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Calculating the Optimal Step in Shift-Reduce Dependency Parsing: From Cubic to Linear Time

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Abstract

We present a new cubic-time algorithm to calculate the optimal next step in shift-reduce dependency parsing, relative to ground truth, commonly referred to as dynamic oracle. Unlike existing algorithms, it is applicable if the training corpus contains non-projective structures. We then show that for a projective training corpus, the time complexity can be improved from cubic to linear.

1 Introduction

A deterministic parser may rely on a classifier that predicts the next step, given features extracted from the present configuration (Yamada and Matsumoto, 2003; Nivre et al., 2004). It was found that accuracy improves if the classifier is trained not just on configurations that correspond to the ground-truth, or ''gold'', tree, but also on configurations that a parser would typically reach when a classifier strays from the optimal predictions. This is known as a *dynamic oracle*.[1]

The effective calculation of the optimal step for some kinds of parsing relies on 'arc-decomposibility', as in the case of Goldberg and Nivre (2012, 2013). This generally requires a projective training corpus; an attempt to extend this to non-projective training corpora had to resort to an approximation (Aufrant et al., 2018). It is known how to calculate the optimal step for a number of non-projective parsing algorithms, however (Gómez-Rodríguez et al., 2014; Gómez-Rodríguez and Fernández-González, 2015; Fernández-González Gómez-Rodríguez, 2018a); see also de Lhoneux et al. (2017).

Ordinary shift-reduce dependency parsing is known at least since Fraser (1989); see also Nasr (1995). Nivre (2008) calls it ''arc-standard parsing.'' For shift-reduce dependency parsing, calculation of the optimal step is regarded to be difficult. The best known algorithm is cubic and is only applicable if the training corpus is projective (Goldberg et al., 2014). We present a new cubic-time algorithm that is also applicable to non-projective training corpora. Moreover, its architecture is modular, expressible as a generic tabular algorithm for dependency parsing plus a context-free grammar that expresses the allowable transitions of the parsing strategy. This differs from approaches that require specialized tabular algorithms for different kinds of parsing (Gómez-Rodríguez et al., 2008; Huang and Sagae, 2010; Kuhlmann et al., 2011).

The generic tabular algorithm is interesting in its own right, and can be used to determine the optimal projectivization of a non-projective tree. This is not to be confused with pseudo-projectivization (Kahane et al., 1998; Nivre and Nilsson, 2005), which generally has a different architecture and is used for a different purpose, namely, to allow a projective parser to produce non-projective structures, by encoding non-projectivity into projective structures before training, and then reconstructing potential non-projectivity after parsing.

A presentational difference with earlier work is that we do not define optimality in terms of ''loss'' or ''cost'' functions but directly in terms of attainable accuracy. This perspective is shared by Straka et al. (2015), who also relate accuracies of competing steps, albeit by means of actual parser output and not in terms of best attainable accuracies.

[1] A term we avoid here, as dynamic oracles are neither oracles nor dynamic, especially in our formulation, which allows gold trees to be non-projective. Following, for example, Kay (2000), an oracle informs a parser whether a step may lead to the correct parse. If the gold tree is non-projective and the parsing strategy only allows projective trees, then there are no steps that lead to the correct parse. At best, there is an optimal step, by some definition of optimality. An algorithm to compute the optimal step, for a given configuration, would typically not change over time, and therefore is not dynamic in any generally accepted sense of the word.

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We further show that if the training corpus is projective, then the time complexity can be reduced to linear. To achieve this, we develop a new approach of excluding computations whose accuracies are guaranteed not to exceed the accuracies of the remaining computations. The main theoretical conclusion is that arc-decomposibility is not a necessary requirement for efficient calculation of the optimal step.

Despite advances in unrestricted non-projective parsing, as, for example, Fernández-González and Gómez-Rodríguez (2018b), many state-of-the-art dependency parsers are projective, as, for example, Qi and Manning (2017). One main practical contribution of the current paper is that it introduces new ways to train projective parsers using non-projective trees, thereby enlarging the portion of trees from a corpus that is available for training. This can be done either after applying optimal projectivization, or by computing optimal steps directly for non-projective trees. This can be expected to lead to more accurate parsers, especially if a training corpus is small and a large proportion of it is non-projective.

2 Preliminaries

In this paper, a *configuration* (for sentence length n) is a 3-tuple (α, β, T) consisting of a *stack* α, which is a string of integers each between 0 and n, a *remaining input* β, which is a suffix of the string $1 \cdots n$, and a set T of pairs (a, a') of integers, with $0 \leq a \leq n$ and $1 \leq a' \leq n$. Further, $\alpha\beta$ is a subsequence of $0\ 1 \cdots n$, starting with 0. Integer 0 represents an artificial input position, not corresponding to any actual token of an input sentence.

An integer a' ($1 \leq a' \leq n$) occurs as second element of a pair $(a, a') \in T$ if and only if it does not occur in $\alpha\beta$. Furthermore, for each a' there is at most one a such that $(a, a') \in T$. If $(a, a') \in T$ then a' is generally called a *dependent* of a, but as we will frequently need concepts from graph theory in the remainder of this article, we will consistently call a' a *child* of a and a the *parent* of a'; if $a' < a$ then a' is a *left child* and if $a < a'$ then it is a *right child*. The terminology is extended in the usual way to include *descendants* and *ancestors*. Pairs (a, a') will henceforth be called *edges*.

For sentence length n, the *initial configuration* is $(0, 1\ 2 \cdots n, \emptyset)$, and a *final configuration* is of the form $(0, \varepsilon, T)$, where ε denotes the empty string. The three transitions of shift-reduce dependency parsing are given in Table 1. By *step* we mean the application of a transition on a particular configuration. By *computation* we mean a series of steps, the formal notation of which uses $\vdash^*$, the reflexive, transition closure of $\vdash$. If $(0, 1\ 2 \cdots n, \emptyset) \vdash^* (0, \varepsilon, T)$, then T represents a tree, with 0 as root element, and T is *projective*, which means that for each node, the set of its descendants (including that node itself) is of the form $\{a, a+1, \ldots, a'-1, a'\}$, for some a and a'. In general, a *dependency tree* is any tree of nodes labelled 0, 1, . . . , n, with 0 being the root.

shift:
$(\alpha,\ b\beta,\ T) \vdash (\alpha b,\ \beta,\ T)$
reduce_left:
$(\alpha a_1 a_2,\ \beta,\ T) \vdash (\alpha a_1,\ \beta,\ T \cup \{(a_1, a_2)\})$
reduce_right:
$(\alpha a_1 a_2,\ \beta,\ T) \vdash (\alpha a_2,\ \beta,\ T \cup \{(a_2, a_1)\})$,
provided $|\alpha| > 0$

Table 1: Shift-reduce dependency parsing.

The *score* of a tree T for a sentence is the number of edges that it has in common with a given gold tree T_g for that sentence, or formally $|T \cap T_g|$. The *accuracy* is the score divided by n. Note that neither tree need be projective for the score to be defined, but in this paper the first tree, T, will normally be projective. Where indicated, also T_g is assumed to be projective.

Assume an arbitrary configuration (α, β, T) for sentence length n and assume a gold tree T_g for a sentence of that same length, and assume three steps $(\alpha, \beta, T) \vdash (\alpha_i, \beta_i, T_i)$, with $i = 1, 2, 3$, obtainable by a **shift**, **reduce_left** or **reduce_right**, respectively. (If $\beta = \varepsilon$, or $|\alpha| \leq 2$, then naturally some of the three transitions need to be left out of consideration.) We now wish to calculate, for each of $i = 1, 2, 3$, the maximum value of $|T'_i \cap T_g|$, for any T'_i such that $(\alpha_i, \beta_i, T_i) \vdash^* (0, \varepsilon, T'_i)$. For $i = 1, 2, 3$, let σ_i be this maximum value. The absolute scores σ_i are strictly speaking irrelevant; the relative values determine which is the optimal step, or which *are* the optimal steps, to reach a tree with the highest score. Note that $|\{i \mid \sigma_i = \max_j \sigma_j\}|$ is either 1, 2, or 3. In the remainder of this article, we find it more convenient to calculate $\sigma_i - |T \cap T_g|$ for each i—or, in other words, gold edges that were previously found are left out of consideration.

We can put restrictions on the set of allowable computations $(\alpha, \beta, T) \vdash^* (0, \varepsilon, T \cup T')$. The *left-before-right* strategy demands that all edges $(a, a') \in T'$ with $a' < a$ are found before any edges $(a, a') \in T'$ with $a < a'$, for each a that is rightmost in α or that occurs in β. The *strict left-before-right* strategy in addition disallows edges $(a, a') \in T'$ with $a' < a$ for each a in α other than the rightmost element. The intuition is that a non-strict strategy allows us to correct mistakes already made: If we have already pushed other elements on top of a stack element a, then a will necessarily obtain right children before it occurs on top of the stack again, when it can take (more) left children. By contrast, the strict strategy would not allow these left children.

The definition of the *right-before-left* strategy is symmetric to that of the left-before-right strategy, but there is no independent *strict right-before-left* strategy. In this paper we consider all three strategies in order to emphasize the power of our framework. It is our understanding that Goldberg et al. (2014) does not commit to any particular strategy.

3 Tabular Dependency Parsing

We here consider context-free grammars (CFGs) of a special form, with nonterminals in $N \cup (N_\ell \times N_r)$, for appropriate finite sets N, N_ℓ, N_r, which need not be disjoint. The finite set of terminals is denoted Σ. There is a single start symbol $S \in N$. Rules are of one of the forms:

- $(B, C) \to a$,
- $A \to (B, C)$,
- $(B', C) \to A\ (B, C)$,
- $(B, C') \to (B, C)\ A$,

where $A \in N$, $B, B' \in N_\ell$, $C, C' \in N_r$, $a \in \Sigma$. A first additional requirement is that if $(B', C) \to A\ (B, C)$ is a rule, then $(B', C') \to A\ (B, C')$, for any $C' \in N_r$, is also a rule, and if $(B, C') \to (B, C)\ A$ is a rule, then $(B', C') \to (B', C)\ A$, for any $B' \in N_\ell$, is also a rule. This justifies our notation of such rules in the remainder of this paper as $(B', _) \to A\ (B, _)$ and $(_, C') \to (_, C)\ A$, respectively. These two kinds of rules correspond to attachment of left and right children, respectively, in dependency parsing. Secondly, we require that there is precisely one rule $(B, C) \to a$ for each $a \in \Sigma$.

$$W_\ell(B, i, i) = \begin{cases} \mathbb{1}, & \text{if } (B, C) \to a_i \\ \mathbb{0}, & \text{otherwise} \end{cases}$$
$$W_r(C, i, i) = \begin{cases} \mathbb{1}, & \text{if } (B, C) \to a_i \\ \mathbb{0}, & \text{otherwise} \end{cases}$$
$$W_\ell(B, C, i, j) = \textstyle\bigoplus_k W_r(B, i, k) \otimes W_\ell(C, k+1, j) \otimes w(j, i)$$
$$W_r(B, C, i, j) = \textstyle\bigoplus_k W_r(B, i, k) \otimes W_\ell(C, k+1, j) \otimes w(i, j)$$
$$W_\ell(C', i, j) = \textstyle\bigoplus_{A \to (D,B),\ (C',_) \to A\ (C,_),\ k} W_\ell(D, i, k) \otimes W_\ell(B, C, k, j)$$
$$W_r(B', i, j) = \textstyle\bigoplus_{(_,B') \to (_,B)\ A,\ A \to (C,D),\ k} W_r(B, C, i, k) \otimes W_r(D, k, j)$$
$$W = \textstyle\bigoplus_{S \to (B,C)} W_\ell(B, 0, 0) \otimes W_r(C, 0, n)$$

Table 2: Weighted parsing, for an arbitrary semiring, with $0 \leq i < j \leq n$.

Note that the additional requirements make the grammar explicitly ''split'' in the sense of Eisner and Satta (1999), Eisner (2000), and Johnson (2007). That is, the two processes of attaching left and right children, respectively, are independent, with rules $(B, C) \to a$ creating ''initial states'' B and C, respectively, for these two processes. Rules of the form $A \to (B, C)$ then combine the end results of these two processes, possibly placing constraints on allowable combinations of B and C.

To bring out the relation between our subclass of CFGs and bilexical grammars, one could explicitly write $(B, C)(a) \to a$, $A(a) \to (B, C)(a)$, $(B', _)(b) \to A(a)\ (B, _)(b)$, and $(_, C')(c) \to (_, C)(c)\ A(a)$.

Purely symbolic parsing is extended to weighted parsing much as usual, except that instead of attaching weights to rules, we attach a score $w(i, j)$ to each pair (i, j), which is a potential edge. This can be done for any semiring. In the semiring we will first use, a value is either a non-negative integer or $-\infty$. Further, $w_1 \oplus w_2 = \max(w_1, w_2)$ and $w_1 \otimes w_2 = w_1 + w_2$ if $w_1 \neq -\infty$ and $w_2 \neq -\infty$ and $w_1 \otimes w_2 = -\infty$ otherwise. Naturally, the identity element of $\oplus$ is $\mathbb{0} = -\infty$ and the identity element of $\otimes$ is $\mathbb{1} = 0$.

Tabular weighted parsing can be realized following Eisner and Satta (1999). We assume the input is a string $a_0 a_1 \cdots a_n \in \Sigma^*$, with a_0 being the prospective root of a tree. Table 2 presents the cubic-time algorithm in the form of a system of recursive equations. With the semiring we chose above, $W_\ell(B, i, j)$ represents the highest score of any right-most derivation of the form $(B, _) \Rightarrow A_1(B_1, _) \Rightarrow A_1 A_2(B_2, _) \Rightarrow^*$

(S, S)	$\rightarrow$	a
S	$\rightarrow$	(S, S)
$(_, S)$	$\rightarrow$	$(_, S)\ S$
$(S, _)$	$\rightarrow$	$S\ (S, _)$

Table 3: Grammar for projective dependency parsing, with $\Sigma = \{a\}$ and $N = N_\ell = N_r = \{S\}$.

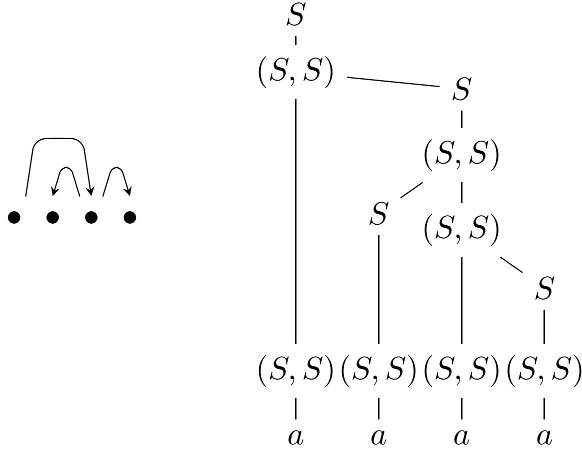


Figure 1: Dependency structure and corresponding parse tree that encodes a computation of a shift-reduce parser.

$A_1 \cdots A_m(B_m, _) \Rightarrow A_1 \cdots A_m a_j \Rightarrow^* a_i \cdots a_j$, for some $m \geq 0$, and $W_r(C, i, j)$ has symmetric meaning. Intuitively, $W_\ell(B, i, j)$ considers a_j and its left dependents and $W_r(C, i, j)$ considers a_i and its right dependents. A value $W_\ell(B, C, i, j)$, or $W_r(B, C, i, j)$, represents the highest score combining a_i and its right dependents and a_j and its left dependents, meeting in the middle at some k, including also an edge from a_i to a_j, or from a_j to a_i, respectively.

One may interpret the grammar in Table 3 as encoding all possible computations of a shift-reduce parser, and thereby all projective trees. As there is only one way to instantiate the underscores, we obtain rule $(S, S) \rightarrow (S, S)\ S$, which corresponds to **reduce_left**, and rule $(S, S) \rightarrow S\ (S, S)$, which corresponds to **reduce_right**.

Figure 1 presents a parse tree for the grammar and the corresponding dependency tree. Note that if we are not given a particular strategy, such as left-before-right, then the parse tree underspecifies whether left children or right children are attached first. This is necessarily the case because the grammar is split. Therefore, the computation in this example may consist of three **shift**s, followed by one **reduce_left**, one **reduce_right**, and one **reduce_left**, or it may consist of two **shift**s, one **reduce_right**, one **shift**, and two **reduce_left**s.

(P, P)	$\rightarrow$	p
(S, S)	$\rightarrow$	s
P	$\rightarrow$	(P, P)
S	$\rightarrow$	(P, S)
S	$\rightarrow$	(S, S)
$(S, _)$	$\rightarrow$	$P\ (S, _)$
$(S, _)$	$\rightarrow$	$S\ (S, _)$
$(_, S)$	$\rightarrow$	$(_, P)\ S$
$(_, S)$	$\rightarrow$	$(_, S)\ S$
$(S, _)$	$\rightarrow$	$P\ (P, _)$

Table 4: Grammar for dependency parsing of $p^k s^{m+1}$, representing a stack of length $k + 1$ and remaining input of length m, with $\Sigma = \{p, s\}$, $N = N_r = N_\ell = \{P, S\}$. The last rule would be excluded for the strict left-to-right strategy, or alternatively one can set $w(i, j) = -\infty$ for $j < i < k$.

For a given gold tree T_g, which may or may not be projective, we let $w(i, j) = \delta_g(i, j)$, where we define $\delta_g(i, j) = 1$ if $(i, j) \in T_g$ and $\delta_g(i, j) = 0$ otherwise. With the grammar from Table 3, the value W found by weighted parsing is now the score of the most accurate projective tree. By backtracing from W as usual, we can construct the (or more correctly, a) tree with that highest accuracy. We have thereby found an effective way to projectivize a treebank in an optimal way. By a different semiring, we can count the number of trees with the highest accuracy, which reflects the degree of ''choice'' when projectivizing a treebank.

4 $\mathcal{O}(n^3)$ Time Algorithm

In a computation starting from a configuration $(a_0 \cdots a_k, b_1 \cdots b_m, T)$, not every projective parse of the string $a_0 \cdots a_k b_1 \cdots b_m$ is achievable. The structures that are achievable are captured by the grammar in Table 4, with P for prefix and S for suffix (also for ''start symbol''). Nonterminals P and (P, P) correspond to a node a_i ($0 \leq i < k$) that does not have children. Nonterminal S corresponds to a node that has either a_k or some b_j ($1 \leq j \leq m$) among its descendants. This then means that the node will appear on top of the stack at some point in the computation. Nonterminal (S, S) also corresponds to a node that has one of the rightmost $m + 1$ nodes among its descendants, and, in addition, if it itself is not one of the rightmost $m + 1$ nodes, then it must have a left child.

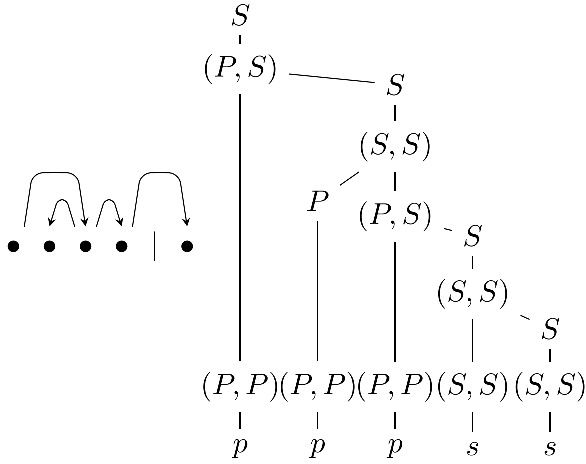
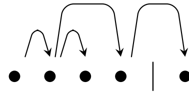


Figure 2: Dependency structure and corresponding parse tree, for stack of height 4 and remaining input of length 1.

Nonterminal (P, S) corresponds to a node a_i ($0 \leq i < k$) that has a_k among its descendants but that does not have a left child. Nonterminal (S, P) corresponds to a node a_i ($0 \leq i < k$) that has a left child but no right children. For a_i to be given a left child, it is required that it eventually appear on top of the stack. This requirement is encoded in the absence of a rule with right-hand side (S, P). In other words, (S, P) cannot be part of a successful derivation, unless the rule $(S, S) \rightarrow (S, P)\ S$ is subsequently used, which then corresponds to giving a_i a right child that has a_k among its descendants.

Figure 2 shows an example. Note that we can partition a parse tree into ''columns'', each consisting of a path starting with a label in N, then a series of labels in $N_\ell \times N_r$ and ending with a label in Σ.

A dependency structure that is not achievable, and that appropriately does not correspond to a parse tree, for a stack of height 4 and remaining input of length 1, is:



Suppose we have a configuration $(a_0 \cdots a_k, b_1 \cdots b_m, T)$ for sentence length n, which implies $k + m \leq n$. We need to decide whether a **shift**, **reduce_left**, or **reduce_right** should be done in order to achieve the highest accuracy, for given gold tree T_g. For this, we calculate three values σ_1, σ_2 and σ_3, and determine which is highest.

The first value σ_1 is obtained by investigating the configuration $(a_0 \cdots a_k b_1, b_2 \cdots b_m, \emptyset)$ resulting after a shift. We run our generic tabular algorithm for the grammar in Table 4, for input $p^{k+1}s^m$, to obtain $\sigma_1 = W$. The scores are obtained by translating indices of $a_0 \cdots a_k b_1 \cdots b_m = c_0 \cdots c_{k+m}$ to indices in the original input, that is, we let $w(i, j) = \delta_g(c_i, c_j)$. However, the shift, which pushes an element on top of a_k, implies that a_k will obtain right children before it can obtain left children. If we assume the left-before-right strategy, then we should avoid that a_k obtains left children. We could do that by refining the grammar, but find it easier to set $w(k, i) = -\infty$ for all $i < k$.

For the second value σ_2, we investigate the configuration $(a_0 \cdots a_{k-1}, b_1 \cdots b_m, \emptyset)$ resulting after a **reduce_left**. The same grammar and algorithm are used, now for input $p^{k-1}s^{m+1}$. With $a_0 \cdots a_{k-1} b_1 \cdots b_m = c_0 \cdots c_{k+m-1}$, we let $w(i, j) = \delta_g(c_i, c_j)$. We let $\sigma_2 = W \otimes \delta_g(a_{k-1}, a_k)$. In case of a strict left-before-right strategy, we set $w(k-1, i) = -\infty$ for $i < k-1$, to avoid that a_{k-1} obtains left children after having obtained a right child a_k.

If $k \leq 1$ then the third value is $\sigma_3 = -\infty$, as no **reduce_right** is applicable. Otherwise we investigate $(a_0 \cdots a_{k-2} a_k, b_1 \cdots b_m, \emptyset)$. The same grammar and algorithm are used as before, and $w(i, j) = \delta_g(c_i, c_j)$ with $a_0 \cdots a_{k-2} a_k b_1 \cdots b_m = c_0 \cdots c_{k+m-1}$. Now $\sigma_3 = W \otimes \delta_g(a_k, a_{k-1})$. In case of a right-before-left strategy, we set $w(k, i) = -\infty$ for $k < i$.

We conclude that the time complexity of calculating the optimal step is three times the time complexity of the algorithm of Table 2, hence cubic in n.

For a proof of correctness, it is sufficient to show that each parse tree by the grammar in Table 4 corresponds to a computation with the same score, and conversely that each computation corresponds to an equivalent parse tree. Our grammar has spurious ambiguity, just as the shift-reduce parser from Table 1, and this can be resolved in the same way, depending on whether the intended strategy is (non-)strict left-before-right or right-before-left, and whether the configuration is the result of a **shift**, **reduce_left**, or **reduce_right**. Concretely, we can restrict parse trees to attach children lower in the tree if they would be attached earlier in the computation, and thereby we obtain

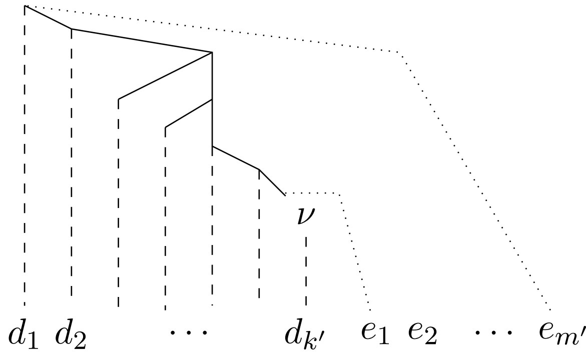


Figure 3: A node ν with label in $N_\ell \times N_r$ translates to configuration $(\bar{d}_1 \cdots \bar{d}_{k'}, \bar{e}_1 \cdots \bar{e}_{m'}, T)$, via its shortest path to the root. The overlined symbols denote the integers between 0 and n corresponding to $d_1 \cdots d_{k'} e_1 \cdots e_{m'} \in p^+ s^*$.

a bijection between parse trees and computations. For example, in the middle column of the parse tree in Figure 2, the (P, S) and its right child occur below the (S, S) and its left child, to indicate the **reduce_left** precedes the **reduce_right**.

The proof in one direction assumes a parse tree, which is traversed to gather the steps of a computation. This traversal is post-order, from left to right, but skipping the nodes representing stack elements below the top of the stack, starting from the leftmost node labeled s. Each node ν with a label in $N_\ell \times N_r$ corresponds to a step. If the child of ν is labeled s, then we have a **shift**, and if it has a right or left child with a label in N, then it corresponds to a **reduce_left** or **reduce_right**, respectively. The configuration resulting from that step can be constructed as sketched in Figure 3. We follow the shortest path from ν to the root. All the leaves to the right of the path correspond to the remaining input. For the stack, we gather the leaves in the columns of the nodes on the path, as well as those of the left children of nodes on the path. Compare this with the concept of *right-sentential forms* in the theory of context-free parsing.

For a proof in the other direction, we can make use of existing parsing theory, which tells us how to translate a computation of the shift-reduce parser to a dependency structure, which in turn is easily translated to an undecorated parse tree. It then remains to show that the nodes in that tree can be decorated (in fact in a unique way), according to the rules from Table 4. This is straightforward given the meanings of P and S described earlier in this section. Most notably, the absence of a rule with right-hand side $(_, P)$ P does not prevent the decoration of a tree that was constructed out of a computation, because a reduction involving two nodes within the stack is only possible if the rightmost of these nodes eventually appears on top of the stack, which is only possible when the computation has previously made a_k a descendant of that node, hence we would have S rather than P.

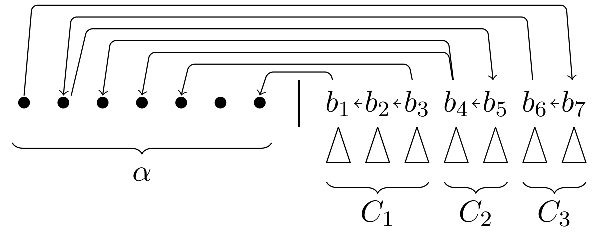


Figure 4: Components C_1, C_2, C_3 partitioning nodes in β, and gold edges linking them to α.

5 $\mathcal{O}(n^2)$ Time Algorithm

Assume a given configuration (α, β, T) as before, resulting from a shift or reduction. Let $\alpha = a_0 \cdots a_k$, $A = \{a_0, \dots, a_k\}$, and let B be the set of nodes in β. We again wish to calculate the maximum value of $|T' \cap T_g|$ for any T' such that $(\alpha, \beta, \emptyset) \vdash^* (0, \varepsilon, T')$, but now under the assumption that T_g is projective. Let us call this value σ_{max}. We define w in terms of δ_g as in the previous section, setting $w(i, j) = -\infty$ for an appropriate subset of pairs (i, j) to enforce a strategy that is (non-)strict left-before-right or right-before-left.

The edges in $T_g \cap (B \times B)$ partition the remaining input into maximal connected components. Within these components, a node $b \in B$ is called *critical* if it satisfies one or both of the following two conditions:

- At least one descendant of b (according to T_g) is not in B.
- The parent of b (according to T_g) is not in B.

Let $B_{crit} \subseteq B$ be the set of critical nodes, listed in order as $b_1, \dots, b_m$, and let $B_{ncrit} = B \setminus B_{crit}$. Figure 4 sketches three components as well as edges in $T_g \cap (A \times B)$ and $T_g \cap (B \times A)$. Component C_1, for example, contains the critical elements b_1, b_2, and b_3. The triangles under b_1, ..., b_7 represent subtrees consisting of edges leading to non-critical nodes. For each $b \in B_{crit}$, $|T_g \cap (\{b\} \times A)|$ is zero or more, or in words,

critical nodes have zero or more children in the stack. Further, if $(a, b) \in T_g \cap (A \times B_{crit})$, then b is the rightmost critical node in a component; examples are b_5 and b_7 in the figure.

Let T_{max} be any tree such that $(\alpha, \beta, \emptyset) \vdash^* (0, \varepsilon, T_{max})$ and $|T_{max} \cap T_g| = \sigma_{max}$. Then we can find another tree T'_{max} that has the same properties and in addition satisfies:

1. $T_g \cap (B \times B_{ncrit}) \subseteq T'_{max}$,
2. $T'_{max} \cap (B_{ncrit} \times A) = \emptyset$,
3. $T'_{max} \cap (B \times B_{crit}) \subseteq T_g$,

or in words, (1) the subtrees rooted in the critical nodes are entirely included, (2) no child of a non-critical node is in the stack, and (3) within the remaining input, all edges to critical nodes are gold. Very similar observations were made before by Goldberg et al. (2014), and therefore we will not give full proofs here. The structure of the proof is in each case that all violations of a property can be systematically removed, by rearranging the computation, in a way that does not decrease the score.

We need two more properties:

4. If $(a, b) \in T'_{max} \cap (A \times B_{crit}) \setminus T_g$ then either:
   - b is the rightmost critical node in its component, or
   - there is $(b, a') \in T'_{max} \cap T_g$, for some $a' \in A$ and there is at least one other critical node b' to the right of b, but in the same component, such that $(b', a'') \in T'_{max} \cap T_g$ or $(a'', b') \in T'_{max} \cap T_g$, for some $a'' \in A$.
5. If $(b, a) \in T'_{max} \cap (B_{crit} \times A) \setminus T_g$ then there is $(b, a') \in T'_{max}$, for some $a' \in A$, such that a' is a sibling of a immediately to its right.

Figure 5, to be discussed in more detail later, illustrates property (4) for the non-gold edge from a_4; this edge leads to b_4 (which has outgoing gold edge to a_5) rather than to b_5 or b_6. It further respects property (4) because of the gold edges connected to b_7 and b_8, which occur to the right of b_4 but in the same component. Property (5) is illustrated for the non-gold edge from b_3 to a_8, which has sibling a_9 immediately to the right.

The proof that property (4) may be assumed to hold, without loss of generality, again involves making local changes to the computation, in particular replacing the b in an offending non-gold edge $(a, b) \in A \times B_{crit}$ by another critical node b' further to the left or at the right end of the component. Similarly, for property (5), if we have an offending non-gold edge (b, a), then we can rearrange the computation, such that node a is reduced not into b but into one of the descendants of b in B that was given children in A. If none of the descendants of b in B was given children in A, then a can instead be reduced into its neighbor in the stack immediately to the left, without affecting the score.

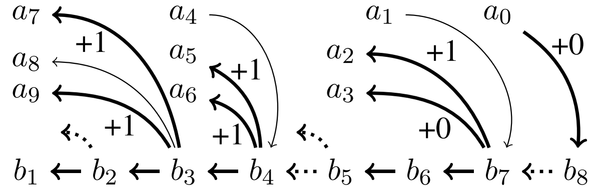


Figure 5: Counting additional gold edges in $A \times B_{crit} \cup B_{crit} \times A$. Gold edges are thick, others are thin. Gold edges that are not created appear dotted.

By properties (1)–(3), we can from here on ignore non-critical nodes, so that the remaining task is to calculate $\sigma_{max} - |B_{ncrit}|$. In fact, we go further than that and calculate $\sigma_{max} - M$, where $M = |T_g \cap (B \times B)|$. In other words, we take for granted that the score can be at least as much as the number of gold edges within the remaining input, which leaves us with the task of counting the additional gold edges in the optimal computation. For any given component C we can consider the sequence of edges that the computation creates between A and C, in the order in which they are created:

- for the first gold edge between C and A, we count +1,
- for each subsequent gold edge between C to A, we count +1,
- we ignore interspersed non-gold edges from C to A,
- but following a non-gold edge from A to C, the immediately next gold edge between C and A is *not* counted, because that non-gold edge implies that another gold edge in $B_{crit} \times B_{crit}$ cannot be created.

This is illustrated by Figure 5. For (b_3, a_9) we count $+1$, it being the first gold edge connected

to the component. For the subsequent three gold edges, we count $+1$ for each, ignoring the non-gold edge (b_3, a_8). The non-gold edge (a_4, b_4) implies that the parent of b_4 is already determined. One would then perhaps expect we count -1 for non-creation of (b_5, b_4), considering (b_5, b_4) was already counted as part of M. Instead, we let this -1 cancel out against the following (b_7, a_3), by letting the latter contribute $+0$ rather than $+1$. The subsequent edge (b_7, a_2) again contributes $+1$, but the non-gold edge (a_1, b_7) means that the subsequent (a_0, b_8) contributes +0. Hence the net count in this component is 5.

The main motivation for properties (1)–(5) is that they limit the input positions that can be relevant for a node that is on top of the stack, thereby eliminating one factor m from the time complexity. More specifically, the gold edges relate a stack element to a ''current critical node'' in a ''current component''. We need to distinguish however between three possible *states*:

- $\mathcal{N}$ (none): none of the critical nodes from the current component were shifted on to the stack yet,
- $\mathcal{C}$ (consumed): the current critical node was 'consumed' by it having been shifted and assigned a parent,
- $\mathcal{F}$ (fresh): the current critical node was not consumed, but at least one of the preceding critical nodes in the same component was consumed.

For $0 \leq i \leq k$, we define $p(i)$ to be the index j such that $(b_j, a_i) \in T_g$, and if there is no such j, then $p(i) = \bot$, where $\bot$ denotes 'undefined'. For $0 \leq i < k$, we let $p_{\geq}(i) = p(i)$ if $p(i) \neq \bot$, and $p_{\geq}(i) = p_{\geq}(i+1)$ otherwise, and further $p_{\geq}(k) = p(k)$. Intuitively, we seek a critical node that is the parent of a_i, or if there is none, of $a_{i+1}, \ldots$ We define $c(i)$ to be the smallest j such that $(a_i, b_j) \in T_g$, or in words, the index of the leftmost child in the remaining input, and $c(i) = \bot$ if there is none.

As representative element of a component with critical element b_j we take the critical element that is rightmost in that component, or formally, we define $R(j)$ to be the largest j' such that $b_{j'}$ is an ancestor (by $T_g \cap (B_{crit} \times B_{crit})$) of b_j. For completeness, we define $R(\bot) = \bot$. We let $P(i) = R(p(i))$ and $P_{\geq}(i) = R(p_{\geq}(i))$. Note that

$$
\begin{aligned}
&\mathbf{score}(i,j,q) = 0 \text{ if } i < 0, \text{ otherwise:}\\
&\mathbf{score}(i,j,q) =\\
&\quad [\mathbf{nchildren}(i) - \Delta(c(i) = P_{\geq}(j) \land q \neq \mathcal{N})] \otimes\\
&\qquad w(i,j) \otimes \mathbf{score}(i-1, i, \tau(i,j,q)) \ \oplus\\
&\quad w(j,i) \otimes \mathbf{score}(i-1, j, q) \ \oplus\\
&\quad [\mathbf{if}\ p(j) = \bot \lor q = \mathcal{C}\ \mathbf{then}\ -\infty\\
&\qquad \mathbf{else}\ \Delta(q = \mathcal{N}) \otimes \mathbf{score}'(i, p(j))]\\
&\mathbf{score}'(i,j) = 0 \text{ if } i < 0, \text{ otherwise:}\\
&\mathbf{score}'(i,j) =\\
&\quad [\mathbf{if}\ p'(i,j) = \bot\ \mathbf{then}\ \mathbf{score}'(i-1, j)\\
&\qquad \mathbf{else}\ 1 \otimes \mathbf{score}'(i-1, p'(i,j))] \ \oplus\\
&\quad \mathbf{nchildren}(i) \otimes \mathbf{score}(i-1, i, \tau'(i,j))\\
&\mathbf{nchildren}(i) = |\{j \mid w(i, j+k) = 1\}|\\
&\tau(i,j,q) = \mathbf{if}\ q = \mathcal{N} \lor P_{\geq}(i) \neq P_{\geq}(j)\ \mathbf{then}\ \mathcal{N}\\
&\qquad\qquad \mathbf{else\ if}\ p_{\geq}(i) \neq p_{\geq}(j)\ \mathbf{then}\ \mathcal{F}\\
&\qquad\qquad \mathbf{else}\ q\\
&\tau'(i,j) = \mathbf{if}\ P_{\geq}(i) \neq R(j)\ \mathbf{then}\ \mathcal{N}\\
&\qquad\qquad \mathbf{else\ if}\ p_{\geq}(i) \neq j\ \mathbf{then}\ \mathcal{F}\\
&\qquad\qquad \mathbf{else}\ \mathcal{C}
\end{aligned}
$$

Table 5: Quadratic-time algorithm.

$R(c(i)) = c(i)$ for each i. For $0 \leq i \leq k$ and $1 \leq j \leq m$, we let $p'(i,j) = p(i)$ if $P(i) = R(j)$ and $p'(i,j) = \bot$ otherwise; or in words, $p'(i,j)$ is the index of the parent of a_i in the remaining input, provided it is in the same component as b_j.

Table 5 presents the algorithm, expressed as system of recursive equations. Here $\mathbf{score}(i,j,q)$ represents the maximum number of gold edges (in addition to M) in a computation from $(a_0 \cdots a_i a_j, b_\ell \cdots b_k, \emptyset)$, where ℓ depends on the state $q \in \{\mathcal{N}, \mathcal{C}, \mathcal{F}\}$. If $q = \mathcal{N}$, then ℓ is the smallest number such that $R(\ell) = P_{\geq}(j)$; critical nodes from the current component were not yet shifted. If $q = \mathcal{C}$, then $\ell = p_{\geq}(j) + 1$ or $\ell = P_{\geq}(j) + 1$; this can be related to the two cases distinguished by property (4). If $q = \mathcal{F}$, then ℓ is greater than the smallest number such that $R(\ell) = P_{\geq}(j)$, but smaller than or equal to $p_{\geq}(j)$ or equal to $\ell = P_{\geq}(j) + 1$. Similarly, $\mathbf{score}'(i,j)$ represents the maximum number of gold edges in a computation from $(a_0 \cdots a_i b_j, b_{j+1} \cdots b_k, \emptyset)$.

For $i \geq 0$, the value of $\mathbf{score}(i,j,q)$ is the maximum (by $\oplus$) of three values. The first corresponds to a reduction of a_j into a_i, which turns the stack into $a_0 \cdots a_{i-1} a_i$; this would also include shifts of any remaining right children of a_i, if there are any, and their reduction into a_i. Because there is a new top-of-stack, the state is updated using τ. The function $\mathbf{nchildren}$ counts the critical nodes that are children of a_i. We define $\mathbf{nchildren}$ in terms of w rather than T_g, as in the case of the right-before-left strategy

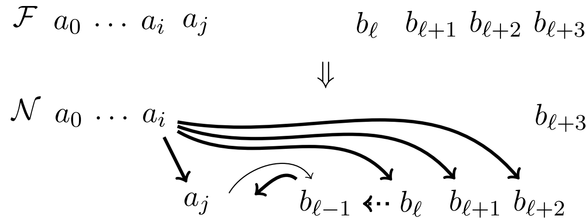


Figure 6: Graphical representation of the first value in the definition of **score**, for the case $q = \mathcal{F}$, assuming $c(i) = P_{\geq}(j) = \ell$ and a_i further has children $b_{\ell+1}$ and $b_{\ell+2}$. Because $q = \mathcal{F}$, there was some other node $b_{\ell'}$ in the same component that was shifted on to the stack earlier and given a (non-gold) parent; let us assume $\ell' = \ell - 1$. We can add 3 children to the score, but should subtract $\Delta(c(i) = P_{\geq}(j) \wedge q \neq \mathcal{N}) = 1$, to compensate for the fact that edge $(b_\ell, b_{\ell-1})$ cannot be constructed, as $b_{\ell-1}$ can only have one parent. If we further assume a_i has a parent among the critical nodes, then that parent must be in a different component, and therefore $\tau(i, j, q) = \mathcal{N}$.

after a **reduce_right** we would preclude right children of a_k by setting $w(k, i) = -\infty$ for $k < i$. The leftmost of the children, at index $c(i)$, is not counted (or in other words, 1 is subtracted from the number of children) if it is in the current component $P_{\geq}(j)$ and that component is anything other than 'none'; here Δ is the indicator function, which returns 1 if its Boolean argument evaluates to true, and 0 otherwise. Figure 6 illustrates one possible case.

The second value corresponds to a reduction of a_i into a_j, which turns the stack into $a_0 \cdots a_{i-1}a_j$, leaving the state unchanged as the top of the stack is unchanged. The third value is applicable if a_j has parent b_ℓ that has not yet been consumed, and it corresponds to a shift of b_ℓ and a reduction of a_i into b_ℓ (and possibly further shifts and reductions that are implicit), resulting in stack $a_0 \cdots a_i b_\ell$. If this creates the first gold edge connected to the current component, then we add $+1$.

For $i \geq 0$, the value of $\mathbf{score}'(i, j)$ is the maximum of two values. The first value distinguishes two cases. In the first case, a_i does not have a parent in the same component as b_j, and a_i is reduced into b_j without counting the (non-gold) edge. In the second case, a_i is reduced into its parent, which is b_j or another critical node that is an ancestor of b_j; in this case we count the gold edge. The second value in the definition of $\mathbf{score}'(i, j)$ corresponds to a reduction of b_j into a_i (as well as shifts of any critical nodes that are children of a_i, and their reduction into a_i), resulting in stack

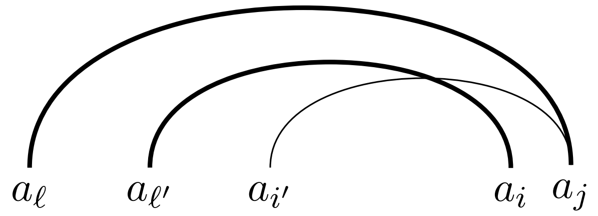


Figure 7: Assuming the thick edges are gold, then the thin edge cannot be gold as well, as the gold tree is projective. A score obtained from a stack $a_0 \cdots a_{i-1}a_i$ is therefore at least as high as a score obtained from a stack $a_0 \cdots a_{i-1}a_j$, unless all of $a_{\ell'+1}, \ldots, a_i$ first become children of a_j via a series of **reduce_right** steps, all producing non-gold edges, and therefore adding nothing to the score. The κ function implements such a series of **reduce_right** steps.

$a_0 \cdots a_{i-1}a_i$. The state is updated using τ', in the light of the new top-of-stack.

The top-level call is $\mathbf{score}(k-1, k, \mathcal{N})$. As this does not account for right children of the top of stack a_k, we need to add $\mathbf{nchildren}(k)$. Putting everything together, we have $\sigma_{max} = M \otimes \mathbf{score}(k-1, k, \mathcal{N}) \otimes \mathbf{nchildren}(k)$. The time complexity is quadratic in $k + m \leq n$, given the quadratically many combinations of i and j in $\mathbf{score}(i, j, q)$ and $\mathbf{score}'(i, j)$.

6 $\mathcal{O}(n)$ Time Algorithm

Under the same assumption as in the previous section, namely, that T_g is projective, we can further reduce the time complexity of computing σ_{max}, by two observations. First, let us define $\lambda(i, j)$ to be true if and only if there is an $\ell < i$ such that $(a_\ell, a_j) \in T_g$ or $(a_j, a_\ell) \in T_g$. If $(a_j, a_i) \notin T_g$ and $\lambda(i, j)$ is false, then the highest score attainable from a configuration $(a_0 \cdots a_{i-1}a_j, \beta, \emptyset)$ is no higher than the highest score attainable from $(a_0 \cdots a_{i-1}a_i, \beta, \emptyset)$, or, if a_j has a parent $b_{j'}$, from $(a_0 \cdots a_i b_{j'}, \beta', \emptyset)$, for appropriate suffix β' of β. This means that in order to calculate $\mathbf{score}(i, j, q)$ we do not need to calculate $\mathbf{score}(i-1, j, q)$ in this case.

Secondly, if $(a_j, a_i) \notin T_g$ and $\lambda(i, j)$ is true, and if there is $\ell' < i$ such that $(a_{\ell'}, a_i) \in T_g$ or $(a_i, a_{\ell'}) \in T_g$, then there are no edges between a_j and $a_{i'}$ for any i' with $\ell' < i' < i$, because of projectivity of T_g. We therefore do not need to calculate $\mathbf{score}(i', j, q)$ for such values of i' in order to find the computation with the highest score. This is illustrated in Figure 7.

Let us define $\kappa(i)$ to be the smallest ℓ' such that $(a_{\ell'}, a_i) \in T_g$ or $(a_i, a_{\ell'}) \in T_g$, or $i-1$ if there is no such ℓ'. In the definition of **score**, we may now replace $w(j,i) \otimes \mathbf{score}(i-1,j,q)$ by:

$$[\textbf{if } w(j,i) = 1 \textbf{ then } 1 \otimes \mathbf{score}(i-1,j,q) \\ \quad \textbf{else if } w(j,i) = 0 \wedge \lambda(i,j) \\ \quad\quad \textbf{then } \mathbf{score}(\kappa(i),j,q) \\ \quad \textbf{else } -\infty]$$

Similarly, we define $\lambda'(i,j)$ to be true if and only if there is an $\ell < i$ such that $(a_\ell, b_{j'}) \in T_g$ or $(b_{j'}, a_\ell) \in T_g$ for some j' with $R(j') = R(j)$. In the definition of $\mathbf{score}'$, we may now replace $\mathbf{score}'(i-1,j)$ by:

$$[\textbf{if } \lambda'(i,j) \textbf{ then } \mathbf{score}'(\kappa(i),j) \textbf{ else } -\infty]$$

Thereby the algorithm becomes linear-time, because the number of values $\mathbf{score}(i,j,q)$ and $\mathbf{score}'(i,j)$ that are calculated for any i is now linear. To see this, consider that for any i, $\mathbf{score}(i,j,q)$ would be calculated only if $j = i+1$, if $(a_i, a_j) \in T_g$ or $(a_j, a_i) \in T_g$, if $(a_j, a_{i+1}) \in T_g$, or if j is smallest such that there is $\ell < i$ with $(a_\ell, a_j) \in T_g$ or $(a_j, a_\ell) \in T_g$. Similarly, $\mathbf{score}(i,j)$ would be calculated only if $\mathbf{score}(i,j',q)$ would be calculated and $(b_j, a_{j'}) \in T_g$, if $(b_j, a_{i+1}) \in T_g$, or if j is smallest such that there is $\ell \le i$ with $(a_\ell, b_{j'}) \in T_g$ or $(b_{j'}, a_\ell) \in T_g$ for some j' such that $b_{j'}$ an ancestor of b_j in the same component.

7 Towards Constant Time Per Calculation

A typical application would calculate the optimal step for several or even all configurations within one computation. Between one configuration and the next, the stack differs at most in the two rightmost elements and the remaining input differs at most in that it loses its leftmost element. Therefore, all but a constant number of values of $\mathbf{score}(i,j,q)$ and $\mathbf{score}'(i,j)$ can be reused, to make the time complexity closer to constant time for each calculation of the optimal step. The practical relevance of this is limited however if one would typically reload the data structures containing the relevant values, which are of linear size. Hence we have not pursued this further.

8 Experiments

Our experiments were run on a laptop with an Intel i7-7500U processor (4 cores, 2.70 GHz) with 8 GB

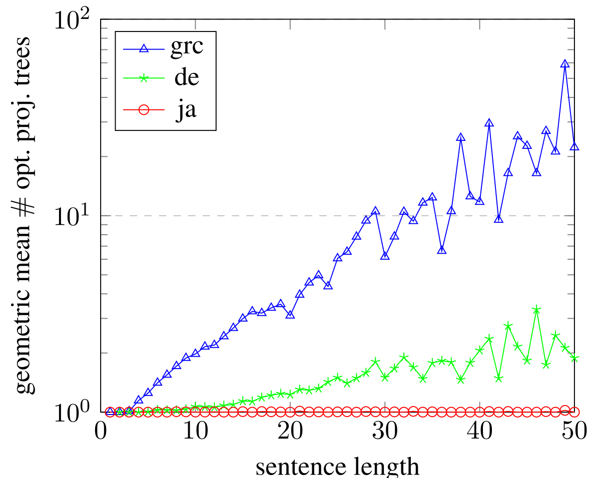


Figure 8: Geometric mean of the number of optimal projectivized trees against sentence length.

of RAM. The implementation language is Java, with DL4J[2] for the classifier, realized as a neural network with a single layer of 256 hidden nodes. Training is with batch size 100, and 20 epochs. Features are the (gold) parts of speech and length-100 word2vec representations of the word forms of the top-most three stack elements, as well as of the left-most three elements of the remaining input, and the left-most and right-most dependency relations in the top-most two stack elements.

8.1 Optimal Projectivization

We need to projectivize our training corpus for the experiments in Section 8.2, using the algorithm described at the end of Section 3. As we are not aware of literature reporting experiments with optimal projectivization, we briefly describe our findings here.

Projectivizing all the training sets in Universal Dependencies v2.2[3] took 244 sec in total, or 0.342 ms per tree. As mentioned earlier, there may be multiple projectivized trees that are optimal in terms of accuracy, for a single gold tree. We are not aware of meaningful criteria that tell us how to choose any particular one of them, and for our experiments in Section 8.2 we have chosen an arbitrary one. It is conceivable, however, that the choices of the projectivized trees would affect the accuracy of a parser trained on them. Figure 8 illustrates the degree of ‘‘choice’’ when projectiving trees. We consider

[2]https://deeplearning4j.org/.
[3]https://universaldependencies.org/.

	pseudo	optimal
grc	91.41	92.50
de	98.89	98.97
ja	99.99	99.99

Table 6: Accuracy (LAS or UAS, which here are identical) of pseudo-projectivization and of optimal projectivization.

two languages that are known to differ widely in the prevalence of non-projectivity, namely Ancient Greek (PROIEL) and Japanese (BCCWJ), and we consider one more language, German (GSD), that falls in between (Straka et al., 2015). As can be expected, the degree of choice grows roughly exponentially in sentence length.

Table 6 shows that pseudo-projectivization is non-optimal. We realized pseudo-projectivization using MaltParser 1.9.0.[4]

8.2 Computing the Optimal Step

To investigate the run-time behavior of the algorithms, we trained our shift-reduce dependency parser on the German training corpus, after it was projectivized as in Section 8.1. In a second pass over the same corpus, the parser followed the steps returned by the trained classifier. For each configuration that was obtained in this way, the running time was recorded of calculating the optimal step, with the non-strict left-before-right strategy. For each configuration, it was verified that the calculated scores, for **shift**, **reduce_left**, and **reduce_right**, were the same between the three algorithms from Sections 4, 5, and 6.

The two-pass design was inspired by Choi and Palmer (2011). We chose this design, rather than online learning, as we found it easiest to implement. Goldberg and Nivre (2012) discuss the relation between multi-pass and online learning approaches.

As Figure 9 shows, the running times of the algorithms from Sections 5 and 6 grow slowly as the summed length of stack and remaining input grows; note the logarithmic scale. The improvement of the linear-time algorithm over the quadratic-time algorithm is perhaps less than one may expect. This is because the calculation of the critical nodes and the construction of the necessary tables, such as p, p', and R, is considerable compared to the costs of the memoized recursive calls of **score** and **score**$'$.

[4] http://www.maltparser.org/.

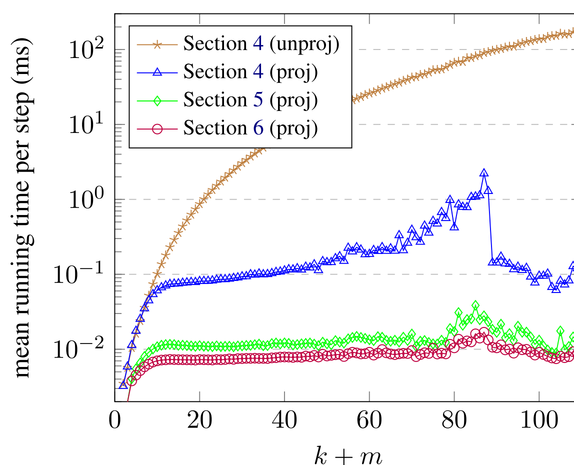


Figure 9: Mean running time per step (milliseconds) against length of input, for projectivized and unprojectivized trees.

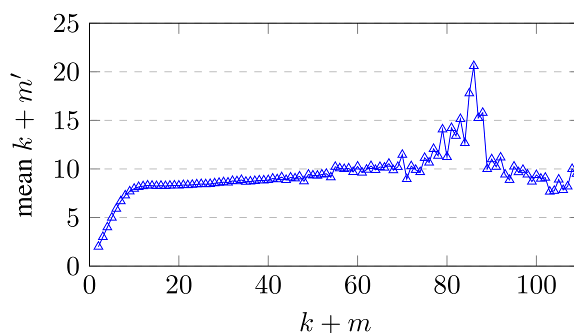


Figure 10: Mean $k + m'$ against $k + m$.

Both these algorithms contrast with the algorithm from Section 4, applied on projectivized trees as above (hence tagged proj in Figure 9), and with the remaining input simplified to just its critical nodes. For $k + m = 80$, the cubic-time algorithm is slower than the linear-time algorithm by a factor of about 65. Nonetheless, we find that the cubic-time algorithm is practically relevant, even for long sentences.

The decreases at roughly $k + m = 88$, which are most visible for Section 4 (proj), are explained by the fact that the running time is primarily determined by $k + m'$, where m' is the number of critical nodes. Because $k + m$ is bounded by the sentence length and the stack height k tends to be much less than the sentence length, high values of $k + m$ tend to result from the length m of the remaining input being large, which in turn implies that there will be more non-critical nodes that are removed before the most time-consuming part of the analyses is entered. This is confirmed by Figure 10.

LAS UAS	(1) subset	(2) all	(3) subset Sec. 6	(4) all Sec. 6	(5) all Sec. 4
de, 13%	71.15	71.33	71.69	**72.57**	72.55
263,804	78.09	78.14	78.96	**79.78**	79.77
da, 13%	69.11	71.42	69.95	72.18	**72.25**
80,378	75.13	76.98	76.30	78.00	**78.21**
eu, 34%	54.69	**58.27**	54.11	57.49	57.81
72,974	67.49	70.07	67.71	70.07	**70.13**
el, 12%	71.62	**72.78**	70.49	72.66	72.34
42,326	77.45	78.34	77.14	**78.91**	78.36
cu, 20%	56.25	59.09	56.31	58.78	**59.52**
37,432	68.08	69.95	69.07	70.10	**70.94**
got, 22%	51.96	55.00	53.44	55.94	**56.20**
35,024	64.48	66.58	65.85	67.85	**68.09**
hu, 26%	52.70	56.20	54.09	57.37	**57.62**
20,166	65.72	68.96	67.55	70.20	**70.30**

Table 7: Accuracies, with percentage of trees that are non-projective, and number of tokens. Only gold computations are considered in a single pass (1,2) or there is a second pass as well (3,4,5). The first pass is on the subset of projective trees (1,3) or on all trees after optimal projectivization (2,4,5). The second pass is on projectivized trees (3,4) or on unprojectivized trees (5).

The main advantage of the cubic-time algorithm is that it is also applicable if the training corpus has not been projectivized. To explore this we have run this algorithm on the same corpus again, but now without projectivization in the second pass (for training the classifier in the first pass, projectivization was done as before). In this case, we can no longer remove non-critical nodes (without it affecting correctness), and now the curve is monotone increasing, as shown by Section 4 (unproj) in Figure 9. Nevertheless, with mean running times below 0.25 sec even for input longer than 100 tokens, this algorithm is practically relevant.

8.3 Accuracy

If a corpus is large enough for the parameters of a classifier to be reliably estimated, or if the vast majority of trees is projective, then accuracy is not likely to be much affected by the work in this paper. We therefore also consider six languages that have some of the smallest corpora in UD v2.2 in combination with a relatively large proportion of non-projective trees: Danish, Basque, Greek, Old Church Slanovic, Gothic, and Hungarian. For these languages, Table 7 shows that accuracy is generally higher if training can benefit from *all* trees. In a few cases, it appears to be slightly better to train directly on non-projective trees rather than on optimally projectivized trees.

9 Conclusions

We have presented the first algorithm to calculate the optimal step for shift-reduce dependency parsing that is applicable on non-projective training corpora. Perhaps even more innovative than its functionality is its modular architecture, which implies that the same is possible for related kinds of parsing, as long as the set of allowable transitions can be described in terms of a split context-free grammar. The application of the framework to, among others, arc-eager dependency parsing is to be reported elsewhere.

We have also shown that calculation of the optimal step is possible in linear time if the training corpus is projective. This is the first time this has been shown for a form of projective, deterministic dependency parsing that does not have the property of arc-decomposibility.

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