on the highway at time t.

N_r	Size of the node population within cell r at the steady-state.
$N_r(z)$	Probability generating function of N_r.
n_t	Total number of nodes in the highway network respectively.
$P_{\Omega_k}(m)$	$\Pr(M_{\Omega_k} = m)$.
$P_{\Omega_k}(z)$	Probability generating function of M_{Ω_k}.
P_A	Probability that a node which is located in cell r-1, remains in its cell within the next Δt seconds.
p_{cc}	Probability that two nodes which are located at adjacent cells will have connectivity at $t+\Delta t$ given that they have connectivity at t.
p_{dc}	Probability that two nodes which are located at adjacent cells will have connectivity at $t+\Delta t$ given that they do not have connectivity at t.
P_e	Probability that cell r from segment s is empty.
$p_f(t)$	Probability that a node has departed from the highway by the time t.
$p_{ji,rs}$	Probability that a node departing from segment S_{ji}, next will receive service from segment S_{rs}.
$P_k(t)$	$\Pr[N_{kj}(t) = k]$.
$P_{mn}(r)$	Probability that a node from s_{mn}'th streams is located in cell r.
p_r	$\Pr(L \leq d \mid$ two nodes are located at consecutive cells r and $r + 1)$.
$p_s(t)$	Probability that a node is still on the highway at time t.
$P_s(m)$	$\Pr(m_s = m$ at the steady state).
$P_s(z)$	Probability generating function of m_s.
$Q(m)$	Pr(cluster consists of m nodes).
Q_j	Number of segments in highway j of the network of highways model.
q_{r+1}	Probability that a node located in cell r + 1 will not have direct communications with any of the nodes in cell r.

q'_{r-1}	Probability that a node located in cell r-1 will not have any direct communications with node population in cell r.
q_s	Probability that a node in a cell of segment s does not have direct connectivity with the population of its preceding cell with mean population of $\bar{\phi}_s$.
r	Cell counter.
r_k	The starting cell of k'th segment in the single highway.
$\bar{R}_{\Omega_k}(1)$	Mean communication unavailability time or the average time that takes for the path Ω_k to return to state one following its departure from this state.
R_{ji}	The length of segment S_{ji}.
R_{kj}	Mathematical notation, defined in (4.5).
$\bar{R}_s(m)$	Cluster mean recurrence time or the average time that it takes for the system to return to cluster state $m_s = m$ following its departure from this state.
$\tilde{s}_{kj}$	Stream of nodes with arrivals and departures at the service point k and j respectively.
S_{ji}	i'th segment of highway j in the network of highways model.
$\lvert s \rvert$	Number of cells in segment s from path Ω_k.
t_{i-1}	The beginning time of the i'th epoch.
T_i	The duration of the i'th epoch.
$T_{\Omega_k}(1)$	Continuous path communication availability time in each visit.
$\bar{T}_{\Omega_k}(1)$	Mean continuous path communication availability time in each visit.
$T_s(m)$	Cluster holding time or the time that the system spends in state $m_s = m$ in each visit.
$\bar{T}_s(m)$	Mean of $T_s(m)$.
u_{r+1}	The number of nodes in cell r+1 that will gain connectivity with at least

one of the nodes in cell r by the time $t+\Delta t$ given that they do not have connectivity with any node in cell r at time t.

$U_{r+1}(z)$ Probability generating function of u_{r+1}.

V_i The speed of a node during the i'th epoch.

V_r Random variable of a node speed in cell r of segment S_{ji}.

x_i The amount of distance covered by a node during i'th epoch.

$X(t)$ The covered distance by a node as a function of time from its arrival point to the network.

$X_n(t)$ The covered distance by a node as a function of time from its arrival point to the network given that there are n epochs during the time interval $(0, t)$.

$X_{n,T_i}(t)$ The covered distance by a node as a function of time from its arrival point to the network given that there are n epochs during the time interval $(0, t)$ and the epoch durations are constant.

Y_r A random variable which denotes the distance of a node in a cell.

α_k Probability that a node departs the highway at the end of segment k–1 in the single highway.

α Probability that a user requests a transmission during a propagation time.

β Exponential distribution parameter for mobility epoch duration.

$\gamma(h_{ji})$ State dependent service rate of the nodes at the service center S_{ji}, given that the population density of the nodes at that service center is h_{ji}.

$\overline{\Gamma}(r)$ Average single hop packet delay.

$\overline{\Gamma}_s$ Average total packet delay for the segment S.

δ_r Number of nodes in cell r that will gain connectivity to a chosen node in cell $r+1$ by the time $t+\Delta t$ given that they do not have connectivity to this node at time t.

$\delta_r(z)$ Probability generating function of δ_r.

$\varepsilon_{x_{n,T_i}(t)}$ Mean of $X_{n,T_i}(t)$.

$\varepsilon_{x_n(t)}$	Mean of $X_n(t)$.
$\varepsilon_{x(t)}$	Mean of $X(t)$
$\theta_{x_n(t)}$	Variance of $X_n(t)$.
$\theta_{x_{n,T_i}(t)}$	Variance of $X_{n,T_i}(t)$.
$\theta_{x(t)}$	Variance of $X(t)$
ξ	Normalized mean cluster size which is seen by a new arrival in the single highway.
ξ'	Normalized mean cluster size which is seen by a random node in the single highway.
λ_k	Nodes' arrival rate to the k'th service point.
λ_{ji}	The external arrival rate of the traffic to the segment S_{ji}.
λ_t	Total arrival rate of nodes to the highway network.
μ	Mean speed of nodes.
$\mu(h_{ji})$	State dependent mean speed of the nodes at the service center S_{ji} given that the population density of the nodes at that service center is h_{ji}.
$\rho_{cc}(s)$	$1-\rho_{cd}(s)$
$\rho_{cd}(s)$	Probability that the node populations of cells r and r+1 of segment s will become disconnected at $t+\Delta t$ given that they have connectivity at t.
$\rho'_{cd}(s)$	Probability that the distances between the population of cells r and r+1 become higher than d.
$\rho^*_{cd}(s)$	Probability that the last cell of segment s with density of $\bar{\phi}_s$ and the first cell of segment $s+1$ with density of $\bar{\phi}_{s+1}$ will become disconnected at $t+\Delta t$ | they have connectivity at t.
$\rho''_{cd}(s)$	Probability that one of the cells r and r+1 becomes empty.
$\rho_{dc}(s)$	$1-\rho_{dd}(s)$

$\rho_{dd}(s)$	Probability that the node populations of cells r and r+1 of segment s will remain disconnected at $t+\Delta t$ given that they do not have connectivity at t.
σ	Standard deviation of the nodes' speed.
τ	Arrival time of a node to the highway.
τ_p	Propagation delay for a cell.
v_0	Maximum speed in the segments of highway network.
$\tilde{\phi}_{kj}$	Mean of node population size in stream $\tilde{s}_{kj}$ at the steady state.
$\phi_{mn}(r)$	Mathematical notation, defined in (4.13).
$\bar{\phi}_s$	Mean of node population size within cells r of segment s at the steady state.
$\bar{\phi}_r$	Mean of node population size within cell r at the steady-state.
φ_{ji}	Mean of node population size in segment S_{ji}.
ψ	Mean of node population size on the single highway.
ω_j	Probability that the cluster consists of the first j cells.
$\omega'_{\ell,i,h}$	Probability that cluster of a random node located in cell i consists of all the cells r such that $\ell \le r \le h$.
ω'_I	Probability that a random node will see the entire node population in a single cluster.
Ω_k	k'th path in the network of highways model.
$\lvert\Omega_k\rvert$	Number of segments in path Ω_k.

Chapter 1

Introduction

1.1 Background

One of the potential technologies for Intelligent Transportation Systems (ITS) is Vehicular Ad hoc NETworks (VANETs). The main goal in implementing this type of networks is to provide safety in transportation through wireless vehicle to vehicle (V2V) and infrastructure to vehicle (I2V) communications. VANETs lack a central instance for network organization and they represent fully distributed and self-organizing networks based on wireless multi-hop communication.

In this chapter, first we review the evolution of wireless networks since introduction of the commercial cellular communication in 1991. Then, VANETs are discussed from the perspective of ad hoc networks. We also discuss the current state of this technology, its standardization and its potential applications.

1.2 Evolution of Wireless Networks

Presently, wireless communications is the leading technology in telecommunications. This technology has shown tremendous amount of growth in a very short time period and has significantly impacted our way of life. Wireless networks have extended the services given to the wireline users to the mobile users. The evolution of wireless communications has been extremely rapid. In about two decades it has gone through four generations. The first generation wireless networks used analog communication with circuit switching technology. In circuit switching (CS), dedicated bandwidth is assigned to each call. The route of the call may consist of several hops and at least one of the hops is through a wireless network. The second generation wireless networks (2G) continued to use circuit switching technology, but they replaced analog channels with digital communication channels. The first two generations met the voice communication needs of mobile users, but they were not suitable for data communications. Since data is bursty, and in circuit switching the bandwidth is dedicated to a call, it results in under utilization of network resources. The third generation wireless networks (3G) have been developed to meet the requirements of the mobile data communication users, and, at the same time, achieve higher network utilizations. The objective has been providing Internet access to the users, such that they can browse web pages and have voice or video sessions while on the move. The third generation networks replaced CS with packet switching (PS) technology that allows dynamic sharing of bandwidth among different applications. This allows statistical multiplexing of users' information and idle users do not consume network bandwidth since they are not generating any information. This technology has allowed introduction of several new services, such as, video conferencing, IP telephony, video and audio streaming services.

Presently, fourth generation cellular networks are in development with the objective of providing high-speed Internet access. It is likely to appear after the successful deployment of 3G systems. 4G is supposed to complement and replace 3G systems in different ways. The preliminary goal of 4G is to provide higher data rates (higher than 2 Mb/sec) and to find new frequency bands for a worldwide standard. However, the main challenge in 4G is to provide a telecommunication environment that many types of wireless and wire-line systems can coexist and communicate with each other. Further, any peer to peer communication media becomes completely transparent to the users [1].

Different standards and technologies have been proposed for 4G. The examples of which are: smart antenna technology, WIMAX topology, ad-hoc networks, OFDM WLAN and etc. As an illustration of what may happen in a 4G communication, we can imagine a user who initiates a high quality video-conferencing from his cell phone with a friend while he is sitting in a café. This café may be served with a WIMAX or WIFI technology. On the other side, his friend may be sitting in a car which is served with Vehicular ad hoc technology or in a plane which is served with the satellite technology. In this example, not only the end points are served with different technologies, but the intermediate networks may be different types such as, Ethernet, IP networks and ATM. Finally, all these changes and topologies are transparent to the end users. Fig. 1.1 shows the evolution in the wireless technology since the introduction of 2G systems and potential applications for 4G systems.

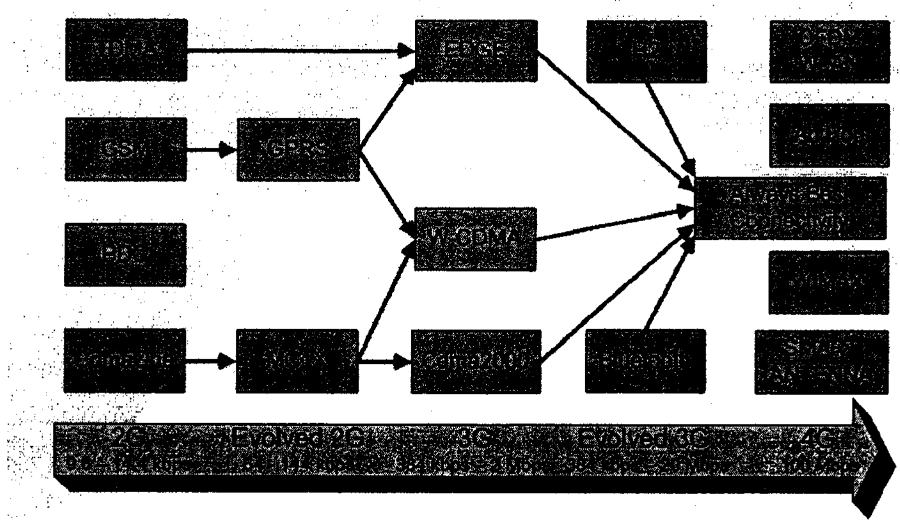


Fig. 1.1. Evolution of the wireless technology since introduction of 2G systems [2].

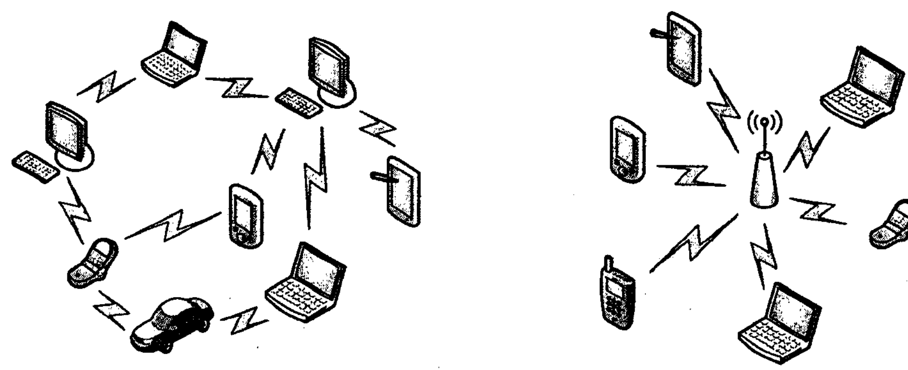
The key for success in the road towards 4G communications is directly related to the success of all the relevant technologies as well as communications among them.

1.3 Ad hoc Networks

Most recently, a new network research area known as ad hoc networks has emerged in wireless communications which is presenting significant challenges to 4G communications. Mobile ad hoc NETworks (MANETs) are spontaneously formed by mobile users; therefore, they do not have cell infrastructure such as, base stations. These are forms of peer-to-peer networks with distributed control. In this type of networks, each node in addition to being the source or destination of the data, cooperates with other nodes for the transportation of the information within the network. Thus, each mobile user acts as a store-and-forward node for the information that receives from its neighbors. The ad hoc networks may be stand-alone or may have interface to wireline networks. Fig.1.2 compares the traditional cellular infrastructure with ad hoc scheme.

There are many applications of ad hoc networks. The future applications can be classified based on the availability of traditional infrastructure-based cellular networks, as explained below [3],

- Infrastructure is not available. In some situations, implementing an infrastructure for communication is not feasible. The example could be deployment of an emergency response network for a search and rescue operation in an area where an earthquake has occurred. In this scenario, the ad hoc network can provide communication among rescue teams on the scene.
- Infrastructure is available but it is inadequate. This is due to a traffic surge in emergency or changes in geographical distribution of traffic. The example is communications among drivers for finding a better route in transportation highways. In this situation, the cellular architecture is inadequate for providing the required information capacity as it is saturated.
- Infrastructure is not necessary. When the traffic is local, the routing is not necessary through network's infrastructure which is external to the location. For example, communications in a meeting room or a construction site can be handled with an ad hoc architecture. Another example is the voice telephony inside a cell which is usually routed through the infrastructure which wastes



(a) (b)

Fig. 1.2. a) Ad hoc architecture versus b) cellular architecture.

the cellular capacity. Voice telephony may be provided through ad hoc communications except when the end users are located at different cells.

Ad hoc networking had originally been proposed for military applications, but recently, many commercial applications have also emerged. Some of the basic services which already implemented are Bluetooth technology, infrared device to device conferencing applications and PC to PC ad hoc communication. Although these technologies work in ad hoc manners, their applications are limited to one-hop communications.

1.4 Vehicular Ad hoc Networks

An emerging new type of ad hoc networks is vehicular ad hoc networks which envision Inter-Vehicle Communications (IVC) or equivalently Vehicle to Vehicle Communication. VANETs are a class of mobile ad hoc networks where the vehicles are the mobile nodes of the network and they communicate with each other through an ad hoc scheme. VANETs have mobility characteristics that distinguish them from other

MANETs and make their behavior fundamentally different. The main differences can be classified as following:

- VANETs have a highly dynamic topology and experience frequent fragmentation due to the fast motions of vehicles. This phenomenon results in small effective network diameter.
- They also have highly dynamic scale and network density. VANETs are the largest ad hoc network ever proposed. Therefore, the issues of stability, scalability, reliability, and security are of great concern.
- The nodes' motions are restricted to a geographical pattern, such as network of highways or city streets.
- The nodes do not have power constraints due to their access to the electrical system of the vehicle.
- The driver behavior is also another important issue. The drivers may react according to the data which they receive from the network and his behavior may influence the network topology.

These characteristics have important implications for design decisions in VANETs. The resulting challenges will be discussed later on. Fig. 1.3 shows a schematic of vehicular communication in the streets of a city.

Although VANETs are the application of MANETs in vehicular environment, for simplicity reasons, we use the word "MANETs" to refer to the ad hoc networks other than vehicular ones. This customary error conforms to the literature and enables the authors to distinguish these two types of services. However we employ "ad hoc networks" to refer to both MANETs and VANETs in this study.

1.4.1 VANETs Applications

The prospective applications of VANETs are categorized into two groups: *safety* and *comfort* applications [4]. The first group is expected to improve the driving safety. The safety applications may be classified into three groups according to their safety natures: assisting, informing and warning. Examples of assisting safety applications are cooperative collision avoidance and lane-changing assistant. Examples of informative safety applications are speed limit or work zone information. Finally, the examples for warning safety applications are obstacle, emergency or road condition warnings.

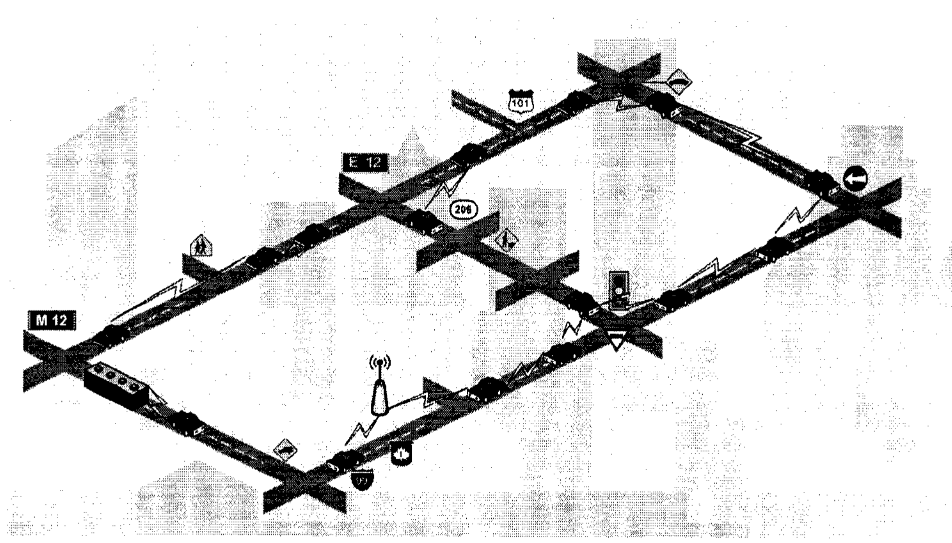
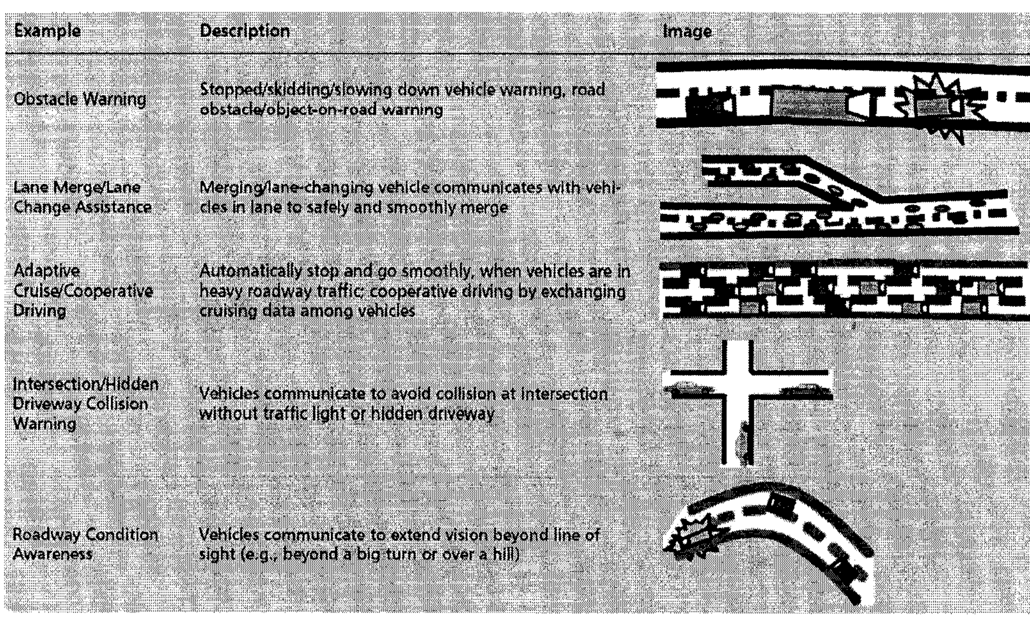


Fig. 1.3. Vehicular Ad hoc Networking.

In safety applications, two types of messages may be exchanged in the network, periodic and event driven (aperiodic). The periodic message exchange (or beaconing) is preventive in nature and its objective is to avoid occurrence of dangerous situations. The beaconing messages may contain information regarding to the position, direction, speed and driver intensions of the sending vehicle. The event driven messages may be generated as a result of a dangerous situation, such as, close by vehicles traveling in high speeds. In this case, the received information may even be used to directly activate an actuator of a safety system [4]. Fig. 1.4 illustrates some safety applications for VANETs.

The comfort application group is expected to improve the passengers' comfort and optimize traffic efficiency. Examples of comfort applications are the traffic information system, alternative route selection, weather information, mobile Internet access and receiving data from commercial vehicles and from roadside about businesses and



Example	Description	Image
Obstacle Warning	Stopped/skidding/slowing down vehicle warning, road obstacle/object-on-road warning	
Lane Merge/Lane Change Assistance	Merging/lane-changing vehicle communicates with vehicles in lane to safely and smoothly merge	
Adaptive Cruise/Cooperative Driving	Automatically stop and go smoothly, when vehicles are in heavy roadway traffic; cooperative driving by exchanging cruising data among vehicles	
Intersection/Hidden Driveway Collision Warning	Vehicles communicate to avoid collision at intersection without traffic light or hidden driveway	
Roadway Condition Awareness	Vehicles communicate to extend vision beyond line of sight (e.g., beyond a big turn or over a hill)	

Fig. 1.4. Examples of VANETs safety applications [12].

enterprises. In general all type of applications which run on top of TCP/IP stack may fall in this group [5]. The important issue in comfort applications is that they should not interfere with safety applications.

1.4.2 Feasibility of VANET Technology

Although vehicular ad hoc networking has only recently been proposed, it has gained a great deal of attention in a very short period of time, and it is expected that its applications will appear in near future.

There are several driving forces behind this rapid evolution of VANETs. The most important driving force is that the implementation of safety applications will provide significant benefits. To illustrate the importance of such applications, we may take a look to the traffic-related fatality statistics. According to Canadian Council of Motor Transport Administration [6], annual traffic-related fatalities in 1984 were approximately 4100 deaths which have been reduced to 2927 in 2000. This improvement was the result of strict driving regulation enforcement such as alcohol consumption restrictions, employment of people awareness programs, and application of passive safety technologies such as seat belts, airbags, car seat and etc. Similar improvements can be observed in the other countries but the present statistics still are not satisfactory. For example, in the United States, there were more than 40,000 deaths and 5 million injuries in 2000. The economic loss resulting from those crashes was estimated to be more than 230 billion dollar in that year [7]. Therefore, there is an immediate need for employment of active safety technologies such as vehicular safety applications.

Besides the above motivations, the auto industry is the second driving force behind this technology which would like to equip the vehicles with them, either for providing safety, gaining extra value or implementing luxury applications. The examples are

Toyota, BMW and Daimler-Chrysler which have launched important projects for VANETs communications.

In general, feasibility of a technology is related to the economic issues. There are two mechanisms which may lead to a successful market for a technology: either a visible added value of that for the costumer or a regulative order which makes the technology a must. For a regulative order, first the effectiveness of the technology has to be proven. This requires that VANETs applications reach to a certain market penetration which it may take several years. Therefore, at the first stage, it is not feasible that a regulative order is issued based on the promised safety application. For the added value approach, a similar problem may exist as a consumer can take advantage of a technology once a certain market penetration is reached [5].

According to [5], it was estimated that in order to make the network usable, at least 10% market penetration is required which by the assumption of equipping 50% of all newly manufactured autos with VANETs' features; it will take more than 3 years to reach this objective. Further we may note that even 10% market penetration is not really sufficient for safety applications. Because, it means that a VANET equipped vehicle will not be able to communicate with 90% of vehicles on the road. In the other words, 10% of customers will pay for a service which it will take them several years to use it effectively.

To solve the above economic problem a strategic idea has been proposed which postpones the safety applications to the future. Thus, the idea is initially to introduce car to infrastructure communication (C2IC) applications which covers the area of comfort applications. In this strategy, some fixed nodes will be installed in the road sides which will provide some comfort applications to the VANETs enabled vehicles. Therefore, all the users will be able to receive some functionality right from the start. Following that, the real applications of VANETs will be made available as soon as the necessary market penetration factor is achieved [5].

1.4.3 VANET Standardization

VANET is a potential service for 4G which is expected to play an important role in future vehicular communications. It has been identified as a key technology for increasing road safety and transport efficiency, and providing Internet access on the move to ensure wireless ubiquitous connectivity. The potential of VANET technology has led to establishment of worldwide research programs and efforts. Next, we mention some of the standardization activities and research projects which have been initiatiated in Europe, US and Japan.

There are several ongoing projects in Europe. Among them, we can mention CarTALK 2000, PROMOTEChauffeur, PReVENT WILLWARN, INVENT VLA, SAFESPOT and finally FleetNet which has been extended to Car2Car and Network on Wheels projects. The problems which are being researched in the above projects are hazard warnings triggered by hazard flashers, studied within Inter-Vehicle Hazard Warning project (IVHW), cooperative driving, and other challenges such as frequency allocation, protocol definition and infrastructure deployment [5].

The Car2Car Communication Consortium [8] is a non-profit organization which aims to increase road traffic safety. Its mission is to create a standard model for inter-vehicle communication systems and propose a realistic deployment strategies as well as business models to accelerate the market penetration [5].

Network on Wheels [9] is a German research project which is funded by the German government. It aims to design different functionalities and standardization across European countries in cooperation with the Car2Car Communication Consortium [5].

In United States, there are several organizations such as Vehicle Safety Communication Consortium (VSC) which is working on the development of different standards, protocols and applications for inter-vehicle and vehicle-to-infrastructure communications since 2002. Among the standards developed for this field is Dedicated Short Range Communication (DSRC) standard which has been developed jointly by IEEE and ASTM (American Society for Testing and Materials) in 2003. In summary, this standard defines wireless communication for Intelligent Transportation Systems (ITS) applications up to 1000 meter transmission ranges at regular highway speeds. It uses a dedicated band at 5.9 GHz and it provides seven 10 MHz wide channels for different applications of which one is restricted to safety communications only [10, 11]. Fig. 1.5 shows the DSRC dedicated band with more details.

A subset of DSRC has been incorporated into IEEE 802.11 WLAN standard, and new standard is referred as IEEE 802.11p. This incorporation has resulted in some modifications in the physical and MAC layers of IEEE 802.11 standard in order to make it supportive for ITS applications. IEEE 802.11p is also referred as Wireless Access for the Vehicular Environment (WAVE[1]) [11].

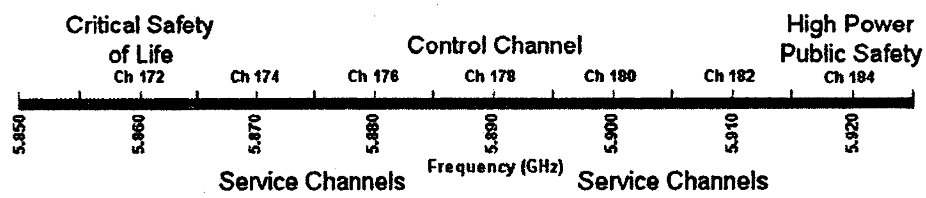


Fig. 1.5. DSRC spectrum band [11].

As another activity in USA, we can refer to Crash Avoidance Metrics Partnership (CAMP) program which is organized by the Department of Transportation Intelligent Vehicle Initiative to accelerate R&D on crash avoidance technologies [5].

In Japan, there are three main initiatives in this area which are organized by the Ministry of Land, Infrastructure and Transportation of Japan (MLIT). These initiatives are Advanced Safety Vehicle (ASV) and Advanced Cruise-Assist Highway Systems (AHS) programs which have been merged into a new program called Smartway. Smartway has several projects such as systems for communication between the road and the vehicles, and a variety of sensors, like short and midrange radars, cameras, ultrasound, infrared and others [5, 12]. An important issue in standardization of VANETs interfaces and protocols is interoperability assurance as vehicles are coming from different vendors. This requires effort to harmonize the works of different standard

[1] WAVE technology is sometimes referred to as CALM (Continuous Air Interface for Long and Medium range) [4].

organizations across the world and propose a wise deployment strategy. Otherwise, having different standards and competing systems would result in decreased market penetration and poor overall system efficiency [5].

1.5 Motivations for the Thesis

The potential of VANETs also bring many new challenges compared to traditional ad hoc networks, which need to be addressed if VANETs to succeed. A very important challenge facing the VANETs functionality is communications connectivity and path availability issues. Since in VANETs, nodes are both mobile as well as carrier of information, the network may not have full communications connectivity all the time. Therefore, the nodes may form several *clusters*, where, a cluster refers to a set of nodes that have direct or indirect communications with each other at a given time[1]. Fig. 1.6 shows a vehicle population which forms two clusters. In ad hoc networking, multi-clustering happens when the minimum distance between the nodes of two or more groups becomes more than the transmission range of a node. At the times that the nodes' population is partitioned into multiple clusters, only intra-cluster communications is possible. Thus, only if the nodes' population forms a single cluster, all the nodes will be able to communicate with each other. The knowledge of network connectivity is needed for successful design of ad hoc communication protocols both for MANETs and VANETs.

[1] In some studies the words "fragmentation" or "Partition" have been used equivalent to "cluster" which was defined here.

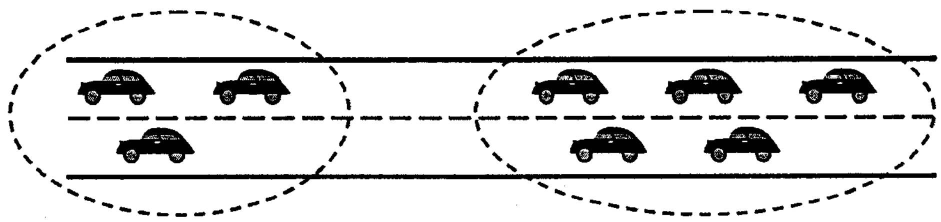


Fig. 1.6. multiple clusters in the vehicle population of a path.

Recently, researchers have addressed the network connectivity issues in ad hoc networks. For MANETs, there are only few analytical approaches which are either too complicated or have too many simplifying and restrictive assumptions which fail to capture the inherent dynamics of the system. For example, in the literature, the connectivity analysis of ad hoc networks has been mostly considered under the assumption of randomly distributed stationary nodes within the service area for both 1-D and 2-D cases. These models fail to capture the effects of user mobility on the performance of the network. Another restrictive assumption in most of these studies is that a constant number of nodes exist in the system while in a realistic situation; the population size of the nodes will be a random variable. Further, some of the results have been presented as upper bounds for the connectivity of the nodes.

More specifically, in the area of performance modeling of connectivity in vehicular ad hoc networking, even fewer analytical works exist compared to those of MANET which they will be described in the following chapter.

A question which may arise is the possibility of specializing available performance studies on MANETs to VANETs, as VANETs can be considered as a special case of MANETs in a 1-D area. The answer to this question is negative since the results in 2-D are not applicable to 1-D because of several inherent differences between them. For example, most of the studies in 2-D ad hoc networks consider mobility models such as, Random Waypoint (RWP) or Random Walk (RW) which are not appropriate for VANETs' environment, since a node's motion involves both speed and direction in RWP and RW. In RWP model, a node alternates between periods of motion and stop; clearly, vehicular movements do not follow such a characteristic. Similarly, in 1-D RW model, a node moves forward and backward at each step with probabilities of p and $1-p$ respectively which is not appropriate to model the nodes' movements in VANETs. Further, nodes in vehicular ad hoc networks experience faster mobility in comparison with the nodes in MANETs which usually have speeds in the order of human's movement with small deviations. Finally, VANETs' services, especially, the safety applications are very sensitive to delay and must meet the upper bound of 100 msec which has been proposed in the DSRC standard. These differences plus others which have been described before in section 1.3 renders adaptation of 2-D results to VANETs impossible.

The highly volatile environment of VANETs make the design of protocol stacks very challenging. The physical layer must conform to the highly variable and frequently unavailable wireless channel. Link layer must adapt itself to the variations in link error rate. MAC layer should minimize the number of collisions in accessing medium to meet the required QoS specifications. In the network layer, an effective routing protocol is needed to determine the best path without flooding or unnecessary bandwidth usage. The transport layer must be able to handle the delay and packet loss in an efficient manner.

Finally, applications need to be modified in order to cope with frequent disconnections and highly variable delay and packet loss. A successful design of such a network requires knowledge of network connectivity which may be obtained either through an analysis or simulation. Presently, there is a lack of analytical studies on the connectivity of VANETs in the literature.

1.6 Major Contributions of the Thesis

In this work, we study statistical properties of the connectivity and communication path availability of vehicular ad hoc networks with user mobility and dynamic node population size. The major contributions of this work are:

- First, we assume a single highway with unidirectional vehicle traffic. The highway is assumed to consist of concatenated segments. The segment borders are the entry and exit points of node traffics. The nodes arrive at the highway through one of the entry points according to a Poisson process. A new arriving node begins to move along the highway according to the assumed mobility model, independent of all the other nodes. The nodes depart from the highway through one of the exit points. We determine the distribution of the distance of a node from its arrival point at the steady-state. We find the distribution of node population within each highway segment by modeling each stream of nodes in the highway as a $M/G/\infty$ queue. Then these results are used to determine the statistical characteristics of a cluster seen by a new arrival or a random node. We present the average cluster size and the probability that all the nodes on the highway will form a single cluster. The probability of single cluster gives the percentage of the time that the node population will have communication connectivity with each other. We also present simulation results that confirm the accuracy of the analysis.

- Then the analysis has been extended to a network of highways. We model the system as a BCMP network of queues with state dependent service rate where segments correspond to the service centers. We obtain the joint distribution of the number of nodes in segments. Then, we model the number of clusters in a segment as a Markovian birth-death process. This allows us to determine the distribution of time that node population in a path spends in the single cluster state and mean recurrence time following the departure from this state. Thus, we derive the distribution of the time that a path in the network is continuously available for communications; as well as, mean duration of the path unavailability time for communications. These results enable the determination of the conditions under which there will be high communication path availability. Finally, mean packet delay is presented for an end to end communication in a path. We give the numerical results which illustrate the effects of mobility on communication path availability and mean packet delay. The results show that the continuous mean communication availability time of a path varies logarithmically with the transmission range. On the other hand, mean delay varies directly with the transmission range. Thus, the transmission range should not be chosen higher than what is needed for acceptable communication path availability in order to maintain an acceptable mean packet delay. In practice, an adaptive transmission range may be required to achieve the right balance between the communication path availability and delay. The results can help to advance solutions to other design problems in VANETs such as, routing algorithms, optimal node transmission range, optimization of cross layer design schemes and MAC protocols.

1.7 Organization of the Dissertation

This thesis consists of six chapters.

In chapter 2, we discuss the main challenges in the area of ad hoc and more specifically in vehicular ad hoc networking. Then, we review previous works on, connectivity and communication path availability analyses.

In chapter 3, the mobility modeling of the work will be presented. We start with reviewing the major mobility models which have been used for studying ad hoc networks in the literature. Then, we derive the statistics of our mobility model. We also find the distribution of a node's distance at the steady state which moves with the given mobility model.

In chapter 4, we define a single highway as our environment modeling. Then, the node population distributions within each segment of the highway will be derived. We study the statistical characteristics of cluster size which is seen by a new arrival or a random node in the highway. Then different performance measures such as average node population size in a cluster and the probability that all the nodes on the highway form a single cluster will be presented. We also present simulation results that confirm the accuracy of the analysis.

In chapter 5, we extend the single highway to a network of highways. The joint distribution of node populations within each segment of highways network will be derived through the application of BCMP theorem. Further, to study the network connectivity, we extend the work of chapter 4 in several directions. We define the time that the system spends in a single cluster state in a path of highway network as the conservative measure of the network availability for communications. Then, we model the number of clusters in the node population of a network path as a Markovian birth–death process. We derive the probability distribution of the number of clusters and determine the means of continuous communication path availability and unavailability times as functions of mobility and traffic arrival parameters. Mean packet delay is presented for an end to end communication in a path. We give numerical results which illustrate the effects of mobility on connectivity and communication delay.

Chapter 6 presents the conclusions and proposes some future works.

Chapter 2

Challenges of Vehicular Ad Hoc Networks

2.1 Introduction

There are many challenges facing the development and deployment of ad hoc networks. In general, these challenges originate from three factors: lack of infrastructure, user mobility and limited power sources. These factors impact the network topology, routing, end to end delay, quality of service, security and scalability. Most of these challenges also apply to the vehicular ad hoc networks. However, because of their distinct characteristics, VANETs have additional requirements. As it has been mentioned in the previous chapter, these differences are mainly due to the VANETs' mobility constraints, high speeds, service requirements, small effective network diameter, and even the behaviors of the drivers.

In section 2.2, we will discuss the main challenges and fundamental aspects of ad hoc networking paradigm. In each sub-section, if applicable, we will also describe the additional challenges encountered by the vehicular ad hoc networks.

The future success of VANETs very much also depends on the development of accurate models of these networks. Therefore, we will also discuss various modeling issues related to VANETs. In section 2.3, we discuss mobility and environment modeling issues in the analysis of ad hoc networks. In sections 2.4-2.5, an in-depth literature review of connectivity and communication path availability will be presented which are the main research problems of this study.

2.2 Challenges of Ad hoc Networks

2.2.1 Network Topology and Stability

An important challenge of ad hoc networks is self-organization and self-maintenance of the infrastructure. The solutions to these challenges should be implemented efficiently such that they are fast and transparent to the users and applications. This requires nodes to go under several topology re-organization processes to discover their neighbours. This can be performed with a periodic transmission of short packets which are called beacons, in order to detect the neighbours' activities. This enables the nodes to discover the set of nodes which are within their direct communication range and further, to determine the communication availability of the other nodes which are more than one-hop distance away. The factors which may impact the number of neighbours of a node are mobility, noise, weather, interference and transmission ranges, of which only the latter one is controllable. Therefore, nodes in ad hoc networks need an intelligent mechanism to control their transmission range for achieving the best topology organization. This mechanism is called topology control and the efficiency of a communication network depends significantly on its topology control algorithms [13]. The topology control algorithm will determine a node's optimal transmission range according to the mobility, interference, routing and energy constraints.

Stability of a specific topology is another concern [13] which may impact the transmission range selection. Stability can be affected by frequent topology changes due to mobility, failure of nodes etc. Therefore, it is also the duty of the topology algorithm to choose the optimal path in a network which lasts longer than the other paths and achieves the best communication availability and stability.

2.2.2 Routing in Ad hoc Environments

Many routing protocols have been introduced for wired and wireless networks in the literature which are not necessarily appropriate for the wireless ad hoc networks because of their dynamic topology. In general, in designing a routing algorithm, different factors should be considered such as, mobility, limited bandwidth and power resources, high error rates, hidden and exposed terminal problems. An effective routing protocol will find a path for the traffic from a source to a destination node in a dynamically changing environment and at the same time achieve quick convergence to optimal routes and provide certain level of QoS [18].

There are numerous protocols offered in the literature that each of them addresses some of the challenges. According to [13], routing protocols for MANETs may be classified based on different criteria such as routing information update mechanisms and use of temporal information for routing. Routing protocols based on the former can be further classified into three major categories: (i) Table driven (proactive) routing protocols such as, DSDV, WRP, FSR etc., (ii) On-demand (reactive) routing protocols such as, AODV, DSR etc, (iii) Hybrid routing protocols ZRP, DDR, CEDAR, ZHLS etc.

In the case of second criteria, the routing protocols use information such as lifetime of wireless links, lifetime of the selected paths and so on to make routing decisions. The protocols within this group can rely on past or future information. Extensive details on all above-mentioned ad hoc routing protocols are given in [19].

The above routing protocols have received a lot of attention during the recent years; however they are not necessarily suitable for VANETs. In vehicular ad hoc environments, application of safety communications introduces further requirements which must be addressed by the routing protocols. Safety applications require sharing traffic and environment conditions among the vehicles to support different safety applications such as, emergency warning, collision avoidance through driver assisting and lane-changing assistant. The nature of these tasks requires fast and reliable communications among the vehicles. To understand the VANETs routing requirements better, let us consider the safety applications in two different scenarios with the path between the sender and receiver vehicles being either available or not.

A path is available when intermediate nodes can be found between source and destination and an end-to-end connection can be established. In this case, the routing requirement is providing the strict latency bound of 100 milliseconds, recommended by

DSRC standard for safety applications. Therefore, the challenge is designing a routing protocol which exchanges the information among the vehicles and establishes the connection almost with a real time configuration.

In the second scenario, when the path is not available, network is not allowed to drop the forwarded message as it may belong again to a safety application. This is different from many applications which have been proposed for MANETs. A proposed solution is "carry and forward" idea, where, if no path exists, a message will be carried by a node until it can be forwarded to a node which is closer to the destination node.

The 'carry and forward' concept was considered in three type of routing algorithms which are suitable for VANETs [5]: opportunistic forwarding, trajectory based forwarding and geographic forwarding.

In opportunistic algorithms, a message is stored and forwarded whenever it is given the opportunity. In geographic forwarding algorithms, the messages are forwarded towards the destination based on node geographical location and based on a simple Cartesian distance model. In trajectory routing the road infrastructure serves as an overlay directed graph, with intersections seen as graph nodes and roads as graph edges. Messages move following predefined trajectories and distance is defined as a graph distance [5].

In this study, we do not consider routing issue directly. However, the derived analytical results for communication path availability and mean packet delay will be helpful in the design of VANET routing protocols in both of the scenarios, discussed above.

Another issue which should be considered by VANET routing protocols is storm prevention in broadcasting. The implementation of a regular broadcasting mechanism in ad hoc networks may lead to a scenario where an excessive number of packets being broadcasted in an area. This situation results in a high level of contention for the transmission medium. This problem also exists in MANETs but it is more critical in VANETs since the nodes move along a 1-D path instead of a 2-D area [20]. A survey of challenges in routing and broadcasting in VANETs are presented in [21, 22].

2.2.3 Security and Privacy Aspects

As ad hoc network topology is decentralized and without infrastructure, therefore, it presents serious security threats for communications. These threats may originate from other networks that coexist with the ad hoc networks and it may take any form such as passive eavesdropping, active impersonation, message replay and distortion [13]. As a result, communication security aspects such as confidentiality, integrity, authentication and non-repudiation have become very important research issues.

Among the topics studied for ad hoc network security are secure routing, key management and intrusion detection. It is very important that the proposed algorithms are distributed and do not rely on centralized authorities. More details on the various aspects of security in ad hoc networks may be found in [13, 23].

The security issue is much more crucial in VANETs as the information which is carried in a vehicular network can affect life-or-death decisions. For example, it is necessary to make sure that the critical information in a safety application cannot be modified, distorted or inserted by an attacker. The privacy of the drivers and passengers must be protected as much as possible and communications should not let the vehicle to be tracked or identified for non-trusted parties [24].

Trust is another issue in security of vehicular ad hoc networks which needs to be carefully assessed and addressed. Trust means whether a node can rely on a message which is received from another node. Trust establishment requires authentication, the implementation of which in real-time is a major challenge in vehicular ad hoc networks [25]. A detailed survey of challenges for VANETs security is given in [26]. Reference [27] also studies the security concerns which exist in VANETs routing protocols. The authors discuss the feasibility of implementing such protocols for vehicular environments especially when they interact with other type of networks.

2.2.4 Capacity Bounds

In mobile ad hoc networks, the nodes can be either the destination or an intermediate node which is functioning as a relay in a communication path. Each node in the network may have several channels at its disposal for communication within its neighborhood. This phenomenon can result in capacity constriction or, sometimes, communication instability. There are many studies in the literature which discuss greediness in ad hoc nodes and how the channels should be distributed such that power constraints can be met and optimum routing is achieved [28]. An important study on capacity bounds in ad hoc networks is the work presented in [29]. In that study, the authors have proposed different capacity metrics such as, transport and throughput capacities for ad hoc networks. Transport capacity is defined as the total product of bit-distance per second that can be transported by the network, and throughput capacity is defined as the maximum common throughput that can be provided to each randomly chosen node in the network. It has been shown that if n nodes are identically randomly located in a disk with area of A square meters, and each node is capable of transmitting W bit per second using a fix transmission range, then the transport and throughput capacities of an arbitrary network will be given by $\Theta(W\sqrt{An})$ and $\Theta(W\sqrt{n})$ respectively. The results of [29] have been used as a foundation by other researchers. For example, [30] and [31] have shown that ad hoc network capacity will improve in the presence of some fixed nodes as in a network with infrastructure and employment of directional antennas respectively.

[32] is the first study on ad hoc network's capacity which considers mobility of the nodes. They demonstrate that the throughput per user increases dramatically when nodes are mobile rather than fixed.

[33] and [34] are among the few works which study the capacity issue in VANETs. In [33], the authors have determined the capacity of control channel in IEEE 802.11p using simulation in vehicle to vehicle and vehicle to infrastructure scenarios. They found that the capacity is significantly reduced if the amount of roadside fixed infrastructure increases within a constant zone.

[34] states that the current IEEE 802.11p MAC protocol has deficiency in supporting safety applications when the network is dense. They propose a novel MAC protocol called VMESH. This protocol employs a distributed beaconing scheme to realize neighborhood awareness and uses dynamic channel resource reservation. Then they found the network throughput by dividing the amount of information successfully transmitted in one reservation by the duration of reservation length.

In fact, capacity study in vehicular environments is still an open issue and needs more consideration.

2.2.5 Power Management

Power management in ad hoc networks is more critical than other wireless communication networks as the nodes in ad hoc networks are used as relays in addition to being source of traffic. Power issues usually are not studied separately as the power consumption depends on other algorithms such as, routing or mobility management. Therefore, they are usually treated as performance parameters in the studies of other algorithms. According to [13], in general, there are three approaches to reduce the energy consumption in MANETs: *power save*, *power control* and *maximum life-time routing*. An example for power saving technique is to put a node into sleep mode whenever there is no packet ready for transmission. In this case, the challenge is to design an efficient mechanism to wake-up the nodes. The other details about the challenges of energy control in ad hoc networks can be found in [13].

In VANETs, contrary to MANETs, the power management is not concerned with energy efficiency, as the nodes have access to the auto power system. The main challenge in VANETs is adaptive power control [5]. It will be shown in this work that increasing the transmission range increases the communication path availability but it also increases the packet delay due to the higher contention in accessing the transmission medium. Further, a new transmission may disrupt another transmission at a distant node due to interference. Higher mean packet delay can cause serious problems to safety applications, especially in a dense traffic scene. Thus, the power management issue in VANETs usually deals with optimal assignment of transmission range.

2.2.6 Physical Layer Aspects

The main concerns in the study of physical layer of ad hoc networks are how to provide better coverage, higher spectrum efficiency and higher data rates. Therefore, many studies can be found on the physical layer topics such as, air interface, antennas and advanced signal processing techniques. Most of these studies try to improve the ad hoc physical layer performances by applying powerful coding and interleaving techniques, channel-state aware decoding, adaptive modulations and etc. An important subject in ad hoc networks is the adaptation of physical layer to different channel conditions. For example, physical layer may need to adapt to the fast changes in SNR which is caused by mobility or unpredictability of medium [4, 13].

The scarcity of bandwidth is even more challenging in VANETs' because of several safety applications. As it was mentioned earlier, two types of safety messages can be transmitted over the network; aperiodic and periodic messages. The former contains information about the environmental hazards and as a result, they require fast and guaranteed delivery. The periodic messages contain information about vehicle position, dynamics and driver intentions and they should be transmitted at a very high frequency for each vehicle. Because of the nature of safety messages, especially aperiodic ones, we also may have scenarios which produce high volume of packet traffic such as, broadcast flooding when a hazard happens in a dense traffic. The limited bandwidth problem in the VANETs can be partially addressed at the physical layer. Different approaches have been proposed in the literature, such as, decreasing the transmission power to reach a recipient in a higher number of hops, employment of directional antenna and using different channels for different types of messages based on their priority [21].

2.2.7 MAC Layer Aspects

MAC protocols in ad hoc networks are responsible for managing and controlling the access to the scarce radio spectrum and therefore to the limited available bandwidth in a fair and non-centralized way. As a result, MAC layer in ad hoc environments need to take into account the dynamic topology caused by problems such as, mobility, scalability, capability of power control, distributed functionality and hidden-exposed terminal problems [13]. The main MAC protocol which has been proposed for ad hoc networks is IEEE 802.11. IEEE 802.11 MAC layer [35] is based on CSMA/CA (Carrier Sense Medium Access with Collision Avoidance) protocol [36]. There are also a number of studies in the literature which propose the TDMA (Time Division Multiple Access) as a contention-free access scheme to the medium [37]. [38] surveys the different contention-based MAC protocols and their variations which have been proposed for ad hoc networking.

Next we will discuss the MAC challenges in VANETs. As it was mentioned before, vehicular ad hoc networks experiences faster network topology changes compared to the other MANETs. On the other hand, they have the advantages of having mobility in a predefined direction and absence of constraints in data storage and power. Another prospectus advantage of VANETs is its potential accessibility to GPS system which will enable the vehicles to obtain location information and achieve network synchronization

[4]. The above advantages and disadvantages create new challenges in the study of MAC protocols for VANETs. On one side, timing synchronization and power availability is not a problem, on the other side, efficient MAC protocols should be designed to reduce the medium access delays. There may be a need to give priority to the message traffic generated by safety applications. Until now, the VANET MAC protocol which has been receiving most attention is IEEE 802.11 and its variations [39].

Next, we will take a look to the DSRC MAC protocol, i.e. IEEE 802.11p which had been introduced in the previous chapter. IEEE 802.11p WAVE MAC protocol is not considered as a new standard but a modification of IEEE 802.11. Basically, traditional IEEE 802.11 is responsible for arranging a group of wireless nodes to establish and keep a corporative communication environment. Further it filters out all transmissions which are coming from outside the group. To join to a group, a node first listens and scans the channel for beacons from an access point and then following a number of handshaking and authentication steps, the communication will be established in a specific channel. This communicating group of nodes is called a Basic Service Set (BSS) and it is identified as the access point by some Service Set Identifications (SSID). The above mechanism is not appropriate for VANET's safety applications since the channel scanning and handshaking takes a long time. Therefore, IEEE 802.11p MAC protocol proposes a new solution which is called "WAVE mode" communication. In this mechanism, the nodes are not required to belong to a specific BSS for communication and they communicate using the same channel. Besides, they are assigned the same identification which is called Basic Service Set Identification (BSSID). It means that two nodes can communicate upon meeting each other without extra overhead and a long handshaking process [11].

2.2.8 Cross Layer Optimization

Recently, protocol design through cross layer optimization has received a lot of attention in the area of wireless network research. In the legacy layered architecture of OSI model, the protocol stack has been divided into a number of layers where each layer provides service to the layer above. This layered architecture does not lend itself to direct communications between the nonadjacent layers and the applications of each layer is optimized according to the information provided from the neighboring layers without considering the requirements of the other layers. This architecture works well with the wireline communications, but in the case of wireless communications, it results in the degradation of the network functionality and its performance. The cause of degradation is mainly related to the variable air channel quality which deteriorates further in the multi hop communication environments of ad hoc networks. In cross layer optimization, the physical and MAC layer knowledge of the wireless medium is shared with higher layers to optimize the functionality of the protocol stack [40].

There are a number of studies for ad hoc networks which employ cross layer protocol optimization. Next we mention to some of the studies related to VANETs. In [41], a cross layered approach is presented for MAC and packet routing protocol to support fast propagation of broadcast messages in VANETs. An algorithm is proposed to find the trade-off between the minimum number of hops which is required to forward a message within a cluster and stability of the connection. In [42], a new Internet access protocol for

VANETs has been introduced which optimizes the physical and MAC layers functionalities simultaneously. This optimization aims to increase the end-to-end throughput while it achieves fairness in bandwidth usage. Reference [43] introduces a cross-layer design between packet routing and MAC of VANETs in order to guarantee a special QoS requirement to the application. In that study, the MAC protocol provides the information about the neighbor node, environmental object and cluster to the routing algorithm to minimize the broadcasting delay.

2.3 Mobility and Environment Modeling

Mobility modeling is an important issue which has a significant impact on performance analysis of the ad hoc networks. It affects main performance measures such as, capacity, delay, throughput, routing and connectivity of ad hoc networks. Selection of an appropriate mobility model can significantly affects the results, both in analytical and simulation studies. Several mobility models have been proposed in the literature, which are widely used in the studies of ad hoc networks, such as, random walk, random waypoint, random direction mobility model and etc. The available mobility models can be classified based on their appropriateness for analytical or simulation studies. We describe some of these models along with the mobility model of this study in the following chapter.

In the analytical studies of ad hoc networks, the mobility models are desired which are general enough to capture the main characteristics of a realistic movement and are simple enough to formulate the steady state topology behavior [14]. However, in simulation-based studies, the models pay more attention to the details of node movements and even they can be classified based on those details. For example, in the simulation of node's mobility in VANETs, the movements are classified into microscopic or macroscopic mobility models. In macroscopic modeling, motion constraints such as, highways, streets, traffic lights, node density, traffic flows and initial location of vehicles will be specified. In microscopic modeling, the movement of each vehicle and its behavior with respect to others will be defined [15]. The importance of the choice of an adequate mobility model and its impact on the performance of the system is discussed in [16] and [17] for MANETs and VANETs respectively. The authors have shown that the mobility model plays a key role in the accuracy of analytical results.

Environment modeling in the analysis of ad hoc networks refers to the definition of the area in which the nodes are allowed or restricted to move. In ad hoc networks the environments can have different shapes. This is in contrast to the cellular communications where the nodes are usually located in a hexagonal area and they are covered by a base station. Deployment of an appropriate environment model depends on ad hoc network services. In studies of MANETs, the environments are usually 2-D, and they are in the form of a circle or rectangle with either closed or open boundaries. In the case of closed boundaries, the nodes may wrap or be absorbed in the boundaries.

In the study of vehicular ad hoc networks, the environment consists of streets or highways with predefined 1-D layouts which restrict the node movements to specific directions. The main difference between the street and highway environments is that the streets are usually considered as objects which have intersections and stopping points while highways are environments with fast mobility. In the simulation based studies, one may consider more complicated environments such as, areas with barriers or hot spots. We may note that this type of environment models is not appropriate for analytical studies because of loss of tractability. In the analysis of ad hoc networks, selection of an appropriate environment is another important issue which can directly impact the results.

2.4 Connectivity in Ad hoc Networks

The ad hoc network topology is dynamic as a result of user mobility. Since the geographical positions of users change with time, the routes within the network may break down routinely. Therefore, connectivity of ad hoc networks is an important consideration that impacts the performance of the network. In VANETs, connectivity issue is even more critical as the network topology is more dynamic because of faster motions of the nodes.

In the glossary of ad hoc networks, "connectivity" is defined as the percentage of time that the nodes in a specific path or area are connected. In the literature, the connectivity analysis of ad hoc networks has been considered mostly under the assumption of randomly distributed stationary nodes within the service area, both for 1-D and 2-D cases. As a result, those models fail to capture the effects of user mobility on the performance of the network. There are several studies in the literature which discuss the importance of mobility in the study of connectivity issue. In [44], it has been shown that performance of mobile ad hoc networks is highly sensitive to nodes mobility. The impact of mobility on the performance of routing algorithms and MAC protocols for ad hoc networks has been studied in [45],[46] respectively. In [47], the effect of mobility on maximum communication distance has been studied by using simulation. In that study, a new parameter, "pause probability" has been defined which is a function of node's speed and pause times. Their simulation results for a constant number of nodes in the network show the effect of this probability on the maximum communication distance. It was found that if pause probability is smaller than 0.5, then the maximum communication distance in the network is about 10 percent higher than that of static node model. For the values of pause probability greater than 0.5, mobility model does not affect this distance, and it is approximately equal for networks with dynamic or static nodes.

Further, majority of the studies, which include an appropriate mobility model, have used simulation as the investigation technique. While simulation has advantages of ease and flexibility, it also has some serious drawbacks. Its most important disadvantage is that the transient properties of simulation may result in wrong interpretations. In [48], simulation implementation problems have been reviewed and some solutions have been proposed. [49] presents a mathematically tractable node mobility model with constant speed, and uses it to show the dependency between mobility and connection stability.

Next, we will present a survey of the literature on connectivity. The available studies will be classified into two groups, with or without mobility model both for VANETs and MANETs.

2.4.1 Connectivity Studies without Mobility Consideration

In this group, the nodes are randomly distributed spatially but they are stationary. In [50], it is assumed that the nodes are dispersed according to a spatial Poisson process within a square area and connectivity of the network has been determined through simulation. They study connectivity based on two measures, distance and power respectively. In the distance-based connectivity model, two nodes may communicate if the distance between them is less than *d*, while in the power based connectivity, two nodes may communicate if the received power is greater than a threshold value. In [50], they have also introduced some parameters to measure scalability of the connectivity in the network. One of these parameters is cluster fragility factor, which expresses cluster robustness as a function of the number of nodes in a cluster. Another parameter, introduced in that study, is the average connectivity of the reachable nodes in h number of hops. This parameter represents the average number of times that a node is connected to another node which is h hops away. They have shown that this parameter tends to have a Gaussian distribution for the distances up to four hops.

In [51], it is stated that depending on the density of nodes in a unit area, power or distance may have more dominant effect on connectivity. In an area with a high node density, interferences and consequently received power is more important than the distances between nodes. Contrary, in a low-density area, the distance will be more dominant, as the interference will be less critical.

References [52], [53] and [54] consider a power-based model for connectivity study. It is assumed that the sender nodes $S_1, S_2\ldots, S_m$ have positions $X_1, X_2\ldots, X_m$ and the receiver node R has the position X_0. If S_i uses power P_i for transmission, the transmission from S_i to R is successful if the ratio of the signal strength to noise plus interferences from the other nodes at the location X_0 is greater than a threshold. Based on this model, an approximate expression for the capacity as a function of the maximum allowable delay has been found in [54]. It is shown that there exists a critical value for delay below which, the capacity does not benefit appreciably from the nodes movement, while above this value, the capacity will increase as a function of motion. [52] also has used this transmission model and it has proposed a routing algorithm which exploits the patterns in the mobility of nodes to provide guarantees on the delay.

In [55], a network has been considered which consists of n nodes located randomly according to a uniform distribution within a disc of unit area and node i is transmitting at a power level that has a transmission range of $r(i)$. They found the necessary and sufficient condition on node transmission range, $r(i)$, for asymptotic connectivity as $n \to +\infty$.

In [56], it is stated that random placement of n nodes within an area may result in unrealistic node concentrations; therefore, the resulting connectivity measures such as, average number of neighbors per node, can have a relatively large variance. Examples of this type of node placements are Poisson or uniformly distributed models. This study employs node dispersion with so called "Inhibition" model to space node positions more regularly. Then, this model has been used to determine the statistical characteristics of parameters such as, area coverage, number of neighbors per node and spatial distribution through simulation. It has been shown that with the new model, these parameters have smaller variances.

In [57], a distance-based model has been considered and a mathematical analysis has been presented for the link distance distribution. The nodes are assumed to be spatially dispersed according to uniform or Gaussian distributions within a rectangular area. In that study, only link distance distribution between two nodes has been studied under the independence assumption from the location of other nodes in the network. Their result applies only to one hop connectivity since in the case of multi-hop connectivity, the location of relay nodes must also be included. Therefore, link distance distributions are not independent of intermediate nodes' location. For example, if node i communicates with node j through node k (two-hop connectivity), their connectivity probability is not independent of node k location. Hence, joint distribution of distances between three nodes is required.

In [58], the case of two-hop connectivity between terminals with identical location distribution has been studied. The resulting joint distribution is given in a general form, which is suitable for computation of connectivity of nodes in a network of randomly dispersed nodes. Then, the joint distribution of link distances for more than two-hop connectivity and its complexity has been discussed.

An important theory, which gives some interesting results on connectivity, is Percolation. This theory may be described by assuming that a large porous stone is immersed in a bucket of water. The problem is finding the probability that the center of the stone is wetted. A two-dimensional model of this stone may be shown as in fig. 2.1. In this figure, the vertices and edges are showing the holes and inner passageways of the stone respectively. Each edge may be open with probability p and close otherwise, independent of all other edges. On immersion of the stone in water, a vertex x inside the stone is wetted if and only if there exists a path from x to some vertex on the boundary of the stone, using open edges only [59].

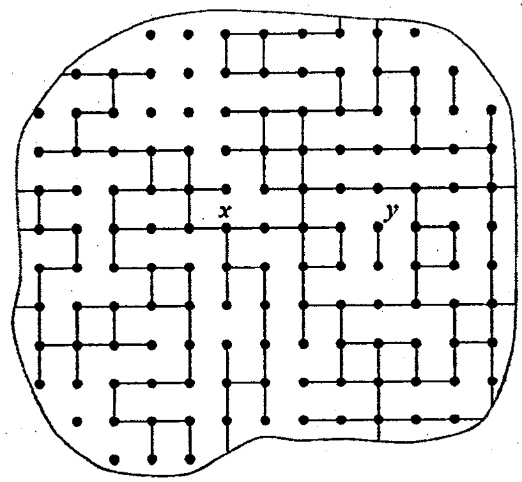


Fig. 2.1. A sketch of the structure of a 2-D porous stone. The lines indicate the open edges. On immersion of the stone in water, vertex x will be wetted by the invasion of water, but vertex y will remain dry [59].

Percolation theory is concerned primarily with the existence of such 'open paths'. The probability that a vertex near the center of the stone is wetted by water permeating into the stone from its surface will behave rather similarly to the probability that this vertex is the end-vertex of an infinite path of open edges. That is to say, the large-scale penetration of the stone by water is related to the existence of infinite connected clusters of open edges. The simulations show that the connected clusters of open edges are isolated and rather small for small values of p. As p increases, the sizes of clusters increase also, and there is a critical value of p, at which there forms a cluster, which pervades the entire picture. In loose terms, as we throw in more and more open edges, there comes a moment when large-scale connections are formed across the lattice [59].

If we consider the holes as nodes and the edges as connection links, in network with unlimited number of nodes, each pair of nodes may have a connection, which is assumed to be independent of all other connections in the network, with probability p. The network may consist of a finite number of clusters with cluster size increasing with probability p. There is a critical value of p, above which all nodes form a single infinite cluster with probability one. In [51], this theory is studied for ad hoc networks. It is assumed that the nodes are dispersed according to a Poisson distribution within a square area. Two cases have been considered which are 1-D and finite 2-D network topologies for VANETs and MANETs respectively. It has been shown that in finite networks, there exists a critical node density (λ_c) instead of critical connection probability. In the range of sub-critical density defined as $\lambda < \lambda_c$, all clusters are bounded surely, while in the supercritical density, defined as $\lambda > \lambda_c$, there exists a unique large cluster to which most of the nodes belong. They also employed Percolation theory for 1-D as a special case to study of connectivity of hybrid networks with some infrastructure fix nodes operating in the network as relays. They have shown that the employment of the fix nodes in VANETs improves the network connectivity drastically while in MANETs the introduction of fix nodes does not significantly increase the connectivity. Besides, in VANETs, the optimal distance between fix nodes on the path which guarantees a specific connectivity probability grows exponentially with the node population density.

Next we continue to review the works on connectivity of VANETs. [60] studies connectivity of a source-destination pair in a VANET. They determine the probability that the nodes in a network with length L forms a single cluster given that a certain number of nodes are randomly and uniformly placed between the source and destination nodes. Further, they have found a formula which shows the relationship between the number of nodes and network connectivity probability.

[61] presents an exact analytical formula for the probability that a VANET with the length L and n stationary nodes composed of at most c clusters. This formula is in a series form and it presents the probability of a single cluster as corresponding to $c=1$.

[62] studies the connectivity probability of a VANET in which the nodes are randomly and non-identically distributed. They used a Gaussian distribution with different means for the nodes' locations and discussed the scenarios where the vehicles are not necessarily uniformly distributed. They have used a curve fitting techniques to optimize the number of nodes required for a certain connectivity probability.

2.4.2 Connectivity Studies with Mobility Consideration

Next, we discuss previous work on connectivity analyses which include user mobility. In [63], the connectivity of an ad hoc network with the random waypoint mobility model for MANETs has been analyzed. In that study, the motions of n nodes within a unit circle have been considered and an approximation for the probability of the nodes forming a single cluster has been derived.

In [64], the connectivity of a 1-D ad hoc network has been studied both with nodes mobile according to random waypoint mobility model and with stationary nodes uniformly distributed in a line. Their interesting result is that when the node population density is high, study of connectivity with a stationary model is a good estimation of real connectivity while when the density is low, the results differ from each other. The study assumes random waypoint mobility model because of its mathematical tractability properties. However, as it was discussed before, it is not an appropriate mobility model for the nodes in VANETs. It has been shown in [65] that the random waypoint produce significantly different node connectivity results in VANETs compare to more realistic mobility models through a simulation.

In [66], simulation is used to determine the minimum transmission range (MTR) that guarantees the network connectivity of VANETs at different traffic densities and road conditions. They have shown that the value of MTR decreases as vehicle density increases up to a critical density and then it remains constant or increases slightly. Therefore, vehicle density increase is not always results in better connectivity.

[67] proposes a new algorithm for optimal transmission range assignment problem. In this algorithm, the vehicles estimate the local node density and traffic condition within the highway through the vehicle speed characteristics. The density estimation is used to assign the transmission range accordingly. They have shown that this algorithm is successful in maintaining the connectivity in highly dynamic networks.

[67] also studies the problem of minimum transmission range in 1-D and 2-D regions for both cases of mobile and stationary nodes. For stationary node scenario, they have derived some bounds for the critical transmission range. Further, they have found that, in 2-D ad hoc networks, reduction of the transmission range from the MTR value results in progressive increase of cluster size, while in 1-D, a modest decrease on the transmission range over the minimum required for connectivity can cause the formation of several

clusters faster and in relatively small sizes. In the mobile scenario with random waypoint mobility model, they study the connectivity from the energy saving point of view through simulation. They have shown that considerable energy savings can be achieved if temporary disconnections can be tolerated in VANETs. Their results show a trade off between energy saving and connectivity percentage.

All these studies assumed a constant number of nodes in their model while in a realistic situation, population size of the nodes will be a random variable. Further, most of them use simulation as a tool for their analysis. In [68], we have studied the connectivity of two-dimensional ad hoc networks with dynamic user population which has demonstrated the significance of user mobility.

2.5 Communication Availability in Ad hoc Networks

Mobile ad hoc networks have dynamic topologies. Because of the node's random mobility, the period of continuous communication availability between two nodes will also be a random variable. In ad hoc networks "communication availability", also called "path availability", "link availability" or "path reliability" refers to the distribution of the time that a communication path is continuously available between two nodes. This definition may be considered as an extension of "link availability" in traditional wireless networks. Link availability is defined as the probability that a wireless link between two mobile nodes remains available at time $t+t_0$, given that the link exists at time t_0.

Connectivity studies which have been presented in the previous section mainly aim to find the percentage of the time that the network will have communications availability.

Therefore, those connectivity parameters are not able to give information regarding the frequency and duration of the visits to the single cluster state. Clearly, the state of the network alternates between communication availability and unavailability. Knowledge of these durations or equivalently, the prediction of connectivity life time will be significant measures in optimization of the performance of ad hoc protocols. In the following, we discuss the works on communication availability in the literature. First, we present the work on the communication availability in MANETs.

In [69-71], the problem of selecting reliable paths that can last as long as possible have been studied. They introduce a prediction-based link availability estimation to measure the path reliability. The estimation is based on predicting a time T_p that a currently available link will last from t_0 by assuming that the two nodes associated with the link keep their current speed and direction unchanged during T_p. Then, they estimate the probability that this link may last to $t_0 + T_p$, by considering possible velocity changes that may occur between t_0 and $t_0 + T_p$. These studies assume a constant number of nodes in the service area.

In [72], they have derived a statistical lower bound for path availability in ad hoc networks with random walk mobility model. [73] has considered the problem from the routing reconstruction point of view. The study has tried to predict the topology changes such that route reconstruction can be done ahead of topology change, so the amount of data loss due to the latency of route reconstruction is minimized.

In [74], a queuing network model has been used to find the probability distribution of the time for path availability. The model uses the data generated from various mobility scenarios through simulation to determine an appropriate distribution for connectivity between two nodes. They also study the effects of velocity, area and pause time on link availability.

In [75], an analytical framework has been developed for calculating the probability of link change. The effect of node's maximum speed on this probability has been studied.

In [76], a new 'Link availability ratio' has been introduced and the effects of different parameters such as, traffic, topology and mobility on the network throughput have been studied.

In [77], an efficient broadcast scheme in mobile ad hoc networks has been studied by determining a small set of forward nodes to ensure full coverage. It has been determined that full coverage is guaranteed if three sufficient conditions, connectivity, link availability, and consistency are met. They have studied the connectivity condition by proposing a minimal transmission range that maintains the connectivity in the virtual network constructed from local views. They also assume another transmission range for neighborhood information collection to form a buffer zone that guarantees the availability of links in the physical network. Finally, they have proposed a mechanism called aggregated local view to ensure consistent local views.

Next, we consider studies that address the path availability in VANETs, which are only few. Reference [78] studies the stability of a path in a VANET and they proposed a scheme to improve the durations of connectivity periods. They group vehicles according to their moving directions, speed range and prospective available digital mapping of roads. Vehicles are grouped according to their velocity vectors. This grouping ensures that the vehicles which belong to the same category are more likely to establish stable single and multi-hop paths as they are moving together. Their proposed stability algorithm, called Receive on Most Stable Group-Path (ROMSGP) is responsible to choose the most stable route using the above scheme. This algorithm decides the most stable link based on computation of the link expiration time of each path. The path which has the highest link expiration time is considered to be the most stable one.

In [79], the authors study the reliability of an established path between a source and a destination node in VANETs. It is assumed that n nodes which are uniformly located between a source and a destination initially form a single cluster and a route exists between them in h hops. They assume that the relay nodes move with a uniform speed in the range of $(0, m)$. Then, they found the average duration of time that it takes for this route to break up. First, they evaluate the individual link reliability by modeling the period of time that a link between two adjacent forwarding nodes remains connected. Then, the reliability of the path between source and destination nodes has been found by adding all possible losses of connectivity among the pair of nodes within the established route. In that study, it is assumed that the source and destination nodes are stationary.

Reference [80] is an experimental study which considers the path availability problem in VANETs for real time communication. The authors evaluate the performance of video streaming in inter-vehicular environments using the 802.11 ad hoc network protocol in two environments of highway and streets. They measured some performance parameters in the presence of link breakdowns for different packet sizes.

Again, all these studies assumed a constant number of nodes in their model while in a realistic situation, population size of the nodes will be a random variable. In chapter 5, we present an analytical study of communication availability between two mobile nodes in a predefined path with random node population size.

2.6 Summary

In this chapter, we have presented the major challenges in the study of ad hoc networks and more specifically in VANETs. We highlighted challenges in VANETs which are mainly due to their mobility constraints, high speeds and even driver's behaviors. Then, we have presented an in depth literature review of the main research topics of this thesis, connectivity and communication availability.

Connectivity and communication availability of ad hoc networks, either in MANETs or VANETs depend on the motion of the nodes, node's speed and movement directions which vary both in space and time. We have shown that though a significant amount of previous work in ad hoc networks exists, only few of them use analytical approaches which include an appropriate mobility model. Further, although VANETs are ad hoc networks in 1-D environment, their performance cannot be evaluated by specializing the results of 2-D MANETs because of the inherent difference between the two environments.

Connectivity and communication availability are two major performance measures which should guide the design of reliable and high performance protocols and applications. They also impact other performance measures such as, network delay and throughput.

Chapter 3

Mobility Modeling

3.1 Introduction

The performance modeling of VANETs is important for their successful development and deployment. The development of protocol stack and routing algorithms require accurate models of VANETs. However, successful modeling of VANETs heavily depend on user mobility model.

In this chapter, first we review the mobility models which have been used in the performance studies of ad hoc networks. We discuss their suitability for modeling user mobility and their analytical tractability. Then we present the mobility model of this study and derive its performance statistics. We show that this mobility model is general enough to capture the main characteristics of a realistic node's movements in a highway and applicable in a macroscopic scale. Further, it is analytically tractable and simple enough to determine the steady state behavior. The results to be derived in this chapter will be used in the following chapters in the analysis of connectivity and communication path availability in VANETs.

3.2. Mobility Models

In the studies of ad hoc networks, deployment of an appropriate mobility model is an important factor on accuracy of the results. The more a model mimics the movements of a mobile user, the more accurate will be results. For example, some models are able to capture changes in speed and direction while others may only capture direction's changes.

The mobility models are classified into two, those suitable for capturing movements of a single user and the other movements of a group. Among the important single user mobility models are random walk, random waypoint, random direction, Gauss-Markov and city section mobility models such as, Manhattan model. Some examples of group mobility models are: Exponentially correlated random mobility model, Column mobility model, Pursue mobility model and Reference point group mobility model [81, 82]. In another classification, mobility models are divided into two groups, those suitable for MANETs and the other ones for VANETs. In mobility models for MANETs, the nodes usually move in a 2-D area while in VANETs, they move in a 1-D environment or a geographically restricted area. In 2-D models, the nodes' movements are considered within a closed region such as, a rectangle or disc, and they either bounce off (reflection), or are absorbed at the boundaries of the region. Absorption corresponds to the departure of a node from the area [83]. Next, we will describe some of the more significant of these models which have been used in the literature frequently.

3.2.1 Random Waypoint Mobility Model

In random waypoint mobility model, a node chooses a uniformly distributed destination within the unit disk, as well as a uniformly distributed speed. The node moves in the direction of the destination with the chosen speed until the destination is reached. Then, it pauses at the destination for a random duration of time according to a probability distribution. At the end of the pause, a new journey begins with a different destination and speed and the process repeats itself. This model is able to capture random movement, speed and pauses [81]. The simulation of the random waypoint and its statistical characteristics, pose a surprising number of challenges, such as, speed decay, changes in the distribution of location and speed as the simulation progresses [84]. In [48], the disadvantages of this model have been discussed and some solutions have been proposed. Among the disadvantages is that the random waypoint model takes a long time to reach to steady state. Further, it suffers from density wave effect which is the clustering of nodes in one part of the simulation area. In the case of the random waypoint mobility model, this clustering occurs near the center of the simulation area. Another deficiency of random waypoint mobility model is that the number of neighbors of a node may change rapidly during a simulation. This problem produces high variability in the performance results [84]. The random waypoint is still a widely used mobility model in the performance analysis of MANETs. [63] and [84] are examples of studies which investigate the node location distribution based on random waypoint model in 2-D area. In [64], the location distribution has been derived for the nodes which move in 1-D area with this mobility model. Although this mobility model has been used by a number of works to investigate the performance of VANETs, it does not mimic the nodes movements in a realistic way. In [85], this deficiency has been studied carefully through a simulation approach. In that work, a software tool, called MOVE, has been presented which generates realistic node's movements for VANET simulations. The authors have shown that the simulation results, when the nodes are moving according to MOVE are significantly different from that of the random waypoint model.

3.2.2 Random Direction Mobility Model

The random direction mobility model was created to overcome density waves in the average number of neighbors produced by the random waypoint mobility model. Under the random direction model, a node chooses a new direction and speed following a random time interval [81]. Upon reaching to the simulation boundary, a node pauses for a fixed amount of time and continue to move based on a predefined rule. Two versions of this mobility model in a closed area have been considered which are random direction with wrap around and random direction with reflection. In the first one, when the mobile node hits the boundary, it reappears at the opposite boundary with the same direction, while in the second one the node is reflected at the point of impact with reverse direction [81]. In [48] properties of random direction model has been studied and its statistical characteristics such as, the stationary distribution of location and direction has been given.

3.2.3 Uniform Mobility Model

This model is introduced in [52] and it is similar to the random direction model. It has the advantage of being analytically tractable. In this model, each of the m nodes moves at speed v inside the unit circular disk. Initial positions of these nodes are chosen to be uniformly distributed within the disk. The initial direction of motion is also chosen to be uniformly distributed in $(0, 2\pi)$. Between two consecutive movements, there is a uniformly distributed pause time. This model reflects the nodes hitting the boundary.

3.2.4 Modified Random Direction Mobility Model

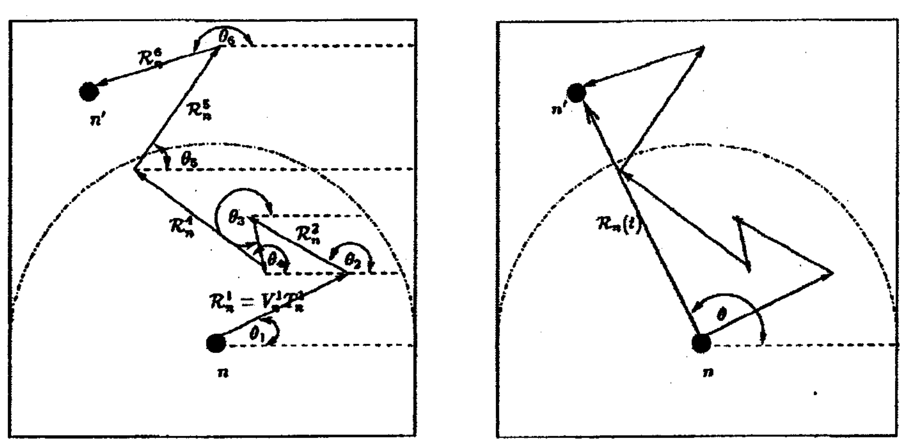
In [86], a mobility model has been introduced which is similar to the random direction mobility model. Based on this model, each node movement consists of a sequence of random length intervals called mobility epochs during which a node moves in a constant direction at a constant speed. The speed and direction of each node varies randomly from epoch to epoch. The epoch durations for node n are i.i.d exponentially distributed with mean $1/\lambda_n$. The direction of movement in each epoch is i.i.d. with uniform distribution over $(0, 2\pi)$. The speed of node n is also i.i.d. with normal distribution and has mean μ_n and variance σ_n^2, which are constant during an epoch.

The number of epochs for node n during an interval of length t has Poisson distribution with parameter λ_n given by $G_n(t)$. $\vec{R}_n^i$ is defined as the epoch random mobility vector and represents the direction and distance traveled by node n during mobility epoch i. It has magnitude $|\vec{R}_n^i| = V_n^i T_n^i$ where V_n^i and θ_n^i are speed and direction of node n during epoch i which has duration T_n^i. A sample path containing six epochs is shown in fig. 3.1a. The distance of node n is denoted by vector $\vec{R}_n(t)$ which represents the relative displacement of node n after t second with respect to the origin. The cumulative distance vector can be expressed as a random sum of the epoch mobility vectors, $\vec{R}_n(t) = \sum_{i=1}^{G_n(t)} \vec{R}_n^i$.

The distance vector corresponding to the epoch mobility vectors of fig. 3.1a is shown in fig. 3.1b. In [86], it has been shown that $|\vec{R}_n(t)|$ has a Raleigh distribution,

$$\mathrm{Prob}(|\vec{R}_n(t)| \le r) = 1 - \exp(\frac{-r^2}{\alpha_n}), \qquad 0 \le r \le \infty$$

where $\alpha_n = (2t/\lambda_n)(\sigma_n^2 + \mu_n^2)$. As it will be explained in section 3.3, modified random direction mobility model provides a framework for the mobility model of our study.



(a) (b)

Fig. 3.1. Modified Random direction mobility model: (a) epoch random mobility vectors (b) Random mobility vector of node n [86].

3.2.5 Random Walk Mobility Model

This is one of the oldest and simplest of the discrete time mobility models. The continuous time version of this model is Brownian motion [87]. Random walk may be defined in 1-D, 2-D or 3-D spaces. In one-dimensional random walk, an object moves randomly one unit to the left or to the right with probabilities p and $1-p$ respectively at each time step [88, 89]. The traveled distance in each time step could be fixed or variable. In two-dimensional random walk each node chooses a direction to north, south, east or west at random and moves some distance in that direction [90, 91]. The two-dimensional random walk model may also be expressed in polar coordinates by choosing the speed and direction in each segment to be uniformly distributed within the range [speedmin, speedmax] and [0, 2π] respectively. Each movement in the random walk corresponds to either a constant time interval t or a constant travel distance d [81]. This definition of random walk is same as random waypoint model if the pause time between the travel segments in the latter is set to zero. In [48] it is stated that the random walk model does not experience the speed decay and density wave problems associated with random waypoint model.

As a simple model, the analysis of random walk is easier but it has some drawbacks as a mobility model. In [92], it is stated that its memoryless and complete randomness makes it unrealistic as a user mobility. For example, the movements of vehicles and people are more or less restricted into paths such as, streets; therefore they are not completely random. Nevertheless, random walk model has been used frequently in performance studies of wireless networks, both in cellular and ad hoc networks [93-95].

3.2.6 Graph Based Mobility Models

In [92], the authors introduced a graph based mobility model to compensate the drawbacks of the random walk model. This model uses a graph to model the movement constraints imposed by the infrastructure. The vertices of the graph represent locations that the users might visit and the edges the links between these locations. In this model, each mobile node is initialized randomly at a vertex in the graph and moves towards another vertex, which is selected randomly as its destination. It has been stated that this model provides a realistic balance between completely deterministic and completely random mobility models. Then, this model was compared with the random walk model by using three important routing algorithms in ad hoc networks, i.e., DSDV, DSR and AODV through simulation. Simulation results show that the spatial constraints in the real world have a significant impact on the performance of ad hoc routing.

A graph based mobility model has been introduced in [96] for VANETs. This simulation based mobility model, called VanetMobiSim is capable of producing realistic vehicular mobility traces for several network simulators. Although these models solve the problem of memorylessness in the random walk, it loses the analytical tractability of that model.

3.2.7 The Stop Sign and Traffic Sign Mobility Models

In [97], two mobility models have been introduced which are suitable for simulation study of VANETs. These mobility models are able to capture various vehicle movements in the city streets. The Stop Sign Model (SSM) and Traffic Sign Model (TSM) simulate the vehicle's mobility behaviour in the presence of stop signs and traffic lights respectively. In SSM, each vehicle waits for a constant period of time, when it arrives to an intersection and then it continues. SSM captures the dependency among the vehicles mobility and their waiting times at intersections.

In TSM, the vehicles can pass or stop at a traffic light with a certain probability and when they stop; their stopping time is random according to a probability distribution. Further, when a vehicle stops, all other ones which arrive to the intersection behind the current vehicle also stop and they remain at standstill until it proceeds. SSM and TSM models capture the vehicle line up at the road intersections which is called clustering effect. The authors believe that the clustering effect at the intersections impacts the performance of routing protocol significantly. Later, using the offered mobility models in a simulation, the authors evaluate different VANETs performance measures such as, delay and throughput at [17]. Although these models pay more attention to the details of vehicular movements, they are not appropriate for analytical studies as they are not mathematically tractable.

3.3 Mobility Model of This Study

In this work, we use an adaptation of the modified random direction model which was introduced in section 3.2.4 to 1-D as a mobility model. In the adapted model, the direction of movement remains constant all the time.

Similar to the modified random direction mobility model, the movement of each node as a function of time consists of a sequence of random intervals called mobility epochs. The epoch durations of each node are i.i.d exponentially distributed with mean $1/\beta$.

During each epoch a node moves at a constant speed chosen independently from a normal distribution with mean μ and variance σ^2. Later on, we also use a state dependent mean speed $\mu(h_{ji})$ where h_{ji} denotes the state of the system. This model has interesting characteristics which make it appropriate for analytical studies of vehicular ad hoc networks.

Next, we will derive probability density function of the distance travelled by a node as a function of the time. Let t_{i-1} denote the beginning time of the i'th epoch, $i \geq 1$, for a node and refer to it as renewal epoch with the assumption that $t_0 = 0$. Since epoch durations are exponentially distributed, the number of renewal epochs over any time interval has a Poisson distribution. We let T_i denote the duration of the i'th epoch and x_i the amount of distance covered by a node during that epoch.

Fig. 3.2 shows the distance of a node from its arrival point to the network as a function of time. Then, we have the following results regarding the statistics of renewal epochs and epoch durations,

Lemma 3.1: Given that there are n epochs during the interval (0, t), the joint pdf of renewal epochs, $t_i, i = 1..n-1$, $0 \leq t_1 \leq .... \leq t_i..t_{n-1} \leq t$, is given by $(n-1)!/t^{n-1}$ [98].

The duration of the i'th epoch is given by,

$$T_i = \begin{cases} t_i - t_{i-1} \ , & 1 \leq i < n \ , \ t_0 = 0 \\ t - t_{n-1} \ , & i = n \end{cases} \tag{3.1}$$

Then, the j'th moment of the random variable T_i is given by,

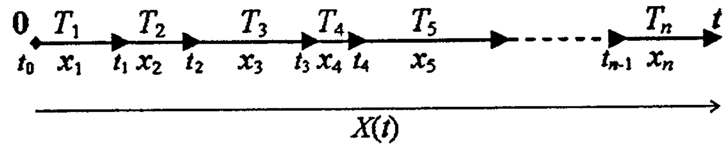


Fig. 3.2. Distance of a node from its arrival service point as a function of time.

$$E(T_i^j \mid n\text{ epochs}) = \frac{(n-1)!}{t^n} \int_{t_{n-1}=0}^{t_n} .. \int_{t_i=0}^{t_{i+1}} .... \int_{t_1=0}^{t_2} (t_i - t_{i-1})^j \, dt_1 ... dt_i ... dt_{n-1}\,, \; 1 \le i \le n, n \ge 2\,, \; t_0 = 0, t_n = t\,. \tag{3.2}$$

After evaluating the first two moments for the few initial values of the number of epochs, we conclude that,

$$E(T_i \mid n\,epochs) = \frac{t}{n}, \quad 1 \le i \le n\,, \tag{3.3}$$

$$E(T_i^2 \mid n\,epochs) = \frac{2t^2}{n(n+1)}, \quad 1 \le i \le n\,. \tag{3.4}$$

Let us define $X(t)$ as the distance of a node as a function of time from its arrival point to the network. $X(t)$ will be denoted by $X_n(t)$ given that there are n epochs during the time interval (0, t), and by $X_{n,T_i}(t)$ if epoch durations are also constant. Then, $X_{n,T_i}(t)$ will be given by $X_{n,T_i}(t) = \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} V_i T_i$ where V_i is the speed of a node during i'th epoch. Since, $X_{n,T_i}(t)$ is given by a linear sum of i.i.d normal random variables with constant coefficients, it will have a normal distribution with mean and variance given by,

$$\varepsilon_{X_{n,T_i}(t)} = E[X_{n,T_i}(t)] = \mu \sum_{i=1}^{n} T_i\,, \tag{3.5}$$

$$\theta_{x_{n,T_i}(t)} = \mathrm{var}[X_{n,T_i}(t)] = \sigma^2 \sum_{i=1}^{n} T_i^2 . \tag{3.6}$$

The mean and variance of $X_n(t)$ will be given by,

$$\varepsilon_{x_n(t)} = \mu \sum_{i=1}^{n} E[T_i], \tag{3.7}$$

$$\theta_{x_n(t)} = \sigma^2 \sum_{i=1}^{n} E[T_i^2]. \tag{3.8}$$

Where corresponding expected values are given by (3.3) and (3.4), therefore,

$$\varepsilon_{x_n(t)} = \mu \sum_{i=1}^{n} \frac{t}{n} = \mu t, \tag{3.9}$$

$$\theta_{x_n(t)} = \sigma^2 \sum_{i=1}^{n} \frac{2t^2}{n(n+1)} = \frac{2\sigma^2 t^2}{(n+1)}. \tag{3.10}$$

Finally, the mean of $X(t)$ is given by,

$$\varepsilon_{x(t)} = \sum_{n=1}^{\infty} \varepsilon_{x_n(t)} \Pr(n \text{ epochs in } (0,t)) \tag{3.11}$$

$$= \sum_{n=1}^{\infty} \mu t \frac{(\beta t)^{n-1} e^{-\beta t}}{(n-1)!} = \mu t \sum_{n=0}^{\infty} \frac{(\beta t)^n e^{-\beta t}}{n!}$$

$$\varepsilon_{x(t)} = \mu t, \tag{3.12}$$

and its variance by,

$$\theta_{x(t)} = \sum_{n=1}^{\infty} \theta_{x_n(t)} \Pr(n \text{ epochs in the interval } (0,t)) \tag{3.13}$$

Substituting (3.10) in (3.13), we have,

$$\theta_{x(t)} = \sum_{n=1}^{\infty} \frac{2\sigma^2 t^2}{(n+1)} \frac{(\beta t)^{n-1} e^{-\beta t}}{(n-1)!} = \frac{2\sigma^2 t^2}{\beta^2 t^2} \sum_{n=1}^{\infty} \frac{n(\beta t)^{n+1} e^{-\beta t}}{(n+1)!}$$

Using the substitution $m = n+1$, we have,

$$\theta_{x(t)} = \frac{2\sigma^2}{\beta^2} (\sum_{m=2}^{\infty} \frac{(m-1)(\beta t)^m e^{-\beta t}}{m!}) = \frac{2\sigma^2}{\beta^2} (\sum_{m=0}^{\infty} \frac{(m-1)(\beta t)^m e^{-\beta t}}{m!} + e^{-\beta t})$$

$$\theta_{x(t)} = \frac{2\sigma^2}{\beta^2} (\sum_{m=0}^{\infty} \frac{m(\beta t)^m e^{-\beta t}}{m!} - \sum_{m=0}^{\infty} \frac{(\beta t)^m e^{-\beta t}}{m!} + e^{-\beta t})$$

The first sigma presents the mean of Poisson distribution and the second one is the sum of probabilities for all values of m which is one, therefore,

$$\theta_{x(t)} = a_c(\beta t - 1 + e^{-\beta t}), \tag{3.14}$$

where $a_c = \frac{2\sigma^2}{\beta^2}$.

We note that central limit theorem is not applicable to the sum of epoch durations as the number of epochs in (3.7) and (3.8) is a random variable itself and not infinity. Therefore, we will assume that the random variable $X(t)$ will still have a normal distribution with the mean and variance given by (3.12) and (3.14) respectively. The accuracy of this assumption will be shown by simulation shortly. Letting $b_{x(t)}(y)$ and $B_{x(t)}(y)$ denote the pdf and cumulative distribution function (CDF) of $X(t)$,

$$b_{x(t)}(y) = \frac{1}{\sqrt{2\pi\theta_{x(t)}}} e^{-(y-\varepsilon_{x(t)})^2 / 2\theta_{x(t)}} \tag{3.15}$$

$$B_{x(t)}(y) = \Pr(X(t) \le y) = \int_0^y b_{x(t)}(\tau)d\tau . \quad (3.16)$$

From (3.12) and (3.14), the mean and variance increase as the time increases. As a result, the pdf shifts to the right, its maximum value shrinks and gradually becomes more flat. Thus the pdf of the distance of a node from its arrival service point approaches to a uniform distribution as the time increases.

The above assumptions are verified by simulation. We have assumed that a node starts to move at time $t = 0$ with the given mobility model in a straight path. The values of mobility parameters have been set to $\beta = 1$, $\mu = 25$ m/sec and $\sigma = 3$. Fig.3.3 shows the simulation and analytical results for the pdf of the distance of a node at three instances, following its arrival to the path with length 10,000 meters at time t=0. The solid lines

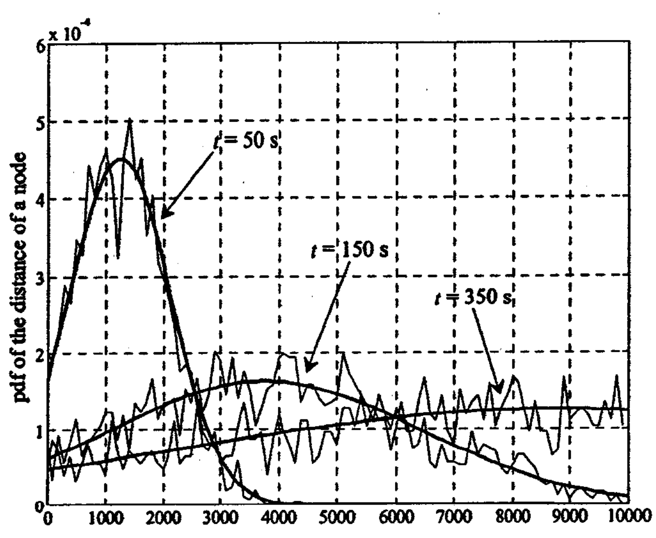


Fig. 3.3. Probability density function of the distance of a node upon starting movement in a path at three different time values of t=50, 150 and 350 s from analysis and simulation for

correspond to analytical result given by (3.15) while the zigzag lines present the simulation results. In this discrete-event simulation, we have recorded the locations of different arrivals to the beginning of path after three time instances 50, 100 and 150 sec. Description of the simulation program of this study is given in appendix A. As may be seen, for each time instance, the node location distribution from simulation follows the normal curve given by (3.15). This justifies our assumption that the random variable $X(t)$ will still have a normal distribution with the mean and variance given by (3.12) and (3.14) respectively. Further, comparing the densities at different time instances, even at t = 50 sec, the peak of the pdf is less than 5×10^{-4}. As the time increases the pdf becomes more flat approaching to a constant value of 10^{-4}. This constant value is the inverse of the length of the path. These results confirm our guess that the pdf of the distance of a node approaches to a uniform distribution.

3.4 Summary

In this chapter, first we have reviewed the mobility models which have been used in the performance studies of ad hoc networks. We have discussed these models in terms of their analytical tractability and their suitableness for different criteria such as 1-D and 2-D movements or analytical and simulation approaches. Then, we presented the mobility model of this study which is an adaptation of modified random direction mobility model to 1-D space. The model may capture the vehicles' movements in network of highways.

3.4 Summary

With the given mobility model, we have shown that the node's location follows a normal distribution as a function of time. Further, it was shown that at the steady state, this pdf approaches to a uniform distribution.

Chapter 4

A Performance Modeling of Connectivity in a VANET in a Single Highway Environment

4.1 Introduction

In this chapter, we study the statistical properties of the communications connectivity of the nodes with user mobility at the steady state. In section 4.2, the environment model of the work will be presented. We define a single highway model as a unidirectional path with multiple lanes. The nodes arrive to the highway through one of the entry points, called service points according to a Poisson process. Then, the nodes begin to move along the highway according to the assumed mobility model independent of all the other nodes. The nodes depart from the highway through one of the exit points. We note that this model makes the size of node population within the highway a random variable as in real scenarios. We introduce partitioning of the highway into segments and define traffic arrivals in section 4.2. Then, we determine the population size of the nodes in the highway segments.

As it was mentioned before, we let 'cluster' to denote the set of nodes in a highway that have connectivity with each other at a given time. This means that all the nodes in the cluster may communicate with each other either directly or indirectly. Clearly for this, necessary but not sufficient condition is that each node has at least one neighbor that it can communicate directly. If no direct or indirect communication is possible between the nodes of two clusters, then, these clusters are non-communicating. The mobile user population may consist of several non-communicating clusters at a given time. The size and membership of clusters change as a function of time due to user mobility. Clearly, the ideal topology is to have a single cluster all the time; however this may not be possible in practice. Thus, we may take the amount of the time that node population spends in the single cluster state as the conservative measure of communication path availability.

In section 4.3 we present the network connectivity analysis. The objective is to study the statistical characteristics of a cluster seen by a cluster-head where a cluster-head refers to a node which we are interested to measure its connectivity. In this study, we take both a new arrival and a random node in the highway as cluster-heads.

Then, we present the distribution of the number of clusters, average node population size in the cluster and the probability that all the nodes on the highway will form a single cluster. We will show that our cluster analysis is not restricted to the applied mobility model.

4.2 The Single Highway Model

In this model, we assume a unidirectional highway with multiple lanes and length of R meters. The model allows vehicles passing of each other. We assume that the highway consists of a number of concatenated segments. The beginning and end of each segment serves as a service point which allows entry and exit of the traffic from the highway. These service points may correspond to the entry and exit ramps in the real highways. The end of the highway serves as the final exit point of the traffic. We let K denote the number of segments in the highway and the segments are numbered as $k = 1,2, \ldots, K$.

We assume that each highway segment is divided into a number of fixed length virtual cells where the length of each cell equals to a node's transmission range, d. We will let I denote the total number of cells in the highway and further the cells will be numbered sequentially as $r = 1 \ldots I$.

We assume that the k'th segment begins at cell r_k, $(r_1 < \ldots r_{k-1} < r_k < r_{k+1} \ldots < I)$, where, $r_1 = 1$ and the length of the k'th segment is given by $\ell_k = (r_{k+1} - r_k)d$ units. Fig. 4.1 shows the single highway model and its segmentation.

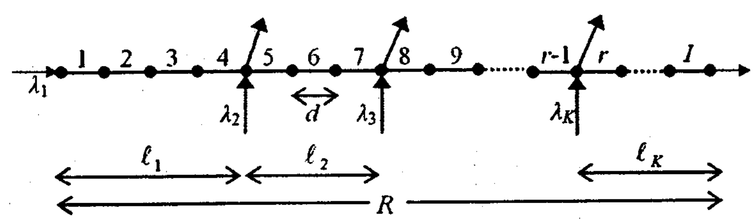


Fig. 4.1. The single highway model

The new nodes arrive at the k'th service point according to a Poisson process with the arrival rate λ_k.. The nodes completing their journey in k-1'st segment continues to the k'th segment with probability α_k or departs from the highway with probability $1-\alpha_k$. We note that a node cannot depart immediately from the service point that it has just arrived. We let $\tilde{s}_{kj}$ denote the stream of nodes with arrivals and departures at the service point k and j respectively. Letting $\tilde{\lambda}_{kj}$ denote the arrival rate of the $\tilde{s}_{kj}$'th stream, then, it is given by,

$$\tilde{\lambda}_{kj} = \lambda_k(1-\alpha_j)\prod_{m=k+1}^{j-1}\alpha_m \,. \tag{4.1}$$

In the above, the product is assumed to have the value of one, if the upper limit is less than the lower limit.

In real scenarios, the number of simultaneous vehicles which may pass each other at any point is restricted by the number of lanes on the highway; however, we assume no restriction on the number of simultaneous vehicles which may pass each other for analytical simplicity. The accuracy of this assumption is tested by simulation in the numerical result section.

4.2.1 Steady State Distribution of a Node's Location

Next, we determine the probability distribution of the distance of a node from its arrival service point at the steady state. Let the pdf and CDF of the distance of a node that belongs to the $\tilde{s}_{kj}$'th stream from its arrival service point to be denoted by $g_{x_{kj}(t)}(y)$ and $G_{x_{kj}(t)}(y)$ respectively. Since the nodes arrive to a service point according to a Poisson process, the arrival time of each node will be uniformly distributed over the time interval (0, t). Given that a node has arrived at time τ, then,

$$g_{x_{kj}(t)}(y) = \frac{1}{t}\int_0^t b_{x(t-\tau)}(y)d\tau = \frac{1}{t}\int_0^t b_{x(\tau)}(y)d\tau, \tag{4.2}$$

$$G_{x_{kj}(t)}(y) = \frac{1}{t}\int_0^t B_{x(\tau)}(y)d\tau, \tag{4.3}$$

where $b_{x(\tau)}(y)$ and $B_{x(\tau)}(y)$ are given by (3.15) and (3.16) respectively. Next, we let $\tilde{g}_{x_{kj}}(y)$ denote the pdf of the distance of a node from the $\tilde{s}_{kj}$'th stream at the steady-state, then,

$$\tilde{g}_{x_{kj}}(y) = \lim_{t\to\infty} g_{x_{kj}(t)}(y) \tag{4.4}$$

where $g_{x_{kj}(t)}(y)$ is given by (4.2). From the remark following (3.16), the steady-state pdf approaches a uniform distribution and $\tilde{g}_{x_{kj}}(y)$ will be given by,

$$\tilde{g}_{x_{kj}}(y) = \frac{1}{R_{kj}}, \quad \text{for } 0 \le y \le R_{kj}. \tag{4.5}$$

where $R_{kj} = \sum_{m=k}^{j} \ell_m$ and ℓ_m is the length of the m'th segment, introduced earlier on.

4.2.2 Distribution of Population Size

Next, we will determine the steady-state distribution of the number of nodes from each stream on the highway. Each stream of nodes may be modeled as an M/G/∞ queue with the service time of a node being the amount of time that it spends on the highway. The service time of a node terminates with its departure from the highway. Let $N_{kj}(t)$ denote the number of nodes from the $\tilde{s}_{kj}$'th stream on the highway at time t. Following the analysis of M/G/∞ queue in [99, 100],

$$P_k(t) = \Pr[N_{kj}(t) = k] = \sum_{n=k}^{\infty} \Pr[N_{kj}(t) = k | n \text{ arrivals in } (0,t)] \frac{e^{-\tilde{\lambda}_{kj} t} (\tilde{\lambda}_{kj} t)^n}{n!}. \tag{4.6}$$

Defining $p_s(t)$ as the probability that a node is still on the highway at time t, then,

$$p_s(t) = G_{x_{kj}(t)}(R_{kj}) = \frac{1}{t} \int_0^t B_{x(\tau)}(R_{kj}) d\tau, \tag{4.7}$$

The probability that a node has departed from the highway by the time t is given by,

$$p_f(t) = 1 - G_{x_{kj}(t)}(R_{kj}) = 1 - \frac{1}{t} \int_0^t B_{x(\tau)}(R_{kj}) d\tau$$

$$= \frac{1}{t} \int_0^t [1 - B_{x(\tau)}(R_{kj})] d\tau. \tag{4.8}$$

Since each node may be on the highway at time t according to independent, identically distributed Bernoulli trials,

$$\Pr[N_{kj}(t) = k | n \text{ arrivals in } (0,t)] = \binom{n}{k} [p_s(t)]^k [p_f(t)]^{n-k}. \tag{4.9}$$

Substituting (4.7), (4.8) and (4.9) in (4.6), we have,

$$P_k(t)=\sum_{n=k}^{\infty}\binom{n}{k}\left[p_s(t)\right]^k\left[p_f(t)\right]^{n-k}\frac{e^{-\tilde{\lambda}_{kj}t}(\tilde{\lambda}_{kj}t)^n}{n!}$$

$$P_k(t)=\sum_{n=k}^{\infty}\binom{n}{k}\left[\frac{1}{t}\int_0^t B_{x(\tau)}(R_{kj})d\tau\right]^k\left[\frac{1}{t}\int_0^t[1-B_{x(\tau)}(R_{kj})]d\tau\right]^{n-k}\frac{e^{-\tilde{\lambda}_{kj}t}(\tilde{\lambda}_{kj}t)^n}{n!}$$

Substituting for the Binomial coefficient gives,

$$P_k(t)=\sum_{n=k}^{\infty}\frac{\left[\frac{1}{t}\int_0^t B_{x(\tau)}(R_{kj})d\tau\right]^k\left[\frac{1}{t}\int_0^t[1-B_{x(\tau)}(R_{kj})]d\tau\right]^{n-k} e^{-\tilde{\lambda}_{kj}t}(\tilde{\lambda}_{kj}t)^n}{k!(n-k)!}$$

Cancelling t^n from the numerator with factors attached to the integrals,

$$P_k(t)=\sum_{n=k}^{\infty}\frac{\left[\tilde{\lambda}_{kj}\int_0^t B_{x(\tau)}(R_{kj})d\tau\right]^k\left[\tilde{\lambda}_{kj}\int_0^t[1-B_{x(\tau)}(R_{kj})]d\tau\right]^{n-k} e^{-\tilde{\lambda}_{kj}t}}{k!(n-k)!}$$

Taking the factors that don't depend on the summation index out of the sum notation,

$$P_k(t)=e^{-\tilde{\lambda}_{kj}t}\frac{\left[\tilde{\lambda}_{kj}\int_0^t B_{x(\tau)}(R_{kj})d\tau\right]^k}{k!}\sum_{n=k}^{\infty}\frac{\left[\tilde{\lambda}_{kj}\int_0^t[1-B_{x(\tau)}(R_{kj})]d\tau\right]^{n-k}}{(n-k)!}$$

After change of variables,

$$P_k(t)=e^{-\tilde{\lambda}_{kj}t}\frac{\left[\tilde{\lambda}_{kj}\int_0^t B_{x(\tau)}(R_{kj})d\tau\right]^k}{k!}\sum_{n=0}^{\infty}\frac{\left[\tilde{\lambda}_{kj}\int_0^t[1-B_{x(\tau)}(R_{kj})]d\tau\right]^{n}}{n!}$$

Substituting the exponential function instead of summation,

$$P_k(t) = e^{-\tilde{\lambda}_{kj}t} \frac{\left[\tilde{\lambda}_{kj} \int_0^t B_{x(\tau)}(R_{kj}) d\tau\right]^k e^{\left[\tilde{\lambda}_{kj} \int_0^t [1 - B_{x(\tau)}(R_{kj})] d\tau\right]}}{k!}$$

$$= e^{\left[-\tilde{\lambda}_{kj}t + \tilde{\lambda}_{kj} \int_0^t [1 - B_{x(\tau)}(R_{kj})] d\tau\right]} \frac{\left[\tilde{\lambda}_{kj} \int_0^t B_{x(\tau)}(R_{kj}) d\tau\right]^k}{k!}$$

$$P_k(t) = e^{-\tilde{\lambda}_{kj} \int_0^t B_{x(\tau)}(R_{kj}) d\tau} \frac{[\tilde{\lambda}_{kj} \int_0^t B_{x(\tau)}(R_{kj}) d\tau]^k}{k!}. \tag{4.10}$$

Thus the probability distribution of the number of nodes from the $\tilde{s}_{kj}$'th stream on the highway at time t is Poisson with parameter $\tilde{\lambda}_{kj} \int_0^t B_{x(\tau)}(R_{kj}) d\tau$. The steady state distribution of the node population is also Poisson with the parameter,

$$\tilde{\phi}_{kj} = \lim_{t\to\infty} \tilde{\lambda}_{kj} \int_0^t B_{x(\tau)}(R_{kj}) d\tau = \lim_{t\to\infty} \tilde{\lambda}_{kj} \int_0^t \int_0^{R_{kj}} \frac{1}{\sqrt{2\pi\theta_{x(\tau)}}} e^{-\frac{(y-\mu\tau)^2}{2\theta_{x(\tau)}}} dy d\tau \tag{4.11}$$

The above integral may be evaluated numerically including for large values of t.

4.2.3 Distribution of the Node Population Size within a Cell

Let N_r denote the size of node population within cell r at the steady-state. Defining $\bar{k}$ as the segment that cell r is located, then, $\bar{k} = \{\max(1,..., j,..., K) \forall r_j < r\}$. N_r also has Poisson distribution with parameter $\bar{\phi}_r$ which is determined from,

$$\bar{\phi}_r = \sum_{m=1}^{\bar{k}} \sum_{n=\bar{k}+1}^{K} \phi_{mn}(r) , \tag{4.12}$$

where,

$$\phi_{mn}(r) = \widetilde{\phi}_{mn} P_{mn}(r), \tag{4.13}$$

With $P_{mn}(r) = \text{Prob(a node from } s_{mn}\text{'th stream is located in cell } r)$ and $\widetilde{\phi}_{mn}$ given by (4.11). From (4.5),

$$P_{mn}(r) = \frac{d}{R_{mn}}, \tag{4.14}$$

where d had been defined as the length of a cell. Then, the probability distribution of the number of nodes within cell r at the steady state is given by,

$$\Pr(N_r = n) = e^{-\overline{\phi}_r} \frac{\overline{\phi}_r^{\,n}}{n!}. \tag{4.15}$$

Clearly, the probability distribution of the number of nodes within the transmission range of a randomly chosen node will also be given by the above Poisson distribution. These are the nodes that may interfere with the transmission of the random node.

4.2.4 Steady State Distribution of a Node's Location within a Cell

Next, we determine the distribution of the distance of a node which is located at cell r from the beginning of the highway at the steady state. Let random variable Y_r denote this distance and $f_{Y_r}(y_r)$ the corresponding pdf of this variable. As before, the cell r is assumed to be located in segment $\vec{k}$, thus a node in this cell may belong to any of the first k streams. Thus, $f_{Y_r}(y_r)$ will be determined by the weighted average of the pdfs of distances of the nodes from different streams given by $\widetilde{g}_{x_{kj}}(y)$ in (4.5). It should be noted that we also have to take into consideration that pdf of the distance for each stream is relative to its service point. Thus,

$$f_{Y_r}(y_r)=\frac{1}{\bar{\phi}_r}\sum_{m=1}^{\bar{k}}\sum_{n=\bar{k}+1}^{K}\phi_{mn}(r)\frac{\tilde{g}_{x_{mn}}(y-R_{1m})}{\tilde{G}_{x_{mn}}(rd-R_{1m})-\tilde{G}_{x_{mn}}[(r-1)d-R_{1m}]}$$

$$=\frac{1}{\bar{\phi}_r}\sum_{m=1}^{\bar{k}}\sum_{n=\bar{k}+1}^{K}\phi_{mn}(r)\frac{\frac{1}{R_{mn}}}{\frac{rd-R_{1m}}{R_{mn}}-\frac{(r-1)d-R_{1m}}{R_{mn}}}$$

$$=\frac{1}{\bar{\phi}_r}\sum_{m=1}^{\bar{k}}\sum_{n=\bar{k}+1}^{K}\phi_{mn}(r)\frac{1}{(rd-R_{1m})-[(r-1)d-R_{1m}]}$$

$$f_{Y_r}(y_r)=\frac{1}{d}, \tag{4.16}$$

where, $(r-1)d \le y_r \le rd$, $1 \le r \le I-1$ and $\tilde{G}_{x_{kj}}(y)$ is the CDF of $\tilde{g}_{x_{kj}}(y)$.

Equation (4.16) states that the distance of a node which is located at cell r is uniformly distributed at the steady state and it is independent from the stream which it belongs to.

4.3 Analysis of Network Connectivity in the Single Highway

In this section, we will determine the network connectivity of a new node arriving at the beginning of the single highway and a random node at the steady state. A new arrival or the random node will be chosen as the cluster-head and the cluster seen by this node will be determined. All the nodes within the cluster will have direct or indirect communications with the cluster-head. It will be assumed that two nodes will be able to communicate directly if $L < d$ where L is the distance between them and d is the constant transmission range of a node. As before, we will assume that the length of the highway, R is an integer multiple of a node's transmission range, d. Clearly, all the nodes within a cell are able to communicate directly as they are within each other's transmission range.

4.3.1 Derivation of the Connectivity Performance Measures for a New Arriving Node

The new arriving node to the beginning of the highway will see the equilibrium distribution of the population size and it will be able to communicate directly with all the nodes within the first cell. Next, we will determine the direct communication probability of two nodes located at two consecutive cells r and $r+1$. Let us define,

$$p_r = \Pr(L \le d \mid \text{two nodes are located at consecutive cells } r \text{ and } r+1).$$

Lemma 4.1: The direct communication probability of two nodes located within two consecutive cells has a constant value of ½ .

Proof: The direct communication probability of the two nodes located at consecutive cells r and $r+1$ respectively is given by,

$$p_r = \Pr(Y_{r+1} - Y_r < d) = \iint\limits_{y_r, y_{r+1}} f_{Y_r}(y_r) f_{Y_{r+1}}(y_{r+1}) dy_r dy_{r+1} \,. \tag{4.17}$$

$$(r-1)d \le y_r \le rd, \quad 1 \le r \le I-1$$

Where Y_r denote the distance of a node which is located at cell r from the beginning of the highway and $f_{Y_r}(y_r)$ the corresponding pdf of this variable. $f_{Y_r}(y_r)$ is given by (4.16) as a uniform distribution. I denotes the total number of cells in the single highway model. Substituting (4.16) in (4.17) and integrating over appropriate intervals; we have,

$$p_r = \int\limits_{y_r=(r-1)d}^{rd} \int\limits_{y_{r+1}=rd}^{d+y_r} \frac{1}{d^2} dy_r dy_{r+1} = \frac{1}{d^2} \int\limits_{y_r=(r-1)d}^{rd} (d + y_r - rd) dy_r \,,$$

after some simplifications,

$$p_r = 1/2,$$

which completes the proof. ∎

In general, it can be proved that the direct communication probability of two nodes located at consecutive cells r and $r+1$ respectively with any arbitrary but identical distributions is always ½[1]. Therefore, the ongoing analysis can be applied to any other mobility model which results to the identical cell location distributions other than uniform.

Next, we will determine probability that the node population of cell $r+1$ will have connectivity with that of cell r given that the latter has nonzero population for $0 < r < I$.

---

[1] *Proof:* Let Y_r denote the i.i.d random variables of a node distance which is located at cell r from the beginning of the highway and $f_{Y_r}(y_r)$ and $F_{Y_r}(y_r)$ the corresponding i.i.d pdf and CDF respectively. We note that, $(r-1)d \le y_r \le rd$, $rd \le y_{r+1} \le (r+1)d$. Therefore,

$$p_r = \Pr(Y_{r+1} - Y_r < d) = \int_{y_r=(r-1)d}^{rd} \int_{y_{r+1}=rd}^{d+y_r} f_{Y_r}(y_r) f_{Y_{r+1}}(y_{r+1}) dy_r dy_{r+1} = \int_{y_r=(r-1)d}^{rd} f_{Y_r}(y_r) dy_r \int_{y_{r+1}=rd}^{d+y_r} f_{Y_{r+1}}(y_{r+1}) dy_{r+1}$$

$$= \int_{y_r=(r-1)d}^{rd} f_{Y_r}(y_r) dy_r [F_{Y_{r+1}}(d+y_r) - F_{Y_{r+1}}(rd)] = \int_{y_r=(r-1)d}^{rd} f_{Y_r}(y_r) F_{Y_r}(y_r) dy_r \quad \{\text{Note: } F_{Y_{r+1}}(rd) = 0 \text{ and } F_{Y_{r+1}}(d+y_r) = F_{Y_r}(y_r)\}$$

We evaluate the above integral with integration by part rule,

$$\int_{y_r=(r-1)d}^{rd} f_{Y_r}(y_r) F_{Y_r}(y_r) dy_r = F_{Y_r}(y_r) F_{Y_r}(y_r) \Big|_{(r-1)d}^{rd} - \int_{y_r=(r-1)d}^{rd} F_{Y_r}(y_r) f_{Y_r}(y_r) dy_r$$

$$\int_{y_r=(r-1)d}^{rd} f_{Y_r}(y_r) F_{Y_r}(y_r) dy_r = 1 - \int_{y_r=(r-1)d}^{rd} F_{Y_r}(y_r) f_{Y_r}(y_r) dy_r$$

$$2 \int_{y_r=(r-1)d}^{rd} f_{Y_r}(y_r) F_{Y_r}(y_r) dy_r = 1$$

$$\int_{y_r=(r-1)d}^{rd} f_{Y_r}(y_r) F_{Y_r}(y_r) dy_r = 1/2 = p_r, \qquad 1 \le r \le I-1.$$ ∎

Let us consider pairs of nodes where the two nodes are not located in the same cell. The populations of two cells will have connectivity, if there is at least a single pair of nodes which have direct communications with each other.

Let us define for $r > 0$,

q_{r+1} = Pr(a node located in cell r + 1 will not have direct communications with any of the nodes in cell r).

C_{r+1} = Pr(node population in cell r + 1 will have connectivity with node population in cell r).

Let us assume that the population size of cell r is nonzero, $N_r > 0$,

$$q_{r+1} = \frac{1}{1-\Pr(N_r = 0)} \sum_{j=1}^{\infty} (1-p_r)^j \Pr(N_r = j). \tag{4.18}$$

Substituting for distribution of N_r from (4.15),

$$q_{r+1} = \frac{(e^{-0.5\bar{\phi}_r} - e^{-\bar{\phi}_r})}{(1-e^{-\bar{\phi}_r})}, \tag{4.19}$$

$$C_{r+1} = 1 - \sum_{j=0}^{\infty} (q_{r+1})^j \Pr(N_{r+1} = j). \tag{4.20}$$

The summation in the above corresponds to the probability that there is not even a single pair of nodes that have direct communications with each other. We note that if cell r+1 is not populated, then, it will also not have connectivity with cell r. From (4.15), (4.19) and (4.20),

$$C_{r+1} = 1 - \sum_{j=0}^{\infty} e^{-\bar{\phi}_{r+1}} \frac{(q_{r+1}\bar{\phi}_{r+1})^j}{j!}$$

$$= 1 - e^{-\bar{\phi}_{r+1}} \sum_{j=0}^{\infty} \frac{(q_{r+1}\bar{\phi}_{r+1})^j}{j!}$$

$$= 1 - e^{-\bar{\phi}_{r+1}} e^{q_{r+1}\bar{\phi}_{r+1}}$$

$$C_{r+1} = 1 - e^{-\bar{\phi}_{r+1}(1-q_{r+1})}, \quad \text{for } 0 < r < I \tag{4.21}$$

Next, let us also define,

C_1 = Pr(node population in cell one will have connectivity with the new arrival)

C_1 is given by,

$$C_1 = 1 - \Pr(N_1 = 0) = 1 - e^{-\bar{\phi}_1}. \tag{4.22}$$

- **Cluster Seen by a New Arriving Node**

Next, we will derive statistics of the cluster seen by a new arrival. If we assume that the cluster consists of the first j cells, this means that in the first j cells, each cell has connectivity to the adjacent cell and the population of j+1'st cell does not have connectivity with that of the j'th cell. Let us define,

ω_j = Pr(the cluster consists of the first j cells).

We assume that connectivity of consecutive cells are independent of each other, therefore ω_j is given by a geometric distribution with a different parameter on each trial. The accuracy of this assumption will be justified in the numerical result section. Thus,

$$\omega_j = (1 - C_{j+1}) \prod_{r=1}^{j} C_r , \quad \text{for} \quad 1 \le j \le I , \tag{4.23}$$

where C_r is given by (4.21), (4.22) and we define $C_{I+1} = 0$. We note that ω_I corresponds to the probability that a new arriving node will see a single cluster.

Next, we will determine the probability distribution of the number of nodes in the cluster. Let us define,

$Q(m) = \Pr(\text{cluster seen by a new arriving node consists of } m \text{ nodes})$.

Then, it is given by,

$$Q(m) = \sum_{j=1}^{I} Q(m \mid \text{cluster consists of } j \text{ cells}) \omega_j . \tag{4.24}$$

Since the number of nodes in each cell has a Poisson distribution with the same parameter given by (4.15),

$$Q(m) = \sum_{j=1}^{I} \omega_j e^{-\sum_{r=1}^{j} \bar{\phi}_r} \frac{(\sum_{r=1}^{j} \bar{\phi}_r)^m}{[1 - \sum_{r=1}^{j} \bar{\phi}_r] m!} . \tag{4.25}$$

Defining $\overline{m}$ as the average number of nodes in the cluster, then, from the above,

$$\overline{m} = \sum_{j=1}^{I} \omega_j \overline{m}_j , \tag{4.26}$$

where, $\overline{m}_j = \sum_{r=1}^{j} \frac{\bar{\phi}_r}{1 - e^{-\bar{\phi}_r}}$. Defining, ξ as the normalized mean cluster size, then,

$$\xi = \frac{\overline{m}}{\psi} , \tag{4.27}$$

where ψ is the mean of the number of nodes on the highway and is given by,

$$\psi = \sum_{r=1}^{I} \bar{\phi}_r . \tag{4.28}$$

4.3.2 Derivation of the Connectivity Performance Measures for a Random Node

Next, the above analysis will be extended to the determination of a cluster seen by a random node. A node from the node population in the highway will be randomly chosen as the cluster head. Letting f_i denote the probability that the random node is located in cell *i*,

$$f_i = \frac{\bar{\phi}_i}{\psi}, \quad i=1,2,\ldots, I. \tag{4.29}$$

The cells that belong to the cluster have connectivity with each other; therefore, they form an uninterrupted chain. We assume that all the cells in the range of, $\ell \le r \le h$, belong to the cluster where ℓ and h denote the two cells at the opposite ends of the chain. Let us define,

q'_{r-1} = Pr(a node located in cell r - 1 will not have any direct communications with node population in cell r).

C'_{r-1} = Pr(node population in cell r - 1 will have connectivity with node population in cell r).

We note that,

$$q'_{r-1} = q_{r+1} \quad , \quad r \ge 2 \tag{4.30}$$

$$C'_{r-1} = 1 - e^{-\bar{\phi}_{r-1}(1-q'_{r-1})}, \quad r \geq 2 \tag{4.31}$$

Let us define $\omega'_{\ell,i,h}$ as the probability that cluster of a random node located in cell i consists of all the cells r such that $\ell \leq r \leq h$, Then $\omega'_{\ell,i,h}$ is given by,

$$\omega'_{\ell,i,h} = (1 - C'_{\ell-1})(1 - C_{h+1}) \prod_{k=\ell}^{i-1} C'_k \prod_{j=i+1}^{h} C_j, \quad 1 \leq \ell \leq i \leq h \leq I. \tag{4.32}$$

In the above, we assume that $C'_k = 0$ for $k \leq 0, C_j = 0$ for $j > I$ and as before we follow the convention that $\prod_{k=i}^{j} A_k = 1$, if $i > j$. Next, let us define $\overline{m}'_{\ell,i,h}$ as the average cluster size for the case under consideration then,

$$\overline{m}'_{\ell,i,h} = \sum_{k=\ell}^{h} \frac{\bar{\phi}_k}{1 - \Pr(N_k = 0)} = \sum_{k=\ell}^{h} \frac{\bar{\phi}_k}{1 - e^{-\bar{\phi}_k}}. \tag{4.33}$$

Let $\overline{m}'$ and ω'_I denote the unconditional average cluster size and the probability that a random node will see the entire node population in a single cluster. Then,

$$\omega'_I = \sum_{i=1}^{I} \sum_{h=i+1}^{I} \sum_{\ell=1}^{i-1} f_i \omega'_{\ell,i,h}, \tag{4.34}$$

$$\overline{m}' = \sum_{i=1}^{I} \sum_{h=i+1}^{I} \sum_{\ell=1}^{i-1} f_i m'_{\ell,i,h} \omega'_{\ell,i,h}. \tag{4.35}$$

As before, the normalized average number of nodes in the cluster is given by,

$$\xi' = \frac{\overline{m}'}{\psi}. \tag{4.36}$$

4.3.3 Application of Connectivity Analysis to a Highway with Two-way Traffic

In the highway under consideration, we have assumed that two VANETs which are independent of each other, will serve to the users traveling in the opposite directions. The system may allow the vehicles to be part of a single VANET independent of the direction of their travels. In this case, node populations in opposite directions will be combined and the integrated system will achieve higher network connectivity. However, connectivity stability of the integrated system will be lower compare to one directional system carrying equivalent amount of traffic.

In the integrated system, the size of the node population within a cell is still given by the Poisson distribution in (4.15) with the appropriate parameter values. As before, the pdf of the location of a node within a cell is given by a uniform distribution, therefore, mean cluster size and probability of a single cluster calculation remain same as in (4.26) and (4.23) respectively.

The integrated system will be useful in analysis of collision avoidance applications and when the network connectivity breaks down during a response to a query. For example, in the case of a sudden hard breaking or accident, a notification will be sent to the vehicles within the immediate proximity. When the network connectivity breaks down, the message can be propagated by the vehicles traveling in the opposite direction until network connectivity is reestablished.

4.3.4 Application of Connectivity Results to the Topological Design of Vehicle to Infrastructure Communications

Vehicle to infrastructure (V2I) communication has been proposed as a complementary service for vehicular networking. This can be done by placing some fix nodes in the highway. These nodes aim to serve both as relay nodes to increase the connectivity of the networks as well as gateways to provide Internet service or other comfort applications to the vehicles. Clearly these nodes cannot be placed with short intervals because of economical issues and performance degradation.

The given connectivity analysis can be used for topological design of fix nodes in the network. A constant node can be considered as a new arrival and the statistics of coverage can be obtained from the given analysis. Therefore optimal distance between two constant nodes can be obtained if we consider the highway between two of them. Further, the average number of nodes which will be served by a fix node can be estimated from the results of cluster mean node population size given in (4.26). These estimates will be helpful in the dimensioning of VANETs.

4.4 Numerical Results

In this section, we present some numerical results regarding the analysis in this chapter, together with simulation results to confirm the accuracy of the approximations in the analysis. The main approximation is the connectivity of consecutive cells being independent of each other.

First, we present results for a single highway such as the one which is shown in fig. 4.1. We assume that length of the highway is R=10.0 kms. As it was stated before, the transmission range of a node, d, is chosen such that there will be an integer number of cells on the highway. We assume that there are three service points which are located at the distances of 1.0, 3.0 and 6.0 kms from the beginning of the highway. Unless otherwise stated, the following traffic arrival and departure rates have been assumed at the service points, λ_1=0.5, λ_2=0.1, λ_3=0.2, λ_4=0.1 nodes/s and α_2= α_3= α_4= 0.7. The epoch rate and standard deviation of the nodes' speed are set to constant values of β=1 and σ=3 respectively.

Fig. 4.2, shows both the analytical and simulation results for mean node population within each cell for a transmission range of d=200 m. As expected, there are discontinuities at the beginning of each segment due to arrivals and departures at service points and node densities in the first and third segments are higher than the other segments. As may be seen, the numerical and simulation results are close to each other. The variations in the node density impact the communications connectivity of the network.

Fig. 4.3, shows the probability distribution of the cluster size seen by a new arriving node to the beginning of the highway from (4.25). The results are given for three transmission ranges, d=100, 200 and 500 m and mean node speed of 25 m/s. It may be seen that for the low transmission range the peak of the distribution is close to zero and drops sharply, thus there is only very small number of nodes in the cluster. For the high transmission range, the peak of the distribution occurs at a large cluster size. The middle transmission range results in a large spread of the cluster size.

Fig.s 4.4 and 4.5, present both simulation and numerical results for normalized mean cluster size from (4.27) and (4.36) seen by a new arriving node and a random node respectively. The results are plotted as a function of the transmission range with mean node speed as a parameter. It may be seen that as the transmission range increases the normalized mean cluster size increases. The increase is steeper at lower speeds because there will be larger node population on the highway. Finally, the numerical and simulation results are close to each other that justifies the approximations in the analysis. Comparison of the corresponding curves from Fig.s 4.4 and 4.5 shows that, for any given transmission range, mean cluster size is slightly higher for a random node than for a new arriving node. This is because the latter cluster is always anchored at the beginning of the highway.

Fig.s 4.6 and 4.7, show the probability that an arriving and a random node will see the entire node population on the highway as a single cluster from (4.23) for $j=I$ and (4.34) respectively. The plots assume the same system parameters as those in Fig.s 4.4 and 4.5 and similar observations apply to these curves. As expected, probability that a new arriving or a random node will see a single cluster is same since in both cases all the nodes will be included in the cluster. It may be seen that at higher speeds a node has to increase its transmission range significantly to achieve a high probability of single cluster. When the single cluster probability is high, since the locations of the nodes are uniformly distributed, probably the nodes will be scattered all over the highway. As a result, the nodes will be able to receive traffic information from any part of the highway.

Next, we present the simulation results about the effect of finite number of lanes on highways which limits the number of vehicles that may pass each other. First, we have measured the average number of times per second that a random node has been overtaken by the other nodes during the simulation in a system with no limitations on the number of overtakes. The measured value of this average is 0.31 overtakes/sec when the mean node speed is 20 m/sec. The average value increases to 0.72 overtakes/sec, when the stream arrival rates are doubled (λ_1=1, λ_2=0.2, λ_3=0.4, λ_4=0.2), while keeping the mean node speed constant. As may be seen this average is not high. Next, we have assumed that the number of lanes on a highway is limited to M. We have modified the simulation such that the number of passes experienced by a node is at most M during an epoch duration.

Fig. 4.8, presents the simulation results for normalized mean cluster size seen by a new arriving node for M = 2 lanes. The results are plotted as a function of the transmission range with node speed as a parameter. Also shown in this figure is the analytical results given before in fig. 4.4, which correspond to unlimited number of lanes on a highway. As may be seen, the simulation and analytical results are close to each other except for some divergence at high speeds and in smaller transmission ranges. However, this divergence drops sharply at higher number lanes or node arrival rates. We also note that a congested network is not interesting from the connectivity point of view, since the network will have full connectivity. Thus, these results justify the assumption of no limitation on the number of passings that a vehicle may experience in the analysis.

The conclusions of this section remain same with longer highways and different parameter values. We also note the close agreement between the numerical and simulation results which validates the approximation of the connectivity of consecutive cells being independent of each other.

4.5 Summary

In this chapter, we have introduced a unidirectional single highway model with multiple lanes where the nodes arrive through one of the highway entry points and depart through one of the exit points. The highway has been divided into concatenated segments which are further divided into virtual cells. We have found the pdf of node population in

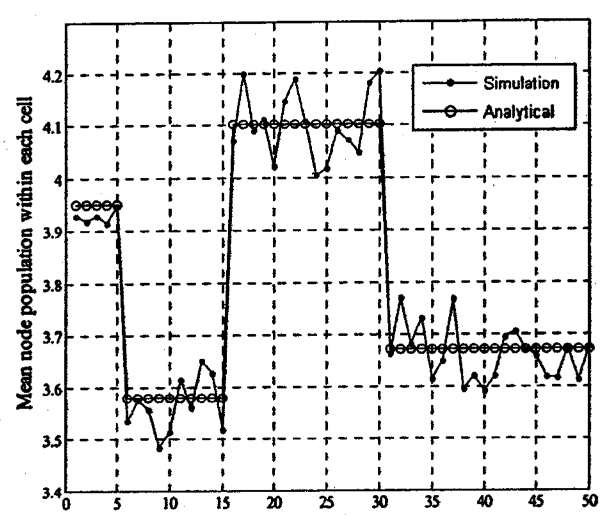


Fig. 4.2. Mean node population within cells for transmission range of *d*=200 m and mean node speed of 25 m/s (90 km/hr).

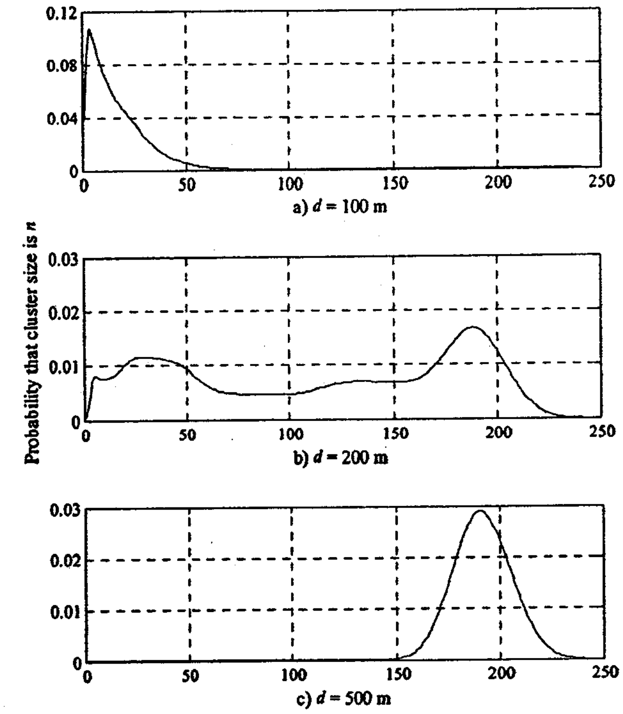


Fig. 4.3. Probability that cluster seen by a new arrival consists of n nodes for three values of transmission ranges of d=100, 200 and 500 m and mean node speed of 25 m/s (90 km/hr).

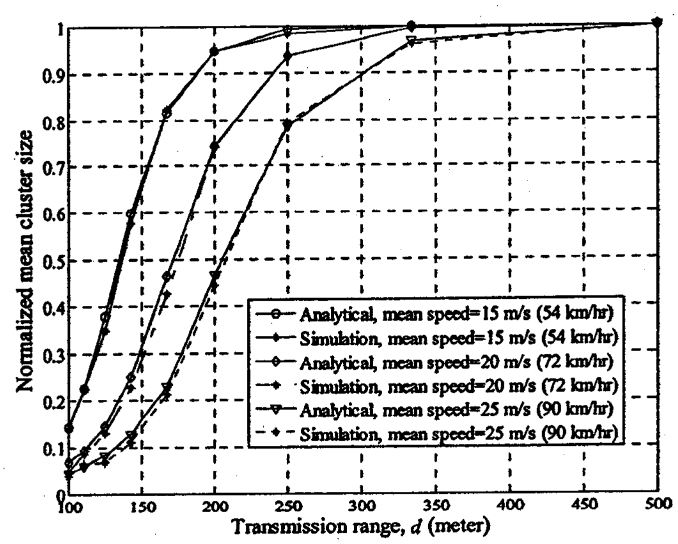


Fig. 4.4. Normalized mean cluster size seen by a new arrival as a function of transmission range with mean node speed as a parameter.

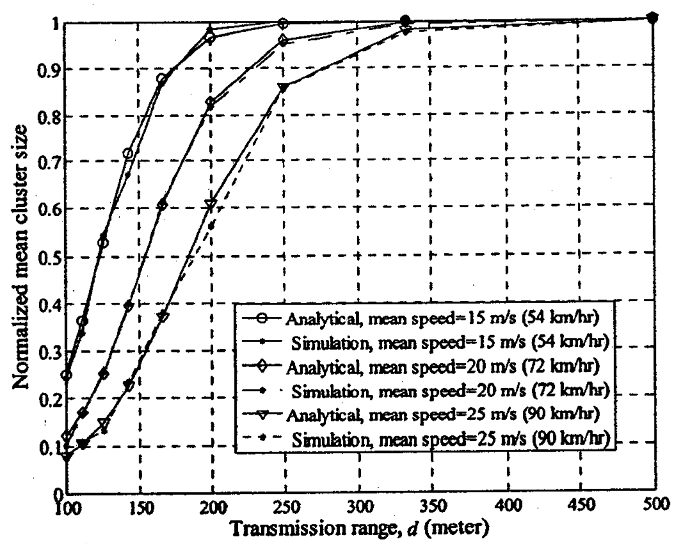


Fig. 4.5. Normalized mean cluster size seen by a random node as a function of transmission range with mean node speed as a parameter.

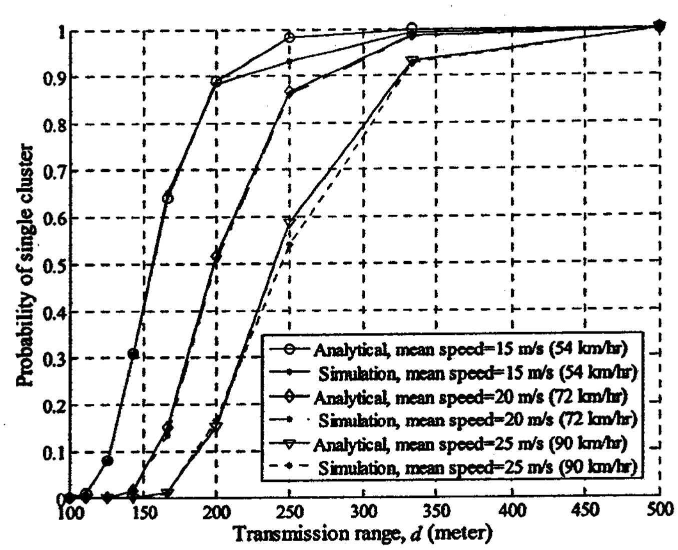


Fig. 4.6. Probability that a new arrival will see a single cluster as a function of transmission range with mean node speed as a parameter.

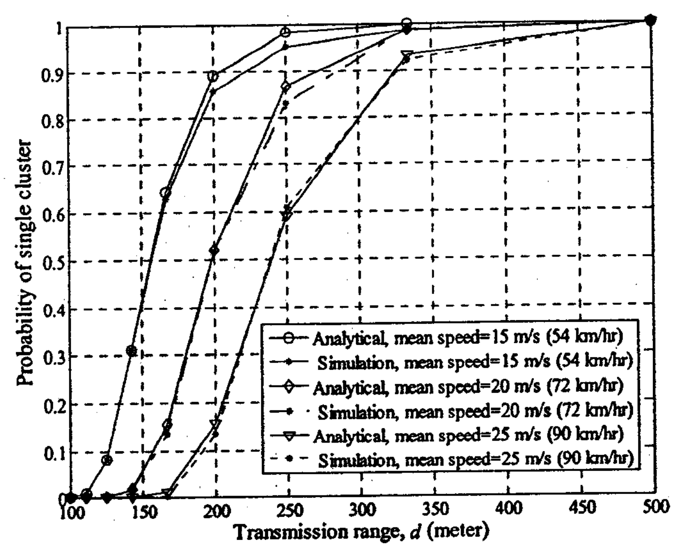


Fig. 4.7. Probability that a random node will see a single cluster as a function of transmission range with mean node speed as a parameter.

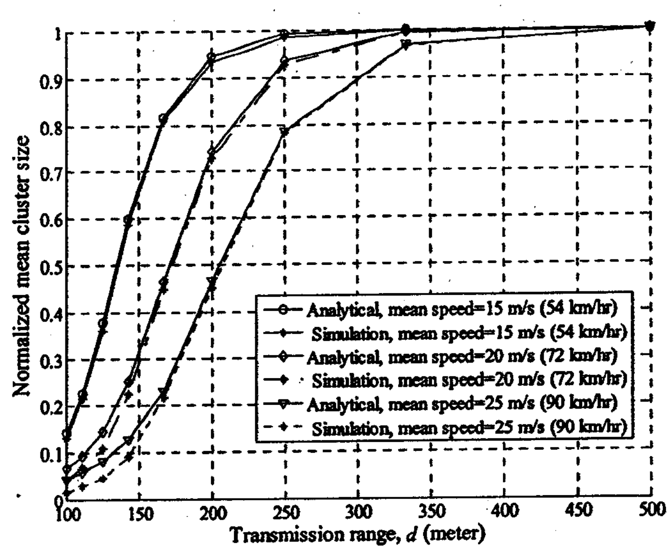


Fig. 4.8. Normalized mean cluster size seen by a new arrival as a function of transmission range from simulation for two-lane highway and from numerical result.

each highway segment and cell. We have shown that node population distribution within a cell follows a Poisson distribution. Clearly, the probability distribution of the number of nodes within the transmission range of a randomly chosen node will also be given by the same Poisson distribution. These are the nodes that may interfere with the transmission of the random node.

Then, we have studied the network connectivity of the mobile nodes in the single highway model. We derived connectivity performance measures such as, mean cluster size and probability that a new arriving node or a random node will see the entire node population in a single cluster. It is seen that, probability of a single cluster is lower at higher node speeds than lower speeds for constant arrival rates. As a result, nodes may have to increase their transmission ranges at higher speeds to achieve a high probability of network connectivity.

In a bidirectional highway, identical but independent VANETs will serve to the users traveling in the opposite directions. The system may allow the vehicles to be part of a single VANET independent of the direction of their travels. The results of the chapter may be combined to give the mean cluster size and probability of a single cluster in such a system. The integrated system will achieve higher network connectivity.

The given analysis is also helpful for route selection applications. Let us assume a newly arriving node to the beginning of two highways is interested in the network connectivity on each of the highways. The user may be interested in finding out traffic congestion information on these highways in order to make a route selection. In the next chapter, we extend the single highway to a network of highways model which enable us to compare the connectivity parameters in different paths.

As explained before, the amount of time the node population spends in the single cluster may be taken as the conservative measure of communications path availability. Thus, single cluster probability gives us the percentage of time that a path is available for communications.

Unfortunately, the analysis of this chapter does not extend to the networks of highways with arbitrary topology. In the next chapter, we will introduce a different approach to analyze such networks.

Chapter 5

An Analysis of Continuous Communication Path Availability Time in VANETs

5.1 Introduction

The objective of this chapter is to study the continuous communication availability time along a path between two nodes of a vehicular ad hoc network with a mobility model. The communication availability refers to the distribution of the time that a communication path is continuously available between two nodes. It is closely related to the cluster formation of the node population in that path. As before, a cluster refers to a group of nodes that can communicate with each other directly or indirectly. At the times that the node population in a path forms multiple clusters only intra-cluster communications will be possible. Thus only if the node population forms a single cluster, all the nodes will be able to communicate with each other. As before, we will take the time that the system spends in single cluster state as the conservative measure of network availability for communications. In chapter 4, we have determined the probability of single cluster in a single highway model. Thus we had obtained the percentage of the time that a path will have communications availability. The main limitation of that work is that it fails to give information regarding the frequency and duration of the visits to the single cluster state. For example, even if the single cluster probability is high, if the duration of the visits is short, it may not be possible for nodes to complete their calls. Clearly, the state of a path alternates between communication availability and unavailability. The durations of continuous communication availability and unavailability will be significant measures which will determine the quality of communications among the users.

This chapter extends the preceding analysis in several directions. First of all, it assumes a *network of highways* with arbitrary topology. As before, each highway consists of a number of concatenated segments where the segment joints are the point of entry and exit for the node traffic. The arrival of the nodes to the network is according to a *traffic matrix* and it follows a Poisson process. Each node moves in the network according to the mobility model with *load dependent speed* and its path is determined by a *routing matrix*.

We model the system as a *BCMP network of queues* with state dependent service where segments correspond to the service centers. We obtain the joint distribution of the number of nodes in segments. Then, we model the number of clusters in a segment as a

Markovian birth-death process. This allows us to determine the distribution of time that node population in a path spends in the single cluster state and mean recurrence time following the departure from this state. Thus, we derive the distribution of the time that a path in the network will be continuously available for communications, as well as, mean duration of the path unavailability time for communications. These results enable the determination of the conditions under which there will be high communication path availability.

The remainder of this chapter is organized as follows:

Section 5.2 introduces the network of highways model. In section 5.3, we find the probabilities of communication connectivity for adjacent cells. Using these probabilities, in section 5.4 we model the number of clusters in a segment as a Markovian birth-death process and determine the distribution of communication availability time in the single cluster state for any segment of the highways' network and mean recurrence time following the departure from this state. In section 5.5 we determine the distribution of the time that any network path will be continuously available to communications, as well as mean duration of the interruption in communications. In section 5.6, we present the mean packet delay analysis within a path of the highway network, when all of its nodes form a single cluster and finally section 5.7 describe the numerical results of the study.

5.2 Network of Highways Model

In the previous chapter, we presented the single highway model. This model enabled us to find the distribution of the node's population within a cell by superposition of node's population of streams which go through that cell. Further, we have shown that with the given mobility model, node location distribution is given by a uniform distribution for any length of highway.

In this section we extend the single highway model to a network of highways. In this model, we assume a network of N highways with arbitrary topology which traffic may be exchanged between the highways through the intersections. The highways are numbered as j=1... N. Fig. 5.1 shows an example network where highways AB, EF and AC have been assigned the numbers j=1, 2 and 4 respectively. Similarly, the highways are assumed to be unidirectional and have multiple lanes that allow vehicles pass each other without any restriction on the number of lanes.

Similar to single highway model, we assume that each highway consists of a number of concatenated segments with the beginning and end of each segment serving as a service point. These service points allow entry and exit of the traffic from the highway and they may correspond to the entry and exit ramps in the real highways. Further, they may serve as an interchange point for traffic among highways.

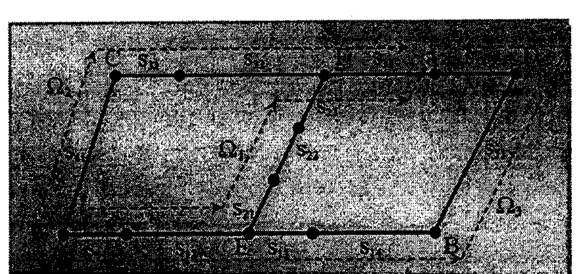


Fig. 5.1. A sample model of highways' network.

We let Q_j denote the number of segments in highway j. The segments in a highway will be numbered by double indices; thus, S_{ji} will refer to the i'th segment of highway j.

Next, we consider a continuous path between any two nodes in the network. We are interested in studying the connectivity of these two nodes along this path. We may define different paths in the network. Let Ω_k denote the k'th path, and $|\Omega_k|$ the number of segments in Ω_k. Each path will be specified by listing its segments and a segment position in the set will be denoted by s.

Ω_1, Ω_2 and Ω_3 are some example paths in the highway network in fig. 5.1, where for example, path Ω_1 is specified as,

$$\Omega_1 = \{S_{11}, S_{12}, S_{21}, S_{22}, S_{23}, S_{33}\}, \tag{5.1}$$

For the above path, we have $|\Omega_1| = 6$ and $1 \le s \le |\Omega_1|$.

5.2.1 Virtual Cell Model of a Segment

As before, we assume that each highway segment is divided into a number of fixed length virtual cells where the length of each cell equals to a node's transmission range, d. We will let $|s|$ denote the number of cells in segment s from path Ω_k and further number the cells sequentially as $r = 1..|s|$.

5.2.2 Joint Distribution of Node Population within the Highways' Segments

For simplicity reasons, we assume that the current epoch of a node will terminate when it departs a segment and it will start in the new segment with a new epoch. This enables us to decouple the amount of time that a node spends in consecutive segments. Assuming that epoch durations are short, this will be an acceptable approximation. This is the only additional approximation in this analysis compared to that in the previous chapter. The accuracy of this assumption will be tested by simulation later on. Therefore, we can model the network of highways as a BCMP network [101], where each highway segment becomes a service center of the infinite server type.

In the following, the terms segment and service center will be used interchangeably and each service center will be identified by the symbol of the highway segment that it serves. The service time at a service center will be the amount of time it takes for a node to travel the corresponding highway segment and it will be proportional to the node speed. Next, let us define the following parameters,

n_{ji} = number of nodes in the service center S_{ji}.

R_{ji} = the length of segment S_{ji}.

φ_{ji} = mean of node population size in service center S_{ji}.

h_{ji} = node population density in service center S_{ji}, i.e., $h_{ji} = \varphi_{ji} / R_{ji}$. h_{ji} gives the mean population per unit length of the segment S_{ji}.

λ_{ji} = the external arrival rate of the traffic to the segment S_{ji}.

$p_{ji,rs}$ = probability that a node departing from segment S_{ji}, next will receive service from segment S_{rs}. If $r = j$ and $s = i+1$, these are two consecutive segments of the same highway. $p_{ji,rs}$ determines the routing matrix of the nodes in the network.

e_{ji} = total arrival rate of the nodes to segment S_{ji}. e_{ji} is determined through the simultaneous solution of the following traffic equations,

$$e_{ji} = \lambda_{ji} + \sum_{rs} e_{rs} p_{rs,ji}, \qquad j = 1,..N, \ i = 1..Q_j. \tag{5.2}$$

The state of the system is determined by a vector that specifies the number of nodes in each highway segment of the network. Let $\vec{n}$ denotes this vector, then,

$$\vec{n} = (n_{11},..n_{1Q_1},...,n_{N1}..n_{NQ_N}).$$

In BCMP queuing model, the nodes may also have state dependent service times. In our system, we will also assume such a service by expressing the nodes' mean speed as a function of segment populations. As the node population size within a segment is changing continuously, we will assume that a node service time depends on mean of node population size in the segment (or equivalently node population density). Next let us define the following symbols,

$\mu(h_{ji})$ = state dependent mean speed of the nodes at the service center S_{ji} given that the population density of the nodes at that service center is h_{ji}.

$\gamma(h_{ji})$ = state dependent service rate of the nodes at the service center S_{ji}, given that the population density of the nodes at that service center is h_{ji}.

$\gamma(h_{ji})$ are given by,

$$\gamma(h_{ji}) = \frac{\mu(h_{ji})}{R_{ji}}. \tag{5.3}$$

From BCMP theorem, the joint distribution of the number of nodes at each service center is given by [101],

$$P(\vec{n}) = Gd(\vec{n})\prod_{j=1}^{N}\prod_{i=1}^{Q_j} g(n_{ji}), \tag{5.4}$$

where G is the normalization constant and $g(n_{ji})$ for a queue with infinite servers is given by,

$$g(n_{ji}) = \frac{1}{n_{ji}!}\prod_{r=1}^{n_{ji}}\left(\frac{e_{ji}}{\gamma(h_{ji})}\right) = \frac{1}{n_{ji}!}\left(\frac{e_{ji}}{\gamma(h_{ji})}\right)^{n_{ji}}. \tag{5.5}$$

$d(\vec{n})$ is a function of total arrivals,

$$d(\vec{n}) = \prod_{i=1}^{n_t}\lambda_t = \lambda_t^{n_t}, \tag{5.6}$$

where λ_t and n_t denote the total arrival rate and total number of nodes in the highway network respectively,

$$\lambda_t = \sum_{j=1}^{N}\sum_{i=1}^{Q_j}\lambda_{ji}, \tag{5.7}$$

$$n_t = \sum_{j=1}^{N}\sum_{i=1}^{Q_j} n_{ji}. \tag{5.8}$$

We assume that $\mu(h_{ji})$ is a decreasing function of h_{ji}, since vehicles' speed decreases as the traffic increases in a highway. For example, it may be chosen as an inverse function,

$$\mu(h_{ji}) = \frac{v_0}{f(h_{ji})} \tag{5.9}$$

where v_0 is the maximum speed in the highway segment, and $f(h_{ji})$ is an increasing function of h_{ji}. With this given function, (5.5) and (5.9) may be written as,

$$\gamma(h_{ji}) = \frac{v_0}{f(h_{ji})R_{ji}}$$

$$g(n_{ji}) = \frac{1}{n_{ji}!}(\frac{f(h_{ji})R_{ji}e_{ji}}{v_0})^{n_{ji}}$$

The steady-state joint distribution of (5.4) at the service centers is given by,

$$P(\bar{n}) = G\lambda_t^{n_t}\prod_{j=1}^{N}\prod_{i=1}^{Q_j}\frac{1}{n_{ji}!}(\frac{f(h_{ji})R_{ji}e_{ji}}{v_0})^{n_{ji}},$$

$$= G\lambda_t^{n_t}\prod_{j=1}^{N}\prod_{i=1}^{Q_j}\frac{1}{n_{ji}!}(\frac{\lambda_t f(h_{ji})R_{ji}e_{ji}}{v_0})^{n_{ji}},$$

$$P(\bar{n}) = G\prod_{j=1}^{N}\prod_{i=1}^{Q_j}\frac{1}{n_{ji}!}(D_{ji}f(h_{ji}))^{n_{ji}}. \qquad (5.10)$$

where,

$$D_{ji} = \frac{\lambda_t e_{ji}R_{ji}}{v_0}, \qquad (5.11)$$

The normalization constant, G in (5.10) may be found as,

$$G^{-1} = \sum_{n_{NQN}=0}^{\infty}\cdots\sum_{n_{ji}=0}^{\infty}\cdots\sum_{n_{11}=0}^{\infty}\prod_{j=1}^{N}\prod_{i=1}^{Q_j}\frac{1}{n_{ji}!}(D_{ji}f(h_{ji}))^{n_{ji}}$$

or interchanging the order of infinite summation and the product,

$$G^{-1} = \prod_{j=1}^{N}\prod_{i=1}^{Q_j}\sum_{n_{NQN}=0}^{\infty}\cdots\sum_{n_{ji}=0}^{\infty}\cdots\sum_{n_{11}=0}^{\infty}\frac{1}{n_{ji}!}(D_{ji}f(h_{ji}))^{n_{ji}} = \prod_{j=1}^{N}\prod_{i=1}^{Q_j}e^{D_{ji}f(\rho_{ji})} \qquad (5.12)$$

Therefore, (5.10) can be written as,

$$P(\vec{n}) = \prod_{j=1}^{N}\prod_{i=1}^{Q_j} \frac{e^{-D_{ji}f(h_{ji})}(D_{ji}f(h_{ji}))^{n_{ji}}}{n_{ji}!} \tag{5.13}$$

From the above, the marginal distribution of node population within the service center S_{ji} follows a Poisson distribution with mean value of $D_{ji}f(h_{ji})$. Since mean node population in segment S_{ji} has been defined earlier on as φ_{ji}, it may be found by solving the following equation,

$$\varphi_{ji} = D_{ji}f(h_{ji}) = D_{ji}f(\varphi_{ji}/R_{ji}). \tag{5.14}$$

For a system with state independent service time, we let $\mu_{ji}(h_{ji}) = v_0$ which reduces (5.13) to,

$$P(\vec{n}) = G\prod_{j=1}^{N}\prod_{i=1}^{Q_j} \frac{e^{-D_{ji}}(D_{ji})^{n_{ji}}}{n_{ji}!}. \tag{5.15}$$

In the above analysis, it is assumed that the speed of each node's motion during a mobility epoch is still constant and chosen independently from a normal distribution with mean $\mu(h_{ji})$ and variance σ^2 in segment S_{ji}. In other words, the density of node population during each epoch is considered as a constant. The simulation results to be presented later on shows the accuracy of this assumption at the steady state.

Next, we determine the node population distribution in a cell of a given path in the highway network. Similar to the single highway model, let random variable Y_r denote the location of a node in cell r from the beginning of its segment. We have shown in section

4.2.4 that with the given mobility model, $f_{Y_r}(y_r)$ has a uniform distribution given by (4.16). Let N_r also denote the number of the nodes in cell r of segment s. Since the location of each node within a cell is uniformly distributed, the node population within each cell will also have a Poisson distribution as the segment node population. As before, let us define $\bar{\phi}_r$ as the mean of node population in cell r of segment s, then,

$$\bar{\phi}_r = \frac{\varphi_{ji}}{|s|}, \tag{5.16}$$

Where φ_{ji} is given by (5.14). We note that $\bar{\phi}_r$ where $r = 1..|s|$ for all the cells of segment s are the same. Therefore, for simplicity reasons, we use the notation $\bar{\phi}_s$ to refer to the mean of node population in cells r of segment s interchangeably.

5.3 Communication Connectivity of Adjacent Cells

In this section, first we will derive the distribution of the distance between two nodes located in adjacent cells and then the connectivity of these two nodes. At the end, we will determine the communication connectivity of adjacent cells.

5.3.1 Distribution of Distance between Two Nodes Located at Adjacent Cells

As before, it is assumed that two nodes are directly connected if their distance is less than the constant transmission range of a node, d. Therefore, all the nodes in a cell are connected as they are within the transmission range of each other. Let us define random variable L as the distance between two nodes located in consecutive cells r and r+1 and its pdf as $f_L(\ell)$. Then, $L = Y_{r+1} - Y_r$ where Y_r has been defined as the random variables of distance of a node located at cell r from the beginning of its segment and its probability density function, $f_{Y_r}(y_r)$ was given by (4.16). Next, we will determine $f_L(\ell)$. First let us determine the following probability,

$$\Pr(L < \ell) = \Pr(Y_{r+1} - Y_r < \ell) = \Pr(Y_{r+1} < Y_r + \ell), \quad 0 < \ell < 2d \tag{5.17}$$

This probability may be found by conditioning on the distance between two nodes, thus for $0 < \ell < d$, the area of integration corresponds to fig. 5.2. Therefore,

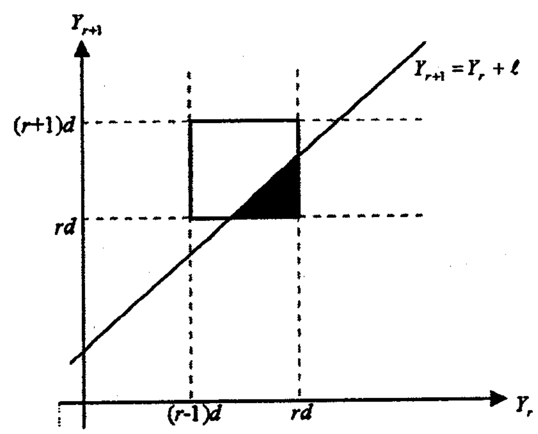


Fig. 5.2. Area of integration for $0 < \ell < d$

$$\Pr(L < \ell | 0 < \ell < d) = \int_{y_r = rd-\ell}^{rd} \int_{y_{r+1}=rd}^{y_r+\ell} f_{Y_r}(y_r) f_{Y_{r+1}}(y_{r+1}) dy_{r+1} dy_r$$

$$\Pr(L < \ell | 0 < \ell < d) = \int_{y_r = rd-\ell}^{rd} \int_{y_{r+1}=rd}^{y_r+\ell} \frac{1}{d^2} dy_{r+1} dy_r = \frac{\ell^2}{2d^2}. \tag{5.18}$$

We note that for $\ell = d$ the above probability corresponds to the probability defined in (4.17). For $d < \ell < 2d$, the area of integration corresponds to fig. 5.3. Therefore,

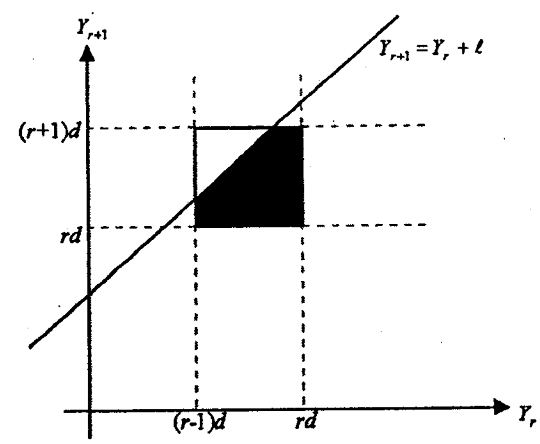


Fig. 5.3. Area of integration for $d < \ell < 2d$

$$\Pr(L < \ell | d < \ell < 2d) = 1 - \int_{y_r=0}^{(r+1)d-\ell} \int_{y_{r+1}=y_r+\ell}^{(r+1)d} f_{Y_r}(y_r) f_{Y_{r+1}}(y_{r+1}) dy_{r+1} dy_r$$

$$= 1 - \int_{y_r=0}^{(r+1)d-\ell} \int_{y_{r+1}=y_r+\ell}^{(r+1)d} \frac{1}{d^2} dy_{r+1} dy_r = 1 - \frac{4d^2 - 4d\ell + \ell^2}{2d^2}$$

$$\Pr(L < \ell | d < \ell < 2d) = \frac{-2d^2 - \ell^2 + 4d\ell}{2d^2}. \tag{5.19}$$

Then, $f_L(\ell)$ can be found by taking derivative of (5.18) and (5.19) wrt ℓ.

$$f_L(\ell) = \begin{cases} \dfrac{\ell}{d^2} & 0 < \ell < d \\ \dfrac{(2d - \ell)}{d^2} & d < \ell < 2d \end{cases} \tag{5.20}$$

Next, we find the conditional probability density function of the distance between two nodes given that their distance is either greater or less than the node transmission range, respectively,

$$f_L(\ell | L < d) = \frac{f_L(\ell)}{\Pr(L < d)} = \frac{\ell/d^2}{0.5} = \frac{2\ell}{d^2} \qquad 0 < \ell < d\,, \tag{5.21}$$

$$f_L(\ell | L > d) = \frac{f_L(\ell)}{\Pr(L > d)} = \frac{(2d-\ell)/d^2}{0.5} = \frac{4d-2\ell}{d^2} \quad d < \ell < 2d\,. \tag{5.22}$$

In above, $\Pr(L > d) = \Pr(L < d) = 0.5$, as it was proved in Lemma 4.1. The conditional densities (5.21) and (5.22) will be used in the evaluation of cells connectivity.

5.3.2 Communication Connectivity of Two Nodes Located at Adjacent Cells within a Short Time Interval

Next, we will determine the conditional communication connectivity of two nodes located in adjacent cells within a small increment of time, Δt, at time t at the steady state. We will obtain the following two conditional connectivity probabilities,

p_{cc} = Pr(two nodes which are located at adjacent cells will have connectivity at $t+\Delta t$ | they have connectivity at t).

p_{dc} = Pr(two nodes which are located at adjacent cells will have connectivity at $t+\Delta t$ | they do not have connectivity at t).

We let random variable V_r denote the speed of a node in cell r of segment S_{ji} where V_r is i.i.d. normally distributed with mean $\mu(h_{ji})$ and variance σ^2. It will be assumed that Δt is small enough such that two nodes will remain in their respective cells. The new location of the node in cell r at time $t+\Delta t$ will be given by $Y_r + V_r\Delta t$. We let L' denote the distance between these two nodes at time $t+\Delta t$, then, it will be given by, $L' = (Y_{r+1} + V_{r+1}\Delta t) - (Y_r + V_r\Delta t)$. Next we find the connectivity probability of two nodes at $t+\Delta t$ given that their distance at time t is $L = \ell$.

$$\Pr(-d < L' < d) = \Pr[-d < (Y_{r+1} + V_{r+1}\Delta t) - (Y_r + V_r\Delta t) < d]$$

$$= \Pr[-d < (Y_{r+1} - Y_r) + (V_{r+1} - V_r)\Delta t < d]$$

$$= \Pr[-d < L + (V_{r+1} - V_r)\Delta t < d]$$

$$\Pr(-d < L' < d | L = \ell) = \Pr[-d < \ell + (V_{r+1} - V_r)\Delta t < d | L = \ell]$$

$$= \Pr[-d - \ell < (V_{r+1} - V_r)\Delta t < d - \ell | L = \ell] \tag{5.23}$$

We note that as V_r is normally distributed with $(\mu(h_{ji}), \sigma^2)$, $(V_{r+1} - V_r)\Delta t$ is also normally distributed with parameters $(0\ ,\ 2\Delta t^2\sigma^2)$. Therefore, (5.23) may be determined as follows,

$$\Pr(-d < L' < d | L = \ell) = \int_{x=-d-\ell}^{d-\ell} \frac{1}{\sqrt{4\pi\Delta t^2\sigma^2}} \exp\left(\frac{-x^2}{4\Delta t^2\sigma^2}\right) dy \tag{5.24}$$

Now, p_{cc} and p_{dc} can be found by unconditioning (5.24) wrt L using the pdfs given in (5.21) and (5.22) respectively,

$$p_{cc} = \int_{\ell=0}^{d} \frac{2\ell}{d^2} \int_{x=-d-\ell}^{d-\ell} \frac{1}{\sqrt{4\pi\Delta t^2\sigma^2}} \exp\left(\frac{-x^2}{4\Delta t^2\sigma^2}\right) dx\, d\ell \tag{5.25}$$

$$p_{dc} = \int_{\ell=d}^{2d} \frac{4d - 2\ell}{d^2} \int_{x=-d-\ell}^{d-\ell} \frac{1}{\sqrt{4\pi\Delta t^2\sigma^2}} \exp\left(\frac{-x^2}{4\Delta t^2\sigma^2}\right) dx\, d\ell \tag{5.26}$$

p_{cc} and p_{dc} may be found by numerical methods since above integrals do not have a closed form.

5.3.3 Communication Connectivity of Adjacent Cell Populations within a Short Time Interval

Next, we will determine the conditional communication connectivity of the node populations of adjacent cells within a very short time interval. Let us define the following conditional connectivity probabilities of two consecutive cells r and r+1 in segment s.

$\rho_{dd}(s)$=Pr(the node populations of cells r and r+1 of segment s will remain disconnected at $t+\Delta t$ | they do not have connectivity at t)

$\rho_{cd}(s)$=Pr(the node populations of cells r and r+1 of segment s will become disconnected at $t+\Delta t$ | they have connectivity at t).

$\rho^{*}_{cd}(s)$=Pr(last cell of segment s with density of $\bar{\phi}_s$ and the first cell of segment $s+1$ with density of $\bar{\phi}_{s+1}$ will become disconnected at $t+\Delta t$ | they have connectivity at t).

Fig. 5.4a and 5.4b show the events which correspond to $\rho_{dd}(s)$ and $\rho_{cd}(s)$ respectively. To find the first probability, let us define the following random variables,

b_r : a Bernoulli variable that assumes values of 1, 0 if two nodes in cells r and r+1 gain connectivity or remain disconnected by the time $t+\Delta t$ given that they do not have connectivity at time t. We note that the success probability of this Bernoulli trial is p_{dc} given in (5.26).

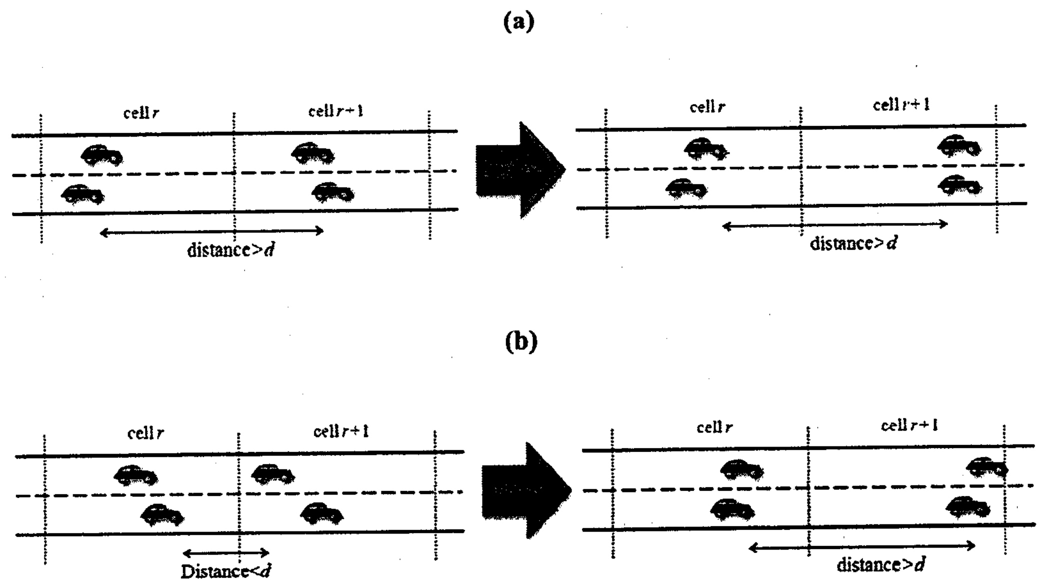


Fig. 5.4. Discontinuity of two consecutive cells r and r+1 at t+Δt given that: a) they do not have connectivity at t, b) they have connectivity at t.

δ_r : the number of nodes in cell r that will gain connectivity to a chosen node in cell $r+1$ by the time t+Δt given that they do not have connectivity to this node at time t.

u_{r+1} : the number of nodes in cell r+1 that will gain connectivity with at least one of the nodes in cell r by the time t+Δt given that they do not have connectivity with any node in cell r at time t.

We note that,

$$\delta_r = \sum_{j=0}^{N_r} b_r \,, \tag{5.27}$$

Where N_r had been defined before as the number of nodes in cell r. Let us also define the following probability generating functions (PGFs),

$$B_r(z) = E[z^{b_r}] = p_{dc}z + 1 - p_{dc} \tag{5.28}$$

$$\delta_r(z) = E[z^{\delta_r}], \tag{5.29}$$

$$U_{r+1}(z) = E[z^{u_{r+1}}], \tag{5.30}$$

$$N_r(z) = E[z^{N_r}] = e^{\bar{\phi}_s(z-1)} \tag{5.31}$$

Next we assume that the random variables b_r are independent of each other. Then, since (5.27) is in the form of a random sum, we have the following relationship,

$$\delta_r(z) = N_r(z)\Big|_{z = B_r(z)} \tag{5.32}$$

Which after simplification,

$$\delta_r(z) = e^{\bar{\phi}_s p_{dc}(z-1)} \tag{5.33}$$

We note that $1-\delta_r(0)$ corresponds to the probability that a node in cell r+1 which does not have connectivity with any of the nodes in cell r at time t, gains connectivity with at least one of the nodes in cell r by the time $t+\Delta t$. Therefore, again, from application of the random sum formula,

$$U_{r+1}(z) = N_{r+1}(z)\Big|_{z = [1-\delta_r(0)]z + \delta_r(0)} \tag{5.34}$$

After substituting for $\delta_r(0)$ and following simplification,

$$U_{r+1}(z) = e^{\bar{\phi}_s(1-e^{-\bar{\phi}_s p_{dc}})(z-1)} \tag{5.35}$$

In the above, $U_{r+1}(0)$ corresponds to the probability that consecutive cells r and $r+1$ remain disconnected at $t+\Delta t$ given that they were disconnected at t. Therefore $\rho_{dd}(s)$ for the cells in segment s is given by,

$$\rho_{dd}(s) = U_{r+1}(0) = e^{-\bar{\phi}_s(1-e^{-\bar{\phi}_s p_{dc}})}, \quad 1 \le r < |s| \tag{5.36}$$

In the random sum given above, it is assumed that the direct connectivity between a pair of nodes in two consecutive cells is independent of any other pair. Therefore, the location distribution of a node stays uniform, independent of the connectivity status of that node. The accuracy of this assumption will be justified in the numerical result section.

Next, we present the probability $\rho_{cd}(s)$. We note that two events may lead to the loss of connectivity between cells r and $r+1$ by the time $t+\Delta t$, given that they had connectivity at time t, which are either the distances between the population of these two cells become higher than d or one of the two cells becomes empty. Let $\rho'_{cd}(s)$ and $\rho''_{cd}(s)$ denote the probabilities of these two events respectively. We assume that the probability of these events occurring simultaneously is zero, thus,

$$\rho_{cd}(s) = \rho'_{cd}(s) + \rho''_{cd}(s) \tag{5.37}$$

Derivation of $\rho'_{cd}(s)$ and $\rho''_{cd}(s)$ are similar to derivation of $\rho_{dd}(s)$ above, except more involved, therefore they are given in the appendix B. From (B.8) and (B.15),

$$\rho_{cd}(s) = \frac{e^{-\bar{\phi}_s P_A} - e^{-\bar{\phi}_s}}{1-e^{-\bar{\phi}_s}} e^{-\bar{\phi}_s(1-P_A)} + \frac{e^{-(1-q_s)(1-\delta_r^d(0))\bar{\phi}_s} - e^{-(1-q_s)\bar{\phi}_s}}{1-e^{-(1-q_s)\bar{\phi}_s}}, \; 1 \le r < |s| \tag{5.38}$$

where q_s denotes the probability that a node in a cell of segment s does not have direct connectivity with the population of its preceding cell with mean population of $\bar{\phi}_s$. This probability has been derived in (4.18) as q_{r+1} for cells r and $r+1$ and it can be rewritten as,

$$q_s = \frac{(e^{-0.5\bar{\phi}_s} - e^{-\bar{\phi}_s})}{(1 - e^{-\bar{\phi}_s})} \tag{5.39}$$

P_A is probability that a node which is located in cell r-1, remains in its cell within the next Δt seconds and it is given by (B.11). We note that in derivation of $\rho^*_{cd}(s)$, two consecutive cells are boundary cells of segments s and $s+1$ which have different population densities of $\bar{\phi}_s$ and $\bar{\phi}_{s+1}$. Therefore, derivation of (5.38) will be modified as follows,

$$\rho^*_{cd}(s) = \frac{e^{-\bar{\phi}_s P_A} - e^{-\bar{\phi}_s}}{1 - e^{-\bar{\phi}_s}} e^{-\bar{\phi}_{s+1}(1-P_A)} + \frac{e^{-(1-q_s)(1-\delta^d_r(0))\bar{\phi}_{s+1}} - e^{-(1-q_s)\bar{\phi}_{s+1}}}{1 - e^{-(1-q_s)\bar{\phi}_{s+1}}} \tag{5.40}$$

This probability will be used later on to determine the communication availability of a path.

5.4 Cluster Analysis of Node Population in a Segment

In this section, we will give a cluster analysis of node population in a segment which will be extended later on to a path. As before, a cluster consists of a number of cells with connectivity which enables the flow of message traffic across the cells. In the following analysis, we are interested in finding the probability distribution of the number of clusters, the communication availability and unavailability in a segment at the steady state. We note that a cluster analysis here is more general than the one in section 4.3 in the previous chapter because that was able to give us only single cluster probability.

5.4.1 Birth Death Modeling of the Number of Clusters in a Segment

Next, we will model the number of clusters in a segment as a birth death process. Let the number of clusters denote the state of a segment. A birth in the cluster population will refer to the breakup of a cluster into two clusters. This will happen if two adjacent cells of a cluster lose their connectivity. We assume that the probability of the breaking up of a cluster simultaneously into more than two clusters is negligible within a very short time interval Δt. A death in the cluster population will refer to the merging of two adjacent clusters. This will happen if the border cells of the two clusters gain connectivity with each other. Whenever a birth or death takes place, the number of clusters will increase or decrease by one respectively. We assume that probability of concurrent births, deaths or birth and death are very small in the order of $o(\Delta t)$. Fig. 5.5, shows birth and death examples in the cluster populations.

We assume that the cluster birth and death will occur at the end of epoch durations. Since epoch durations are exponentially distributed, the time interval between births and deaths follow an Erlang distribution, however, the analysis will assume that this interval is exponentially distributed for mathematical convenience. This assumption will be justified by simulation results later on. Next, let us make the following definitions,

m_s = number of clusters in segment s, assuming nonzero node population, the range of m_s is given by $1 \le m_s \le |s|$.

$a_s(m)$ = birth rate when the cluster state is $m_s = m$.

$b_s(m)$ = death rate when the cluster state is $m_s = m$.

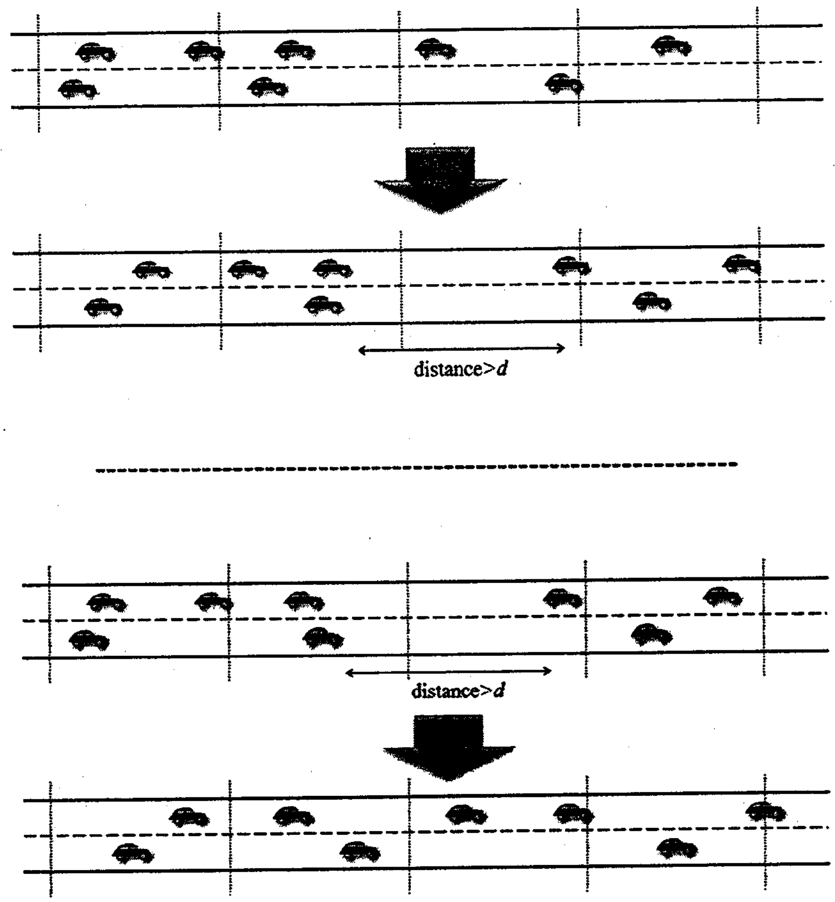


Fig. 5.5. Birth and death examples in the cluster populations

$P_s(m) = \Pr(m_s = m$ at the steady state), $1 \leq m \leq |s|$.

As explained before, the adjacent cells in a cluster have connectivity with each other. We note that if a cell is empty, then, it cannot have connectivity with the adjacent cells, thus, empty cells cannot be part of any cluster. The adjacent clusters are separated from each other either by empty cells or their border cells do not have connectivity with each other. The probability that cell r from segment s will be empty is given by $P_e = e^{-\bar{\phi}_s}$. The probability of having consecutive empty cells is very small even under light loading and it will be neglected here. Let E denote the number of empty cells given that there are m clusters, then, its maximum value will be $m-1$. Thus E clusters will be separated by empty cells, while $m-E$ of them by border cells that do not have connectivity with each other. The probability distribution of the number of empty cells will be given by Binomial distribution. When the cluster population size of segment s is in state $m_s = m$, a birth occurs in the next Δt second if one of the clusters splits into two parts. This will happen, if in a cluster, a cell's connectivity is lost. Therefore, the birth rate at the state $m_s = m$ is given as,

$$a_s(m) = \frac{1}{\Delta t}\sum_{E=0}^{m-1}\binom{m-1}{E} P_e^{\,E}(1-P_e)^{m-1-E}\rho_{dd}(s)^{m-E-1}\rho_{cc}(s)^{|s|-m-E-1}\rho_{cd}(s)(|s|-m-E), \quad 1 \le m < |s| \tag{5.41}$$

In the above, E clusters are separated by empty cells and $m-1-E$ of them by border cells without connectivity. $\rho_{cd}(s)$ is the probability that a cell connectivity will be lost and $\rho_{cc}(s) = 1-\rho_{cd}(s)$. The factor $(|s|-m-E)$ corresponds to the total number of cell connectivities.

Next, we present the death rate. At state m, a death occurs in the next Δt second, if any of the two adjacent clusters merge into a single cluster. Therefore, the death rate at the state $m_s = m$ is given by,

$$b_s(m) = \frac{1}{\Delta t}\sum_{E=0}^{m-1}\binom{m-1}{E} P_e^{\,E}(1-P_e)^{m-1-E}\rho_{dd}(s)^{m-E-2}\rho_{cc}(s)^{|s|-m-E}\rho_{dc}(s)(m-E-1), \quad 1 < m \le |s| \tag{5.42}$$

where $\rho_{dc}(s) = 1-\rho_{dd}(s)$.

Fig. 5.6, shows the state transition diagram of the birth-death model of cluster size in segment s.

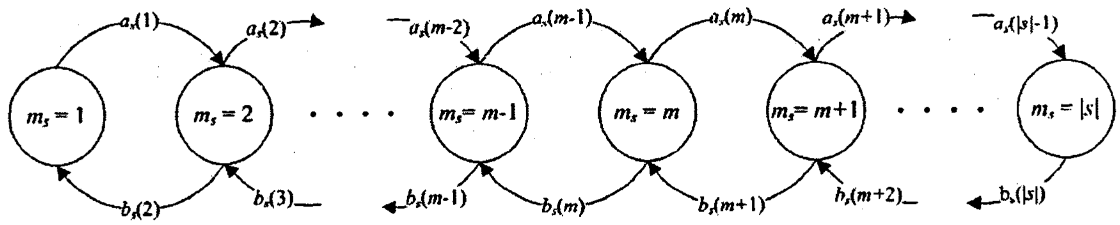


Fig 5.6. Birth-death state transition diagram of cluster size

Since we have modeled the cluster population size as a birth-death process, then, the distribution of the number of clusters at the steady state is given by the product form solution [100],

$$P_s(m) = P_s(1)\prod_{i=1}^{m-1}\frac{a_s(i)}{b_s(i)}, \qquad 1 < m \le |s| \tag{5.43}$$

where $P_s(1)$ is found from the normalization condition.

5.4.2 Derivation of Cluster Performance Measures for a Segment

In this subsection, we will derive mean holding and recurrence times for a given cluster state in segment s. Let us define,

$T_s(m)$ = cluster holding time or the time that the system spends in state $m_s = m$ in each visit and its mean value as $\overline{T}_s(m)$.

$f_{T_s(m)}(t)$ = pdf of the random variable $T_s(m)$.

$\overline{R}_s(m)$ = cluster mean recurrence time or the average time that it takes for the system to return to cluster state $m_s = m$ following its departure from this state.

First, let us determine the following probability,

$$\Pr(T_s(m) < t) = 1 - \Pr(T_s(m) > t)$$

$$= 1 - \Pr(\text{no birth or death before } t)$$

$$= 1 - \Pr(\text{no birth before time } t)\Pr(\text{no death before time } t)$$

$$= 1 - (e^{-a_s(m)t})(e^{-b_s(m)t})$$

$$\Pr(T_s(m) < t) = 1 - e^{-(a_s(m)+b_s(m))t} \tag{5.44}$$

Taking the derivative of the above gives the pdf of cluster holding time as,

$$f_{T_s(m)}(t) = \frac{1}{a_s(m)+b_s(m)} e^{-(a_s(m)+b_s(m))t} \tag{5.45}$$

which has an exponential distribution. Thus the cluster state mean holding time in state $m_s = m$,

$$\overline{T}_s(m) = \frac{1}{a_s(m)+b_s(m)}, \quad 1 \le m < |s| \tag{5.46}$$

We note that $\overline{T}_s(1)$ presents the mean communication availability time for segment s.

Next, we present mean recurrence time in state $m_s = m$, $\overline{R}_s(m)$. From [102], the mean recurrence time for a Markovian birth-death process is given by,

$$\overline{R}_s(m) = \frac{\overline{T}_s(m)}{P_s(m)} - \overline{T}_s(m) = \frac{\overline{T}_s(m)(1-P_s(m))}{P_s(m)}, \quad 1 \le m < |s| \tag{5.47}$$

5.5 Cluster Analysis of Node Population in a Path

In this section, we will extend the cluster analysis of the previous section to the node population in a path and determine the continuous communication availability time. Further, without loss of any generality, we assume that the two nodes which we are interested in determining their communication availability, are located in the first and the last cells of path Ω_k.

As discussed before, we will take the continuous amount of time that the node population spends in the single cluster state as the communication availability time. Let us define,

M_{Ω_k} = number of clusters in the node population of path Ω_k.

$P_{\Omega_k}(m) = \Pr(M_{\Omega_k} = m)$.

$T_{\Omega_k}(1)$ = continuous communication availability time in each visit and its mean value as $\overline{T}_{\Omega_k}(1)$.

$\overline{R}_{\Omega_k}(1)$ = mean communication unavailability time or the average time that takes for the path Ω_k to return to state one following its departure from this state.

First, we will determine the number of clusters in the path. We note that the number of clusters in path Ω_k may be less than sum of the number of clusters in its segments because of cluster merging, i.e.,

$$M_{\Omega_k} < \sum_{i=1}^{|\Omega_k|} m_i$$

If the border cells of two adjacent segments have connectivity with each other, then, the two clusters containing these cells will merge into a single cluster. Let g_s denote the probability that node populations of the border cells of consecutive segments s and $s+1$ will have connectivity with each other. In chapter 4, this probability has been derived for the cells r and $r+1$ in (4.21) and for the border cells of segments s and $s+1$ with population densities of $\bar{\phi}_s$ and $\bar{\phi}_{s+1}$, it can be written as,

$$g_s = 1 - e^{-\bar{\phi}_{s+1}(1-q_s)} \tag{5.48}$$

Where q_s is given by (5.39). Let us define b_s as independent Bernoulli variables that assume values of 1, 0 with probabilities g_s or $1-g_s$ respectively. Then the number of clusters in path Ω_k is given by,

$$M_{\Omega_k} = \sum_{s=1}^{|\Omega_k|} m_s - \sum_{s=1}^{|\Omega_k|-1} b_s \tag{5.49}$$

Let us also define the following PGFs,

$$P_{\Omega_k}(z) = E[z^{M_{\Omega_k}}] \tag{5.50}$$

$$P_s(z) = E[z^{m_s}] \tag{5.51}$$

$$B_s(z) = E[z^{b_s}] = g_s z + (1-g_s) \tag{5.52}$$

(5.49) gives the relationship between the above PGFs as,

$$P_{\Omega_k}(z) = \frac{\prod_{s=1}^{|\Omega_k|} P_s(z)}{\prod_{s=1}^{|\Omega_k|-1} B_s(z)} = \frac{\prod_{s=1}^{|\Omega_k|} P_s(z)}{\prod_{s=1}^{|\Omega_k|-1} [g_s z + (1-g_s)]} \tag{5.53}$$

In the above PGF, numerator and denominator are in the form of polynomial expressions and $P_{\Omega_k}(m)$ can be found by taking derivatives of (5.53) with numerical methods through the following formula,

$$P_{\Omega_k}(m) = \frac{1}{m!}\frac{d^m P_{\Omega_k}(z)}{dz^m}\bigg|_{z=0} \tag{5.54}$$

Next, we will determine mean communication availability time for path Ω_k. Let us assume that there is a single cluster in path Ω_k. This means that each segment of the path contains a single cluster and all the border cells of adjacent segments have connectivity with each other. Let us define $A_{\Omega_k}(1)$ as the cluster birth rate of path Ω_k when it is in single cluster state. A birth may occur as a result of breakup of the cluster in any of the segments or disconnection of any two consecutive border cells. As before, we assume that the probability of concurrent birth is negligible. As a result single cluster birth rate may be written as,

$$\Delta t A_{\Omega_k}(1) = \Delta t \sum_{i=1}^{|\Omega_k|} a_i(1) + \sum_{i=1}^{|\Omega_k|-1} \rho^*_{cd}(i) \tag{5.55}$$

Where $\rho^*_{cd}(i)$ is given by (5.40).

Similar to (5.44), if we assume that the time that the path will spend in the single cluster state is exponentially distributed,

$$\Pr(T_{\Omega_k}(1) > t) = e^{-A_{\Omega_k}(1)t}$$

$$\Pr(T_{\Omega_k}(1) < t) = 1 - e^{-A_{\Omega_k}(1)t}$$

$$= 1 - e^{-[\sum_{i=1}^{|\Omega_k|} a_i(1) + \frac{1}{\Delta t} \sum_{i=1}^{|\Omega_k|-1} \rho_{cd}^{*}(i)]t} \tag{5.56}$$

Therefore, the mean communication availability time for path Ω_k is given as,

$$\overline{T}_{\Omega_k}(1) = \frac{1}{\sum_{i=1}^{|\Omega_k|} a_i(1) + \frac{1}{\Delta t} \sum_{i=1}^{|\Omega_k|-1} \rho_{cd}^{*}(i)} \tag{5.57}$$

Mean communication unavailability time, $\overline{R}_k(1)$, can be found with a similar approach to (5.47) as following,

$$\overline{R}_{\Omega_k}(1) = \frac{\overline{T}_{\Omega_k}(1)(1 - P_{\Omega_k}(1))}{P_{\Omega_k}(1)} \tag{5.58}$$

5.6 Mean Packet Delay

In this section, first, we will determine the mean packet delay in a segment and, then, in a path. Assuming that a segment is in a single cluster state, then, the packet delay will determine the amount of time it takes to transfer a packet from the first to the last cell of the segment. The packet will advance by one cell at a time which will be referred to as a hop. The numbers of hops that a packet needs to travel will be equal to the number of cells in the segment. Next we will determine single hop mean packet delay.

We will assume that nodes use CSMA/CA MAC protocol [103] to communicate with each other, which is the proposed standard for VANET's applications. There have been various studies which analyze the performance of CSMA/CA protocol under different assumptions and we will make use of the readily available results in the literature. Most of these studies have considered a Markov chain approach to determine network throughput and delay [104-106 and the references therein]. We will use the results from [106] to determine single hop mean packet delay, because of their simplicity. That study assumes a constant packet transmission time which equals to J in units of the channel propagation delay (τ_p). It also assumes that each node has at most a single packet to transmit at anytime. We will make the same assumption; it will be assumed that when a packet is transmitted from one cell to the next, it will be received by a node with no other packets. Assuming j nodes in cell r, the average single hop packet delay from [106] is given by,

$$\overline{\Gamma}(r \mid N_r = j) = [\frac{ja[J(1-x)+1]-Jy}{Jy}]J\tau_p \tag{5.59}$$

where,

$$x = (1-\alpha)^j \tag{5.60}$$

$$y = j\alpha(1-\alpha)^{j-1} \tag{5.61}$$

In (5.59), the fraction in square brackets is the average number of transmissions until a successful transmission occurs, $J\tau_p$ is the packet transmission time where τ_p is the propagation delay for a cell. a was defined as probability that an idle node will receive a new packet for transmission during packet transmission time and it is the parameter which specifies a node's packet arrival rate. Finally, α is the probability that a user requests a transmission during a propagation time, i.e. $\alpha = a / J$.

Next, we uncondition the average single hop packet delay wrt the cell node population,

$$\overline{\Gamma}(r) = \sum_{j=0}^{\infty} \overline{\Gamma}(r \mid N_r = j) \frac{e^{-\overline{\phi}_s} \overline{\phi}_s^{\,j}}{j!} \tag{5.62}$$

Substituting (5.59) in (5.62), we have,

$$\overline{\Gamma}(r) = \sum_{j=0}^{\infty} [\frac{ja[J(1-x)+1] - Jy}{Jy}] J\tau_p \frac{e^{-\overline{\phi}_s} \overline{\phi}_s^{\,j}}{j!},$$

where $a = \alpha J$. Then, we factor $J\tau_p$ out,

$$\overline{\Gamma}(r) = J\tau_p \sum_{j=0}^{\infty} [\frac{j\alpha[J(1-x)+1] - y}{y}] \frac{e^{-\overline{\phi}_s} \overline{\phi}_s^{\,j}}{j!},$$

distributing the summation, we have,

$$\overline{\Gamma}(r) = [\sum_{j=0}^{\infty} \frac{j\alpha[J(1-x)+1]}{y} \frac{e^{-\overline{\phi}_s} \overline{\phi}_s^{\,j}}{j!} - \sum_{j=0}^{\infty} \frac{e^{-\overline{\phi}_s} \overline{\phi}_s^{\,j}}{j!}] J\tau_p,$$

second summation in above is the infinite sum of a Poisson distribution and it equals one,

$$\overline{\Gamma}(r) = [\sum_{j=0}^{\infty} \frac{j\alpha[J(1-x)+1]}{y} \frac{e^{-\overline{\phi}_s} \overline{\phi}_s^{\,j}}{j!} - 1] J\tau_p,$$

distributing the summation again, we have,

$$\overline{\Gamma}(r) = [\sum_{j=0}^{\infty} \frac{j\alpha J(1-x)}{y} \frac{e^{-\overline{\phi}_s} \overline{\phi}_s^{\,j}}{j!} + \sum_{j=0}^{\infty} \frac{j\alpha}{y} \frac{e^{-\overline{\phi}_s} \overline{\phi}_s^{\,j}}{j!} - 1] J\tau_p,$$

next, we replace x and y with their equations according to (5.60) and (5.61),

$$\overline{\Gamma}(r) = [\sum_{j=0}^{\infty} \frac{J[1-(1-\alpha)^j]}{(1-\alpha)^{j-1}} \frac{e^{-\overline{\phi}_s} \overline{\phi}_s^{\,j}}{j!} + \sum_{j=0}^{\infty} \frac{1}{(1-\alpha)^{j-1}} \frac{e^{-\overline{\phi}_s} \overline{\phi}_s^{\,j}}{j!} - 1] J\tau_p$$

distributing the first summation and some factorization, we have,

$$\overline{\Gamma}(r)=[(1-\alpha)J\sum_{j=0}^{\infty}\frac{1}{(1-\alpha)^j}\frac{e^{-\overline{\phi}_s}\overline{\phi}_s^{\,j}}{j!}-\sum_{j=0}^{\infty}J(1-\alpha)\frac{e^{-\overline{\phi}_s}\overline{\phi}_s^{\,j}}{j!}+(1-\alpha)\sum_{j=0}^{\infty}\frac{1}{(1-\alpha)^j}\frac{e^{-\overline{\phi}_s}\overline{\phi}_s^{\,j}}{j!}-1]J\tau_p$$

$$=[(1-\alpha)(1+J)\sum_{j=0}^{\infty}\frac{1}{(1-\alpha)^j}\frac{e^{-\overline{\phi}_s}\overline{\phi}_s^{\,j}}{j!}-J(1-\alpha)\sum_{j=0}^{\infty}\frac{e^{-\overline{\phi}_s}\overline{\phi}_s^{\,j}}{j!}-1]J\tau_p$$

second summation is the sum of the probabilities of a Poisson distribution, therefore,

$$\overline{\Gamma}(r)=[(1-\alpha)(1+J)\sum_{j=0}^{\infty}\frac{1}{(1-\alpha)^j}\frac{e^{-\overline{\phi}_s}\overline{\phi}_s^{\,j}}{j!}-J(1-\alpha)-1]J\tau_p$$

$$=[(1-\alpha)(1+J)e^{-\overline{\phi}_s}\sum_{j=0}^{\infty}(\frac{\overline{\phi}_s}{1-\alpha})^j\frac{1}{j!}-J(1-\alpha)-1]J\tau_p$$

$$=[(1-\alpha)(1+J)e^{-\overline{\phi}_s}e^{\frac{\overline{\phi}_s}{1-\alpha}}-J(1-\alpha)-1]J\tau_p$$

$$\overline{\Gamma}(r)=[(1-\alpha)(1+J)e^{\frac{\overline{\phi}_s\alpha}{1-\alpha}}-J(1-\alpha)-1]J\tau_p \tag{5.63}$$

Since all the cells in a segment has the same node density, the average total packet delay for the segment s is given by,

$$\overline{\Gamma}_s=|s|\overline{\Gamma}(r) \tag{5.64}$$

Next, we will determine end to end average packet delay for the path Ω_k assuming that it is in the single cluster state. This delay is given by the sum of the average packet delays for the segments along the path. Let $\overline{D}_{\Omega_k}$ denote this delay, then,

$$\overline{D}_{\Omega_k}=\sum_{s=1}^{|\Omega_k|}\overline{\Gamma}_s \tag{5.65}$$

5.7 Numerical Results

In this section, we present some numerical results regarding the analysis in this chapter, together with some simulation results to confirm the accuracy of approximations. The main approximations are independent nodes connectivity within two consecutive cells presented in section 5.3.3 and nodes epoch time termination upon its arrival to a new segment, stated at the beginning of the section 5.2.2.

Next we specify the parameters' value for the mobility and environment models. We assume that the arrival rate of the nodes to the network of fig. 5.1, is given by the following matrix,

$$\lambda = \begin{bmatrix} \lambda_{11} & 0 & 0 & 0.4 & 0.2 \\ 0.12 & 0.12 & 0.12 & 0 & 0 \\ 0 & 0.12 & 0 & 0 & 0 \\ 0.12 & 0 & 0.12 & 0 & 0 \end{bmatrix} \tag{5.66}$$

where the entry (i, j), λ_{ji}, specifies the arrival rate of the nodes to the highway segment S_{ji}. The number of columns in the matrix is determined by the highway which has the largest number of segments. λ_{11} is set as a parameter and presents the traffic arrival rate to the first segment of the highway one. We set $p_{ji,j'i'} = 0.8$ for consecutive segments except for $p_{1213} = p_{2332} = p_{2333} = p_{12,21} = 0.4$, $p_{1451} = p_{41,31} = 1$ and $p_{ji,j'i'} = 0$ for all nonconsecutive segments. The state dependent mean node speed (5.9), is chosen as the decreasing function of,

$$\mu(h_{ji}) = \frac{v_0}{h_{ji}^{\;0.1}}. \tag{5.67}$$

We note that a real function for state dependent mean node speed can be derived by studying the statistics of a real highway. Thus the selection of above function is to carry the numerical result purpose. The parameter v_0 corresponds to the nodes maximum allowed mean speed. Finally, epoch rate and the standard deviation of a node's speed are set to constant values of $\beta=1$ and $\sigma=3$ respectively.

First, we will determine the communication availability between two nodes located at the end cells of the path Ω_1 defined in (5.1). This path is shown in Fig. 5.7 with more details. We assume that lengths of the segments in this path are set to $[R_{11}, R_{12}, R_{21}, R_{22}, R_{23}, R_{33}]=[1,2,1,2,3,1]$ kms. As before, we choose the transmission range of a node, d, such that there will be an integer number of cells within the highway segments. Initially, we set λ_{11} to the value of 0.5 nodes/sec. From the simultaneous solution of (5.2) with the given values, the vector of the total arrival rates to the segments located on this path is given by,

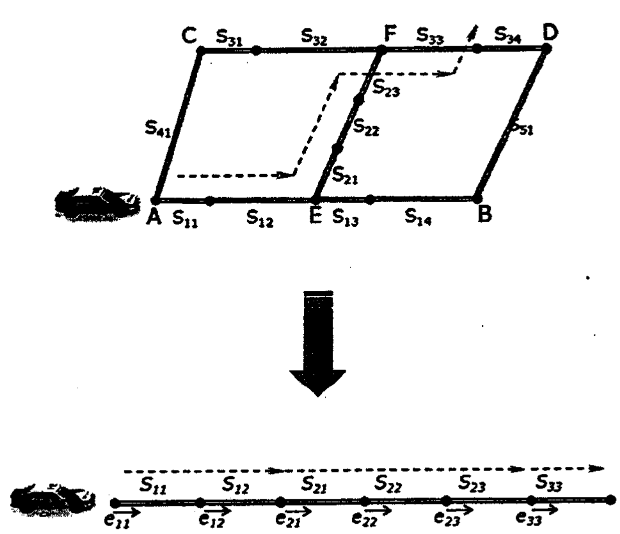


Fig. 5.7. Segmentation of path Ω_1 in the highway network

$$[e_{11}, e_{12}, e_{21}, e_{22}, e_{23}, e_{33}] = [0.5, 0.5, 0.32, 0.376, 0.42, 0.36] \tag{5.68}$$

In the following, the presented results will always be analytical, unless otherwise indicated. First, we show the effect of state dependent in comparison to independent mean node speed. Fig. 5.8 presents the mean node population for joint distributions of (5.13) and (5.15) within each of the 50 cells in the path Ω_1 when the transmission range is d = *200* m and $v_0 = 25$ m/sec for the two cases. In the independent case, the mean node speed is constant and it has been set to the maximum of dependent node speed, $v_0 = 25$ m/sec. Mean node population for the dependent case has been found by solving (5.14) for the given parameter values. As expected, there are discontinuities at the beginning of each segment. As may be seen mean node density in a cell with dependent speed is higher than that of independent speed, since the node density is inversely proportional to the mean speed. This figure shows the importance of considering a state dependent mean speed in the connectivity analysis of ad hoc networks as the variations in the node density impact the performance parameters.

Fig. 5.9 presents both the analytical and simulation results for the probability distribution of the number of clusters in the path for states from (5.54). The results have been plotted for two values of maximum allowed mean speed $v_0 = 20$, 25 m/sec (72, 90 km/hr). The node transmission range and λ_{11} are set to d = 200 m and $\lambda_{11} = 0.5$ nodes/sec respectively. The maximum number of clusters that may occur in a path equals to the number of virtual cells, which is 50. As may be seen, the probabilities of observing more than six clusters have negligible values for this example. Further, the probability of observing a single cluster is higher for slower than the faster node speed because, in the former case, the node density will be higher in the path. The presented results show that the analytical and simulation results are in close agreement. This validates two main approximations in the analysis which have been mentioned earlier on.

Figs. 5.10, 5.11 and 5.12 present the analytical and some simulation results for probability of observing a single cluster, continuous mean communication availability and unavailability times. The simulation results have been marked with asterisk. The results have been plotted as a function of transmission range with the maximum mean speed as a parameter. From fig. 5.10, as the node speed increases, the transmission range has to increase significantly in order to maintain a constant probability of single cluster. Fig. 5.11 shows that the continuous mean communication availability time increases logarithmically with transmission range for any given value of maximum node speed. On the other hand, from Fig. 5.12 mean communication unavailability time decreases rapidly with the transmission range for any given value of maximum node speed. At any given transmission range, mean communication availability is higher at the lower speeds and the opposite is true for mean communication unavailability. For the path under consideration, it may be seen that the mean communication availability time will be in the order of ten of seconds, while unavailability time less than ten seconds for transmission range of 250m. It is clear that this will be minimum transmission range in order to have periods of time, long enough, that provides stable communications.

We note that the results may be plotted with arrival rates or another mobility variable as a parameter instead of maximum mean speed v_0. Figs. 5.13, 5.14 and 5.15 present again the analytical results for probability of observing a single cluster, mean communication availability and unavailability times. This time, the results are plotted as a function of the transmission range with node arrival rate λ_{11} as a parameter for a fixed value of the maximum mean speed $v_0 = 25$ m/s. Since the effect of increasing arrival rate is to increase the node density, increasing the arrival rate has the same effect as decreasing the mean node speed. Therefore, comments identical to that given above for figs. 5.10, 5.11 and 5.12 will apply here with the highest arrival rate corresponding to the lowest mean speed.

In practice there may be multiple communication paths between a source and a destination. For example, let us assume that in the model of highway network given in fig. 5.1, a vehicle located at point A would like to communicate with a vehicle at point X. As may be seen, there are three paths between these two service points, Ω_1, Ω_2 and Ω_3. As a result, we need to determine the communication availabilities of these three paths. Let us assume that the segments lengths of the paths Ω_2 and Ω_3 are $[R_{11}, R_{12}, R_{13}, R_{14}, R_{51}, R_{34}]=[1,2,1,2,3,1]$ and $[R_{41}, R_{31}, R_{32}, R_{33}]=[6,1,2,1]$ km respectively while the other parameter values remain same as before. Figs. 5.16, 5.17 and 5.18 present the analytical results for probability of observing a single cluster, mean communication availability and unavailability times for the three paths. The results have been plotted as a function of transmission range. It may be seen that for any given transmission range; path Ω_2 has the highest mean availability time and should be the path of choice for communications. The question is, in practice, how a user may find out about this. A possible solution may be that the vehicles keep track of the recent communication availability of the paths through exchange of short control packets. Further, this information can be obtained from some potential sensor networks installed in the highways. These types of sensor networks are responsible for monitoring the traffic condition in the highways and they are aware of arrival rates, speed and other mobility related parameters.

Figs. 5.19 and 5.20 present the average packet delay over the path Ω_1, when it is available, for node's maximum mean speeds of $v_0 = 30$ and 25 m/sec respectively. The results have been plotted as a function of the input packet arrival rate of a node (a) in the path with the transmission range as a parameter and the node arrival rate to the first segment of the path as $\lambda_{11} = 0.5$ nodes/sec. The packet transmission time is set to 20 times of propagation delay, J=20 as in the original paper [106], and propagation delay, τ_p in (5.63) has been chosen as the amount of time it takes for light to travel a cell's length, d.

The five curves in each figure correspond to transmission ranges of d=100, 125, 166, 250 and 500 m. It may be seen that for a given maximum mean speed and packet input rate, average packet delay increases with the value of the transmission range because of the increased contention for channel access due to higher node population in the cells. While mean packet delay varies directly with the transmission range, it has been shown above that mean path availability varies directly with the transmission range. Thus the transmission range for high path availability should be chosen in such a way that the mean delay remains acceptable. Same conclusion is also true for a constant transmission range, when maximum mean speed decreases.

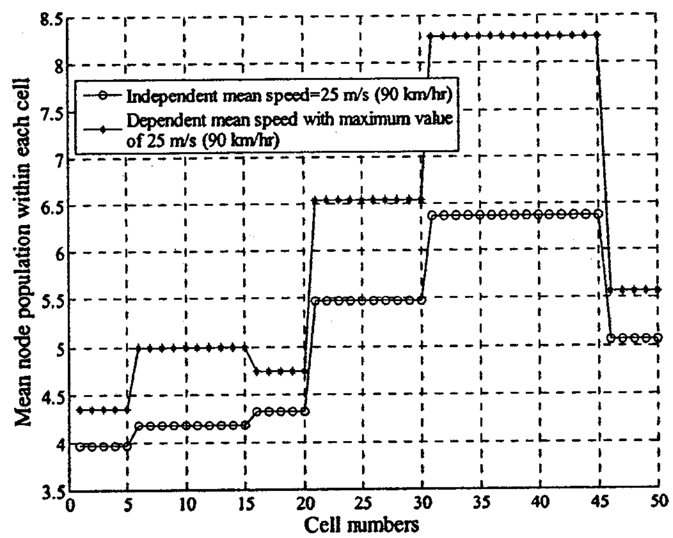


Fig. 5.8. Mean node population within cells of the path Ω_1 for transmission range of d=200 meters.

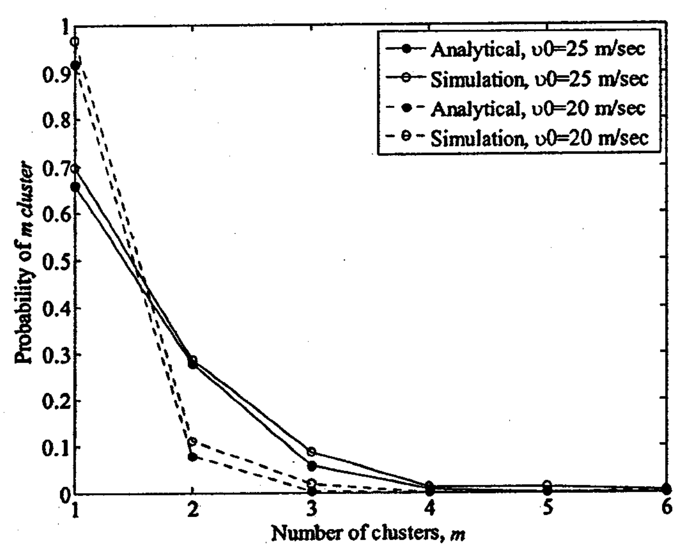


Fig. 5.9. Probability distribution of the number of clusters, for transmission range of d=200 m and maximum mean node speed of $v_0 = 20$ m/sec (72 km/hr) and $v_0 = 25$ m/sec (90 km/hr).

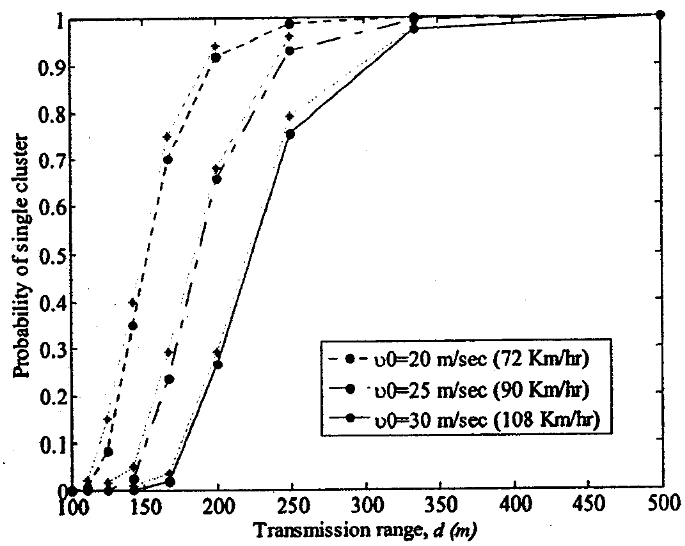


Fig. 5.10. Probability of observing a single cluster along the path Ω_1, as a function of the transmission range, with maximum mean speed (v_0) as a parameter.

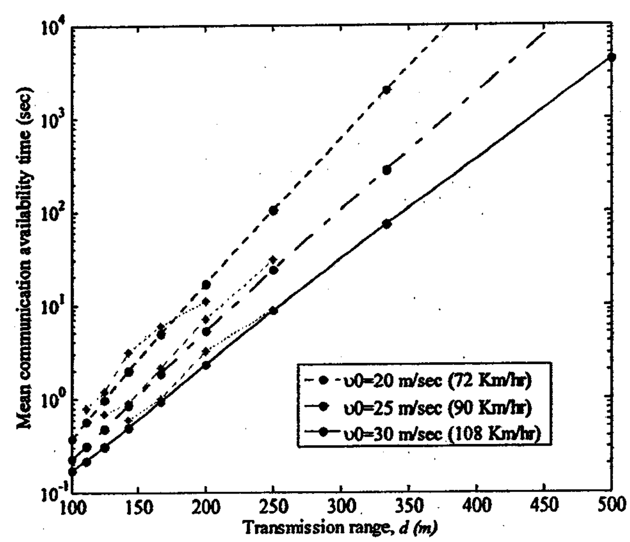


Fig. 5.11. Mean communication availability time for the path Ω_1, as a function of the transmission range, with maximum mean speed (v_0) as a parameter.

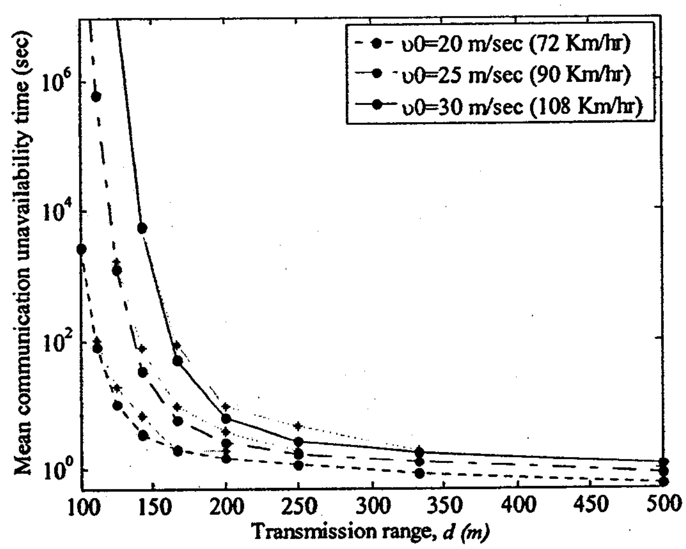


Fig. 5.12. Mean communication unavailability time for the path Ω_1, as a function of the transmission range, with maximum mean speed (v_0) as a parameter.

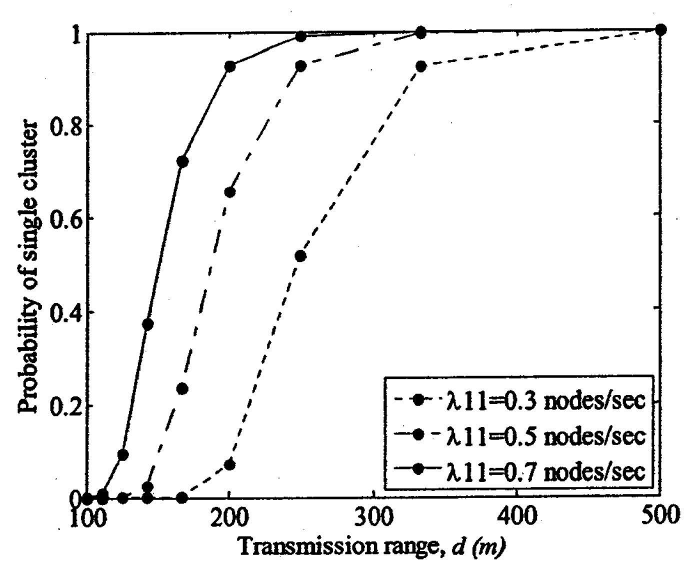


Fig. 5.13. Probability of observing a single cluster along the path Ω_1, as a function of the transmission range, with arrival rate λ_{11} as a parameter and maximum mean speed $v_0 = 25$ m/s.

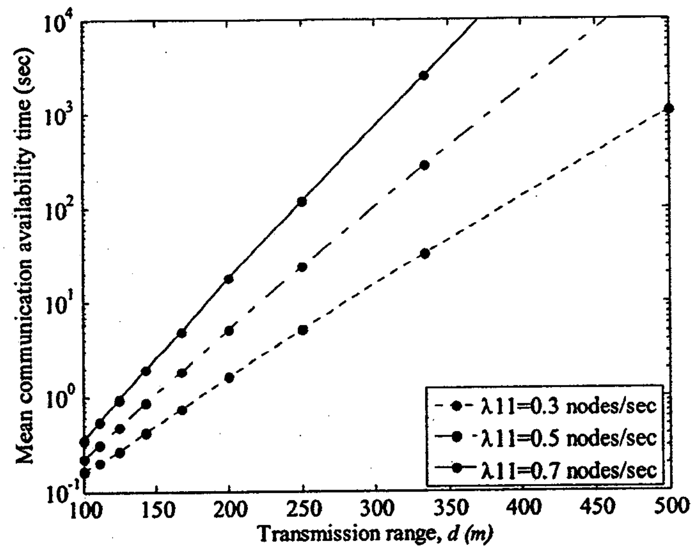


Fig. 5.14. Mean communication availability time for the path Ω_1, as a function of transmission range, with arrival rate λ_{11} as a parameter and maximum mean speed $v_0 = 25$ m/s.

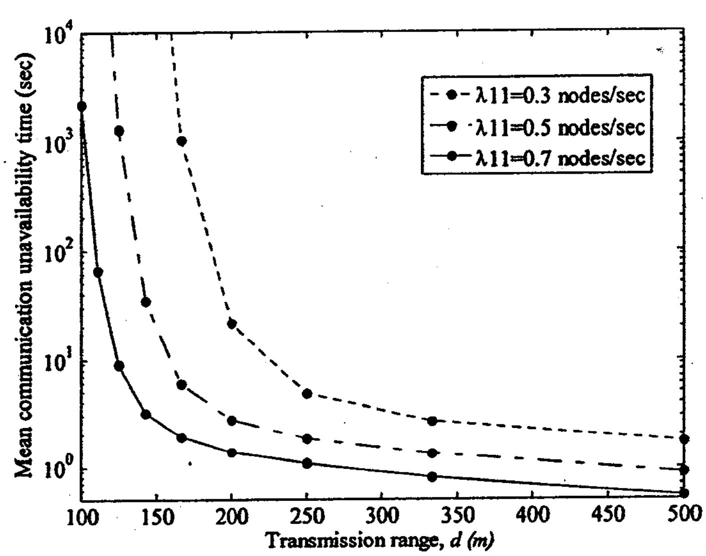


Fig. 5.15. Mean communication unavailability time for the path Ω_1, as a function of the transmission range, with arrival rate λ_{11} as a parameter and maximum mean speed $v_0 = 25$ m/s.

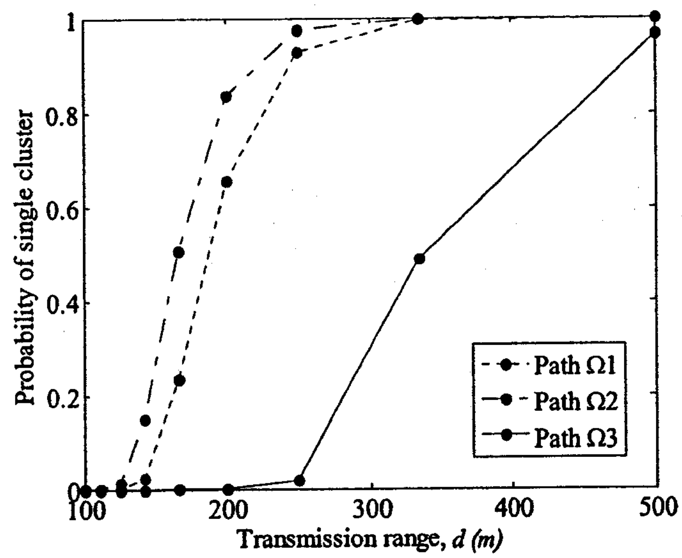


Fig. 5.16. Probability of observing a single cluster along the three paths Ω_1, Ω_2 and Ω_3, as a function of the transmission range.

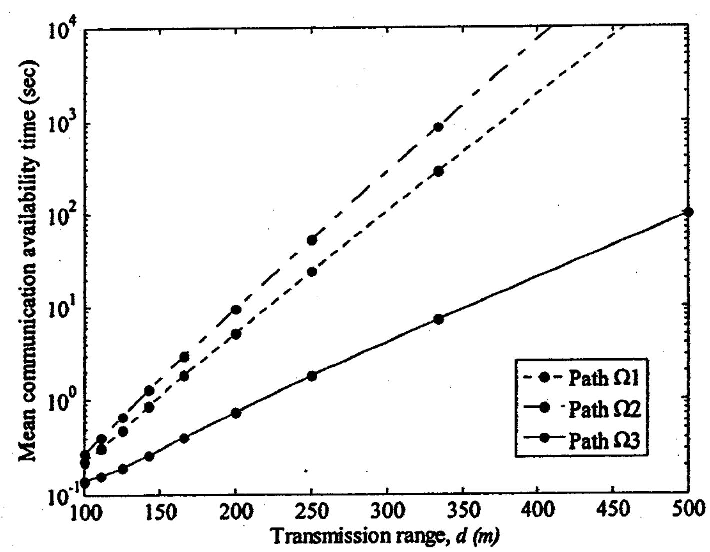


Fig. 5.17. Mean communication availability time for the three paths Ω_1, Ω_2 and Ω_3, as a function of the transmission range.

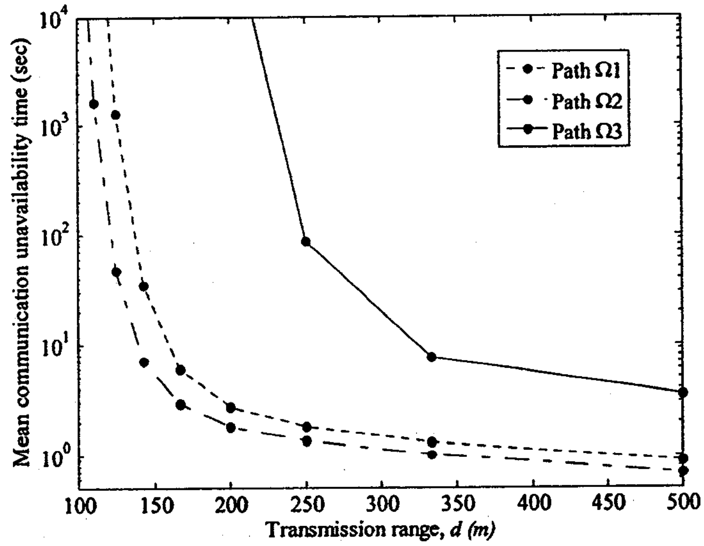


Fig. 5.18. Mean communication unavailability time for the three paths Ω_1, Ω_2 and Ω_3, as a function of the transmission range.

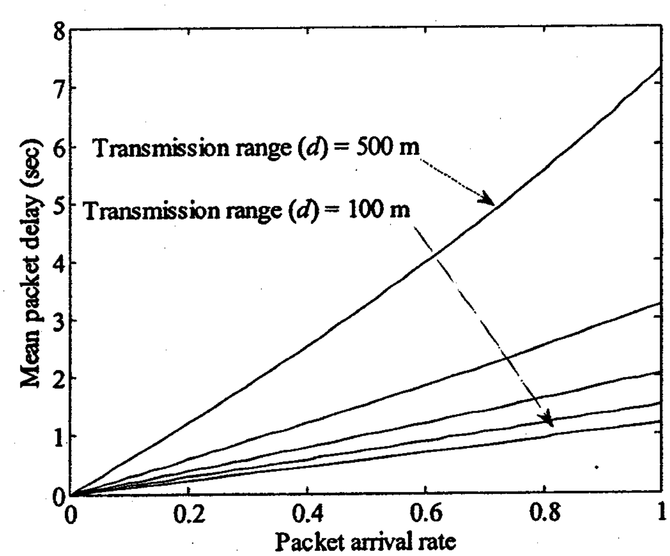


Fig. 5.19. Average packet delay in path Ω_1 as a function of node packet arrival rate with transmission range (*d*) as a parameter. Maximum mean speed is $v_0 = 30$ m/s and node arrival rate is $\lambda_{11} = 0.5$ nodes/s.

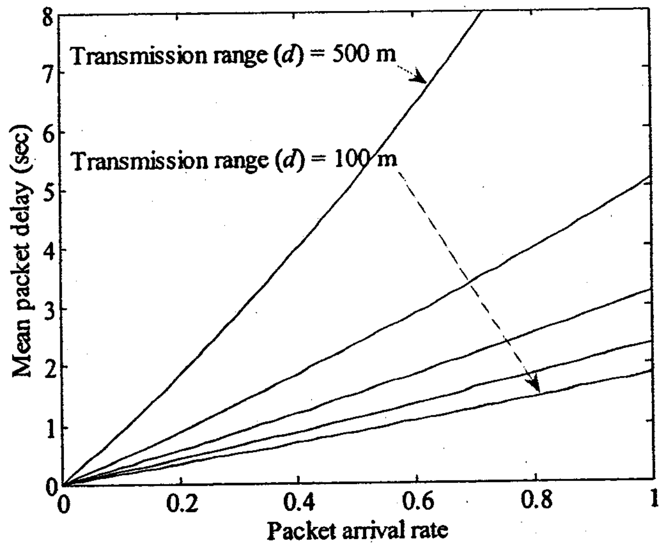


Fig. 5.20. Average packet delay in path Ω_1 as a function of node packet arrival rate with transmission range (d) as a parameter. Maximum mean speed is $v_0 = 20$ m/s and node arrival rate is $\lambda_{11} = 0.5$ nodes/s

5.8 Summary

In this chapter, we studied continuous communication path availability in vehicular ad hoc networks. We have defined a network of highways as our environment model. Then, we have determined the joint distribution of node populations in the highway segments by modeling the system as a BCMP network of queues with state dependent service times. Following that, we derived the distribution of number of clusters in the node population of a path through a birth-death process analysis. This model enabled us to determine the mean of continuous communication path availability and unavailability times. At the end, we have derived the mean packet delay for a path in the highway network.

We have shown that the mean path availability time as a function of nodes' transmission range is logarithmic. The numerical examples show that continuous mean availability time may be on the order of tens of seconds while continuous mean unavailability time an order of magnitude less than that when the transmission range is about 250m. It has been determined that mean packet delay varies directly with transmission range due to increased contention for channel access. Therefore, the transmission range should be high enough for acceptable communication availability but low enough for acceptable mean packet delay. We also present simulation results that confirm the accuracy of the results.

In this analysis, we assumed unidirectional highways, while similar to the remark given at the end of chapter 4, the effect of identical VANETs which serve to the users traveling in the opposite directions in highways can be considered. The system may allow the vehicles to be part of a single communication system independent of the direction of their travels. Therefore, the results may be combined to give communication availability in such a system. The integrated system will achieve higher communication availability. However, cluster stability of the integrated system will be lower compare to one directional system carrying equivalent amount of traffic.

Chapter 6

Conclusion & Future Work

6.1 Conclusions

In this work, we have studied the network connectivity and continuous communication path availability of the mobile nodes in VANETs at the steady state, in the presence of a mobility model.

We assume that a network path will have communication connectivity only when the node population in the path forms a single cluster. This gives us percentage of time that a path will be available for communications. However, a high value of this performance measure is not sufficient to ensure that a call between two users may be completed successfully. We also need to know duration of continuous communication path availability. The objective of this thesis has been to quantify these two measures.

First, we have assumed a single highway with several traffic entry and exit points. We have also assumed that the flow of traffic is unidirectional. The arrival and departure of the nodes from the highway make the size of node population a random variable as in the real scenarios. Further, we assume that the highways have multiple lanes that allow passing of any number of vehicles each other simultaneously. Then, we determined the probability distributions of a node's location and size of node population on the highway as well as within the transmission range of a random node at the steady state. The latter distribution will be useful in designing effective MAC protocols for VANETs. Afterwards, we have derived the probability that a new arriving node or a random node will see entire node population in a single cluster. It is seen that, probability of a single cluster is lower at higher node speeds than lower speeds for constant arrival rates. As a result, nodes may have to increase their transmission ranges at higher speeds to achieve a high probability of network connectivity.

Then, we studied continuous communication path availability in vehicular ad hoc networks in a network of highways. We determined the joint distribution of node populations in highway segments by modeling the system as a BCMP network of queues with state dependent service times. Then, we derived the distribution of number of clusters in the node population of a path through a birth-death process analysis. From this we determine mean communication path availability and unavailability times and, then, mean packet delay for a path. The numerical results show that for a given maximum mean speed and packet input rate, average packet delay and communication path availability time increase with the value of the transmission range. Thus the transmission range for high path availability should be chosen in such a way that the mean delay remains acceptable. We have also presented simulation results that confirmed the accuracy of the analysis.

In the highway models, identical VANETs will serve to the users traveling in the opposite directions. The system may allow the vehicles to be part of a single communication system independent of the direction of their travels. The results of this chapter may be combined to give the mean cluster size and probability of a single cluster in such a system. The integrated system will achieve higher network connectivity and such a system will be needed for collision avoidance. However, cluster stability of the integrated system will be lower compare to one directional system carrying equivalent amount of traffic.

Finally, this work may be helpful in studying the optimal node transmission range assignment, routing algorithms, optimization of cross layer design schemes and MAC protocols in VANETs.

6.2 Recommended Future Research

The result of this work can be used to improve further studies in the area of vehicular ad hoc networking. Next, we mention some of them.

6.2.1 Determination of Optimal Node Transmission Range in the Presence of Channel Model

The given analysis has not taken into consideration the model of air channel; therefore, the effects of fading and interference have not been included. We may study the effects of these parameters by defining a transmission range $d(h)$ as a function of channel state h which mainly depends on the number of other transmissions and consequently node population in a cell. As a result, the length of a cell will become a random variable.

6.2.2 Routing Algorithms

The results of this work may be used to study the performance of different routing algorithms for VANETs. Multi-clustering effect and introduction of safety applications pose new challenges for vehicular ad hoc networking. These issues have only been addressed in the literature through simulation. The derived results for durations of continuous path availability and unavailability and mean packet delay when the path is available, provide new tools for analytical study of routing algorithms.

6.2.3 VANETs Throughput

Throughput of VANETs depends on the number of nodes in a cell as they are carrier of the messages. Higher number of nodes in the path results in higher contention to the channel and decreases the throughput, while lower number of nodes may increase multi-clustering which consequently causes network outage.

For example, [34] which has been reviewed in chapter 2, proposes a novel MAC protocol, called VMESH. This protocol employs a distributed beaconing scheme to make neighborhood awareness and dynamic channel resource reservation. Then, they find the network throughput by dividing the amount of information successfully transmitted in one reservation by the duration of reservation interval. However, they assume the nodes are always in a single cluster state and they neglect durations of communication path unavailability which affects the throughput. Therefore, throughput calculation can be improved by considering the proposed birth-death Markovian model of the number of clusters.

6.2.4 Cross Layer Optimization of VANETs Data link and Physical Layers

Ad hoc networks such as VANETs and Sensor networks have posed new challenges for the design of effective protocol stacks. MAC protocols in ad hoc networks are mainly responsible for management and control of the scarce radio spectrum in a fair and non-centralized way. Therefore, MAC layer in ad hoc networks should be adaptive to the changing conditions in other layers such as physical. In VANETs, the problem of designing an effective MAC protocol is even more challenging as these types of networks experience faster topology change, compare to the MANETs. Vehicular ad hoc networks require an intelligent cross layered functionality to improve the total network performance.

A possible future study can be design of effective network layer and data link layer protocols that take into consideration physical layer requirements. In other words, feedback from physical layer on channel quality can be shared with upper layers which may be used in optimization of various algorithms such as access mechanisms, prioritizing, scheduling and minimum transmission range assignment.

Although cross layer optimization techniques have received a lot of attention in the literature recently, they may cause some system performance degradation themselves. The main reason is introduction of some functional algorithms between the layers which must frequently adjust some parameters in those layers; therefore, extra care must be taken into account.

References:

[1] A. Bria, F. Gessler, O. Queseth, R. Stridh, M. Unbehaun, et. al., "4th-Generation Wireless Infrastructures: Scenarios and Research Challenges", *IEEE Personal Communications*, vol. 8, issue 6, pp. 25 - 31, Dec. 2001.

[2] J. Lind, "Infrastructure evolution – the machines behind seamless mobility", *Center for Information and Communication Research, Stockholm School of Economics,* Sep. 2001.

[3] F.Y Loo, "Ad hoc network: prospects and challenges", *Aoyama Morikawa Laboratory report*, Rinkou Paper, Jan 2004.

[4] T S. Yousefi, M. S. Mousavi, M. Fathy, "Vehicular ad hoc networks (VANETs) challenges and perspectives", *in proc. Of 6th IEEE International Conference on ITS Telecommunications Proceedings*, pp. 761 - 766, June 2006.

[5] J. Jakubiak, Y. Koucheryavy, "State of the Art and Research Challenges for VANETs", *5th IEEE Consumer Communications and Networking Conference, CCNC*, pp. 912-916, Jan. 2008.

[6] The Canadian council of Motor Transport Administrators, "Canadian motor vehicle traffic collision statistic", Oct. 2004.

[7] National Highway Traffic Safety Administration, NHTSA, "The Economic Impact Of Motor Vehicle Crashes 2000", May 2002.

[8] Car2Car Communication Consortium, http://www.carto-car.org , date accessed : Jan. 2008.

[9] Network on Wheels project (NoW), http://www.network-on-wheels.de , date accessed : Jan. 2008.

[10] ASTM E2213-03, "Standard specification for telecommunications and information exchange between roadside and vehicle systems – 5 GHz band dedicated short range communications (DSRC) medium Access Control (MAC) and physical layer (PHY) specifications," *ASTM Int'l*, july 2003.

[11] D. Jiang, L. Delgrossi, "IEEE 802.11p: Towards an International Standard for Wireless Access in Vehicular Environments", *IEEE Vehicular Technology Conference, VTC*, pp. 2036-2040, May 2008.

[12] W. Chen, S. Cai, "Ad hoc peer-to-peer network architecture for vehicle safety communications", *IEEE Communications Magazine,* vol. 43, issue 4, pp. 100-107, April 2005.

[13] L. Gavrilovska, V. Atanasovski, "Ad Hoc Networking Towards 4G: Challenges and QoS Solutions", *7th International Conference on Telecommunications in Modern Satellite, Cable and Broadcasting Services,* vol. 1, pp. 71 – 80, 28-30 Sept. 2005.

[14] D.N. Alparslan, K.A. Sohraby, "Generalized Random Mobility Model for Wireless Ad Hoc Networks and Its Analysis: One-Dimensional Case", *IEEE/ACM Transactions on Networking,* vol. 15, issue 3, pp. 602 – 615, June 2007.

[15] H. Jerome, F. Fethi, B. Christian, "Mobility Models for Vehicular Ad Hoc Networks: A Survey and Taxonomy", *Research report RR-06-168, Institut Eurecom, Department of Mobile Communications*, March 2006.

[16] D. Yu, L. Hui, "Influence of mobility models on node distribution in ad hoc networks," *in Proc. Of International Conference on Communication Technology*, vol. 2, pp. 985-989, April 2003.

[17] A. Mahajan, N. Potnis, K. Gopalan, A.I.A. Wang, "Evaluation of Mobility Models for Vehicular Ad-Hoc Network Simulations", *in the Proceedings of the 2nd IEEE International Workshop on Vehicular Technology*, pp. 347 - 353, 2006.

[18] Z. Zhensheng, "Routing in intermittently connected mobile ad hoc networks and delay tolerant networks: overview and challenges", *IEEE Communications Surveys & Tutorials*, vol. 8, issue 1, pp. 24-27, First Quarter 2006.

[19] C. Siva Ram Murthy and B. S. Manoj, Ad Hoc Wireless Networks: Architectures and Protocols, *Prentice Hall Communications Engineering and Emerging Technologies Series*, 2004. Page 177.

[20] N. Wisitpongphan, O.K. Tonguz, J.S. Parikh, P. Mudalige, F. Bai, V.Sadekar, "Broadcast storm mitigation techniques in vehicular ad hoc networks", *IEEE Wireless Communications*, vol. 14, issue 6, pp. 84 - 94, Dec. 2007.

[21] J.J. Blum, A. Eskandarian, L.J. Hoffman, "Challenges of intervehicle ad hoc networks", *IEEE Transactions on Intelligent Transportation Systems*, vol. 5, issue 4, PP. 347 – 351, Dec. 2004.

[22] J. Chennikara-Varghese, W. Chen, O. Altintas, S. Cai, "Survey of Routing Protocols for Inter-Vehicle Communications", *Third Annual International Conference on Mobile and Ubiquitous Systems: Networking & Services*, pp. 1-5, July 2006.

[23] A. Sikora, "Design challenges for short-range wireless networks", *IEE Proceedings of Communications*, vol. 151, issue 5, pp. 473-479, Oct. 2004.

[24] M. Raya, J.P. Hubaux, "The Security of Vehicular Ad Hoc Networks", *3rd ACM Workshop on Security of Ad hoc and Sensor Networks*, 2005.

[25] P. Wex, J. Breuer, A. Held, T. Leinmuller, L. Tim, "Trust Issues for Vehicular Ad Hoc Networks", *IEEE Vehicular Technology Conference*, pp. 2800-2804, May 2008.

[26] B Parno, A Perrig, "Challenges in Securing Vehicular Networks", *Proceedings of ACM HotNets-IV*, 2005.

[27] I. Broustis, M. Faloutsos, "Routing in Vehicular Networks: Feasibility, Modeling, and Security", accepted for publication, *International Journal of Vehicular Technology*, Article ID 267513, 8 pages, 2008. doi:10.1155/2008/267513.

[28] C. Comaniciu, H.V. Poor, "On the capacity of mobile ad hoc networks with delay constraints", *IEEE Transactions on Wireless Communications*, vol. 5, issue 8, pp. 2061-2071, Aug. 2006.

[29] P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks", *IEEE Transactions On Information Theory*, vol. 46, pp. 388-404, 2000.

[30] J. Zhang, S. Liew, "Capacity Improvement of Wireless Ad Hoc Networks with Directional Antenna", *IEEE 63rd Vehicular Technology Conference, VTC*, vol. 2, pp. 911 – 915, spring 2006.

[31] J. Cho, S. Kim, S. Chong, "Infrastructure Support Increases the Capacity of Ad Hoc Wireless Networks", *IEEE 62nd Vehicular Technology Conference, VTC*, vol. 4, pp. 2663 – 2668, Sept. 2005.
[32] M. Grossglauser, D.N.C Tse, "Mobility Increases the Capacity of Ad Hoc Wireless Networks", *IEEE/ACM Transactions on Networking*, vol 10, Issue 4, pp. 477 – 486, Aug. 2002.
[33] L. Jun, J. McNew, R. Trerotola, "Wireless LAN Extensions for Vehicular Environments and the Control Channel Capacity", *IEEE 60th Vehicular Technology Conference, VTC*, vol. 5 pp. 3210 – 3213, Sept. 2004.
[34] Z. Yunpeng, S. Zang, L. Stibor, B. Walke, H. Reumerman, A. Barroso, "A Novel MAC Protocol for Throughput Sensitive Applications in Vehicular Environments", *IEEE 65th Vehicular Technology Conference, VTC*, pp. 2580 – 2584, April 2007.
[35] A. Chandra, V. Gummalla, and J. O. Limb, "Wireless Medium Access Control Protocols", *IEEE Communications Surveys*, pp. 2-15, 2000.
[36] Z. Yao, P. Fan, Z. Cao, "An enhanced CSMA-CA mechanism for multihop ad hoc networks", 5th *IEEE International Symposium on Multi-dimensional Mobile Communications*, vol. 2, pp. 966-970, Aug 2004.
[37] W. Li, J. Wei, S. Wang, "An Evolutionary-Dynamic TDMA Slot Assignment Protocol for Ad Hoc Networks", *IEEE Wireless Communications and Networking Conference, WCNC*, pp. 138 – 142, March 2007.
[38] F. Borgonovo, L. Campelli, M. Cesana, L. Coletti, "MAC for ad-hoc intervehicle network: service and performance", *IEEE Vehicular Technology Conference, VTC*, vol.5, pp. 2789-2793, October 2003.
[39] H. Menouar, F. Filali, M.A. Lenardi, "survey and qualitative analysis of MAC protocols for vehicular ad hoc networks, *IEEE Wireless Communications*, vol. 13, issue 5, pp. 30-35, Oct. 2006.
[40] E. M. Royer, S. J. Lee, C. E. Perkins, "The effects of MAC protocols on ad hoc networks communication,", *IEEE Wireless Communications and Networking Conference, WCNC*, pp. 543-548, Sep. 2000.
[41] L., Bononi, M. Di Felice, "A Cross Layered MAC and Clustering Scheme for Efficient Broadcast in VANETs", *IEEE Internatonal Conference on Mobile Ad hoc and Sensor Systems, MASS*, pp. 1-8, Oct. 2007.
[42] G. Korkmaz, E. Ekici, F. Özgüner, "A Cross-Layer Multihop Data Delivery Protocol With Fairness Guarantees for Vehicular Networks", *IEEE Transactions on Vehicular Technology*, vol. 55, May 2006.
[43] B. Wiegel, Y. Gunter, H. Grossmann, "Cross-Layer Design for Packet Routing in Vehicular Ad Hoc Networks", *IEEE 66th Vehicular Technology Conference, VTC*, pp. 2169 - 2173, Sept. 2007.
[44] S.A. Jafar, "Too much mobility limits the capacity of wireless ad hoc networks," *IEEE Transactions on Information Theory*, vol. 51, No. 11, pp. 3954-3965, Nov. 2005.
[45] M. Bhatt, R. Chokshi, S. Desai, et. al., "Impact of mobility on the performance of ad hoc wireless networks," *IEEE Vehicular Technology Conference*, vol. 5, pp. 3025-3029, Oct. 2003.

[46] L. Xiaolong, D.P. Agrawal, Z. Qing-An, "Impact of mobility on the performance of mobile ad hoc networks," *Wireless Telecommunications Symposium*, pp. 154-160, 2004.
[47] Z. Guanghui, L. Jiandong, C. Yanhui, L. Jing, "Effect of mobility on the critical transmitting range for connectivity in wireless ad hoc networks," *19th International Conference on Advanced Information Networking and Applications*, vol. 2, pp. 9-12, March 2005.
[48] J. L. Boudec, M.Vojnovic, "Perfect Simulation and stationary of a class of mobility models," In *Proceedings of IEEE INFOCOM*, vol. 4, pp. 2743-2754, March 2005.
[49] C. Sungsoon, H. Cho, J.P. Hayes, "Impact of mobility on connection in ad hoc networks," *Wireless Communications and Networking Conference*, vol. 3, pp. 1650-1656, March 2005.
[50] C. Vargas – Rosales, A. Lopez, "Evaluating connectivity in wireless ad-hoc networks", *IEEE Communication Society*, 2004.
[51] O. Dousse, P.Thiran, M. Hasler, "Connectivity in ad hoc and hybrid networks", *In Proc. Of IEEE INFOCOM*, New-York, June 2002.
[52] N. Bansal, Z. Liu, "Capacity, delay and mobility in wireless Ad-Hoc Networks", *Twenty-Second Annual Joint Conference of the IEEE Computer and Communications Societies, INFOCOM*, vol. 2, pp. 1553 – 1563, April 2003.
[53] O. Dousse, P.Thiran, F. Baccelli, "Impact of interferences on connectivity in ad hoc networks", *IEEE/ACM Transactions on Networking*, vol. 13, issue 2, pp. 425-436, April 2005.
[54] E. Perevalov, R. Blum, "Delay limited capacity of ad hoc networks: Asymptotically optimal transmission and relaying strategy", *Twenty-Second Annual Joint Conference of the IEEE Computer and Communications Societies, INFOCOM*, vol. 2, pp. 1575 – 1582, April 2003.
[55] P. Gupta, P.R. Kumar, "Critical power for asymptotic connectivity", *In the Proceedings of 37th IEEE Conference on Decision and Control*, vol.1, 1998.
[56] L. E. Miller, "Distributional properties of inhibited random positions of mobile radio terminals", *Conference on Information Science and Systems, Princeton University*, March 20-22, 2002.
[57] L. E. Miller, "Distribution of link distances in a wireless network", *Journal of Research of the National Institute of Standards and Technology*, vol. 106, no. 2, March-April 2001.
[58] L. E. Miller, "Joint distribution of link distances", *Conference on information science and systems, The John Hopkins University*, March 12-14, 2003.
[59] G. Grimmett, "Percolation", *Springer*, Second Edition, 1999.
[60] C. H. Foh, B. S. Lee, "A closed form network connectivity formula for one-dimensional MANETs", *IEEE International Conference on Communications*, vol 6, pp. 20-24, June 2004.
[61] A. Ghasemi, S. Nader-Esfahani,, "Exact Probability of Connectivity in One-Dimensional Ad Hoc Wireless Networks", *IEEE Communications Letters*, vol. 10, issue 4, pp. 251 – 253, Apr 2006.

[62] J. Li, L.L.H. Andrew, H. F. Chuan, M. Zukerman, M.F. Neuts, "Meeting Connectivity Requirements in a Wireless Multihop Network", *IEEE Communications Letters*, vol. 10, issue 1, pp. 19 - 21, Jan. 2006.

[63] E. Hyytiä, P. Lassila, J. Virtamo, "Spatial node distribution of the random waypoint mobility model with applications", *IEEE Transactions on Mobile Computing*, vol 5, pp. 680-694, June 2006.

[64] C. H. Foh, G. Liu, B. S. Lee, B. Seet, et. al., "Network connectivity of one-dimensional MANETs with random waypoint movement", *IEEE Communication Letters*, vol. 9, pp. 31-33, Jan 2005.

[65] I.W.H. Ho, K. K. Leung, J. W. Polak, R. Mangharam, "Node Connectivity in Vehicular Ad Hoc Networks with Structured Mobility", *32nd IEEE Conference on Local Computer Networks, LCN*, pp. 635 - 642, Oct. 2007.

[66] M.M. Artimy, W.J. Phillips, W. Robertson, "Connectivity with static transmission range in vehicular ad hoc networks", In *Proc. of the 3rd Annual Communication Networks and Services Research Conference, 2005*, pp. 237-242, 16-18 May 2005.

[67] P. Santi, D. M. Blough, "The critical transmitting range for connectivity in sparse wireless ad hoc networks", *IEEE Transactions on Mobile Computing*, vol 2, pp. 25-39, Jan-March 2003.

[68] M.K.M. Ali, M. Khabazian, "A performance modeling of connectivity in ad hoc networks", *IEEE Wireless Communications and Networking Conference, WCNC*, vol. 1, pp. 361 - 366, 2006.

[69] J. Shengming, H. Dajiang, R. Jianqiang, "A prediction-based link availability estimation for mobile ad hoc networks", *IEEE INFOCOM Proceedings*, vol. 3, pp. 1745-1752, April 2001.

[70] H. Dajiang, R. Jianqiang, "A prediction-based link availability estimation for routing metrics in MANETs", *IEEE/ACM Transactions on Networking*, vol. 13, pp. 1302-1312, Dec. 2005.

[71] J. Shengming, "An enhanced prediction-based link availability estimation for MANETs", *IEEE Transactions on Communications*, vol. 52, pp. 183, Feb 2004.

[72] D. Yu, L. Hui, " Lower bound on path availability in ad hoc network", 57th IEEE Semiannual Vehicular Technology Conference, vol. 3, pp. 1758-1762, April 2003.

[73] B.H. Sathyaraj, R.C. Doss, "Route maintenance using mobility prediction for mobile ad hoc networks", *IEEE International Conference on Mobile Adhoc and Sensor Systems Conference*, pp. 6, Nov. 2005.

[74] V. Jain, M. Jain, "Queuing network model for link and path availability of ad hoc networks Wireless and Optical Communications Networks", 2006 *IEEE IFIP*, pp.5, April 2006.

[75] T. Quang-My, A. Dadej, S. Perreau, "Link Change and Generalized Mobility Metric for Mobile Ad-hoc Networks", *Military Communications Conference, MILCOM*, pp. 1-7, Oct. 2006.

[76] R. Sharma, R. Bhagavathula, K. Namuduri, R. Pendse, "Link availability and its effects on the capacity of mobile ad hoc wireless networks, an analytical approach", *IEEE Conference on Local Computer Networks*, pp. 295-299, Nov. 2006.

[77] W. Jie, D. Fei, "Efficient broadcasting with guaranteed coverage in mobile ad hoc networks", *IEEE Transactions on Mobile Computing*, vol. 4, pp. 259-270, May-June 2005.
[78] T. Taleb, E. Sakhaee, A. Jamalipour, K. Hashimoto, N. Kato, Y. Nemoto, "A Stable Routing Protocol to Support ITS Services in VANET Networks", *IEEE Transactions on Vehicular Technology*, vol. 56, issue 6, pp. 3337 - 3347, Nov. 2007.
[79] J. W. Tantra, C. H. Foh, D. Qiu, "On Link Reliability in Wireless Mobile Ad Hoc Networks", *IEEE Vehicular Technology Conference, VTC*, pp. 1 - 5, Sept. 2006.
[80] P. Bucciol, E. Masala, N. Kawaguchi, K, Takeda,, J.C. De Martin, "Performance Evaluation of H. 264 Video Streaming over Inter-Vehicular 802.11 Ad Hoc Networks", *IEEE 16th International Symposium on Personal, Indoor and Mobile Radio Communications, PIMRC*, vol. 3, pp. 1936-1940, Sept. 2005.
[81] T. Camp, J. Boleng, V. Davis, "A survey of mobility models for ad hoc network Research", *Wireless Communication & Mobile Computation (WCMC)*, vol 2, no. 5, 2002.
[82] G. Lin, G. Noubir, "Mobility models for ad hoc network simulation", *Twenty-third Annual Joint Conference of the IEEE Computer and Communications Societies, INFOCOM*, vol. 1, March 2004.
[83] B. Jabbari, Y Zhou, F. Hillier, "Simple random walk models for wireless terminal movement", *IEEE Vehicular Technology Conference*, vol. 3, pp. 1784 – 1788, 1999.
[84] W.Navidi, T. Camp, "Stationary distributions for the random waypoint model", *IEEE Trans. On Mobile Computing*, vol. 3, 2004.
[85] F.K. Karnadi, M. H. Zhi, L. Kun-chan, "Rapid Generation of Realistic Mobility Models for VANET", *IEEE Wireless Communications and Networking Conference, WCNC*, pp. 2506 - 2511, March 2007.
[86] A. B. McDonald, T. Znati, "A mobility-based framework for adaptive clustering in wireless Ad-Hoc networks", *IEEE Journal on Selected Areas in Communication*, vol. 17, No 8, August 1999.
[87] H. Qiang, G. M. Raymond, J. B. Bassingthwaighte, "On two-dimensional Brownian motion and fractional Brownian random field", *Journal of Phys. A:Math. Gen.*, vol.31, P.527-535, 1998.
[88] E. Weisstein, *Random Walk, Wolfram Research*, http://mathworld.wolfram.com /RandomWalk.html, accessed date: Oct. 2004.
[89] Karlin and Taylor, "A first course in stochastic processes", *Academic Press*, second edition, 1975.
[90] F. Spitzer, "Principles of Random Walk", *The University Series in Higher Mathematics*, D. Van Nostrand Company, INC., 1964.
[91] W. Feller, "An Introduction to probability theory and its application", *John Wiley*, 3d ed., 1968.
[92] J. Tian, J. Hahner, C. Becker, et al., "Graph-Based mobility model for ad hoc network simulation", *in Proceedings of IEEE 35th Annual Simulation Symposium*, pp. 337-344, April 2002.

[93] K. Chiang, N. Shenoy, "A 2-D random-walk mobility model for location-management studies in wireless networks", *IEEE Transactions on Vehicular Technology*, vol. 53, issue: 2, pp. 413-424, March 2004.
[94] I.F. Akyildiz, L. Yi-Bing, L. Wei-Ru, C. Rong-Jaye, "A new random walk model for PCS networks", *IEEE Journal on Selected Areas in Communications*, vol. 18, issue 7, pp. 1254-1260, July 2000.
[95] K. Chiang; N. Shenoy, "A random walk mobility model for location management in wireless networks", *12th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, vol. 2, pp. E-43 - E-48, Sept. 2001.
[96] J. Härri, F. Filali, C. Bonnet, M. Fiore, "VanetMobiSim: Generating Realistic Mobility Patterns for VANETs", *in Proceedings of the 3rd International Workshop on Vehicular Ad hoc Networks*, pp. 96 - 101, 2006.
[97] A. K. Saha, D.B. Johnson, "Mobility Models for Vehicular Ad Hoc Network Simulations", *in proceedings of the 1st ACM International Workshop on Vehicular Ad hoc Networks*, pp. 91 - 97, 2004.
[98] D.R. Cox, P.A.W. Lewis, *The Statistical Analysis of Series of Events*, pp. 27, John Wiley, 1966.
[99] D. Gross, C.M. Harris, *Fundamentals of Queueing Theory*, John Wiley & Sons, 1974.
[100] L. Kleinrock, *Queueing Systems*, vol. 1, John Wiley & Sons, 1975.
[101] J. F. Hayes, T.V. J. Ganesh Babu, *Modeling and Analysis of Telecommunications Networks*, pp.150, John Wiley, 2004.
[102] R. A. Howard, *Dynamic Probabilistic Systems, Volume II: SemiMarkov and Decision Processes*, pp. 735, John Wiley, 1971.
[103] IEEE Computer Society LAN MAN Standards Committee, ed., *IEEE Standard for Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications*, IEEE Std 802.11-1999, The Institute of Electrical and Electronics Engineers, NY, 1999.
[104] T.S. Ho, K.C. Chen, "Performance analysis of IEEE 802.11 CSMA/CA medium access control protocol", 7th *IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, vol. 2, pp. 407-411, 1996.
[105] Z. Hadzi-Velkov, B. Spasenovski, "An analysis of CSMA/CA protocol with capture in wireless LANs", *IEEE Wireless Communications and Networking*, vol.2, pp. 1303-1307, March 2003.
[106] F. Gebali, A.J. Al-Sammak, "Modeling CSMA/CA protocol for wireless channels that use collaborative codes modulation", *IEE Proceeding on Communications*, vol. 153, pp. 405-409, June 2006.

Appendix A

Simulation Program Description

In this section, we describe the simulation software which is used to verify the analysis introduced in the previous sections. This is an event-driven simulation program written in the MATLAB software environment. We have performed multiple independent simulation runs and each run terminates after a specified number of nodes goes through the network. In each run, the statistics have been collected after a warm up period in order to guarantee that the system has reached to the steady state. At the end, the collected statistics has been averaged over multiple runs. The arrival of the nodes in each stream is according to a Poisson process and each node travels through the network according to the mobility model.

Fig. A.1 presents the flow chart of the simulation algorithm. In this algorithm an array, called Event_scheduler keeps tracks of all future events. Two types of events have been defined. Type 1 corresponds to the arrival of a new node and Type 2 to the termination of an epoch. The node information is stored in an array called Node_array. This information consists of node and stream IDs, present epoch duration, node speed and last recorded location of the node. Whenever a type 1 event happens, a new entry for the new node will be made in the Node_array while whenever a type 2 event happens, location for the specific node will be updated in the Node_array.

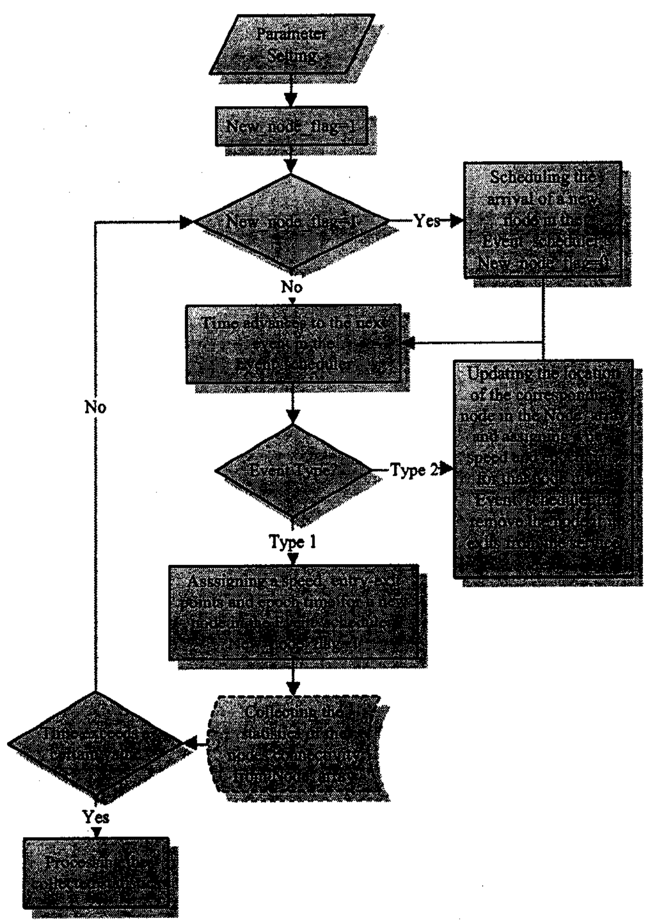


Fig. A.1. Flowchart of the simulation.

Appendix B

Derivation of $\rho_{cd}(s)$

In this appendix, we derive probability $\rho_{cd}(s)$ which has been defined in section 5.3 as,

$\rho_{cd}(s)$=Pr(the cells r and r+1 of segment s will become disconnected at $t+\Delta t$ | they have connectivity at t).

$\rho_{cd}(s)$ has been expressed as the sum of two probabilities, $\rho'_{cd}(s)$ and $\rho''_{cd}(s)$ in (5.37). These probabilities have been defined as,

$\rho'_{cd}(s)$=Pr(the distance between node populations of cells r and r+1 will be larger than d at $t+\Delta t$ | they have connectivity at t), where d is the node transmission range.

$\rho''_{cd}(s)$ = Pr(the node populations of either cell r or r+1 will become zero at $t+\Delta t$ | they have connectivity at t).

Next we determine each of these probabilities.

- **Derivation of $\rho'_{cd}(s)$**

We note that if the adjacent cells r and $r+1$ have connectivity with each other at any time, then, their node populations must be nonzero. Let $\tilde{N}_r$ denote the node population of cell r of segment s, then, it will be given by a normalized Poisson distribution with PGF,

$$\tilde{N}_r(z) = \frac{e^{\bar{\phi}_s(z-1)} - e^{-\bar{\phi}_s}}{1 - e^{-\bar{\phi}_s}} \qquad 1 \le r \le |s| \tag{B.1}$$

The node population of each of the two cells may be divided into two groups, those nodes which have direct connectivity with at least a single node of the other cell and those which have indirect connectivity. Let us refer to these subpopulations in cells r and $r+1$ as direct and indirect groups and denote their sizes with random variables $\tilde{N}_r^d$ $[\tilde{N}_{r+1}^d]$ and N_r^i $[N_{r+1}^i]$ respectively. We note that direct group population sizes must be nonzero because of connectivity assumption between cells. Further, probability that in segment s, a node of cell $r+1$ will not have direct connectivity with cell r is given by (5.39). Therefore, each node will belong to direct and indirect groups with parameters $(1-q_s)\bar{\phi}_s$ and $q_s\bar{\phi}_s$ respectively. From (B.1), the PGFs of the population sizes of the two groups of cell $r+1$ will be given by,

$$\tilde{N}_{r+1}^d(z) = \frac{e^{(1-q_s)\bar{\phi}_s(z-1)} - e^{-(1-q_s)\bar{\phi}_s}}{1 - e^{-(1-q_s)\bar{\phi}_s}} \tag{B.2}$$

$$\tilde{N}_{r+1}^i(z) = e^{q_s\bar{\phi}_s(z-1)} \tag{B.3}$$

Same conclusion is also valid for the populations of cell r.

Now, we are ready to determine $\rho'_{cd}(s)$, which may be expressed as,

$$\rho'_{cd}(s) = \prod_{i=1}^{4} a_i \tag{B.4}$$

where, conditional probabilities a_i have been defined below,

a_1= Pr(direct population of cell r+1 ($\tilde{N}^d_{r+1}$) will lose its connectivity to the direct population of cell r ($\tilde{N}^d_r$) by the time t+Δt | they have connectivity at t).

a_2= Pr(indirect population of cell r+*1* (N^i_{r+1}) will remain disconnected to the indirect population of cell r (N^i_r) at t+Δt | they do not have connectivity at t).

a_3= Pr(direct population of cell r+*1* ($\tilde{N}^d_{r+1}$) will remain disconnected to the indirect population of cell r (N^i_r) at t+Δt | they do not have connectivity at t).

a_4= Pr(indirect population of cell r+*1* (N^i_{r+1}) will remain disconnected to the direct population of cell r ($\tilde{N}^d_r$) at t+Δt | they do not have connectivity at t).

Next, we will give derivation of the first of these probabilities. Let us define following random variables and PGFs,

δ^d_r : the number of nodes from direct population of cell r that will continue to have connectivity to at least a member of direct population of cell r+1 at t+Δt, given that they had connectivity at time t. Let us denote its PGF as $\delta^d_r(z) = E[z^{\delta^d_r}]$.

u^d_{r+1} : the number of nodes from direct population of cell r+1 that keep their connectivity with the direct population of cell r at t+Δt given that they had connectivity at time t. Let us define its PGF as, $U^d_{r+1}(z) = E[z^{U^d_{r+1}}]$.

We note that each node will continue to have connectivity with probability p_{cc} given by (5.25). Then, the PGF $\delta_r^d(z)$ is given by,

$$\delta_r^d(z) = \tilde{N}_r^d(z)\Big|_{z = p_{cc}z + 1 - p_{cc}}$$

$$\delta_r^d(z) = \frac{e^{(1-q_s)\bar{\phi}_s p_{cc}(z-1)} - e^{-(1-q_s)\bar{\phi}_s}}{1 - e^{-(1-q_s)\bar{\phi}_s}}$$

$$\delta_r^d(0) = \frac{e^{-(1-q_s)\bar{\phi}_s p_{cc}} - e^{-(1-q_s)\bar{\phi}_s}}{1 - e^{-(1-q_s)\bar{\phi}_s}} \tag{B.5}$$

We note that $1 - \delta_r^d(0)$ gives the probability that a node in direct population of cell $r+1$ keeps its connectivity with the nodes at cell r at $t+\Delta t$. Therefore,

$$U_{r+1}^d(z) = \tilde{N}_{r+1}^d(z)\Big|_{z = [1 - \delta_r^d(0)]z + \delta_r^d(0)} \tag{B.6}$$

$$U_{r+1}^d(z) = \frac{e^{(1-q_s)(1-\delta_r^d(0))\bar{\phi}_s(z-1)} - e^{-(1-q_s)\bar{\phi}_s}}{1 - e^{-(1-q_s)\bar{\phi}_s}} \tag{B.7}$$

We note that a_1 is given by $a_1 = U_{r+1}^d(0)$. The other three probabilities may be derived similarly but they will not be given here. The numerical results show that the values of the other three probabilities are approximately one with less than 1% error. Therefore, we have,

$$\rho'_{cd}(s) \sim U_{r+1}^d(0). \tag{B.8}$$

- **Derivation of $\rho''_{cd}(s)$**

Next, we will determine $\rho''_{cd}(s)$, the probability that cell r or $r+1$ will become empty at $t+\Delta t$, given that they were occupied and had connectivity to each other at time t. As explained before, the probability of two consecutive cells with nonzero populations becoming empty within a very short period of time is negligible. Therefore, we need to determine only the probability that one of these cells will become empty. Let us determine this probability for the cell r. Given that cell r is occupied at time t, it will become empty if all of its nodes travel outside the boundary of this cell and there are no arrivals to it from the cell r-1 within the next Δt seconds. Since, these two events are independent of each other, $\rho''_{cd}(s)$ may be expressed as,

$$\rho''_{cd}(s) = \Pr(\text{cell } r \text{ will empty by the time } t+\Delta t \mid \text{it is occupied at time } t)$$

$$= \Pr(b_A = 0)\ \Pr(b_o = 0) \tag{B.9}$$

where,

b_A = number of arrivals from cell r-1 within next Δt.

b_o = number of occupants of cell r within next Δt.

Next, we will determine each of the above probabilities. We let $B_A(z)$ and $B_o(z)$ denote the PGFs of the random variables b_A and b_o respectively. First, let us find the probability of no arrival from cell r-1 within next Δt. Again, letting Y_{r-1} denote the location of a node in cell r-1 at time t, this node will still be in its present cell within the next Δt seconds, if $Y_{r-1} + V_{r-1}\Delta t < (r-1)d$. Let P_A denote the probability of this event, then,

$$P_A = \Pr(Y_{r-1} + V_{r-1}\Delta t < (r-1)d)$$

$$= \Pr(V_{r-1}\Delta t < (r-1)d - Y_{r-1}). \tag{B.10}$$

We note that $V_{r-1}\Delta t$ is a normally distributed random variable with mean $\mu(h_{jl})\Delta t$ and variance $\sigma^2\Delta t^2$ and Y_{r-1} is uniformly distributed given by equation (4.16). Thus,

$$P_A = \int_{y_{r-1}=(r-2)d}^{(r-1)d} \frac{1}{d} \int_{v^*=0}^{(r-1)d-y_{r-1}} f_{v^*}(v^*) dv^* dy_{r-1}, \tag{B.11}$$

where $f_{v^*}(v^*)$ denotes a normal distribution with the given parameters. We may evaluate this probability with the numerical methods. Then, the desired probability is given by,

$$\Pr(b_A = 0) = B_A(z)\big|_{z=0}$$

where,

$$B_A(z) = N_{r-1}(z)\big|_{z=(1-P_A)z+P_A}$$

or

$$\Pr(b_A = 0) = e^{-\bar{\phi}_r(1-P_A)} \tag{B.12}$$

Next we find probability that all of the nodes in cell r leave this cell by the time $t+\Delta t$. A node will leave cell r if $Y_r + V_r\Delta t > rd$, therefore,

$$\Pr(Y_r + V_r\Delta t < rd) = 1 - \Pr(Y_r + V_r\Delta t > rd)$$

$$= 1 - P_A \tag{B.13}$$

Thus the probability that cell r will become empty may be found as above,

$$B_o(z) = \tilde{N}_r(z)\Big|_{z=P_A z+(1-P_A)} = \frac{e^{\bar{\phi}_s P_A (z-1)} - e^{-\bar{\phi}_s}}{1-e^{-\bar{\phi}_s}}$$

$$\Pr(b_o = 0) = B_o(z)\Big|_{z=0} = \frac{e^{-\bar{\phi}_s P_A} - e^{-\bar{\phi}_s}}{1-e^{-\bar{\phi}_s}} \tag{B.14}$$

Substituting (B.12) and (B.13) in (B.9), we find $\rho''_{cd}(s)$ as,

$$\rho''_{cd}(s) = \frac{e^{-\bar{\phi}_s P_A} - e^{-\bar{\phi}_s}}{1-e^{-\bar{\phi}_s}} e^{-\bar{\phi}_s (1-P_A)} \tag{B.15}$$

Substituting (B.8) and (B.15) in (5.37), finally, we determine the probability $\rho_{cd}(s)$ given by (5.38).