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The “Merger Paradox” and Bertrand Competition with Equally-sized Firms

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Abstract

This essay models profit and price effects of horizontal mergers among equally capacity-constrained firms, in a homogenous product market à la Bertrand. We demonstrate that horizontal mergers that create a larger market rival are always non-harming, in contrast to the provisions of homogenous good symmetric Cournot models. The basic symmetric three-firm model and the general oligopoly one show that post-merger size configurations that allow for mixed strategy equilibria are always profitable for all firms, i.e. there is no free-rider problem by the outsiders. If mixed strategy equilibria arise post-merger then price increases since firms randomize their strategy over a higher price range (price range extrema are strictly greater than those prior to the merger). Moreover, mergers relax competitive constraints since aftermath firms compete less fiercely (post-merger distributions are stochastically dominated by the pre-merger ones).

1. Introduction

In the long history of theoretical economic literature the characteristics of market structure have been a constant concern for economists. The analysis of competition in noncompetitive settings have given rise to a number of alternative theories, formalized game theoretically, to constitute the various modern theories of oligopoly behavior. The main body of modern industrial economics research informs a set of different games that do not represent competing theories, but rather models relevant in different industries or circumstances. The investigation of noncooperative output, pricing, and investment policies inevitably intertwines several neighboring topics such as collusion and cartels, entry deterrence, and product differentiation. Thus, oligopoly theory informs how an industrial economist views and interprets reality and the conclusions of any theoretical work in applied microeconomics. The fundamental strategy choice affects the conclusions not only of the theoretical strategic interaction models, but its applications to more specific settings.

As a clear example of the diversity of the conclusions that can be reached studying the same phenomenon under the light of different oligopolistic settings, merger activity has received great interest in the theoretical industrial economics research. Yet, despite the conspicuous literature on the subject, no unanimous consent has been achieved on the unilateral effects of horizontal combinations of firms in closely related markets. In oligopoly models à la Cournot, Salant Switzer and Reynolds

1 Sir Thomas Moore coined the term oligopoly in his “Utopia” (1517) sensing small number rivalry may not necessarily lead to a fall in price to the competitive level. The reference to Moore relies on Schumpeter (1954, p. 305).

(1983) find that horizontal mergers without cost synergies are not beneficial to merging firms unless they involve the vast majority of market participants. On the contrary, nonmerged firms gain from the merger, free riding the price increase by the merged firm. Perry and Porter (1985), Farrell and Shapiro (1990), Levin (1990), and McAfee and Williams (1992) propose different approaches for the resolution of the paradoxical result within models of quantity competition. The approach in Daughety (1990) assumes that the merger changes the rules of the game. The model analyzes competition à la Stackelberg with multiple followers and leaders in which a merger between two followers allows the new firm to become a leader.

In oligopolies à la Bertrand with product differentiation, Deneckere and Davidson (1985), Braid (1986), Reitzes and Levy (1990), and Levy and Reitzes (1992) attain profitability of horizontal mergers for insiders, although confirming the higher gains for outsiders and the overall industry-wide price increase. On these grounds, horizontal merging would hardly be explainable since a firm always prefers to remain an outsider. In computational models à la Bertrand-Edgeworth, Froeb, Tschantz and Crooke (2003), Higgins, Johnson and Sullivan (2004), and Choné and Linnemer (2010) find that horizontal mergers have no real effect on price, quantity, consumer surplus and total welfare. Froeb et al. (2003) investigate a merger model with capacity-constrained price setters engaging in the parking lots business. The model shows price increase for the non-merged firms, as expected, but not for the merged firms whose capacity constraints are in fact binding and their price does not levitate. They show this latter effect prevails on the former and the merger depresses price. Thus, the net merger effect on price, consumer surplus, and total welfare is null.

Our essay models horizontal mergers, i.e. mergers among close competitors, in noncooperative static pricing games among a fixed number of active capacity-constrained oligopolists. The aim of this dissertation is to characterize theoretically and investigate thoroughly price and profit effects of horizontal mergers, in a static framework of equally-sized suppliers of a homogenous good with Bertrand competition. Choosing price as strategy rather than quantity produces several appealing

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3 Empirical evidence on the impact of horizontal mergers on outsiders upholds the idea that outsiders are harmed by horizontal combinations of rivals, see Eckbo (1983), Stillman (1983), and Banerjee and Eckard (1998).

4 Kamien and Zang (1990) notice that encouraging mergers, short of participation, makes it hard to prevent defections from a proposed merger, even if it is beneficial.

5 Static oligopoly theory provides predictions about short-run firms’ behavior, taking as given the the set of firms competing in the market. Long-run industry analysis can be enriched by entry and exit considerations as in Werden and Froeb (1998), Cabral (2003), Spector (2003), and Davidson and Mukherjee (2007).

6 Whereas the horizontal mergers literature focuses on Cournot competition or differentiated product Bertrand competition, very little theoretical advance has been made in the theoretical investigation of horizontal mergers in homogenous product Bertrand competition. The exceptions are the work of Hirata (2009) in the context of asymmetric triopoly and more recently
results in horizontal merger analysis with capacity constraints. Horizontal mergers, even in absence of cost synergies, ought to be profitable since they increase market power for the merged firm. The symmetric premerger competitors merge to create a bigger firm whose size ought to gain an advantage to the merged firm with respect to outsiders. The results of this work hinge on the dimensional assumptions on firms’ size to determine the nature of equilibrium. As in Kreps and Scheinkman (1983), if capacities are unlimited, equilibrium will be à la Bertrand, i.e. marginal cost pricing, independent of the number of firms or the elasticity of aggregate demand. If capacities are set small enough, i.e. not greater than its best response quantity when the rivals supply their full capacity, each firm simply sets the price that markets all production. For intermediate levels of capacity, equilibrium has to be characterized in mixed strategies.

Horizontal mergers affect equilibrium characteristics by modifying the capacity configuration of the market post-merger. Different dimensional characteristics of the merged firm with respect to the remainder of the industry might alter firms’ price incentives. If individual firms’ capacity is so large to satisfy all forthcoming demand at marginal cost either individually or collectively then horizontal mergers do not affect equilibrium price and profits. Capacity is so large that even pre-merger it drives price down to marginal cost. Analogously, if firms’ capacity is so low (firms are in fact capacity-constrained), that is if firms charge the price that markets aggregate production, then equilibrium post-merger is unaltered by the merger and no effect is exerted by the merger itself. This means that if post-merger firms’ capacity is so big or so small to allow for a pure strategy equilibrium with the same characteristics as those prior to the merger, then horizontal mergers do not have effects on both price and profits (both industry-wide and individually with no distinction between merged and non-merged firms).

However, if the merged firm gains sizable dimensional advantage, post-merger equilibrium in mixed strategies emerges and post-merger firms will randomize their price offers over a range that is strictly higher than the pure strategy equilibrium premerger. Hence, horizontal mergers alter market structure and are always profitable for both merged and non-merged firms.

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the work of Gang Li (2013). The latter work refers to the first essay of the Ph.D. dissertation of the scholar whose work parallels ours in many regards. The study of the symmetric triopoly and oligopoly in Gang Li is shorten of the original Bertrand position, focusing on the pure strategy region where equilibrium is à la Cournot and the mixed strategy one. The extension to the symmetric oligopoly case considers only the merger of two firms among the n participants, in the same equilibrium regions observed in the triopolistic industry. Our model instead considers the full span of capacities, for both the triopoly and the oligopoly, and extends the number of merger partners from two to m, for a more general horizontal mergers analysis in symmetric oligopolies.

Mixed strategy equilibria model the volatility of price offers among suppliers of a homogenous good. The relevance of the analysis of mixed strategy equilibrium in this essay may be paralleled to the investigation of price competition in price dispersion models à la Varian (1980). These models’ findings, allowing for mixed strategy equilibria, do not match the
Mixed strategies equilibria are characterized by higher price than pre-merger and higher single-firm as well as industry profits. Pre-merger mixed strategy equilibria never revert to a pure strategy equilibrium. If capacities fall in the mixed strategies region then firms mix their strategies post-merger on a higher than premerger price range. Not only mergers induce a price increase in this framework, but post-merger price distributions stochastically dominate pre-merger ones as well. Horizontal mergers increase market concentration and so doing internalize the negative externality that more aggressive pricing have on rival suppliers. Then, horizontal combinations of firms in a static price-setting game create a price-increasing effect (that translates into an actual price increase) and allow firms to undercut a certain price level with lower probability than pre-merger.

This essay is organized as follows. Chapter 2 surveys the most preeminent literature of horizontal mergers introducing the theoretical analysis of the “merger paradox” and the strand of literature both in Cournot and Bertrand competition that followed. Chapter 3 gives the interpretative tools for the analysis of the following chapters, illustrating the literature on price-setting oligopolies and their results. Chapter 4 studies the basic symmetric triopoly, whose equilibrium characteristics pre and post-merger is characterized completely to address price and profit effects of horizontal mergers of any two firms in the industry. Chapter 5 extends the reach of the previous chapter, investigating horizontal merger effects in a symmetric oligopoly and a linear demand function example. Chapter 6 concludes.

2. The Trouble with Horizontal Mergers

Mergers represent the ultimate form of collusion: when two firms merge, they combine their assets in a new entity, whose constituent firms lose their independent identities. The simplest class of models in which is possible to characterize, analytically and formally, the consequences of takeovers among sellers are static oligopoly models. Most frequently mergers can be modeled as an exogenous change in market structure. According to the game theoretic approach, mergers correspond to a move from a Nash equilibrium in a given coalition structure to a coarser one. Theoretical approaches do not easily encompass welfare considerations, except in a very partial-equilibrium setting. Due to these difficulties, the positive side of the analysis has received more weight than the normative. The subject of mergers is particularly broad, involving different definitions for different phenomena. Although all are worthy of scrupulous analysis, we limit ourselves to horizontal mergers, i.e. mergers among firms with substantial overlap of business.

provisions of the Law of One Price, and sustain the multiplicity of price levels through time in a market with heterogeneously informed consumers and homogenous product.

8 There is a fairly large literature that tries to endogenize the set of mergers occurring in a market absent any antitrust constraint. See Kamien and Zang (1990), Bloch (1996), Yi (1997), and Gowrisankaran and Holmes (2004).
The raise of economists' and competition agencies’ concern with horizontal combinations has its roots in the relationship between industry concentration and pricing policy. A more concentrated market is presumed to give way of unilateral price increases by the competing firms. Fixing rival prices or outputs, a merger among sellers of substitute goods will lead to internalize the negative externality that more aggressive pricing or output choices have on the merger partners. Further “regularity” conditions are needed for a merger to effectively translate the price-increasing effect in a factual price increase. Absent efficiency gains, a merger raises price under fairly weak regularity conditions in the homogenous product Cournot model.

In differentiated price competition models, for a merger to rise industry-wide price it is necessary that the diversion ratio (the recipient of the lost share of individual firms’ sales, due to a product’s price increase, is the merged firm) creates an incentive for the merged entity to increase the price of any one of its products holding fixed the prices of all of its other products and the prices of rivals. Thus, holding rival prices fixed at any levels, it is sufficient to show that the merger causes the merged firm to raise all of its prices to ensure a price increase. This implies that the best responses are “upward sloping”. The latter condition implies that the merged firm’s price increases lead rivals to increase their prices, which in turn causes the merged firm to further increase its own prices, and so on. Price distortion is not the only presumed effect of a decrease in market concentration. Fewer competitors are presumed to collude more easily, fixing output shares and prices more successfully than in larger and more competitive industries.

This chapter surveys the most important and established results in horizontal merger analysis, both theoretical and empirical, to investigate the emergence of the “merger paradox” and the most relevant studies that followed. Section 2.1 introduces the unilateral effects approach, i.e. the investigation of unilateral post merger price increases. In subsection 2.1.1 the Salant et al. (1983) “merger paradox” is presented, as well as alternative models to overcome such result. Models that encompass welfare considerations in quantity-setting oligopolies are considered in subsection 2.1.2. Section 2.2 presents the “coordinated effects” approach, i.e. investigation of mergers on collusive behavior. The concluding third 2.3 and 2.4 are devoted to a review of the empirics of horizontal mergers and the theory of merger waves.

2.1. The Economics of Horizontal Mergers: The Unilateral Effects Approach

Increased concentration through mergers among direct competitors arises concern since they may lead to anticompetitive price increases; either because the merged entity unilaterally raises prices

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9 For the economics of antitrust and regulation that encompass welfare evaluations, see Motta (2004), Whinston (2006), and Buccirossi (2008). For a survey on the economic principles of antitrust, see Kaplow and Shapiro (2007).
from pre-merger levels (so-called unilateral effects) or because the increase in concentration enhances the prospects for successful collusion (coordinated effects). Nevertheless, it is not at all clear that the acquiring firm benefits from its efforts. This problem was recognized long ago by Stigler (1950) who noted that the promoter of a merger might expect to receive every form of encouragement from other firms, short of participation. The underlying idea of theories of unilateral effects is that the merged firm will have an incentive to raise its price post-merger, because of the elimination of direct competition among the firms that have merged. The idea that a firm with a large share will have more market power, thus charging a higher price still lower than the monopoly price, has been very influential in horizontal merger enforcement. So has the idea that margins are higher in more concentrated industries. The examination of specific oligopoly models makes it possible to quantify horizontal mergers effects in order to identify the mergers that are most likely to have significant price effects and thus cause significant harm to consumers. Moreover, quantification allows estimating the merger efficiencies necessary to offset the loss of competition and thereby be cleared by competition authorities according to the consumer surplus or total welfare standards.

2.1.1. Cournot Competition and the “Merger Paradox”

Although awaited, mergers need not be profitable for the insider firms. Salant, Switzer and Reynolds (1983), notice that horizontal mergers\textsuperscript{11} in homogeneous product Cournot-Nash industry might be unprofitable. That is, mergers can increase industry profit and, at the same time, reduce profitability of the merged firms below the sum of individual firms’ premerger profits. The reduction in the number of firms, that implies an output restriction, raises price. The merging firms reduce their output because they internalize more of the effect of their output on price than they did previously. In turn, non-merging firms raise output somewhat, leading the merging firms to cut output further. At the new equilibrium, price is higher. But the merged firm’s combined share of total industry profits is lower; after all, outsider firms’ quantities rise and the output of the merging firms falls. Salant et al. (1983) consider the case of symmetric oligopolists facing linear decreasing demand \( p = a - bX \) and constant marginal costs. Under these assumptions, industry equilibrium is symmetric, i.e. each firm produces the same output and earns the same profit. More formally, suppose that there are initially \( n \) firms and that each faces price \( p = c + \frac{a-c}{b(n+1)} \), sells quantity \( X_i = \frac{a-c}{b(n+1)} \), and earns profits

\textsuperscript{10} A price increase is often a proxy for other forms of anticompetitive effects, e.g. a reduction in product quality and service

\textsuperscript{11} Following Stigler’s (1966) definition a horizontal merger is defined as a binding agreement to maximize profits jointly by correlating their strategies and making side payments, if necessary. The merged firm is then treated as a multiplant Cournot player engaged in a noncooperative game against the remainder of the industry.
When firms merge, for \( m < n \), it will be the case that, choosing \( X_1, ..., X_n \) to maximize profits

\[
\Pi(n) = \frac{1}{b} \left( \frac{a-c}{(n+1)} \right)^2.
\]

Where \( X_m \) is the total output of the post merger firm. Substituting in the equation of the inverse demand curve

\[
p - c = b \left[ \left( \frac{a-c}{n} \right) - (X_m + X_{m+1} + ... + X_n) \right]
\]

The post merger firm reaction (best-response) function is

\[
2X_m + X_{m+1} + ... + X_n = (a-b)/n
\]

This is the best response of a single firm in an \( n - m + 1 \)-firm Cournot oligopoly. The best response functions of the independent firms are unchanged by the merger, leaving the post merger equilibrium as it would be in the case of a Cournot game with \( n - m + 1 \) equally sized firms.\(^{12}\) These are standard conditions stating that the profit of each firm, as well as total industry profit, increases as the number of firms is reduced. Under these specific conditions on demand and cost functions, the merger is profitable if and only if

\[
\left( n + 1 \right)^2 \geq m \left( n - m + 2 \right)^2
\]

Inequality (1.4) states that the profit to the single merged firm can be less than the sum of the pre-merger profits of its constituent firms. This occurs depending on the values of \( m \) and \( n \). Simplifying inequality (1.4) leads to

\[
\left( 1 - m \right) \left[ m^2 - (3 + 2n)m + n^2 + 2n + 1 \right] \geq 0
\]

Since a merger will occur among more than two firms, the first term will be negative and the relevant inequality turns out to be

\[
m^2 - (3 + 2n)m + n^2 + 2n + 1 \leq 0
\]

Whose solution is

\[
\frac{3 + 2n - \sqrt{5 + 4n}}{2} \leq m \leq \frac{3 + 2n + \sqrt{5 + 4n}}{2}
\]

\(^{12}\) The symmetric treatment stems from the assumption of constant, identical marginal and average cost. The increase in industrial concentration increases outsiders’ profits while the coalition has to divide these profits \( m \) ways, increasing the possibility of horizontal mergers to be unprofitable.
Since, necessarily $m \leq n$, then

$$\frac{3 + 2n - \sqrt{5 + 4n}}{2} \leq m \leq n \quad (2.8)$$

Let’s consider the case is of an almost market-monopolizing merger involving $n - 1$ firms, and solve the inequality for such a case. Then inequality becomes

$$\frac{3 + 2n - \sqrt{5 + 4n}}{2} \leq n - 1 \Rightarrow 5 \leq \sqrt{5 + 4n} \quad (2.9)$$

$$25 \leq 5 + 4n \Leftrightarrow n \geq 5 \quad (2.10)$$

this partial result may be generalized considering a generic number of firms $s$

$$\frac{3 + 2n - \sqrt{5 + 4n}}{2} \leq n - s \Rightarrow 3 + 2s \leq \sqrt{5 + 4n} \quad (2.11)$$

$$9 + 12s + 4s^2 \leq 5 + 4n \Leftrightarrow n \geq 1 + 3s + s^2 \quad (2.12)$$

The condition associated to horizontal merger profitability is quite restrictive, imposing a high number of participants for the merger to be profitable. For instance, a merger to duopoly in a three-firm market will not be profitable in such a setting. The argument may be repeated and extended leading to the fact that for a merger to be profitable the 80% of market participants need to take part in the merger. The exception to such a finding occurs when oligopolists merge into monopoly.

In this simple model, a merger does not lead to a bigger and fiercer competitor in the market in any sense. Furthermore, not only the merger may be at least non profitable for the constituent firms, but it is ultimately profitable for the outsiders.\textsuperscript{13} Note that this model is not a dominant-firm and competitive-fringe model: firms are not price takers. In the new symmetric equilibrium, each firm shares the output restriction equally. The peculiar result obtains because there is nothing to distinguish the merged firm from the $n - m$ non merged firms: all have access to the same technology and therefore, all firms produce $1/(n - m + 1)$ of the industry output. However, this model violates the intuitive notion of what horizontal mergers are all about. Starting with $n$ identical firms of which $m$ merge, we expect the result to be $n - m$ small old firms and one large new firm, that is to say the merged firm would be expected to gain in capacity rather than scaling-down its constituents’ capacity. The new firm should have access to the combined productive capacity of the merger partners. Moreover, the model doesn’t consider capacity constraints, i.e. assumes constant returns to scale, as well as does not assume cost differences among firms and product differentiation.

\textsuperscript{13} Here the merger can be seen to have produced a public good, i.e. the high price level, that the non-merging firms free-ride, earning higher profits than pre merger by selling the same output at higher price.
Accordingly, a theory that plausibly explains mergers that actually occur requires that the merging firms own assets that can be usefully combined. In two different works, i.e. different competitive frameworks, both Deneckere and Davidson (1985), and Perry and Porter (1985), reverse the Salant et al. results and find that an incentive to merge would rise by introducing different notions of size.

Deneckere and Davidson (1985) consider the case of price-setting firms producing symmetrically differentiated products. In their model, when firms merge the merged firm can continue to manufacture the entire product line of each constituent firm. That is to say, the merged firm can always duplicate the actions by its members before they joined, without altering the characteristics of their products. Firms own exclusive technology for production, i.e. patents, and operate at constant and identical average cost. Under these assumptions, the Salant et al. results can be reversed thanks to a particular feature of the price setting game: best responses are upward sloping. This implies that an initial price increase of the coalition will be followed by an increase on the part of outsiders. Coalition then raises price and so on. It must be reminded that the game is a static game, not to be confused with multi-stage repeated games. Settlement will lead to higher price level and all firms will be better off. Hence, under fairly reasonable demand schedule, mergers become always beneficial to existing members and become more and more profitable as the size of the aggregation increases.

Perry and Porter (1985) choose a different route to address the merger paradox, analyzing the cost side. In their model, each firm produces a homogeneous product and owns some fraction of a tangible asset of the capital factor, whose industry fixed supply is normalized to one. So they can address the asymmetries caused by the merger of subsets of firms. When firms merge, the newly created firm controls the assets belonging to each of its constituents and faces a new maximization problem because of its altered cost structure and strategic considerations. It is assumed that the long-run technology is subject to constant returns but that, due to the presence of the fixed factor, any single firm's short-run marginal-cost curve is upward sloping. A large firm, holding a higher fixed factor share, can therefore produce the same output at lower cost than a smaller firm. Two cases are considered, a dominant-firm and competitive-fringe model and an oligopoly with large and small firms. The results from the first model are very similar to those obtained in d'Aspremont, Jacquemin, Jaskold-Gabszewicz and Weymark (1983). Perry and Porter, for instance, find that there is always an incentive for a dominant firm to form out of the fringe. Whether or not additional firms merge depends on the precise demand and cost parameters. In general, however, there will be an


15 This suppresses de novo entry into the industry. The fixed-factor assumption is a limiting case of upward sloping factor supply curve. By fixing the size of the industry it is easy to examine changes in the number of firms and their sizes.
equilibrium number of merged firms, i.e. a stable cartel successfully forms. The second model, with asymmetries in dimensions of firms, is a closer analogy to the Salant et al. setting. With increasing marginal costs, however, the incentive to merge usually remains. The incentive to merge depends upon the resolution of two forces. First, a merger results in a price raise, benefiting all firms, and second, the output reduction of the merged entity declines relative to that of its partners prior to the merger. The price increase benefits all firms and can be often sufficient to compensate for the output reduction of the merged firm, so to increase profits. Although a merger incentive need not necessarily follow, the output decrease is less severe than the Salant et al. result. As a matter of fact, Salant et al. ought to be considered essentially the limiting case of the more general Perry and Porter model, obtained as the marginal-cost curve flattens.

2.1.2. Horizontal Mergers Are Worth a Defense: Static Efficiencies

Antitrust authorities work, amongst the other interests and objectives, to prevent monopolization of the market and render oligopolistic coordination more difficult. Although a standalone reason for mergers to be scrutinized severely, horizontal concentration, via merger, is not only inspired by twisting the terms of trade to gain higher profits, but a few motives may be adduced. Our brief treatment is devoted to the recognition of specific areas of inspection that are considered at length in devoted surveys and articles. We will mention the static economies of scale, scope and product variety and the dynamic ones of learning and technical change, as the most effectively studied and illustrated.

Economies of scale have long been proposed as an antitrust defense, inspired by Williamson’s (1968) seminal contribution.\textsuperscript{16} Economies of scale are said to exist whenever long-run average cost declines with output, because of set-up costs or marginal cost decline as production expands. When economies occur, there is a tradeoff between cost reduction and output restriction as the number of firms varies. The net effect would depend on several factors, e.g. own and cross-price elasticities of demand, own and cross-output elasticities of total cost, degree of collusion in the market, and whether the nature of collusion changes with the number of firms. Upon sufficient assumptions about demand, cost, and collusion, can be obtained partial-equilibrium formulas for the sizes of cost savings and surplus reductions. This is not our intention here. Rather, we simply wish to emphasize that, when scale (and scope) economies exist, under fairly comprehensive assumptions, few firms may be preferred to many and market structure would depart from perfect competition.

\textsuperscript{16} Static efficiencies comprise economies of scope, as well. Economies of scope arise when joint production of two or more products is cheaper than the cost of producing the products separately. For the formal analysis of economies of scope, see Panzar and Willig (1981) and Waterson (1983).
Whether or not mergers are beneficial under these circumstances, however, is a very different issue. Mergers occur between existing firms whose cost structures are not completely flexible, when two of them merge, it is not clear if cost reductions can be realized ex post. Welfare effects evaluation in a static context belong to this stream of economic analysis of oligopolistic competition. The so-called “efficiency defense” has been emphasized by Williamson (1968), in a simple partial equilibrium model of pre-merger competitive equilibrium. The need to reconcile reduction of competition and productivity gains by merger, led Williamson to the result of small cost reductions may more than compensate twisting terms of trade, i.e. price increase, enhancing total welfare. In Whinston (2007, p. 2347) the formal argument is presented as follows: welfare reduction due to an infinitesimal increase in price from the competitive price is of second-order, i.e. has a zero derivative, while the welfare increase from an infinitesimal decrease in cost is of first-order, i.e. has a strictly positive derivative.

The model is not free of doubtful assumptions and is worthy of careful scrutiny. The starting assumption of competitive economy, i.e. price equals marginal cost, is quite strong. Then Williamson glosses over different cost structures among firms, considering a unique marginal cost prevailing in the oligopolistic market, which often show empirical differences among firms’ marginal cost levels, both before and after the merger. Since cost gains are likely to occur to merged firms, unless we consider industry-wide aggregations, this assumption may show itself critical for welfare evaluation. Moreover, Williamson object is aggregate surplus maximization, while antitrust policy raises questions of distribution. Many enforcement agencies focus on consumer surplus standard and not on aggregate one to evaluate mergers. Last, but not least, Williamson considers price as the only competitive variable. In practice, firms compete over capacity, investment, R&D, product quality depending on the market structure a merger might affect.

The argument has been then developed and deepened by Farrell and Shapiro (1990). In their model à la Cournot under quite general demand and cost stability assumptions sufficient conditions for profitable mergers to raise total welfare are provided. Mergers are shown to raise price unless cost synergies are generated among merging firms, i.e. considerable economies of scale or learning. For price to fall, the merged firm’s marginal cost at the pre-merger joint output of merging firms, must be below the marginal cost of the more efficient merger partner. If the merged firm had the same marginal cost as the most efficient merger partner, then a marginal increase in output would have the same incremental cost and would be sold at the same price for the two firms. Perhaps price reduction would be more costly for the merged firm than for the more efficient merger partner, since merged firm sells more. Considering the price incentives of the firms, as the more efficient merger partner did not find it worthwhile to further increase its output before the merger, so it will the merged firm.

17 For related analyses, see Levin (1990) and McAfee and Williams (1992).
Hence, for the merged firm to increase its output above the pre-merger level, and consequently reduce price, it must have a lower marginal cost than the more efficient merger partner.

Let a horizontal merger in Cournot industry be formalized as the replacement of two pre-existing firms (whose respective cost functions are \( C_1(X_1) \) and \( C_2(X_2) \)) with a new firm whose cost function is \( C_{12}(X_{12}) \). Farrell and Shapiro define the merger to raise no synergies if the merger simply allows the merging firms to rationalize output between their existing operations or facilities. The merged firm solves the problem \( C_{12}(X_{12}) = \min_{X_1,X_2}[C_1(X_1) + C_2(X_2)] \) subject to \( X_{12} = X_1 + X_2 \). Define the two merging firms’ pre-merger outputs as \( ̅X_1 \) and \( ̅X_2 \), with \( ̅X_2 \geq ̅X_1 \), and the pre-merger price as \( P = ̅P \). Pre-merger Cournot equilibrium condition obtained by firms’ optimization problem is \( ̅P - MC_i = \frac{S_i}{|e_i|} \), where is the \( MC_i \) firm \( i \) marginal cost, \( S_i \equiv X_i/X \) defines firm \( i \) market share, and \( e_i \) market elasticity of demand. Larger firms have higher markups (the LHS of equilibrium condition), so firm 1’s marginal cost in the pre-merger equilibrium, \( ̅MC_1 = MC_1(̅X_1) \), at least evens firm 2’s, \( ̅MC_2 = MC_2(̅X_2) \). Let the merged firm’s marginal cost at the combined pre-merger output be \( ̅MC_{12} = MC_{12}(̅X_1 + ̅X_2) \). Without synergies, the merged firm’s ability to rationalize production between its existing operations (by equating the marginal cost of production in the two operations) is not sufficient to offset the incentive to raise price that results from combining the ownership interests of the two operations. For a horizontal merger to lead to a reduction rather than an increase in price it is necessary to realize synergies that Farrell and Shapiro quantify in a very general necessary and sufficient condition: a merger reduces price if and only if

\[
MC_2 - MC_{12} > ̅P - MC_1.
\] (2.13)

Since \( MC_2(̅X_2) \leq MC_1(̅X_1) < ̅P \), that follows from the firms’ optimization problem first-order conditions and the fact that \( ̅X_2 \geq ̅X_1 > 0 \). That is, the merger will reduce price if and only if the merged firm marginal cost (at the pre-merger combined output) is lower than the marginal cost of the more efficient firm (at its own pre-merger output) to a greater extent than the difference between pre-merger price and the marginal cost of the smaller, less efficient firm (at its own pre-merger output). This inequality can be expressed in proportion to the pre-merger price as \( MC_2 - MC_{12}/̅P > ̅P - MC_1/̅P = S_1/|e_1| \). This expresses the pre-merger relationship between firm 1’s margin (or market power) and its share. In industries with moderate to large pre-merger margins the condition becomes very strict to be satisfied. Using these general results, Froeb and Werden (1998) provide calculations that relate the required magnitude of the synergies to the pre-merger shares of the merging firms. In the symmetric case, they show that the proportionate reduction in marginal cost necessary for price not to rise is equal to \( S/(|e_1| - S) \), where \( S \) is the pre-merger market share of each merging firm.
The study of welfare effects, presuming that proposed mergers are profitable for the parties and, implicitly, that private gains for merging parties translate into social gains, is more complex and depends upon the cost structures of both the merged firm and the merging partners. Since welfare measures require a great deal of information to be assessed, Farrell and Shapiro introduce differential techniques to study the external effects on consumers and outsider firms. If the externality is positive, so it will be aggregate surplus variation. To overcome the difficulty in directly signing the mathematical effect, the model considers a small change in joint profits, which is called “infinitesimal mergers”. Farrell and Shapiro show that in this case the external effect is non-negative if the market share of the merging firms is less than an elasticity-adjusted Herfindahl–Hirschman index of the non-merging firms. They also find that mergers increasing output are welfare enhancing, as well as those that increase the Herfindahl index. Absent synergies, a merger rises price because merged firm joint output falls, since total output will move in the same direction as merged firm’s output. They consider examples of policy implications within this framework, as well. The first considers moderate size mergers, under reasonably general conditions, that rise price and are privately profitable, increase welfare. The second considers an open economy, and obtains under suitably general conditions, that imposing or tightening an import quota when the import share is low, increases welfare.

2.1.3. Horizontal Mergers Are Worth a Defense: Dynamic Efficiencies

As most analyses focus on static models of oligopolistic interaction, a stream or research has opposed to the one shot static economies, the dynamic ones. These occur over time and, as it is the case of intertemporal problems, the issues are even more complex. Dynamic effects are learning and technical change. Learning is said to occur when unit cost declines with cumulative output. In assessing competition policy, it is important whether learning occurs within an industry or within a firm. When the benefits of learning cannot be captured by firms, there is no incentive to have fewer firms in an industry. The standard economic analysis assumes learning as firm specific, see Spence (1981) and Tirole and Fudenberg (1983). The special dynamic issues that arise as a consequence of learning, such as the possibility of strategic behavior, are beyond the scope of this chapter. Under the lens of competition policy, learning bears a resemblance to scale economies. The calculation of the sizes of these effects is even more complex than with static economies but the same qualitative conclusions hold.

18 The Herfindahl–Hirschman Index (HHI) of market concentration, commonly used in antitrust analysis, especially of horizontal mergers is defined as $H = \sum_i S_i^2$. The industry-wide average, output-weighted, price-cost margin gives the definition of the elasticity-adjusted Herfindahl–Hirschman index: $PCM \equiv \frac{\sum_{i=1}^n S_i \frac{p-MC_i}{p}}{n} \equiv \frac{\sum_i S_i^2}{|E_d|}$. 

16
Technical change, which expands the production possibilities available to society, is perhaps the most important dynamic economy. Few economists would disagree with the claim that the dynamic-efficiency gains from continuing innovation far outweigh the static gains from marginal-cost pricing. If markets where innovation is frequent, might be characterize by less vigilant competition policy, monopoly rents would be constantly eroded with the introduction of new, innovative, products and processes. The debate about the relationship between market structure and innovation has been going on at least since the time of Schumpeter (1954) and yet economists still hold diametrically opposed views on the subject, see a relevant discussion on these issues in Nelson and Winter (1982). Dasgupta and Stiglitz (1980) build a theoretical model that confirms many intuitive notions concerning the relationship between market structure and innovation. In their model, at least for low levels of concentration, there is a positive correlation between concentration and R&D effort. In addition, they find that there may be excessive duplication of effort in an unregulated economy, i.e. resources would be wasted with respect the little cost reduction obtained. As final remark, they find out a pure monopolist underinvests in R&D. However, if empirical evidence were to suggest a positive correlation between market concentration and innovation, such a correlation would not reveal the direction of causality. A school of thought reads the causality relation from costs to market power: Low-cost firms increase their market shares at the expense of less efficient firms. As a result, low-cost firms have large market shares and high concentration is a mark of efficiency.19

2.2. The Economics of Horizontal Mergers: The Coordinated Effects Approach

Mergers pose a risk to competition since increase the likelihood that a collusive outcome prevails post merger. Collusion is assumed to be easier to achieve and sustain when there are fewer suppliers in the industry. Therefore, reducing the number of competitors tends to facilitate collusive behavior. This idea underlies what is referred to as the “structural presumption”, i.e. increases in concentration lead to less competitive interactions. The structural presumption idea guided industrial organization economists’ research devoted to validating empirically the core idea of the structure-conduct-performance paradigm: highly concentrated markets tend to have higher prices and higher profits, causing greater harm to consumers, than competitive market structures, ceteris paribus. As a matter of fact, coordinated-effects theories of harm from horizontal mergers led merger analysis until the mid 1980s, at least.

19 Demsetz (1973) pointed out that a positive correlation would also arise if some firms were more efficient than competitors, and if the more efficient firms had large market shares. Thus, market concentration would result from the presence of larger and more efficient firms.
Overall, economists have grown less confident over the past several decades in stating that there is a systematic relationship between market concentration and market performance, at least over the range of market structures in which there are more than two or three firms. Even so, the Salinger’s (1990, p. 287) cautionary assertion bears repeating today:

“First, despite the well-known problems with this literature, it continues to affect antitrust policy. The inappropriate inferences used to justify an active antitrust policy have given way to equally incorrect inferences that have been used to justify a relaxed merger policy. Second, the alternative to cross-industry studies is to study specific industries. . . . [I]t is important to realize that it was the failure of studies of individual industries to yield general insights that made cross-industry studies popular.”

Nowadays, there is relatively little formal theory exploring the implications for merger policy of the relationship between collusion and market concentration. Hence, as instrument of antitrust enforcement it might suffer from inconsistency, at least. Notably, contributions by Compte, Jenny, and Rey (2002) and Vasconcelos (2005) investigate how the distribution of capacities affects the ability of the firms to sustain collusion in price-setting and quantity-setting supergames. Compte, Jenny and Rey (2002) consider the effects of horizontal mergers among asymmetrically capacity-constrained oligopolists in a repeated Bertrand model; Vasconcelos (2005) studies a repeated Cournot game in which firms’ cost functions \( C_i(X_i, k_i) \) depend both upon their output \( X_i \) and capital \( k_i \) (a merger of firms \( i \) and \( j \) leads to a merged firm with capital \( k_i + k_j \)). Similarly, in Kuhn’s (2004) repeated price setting model, horizontal mergers join the symmetrically differentiated product lines of the merging firms. These papers consider equilibria in which each firm’s profit along any equilibrium path is a constant share of aggregate profit. Davidson and Deneckere (1984) present a quantity-setting supergame and then a capacity-constrained price-setting supergame. First, they consider the equilibrium solutions of an infinitely repeated firm symmetric \( n \)-Cournot game, with linear demand and constant marginal and average cost. A first result is the higher post merger price (lower post merger output), which decreases consumer surplus. To study collusion they consider the infinitely repeated stage game enforced by trigger strategies, i.e. each firm produces its share of cartel output charging cartel price, till one firm increases output above its quota. When cheating occurs, economy turns to Cournot-Nash equilibrium and future collusion is no longer achievable. As \( m \) firms aggregate, they reduce output in each plant

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\[ \text{20 The inability of the firms to tacitly collude in a credible fashion, in repeated interaction models, rests on the exogenous terminal date at which rivalry ends. Such limitations have been evident at least since Stigler’s (1964) classic paper of oligopoly as a problem of policing tacitly collusive agreements.} \]

\[ \text{21 This simplifies equilibrium analysis since the set of subgame perfect equilibrium values for the firms is one-dimensional.} \]
and make it attractive for outsiders to free ride, increasing their own production. Industry equilibrium then moves closer to the cartel solution, increasing discount factor. Thus, defection weakens punishment lowering future losses, and renders collusion less sustainable. The results of the price-setting supergame with capacity constraints and efficient rationing are the same. The gains from cheating become more attractive since punishment declines after the merger. Thus, collusion would be less likely, as temptation to defect cartel arrangement increases. Horizontal mergers create a coalition whose insiders gain from mergers, but confirm that outsiders gain more from the collusive agreement than their rivals that actually collude.

Kovacic et al. (2006) propose a novel way to quantify the coordinated effects dangers when suppliers bid for customer’s patronage. They propose measuring the effects of incremental collusion, i.e. collusion between two firms, before and after the proposed merger. They show how this calculation can be performed in a particular bidding model. Baker (2002) has emphasized the role of maverick firms in destabilizing or preventing collusion and thus the particular dangers that arise when a merger eliminates such a firm (an idea embraced in the US Merger Guidelines as well). Collusion theory indicates that reaching and sustaining an agreement may be difficult if one of the firms expects to gain significant market share in the absence of collusion. Therefore, firms with strategies, products, or costs that are distinct from those of their rivals, as well as firms that are optimistic and growing rapidly, perhaps because they recently entered the market, are obvious candidates to be mavericks. Furthermore, merger may actually create a new maverick.

2.3. The Empirics of Horizontal Mergers

There are thus several theories that attempt to explain the stylized facts. Mergers need not be profitable in several studies analyzing profits, synergies or forms of internal inefficiency, including managerial and financial aspects. According to the Thomson Financial Securities Data, 87,804 mergers and acquisitions were recorded for Europe in the period 1993–2001. In monetary terms, the total value of these deals adds up to US$ 5.6 trillion, when the nominal GDP of the European Union was US$ 16.8 trillion in 2007 (according to IMF statistics). Given these numbers, considering only the EU, a comprehensive and extended empirical record should be expected to account for the actual effects of mergers. Somewhat surprisingly, there is no clear and definitive body of evidence. To some extent, this reflects a lack of data: even in those cases where one can accurately measure the prices charged before and after a merger, it may be hard to attribute price changes to the merger rather than to other changes in industry conditions. Also, the effects of a merger may arise in non-price dimensions such as product quality, customer service, or innovation. Furthermore, if merger

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22IMF figures are drawn from Belleflamme and Peitz (2010). For more surveys on merger empirics, see Golbe and White (1988) and more recently Pautler (2003), Whinston (2006), and Werden and Froeb (2008).
enforcement policy works smoothly, the mergers most likely to have large adverse price effects are never proposed or are blocked on antitrust grounds. In the remainder of the chapter, we examine several distinct methods for identifying and measuring the effects of mergers. In evaluating this evidence, we shall remind that over the past 25 years in the US, only about 2% to 4% of the mergers reported every year under the Hart-Scott-Rodino Act were considered to raise sufficient antitrust issues to warrant a second request from the Federal Trade Commission (FTC) or the Department of Justice (DOJ). This implies that data on the effects of all mergers may not reflect the effects of the major horizontal mergers that are most likely to be scrutinized by authorities.

2.3.1. Stock Market Prices

One way to measure the effects of mergers is to study the stock market performance of the merging firms. Usually, this is done using an event study around the time of the announcement of the merger. Andrade, Mitchell, and Stafford (2001) provide an excellent introduction to this widespread approach in the finance literature. The advantage of this approach is the reliance on detailed and accurate stock market data. However, by its nature, this approach cannot distinguish between favorable stock market returns based on efficiencies versus market power. The stock market being affected by the market sentiment and other behavioral phenomena, this approach measures the expectations of investors about merger effects, rather than the actual merger effects. Furthermore, this literature is not focused on horizontal mergers. The finance literature addresses a more general question: whether mergers and acquisitions produce wealth for shareholders or do they reflect managerial hubris. Finally, event studies do not readily disentangle predicted effects of the merger and other information that may be signaled by the announcement. Andrade et al. (2001) report abnormal negative returns for acquiring firms, based on 1864 deals from the 1990s: 1.0% during a three-day window around the announcement and 3.9% during a longer window from 20 days prior to the announcement through closing of the deal. However, target firms showed a 16% abnormal positive return during the three-day window. The combined firms gained about 1.5% over the short or longer window. Further several studies report negative abnormal returns over the three to five years following the completion of mergers, Andrade et al. state: (p. 112):

23 In principle, a merger that would lead to synergies and lower prices would depress the stock market value of rivals, while an anticompetitive merger that would lead to higher prices through unilateral or coordinated effects would boost the stock market value of rivals.

24 Managerial aggrandissement and executives’ self interest in power and control might lead to overvaluation of the target by bidding firms. Within these boundaries, Roll (1986) proposed the “Hubris hypothesis”: his bold view on merger activity interprets takeovers as moved by managers’ hubris, rather than pure economic gains for the firms they manage. By paying a premium, for the acquired firms, managers impose their own valuation of the target, on the one the (assumed efficient) markets had expressed.
“In fact, some authors find that the long-term negative drift in acquiring firm stock prices overwhelms the positive combined stock price reaction at announcement, making the net wealth effect negative.”

However, Andrade et al. are skeptical of these results, disputing the reliability of these longer-term studies, given that it is hard to understand what the “normal” return should be over these longer periods of time. In the end, Andrade et al. state (p. 117):

“We are inclined to defend the traditional view that mergers improve efficiency and that the gains to shareholders at merger announcement accurately reflect improved expectations of future cash flow performance. But the conclusion must be defended from several recent challenges.”

Andrade et al. uphold the view of efficiency enhancing mergers, although recognizing the potential challenges of this approach. For instance, the source of the stock market gains to the combined firms from mergers has not been identified. In the case of horizontal mergers, at least, those gains could come from enhanced market power. Moreover, acquiring firms do not seem to benefit from mergers, an uncomfortable fact for those who believe in a reasonably efficient stock market.

2.3.2. Accounting measures of firm performance

A second method for measuring the effects of mergers is to study accounting data for the firms involved, looking for changes in variables such as rates of return, cash flows, or profit margins. Ravenscraft and Scherer (1987, 1989), using widely cited FTC Line of Business Data, gather mixed evidence,²⁵ reaching rather negative conclusions on their observations and results (collected for just three years, from 1974–1976): the relevant parts of the mergers and acquisitions they study were unsuccessful, leading to a decline in the post-merger profitability of the acquired line of business. Their contributions uphold the view of excessive managerial zeal about acquisitions. However, their prime investigation subjects are conglomerate mergers, not horizontal mergers; thus, much of their evidence is not directly relevant to horizontal merger control policy. In particular, they find that horizontal mergers tended to be more profitable than conglomerate mergers. However, this result does not distinguish between increased market power and post-merger greater synergies in horizontal combinations.

Healy, Palepu, and Ruback (1992) examine post-merger operating performance for the fifty largest mergers that took place from 1979 to 1984. They find that the merged firms exhibited improved

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²⁵ This is also consistent with the event-study analysis of stock price returns, which finds wide variation in how the market evaluates announced mergers. At the same time, as the case studies in Kaplan (2000) document, a merger’s performance may result very different from the stock market’s initial forecast.
operating performance, as measured by operating cash flows, with respect to their industry peers. They attribute these gains to increased operating efficiency. Along similar lines, Lichtenberg and Siegel (1987) and McGuckin and Nguyen (1995) have examined data from the years following the conglomerate merger wave of the 1960s.²⁶ Lichtenberg and Siegel examine the effect of ownership changes on statistically estimated total factor productivity²⁷ at the plant-level using the Census Bureau’s Longitudinal Establishment Data (LED) for the years 1972–1981. They reported that acquired plants had lower productivity than industry averages prior to acquisition. At the same time, registered productivity increases after the acquisition brought acquired plants almost up to the industry average. These results may overturn the Ravenscraft and Scherer’s inefficient conglomerate mergers. The LED database, however, contains primarily large plants. McGuckin and Nguyen (1995) study the same question using instead the Census Bureau’s Longitudinal Research Database (LRD) for the years 1977–1987. They restrict attention to mergers occurring between 1977 and 1982 and focus on the food manufacturing industry (SIC 20). This sample includes many more small plants than in Lichtenberg and Siegel’s analysis. It also includes plants that only operated during part of the sample period (an “unbalanced panel”), while Lichtenberg and Siegel used a balanced panel. However, instead of a measure of total factor productivity most of their analysis uses labor productivity (the average product of labor relative to the industry average product), which can be affected by shifts in the mix of inputs. In contrast to Lichtenberg and Siegel, McGuckin and Nguyen find that acquired plants have above-average productivity prior to acquisition, although they find that this is not true when they restrict attention to large plants like those studied by Lichtenberg and Siegel. Like Lichtenberg and Siegel, they find post-merger productivity improvements.

2.3.3. Case Studies

A third approach is to study specific mergers, tracking the firms or industries involved, looking at industry-wide measures (and not individual accounting data) such as prices, output, product quality, or R&D intensity. In principle, one can also try to measure the impact of a merger on rivals or customers. Kaplan (2000) provides a useful collection of case studies of mergers in a diverse set of industries, including hospitals, tires, banks, pharmaceutical drugs, airlines, and oil field services. These studies illustrate the great variety of patterns that arise in merger analysis, the important role of mergers as a means by which industry participants adjust to changing market conditions (making it

²⁶ The corporate merger wave raised in 1965, picked three years later, and faded in 1972. Acquiring firms were on average much larger than other companies, this may imply internal growth rates of the largest companies were less than the growth in assets for the manufacturing sector as a whole, see for this idea White (1981).

²⁷ Total factor productivity is determined in much of their work as the residual from estimation of a Cobb–Douglas production function.
especially hard to distinguish the effects of mergers from other industry changes, especially once one recognizes that firms self-select to participate in mergers), and the risks as well as opportunities associated with mergers.

Airline mergers have received great attention, in no small part because good data on fares are available and one can use fares on other routes as a good benchmark when measuring the effects of mergers on fares. Borenstein (1990), Werden, Joskow, and Johnson (1991), and Peters (2003) study two mid-1980s airline mergers, approved by the Department of Transportation over the objections of the DOJ: the merger of Northwest Airlines (NW) with Republic Airlines (RC), and the merger of Trans World Airlines (TWA) with Ozark Airlines. These mergers raised significant antitrust issues because they combined directly competing hubs: Northwest and Republic both had hubs at Minneapolis, and TWA and Ozark both had hubs at St. Louis. Both mergers began in 1985; their final agreements were reached in the first quarter of 1986, and cleared in the third quarter of 1986.

Borenstein (1990) reveal very different experiences following the two mergers. The NW/RC merger caused significant fare increases, whereas the TWA/Ozark merger did not. Borenstein pointed that prices also increased on routes in which NW and RC did not compete prior to the merger. This could reflect a price-constraining effect due to the threat of potential entrants prior to the merger, increased market power arising from domination of the hub airport after the merger, or in the case of markets in which NW and RC faced actual competitors, the effects of increased levels of multimarket contact with competitor airlines. Borenstein also notes that the prices of other airlines on these routes displayed a pattern very similar to the pattern seen for the merging firms. Werden, Joskow, and Johnson (1991) found that the Northwest/Republic merger raised fares by about 5% and the TWA/Ozark merger raised fares by about 1.5%, and that both mergers led to significant service reductions.

Kim and Singal (1993) examine fourteen mid-1980s airline mergers. They compare price changes on the routes served by the merging firms with price changes on other routes of the same distance and conclude that any efficiency gains in mergers between rival airlines were more than offset by enhanced market power, leading to fares that averaged 10% higher after six to nine months. Fare increases were especially large for mergers involving airlines in bankruptcy, which had unusually low (perhaps unsustainably low) pre-merger fares. Examining price changes for the merging firms, relative prices rose by an average of 3.25% over the full sample period in mergers involving firms that were not financially distressed. They rose substantially more (26.25%) in mergers involving a financially distressed firm. Peters (2003) focusing on the evaluation of merger simulation techniques also documents the service changes and entry events that followed six of these mergers. Peters shows that flight frequency tended to decrease in markets that initially were served by both merging firms,
and increase in markets that initially were served by only one of the merging firms. The mergers also led to entry, although changes in the number of rivals were only statistically significant for three of the mergers.

The banking industry is another industry in which good price data are available and many horizontal mergers have occurred, making it possible to measure the price effects of horizontal mergers. Prager and Hannan (1998) study the effects of major horizontal mergers in the U.S. banking from January 1992 through June 1994. Prager and Hannan examine the change in interest rates paid on deposits for NOW accounts (interest-bearing checking accounts), MMDA accounts (personal money market deposit accounts), and 3MOCF accounts (three-month certificates of deposit). Hannan and Prager separately examine the effects of “substantial horizontal mergers” in which the Herfindahl–Hirschman index in the affected market increases by at least 200 points to a post-merger value of at least 1800, and “less substantial mergers”, in which the Herfindahl–Hirschman index increases by at least 100 points to a post merger value of at least 1400 and which were not “substantial mergers”. Their price data are monthly observations on deposit interest rates from October 1991 through August 1994. The results indicate that substantial mergers reduce the rates that banks in a market offer. This effect is largest for NOW accounts (approximately a 17% reduction in rates), for which customers arguably have the strongest attachment to local banks, and least for three-month CD’s (less than 2% reduction in rates, and not statistically significant).

Notably, however, Prager and Hannan find that less substantial mergers increase rates paid in the market. One possible interpretation of this difference is that these mergers involve efficiencies (which allow banks, in the absence of other effects, to increase their rates), but the effects of these efficiencies on prices are more than offset by an increase in market power for substantial mergers. Unlike in Kim and Singal (1993), the direction of these effects is the same in the pre and post-merger period. Finally, although the results in Table 36.8 do not distinguish between the price changes for merging firms and their rivals, Prager and Hannan find that these two groups had similar price effects, paralleling the Borenstein (1990) and Kim and Singal (1993) findings on this point. In a recent paper, Focarelli and Panetta (2003) study bank mergers in Italy during the years 1990–1998 and their effects on deposit rates. Like Kim and Singal (1993) and Prager and Hannan (1998), they separately look at announcement (which they call “transition”) and completion periods. Focarelli and Panetta look at a much longer time period after the merger when examining the completion period (for each merger, they consider the effects until the end of their sample), arguing the necessity of a long time period to realize efficiencies post-merger. Like Kim and Singal they find evidence of market power effects during the announcement/transition period as deposit rates fall during this period.
period. However, they find that in the long run these mergers increased deposit rates. Thus, in this case, the price-reducing effects of merger-related efficiencies seem to have dominated the price-increasing effects of increased market power. They find that substantial horizontal mergers reduce the deposit interest rates offered by the merging banks.

Some recent studies have been done as well in other industries in which price data are available. Horizontal mergers are explicitly studied is Pesendorfer (2003), investigating a horizontal merger wave in the paper industry during the mid 1980s. Rather than estimating productivity directly, Pesendorfer tries to infer pre- and post-merger productivity using the firms’ capacity choices. Marginal costs are inferred from the Cournot profit-maximization first-order conditions for capacity choice. This idea has been found not entirely convincing in his application. First critique involves the investment first order conditions Pesendorfer uses. These are entirely static, while investment choices are likely to be affected by dynamic considerations. Second, his procedure relies on an assumed investment cost function (this might not be necessary if one instead has panel data). Finally, one cannot distinguish whether the changes in marginal cost he derives reflect shifts of the plant’s marginal cost function or movements along an unchanging function.

Taylor and Hosken (2004) study the effects of a 1997 joint venture that combined the refining and retail gas station operations of the Marathon and Ashland oil companies. Specifically, they examine retail and wholesale price changes in Louisville, Kentucky, a city where this merger raised concentration significantly (the wholesale Herfindahl–Hirschman index increased from 1477 to 2263; the retail index increased by over 250, ending up in the 1500–1600 range). They conclude that there is no evidence that the merger caused either wholesale or retail prices to increase. In contrast, Hastings (2004) finds that rivals’ prices increased following ARCO’s 1997 acquisition (through long-term lease) of 260 stations from Thrifty, an unbranded retailer. Vita and Sacher (2001) document large price increases arising from a 1990 merger between the only two hospitals in the city of Santa Cruz, California. The acquirer in this case was a non-profit hospital.

There is one important caveat to the interpretations we have been giving to observed price changes in these studies: homogeneity of the product market is a crucial assumption, i.e. product remains unchanged post-merger. An alternative explanation for price increases or decreases instead may be that the merger led to changes in the quality of the merged firms’ products. Thus, rather than market power, price increases may reflect quality improvements; and rather than cost reductions, price decreases may reflect quality degradation buying cheaper inputs of production. Many of the papers surveyed above document patterns that tend to rule out such interpretations of their findings. For instance, the price increases during the Kim and Singal (1993) announcement period are unlikely to come from quality improvements. Likewise, Focarelli and Panetta (2003) explicitly examine and reject the hypothesis that the long-run increases in merging banks’ interest rates that they document
are due to quality degradation. In summary, the literature documenting price effects of mergers has shown that mergers can lead to either price increases or decreases, in keeping with the central market power versus efficiency trade-off.

2.4. Riding the Wave

The Reagan era has widely been pointed by newspapers as the most financially intense for horizontal takeovers. Nonetheless, recent empirical studies show that even greater merger activity occurred at earlier times. It is not so easy to find consistent data on merger activity, especially in the first periods, except for mining and manufacturing, industries declining over time in the U.S. economy. Moreover, early data sources report high-valued transactions, rendering merger activity measures downward biased, ignoring mergers among small firms. Stock market booms seem to have fostered merger activity, more precisely, extensive aggregations took place close to the turn of the twentieth century, then again in the late 1920’s, the third in the 1960’s, the fourth in 1980’s and 1990’s.

Stigler (1950) defined the first wave as the “merger to monopoly” movement, form 1893 to 1903. The 1920’s was baptized by Stigler (1950) as the “merger to oligopoly” movement, pointing to the follow-up mergers in the industries involved in the first merger wave, transforming declining-dominant -firm markets into oligopoly. Taking off in 1926, it landed after the stock market crash in October 1929. Consumer goods innovations, e.g. cheaper automobiles and commercial radio, paired the financial market innovations, supplying an insatiable demand with new securities. The merger movement of the second part of the century had very different characteristics as it was conglomerate in nature, it created either conglomerate businesses or holding companies owning several firms selling in different product markets. The most recent waves of takeovers, the Deal Decade eighties and the nineties, prove themselves unparalleled both in the number of mergers and in the nominal value of mergers, i.e. not adjusted to inflation. In fiscal 1997, the justice department received 3,702 merger notifications, more than doubling the 1992 record of 1,589. The nominal value of mergers increased from nearly $200 billion in 1990, to over $900 billion in 1997.

The fact that such takeovers happen in waves gave rise to an almost independent stream of research to detect the determinants, both financial and behavioral, of these phenomena. Research shows that a few factors are needed for a merger wave. Shleifer and Vishny (2003) and Rhodes-Kropf and Viswanathan (2004) show merger activity as positively correlated with high stock market valuations, developing models that result in merger waves following managerial timing in overvaluation of their firms. Mitchell and Mulherin (1996) present a more neoclassical explanation of waves as market response industry shock, e.g. deregulation, technology or preferences, disturbing the status quo. Such shocks require large scale reallocation of assets, leaving some weaker that others, so there is room for
merger deals. Managers' payoffs can amplify the wave, as competing executives try to emulate their rivals by surging to industry leaders outperforming industry laggards.

The fact that bouts of intense merger activity happen at the same time across many different industries suggests microeconomic shocks are perhaps necessary, but not sufficient for a merger wave. Harford (2005) shows to what extent capital liquidity, e.g. reduction of financial constraints or corporate savings, accommodate such a reallocation of assets. Testing the neoclassical hypothesis, in which industries reorganize to shocks by a stream of takeovers; and the behavioral model in which rational managers take advantage of consistent pricing errors to buy real assets with overvalued stocks. The tests support such a neoclassical view, modified to embrace macroeconomic determinants. Such capital market variables demonstrate how the positive correlation has been misattributed to behavioral misevaluation factors. Although industry-specific shocks are cause of mergers not necessarily propagate to a merger wave. The macro-level liquidity, reducing transaction costs, justifies the clustering of mergers in a wave that behavioral factors alone would not.

Whenever firms get so big they cannot be allowed to fall over, concerns may arise. Being too big to fail, allows government intervention in giants support, affecting industry-wide price incentive, i.e. creating a distortion and pushing the industry farther from perfect competition. This may be seen as a rent seeking behavior on the side of firms alienating objective function from profits to governmental subsidy maximizing. Post Global Financial Crisis empirical research may point the lens on this peculiar aspect and investigate the role of State intervention by National Governments, Central Banks and financial Authorities, to prevent the contagion effect and save national economies from timeless paralysis.

3. Symmetric Pricing Games

This chapter investigates the different oligopolistic settings studied in the literature of capacity-constrained price competition. Homogeneous product oligopoly models à la Bertrand-Edgeworth have long been challenging for economists' intellects since Levitan and Shubik’s (1972) modern approach to Bertrand-Edgeworth competition. Thorough equilibrium characterization will be necessary to address horizontal merger analysis in the following chapter. Much of the literature we survey in this chapter considers symmetrically-sized firms, focusing on symmetric equilibria. Levitan and Shubik (1972) calculated the equilibrium in mixed strategies for the efficient rationing rule in the special case of symmetric capacities.  

29 Induction from symmetric frameworks to asymmetric ones is troublesome and the bulk of economic analysis does not necessarily extend to the finite asymmetric n-player game. Mixed strategy equilibria in price competition have reached only specific results from specific assumptions and asymmetric oligopolies represent an extensively unknown field of economic
Under the same assumptions Kreps and Scheinkman (1983) characterized the duopolistic mixed-strategy equilibrium for asymmetric capacities in a two-stage capacity and pricing game. Between asymmetric players the largest firm has a stronger incentive to raise price since its residual demand is the largest. This property of duopoly can be easily extended to oligopoly. Further extensions of the Kreps and Scheinkman’s construction have been considered by Osborne and Pitchick (1986) for non-concave demand functions, as well as by Deneckere and Kovenock (1996) for non-identical unit cost structures between the duopolists. Brock and Scheinkman (1985) consider a repeated price game in a general oligopoly characterizing equilibrium payoffs in the stage game assuming symmetry among oligopolists’ dimension. Furthermore, symmetric games have received full attention and complete formalization for the efficient rationing rule of residual demand in Davidson and Deneckere (1984), Vives (1986), and De Francesco and Salvadori (2010b).

Davidson and Deneckere (1984) investigate a price-setting supergame to study pro-collusive effects among active sellers. The symmetric oligopoly is assumed of a given number of equally capacity-constrained competitors some of which merge to create a bigger firm, whose size is a proper multiple of the merging firms. The post-merger oligopoly is a particularly asymmetric market constituted of one large competitor acting as a size leader and the unchanged remainder of the industry. Mixed strategy equilibria are then characterized when the post-merger capacity configuration falls in the mixed strategy region. While Davidson and Deneckere (1984) restricted equilibrium investigation to symmetric equilibria, in a related work De Francesco and Salvadori (2010b) further characterized the mixed strategy region. In the finer partitioning of the mixed strategy region, they recognize the possibility of the rise of a multiplicity, sometimes a continuum, of equilibria. Nonetheless, they show the uniqueness of the payoff of each firm in any equilibrium. Vives (1986) shows how in a symmetric oligopoly the support of equilibrium prices converges to the competitive price.

This preliminary presentation will be needed to understand how deeper investigation of equilibrium properties is required to set solid grounds on the upcoming research of mergers in a specific oligopolistic context as the one studied in this essay. More specific and analytical presentation of merger theory will be studied and applied in the next chapter. Section 1 introduces the properties of models à la Bertrand-Edgeworth. Section 2 and 3 present the symmetric and almost symmetric oligopolies. Section 4 and 5 introduce horizontal mergers modeling in price-setting oligopolies, providing a full characterization of post merger equilibria.

theory. Hirata (2008) and De Francesco and Salvadori (2010a) have presented characterization of the mixed strategy region in triopoly. Full characterization of the mixed strategy region in triopoly is supplied in De Francesco and Salvadori (2013b). These latter models introduce concerns about extension of asymmetric duopoly results to asymmetric markets.
3.1. Oligopoly, Oligopolies

Assume an \(n\)-firm oligopoly whose set of players \( l = \{1, ... , n\} \) is fixed and exogenous, i.e. entry and exit is not allowed in this model. The \( n > 2 \) firms produce a homogeneous good, at equal constant average and marginal cost \( c \), set to zero without loss of generality, up to capacity. Neither installation costs of the existing capacity nor fixed costs are considered in this analysis. Define total capacity as \( K = \sum_{i}^{n} k_i \) and assume the subset of capacity space where, without loss of generality, \( k_1 \geq \cdots \geq k_n \).

Consumers preferences determine the following demand function \( X = D(p) \), where \( X \) refers to total market demanded quantity and \( p \) to market price. Market demand is positive for \( p \in (0, \bar{p}) \), where if \( p > \bar{p} \), then \( D(p) = 0 \). Furthermore, demand function is assumed continuous, decreasing and concave in \( \bar{p} \). Conversely, the inverse demand function is defined for the decreasing part of the demand function such as \( p = D^{-1}(X) = X(p) \), in \( X \in (0, D(0)) \).

Homogeneity of the product market implies no perceived difference among the sellers. Thus, we assume consumers buy first from the cheapest supplier, absent income effects. When the lowest priced firm cannot satisfy the market, residual demand is left for the other competitors. The amount of sales of the higher priced firms depends on the form of their contingent demand curve, i.e. the pool of customers that remains to be served. In this dissertation, any rationing of the forthcoming demand follows the efficient or surplus maximizing rule. Consequently, let \( \Omega(p) \) be the set of firms charging price \( p \), and write the residual demand for firms setting \( p \) as \( \max\{0, D(p) - \sum_{j: p < p_j} k_j\} = Y(p) \). If \( \sum_{i \in \Omega(p)} k_i > Y(p) \), the residual demand forthcoming to any firm \( i \in \Omega(p) \) is a fraction \( \alpha_i(\Omega(p), Y(p)) \) of \( Y(p) \), namely \( D_i(p_1, ..., p_n) = \alpha_i(\Omega(p), Y(p))Y(p) \). Our analysis does not depend on the specific assumptions on \( \alpha_i(\Omega(p), Y(p)) \). For instance, it is consistent with \( \alpha_i(\Omega(p), Y(p)) = k_i/\sum_{r \in \Omega(p)} k_r \), as well as with evenly sharing residual demand assumption, among firms in \( \Omega(p) \).

Additionally, when firms set the same price, demand will distribute proportionally to individual capacities. These two assumptions allow writing sales and profits of firm \( i \) as \( X_i(p_i, p_{-i}) = \max\{0, D(p) - \sum_{j: p < p_j} k_j\} \frac{k_i}{\sum_{r: p_r > p_i} k_r} \) and \( \Pi_i(p_i, p_{-i}) = p_i \min\{q_i(p_i, p_{-i}), k_i\} \).

\[ \begin{align*}
30 \text{ An alternative rationing rule, i.e. proportional rationing, allows writing residual demand as } & \frac{\partial(p_i)}{\partial(p_j)} [D(p_i) - k_j], \text{ for } p_i > p_j. \text{ For this rationing unsatisfied demand for lower-priced firms turns to higher-priced ones only at discounted rates, since customers exit the market if not able to purchase at the desired price. For research with this rationing, see Allen and Hellwig (1986, 1993), Dasgupta and Maskin (1986b), and Cheviakov and Hartwick (2005). For further research with non efficient rationings, see Chowdhury (2003, 2008). }
\end{align*} \]

\[ \begin{align*}
31 \text{ In this case, } & \alpha_i(\Omega(p), Y(p)) = \min\{k_i/Y(p), \beta(p)\} \text{ where } \beta(p) \text{ is the solution in } \alpha \text{ of equation } \sum_{i \in \Omega(p)} \min\{k_i/Y(p), \alpha\} = 1. \text{ Let } M \in \Omega(p) \text{ and } k_M \geq k_i, \text{ each } i \in \Omega(p). \text{ Then, } \sum_{i \in \Omega(p)} \min\{k_i/Y(p), \alpha\} \text{ is increasing in } \alpha \text{ over the range } [0, k_M/Y(p)] \text{ and equal to } \sum_{i \in \Omega(p)} \min\{k_i/Y(p)\} > 1 \text{ for } \alpha = k_M/Y(p). \end{align*} \]
The static $n$-firm price-setting oligopoly model can be represented as a one shot pricing game, i.e. firms name their price simultaneously and non-cooperatively. All firms have perfect computational abilities, act with perfect rationality and foresight, and know with certainty that the same is true for all of their competitors. Formally, we write the normal form of a simultaneous move game when we consider only nonrandom strategy choices $\Gamma_N = \{I, \{P_i\}, \{\Pi_i(p_i, p_{-i})\}\}$. Where $I$ denotes the finite set of players, $P_i$ denotes the set of strategies of each individual player $i$, and $\Pi_i(p_i, p_{-i})$ the payoff function assigning the profit levels associated with the outcome arising from oligopolists’ strategies $p = (p_1, p_2, \ldots, p_n)$, with $p_i \in P_i$ and $p_{-i} \in P_{-i}$ where $P_i, P_{-i} \in [c, \bar{p}]$. We define the profile of pure strategies for any player $i$’s opponent by $p_{-i} = (p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n)$. Let $P = P_1 \times \ldots \times P_i$ and $P_{-i} = P_1 \times \ldots \times P_{i-1} \times P_{i+1} \ldots \times P_n$. To solve for equilibrium strategy, i.e. equilibrium price, we solve for the Nash equilibrium concept. A strategy profile $p = (p_1, \ldots, p_n)$ constitutes a Nash equilibrium of game $\Gamma_N = \{I, \{P_i\}, \{\Pi_i(p_i, p_{-i})\}\}$ if for every $i \in I$, $
abla_i(p_i, p_{-i}) \geq \Pi_i(p_i', p_{-i})$ for all $p_i' \in P_i$. Informally, a set of strategies is a Nash equilibrium if no player can improve by unilaterally changing her strategy.

According to the assumptions on industry capacity, it is possible to address pure strategy equilibria more formally. Let $p^c$ indicate the competitive equilibrium price, the solutions in the pure strategies space, for null marginal cost, are as follows

$$p^c = \begin{cases} 0, & K \geq D(0) \\ p(K), & D(0) \geq K \end{cases}$$

In capacity-constrained oligopolies, pure strategy equilibria differ according to individual firm size vis-à-vis industry size.

If $k_i \geq D(0), \forall i \in I$, in every Nash equilibrium at least two firms set marginal cost pricing, while the rivals charge whatever price not lower than marginal cost. Whenever two firms price at marginal cost, each firm $i$ of the $n - 2$ firms left will be indifferent on charging any other price at least equal to $c$, since it would still earn null profits. At the same time, each of the $j$ firms pricing at marginal cost, has no incentive to price differently. Pricing higher, as well as pricing lower, would gain respectively either null or negative profits to the firm, and do not represent equilibrium positions. By way of contradiction, assume no one, or just one firm charges marginal cost price to show there exists no equilibrium. If only firm $i$ adopts marginal cost pricing, then it would be optimal to increase price up to a slightly lower price than the minimum price charged by the $n - 1$ rival firms. Hence, firm $i$ would supply the entire market demand and earn positive profits. If no firm charges marginal cost pricing, then at least one has incentive to undercut rivals to a lower price than the minimum price to serve the whole market demand.
This assumption is particularly strong, in fact gives rise to the standard Bertrand result. It considers firms that are individually so big to supply the whole forthcoming demand at marginal cost. This is not the only case in which this result arises in such games. It is sufficient assuming the \( n - 1 \) smallest firms to satisfy the whole forthcoming demand at marginal cost, i.e. the sum of all firms’ capacities but the largest, can supply all the demand at marginal cost pricing. Following this second assumption, if each firm owns a capacity \( k_i \) such that \( \sum_{i=2}^{n} k_i \geq D(0) \), then the equilibrium will be in pure strategies. The sum of individual capacities allow the firms to serve market demand at any pure strategy equilibrium price \( p \geq c \). The following proposition establishes the necessary and sufficient conditions for the existence of a pure strategy equilibrium and its uniqueness.

**Proposition 3.1**

(i) \( p_i = p^c, \forall i \) is an equilibrium if and only if either

\[
K - k_1 \geq D(0) \text{ if } D(0) \leq K \tag{3.1}
\]

Or

\[
k_1 \leq -p^c[D'(p)]_{p=p^c} \text{ if } K < D(0) \tag{3.2}
\]

(ii) Whenever either one or the other holds, all firms earn competitive profits at each pure strategy equilibrium and \( p^c \) is the unique equilibrium if \( K < D(0) \).

Proof: The two points will be shown in turn,

(i) If \( D(0) \leq K \), charging \( p^c = 0 \) is a best response of firm \( i \) to rivals charging \( p^c \) if and only if \( \sum_{j \neq i} k_j \geq D(0) \). This holds for each \( i \) if and only if \( \sum_{j \neq 1} k_j \geq D(0) \). If \( D(0) > K \), charging \( p^c \) is a best response of firm \( i \) to rivals charging \( p^c \), if and only if \( \left[d\left[p(D(p) - \sum_{j \neq i} k_j)\right]/dp\right]_{p=p^c} \leq 0 \). This is true for each \( i \) if and only if \( k_1 \leq -p^c[D'(p)]_{p=p^c} \).

(ii) In order to show uniqueness, it is left scrutinize strategy profiles such that \( \hat{p} > p^c \), where \( \hat{p} = \max\{p_1, \ldots, p_n\} \). Assume first \( D(\hat{p}) - \sum_{j: p_j < \hat{p}} k_j > 0 \). If \( \# \Omega(\hat{p}) > 1 \), then at least some firm charging \( \hat{p} \) serves a residual demand (strictly) lower than its capacity. It would raise its profits quoting \( \hat{p} \), a slightly lower price than \( \hat{p} \), producing the quantity \( \min\{k_i, D(\hat{p} - \epsilon) - \sum_{j: p_j < \hat{p}} k_j\} \) rather than the lower \( D(\hat{p}) - \sum_{j: p_j < \hat{p}} k_j \). If \( \# \Omega(\hat{p}) < n \), any firm \( j \notin \Omega(\hat{p}) \) is marketing its entire capacity, though not responding optimally: raising its price keeping it below \( \hat{p} \), would gain the firm higher profits selling all capacity. Next assume \( D(\hat{p}) - \sum_{j: p_j < \hat{p}} k_j \leq 0 \). To optimally react to rivals and charge a lower price than the lowest \( \hat{p} \), then it must be \( \hat{p} = 0 \) and \( \sum_{j: p_j = \hat{p}} k_j \geq D(0) \) (the latter requires \( K \geq D(0) \)). Firms will earn the
competitive null profits. But then, in order for each firm \( j \) charging \( p \) to have also made a best response, it must be \( \sum_{k \neq j} k_s \geq D(0) \). Q.E.D.

Thus, pure strategy equilibria in which \( p > p^c \) may only exist if inequalities (3.1) hold, so the set of equilibria being any strategy profile such that \( \sum_{k \neq j} k_s \geq D(0) \) for each \( j \), such that \( p_j = 0 \). Informally, since it will not increase their profits, firms can choose neither a lower price, nor a higher one than the null marginal cost, i.e. the standard Bertrand result. If inequalities (3.2) hold then a unique pure strategy equilibrium exists and \( p^c > 0 \). The term \(-p^c[D'(p)]_{p=p^c} \) in condition (3.2) can also be interpreted as the Cournot-Nash best response quantity in capacity-unconstrained quantity competition. Assuming \( \forall i \), \( \left[ \frac{d}{dp} \left(pD(p) - \sum_{j \neq i} k_j \right) \right]_{p=p^c} \leq 0 \) is equivalent to the one, \( \forall i \frac{d}{dx_i} X_i p(Q) \geq 0 \) for \( x_i = k_i \) and \( x_j = k_j \), each \( j \neq i \). Thus, condition (3.2) identifies the region of the \((k_1, ..., k_n)\) capacity space in which each firm’s capacity is not greater than its best response quantity when the rivals produce at full capacity.\(^{32}\) As a consequence, the existence of a pure strategy equilibrium depends on industry capacity and the individual capacity of each firm. Either inequalities (3.1) or (3.2) hold if and only if

\[
k_1 \leq \max \left\{ K - D(0), -p^c[D'(p)]_{p=p^c} \right\} \quad (3.3)
\]

Inequality (3.3) states that the largest firm must be sufficiently small with respect to industry capacity, in order for a pure strategy equilibrium to exist. Whenever inequalities (3.1) or (3.2) are violated, pure strategy equilibrium does not exist while mixed strategy equilibria do. That is to say that mixed strategy equilibrium exists if

\[
k > \max \left\{ K - D(0), -p^c[D'(p)]_{p=p^c} \right\} \quad (3.4)
\]

In the mixed strategy region equilibrium always exists satisfying the sufficient conditions for existence of Theorem 5 in Dasgupta and Maskin (1986a). In the theory of games, a player is said to play a mixed strategy whenever chooses randomly over a set of available actions. Formally, a mixed strategy is a probability distribution assigning to each available action a likelihood of being selected. A pure strategy is a particular case of mixed strategy in which the mass of the distribution function is concentrated at one point. Formally, we write the normal form of a simultaneous move game \( \Gamma_N = [I, (\Delta P_j), \{\Pi_i(\phi_i, \phi_{-i})\}] \) allowing for players randomizing over pure strategies. We let the mixed strategy equilibrium profile as \((\phi_1(p), ..., \phi_n(p)) = (\phi_i, \phi_{-i})\). We define the mixed strategy profile of any player \( i \)'s opponent mixed strategy \( \phi_{-i} = (\phi_1, ..., \phi_{i-1}, \phi_{i+1}, ..., \phi_n) \). By definition,

\(^{32}\) The region is namely the region bounded above by the lower envelope of the Cournot best-response functions. Note that the assumptions on capacity do not guarantee the uniqueness of the Cournot equilibrium. Uniqueness would be assured if \( D'(p) + pD''(p) < 0 \) on \((0, P(0))\), see Deneckere and Kovenock (1992).
\( \phi_i = \Pr(p_i < p) \), i.e. the probability that firm \( i \) sets a lower price than \( p \). Mixed strategies bear the common properties of cumulative distribution functions such as, \( p \in S_i \) when \( \phi_i(p) \) is increasing in \( p \), i.e. \( \phi_i(p + h) > \phi_i(p - h) \), for any \( h \in (0, p) \), whereas \( p \not\in S_i \) if \( \phi_i(p + h) = \phi_i(p - h) \) for some \( h > 0 \). Furthermore, \( \phi_i(p) \) is discontinuous at any \( p^o \), such that \( \Pr(p_i = p^o) > 0 \). We write expected equilibrium profits, \( \Pi^*_i = \Pi(\phi_i, \phi_{-i}) \) when the equilibrium mixed strategies \( (\phi_i, \phi_{-i}) \) are played. Similarly, firm \( i \)'s expected profits whenever it plays any price \( p \) and the others the equilibrium mixed strategy \( \phi_{-i} \), can be written \( \Pi_i = \Pi(p, \phi_{-i}) \). Unless otherwise proved, the equilibrium mixed strategies vector is not necessarily unique. Furthermore, let \( S_i = S_i(\phi_i(p)) \) the support of equilibrium mixed strategies and let \( p_m^{(i)} \) and \( p_M^{(i)} \) the infimum and supremum of \( S_i \), respectively. We define \( p_m = \min p_m^{(i)} \) and \( p_M = \max p_M^{(i)} \) together with sets \( M = \{ i : p_M^{(i)} = p_M \} \) and \( L = \{ i : p_M^{(i)} = p_m \} \). It is then straightforward, \( \Pi^*_i \geq \Pi(p, \phi_{-i}), \forall p > 0 \) as well as \( \Pi^*_i = \Pi(p, \phi_{-i}) \), almost everywhere, for \( p \in S_i \).33

In Kreps and Scheinkman (1983) the following mixed strategies equilibria general properties have been found for duopoly. Mixed strategy equilibrium is characterized by \( p_M^{(1)} = p_M^{(2)} = p_M = \arg \max_p p(D(p) - k_2) \) if \( k_1 > k_2 \), whereas \( \phi_1(p_M) = \phi_2(p_M) = 1 \) if \( k_1 = k_2 \), then \( \Pi^*_j = p_m(D(p_M) - k_j) \) for \( j \) such that \( k_j = k_1 \). Recent generalizations to oligopoly can be summarized in the following proposition. These results are obtained as a straightforward generalization of duopoly findings and demonstrated in De Francesco and Salvadori (2010a), as well as other authors in sometimes different, though analogous frameworks.

**Proposition 3.2**

1) \( p_M = \arg \max_p p(D(p) - \sum_{j \neq 1} k_j) \).

2) \( p_M^{(i)} = p_M \forall i : k_i = k_1 \).

3) \( \Pi^*_i = \max_p p(D(p) - \sum_{j \neq 1} k_j) \forall i : k_2 = k_i \).

4) \( p_m = \max \{ \bar{p}, \tilde{p} \} \), where we define \( \bar{p} = \Pi^*_i/k_1 \) and \( \tilde{p} \) as the lower solution of the equation in \( p \)

\[ \Pi^*_i = pD(p); \quad \text{if} \quad k_1 = k_2, \quad p_M^{(i)} = p_m = \bar{p}, \forall i : k_i = k_1. \]

5) \( \phi_1(p_M^{(i)}) = 1, \forall i : k_i < k_1 \).

---

33 It might exist at most a finite set of isolated points \( p^o \in S_i \) (such that \( \phi_i(p^o + \epsilon) - \phi_i(p^o - \epsilon) > 0 \), any \( \epsilon > 0 \) small enough), hence \( \Pi^*_i < \Pi(p^o, \phi_{-i}(p^o)) \).
6) If \( k_1 > k_2 \), \( \phi_1(p_M) < 1 \). If \( k_1 = k_2 \), \( \phi_i(p_M^{(i)}) = \phi_i(p_M) = 1 \) and \( \phi_i(p) = \phi_i(p) \forall p \in [p_m, p_M] \) and \( \forall i : k_i = k_1 \).

7) If \( k_1 > k_i = k_j \), then \( \Pi_i^* = \Pi_j^* \) and necessarily \( \phi_i(p) = \phi_j(p) \forall p \in S_i \cap S_j \) unless \( \Pi_i^*(p, \phi_{-i}) \) were independent of \( \phi_j(p) \) (in such a case even \( \Pi_i^*(p, \phi_{-j}(p)) \) is not independent of \( \phi_i(p) \)).

8) \( \Pi_i^* = p_m k_2 \forall i : k_i = k_2 \).

9) \( p_m > p(\sum_{j \in L} k_j) \).

10) \( \Pi_i^* = \Pi_i^*(p, \phi_{-i}) \forall p \in S_i \), \( \forall i \) and then \( \Pr(p_i = p) = 0 \forall p \in (p_m, p_M) \).

11) \#L \geq 2 \text{ and } \#M \geq 2.

12) \( \forall p \in (p_m, p_M), \#\{i : p \in S_i\} \neq 1 \).

Among the results of Proposition 3.2, in every mixed equilibrium the highest-capacity firms earn Stackelberg follower’s profits, i.e. profits that the firm makes when it reacts optimally to the other’s output (assumed up to full capacity). The result follows from Boccard and Wauthy (2000) and De Francesco (2003), who develop a two-stage model with a finite number of asymmetric firms. Only the largest firm’s equilibrium payoff and strategic profile is examined in the price competition game, lacking of full characterization of the equilibrium strategic profile.\(^{34}\) Moreover \( \phi_i(p) \) is continuous \( \forall p \in S_i \), and the supremum \( p_M \) is charged with positive probability only by the largest firm, firm 1 in the case that \( k_1 > k_2 \). The definitions of \( p_m \) and \( p_M \) also make it possible to characterize the mixed strategies region where inequalities (3.4) and \( k_1 \geq \cdots \geq k_n \geq 0 \) hold. Inequality (3.4) can be substituted with

\[
p(K) < p_m
\]

Indeed, if \( k \leq K - D(0) \) then \( p_m = P(K) = 0 \); whereas, if \( k \leq -p^c[D'(p)]_{p=p^c} \) then \( p_m = p_M = P(K) \geq 0 \). Conversely, if inequality (3.4) holds, then inequality (3.4’) holds as well. Finally, note that in the region where inequalities (3.4) and \( k_1 \geq \cdots \geq k_n \geq 0 \) hold the extrema of equilibrium mixed strategies supports are defined in points 1 and 4 for the supremum and infimum respectively.

\(^{34}\) The analysis under convex costs is similar: full characterization of equilibrium is not available, although the existence of mixed strategy equilibrium is proved in Dixon (1984) and Maskin (1986). Additional assumptions are studied in Dixon (1987) and Chowdhury (2008).
3.2. Symmetric Oligopoly

Vives (1986) studies symmetric equilibria in a price game with symmetric firms, under efficient rationing. Vives (1986) demonstrates that the supports of the symmetric mixed strategies Nash equilibria converge, in distribution, to the unique competitive price. Although in Vives (1986) existence of asymmetric equilibria is not excluded, it is not yet investigated. Uniqueness of symmetric equilibrium is shown in De Francesco and Salvadori (2010b). Drawing on the conclusions of Proposition 3.2 allows to show the necessary symmetry of equilibrium and, therefore, its uniqueness.

Proposition 3.3 Let \( k_i = k, \forall i \in \{1, \ldots, n\} \). Then, there exist the unique symmetric equilibrium

\[
\phi_i(p) = \left[ \frac{(p-p_m)k}{p(K-D(p))} \right]^{\frac{1}{n-1}}, \forall i \in \{1, \ldots, n\}, p \in [p_m, p_M]
\]  

(3.5)

Where \( \Pi_i = \max_p p(D(p) - (n-1)k) \), \( p_M = \arg\max_p p(D(p) - (n-1)k) \) and \( p_m = \Pi_i / k \).

Proof: From Proposition 3.2 it follows: equilibrium is symmetric (point 6), expected equilibrium profit is unique (3), as well as unique are supports’ extrema \( p_m \) and \( p_M \), (2 and 4) where \( p_m = \Pi_i / k \), independent of the existence of a multiplicity of equilibria. It follows that it exists a neighborhood of \( p_m \) in which

\[
p_m = \prod_j \phi_j(p) p(D(p) - (n-1)k) + \left[ 1 - \prod_j \phi_j(p) \right] pk = \prod_j \phi_j(p) p(D(p) - K) + pk
\]

\( \forall i \in \{1, \ldots, n\}, p \in [p_m, p_M] \), where equality (3.4) holds. Thus, it exists an equilibrium in which \( S_i = S_i(\phi_i(p)) = [p_m, p_M], \forall i \in \{1, \ldots, n\} \). In (3.4) the RHS is strictly increasing \( \forall p \in (p_m, p_M) \).

Uniqueness of equilibrium can be found showing that no other symmetric equilibrium exists. In such hypothetical equilibria, equation (3.4) may not hold if \( p^o \notin S_i \cap \ldots \cap S_n \). It must then be \( \phi_j(p) = 0 \) in \( (p^o, p^{oo}) \), i.e. \( \Pr(p_i \in (p^o, p^{oo})) = 0 \). This would imply,

\[
\Pi_i(p, \phi_{-i}) = p \prod_{S \neq i} \phi_k(p^o) p(D(p^o) - K) + pk > \Pi_i = p^o \prod_{S \neq i} \phi_k(p^o) p(D(p^o) - K) + p^o k
\]

35 “Given \( n \) firms and restricting attention to symmetric equilibria …” Vives (1986, p. 114).

36 It is straightforward to find that \( \phi_j(p) > 0 \) if and only if \( p^2D(p) + p_m(K-D(p) - pD'(p)) > 0 \). Since \( K-D(p) - k < pD'(p) \), then \( K-k-D(p) + \delta = pD'(p), \delta > 0 \). Rearranging, \( [\Pi_i - p(D(p) - (n-1)k)] + [(p - p_m)\delta] > 0, \) that is true since each term in brackets is positive.
In a right neighborhood of $p^0$ profit function will be increasing, contradicting assumptions. Q.E.D.

In Vives (1986) only the existence of mixed strategy equilibrium is defined, assuming symmetric equilibria, for equally sized firms. Upholding Vives (1986) result, in De Francesco and Salvadori (2010b) is offered the necessary proof of uniqueness of the symmetric equilibrium. Assuming symmetric equilibria unfolds not very restrictive since the symmetric equilibrium is the unique equilibrium of the game. Uniqueness of the symmetric equilibrium holds not only for the equally-sized firm game, but also for the game in which the capacity of the largest firm is sufficiently close to the smallest one, i.e. an “almost symmetric oligopoly”. In an almost symmetric oligopoly, $k_1 \geq \cdots \geq k_n$ and $D(p_M) \geq k_1 + \ldots + k_{n-1}$. Such oligopoly is investigated and characterized in De Francesco and Salvadori (2013a). In the closing section of this chapter we shall consider an almost symmetric triopoly and show the necessary proof for the existence of the equilibrium mixed strategies.

Uniqueness of the symmetric equilibrium holds not only for the equally-sized firms game, but for the game in which the capacity of the largest firm is sufficiently close to the smallest one, i.e. the “almost symmetric oligopoly”. In an almost symmetric oligopoly size differences should follows, $k_1 \geq \cdots \geq k_n$ and $D(p_M) \geq k_1 + \ldots + k_{n-1}$. The last equation can be rewritten as $D(p_M) - (k_2 + \ldots + k_n) \geq k_1 - k_n$ that holds for capacities that are quite close to one another, i.e. for capacity regions that are sufficiently close to the symmetric case. The sufficient conditions of existence of the unique equilibrium obtained for the mentioned size configurations, as well as those matching somewhat weaker conditions is referred to De Francesco and Salvadori (2013a).

3.3. One Large Firm and Small Symmetric Firms Oligopoly

Besides Vives (1986) and De Francesco and Salvadori (2010b), Davidson and Deneckere (1984) investigate the stage game of a symmetric $n$-firm oligopoly of a pricing supergame, to detect pro-collusive effects of horizontal mergers. It is shown that in a price-setting supergame a higher degree of concentration in the market does not necessarily lead to higher sustainability of collusion. On the contrary, somewhat counterintuitive, collusion will be less sustainable since punishment is expected softer. Although the coordinated effects approach is not the aim of this thesis, Davidson and Deneckere’s characterization of the equilibrium mixed strategies and profits will be necessary to investigate the effects on profitability and industry price of horizontal mergers.

The model considers horizontal mergers occurrence to gain market power and allow merged firm to twist the terms of trade to its advantage. Opposed to quantity competition models, where the merged

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37 Formally in a neighborhood of $p^0$, $d\Pi_i(p, \phi_{-i}(p^0))/dp = d\Pi_i(p, \phi_{-i}(p^0))/dp = \Pi_{x \neq i} \phi_x(p^0) [D(p) - (n - 1)k + pD'(p)] + [1 - \Pi_{x \neq i} \phi_x(p^0)]k > 0$. 
entity is equally-sized with respect to market competitors, the merged firm combines constituents’ assets. Furthermore, its capacity will be a (proper) multiple of the symmetric insider firms’ capacity. Thus, the merged firm’s capacity reads \( k_1 = mk_i \), where \( k_i = k, \forall i \in \{1, \ldots, n\} \), for 1 indicates the merged and largest firm. Outsiders are left with the existing symmetric capacities \( k_i = k \), for \( i = \{2, \ldots, n - m + 1\} \). Any rationing follows the efficient rule, absent income effects. If firms charge the same price, demand distributes proportionally to its capacity. Homogeneous good production proceeds at the same constant, null, marginal and average cost, up to capacity for all firms. Demand function is assumed linear \( D(p) = a - bp \) if \( p \in (0, a/b) \), zero otherwise. Firms’ sales can be written as \( \min \{mk, D(p)\} \) and \( \min \{k, D(p)\} \) for the merged and each small symmetric firm, respectively. It is important to note that Davidson and Deneckere focus on symmetric equilibria, i.e. smaller firms play the same equilibrium strategy profiles. Existence of both the pure and mixed strategy equilibria is demonstrated, whilst uniqueness of the pair of distribution functions \( (\phi_1(p), \phi_2(p)) \) is not, referring to previous work of the two authors. 38

Depending on the relative values of \( n, m \) and \( k \) the nature of the equilibrium would either be in pure or mixed strategies. Three different regions arise post-merger, conditionally on merger size and forthcoming demand vis-à-vis industry capacity. In the following theorem, Davidson and Deneckere define \( \lambda(k) = \left[a \left(1 + \sqrt{\theta(2-\theta)}\right)\right]^{1/2m} \), with \( \theta \equiv (n-m)k/a \), for \( c = 0 \), to address the three equilibrium regions.

**Theorem 3.1** For each pair \((m, k)\), \(1 \leq m \leq n - 1\), the price-setting game with capacity constraints has a unique symmetric Nash equilibrium:

(i) If \( k \geq a/(n - m) \) the equilibrium is in pure strategies, all firms charge \( p = c = 0 \) and earn zero profits, accordingly.

(ii) If \( k \leq a/(n + m) \) the equilibrium is pure strategies, all firms charge \( p = (a - nk)/b \) and profits for the merged and symmetric firms are, respectively, \( \Pi_1^* = (a - nk)mk/b \) and \( \Pi_2^* = (a - nk)k/b \).

(iii) If \( a/(n + m) \leq k \leq a/(n - m) \) and \( \lambda(k) \leq 0 \) the equilibrium is in mixed strategies and:

\[
p_m = \left[a - a\sqrt{\theta(2-\theta)}\right]/2b, \quad p_M = [a - (n - m)k]/2b;
\]

38 For the choice of symmetric equilibria, see “Note that we restrict our search for equilibrium to symmetric cones, i.e. equilibria that treat outsiders symmetrically.” in Davidson and Deneckere (1984, p. 123, n. 10). For what concerns uniqueness, see “The argument leading to uniqueness is rather involved; the interested reader is referred to Davidson (1982) and Deneckere (1983)”, Davidson and Deneckere (1984, p. 129).

39 When \( m = 1 \) this region is empty since then \( \lambda(k) \geq 0 \) for all \( k \) in the mixed strategy region.
\[ \Pi_1^* = \left[ a - (n - m)k \right]^2 / 4b, \quad \Pi_2^* = \left[ a - a\sqrt{\theta(2 - \theta)} \right] / 2bk; \]

\[ \phi_1(p) = \begin{cases} 
 0, & p < p_m \\
 1 - \frac{n^2}{kp}, & p_m < p < p_M \\
 1, & p > p_M 
\end{cases} \]

\[ \phi_2(p) = \begin{cases} 
 0, & p < p_m \\
 \frac{1}{(n-m)k} \left( a - bp - \frac{n^2}{p} \right), & p_m \leq p \leq p_M \\
 1, & p > p_M 
\end{cases} \]

(iv) If \( a/(n + m) \leq k \leq a/n - m \) and \( \lambda(k) > 0 \) the equilibrium is in mixed strategies and

\[ p_m = \left[ a - (n - m)k \right]^2 / 4mbk, \quad p_M = \left[ a - (n - m)k \right] / 2b \]

\[ \Pi_1^* = \left[ a - (n - m)k \right]^2 / 4b, \quad \Pi_2^* = \left[ a - (n - m)k \right]^2 / 4mb. \]

Proof: The demonstration of a pure strategy equilibrium existence, i.e. points (i) and (ii) of Theorem 3.1, is in fact the same as the proof of proposition 3.1. Thus, we refer to section 3.1 to show existence and uniqueness of a pure strategy equilibrium in the specific case of the linear demand function. It is necessary to show the pair of mixed strategies \((\phi_1(p), \phi_2(p))\) is indeed a pair of equilibrium mixed strategies. The argument for uniqueness is, nonetheless, referred to Davidson (1982) and Deneckere (1983). Let \( \hat{p} = \max\{0 \cdot (a - nk)/b\} \) and \( p^* \) any equilibrium price. The following lemma restricts the set of equilibrium pure strategies. In the mixed strategy region, \( a/(n + m) \leq k \leq a/(n - m) \), equilibrium profits \( \Pi_i^* \) must be constant, almost everywhere, across the support of equilibrium mixed strategies \( \phi_i(p) \). Almost everywhere \( \phi_i(p) = \phi_i \) is continuous at \( p \), follows

\[ \Pi_1^* = p \left[ \sum_{j=0}^{n-m} C_j^{n-m} \phi_1^j \left( 1 - \phi_2^j \right)^{n-m-j} \min^+\{a - bp - jk, mk\} \right] \]

\[ \Pi_2^* = p \left\{ \sum_{j=0}^{n-m-1} C_j^{n-m-1} \phi_1^j \left( 1 - \phi_2^j \right)^{n-m-j} \min^+\{a - bp, mk\} \times \left[ \left( 1 - \phi_2^j \right) \min^+\{a - bp - jk, k\} \right] \right. \]

\[ + \left. \phi_2 \min^+\{a - bp - (j + m)k, k\} \right\} \]

Where \( C_j \) indicates the cost function of firm \( j \). The following lemmas simplify the above expression for individual profits.

**Lemma 3.1** \[ \min^+\{a - bp - jk, k\} = k, \forall p \in [p_m, p_M] \text{ and } \forall j = 0, ..., n - m - 1. \]

Proof: \[ \min^+\{a - bp - jk, k\} \geq \min^+\{a - bp_M - (n - m - 1)k, k\} = \min^+\left\{ \frac{a-(n-m)k}{2} + k, k \right\} = k. \text{ The last equality follows since } k \leq a/(n - m) \text{ in mixed strategies.} \]
We have seen that if the largest firm charges a higher price then, smaller firms market all their production. The next lemma shows that when $\lambda(k) \leq 0$, smaller firms do not sell anything if the larger firm charges a lower price.

**Lemma 3.2** If $\lambda(k) = \left[ a \left( 1 + \sqrt{\theta(2-\theta)} \right) \right]/2m - k \leq 0$ then $\min^+\{a - bp - (j + m)k, k\} = 0, \forall p \in (p_m, p_M), \forall j = 0, \ldots, n - m - 1$.

Proof: $\min^+\{a - bp - (j + m)k, k\} \leq \min^+\{a - bp_m - mk, k\}$, where $p_m$ is the one pointed in region (iii) of Theorem 3.1. the last expression is negative whenever $p_m \geq (a - mk)/b$, that is $\lambda(k) \leq 0$.

Similarly, we can prove that in region (iii) the large firm never sells at capacity, i.e. $\min^+\{a - bp - jk, mk\} = a - bp - jk, \forall p \in (p_m, p_M), \forall j = 0, \ldots, n - m$. In region (iii) the expressions for the equilibrium profits become $\Pi_1^* = p(a - bp - (n - m)k\phi_2)$ and $\Pi_2^* = pk(1 - \phi_1)$. For all $p$ in the support. In region (iii) the expressions for the equilibrium profits become

$$\Pi_1^* = p\left[ \sum_{j=0}^{n-m} c_j^{n-m} \phi_2^j (1 - \phi_2)^{n-m-j} \min^+\{a - bp - jk, mk\} \right]$$  \hspace{1cm} (3.6)

$$\Pi_2^* = p \left[ (1 - \phi_1)k + \phi_1 \sum_{j=0}^{n-m-1} c_j^{n-m-1} \phi_1^j (1 - \phi_1)^{n-m-j-1} \min^+\{a - bp - (j + m)k\} \right]$$  \hspace{1cm} (3.7)

The pair of equilibrium mixed strategies $(\phi_1, \phi_2) = (\phi_1(p), \phi_2(p))$ constitute the unique equilibrium mixed strategies pair that solve both A.1’ and A.2’ and maintain expected profits constant throughout the support. It is straightforward to check that profits are decreasing off the support. Thus, all equilibrium conditions are satisfied. Q.E.D.

The two pure strategy regions and the mixed strategy one can be described as follows,

i. In terms of general oligopoly the first region is defined by $K - k_1 \geq D(0)$. A pure strategy equilibrium à la Bertrand arises in this first region. More precisely, this is a symmetric equilibrium, in which all firms set a price equal to the null marginal cost and make zero net profits. Generally, asymmetric strategies perhaps exist in which firms charge a positive price though selling null quantity. Anyhow, profits for each firm are null.

ii. The second region is defined by the general oligopoly conditions $D(0) > K$ and $k_1 + p(K)[D'(p)]_{p=p(K)} \leq 0$, or $k \leq a/(n - m)$. The two conditions state that firms sell all their production at market clearing price for aggregate capacity, and that such a price represents the unique Nash equilibrium of the game.
In both regions in which the pure strategy equilibrium exists, the largest firm’s capacity must be necessarily small enough with respect to industry capacity, i.e. $k_1/K \leq 1 - a/K$ or $k_1/K \leq a/K - 1$, respectively in region (i) and (ii).

iii. The third is the mixed strategies region, where $a/(n + m) \leq k \leq a/n - m$, if and only if $k_1/K \geq [1 - a/K]$ . Davidson and Deneckere present two distinct areas (iii) and (iv) according to the size of the largest firm in the post merger $n - m + 1$-oligopoly. In Davidson and Deneckere (1984) region (iii) is the one in which $D(p_m) \leq k_1$, whilst in region (iv) demand and capacity are related by the inequality $D(p_m) > k_1$.

Notably, point 8 of Proposition 2 states that firm 2’s expected profits are $\Pi_2^* = p_m k_i$. Since smaller firms have all the same capacity as firm 2, then they all earn the same profits. This finding elucidates on the generality of Davidson and Deneckere’s (1984) results, whose profit characterization is not in any way limited, by the restriction to symmetric equilibria. Davidson and Deneckere (1984) model characterizes mixed strategies equilibrium in the case of a peculiar oligopoly setting, in which $m$ symmetric firms merge to create a bigger firm, whose size is a proper multiple of the symmetric oligopolists’ one. Davidson and Deneckere (1984) just inspect symmetric equilibria, i.e. equilibria that arise as the symmetric firms play the same mixed strategy. This constitutes the unique, symmetric equilibrium with the features exposed in the previous paragraph. Besides a symmetric equilibrium, De Francesco and Salvadori (2010b) further investigated whether asymmetric equilibria may exist with firms playing different equilibrium strategies in symmetric oligopolies. Furthermore, investigation involved whether asymmetric equilibria bear different properties than symmetric ones, i.e. equilibrium profits may be different according to the different equilibria. In De Francesco and Salvadori (2010b) is presented a finer partition of the mixed strategies region to investigate asymmetric equilibria. Davidson and Deneckere (1984) mixed strategy region is divided into two sub regions, according to merged firm dimension with respect to industry capacity. The two sub regions differ according to the dimension of the merged and largest firm. In terms of price the two sub regions can be addressed as either $p_m \geq p(k_1)$, or $p_m < p(k_1)$. In De Francesco and Salvadori (2010b) we find them characterized as

A. $D(p_m) \leq k_1$;
B. $D(p_m) > k_1$.

In sub region A it holds $p_m = \bar{p} = \frac{1}{2b} \left[ a - \sqrt{a^2 - 4b\Pi_1} \right]$. Conditions on largest firm’s capacity constrain small symmetric firms to set a lower price than the merged firm in order to sell a positive quantity. In $[p_m, p_M]$ the largest firm can meet all forthcoming demand, consequently small
firms’ expected profits will be $\Pi_i^*(p, \phi_i) = (1 - \phi_i(p))pk_i, \forall i > 1$. Since $\Pi_j^* = p_mk_j \forall j > 1$, then equilibrium mixed strategy for firm 1 will be

$$\phi_1(p) = \frac{p-p_m}{p} \tag{3.8}$$

The equation $\Pi_i^* = \Pi_i(p, \phi_{-1})$ in the unknowns $(\phi_2, ..., \phi_n)$ represents a degree of freedom in the system of equilibrium equations, leaving indeterminate the small firms’ strategy profiles. Assuming symmetry among small firms’ strategies, largest firm’s profit function can be written as

$$\Pi_i^* = \sum_{s=0}^{n-1} \binom{n-1}{s} p (1 - \phi_j)^s (1 - \phi_j)^s [D(p) - (n - 1 - s)k]$$

$$= pD(p) - \sum_{s=0}^{n-1} \binom{n-2}{s} \phi_j^{n-2-s} (1 - \phi_j)^s \phi_j (n - 1)pk$$

And therefore

$$\phi_j = \frac{pD(p) - \Pi_i^*}{(n-1)pk} \tag{3.9}$$

If small firms do not necessarily behave symmetrically then, analogously, firm one’s equilibrium profits will be written as

$$\Pi_1^* = pD(p) - pk \sum_{j=2}^{n} \phi_j \tag{3.10}$$

This equation constitutes a further constraint on $\phi_j$, besides the constraints of mixed strategies properties, i.e. a mixed strategy is continuous, nonnegative and non decreasing in $[p_m, p_M]$.

Thus, in sub region 3A Davidson and Deneckere (1984) find the conditions for the unique symmetric equilibrium with the characteristics defined in Theorem 3.1. In sub region 3.B Davidson and Deneckere (1984) only determine the two extrema of the support and equilibrium profits. The existence of equilibrium mixed strategies $\phi_i(p)$ and $\phi_i(p) = \phi_j(p)$, for $i, j = 2, ..., n$ is not yet shown. Though a complete proof cannot be found, in De Francesco and Salvadori (2010b) a finer partition of the second sub region is presented. Furthermore, De Francesco and Salvadori reckon that whenever an asymmetric solution does not exist, then the symmetric one certainly exists. On the contrary, if an asymmetric solution exists, then a symmetric solution does not necessarily exist. Sub region iii.B can be split among B_a, B_b and B_c. Such areas are characterized by

a. $p_M \geq p(k_1)$ if and only if $m > a/3k + n/3$.  

---

That is to say, $\forall p \in (p_m, p_M)$ it is true $\phi_j' > 0$ for at least some firm $j > 1$, and necessarily $\phi_j(p_m) = 0$, and $\phi_j(p_M) = 1, \forall j > 1$. 

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41
b. \( p(k_1 + (n + 2)k) < p_M \leq p(k_1) \) if and only if \( a/k - n + 2 < m \leq a/3k + n/3. \)

c. \( p(k_1 + (n + 2)k) \geq p_M \) if and only if \( a/k - n + 2 \geq m. \)

Proposition 3.4 allows us to find a unique equilibrium in area \( c \) of the sub region \( \text{iii.B} \) in the interval \([p_m, p_M]\), where

\[
\phi_1(p) = \frac{k}{k_1} \left[ \frac{(p-p_m)k_1}{p(k-D(p))} \right]^{\frac{1}{n-1}} \text{ and } \phi_2(p) = \cdots = \phi_n(p) = \left[ \frac{(p-p_m)k_1}{p(k-D(p))} \right]^{\frac{1}{n-1}}. \quad (3.11)
\]

The only difference between areas \( \text{iii.Ba} \) and \( \text{iii.Bb} \) is the presence of the interval \([p(k_1), p_M]\) in the first area but not in the second. In such interval the large firm is capable to satisfy the forthcoming demand, then the small firms can sell a positive quantity only undercutting the large one. Therefore, equation (3.8) holds only in the interval \([p(k_1), p_M]\). Moreover, in the interval \([p(k_1), p_M]\) exists a symmetric equilibrium in which (3.8) holds. As well as in the interval \([p(k_1), p_M]\) a continuum of asymmetric equilibria exists, in which \( \phi_j(p) \) satisfy (3.9).\(^{41}\) In sub regions \( \text{iii.A} \) and \( \text{iii.Ba} \) there exists a continuum of equilibria. Nonetheless, De Francesco and Salvadori (2010b) show that equilibrium profits are unique regardless uniqueness of equilibrium mixed strategy.

4. Horizontal Mergers Theoretical Analysis in Symmetric Triopolies à la Bertrand-Edgeworth

In this chapter we investigate horizontal mergers between two symmetrically capacity-constrained price setting firms in a symmetric three-firm market. Horizontal mergers have received great attention in the last decades since Salant, Switzer and Reynolds (1983), noticed curiously that the effects of horizontal combinations among quantity-setting sellers of a homogeneous good might not be those intuitively awaited. In fact, in a Cournot-Nash industry horizontal mergers can affect equilibrium increasing industry profit, while decreasing the profitability of merging firms below the sum of their pre-merger profits.

The model considers symmetric oligopolists facing linear demand and constant, identical marginal and average cost. Following these assumptions the equilibrium is symmetric, i.e. every firm produces the same output amount and earns the same profit. To recall the Salant et al. (1983) results, assume that the market is made of \( n \) firms each earning \( \Pi(n) \). If two firms merge, then higher concentration leads to higher profits per firm as well as industry-wide. That is, \( \Pi(n) < \Pi(n-1) \) and \( n\Pi(n) < (n-1)\Pi(n-1) \). Although in Salant et al. (1983) it is also true that \( \Pi(n-1) < 2\Pi(n) \), i.e. the

\(^{41}\) \( \phi_j(p) \) must also be continuous, non negative and right increasing. Such constraints ensure continuity of \( \phi_j(p) \) in \( p(k_1) \), since \( \lim_{p \to p(k_1)^+} \phi_j(p) = \lim_{p \to p(k_1)^+} \phi_j(p) \).
survival organization’s profits is lower than the value of its constituents’ profits pre-merger. In the new equilibrium each firm produces $1/(n-1)$ of the total market production, each firm shares equally the output restriction due to the reduced number of active players in the market. The peculiar result obtains because the merged firm cannot be distinguished from the nonmerged firm as it is itself a symmetric player in the oligopoly. At the same time, unprofitable mergers for the merging parties demonstrate profitable for the nonmerged firms. The horizontal merger allows the merging firms to internalize the premerger rivalry, driving the merged entity to reduce output, increasing market price, to the sole benefit of outsiders. As a result, the takeover generates a positive external effect on the other firms. As a response to the merger, the other firms increase output, at expense of the merging firms’ profit.\(^{42}\)

However, a horizontal merger is argued to gain market power to the merged firm since allegedly, a larger firm can easily twist the terms of trade to its advantage, securing a higher profit than if each merging party acted independently. Further research examined the problem of characterization of profitable mergers, for instance Baik (1995) studies horizontal mergers in a homogeneous product market, introducing sunk capacity costs in the maximization problem of price setting firms. Unlike much of the previous literature, Baik (1995) finds that horizontal mergers always harm outsiders, and do not reduce joint profits of insiders, for certain values of the fixed costs schedule. Moreover, Deneckere and Davidson (1985) show that the Salant et al. (1983) result is reversed for pricing competition. The reason is that a merger causes the participants collectively to behave less aggressively; behavior that benefits price-setters, but not quantity-setters. Perry and Porter (1985) and Farrell and Shapiro (1990) examine mergers in the presence of increasing marginal costs, i.e. when firms own some industry-specific capital. Introducing decreasing returns to scale technology can be considered as well an example of capacity constraint. When firms engage in price competition with capacity constraints horizontal merger analysis describes quite satisfactorily the incentives to merge and the unilateral effects of the takeover. This chapter provides a theoretical analysis of static horizontal mergers among three equally capacity-constrained sellers of a homogeneous good that engage in price competition. It presents a taxonomy of horizontal takeover’s effects depending on the region of capacity in which the equilibrium exists. The results of this examination can be generalized quite intuitively to the more general symmetric oligopoly analysis of the following chapter.

\(^{42}\) The Salant et al. (1983) peculiar result and the difficulties in providing a satisfactory model of horizontal mergers consistent with the profitability of mergers has raised the so-called “merger paradox”. Pepall, Richards, and Norman (1999) define the merger paradox as: “What may be surprising to you is that it is, in fact, quite difficult to construct a simple economic model in which there are sizable profitability gains for the firms participating in a horizontal merger that is not a merger to monopoly. This difficulty is what we call the merger paradox.”
Section 1 lists the assumptions and provides the equilibrium characteristics, both in pure and mixed strategies, in the reference symmetric triopoly model. Section 2 investigates the effects of horizontal mergers in a model in which firms’ capacities allow pure strategy equilibria à la Bertrand pre-merger. Both pure and mixed strategy equilibrium configurations are investigated post-merger. Section 3 investigates the effects of horizontal mergers in a pre-merger pure strategy equilibrium in which firms charge the aggregate capacity-clearing price. Pure and mixed strategy equilibrium configurations are investigated post-merger. The concluding section investigates the effects of horizontal mergers if a mixed strategy equilibrium exists pre-merger.

4.1. Preliminaries

Suppose three identical firms \( i \in I = \{1,2,3\} \) compete in the market and no entry or exit is allowed in the model. Equally-sized oligopolists produce a homogeneous good under constant, and null, marginal and average cost up to capacity. Neither installation costs of the existing capacity nor fixed costs are considered in this analysis. Firms name their price simultaneously and non-cooperatively. We assume all firms owns the same capacity and is constrained at \( k_i = k \), for \( i \in I \), and we define total capacity as \( K = \sum_{i}^{3} k_i = 3k \). Consumers preferences determine the demand function \( X = D(p) \), where \( X \) is market demanded quantity and \( p \) market price. Market demand is positive for \( p \in (0,\bar{p}) \), where if \( p > \bar{p} \), then \( D(p) = 0 \). Furthermore, demand function is assumed continuous, strictly decreasing and such that \( pD(p) \) is strictly concave in \( (0,\bar{p}) \). For some propositions demand function is assumed concave in \( (0,\bar{p}) \). Conversely, the inverse demand function is defined for the decreasing part of the demand function such as \( p = D^{-1}(X) = X(p) \), in \( X \in (0,D(0)) \). Any rationing of the forthcoming demand by the capacity-constrained firms follows the efficient rule whose characteristics we refer to the first section of the previous chapter. According to the assumptions on individual and industry capacity, the simultaneous move game has a unique equilibrium either in pure or mixed strategies.

Pure strategy equilibria à la Bertrand, in which firms charge marginal cost price, arise if \( k_i \geq D(0), \forall i \in I \). In every Nash equilibrium at least two firms set marginal cost pricing, while the rival charges whatever price not lower than marginal cost. Whenever any two firms price at marginal cost, the other competitor will be indifferent on charging any other price at least equal to the null marginal cost, since it would still earn zero profits. At the same time, each of the \( j \) firms charging marginal cost price, has no incentive to name a different price, neither higher nor lower. This is not necessarily the only case in which Bertrand equilibria arise in capacity-constrained price competition. Recalling previous chapter’s results, a pure strategy equilibrium exists when firms are individually capacity-constrained if and only if

\[
k \leq \max \{3k - D(0), -p^c[D'(p)]_{p=p^c}\}
\] (4.1)
Inequality (4.3) states that for a pure strategy equilibrium to exist a symmetric player must be sufficiently small-sized with respect to industry capacity. Let \( p^e \) indicate the competitive equilibrium price. If \( 2k \geq D(0) \) then the triple \( p^e = p_i = 0, \forall i \) represents the unique pure strategy equilibrium; if \( K < D(0) \) and \( k \leq -p^e[D'(p)]_{p=p^e} \); then pure strategy equilibrium is the triple \( p^e = p_i = p(3k) > 0, \forall i \). Thus, condition \( k \leq -p^e[D'(p)]_{p=p^e} \) identifies the region of the \((k_1, k_2, k_3)\) capacity space in which each firm’s capacity is not greater than its best-response quantity when the rivals produce at full capacity.\(^{43}\) Whenever a pure strategy equilibrium does not exist it might well be the case that mixed strategy equilibria do. Mixed strategy equilibria exist if

\[
D(0)/3 > k > -p(3k)[D'(p)]_{p=p(3k)}
\]  

(4.2)

where \( \bar{k} = f(K) = -p(3k)D'(p)]_{p=p(3k)} \). Inequality (4.4) identifies the region in which a mixed strategy equilibrium exists if \( K < D(0) \). The capacity threshold above which firms are allowed to randomize over their price offers is not explicitly defined; as \( k \) appears on the left hand side as well as the right hand side of inequality (4.4). In the following proposition we formalize the result for the capacity threshold \( \bar{k}^* \) as follows

**Proposition 4.1** There exist a unique value of capacity \( \bar{k}^* \) such that a mixed strategy equilibrium exists if \( k > \bar{k}^* \).

Proof: We shall recall the Cournot-Nash best response function \( \forall i \frac{\partial}{\partial q_i} q_i P(Q) \geq 0 \) for \( q_i = k_i \) and \( q_j = k_j \), each \( j \neq i \).\(^{44}\) The first-order conditions \( k_i P(k_i + \sum_{j \neq i} k_j) = 0 \) for firm \( i \) can be written as

\[
P(k_i + \sum_{j \neq i} k_j) + k_i P'(k_i + \sum_{j \neq i} k_j) = 0.
\]

Rearranging \( k_i = -P(k_i + \sum_{j \neq i} k_j)/P'(k_i + \sum_{j \neq i} k_j) \),

\[
k_i = -\frac{-P(k_i + \sum_{j \neq i} k_j)}{D'(k_i + \sum_{j \neq i} k_j)}.
\]

The relation between \( k_i = k, \forall i \) and industry capacity \( K = k_i + \sum_{j \neq i} k_j = 3k \) is obtained differentiating both sides in total capacity. Differentiation yields

\[
dk_i/dK = -P'(k_i + \sum_{j \neq i} k_j)D'(k_i + \sum_{j \neq i} k_j) + \frac{-P(k_i + \sum_{j \neq i} k_j)D''(k_i + \sum_{j \neq i} k_j)P'(k_i + \sum_{j \neq i} k_j)}{D'(k_i + \sum_{j \neq i} k_j)}.
\]

And given the assumptions on concavity of demand function, \( dk_i/dK = -P'(k_i + \sum_{j \neq i} k_j)[D'(k_i + \sum_{j \neq i} k_j) - P(k_i + \sum_{j \neq i} k_j)D''(k_i + \sum_{j \neq i} k_j)] \leq 0 \). The shape of the relation is not yet defined as it depends upon the shape of demand function. Thus, the capacity threshold \( \bar{k}^* \) is the value that identifies in the \((k, K)\)-space the intersection between the negatively sloped function

\(^{43}\) The region is namely the region bounded above by the lower envelope of the Cournot best-response functions. Note that the assumptions on capacity do not guarantee the uniqueness of the Cournot equilibrium. Uniqueness would be assured if \( D'(p) + pD''(p) < 0 \) in \((0, P(0)) \), see Deneckere and Kovenock (1992).

\(^{44}\) The best-response function written in terms of quantity competition is equivalent to the one in terms of price and residual demand \( \forall i, \left[ d[D(D(p) - \sum_{j \neq i} k_j)]/dp \right]_{p=p^e} \leq 0 \).
$k = f(K)$ and the positively sloped ray through the origin $k = K/3$.\textsuperscript{45} Hence, a mixed strategy equilibrium exists if $k > \tilde{k}$ that is true if $k > \hat{k}$. \textbf{Q.E.D.}

In the mixed strategy region an equilibrium always exists satisfying the sufficient conditions for existence of Theorem 5 in Dasgupta and Maskin (1986). We follow Vives (1986) to characterize mixed strategy equilibrium profiles for the symmetric triopoly. The features of the equilibrium distributions are the ones described in the second section of the previous chapter. The definitions of $p_m$ and $p_M$, the infimum and supremum of $S_i = S_i\{\phi_i(p)\}$, the support of equilibrium mixed strategies, follow accordingly. Thus, the mixed strategies region can be addressed as $p(3k) < p_m$. Recall, $p_M = \arg\max_p p(D(p) - 2k)$ and $p_m = \max\{\bar{p}, \bar{p}\}$; we let $\bar{p} = \Pi_i^*/k$ and $\bar{p}$ as the lower solution of the equation in $p : \Pi_i^* = p D(p)$ ($\Pi_i^*$ reads largest firm’s profits, in this case any symmetric competitor pre-merger). Equilibrium mixed strategies of all firms are assumed symmetric, i.e. all firms adopt the same equilibrium strategy, $\phi_i(p)$, for $\forall i \in I$ obtained in Vives (1986). Pre-merger equilibrium mixed strategy for the three-firm industry is $\phi_i(p) = \sqrt[\bar{p} - p_m]{k}, \forall i \in I, p \in S_i$.

\textbf{4.2. Horizontal Mergers Analysis in Symmetric Triopoly: Premerger Pure Strategy Equilibrium \textit{à la} Bertrand}

The proposed merger may involve any of the three symmetric firms, though we suppose 1 and 2 as merging firms and the third as unchanged outsider (post merger firm B). This merger leads to a duopoly in which the merged firm (post merger firm A) owns twice the capacity of its equally sized constituents.\textsuperscript{46} Assume the pre-merger equilibrium is a pure strategy equilibrium in which firms charge a null competitive price and earn zero profits accordingly. Horizontal mergers might either leave firms play pure strategy null equilibrium price or create a larger market player whose size allows to quote a higher price on the market than the competitive one, i.e. the equilibrium turns to mixed strategies.

\textsuperscript{45} The existence of an intersection draws on the fact that $dk_i/dK = -P(k_i + \sum_j \alpha_i k_j)D(k_i + \sum_j \alpha_i k_j) - P(k_i + \sum_j \alpha_i k_j)D(k_i + \sum_j \alpha_i k_j) \leq 0$ can be rewritten and manipulated to obtain $dk_i/dK = -[1 + P(k_i + \sum_j \alpha_i k_j)D(k_i + \sum_j \alpha_i k_j)]/D(k_i + \sum_j \alpha_i k_j) \leq 0$. In order for the latter inequality to hold it must be, $1 + P(k_i + \sum_j \alpha_i k_j)D(k_i + \sum_j \alpha_i k_j)/D(k_i + \sum_j \alpha_i k_j) \geq 0$ and consequently the fact that $P(k_i + \sum_j \alpha_i k_j)D(k_i + \sum_j \alpha_i k_j)/D(k_i + \sum_j \alpha_i k_j) \geq -1$ implies the slope of the function $k = f(K)$ is finite.

\textsuperscript{46} Merging firms are assumed throughout the dissertation to quote the same price with probability one. Alternative to the post merger profit-maximization approach is to assume that a merger allows the new firm to use strategies that were not available pre-merger, see Creane and Davidson (2004) and Huck, Konrad and Müller (2004).
If $k \geq D(0)$ in every Nash equilibrium at least two firms set marginal cost pricing, while the rival charges any price not lower than marginal cost. The uniqueness argument is straightforward and referred to the previous chapter. In this capacity region, firms are individually so big to supply the whole forthcoming demand at marginal cost, i.e. equilibrium is the one à la Bertrand. If $k \geq D(0)$ holds, then necessarily $2k \geq D(0)$ and any merger has no real effect. This result stems from the fact that the merged firm will supply all forthcoming demand, as well as the outsider, at the lowest market price, i.e. marginal cost; this position constitutes the unique equilibrium both pre and post merger. The unique post-merger equilibrium profits are the null profits (the same as pre-merger).

Whenever individual firms’ capacity is not big enough to supply all forthcoming demand at marginal cost, then the horizontal merger might create a duopoly in which size asymmetries between the two players create an incentive to quote a higher price than marginal cost. If $2k \geq D(0)$, equivalently $k \geq D(0)/2$ then the outsider is capacity constrained while the merged firm is big enough to supply the forthcoming demand at marginal cost. This allows for different pricing incentives of the duopolists. That is, if $k < D(0) \leq 2k$ then post merger equilibrium is in mixed strategies. In mixed strategies, duopolists mix their price offers in the post merger price interval $(p_L, p_H)$. The extrema of the support of the mixed strategies are defined as in the previous chapter. Given the assumptions on firms’ dimensions, we can write $p_H = \arg\max_p p(D(p) - k)$ and $p_L$ is the lower solution to $\max_p p(D(p) - k) = p D(p)$. Equilibrium payoffs of the game are $\Pi_A^* = p_L D(p_L)$ and $\Pi_B^* = p_L k$, respectively for the merged firm and outsider. Merged firm’s equilibrium mixed strategy $\phi_A$ will be continuous and increasing over $(p_L, p_H)$, due to concavity of profit function. Outsider’s $\phi_B$ will be continuous and increasing over $(p_L, p_H)$, since it has an incentive to undercut the rival, it will never reach the supremum. Since the merged firm charges the supremum of equilibrium supports with positive probability then its mixed strategy $\phi_A$ everywhere dominates $\phi_B$ over $(p_L, p_H)$. This acknowledged property of duopoly points to the different pricing incentives of the two firms: the smallest firm undercut price with higher probability given the larger size of the rival.

**Proposition 4.2** If $D(0)/2 < k < D(0)$, post merger equilibrium is in mixed strategies where the range of equilibrium prices is strictly greater than the null pre-merger price. Thus, the merger is profitable for the merging parties and for the outsider.

Proof: Horizontal merger is profitable for the outsider if $\Pi_B^* = p_L k > 0$, given null pre-merger profits any horizontal merger that allows the outsider to play a mixed strategy is profitable. Analogously, merged firm’s profits $\Pi_A^* = p_L D(p_L) > 0$ and horizontal mergers are profitable for insiders. Q.E.D.

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47 For further formalization see Theorem 1, point (a) in De Francesco and Salvadori (2013b), where the definition of the mixed strategies support extrema and the firms’ equilibrium profits can be found.
Although it is immediate to notice that the horizontal merger leads to a price increase from the null competitive equilibrium price pre-merger, we are left defining the equilibrium mixed strategies of the two firms post-merger. We shall consider as equilibrium mixed strategies those probability distributions that maximize expected profits at each price level \( p \in S_i \) when the rival plays the equilibrium mixed strategy. The expected profits of any firm \( i \in I \) when it names with certainty any price \( p \in S_i \), whilst the rival \( j \in I \) plays the equilibrium mixed strategy can be written as

\[
\Pi_i = \Pi(p, \phi_j) = p \min \{ \max \{ 0, D(p) - k_j \}, k_i \} \phi_j + p \min \{ D(p), k_i \} (1 - \phi_j), \text{ a.e. in } S_i \quad (4.3)
\]

Equilibrium profits and mixed strategy equilibrium profiles for an asymmetric duopoly are the ones found in Kreps and Scheinkman (1983). The equilibrium mixed strategies are found in the specific case in which the largest firm doubles the dimension of the smaller firm considering that profits shall be maximized and constant along the mixed strategies support. Given the assumption on capacity \( D(p) \leq 2k \) and the expected equilibrium profits for the merged firm and the outsider shall be \( \Pi_A^* = p_l D(p_l) \) and \( \Pi_B^* = p_l k_l \), respectively. We obtain the outsider mixed strategy from the expression of merged firm’s profits \( \phi_B = [pD(p) - p_l D(p_l)]/pk_l \). Since \( D(p) \leq 2k \), the outsider can only sell a positive quantity undercutting the larger one, that is \( \Pi_B = p \min \{ D(p), k \} (1 - \phi_A) \).

The merged firm’s equilibrium strategy yields \( \phi_A = (p - p_l)/p \).

We have shown in this first section that horizontal mergers among capacity unconstrained firms have no real effect on both firms and consumers. Whenever the single capacity of market rivals is so high to satisfy the whole forthcoming demand, the equilibrium recreates the standard Bertrand result both pre and post-merger. If firms are not individually so large but the combined capacity of two firms is sufficient to supply all forthcoming demand at null marginal cost, then an incentive to set a higher price than the competitive one arises. Horizontal mergers combine the capacity of any two of the three competitors to form a larger firm capable to fulfill market demand at marginal cost. Thus, horizontal takeovers allow competing firms to play mixed strategies in the post-merger duopoly, quoting a higher price than the competitive one raising profits for the duopolists.

### 4.3. Horizontal Mergers Analysis in Symmetric Triopoly: Premerger Positive Price Pure Strategy Equilibrium

The proposed merger may involve any of the three symmetric firms, though we suppose 1 and 2 as merging firms and the third as unchanged outsider (post merger firm A). This merger leads to a duopoly in which the merged firm (post merger firm B) owns twice the capacity of its equally sized constituents. Assume pre-merger equilibrium is in pure strategies, i.e. \( k \leq \bar{k}^* \) where \( \bar{k} = -p(K)D'(p) \big|_{p=p(K)} \) and \( \bar{k}^* = \arg\max_X X D^{-1}(X + 2\bar{k}) \) identifies the value of the threshold above which mixed strategies are played following the analysis in the preliminary section 4.1. Thus, pre-
merger equilibrium price is the positive competitive price \( p^c = p(3k) > 0 \) and every symmetric firm sells its full capacity, earning equilibrium profits \( \Pi_i^t = p(3k)k, \) for \( i = \{1, 2, 3\} \). A horizontal merger might either leave firms play pure strategies or create a larger market player whose size allows to quote a higher price on the market than the competitive one, i.e. equilibrium turns to mixed strategies. Since horizontal merging creates a larger firm, whose size doubles the capacity of the two participating firms, the threshold for a pure strategy equilibrium to exist changes accordingly. Inequality (4.4) must hold in this area of capacities for a pure strategy equilibrium pre-merger to exist. Inequality (4.4) identifies the region in which a mixed strategy equilibrium exists if \( K < D(0) \). The capacity threshold above which firms are allowed to randomize over their price offers is not explicitly defined; as \( k \) appears on the left hand side as well as the right hand side of inequality (4.4).

In the following proposition we formalize the result for the post-merger capacity threshold \( \tilde{k}^{**} \) as follows

**Proposition 4.3**  
There exist a unique value of capacity \( \tilde{k}^{**} \) such that a pure strategy equilibrium exists if \( k \leq \tilde{k}^{**} \), that is true if \( k \leq \tilde{k}^*/2 \).

Proof: We shall recall the Cournot-Nash best response function \( \forall i \frac{\partial}{\partial q_i} q_i P(Q) \geq 0 \) for \( q_i = k_i \) and \( q_j = k_j \), each \( j \neq i \).\(^{48}\) Taking the first-order conditions and rearranging similarly to what has been done in section 4.2 we obtain \( k_i = -P'(k_i + \sum_{j \neq i} k_j)D'(k_i + \sum_{j \neq i} k_j) \). The relation between \( k_i = k_i, \forall i \) and industry capacity \( K = k_i + \sum_{j \neq i} k_j = 3k \) is obtained differentiating both sides in total capacity. Differentiation and the assumptions on concavity of demand function yield \( dk_i/dK = -P'(k_i + \sum_{j \neq i} k_j)[D'(k_i + \sum_{j \neq i} k_j) - P'(k_i + \sum_{j \neq i} k_j)D''(k_i + \sum_{j \neq i} k_j)] \leq 0 \). The shape of the relation is not yet defined as it depends upon the shape of demand function. Thus, the capacity threshold \( \tilde{k}^{**} \) is the value that identifies in the \((k, K)\)-space the intersection between the negatively sloped function \( k = f(K) \) and the positively sloped ray through the origin \( k = 2/3 K \).\(^{49}\) In terms of total capacity \( \tilde{k}^{**} \) is obtained as \( \tilde{k}^{**} : 2/3 K = \tilde{k}(K) \). Post-merger threshold must be lower than pre-merger capacity threshold \( \tilde{k}^* \leq 2\tilde{k}^{**} \). Thus, a pure strategy post-merger equilibrium is played if \( k \leq \tilde{k}^*/2 \). Q.E.D.

If \( k \leq \tilde{k}^*/2 \) post-merger equilibrium is in pure strategies. No other equilibrium can be found since each firm hardly has incentive either to cut price below or to raise it above the competitive price.

\(^{48}\) The best-response function written in terms of quantity competition is equivalent to the one in terms of price and residual demand \( \forall i, [d[p(D(p) - \sum_{j \neq i} k_j)]/dp]_{p=p^c} \leq 0 \).

\(^{49}\) The existence of an intersection can be shown as in section 4.1. The slope of the function \( k = f(K) \) is finite and it necessarily intersects the line \( k = 2/3 K \).
It would not be an equilibrium charging a lower price, since the firm would not be able to produce any additional output to meet increased demand. Nor it is charging a higher price since it would reduce its own sales and profits. The equilibrium profits are \( \Pi_i^e = p(3k)k \), for \( i = \{1,2,3\} \). That is, equilibrium post merger has the same characteristics as the pre-merger one: if \( k \leq \hat{k}^*/2 \) holds, the equilibrium post merger will be played in pure strategies and firms will charge the competitive price, \( p^c = p(3k) > 0 \). Equilibrium profits for each firm will be \( \Pi_A^e = p(3k)2k \) and \( \Pi_B^e = p(3k)k \), for the merged and outsider firm, respectively. The merger does not alter equilibrium profits of competitors: neither for the insider firms, whose joint profits even the sum of insiders’ pre-merger profits, since \( \Pi_A^e = p(3k)2k = 2\Pi_i^e = 2p(3k)k \); nor for the outsider whose profits rest at the pre-merger level. Therefore, if a pure strategy equilibrium exists both pre and post merger, horizontal mergers do not have real effects on either firms or consumers.

More theoretically attractive is the case in which horizontal mergers turn the pure strategy equilibrium pre-merger into a mixed strategy one. If \( \hat{k}^* \geq k \geq \hat{k}^*/2 \) then post merger equilibrium is in mixed strategies. In mixed strategies, duopolists mix their price offers in the post merger price interval \( (p_L, p_H) \). The supremum of the post merger price range is determined by the largest firm, monopolizing residual demand as the smallest firm charges a lower price, i.e. \( p_H = \arg\max_p p(D(p) - k) \). Largest firm will earn profits \( \Pi_A^e = p_H(D(p_H) - k) \). The infimum is set by the larger firm perhaps undercutting the small one while earning profits \( \Pi_A^e \). Then, \( p_L = \max \{\bar{p}, \tilde{p}\} \), where we define \( \bar{p} = \Pi_i^e/k \) and \( \tilde{p} \) as the lower solution of equation \( \Pi_A^e = p D(p) \). In turn, \( p_L = \bar{p} \) if \( D(p_L) > k_A \) or \( p_L = \tilde{p} \), otherwise. In turn, \( p_L = \bar{p} \) if \( D(p_L) > k_A \) or \( p_L = \tilde{p} \), otherwise. Merged firm’s equilibrium mixed strategy \( \phi_A \) will be continuous and increasing over \( (p_L, p_H) \), due to concavity of profit function. Outsider’s equilibrium mixed strategy \( \phi_B \) is continuous and increasing over \( (p_L, p_H) \); since it has an incentive to undercut the rival \( \phi_B \) will never reach the supremum. Post-merger equilibrium mixed strategies extrema must satisfy \( p(3k) < p_L < p_H < p(k) \), since the merged firm will neither set \( p_H \) over outsider’s capacity-dumping price, to sell null output; nor set \( p_L \) lower than the competitive price. Moreover, since largest firm charges the supremum of equilibrium supports with positive probability then its mixed strategy \( \phi_A \) everywhere dominates \( \phi_B \) over \( (p_L, p_H) \). That is, the smaller firm sets a lower price than any price \( p \), with higher probability than the larger competitor. This suggests that the smaller firm competes more aggressively than the large firm, charging a lower price than the rival, with higher probability.

It is straightforward to show that merger is profitable for the outsider firm if post-merger equilibrium mixed strategies are played. Premerger individual firm earns pure strategy equilibrium profits \( \Pi_i^e = p(3k)k \), whilst post merger outsider profits result \( \Pi_B^e = p_L k \). Since \( p_L > p(3k) \), for a mixed strategy equilibrium to be played, then outsider firm earns higher profits post merger. Profitability analysis of horizontal merger for the merged firm is less straightforward as it depends on its size vis-
à-vis market demand at $p_L$. If $p_L \leq p(2k)\ (D(p_L) > 2k)$, then equilibrium post merger profits read $\Pi^*_A = p_L 2k$. Comparing post and pre-merger profits $p_L 2k > 2p(3k)k$, the merger is profitable for the merged firm. On the contrary, if $p_L > p(2k)\ (D(p_L) < 2k)$, then merged firm’s profits are $\Pi^*_A = p_L D(p_L)$, since $p_L = \Pi^*_A / D(p_L)$. Merger impact is then non obvious and profitability for insiders needs further investigation.

**Proposition 4.4** If $\tilde{k}^* \geq k > \tilde{k}^*/2$, post-merger equilibrium is in mixed strategies whose equilibrium price range is strictly higher than premerger price. Thus, merger is profitable for insiders.

Proof: For the above reasoning, to show horizontal mergers profitability for insiders, it suffices to investigate the case in which $p_L > p(2k)$. Since $p(2k) > p(3k)$, if $p_L D(p_L) > p(2k)2k$, considered $p(2k)2k > p(3k)2$, then $p_L D(p_L) > p(3k)2k$, then the merger is profitable. Merged firm’s expected profits are also equal to $p_H(D(p_H) - k)$. Optimal choice $p_H$ for merged firm yields first order conditions $D(p_H) - k + p_H D'(p_H) = 0$, that is $D(p_H) + p_H D'(p_H) = k > 0$. The expression $D + pD'(p) = 0$ is the first derivative of the concave function $pD(p)$ in $p$, that is increasing only to the left of its maximum, i.e. $p < p_H$. This implies in turn, $p_L < p < p_H$ and $p_L > p(2k)$. Q.E.D.

If capacities are small enough, i.e. every firm produces at most its Cournot-Nash best response quantity, then an equilibrium in pure strategies arises in which firms charge a positive price and produce at full capacity. In this capacity configuration, capacities represent a quantity commitment for firms therefore capacity constraints are binding. Horizontal mergers can relax such constraints creating a bigger market player whose capacity is sufficiently high to play a mixed strategy. In mixed strategies an incentive to charge a higher price than the competitive one arises and quantities are randomized over a range below their actual capacity. Thus, if a mixed equilibrium arises post merger, industry-wide price increases. Furthermore, when mixed strategies arise post merger, horizontal mergers are profitable for both outsiders and insiders.

4.4. **Horizontal Mergers Analysis in Symmetric Triopoly: Premerger Mixed Strategy Equilibrium**

The proposed merger may involve any of the three symmetric firms, though we suppose 1 and 2 as merging firms and the third as unchanged outsider (post-merger firm A). This merger leads to a duopoly in which the merged firm (post-merger firm B) owns twice the capacity of its equally sized constituents. A mixed strategy equilibrium premerger occurs whenever $k > \tilde{k}^*$. A mixed strategy equilibrium post-merger occurs if $k > \tilde{k}^*/2$. Since $k > \tilde{k}^* > \tilde{k}^*/2$ then a mixed strategy equilibrium post-merger necessarily follows from a mixed strategy equilibrium pre-merger, in the assumptions of our model. Equilibrium mixed strategies before the takeover have the characteristics specified in the opening section. More specifically the supremum is defined as $p_M = \arg\max_p p(D(p) - 2k)$, which

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implies in turn \( D(p_M) > 2k \) and \( p_M < p(2k) \). This allows to state that the open set \((p_m, p_M)\) is a subset of \((p(3k), p(2k))\). The same properties hold for equilibrium mixed strategies post merger, as firms randomize their strategy over the post-merger price range equilibrium mixed strategy support \((p_L, p_H)\). Post-merger mixed strategies are defined accordingly to what already showed in section 4.2. Merged firm’s expected profits for any price \( p \in S_i \) are \( \Pi^*_A(p, \phi_B) = \phi_B p[D(p) - k] + (1 - \phi_B(p)) p \min(2k, D(p)) = p_H[D(p_H) - k] \); and the outsider’s expected profits as \( \Pi^*_B(p, \phi_A) = \phi_A \max \{D(p) - 2k, 0\} + (1 - \phi_A) pk = p_l k \). To analyze horizontal merger profitability it is necessary to investigate the effect the horizontal merger exerts on the extrema of equilibrium mixed strategies support. Post-merger price range extrema result higher than the pre-merger ones that implies mergers are profitable for both duopolists.

**Proposition 4.5** If \( k > \tilde{k} > \frac{k^*}{2} \) the supremum of the equilibrium mixed strategies’ support is lower than the post-merger one, i.e. \( p_M < p_H \).

**Proof:** Mixed strategies support suprema are \( p_H = \arg\max_p p(D(p) - k) \) while \( p_M = \arg\max_p p(D(p) - 2k) \), respectively pre- and post-merger. To obtain the result we need

\[
D(p) - k + p \frac{dD}{dp} \bigg|_{p=p_H} > D(p) - 2k + p \frac{dD}{dp} \bigg|_{p=p_M}
\]

that is

\[
D(p) + p \frac{dD}{dp} \bigg|_{p=p_H} - D(p) + k - p \frac{dD}{dp} \bigg|_{p=p_M} > 0.
\]

Rearranging yields, \( p \left( \frac{dD}{dp} \bigg|_{p=p_H} - \frac{dD}{dp} \bigg|_{p=p_M} \right) > -k \). Since \( 0 > -k \), then \( 0 > p \left( \frac{dD}{dp} \bigg|_{p=p_H} - \frac{dD}{dp} \bigg|_{p=p_M} \right) > -k \) only if \( p \left( \frac{dD}{dp} \bigg|_{p=p_H} - \frac{dD}{dp} \bigg|_{p=p_M} \right) < 0 \), that is true if and only if \( p_H < p_M \), due to concavity of demand function. Q.E.D.

Showing the supremum of equilibrium mixed strategies post-merger is greater than the premerger one is not sufficient to conclude that merger is profitable for insiders. Premerger each triopolist earns profits \( \Pi^*_t(p, \phi_{-i}) = \max_p [D(p) - 2k], \forall i \in \{1,2,3\} \). In order to be profitable for insiders, the merged entity’s profits, i.e. \( p_H[D(p_H) - k] \), must be greater than merging firms’ joint profits pre-merger, i.e. \( 2p_M[D(p_M) - k] = 2p_m k \).

**Proposition 4.6** If \( k > \tilde{k} > \frac{k^*}{2} \), the merger is profitable for the merged firm.

**Proof:** Notice that each firm’s profits increases as outsiders’ capacity decreases, i.e. \( d\Pi^*_i(p, \phi)/d \sum_{j \neq i} k_j = -p < 0 \). That is, if \( p(D(p) - k) > 2p(D(p) - 2k) \), it follows \( D(p) < 3k \).

That is true \( \forall p \) in both supports since in mixed strategies an incentive arises to charge a price higher than the competitive one, i.e. \( p > p(3k) \). Therefore, \( p_M(D(p_M) - k) > 2p_M(D(p_M) - 2k) \). Since \( p_H \) solves optimization of firm’s profits, it implies in turn, \( p_H(D(p_H) - k) > p_M(D(p_M) - k) > p_M(D(p_M) - 2k) \). This leads to \( \Pi^*_A > \Pi^*_t + \Pi^*_j \) for pre-merger, \( j \in \{1,2\} \). Q.E.D.
Hence, horizontal mergers when mixed strategy equilibria arise post-merger are profitable for insider firms, increasing their profits above premerger level.

**Proposition 4.7** If $k > \bar{k} > \bar{k}^*/2$, then $p_L > p_m$ and mergers are profitable for the outsider.

Proof: Note that $p_m = \Pi^*_A/k$ and $p_L = \Pi^*_A/\min\{2k, D(p)\}$. Two cases can be considered, either $p_L < p(2k)$ or $p_L > p(2k)$. In the first, $p_L = \Pi^*_A/k > p_m$, by Proposition 4.5 it is $\Pi^*_A > 2\Pi^*_A$. In the second, $p_L = \Pi^*_A/D(p_L) > \Pi^*_A/2k$ since $D(p_L) < 2k$. Thus, in both cases $p_L > p_m$ holds. Since postmerger outsider’s profits are $\Pi^*_B = p_L k$, then outsiders’ profits are higher than premerger. Q.E.D.

Both Proposition 4.5 and Proposition 4.7 show a rightwards shift of equilibrium mixed strategies supports post-merger. Both extrema post-merger are greater than the corresponding pre-merger ones, since the supremum prior to the merger is necessarily to the left of $p(2k)$, i.e. $p_m < p(2k)$. However, $p_L$ and $p_H$ can individually be on either sides of $p(2k)$, in the assumptions of the model. Post-merger mixed strategies profiles need to be obtained and characterized in order to derive the effect of horizontal merger on price level. The characteristics of equilibrium mixed strategies profile $(\phi_A(p), \phi_B(p))$ depend on the relative positions of $p_L$, $p_H$, and $p(2k)$ within the interval $(p(3k), p(k))$. Nevertheless, $(\phi_A(p), \phi_B(p))$ lie at the right of the pre-merger mixed strategy equilibrium profile, shifting accordingly to the shift of the supports. To find the post merger equilibrium mixed strategies we shall consider the condition of optimality of profits in mixed strategies, i.e. equilibrium profits are maximized and constant for any price on the support. Firms’ profits characterization post-merger follows from Kreps and Scheinkman (1983) for the asymmetric duopoly. Three cases differ according to the relative position of the three prices $p_L$, $p_H$ and $p(2k)$.

That is, $p_L > p(2k)$, $p_L < p(2k) < p_H$, and $p_H < p(2k)$; we shall study in turn.

If $p(2k) < p_L < p_H$ then $D(p_L) < 2k$. In this sub region of equilibrium mixed strategies firms’ expected profits are $\Pi^*_A = p_L D(p_L)$ and $\Pi^*_B = p_L k$, for the merged firm and the outsider, respectively. Expected equilibrium profits for firm B can be written as $\Pi^*_B(p, \phi_A) = (1 - \phi_A(p)) p k$. And substituting for $\Pi^*_B = p_L k$, merged firm’s equilibrium mixed strategy will be $\phi_A(p) = (p - p_L)/p$. Outsider’s equilibrium mixed strategy is $\phi_B(p) = (p D(p) - \Pi^*_1)/pk = (p D(p) - p_L D(p_L))/pk$.

If $p_L < p(2k) < p_H$ then $D(p_L) > 2k$. In this sub region of equilibrium mixed strategies firms’ expected profits are $\Pi^*_A = p_L 2k$ and $\Pi^*_B = p_L k$, for the merged firm and the outsider, respectively. This sub region requires the specification of two smaller subsets as equilibrium mixed strategies change if prices are randomized to the left or to the right of $p(2k)$. As price ranges within $[p_L, p(2k)]$, then demand satisfies the inequality $D(p_L) > D(p) > 2k$. To obtain the equilibrium
mixed strategies we write profits as \( \Pi_A = p_L 2k = p(D(p) - k)\phi_B + p 2k(1 - \phi_B) \) and \( \Pi_B = p_L k = p(D(p) - 2k)\phi_A + pk(1 - \phi_A) \). Thus, equilibrium mixed strategies pair is,

\[
\phi_A(p) = k(p - p_L)/p(3k - D(p)) \quad \text{and} \quad \phi_B(p) = 2k(p - p_L)/p(3k - D(p)). \tag{4.4}
\]

Analogously, as price ranges within \([p(2k), p_H]\), then \(2k > D(p) > D(p_H) > k\), meaning that in the relevant price range firm A can supply all forthcoming demand at relevant price \(p\). Thus, equilibrium mixed strategy pair is,

\[
\phi_A(p) = (p - p_L)/p \quad \text{and} \quad \phi_B(p) = [pD(p) - \Pi^*_1]/pk = [pD(p) - p_L 2k]/pk. \tag{4.5}
\]

The price ranges can be written as closed intervals since both \(\phi_A(p)\) and \(\phi_B(p)\) are continuous at \(p(2k)\). To show this we shall take the left limit of the probability distribution functions (4.6), and the right limit of (4.7).

For \(\phi_A(p)\) it is,

\[
\lim_{p \to p(2k)^-} \phi_A(p) = \lim_{p \to p(2k)^-} \frac{k(p - p_L)}{p(K - D(p))} = \frac{k(p(2k) - p_L)}{p(2k)(3k - D(p(2k)))} = \frac{(p(2k) - p_L)}{p(2k)}
\]

\[
\lim_{p \to p(2k)^+} \phi_A(p) = \lim_{p \to p(2k)^+} \frac{(p - p_L)}{p} = \frac{(p(2k) - p_L)}{p(2k)}
\]

Fig.4.1. Equilibrium Mixed Strategies Supports Pre and Post Merger
For \( \phi_B(p) \) it is,

\[
\lim_{p \to p(2k)^-} \phi_B(p) = \lim_{p \to p(2k)^-} \frac{2k(p - p_l)}{p(K - D(p))} = \frac{2k(p(2k) - p_l)}{p(2k)(K - D(p(2k)))} = \frac{2(p(2k) - p_l)}{p(2k)}
\]

\[
\lim_{p \to p(2k)^+} \phi_B(p) = \frac{p(2k)D(p(2k)) - p_l2k}{p(2k)k} = \frac{2(p(2k) - p_l)}{p(2k)}
\]

Finally, if \( p_l < p_H < p(2k) \) then, \( D(p_l) > 2k \). Each firm’s expected profits are \( \Pi^*_A = 2p_lk \) and \( \Pi^*_B = p_lk \). Solving for equilibrium mixed strategies yields \( \phi_A(p) = k(p - p_l)/p(3k - D(p)) \) and \( \phi_B(p) = 2k(p - p_l)/p(3k - D(p)) \).

To show horizontal mergers unilateral effects, pre- and postmerger equilibrium mixed strategies have to be compared. Equilibrium mixed strategy in the pre-merger symmetric game is \( \sqrt{(p - p_m)k}/p(K - D(p)) \).

**Proposition 4.8** In the mixed strategy region, \( k > \tilde{k} > \tilde{k}/2 \), post merger price distributions \( \phi_A(p), \phi_B(p) \) have strict first order stochastic dominance over pre-merger price distribution \( \phi_i(p) \). That is to say, pre-merger firms have strictly higher probability to sell below any particular price than either of the post merger firms does, except at \( p_H \). Consequently, merger is profitable for both merger insiders and outsider.

Proof (Sketch): To show unilateral effects we shall consider only price ranges in which the pre and post-merger distributions overlap. Following Kreps e Scheinkman (1983), outsider firm is smaller than the merged firm then \( \phi_B > \phi_A \). First, consider the price configurations \( p_l < p < p(2k) < p_H \) and \( p_H < p < p(2k) \). The mixed strategy profile depends on the value of \( p_l \). If \( p_l > p_M \) there is no overlap, as well as if \( p_l < p_M < p \) the relevant price \( p \) does not fall in the overlap. Thus, in both cases pre-merger mixed strategy \( \phi \) is stochastically dominated by post-merger distributions \( (\phi_A, \phi_B) \) in the relevant price range. Consider the price configurations \( p_m < p_l < p < p_M < p_H < p(2k) \) and \( p_m < p_l < p < p_M < p(2k) < p_H \). Pre and post merger equilibrium mixed strategies are \( \phi = \sqrt{(p - p_m)k}/p(3k - D(p)) \), \( \phi_A = k(p - p_l)/p(3k - D(p)) \), and \( \phi_B = 2k(p - p_l)/p(3k - D(p)) \), respectively.

It is straightforward to see that \( \phi_A < \phi^2 < \phi \) for any \( p \in (p_L, p_H) \), i.e. merged firm behaves less aggressively than any other pre-merger firm (undercutting price with lower probability). At the same time \( \phi_B = 2\phi_A < 2\phi^2 \leq \phi \), more precisely if \( \phi \leq 1/2 \) then \( \phi_B = 2\phi_A < 2\phi^2 = 2\phi \phi < \phi \), i.e. \( \phi_B \) is less than \( \phi \) at any price. If \( \phi > 1/2 \) comparing pre and post merger strategies is not immediate and involves the comparison between the slopes of the distributions of \( \phi_B \) and \( \phi \). It can be shown that the slope of \( \phi_B \) is everywhere greater than that of \( \phi \), in the overlap. That is \( \phi_B \), is everywhere smaller than \( \phi \) at any price level in the relevant price range. Second, consider the price
configurations $p_L < p(2k) < p < p_H$. The post merger mixed strategies are $\phi_A(p) = (p - p_L)/p$ and $\phi_B(p) = (pD(p) - p_L 2k)/pk$. Also in this price range there is no overlap between the distributions of $\phi_B$ and $\phi$; thus, $\phi$ is stochastically dominated by $\phi_B$. Third, consider the price configurations $p_L > p(2k)$. Post-merger mixed strategies are $\phi_A(p) = (p - p_L)/p$ and $\phi_B(p) = (pD(p) - p_L D(p_L))/pk$. No overlapping between the distributions produce the same results as the previous price configurations, i.e. $\phi_B$ stochastically dominates $\phi$. Q.E.D.

If mixed strategy equilibria hold pre and post merger, then horizontal mergers reduce competition among oligopolists and favor unilateral price increase by the merged and outsider firms. Equilibrium mixed strategy support post merger shifts rightwards, while the probability for firms to price at any price $p$ is lower than the probability to do the same pre-merger. Thus, price increases post merger. For the outsider firm the effect of the takeover is the same; note that the smaller firm is supposed to compete more aggressively undercutting with higher probability than the largest firm, due to its lower size. Nonetheless post merger the smallest firm increases its support’s infimum and undercut the rival with lower probability than it did pre-merger, relaxing the terms of competition.

4.5. Concluding Remarks

This basic triopoly model investigates merging as creating a new larger firm whose size determines the nature of equilibrium post-merger. If individual capacity is so big to supply all forthcoming demand at marginal cost, $k \geq D(0)$, then it must be $2k \geq D(0)$ and any merger has no real effect. The marginal cost pricing pure strategy equilibrium will be played by all the players both pre and post merger. If individual firms’ dimension do not allow to quote marginal cost as equilibrium price, $k < D(0)$, the merger of two firms might be enough to satisfy demand at marginal cost ($2k \geq D(0)$).

If the latter is true, then horizontal mergers transform pure strategy equilibrium into a mixed strategy one. If single capacity satisfies the inequality $D(0) > k > D(0)/2$ then both the merged and the outsider firm play a mixed strategy, quoting a strictly positive price on the market. Thus, industry-wide price increases and the horizontal merger allows merging firms to increase profits. Whenever $k \leq \tilde{k}^*/2$ firms’ capacity constraints are binding both pre- and postmerger and horizontal mergers do not alter the pure strategy equilibrium $p^c = p(3k)$. If individual size is not so small, i.e. $\tilde{k}^* \geq k > \tilde{k}^*/2$, then post-merger equilibrium is in mixed strategies. Firms randomize over a price range whose lower bound is strictly greater than the unique pre-merger equilibrium price and earn higher profits than the competitive pre-merger ones.

50 The full proof, upheld by intuitive graphical analysis, is referred to Gang Li (2013). Besides the formal proof, we shall consider the qualitative effects of horizontal merging between two of the three rival firms.
The mixed strategy equilibrium pre-merger arises in the intermediate region of capacities, i.e. $k > \tilde{k}^*$. The post-merger equilibrium will necessarily be in mixed strategies and firms mix their price offers over a higher price range. Unilateral effects do not only account for a higher price post-merger, since the takeover affects post-merger price distributions as well. Post-merger merger firms undercut price with lower probability than pre-merger, loosening the terms of competition. With three symmetric price setters a merger to duopoly becomes profitable for insiders firms, if mixed strategy equilibrium prevails post-merger. Contrarily to the predictions of the Salant et al. (1983) result, whose symmetric treatment stems from the assumption of quantity competition, linear demand and constant, identical marginal and average cost, a merger to duopoly in a three-firm price-setting market is ultimately profitable.

\[ p_m = \arg\max_p p(D(p) - 2k) \text{ and } p_m = \Pi_l' / k \]

---

**Figure 4.2.** The Nature of Post-Merger Equilibrium as a Function of Capacity in the Symmetric Triopoly Model
5. Horizontal Mergers Theoretical Analysis in Symmetric Oligopolies à la Bertrand-Edgeworth

In this chapter we extend the previous chapter triopolistic industry analysis to horizontal mergers in symmetric oligopoly. We characterize formally horizontal combinations in a symmetric oligopoly to present a taxonomy of the takeover effects depending on the region of capacity in which the equilibrium lies. The merged firm still constitutes as a larger firm whose size is a proper multiple of merging firms’ capacities. In the symmetric oligopoly model the study of horizontal mergers among \( m \) symmetrically capacity-constrained price setting firms differs from the analysis of triopoly according to the relative values of \( n, m \) and \( k \). The dimensional characteristics of the pre-merger competitors and the merged firm characterize the equilibrium mixed strategies differently from the previous chapter analysis. The relative characteristics of merged firm and the remainder of the industry allow writing different equilibrium profiles and a richer taxonomy of horizontal merger effects.

Then, the chapter also investigates a more specific example of oligopoly with linear demand as it is characterized in Davidson and Deneckere (1984 and De Francesco and Salvadori (2010a). Assuming linear demand and efficient rationing, Davidson and Deneckere (1984) provide a full characterization of equilibria when capacities fall in the mixed strategies region, restricting attention to symmetric equilibria. De Francesco and Salvadori (2010a) deepen the earlier Davidson and Deneckere (1984) contribution and present a finer partition of the mixed strategy region. Davidson and Deneckere (1984) find that mergers are never disadvantageous in the static price game, profits increase in the size of the coalition of the merged and largest firm for all values of the size of the smallest firms (symmetric outsiders). Merged firm constitutes as an attempt to earn monopoly profits by the coalition, in this way it may successfully raise industry prices, benefiting outsiders that even keeping their price, sell more and earn higher profits. Thus, merger effects are the same as in the triopoly model. Mixed strategy equilibria allow firms to earn higher post-merger profits, by randomizing over a price range that is strictly greater than the pre-merger price.

We will study and deepen the Davidson and Deneckere (1984) contribution to assess merger profitability and unilateral price effects in the same linear demand function oligopoly. The section 1 lists the assumptions and provides the equilibrium characteristics, both in pure and mixed strategies for the general symmetric oligopoly. Section 2 investigates the effects of horizontal mergers in a model in which firms’ capacities allow pure strategy equilibria à la Bertrand pre-merger. Section 3 investigates the effects of horizontal mergers in a pre-merger pure strategy equilibrium, in which firms charge the aggregate capacity-clearing price. Section 4 studies mergers in a linear demand oligopoly à la Davidson and Deneckere (1984).
5.1. Preliminaries (in oligopoly)

Suppose \( n \) identical firms \( i \in I = \{1, ..., n\} \) with \( n \geq 3 \), compete in the market and no entry or exit is allowed in the model. Equally sized oligopolists produce a homogeneous good under constant, and null, marginal and average cost up to capacity. Neither installation costs of the existing capacity nor fixed costs are considered in this analysis. Firms name their price simultaneously and non-cooperatively. We assume the symmetric capacity case, i.e. \( k_1 = k_2 = \cdots = k_n = k > 0 \), all firms own the same capacity and we define total capacity as \( K = \sum_i^n k_i = nk \). Consumers preferences determine the demand function \( X = D(p) \), where \( X \) is market demanded quantity and \( p \) market price. Market demand is positive for \( p \in (0, \bar{p}) \), where if \( p > \bar{p} \), then \( D(p) = 0 \). Furthermore, demand function is assumed continuous, strictly decreasing and such that \( pD(p) \) is strictly concave for \( p \in (0, \bar{p}) \). For some propositions demand function can be assumed concave in \( \bar{p} \). Conversely, the inverse demand function is defined for the decreasing part of the demand function such as \( p = D^{-1}(X) = X(p) \), in \( X \in (0, D(0)) \). Any rationing of the forthcoming demand by the capacity-constrained firms follows the efficient rule, whose characteristics we refer to the third chapter.

According to the assumptions on individual and industry capacity, the simultaneous move game has an equilibrium either in pure or mixed strategies. Any pure strategy equilibrium can be found as a straightforward generalization of the result in duopoly. Whenever firms have a sizeable dimension, i.e. \( k \geq D(0), \forall i \in I \), in every Nash equilibrium at least two firms set marginal cost pricing, while the rivals charge whatever price not lower than marginal cost. Whenever two firms price at marginal cost, each firm of the \( n - 2 \) firms left will be indifferent on charging any other price at least equal to \( c \), since it would still earn null profits. At the same time, each of the \( j \) firms pricing at marginal cost, has no incentive to price differently. Pricing higher, as well as pricing lower, would gain respectively either null or negative profits to the firm, and do not represent equilibrium positions. Uniqueness argument follows previous chapter’s analysis. Besides the previous extreme assumption on single firms’ size that resembles Bertrand’s analysis, in oligopoly a pure strategy equilibrium at which firms charge null marginal cost might arise if the combined capacity of all the firms but the largest one can supply the forthcoming demand at marginal cost. In the symmetric player game it suffices that the combined capacity of all firms but one to supply forthcoming demand at marginal cost for a pure strategy equilibrium à la Bertrand to arise. Thus, the \( n \)-tuple \( p_i = 0 \ \forall i \) is a pure strategy equilibrium if \( (n - 1)k \geq D(0) \).

A unique positive pure strategy equilibrium arises for other capacity configurations, as well. If \( K < D(0) \), firms are in fact capacity constrained. If firms’ capacity equals at most their best response function, i.e. \( k \leq -p(nk)[D'(p)]_{p=p(nk)} \) then a pure strategy equilibrium arises. Firms are
sufficiently small and charge the price that clears aggregate capacity (competitive price \( p(nk) \)). The conditions for the existence of a pure strategy equilibrium can be summarized as follows

\[
k \leq \max \left\{ (n - 1)k - D(0), -p(nk)[D'(p)]_{p=p(nk)} \right\}
\]  

(5.1)

Inequality (5.1) states that any capacity-constrained symmetric competitor (pre-merger) must be sufficiently small with respect to industry capacity, in order for a pure strategy equilibrium to exist in capacity-constrained oligopoly. Whenever inequality (5.1) is violated, a pure strategy equilibrium does not exist while mixed strategy equilibria do. That is to say that a mixed strategy equilibrium exists when \((n - 1)k < D(0)\) if capacity satisfies the inequality

\[
k > -p(nk)[D'(p)]_{p=p(nk)}
\]  

(5.2)

where \( \hat{k} = f(K) = -p(nk)[D'(p)]_{p=p(nk)} \). The capacity threshold above which firms are allowed to randomize over their price offers is not explicitly defined; as \( k \) appears on the left hand side as well as the right hand side of inequality (5.2). In the following proposition we formalize the result for the capacity threshold \( \hat{k}^* \) as follows

**Proposition 5.1** There exists a unique value of capacity \( \hat{k}^* \) such that a mixed strategy equilibrium exists if \( k > \hat{k}^* \).

Proof: We shall recall the Cournot-Nash best response function, take the first-order conditions, and obtain the relation between \( k_i = k, \forall i \) and industry capacity \( K = k_i + \sum_{j \neq i} k_j = nk \) differentiating both sides in total capacity, analogously to Proposition 4.1. Differentiation, given the assumptions on concavity of demand function, yields \( dk_i/dK = -P'(k_i + \sum_{j \neq i} k_j)D'(k_i + \sum_{j \neq i} k_j) - P(k_i + \sum_{j \neq i} k_j)D''(k_i + \sum_{j \neq i} k_j) \leq 0 \). The shape of the relation depends upon the shape of demand function. The capacity threshold \( \hat{k}^* \) is the value that identifies in the \((k, K)\)-space the intersection between the negatively sloped function \( \hat{k} = f(K) \) and the positively sloped ray through the origin \( k = K/n \). An intersection exists since \( dk_i/dK = -[1 + P(K)D'(K)/D'(K)] \leq 0 \). Thus, \( 1 + P(K)D'(K)/D'(K) \geq 0 \), and \( P(K)D''(K)/D'(K) \geq -1 \). This implies the slope of \( \hat{k} = f(K) \) is finite. Hence, a mixed strategy equilibrium exists if \( k > \hat{k} \) that is true if and only if \( k > \hat{k}^* \). Q.E.D.

In the mixed strategy region an equilibrium always exists satisfying the sufficient conditions for existence of Theorem 5 in Dasgupta and Maskin (1986a). We follow Vives (1986) to characterize mixed strategy equilibrium profiles for the symmetric oligopoly. We let the mixed strategy equilibrium profile follow the specification of the previous chapter. The definitions of \( p_m \) and \( p_M \), the infimum and supremum of \( S_i = S_i(\phi_i(p)) \), the support of equilibrium mixed strategies, follow
accordingly. Hence, inequality (5.2.) can be substituted with \( p(K) < p_m \) to address the mixed strategy region. Indeed, if \( k \leq K - D(0) \) then \( p_m = P(K) = 0 \); whereas, if \( k \leq -p(nk)[D'(p)]_{p=p(nk)} \) then \( p_m = p_M = P(K) \geq 0 \). Finally, note that in the region where inequalities (5.2) hold the extrema of equilibrium mixed strategies supports are defined \( p_M = \arg\max_p p(D(p) - (n - 1)k) \), \( p_m = \Pi_i^* / k \), where \( \Pi_i^* \) refers to the profits of the largest firm (in this case any symmetric competitor pre-merger). Equilibrium mixed strategies of all firms are assumed symmetric, i.e. all firms adopt the same equilibrium strategy, \( \phi_i(p) \), for \( \forall i \in I \). This assumption allows us to write firm \( i \) pre-merger equilibrium mixed strategy as in Vives (1986).

5.2. **Horizontal Mergers Analysis in Symmetric Oligopoly: Premerger Pure Strategy Equilibrium à la Bertrand**

The investigated merger would take place among \( m \) symmetric firms, leaving outside at least two firms, in order to consider a more general case than the previous triopoly model where only one took the outsider role. Therefore, the proposed horizontal mergers would involve any \( m \geq 2 \) merging firms in a market of \( n \geq 4 \) firms. This merger leads to an oligopoly in which the merged firm (post-merger firm A) owns \( m \)-times the capacity of its equally sized constituents. Any symmetric firm is referred to as firm B in the post-merger setting. Assume the pre-merger equilibrium is a pure strategy equilibrium in which firms charge a null competitive price and earn zero profits accordingly.

Horizontal mergers might either leave firms play the pure strategy null equilibrium price or create a larger market player whose size allows to quote a higher price on the market than the competitive one, i.e. the equilibrium turns to mixed strategies. Individual firms’ capacity may either be big enough to supply all forthcoming demand at marginal cost or not. If they are of considerable dimension a pure strategy equilibrium arises in which firms set the null marginal cost pricing. When single firms cannot supply all forthcoming demand, horizontal mergers might either create a coalition whose size still supplies the whole forthcoming demand at marginal cost; or create an oligopoly in which size asymmetries between the merged firm and the other symmetric players create an incentive to quote a higher price than marginal cost. Postmerger pure strategy (null) marginal cost price equilibrium arises if \( K - k_A \geq D(0) \), for \( D(0) \leq K \). Since \( k_A = mk \) then postmerger pure strategy equilibrium arises if \( (n - m)k \geq D(0) \), i.e. if \( k \geq D(0)/(n - m) \). That is, pure strategy equilibrium arises both pre- and postmerger if \( k \geq D(0)/(n - m) \geq D(0)/(n - 1) \), since the merger involves more than two firms. For this region of capacity horizontal mergers do not affect neither industry-wide price nor single firms’ profits since all competitors charge marginal cost price and earn zero profits.
If individual firms’ capacity is not big enough to supply all forthcoming demand at marginal cost, then the horizontal merger might create an oligopoly in which size asymmetries between the merged firm and the remainder of the industry create an incentive to quote a higher price than marginal cost. A pure strategy post-merger equilibrium in which firms charge the null marginal cost fails to exist if \((n - m)k < D(0)\). Thus, the post-merger size configuration for the individual capacity to allow for mixed strategies follows \(D(0)/(n - m) > k \geq D(0)/(n - 1)\). This allows for different pricing incentives of the oligopolists. In mixed strategies, oligopolists mix their price offers in the post merger price interval \((p_L, p_H)\). Given this capacity configuration, contrarily to the three-firm market in which the merged firm could supply all the forthcoming demand in post-merger mixed strategies, the m-firm merger creates a sizeable market player whose dimension still constraints its profits. The extrema of the support of the mixed strategies are defined as in the third chapter. Given the assumptions on firms’ dimensions, we can write \(p_H = \arg\max_p p(D(p) - (n - m)k)\) and \(p_L = \max \{\bar{p}, \bar{p}\}\), where we define \(\bar{p} = \Pi^I/k\) and \(\bar{p}\) the lower solution of the equation \(\max_p p(D(p) - k) = p D(p)\). In turn, \(p_L = \bar{p}\) if \(D(p_L) > ka\) or \(p_L = \bar{p}\), otherwise. Outsiders’ equilibrium profits reads \(\Pi^*_B = p_L k\). Merged firm’s equilibrium mixed strategy \(\phi_A\) will be continuous and increasing over \((p_L, p_H)\), due to concavity of profit function, whilst outsider’s \(\phi_B\) will be continuous and increasing over \((p_L, p_H)\). Since the merged firm charges the supremum of equilibrium supports with positive probability then its mixed strategy \(\phi_A\) everywhere dominates \(\phi_B\) over \((p_L, p_H)\). This acknowledged property of duopoly points to the different pricing incentives of the differently sized oligopolists: the smallest firms charge a lower price with higher probability given the large size of the rival.

**Proposition 5.2** If \(D(0)/(n - m) > k \geq D(0)/(n - 1)\), post merger equilibrium is in mixed strategies where firms charge a strictly positive price within the mixed strategies support. Thus, the merger is profitable for the merging parties and the outsiders.

Proof: Horizontal merger is profitable for any outsider if \(\Pi^*_B = p_L k > 0\), given null pre-merger profits any horizontal merger that allows the outsider to play a mixed strategy is profitable. Analogously, merged firm’s profits either \(\Pi^*_A = p_L D(p_L)\) or \(\Pi^* = p_L mk\) are both strictly positive and horizontal mergers are profitable for insiders. Q.E.D.

Although it is immediate to notice that the horizontal merger leads to a price increase from the null competitive equilibrium price pre-merger, we are left defining the equilibrium mixed strategies of the two firms’ dimensions post-merger. Any outsider is assumed to play the same mixed strategy post-merger. We shall consider as equilibrium mixed strategies those probability distributions that maximize expected profits at each price level \(p \in S_i\) when the rival plays the equilibrium mixed strategy. Equilibrium profits for an asymmetric oligopoly are the ones found as a straightforward
The generalization of Kreps and Scheinkman’s (1983) results in a duopoly. The equilibrium mixed strategies are found in the specific case in which the largest firm has a size equal to a proper multiple of smaller firms’ size. Profits shall be maximized and constant along the mixed strategies support.

The characterization of dimensional characteristics in this oligopoly determines the different mixed strategy profiles. Given the premerger capacity configuration \((n-1)k \geq D(0)\) that implies \((n-1)k > D(p), \forall p > 0\) and more specifically \((n-1)k > D(p_L)\). Thus, \(p_L > p((n-1)k)\) holds. Mixed strategy profiles depend on the capacity of the merged firm, whether or not it can supply all forthcoming demand at \(p_L\). Mixed strategy are sensitive to the different capacity setting. Conditional upon the merged firm’s dimension with respect to the remainder of the industry, the following inequalities hold: either \((n-1)k > mk > (n-m)k > k\) or \((n-1)k \geq (n-m)k \geq mk > k\). If \(m \geq n/2\) then the market is dimensionally characterized by the first chain of inequalities; by the second otherwise. Accordingly, we are allowed to write either

I. \(p(mk) < p((n-m)k)\), or

II. \(p((n-m)k) \leq p(mk)\).

We shall also consider the mixed strategies support upper bound \(p_H\). Since \(p_H = \arg\max_p p(D(p) - (n-m)k)\) then \(D(p_H) \geq (n-m)k\), that implies \(p_H \leq p((n-m)k)\). Hence, in configuration (I)

\[
\begin{align*}
\text{a)} & \quad p_L \leq p(mk) < p_H \leq p((n-m)k); \\
\text{b)} & \quad p(mk) \leq p_L < p_H \leq p((n-m)k); \\
\text{c)} & \quad p_L < p_H \leq p(mk) < p((n-m)k).
\end{align*}
\]

In configuration (II) it holds \(p_L < p_H < p((n-m)k) \leq p(mk)\).

If capacity configuration is the (I.a) one, i.e. \(p_L < p(mk) < p_H\) then \(D(p_L) > mk > D(p_H)\). In this sub region of equilibrium mixed strategies firms’ expected profits are \(\Pi^*_A = p_L mk\) and \(\Pi^*_B = p_I k\), for the merged firm and the outsider, respectively. This sub region requires the specification of two smaller sub sets as equilibrium mixed strategies change if prices are randomized to the left or to the right of \(p(mk)\). As price ranges within \([p_L, p(mk)]\), then demand satisfies the inequality \(D(p_L) > D(p) > mk\). The symmetric outsider profits can be written taking in consideration that a symmetric outsider can undercut the merged firm, the other symmetric competitors, or both; or rather price higher than any other symmetric competitor and the merged firm. The expected profit formula follows from the previous chapter. Therefore, merged firm’s equilibrium mixed strategy \(\phi_A(p) = \frac{1}{m} \sqrt{\frac{mk(p-p_L)}{p(nk-D(p))}}\), as in De Francesco and Salvadori (2010b). The mixed strategy profile stems from the adherence of the mixed strategy region characteristics in sub region 3Bc with the one above.
Exploiting the equilibrium profits for the merged firm $\Pi_A(p, \phi_B) = p \min\{\max\{0, D(p) - (n-m)k\}, mk\} \phi_B^{n-m} + p \min\{D(p), mk\}(1 - \phi_B^{n-m})$ the equilibrium mixed strategy for any symmetric outsider can be written as follows $\phi_B(p) = \frac{\frac{n-m}{m} \frac{mk(pL-p)}{p(D(p)-k)}}{\sqrt{D(p)-k}}$ in $[p_L, p_H]$.

Analogously, as price ranges within $[p(mk), p_H]$, then $mk > D(p) > D(p_H) > k$, meaning that in the relevant price range firm A can supply all forthcoming demand at relevant price $p$. Thus, the small equally-sized firms can sell a positive output only undercutting the large one. That is, firms no longer set simultaneously their strategies. For firm B profits are $\Pi_B(p, \phi_A) = pLk = (1 - \phi_A(p))pk$. Accordingly, $\phi_A(p) = \frac{(p - p_L)}{p}$. The system $\Pi_A = \Pi'_A(p, \phi_{-A})$ has a degree of freedom in the set of equilibrium mixed strategies of the outsider firms $\phi_{-A}$. Therefore, small symmetric firms are not uniquely defined. Assuming a symmetric equilibrium, i.e. symmetric firms play the same mixed strategy, any outsider’s equilibrium mixed strategy can be written as $\phi_B(p) = \frac{[pD(p) - \Pi'_A]}{pk} = \frac{[pD(p) - pLmk]}{(n-m)pk}$. Similarly to what already done for the reference triopoly case, price intervals are considered closed since both distributions are continuous at $p(mk)$.

Taking the limit as it has been done in the previous chapter shows this. If capacity configuration is the (I.b) one, i.e. $p(mk) < p_L < p_H < p((n-m)k)$ that is in terms of demand $D(p_L) < mk$. In this sub region of equilibrium mixed strategies firms’ expected profits are $\Pi_A = pL D(p_L)$ and $\Pi_B = pLk$, for the merged firm and any outsider, respectively. Expected equilibrium profits for firm B can be written as $\Pi'_B(p, \phi_A) = pLk = (1 - \phi_A(p))pk$. Merged firm’s equilibrium mixed strategy profile is $\phi_A(p) = \frac{(p - p_L)}{p}$. Assuming a symmetric behavior for the small outsider firms, any symmetric firm’s equilibrium mixed strategy profile can be written $\phi_B(p) = \frac{(pD(p) - pL D(p_L))}{(n-m)pk}$.

If capacity configuration is either the (I.c) or the (II) the same profile of strategies can be written, since in both subregions the inequality $p_L < p_H < p(mk)$ holds. In terms of demand $D(p_L) > D(p_H) > mk$. Each firm’s expected profits are $\Pi_A = mpLk$ and any outsider’s $\Pi'_B = pLk$. Solving for equilibrium mixed strategies yields $\phi_A(p) = \frac{1}{m} \frac{n-m}{m} \frac{mk(p_L-p)}{p(nk-D(p))}$ and $\phi_B(p) = \frac{n-m}{m} \frac{mk(pL-p)}{\sqrt{p(D(p)-k)}}$, in $[p_L, p_H]$ the same distributions as in capacity configuration (I.a).

We have shown in this first section that horizontal mergers among capacity unconstrained firms have no real effect on both firms and consumers. Whenever the single capacity of market rivals is so high to satisfy the whole forthcoming demand, the equilibrium recreates the standard Bertrand result both pre and post merger. If firms are not individually so large, but the collective capacity of all but one is enough to set price at marginal cost then mergers allow either to charge the pure strategy marginal cost price or randomize firms’ price offer on a strictly positive price range. If pure strategy equilibria arise post-merger then the horizontal aggregation has no real effect. If horizontal takeovers allow competing firms to play mixed strategies in the post-merger oligopoly, firms quote a higher price.
than the competitive one raising profits for both the merged firm and any symmetric outsider, from the null pre-merger ones.

5.3. **Horizontal Mergers Analysis in Symmetric Oligopoly: Premerger Positive Price Pure Strategy Equilibrium**

Assume the individual capacity of the pre-merger firms allow a pure strategies equilibrium, i.e. $k \leq \hat{k}^*$. Pre-merger equilibrium price is $p(nk)$ and each symmetric firm earns $\Pi_i^* = p(nk)k$, for $i = \{1, \ldots, n\}$. A horizontal merger among $m$ firms affect equilibrium strategies post merger, as it might either leave firms play pure strategies or create a larger market player whose size allows to quote a higher price on the market than the competitive one, i.e. the equilibrium would be in mixed strategies. The merged firm is hereby indexed as firm A and the generic symmetric small-sized player is indexed B. In order to derive post merger equilibrium capacity threshold for the existence of pure strategy equilibrium we recall inequality (5.1) as follows

$$k_1 \leq -p^c[D'(p)]_{p=p^c} \equiv \hat{k} \text{ if } K \leq D(0)$$

and reconsider pure strategy equilibrium requirements at the light of the post merger capacity changes. Merger creates a larger firm, whose size is a proper multiple of the individual capacity of the merging firms, i.e. $k_A = mk$. Merger alters residual demand and modifies outsider competitors’ capacities from $(n-1)k$ to $(n-m)k$. Respectively, $(n-1)k$ indicates aggregate competitors’ capacity that each individual firm faces pre-merger and $(n-m)k$ the outsider’s capacity post-merger. Hence, threshold for a pure strategy equilibrium changes accordingly. Inequality (5.3) identifies the region in which a mixed strategy equilibrium exists if $K < D(0)$. The capacity threshold above which firms are allowed to randomize over their price offers is not explicitly defined; as $k$ appears on the left hand side as well as the right hand side of inequality (5.5). In the following proposition we formalize the result for the post-merger capacity threshold $\hat{k}^{**}$ as follows

**Proposition 5.3** There exist a unique value of capacity $\hat{k}^{**}$ such that a pure strategy equilibrium exists if $k \leq \hat{k}^{**}$, that holds if $k \leq \hat{k}^*/m$.

Proof: We shall recall the Cournot-Nash best response function, take the first-order conditions and rearranging similarly to what has been done in section 5.2. We obtain the relation between $k_i = k_i, \forall i$ and industry capacity $K = k_i + \sum_{j \neq i} k_j = nk$ differentiating both sides in total capacity. Differentiation and concavity of demand function assumptions yield $dk_i/dK = -P(k_i + \sum_{j \neq i} k_j)D'(k_i + \sum_{j \neq i} k_j) - P(k_i + \sum_{j \neq i} k_j)D'(k_i + \sum_{j \neq i} k_j) \leq 0$. The shape of the relation is not yet defined as it depends upon the shape of demand function. Thus, the capacity threshold $\hat{k}^{**}$ is
the value that identifies in the \((k, K)\)-space the intersection between the negatively sloped function 
\( \hat{k} = f(K) \) and the positively sloped ray through the origin \( k = m/nK \). The intersection exists since the slope of \( \hat{k} (K) \) is finite and ranging between \((-1,0)\). In terms of total capacity \( \hat{k}^* \) is obtained as \( \hat{k}^* = m/n K = \hat{k} (K) \). Post-merger threshold must be lower than pre-merger capacity threshold \( \hat{k}^* \leq m \hat{k}^* \). Thus, a pure strategy post-merger equilibrium is played if \( k \leq \hat{k}^*/m \). Q.E.D.

If inequality \( k \leq \hat{k}^*/m \) holds, the equilibrium postmerger will be played in pure strategies and firms will charge the aggregate capacity-exhausting equilibrium price \( p(nk) \). Post-merger profits for the merged firm are \( m \)-times the individual constituents’ profits pre merger \( \Pi_i^* \), i.e. \( \Pi_A^* = p(nk)mk = m\Pi_i^* \). Joint profits are unchanged pre- and post merger, as well as outsider’s profits. Though clearly profitable for the merged firm, the takeover does not alter profits of all competitors. Thus, horizontal mergers in pure strategy equilibria do not have real effects on either firms or consumers.

If individual post-merger capacity \( k \in (\hat{k}^*/m, \hat{k}^*) \), then horizontal mergers lead to a mixed strategy equilibrium. Similarly to the three-firm market, oligopolists will mix their price offers in the interval \((p_L, p_H)\), a subset of either the price interval \( (p(nk), p((n - m)k)) \). The supremum of the post merger price range is determined by the largest firm, monopolizing residual demand as the smallest firm charges a lower price, i.e. \( p_H = \arg\max_p p[D(p) - (n - m)k] \). Largest firm will earn profits \( \Pi_A^* = p_H(D(p_H) - (n - m)k) \). The infimum is set by the larger firm earning profits \( \Pi_A^* \). Then, \( p_L = \max \{\hat{p}, \bar{\hat{p}}\} \), where we define \( \hat{p} = \Pi_i^*/k \) and \( \bar{\hat{p}} \) as the lower solution of equation \( \Pi_A^* = pD(p) \).

In turn, \( p_L = \hat{p} \) if \( D(p_L) > mk \) or \( p_L = \bar{\hat{p}} \), otherwise. It is straightforward to show that the merger is profitable for the outsider firm if post-merger equilibrium mixed strategies are played. Pre-merger individual firm earns pure strategy equilibrium profits \( \Pi_i^* = p(nk)k \), whilst post-merger outsider profits result \( \Pi_B^* = p_L k \). Since \( p_L > p(nk) \), for a mixed strategy equilibrium to be played, then outsider firm earns higher profits post merger. Profitability analysis of horizontal mergers for the merged firm is less straightforward as it depends on its size vis-à-vis market demand at \( p_L \). If \( p_L \leq p(mk) (D(p_L) > mk) \), then equilibrium post merger profits read \( \Pi_A^* = p_L mk \). Comparing post and pre merger profits \( p_L mk > mp(nk)k \), the merger is profitable for the merged firm. On the contrary, if \( p_L > p(mk) (D(p_L) < mk) \), then merged firm’s profits are \( \Pi_A^* = p_L D(p_L) \), since \( p_L = \Pi_A^*/D(p_L) \). Thus, horizontal merger need further investigation and the consequences are non obvious.

**Proposition 5.4**  If \( \hat{k}^* \geq k > \hat{k}^*/m \), post-merger equilibrium is in mixed strategies where the range of equilibrium prices is strictly higher than the premerger price. Thus, the merger is profitable for the merging parties.
Proof: For the above reasoning, to show horizontal mergers profitability for insiders, it suffices to investigate the case in which \( p_i > p(mk) \). Since \( p(mk) > p(nk) \), if \( p_i D(p_L) > p(mk) mk \), considered \( p(mk) mk > p(nk) mk \), then \( p_i D(p_L) > p(mk) mk \), then the merger is profitable. Merged firm’s expected profits are also equal to \( p_H D(p_H) - (n - m) k \). Optimal choice \( p_H \) for merged firm yields first order conditions \( D(p_H) - (n - m) k + p_H D'(p_H) = 0 \), that is \( D(p_H) + p_H D'(p_H) = (n - m) k > 0 \). The expression \( D + pD'(p) = 0 \) is the first derivative of the concave function \( pD(p) \) in \( p \), that is increasing only to the left of its maximum, i.e. \( p < p_H \). This implies in turn, \( p_L < p < p_H \) and \( p_L > p(mk) \). Q.E.D.

This symmetric oligopoly model produces the same results as the triopoly one, whose findings can be extended straightforwardly. If mixed equilibria prevail post merger, equilibrium profits are higher for both merging firms and outsiders. Horizontal mergers lead to price increase since the infimum of the mixed strategies support is higher than the pre-merger pure strategy equilibrium.

5.4. **Horizontal Mergers Analysis in Symmetric Oligopoly: Premerger Mixed Strategy Equilibrium**

Firms play mixed strategy equilibrium profiles, if \( k > \tilde{k}^* \). To a mixed strategy equilibrium pre merger necessarily follows a mixed strategy equilibrium post merger, since \( k > \tilde{k}^* > \tilde{k}^*/m \). We can simply consider the effect of the change in the rivals’ total capacity to observe post-merger changes. In the standard symmetric model outsiders’ capacity is only \( (n - 1)k \), reduced to \( (n - m) k \) post merger, since \( (n - 1)k > (n - m) k \), so that residual demand is higher. Outsiders’ capacity affect price negatively, reducing residual demand. The supremum of mixed strategies support increases as residual demand decreases, i.e. outsiders’ aggregate capacity is lower, as \( \frac{dp_H}{d\sum_{k-i}} = \frac{1}{2D + pD^*} < 0 \).

Intuitively, reduced outsiders’ capacity exerts a weaker competitive externality on the merged firm. We shall see how it affects equilibrium conditions in the remainder of the section. Post-merger equilibrium in mixed strategies resembles the one already seen. Supremum of mixed strategy equilibria is defined \( p_H = \operatorname{argmax}_p pD(p) - (n - m) k \), infimum \( p_L = \Pi^*_A/mk \).

**Proposition 5.5** If \( k > \tilde{k}^* > \tilde{k}^*/m \), the extrema of the mixed strategies support are greater than their respective pre-merger values and the merger is profitable for all firms.

Proof: Following the reasoning of inequality (5.6) we can conclude \( p_H > p_M \). Since capacities are defined as we have seen, \( p(nk) < p_L \). Thus, \( pD(p) - (n - m) k > m pD(p) - (n - 1)k \). That leads to, \( p_H D(p_H) - (n - m) k > p_M D(p_M) - (n - m) k \), and finally \( p_H D(p_H) - (n - m) k > m p_M D(p_M) - (n - 1)k \). The merged firm profits are greater than those of the joint
profits of the \( m \) pre-merger symmetric firms. This implies, \( p_L m k > p_m m k \), or equivalently \( p_L > p_m \). That is to say outsiders earn higher equilibrium profits, charging a higher post-merger price. Q.E.D.

Similarly to section 4.3 equilibrium strategies depend on the position of \( p((n-1)k) \) with respect to the post-merger extrema \( p_L \) and \( p_H \). The dimensional characteristics of the post-merger mixed strategy region conditional upon the merged firm’s dimension with respect to the remainder of the industry can be written either

I. \((n-1)k > mk > (n-m)k > k \) if \( m \geq n/2 \) or
II. \((n-1)k \geq (n-m)k \geq mk > k \), otherwise.

Accordingly, we are allowed to write the following partitioning in capacity configuration (I)

a. \( p_L \leq p((n-1)k) \leq p(mk) < p_H \leq p((n-m)k) \);
b. \( p((n-1)k) \leq p_L < p(mk) < p_H \leq p((n-m)k) \);
c. \( p((n-1)k) < p(mk) \leq p_L < p_H \leq p((n-m)k) \);
d. \( p((n-1)k) < p_L < p_H < p(mk) < p((n-m)k) \);
e. \( p_L \leq p((n-1)k) < p_H < p(mk) < p((n-m)k) \);
f. \( p_L < p_H < p((n-1)k) < p(mk) < p((n-m)k) \).

And finally, we are allowed to write the following partitioning in capacity configuration (II)

a. \( p_L \leq p((n-1)k) < p_H \leq p((n-m)k) \leq p(mk) \);
b. \( p((n-1)k) < p_L < p_H \leq p((n-m)k) \leq p(mk) \);
c. \( p_L < p_H < p((n-1)k) < p((n-m)k) \leq p(mk) \).

For the characteristics of the dimension of the merged firm, if inequalities (I.b) or (I.c) hold, and the one in (II.b), the profile of equilibrium mixed strategies follows those of section 5.2. In the following proposition, the logic is the same as the one underlying Proposition 4.8 and the results (though partial) are the same. Post-merger price is higher than pre-merger and the probability for any firm to undercut any price level \( p \) is lower post-merger than the pre-merger.

**Proposition 5.6** If \( k > \hat{k}^* > \hat{k}^*/m \), post-merger mixed strategies \( (\phi_A(p),\phi_B(p)) \) have strict first-order stochastic dominance over pre-merger mixed strategy \( \phi \), i.e. pre-merger firms have strictly higher probability to sell below any price \( p \) than any of the post merger firms (except at \( p_H \)).

Proof (Sketch): To show first-order stochastic dominance we shall consider those price ranges in which the price distributions overlap. The reasoning follows the proof of Proposition 4.1. Following the generalization of the Kreps e Scheinkman’s (1983) finding for duopoly, outsider firm is smaller
than the merged firm then $\phi_B > \phi_A$. Thus, it is sufficient to confront post-merger price distribution of the smaller (symmetric) firms with the pre-merger one. First, consider $p_M \leq p((n - 1)k)$ and if $p((n - 1)k) < p_l$ it implies $p_M < p_l$ and there is no overlap in the configurations (I.b), (I.c), (I.d) and (II.b). For the missing capacity configurations we analyze the representative and sufficiently explicative case in which capacity falls in (I.f). Similar conclusions hold for the other specifications.

In (I.f) there is no overlap if $p_M < p_l < p_H$, otherwise $p_l < p_M < p_H$ the post-merger profile of the mixed strategies follow from section 5.2. Whenever this reasoning holds the same characterizes the can be written $\phi_A(p) = \frac{1}{m} \left[ \frac{mk(p_l-p)}{p(D(p)-K)} \right]^{\frac{1}{n-m}}$ and $\phi_B(p) = \left[ \frac{mk(p_l-p)}{p(D(p)-K)} \right]^{\frac{1}{n-m}}$. Recall the pre-merger distribution for the symmetric player in the symmetric equilibrium $\phi(p) = \left[ \frac{(p-p_m)k}{p(K-D(p))} \right]^{\frac{1}{n-1}}$, $\forall i \in \{1, ..., n\}, p \in (p_m, p_M)$. Since $\phi_B = m\phi_A, \phi_B > \phi_A$ and in order for $\phi_A < \phi$ it must be $\phi_B < \phi$. For the price range $(p_M, p_L)$ in the relevant capacity configuration stochastic dominance can be shown comparing the slopes $\frac{d\phi_B}{dp}$ and $\frac{d\phi}{dp}$ of the distributions. The premerger distribution slope can be written $\frac{d\phi}{dp} = \frac{1}{n-1} \left[ \frac{k(p_m-p)}{p(D(p)-K)} \right]^{\frac{1}{n-1}} \frac{p(k(D(p)))-k(p_m-p)(K-D(p)+pD(p))}{p^2(D(p)-K)^2}$. The postmerger distribution slope $\frac{d\phi_B}{dp} = \frac{m}{n-m} \left[ \frac{mk(p_l-p)}{p(D(p)-K)} \right]^{\frac{1}{n-m}} \frac{p(k(D(p)))-k(p_l-p)(K-D(p)+pD(p))}{p^2(D(p)-K)^2}$. In order to show post-merger stochastic dominance it must be $\frac{d\phi_B}{dp} > \frac{d\phi}{dp}$. This is true component by component for the two expressions above: the first term $\frac{m}{n-m} > \frac{1}{n-1}$ since $m \geq n/2$. Analogously, the roots $\frac{1}{n-m} < \frac{1}{n-1}$. We can rewrite the radical as $\frac{k(p-p_m)}{p(K-D(p))} > \frac{mk(p_l-p)}{p(K-D(p))}$, then it should be $p - p_m > m(p - p_l)$. From our assumptions it is true $\phi_B(p_M) < \phi(p_M) = 1$, therefore $\frac{k(p_M-p_M)}{p(K-D(p))} > \frac{mk(p_M-p_L)}{p(K-D(p))}$ that implies $p_M - p_m > m(p_M - p_L)$. The difference $p_M - p_m - m(p_M - p_L) > 0$ depends negatively on $p_M$. If price decreases from the upper bound the inequality $p_M - p_m > m(p_M - p_L)$ is confirmed for any price in $(p_l, p_M)$. In the relevant price range the postmerger distribution is steeper than the pre-merger one. Thus, the equilibrium pair $(\phi_A(p), \phi_B(p))$ stochastically dominate the pre-merger $\phi$. Q.E.D.

The general symmetric oligopoly setting confirms the symmetric triopoly model findings: mixed strategies relax competitive constraints and allow quoting a higher price on the market than any pure strategy equilibrium. Any horizontal merger that allows oligopolists to play mixed strategies is profitable for the merging parties and outsiders. Horizontal merger lead to a price increase postmerger and if oligopolists play pre-merger mixed strategies then post-merger competition softens

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51 If this is true then the two distributions do not cross since $p_m < p_l < p_M < p_H$. 

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as postmerger distributions assign a lower probability to undercut a certain price level than pre-merger.

\[
\begin{array}{|c|c|c|}
\hline
\text{Premerger Pure Strategy} & \text{Premerger (Symmetric) Mixed Strategy} & \text{Premerger Pure Strategy} \\
\text{Equilibrium} \ p^c = p(nk) & \text{Equilibrium} \ \phi = \frac{(p^c-p_m)k}{\phi(k-D(p))} & \text{Equilibrium} \ p^c = 0 \\
\hline
\end{array}
\]

\[ k \]

\[ 0 \quad \hat{k}^*/m \quad \hat{k}^* \quad D(0)/(n-m) \quad D(0)/(n-1) \]

\[ \text{Postmerger Pure Strategy Equilibrium} \\
p^c = p(nk) \]

\[ \text{Postmerger Mixed Strategy Equilibrium} \\
p_L = \begin{cases} \frac{\Pi_A}{mk}, & \text{if } p_L \leq p(mk) \\ \frac{\Pi_A}{D(p_L)}, & \text{if } p_L > p(mk) \end{cases} \]

\[ p_H = \arg\max_p p(D(p) - (n-m)k) \]

\[
\text{Postmerger equilibrium mixed strategies } (\phi_A(p), \phi_B(p)) \text{ depend upon merged firm’s dimension with respect to the remainder of the industry}
\]

**Figure 4.3.** The Nature of Postmerger Equilibrium as a Function of Individual Capacity in the Symmetric Oligopoly Model

### 5.5. Horizontal Mergers Analysis in a Linear Demand Function Symmetric Oligopoly

The linear demand model is the Davidson and Deneckere (1984) \( n \)-firm oligopoly whose market demand function is linear \( D(p) = a - bp \) if \( p \in (0, a/b) \), zero otherwise. Although conceived for different purposes rather than private profitability analysis of static mergers, the model gives fair understanding of merger incentives in a price setting oligopoly. Studying pro collusive effects, Davidson and Deneckere (1984) find merger profitable for insiders, though they do not show it.\(^52\) We study Davidson and Deneckere’s contribution in a different framework, i.e. assuming the unilateral effects approach rather than the coordinated effects one. Thus, we can show horizontal mergers are always profitable for both outsiders and insiders. Moreover, increased market concentration allows for a higher market power gained by the merged firm that translates into higher price level to consumer detriment. We shall only consider the pre and post merger mixed strategy equilibria as the analysis produces the most interesting results. Pre-merger symmetric equilibrium characteristics follow the specifications in Vives (1986) and De Francesco and Salvadori (2010b). Pre-merger

\(^52\) Several results are enlisted but not shown in the reference paper, see Davidson and Deneckere (1984, p. 124).
symmetric firms earn equilibrium profits $\Pi^*_i = \frac{a - (n - 1)k}{4b}, \forall i = \{1, \ldots, n\}$. The infimum and supremum of equilibrium mixed strategies supports are defined $p_m = \Pi^*_i / k = \frac{a - (n - 1)k}{4bk}$ and $p_M = \frac{a - (n - 1)k}{2b}$, respectively. The equilibrium mixed strategy pre-merger is $f_i(p) = \left(\frac{p - p_m}{p(K - D(p))}\right)^{\frac{1}{n-1}}$, $\forall i = \{1, \ldots, n\}$. The investigated merger would take place among $m$ symmetric firms, creating a larger firm whose size is a proper multiple of insider firms, $k_A = mk$. Outsiders are left with their existing capacity $k$. Merged firm will be indexed firm A and the generic symmetric outsider will be indexed firm B. Post-merger equilibrium values of profits and supports’ extrema change according to the dimension of the merged firm: whether it is sufficiently big to satisfy all upcoming demand at the infimum of the support of the mixed strategies, or not. If $p_L \geq p(k_1)$, that is $D(p_L) \leq k_1$, then merged firm’s profits read $\Pi_A^* = \frac{a - (n - m)k}{4b}$, outsider firms’ profits read $\Pi_B^* = \left[a - \sqrt{a^2 - 4b \Pi_1^*}\right]k/2b$. Extrema of the mixed strategies support read $p_L = \left[a - \sqrt{a^2 - 4b \Pi_1^*}\right]/2b$ and $p_H = \frac{a - (n - m)k}{2b}$. If $p_L < p(k_1)$, that is $D(p_L) > k_1$, then, merged firm’s profits read $\Pi_A^* = \frac{a - (n - m)k}{4b}$, outsider firms’ profits read $\Pi_B^* = \left[a - (n - m)k\right]/4bm$. Extrema of the mixed strategies support read $p_L = \left[a - (n - m)k\right]2/4mbk$ and $p_H = \text{argmax}_p p(D(p) - (n - m)k) = \left[a - (n - m)k\right]/2b$.

Horizontal mergers are profitable for merged firms, and it is straightforward to see that it is increasingly so in the number of participants. Horizontal merger will be profitable if merged firm’s profits are greater than the sum of the merger partners’ individual pre-merger profits. i.e. $\frac{a - (n - m)k}{4b} > m \frac{a - (n - 1)k}{4b}$. Thus, $\left[a - (n - m)k\right] > m \left[a - (n - 1)k\right]$ and carrying out the square it becomes, $a^2(1 - m) + 2a[m(n - 1) - (n - m)]k + [(n - m)^2 - m(n - 1)^2]k^2 > 0$. The necessary manipulations lead to $a^2(1 - m) - 2am(1 - m)k + (n^2 - m(1 - m)k^2 > 0$. Dividing by $(1 - m) < 0$, since $m \geq 2$, reverses the inequality to $a^2 - 2amk + (n^2 - m)k^2 < 0$. Inequality solves for $a/(n + \sqrt{m}) < k < a/(n - \sqrt{m})$ which is always true in the mixed strategy region, i.e. $a/(n + m) < a/(n + \sqrt{m}) < k < a/(n - \sqrt{m}) < a/(n - m)$. It is immediate to derive price effects that a merger has in the mixed strategy region. Horizontal mergers raise prices since both equilibrium mixed strategies supports extrema are higher post merger than pre merger. As market concentration increases, the support of equilibrium mixed strategies shrinks and moves up and rightwards, as the merged firm concentrates more and more mass at the upper endpoint of its support. If $p_L \geq p(k_1)$, then the above inequality translating the profitability conditions of mergers, can be replicated for the lower bound of the mixed strategies supports, if $\left[a - (n - m)k\right] > m \left[a - (n - 1)k\right]$ then $p_L > p_m$. If $p_L < p(k_1)$ the same
conclusions hold. For the upper bound of equilibrium mixed strategies support it must be $p_H = [a - (n - m)k]/2b > [a - (n - 1)k]/2b = p_M$ that is true for any $m > 1$.

What is left to show is profitability of mergers for the outsider firms. The horizontal merger leads to a more concentrated market whose players charge a higher price. Though, contrarily to Salant et al. (1983) result, non merged firms no longer gain more from the merger than merging partners. Analytically, the same inequality must hold for the insiders as well as for the outsiders. In order to derive the effect of horizontal mergers on price level we shall consider post-merger mixed strategies profiles. We shall just recall De Francesco and Salvadori’s (2010b) full characterization of equilibrium mixed strategies $(\phi_1(p), \phi_2(p))$. Substituting for the linear demand function parameters we can obtain the example-specific mixed strategies, and allow for the comparison between pre and post merger distributions. Mixed strategy profiles are the ones in Theorem 3.1 obtained in Davidson and Deneckere (1984) and described in chapter 3. Moreover, De Francesco and Salvadori (2010b), despite the rise of multiple equilibria (sometimes a continuum), show that equilibrium profits are unique regardless the non-uniqueness of equilibrium mixed strategies.

The linear demand function model confirms the general oligopoly model results. Horizontal merging among $m$ of the $n$ competitors relaxes the terms of competition by allowing competitors to charge a higher price than pre-merger. Moreover, the probability for the post-merger firms to quote a lower price than any price $p$ is lower post-merger than pre-merger.

5.6. Concluding Remarks on Horizontal Mergers in Symmetric Oligopolies

The symmetric oligopoly model extends the findings of the basic triopoly model considering the various capacity configurations that arise in an $n$-firm model. If firms are capacity-unconstrained and their dimension allow them to satisfy all forthcoming demand at marginal cost then the model reproduces the standard Bertrand result. Firms play a pure strategy equilibrium quoting marginal cost as equilibrium price and earn zero profits. Horizontal mergers do not alter the equilibrium and post-merger equilibrium characteristics are the same as pre-merger. The standard Bertrand result can also be replicated in oligopoly if all but the largest oligopolist can satisfy upcoming demand at marginal cost, i.e. $(n - 1)k \geq D(0)$. In this model the largest oligopolist is any equally-sized rival firm pre-merger, and becomes the merged firm as soon as the takeover takes place.

If horizontal mergers create a firm whose size allows respecting the inequality $k \geq D(0)/(n - m) \geq D(0)/(n - 1)$, presumably if the merger is sufficiently small, then horizontal merging does not change equilibrium features and null price pure strategy equilibrium represents the unique equilibrium. On the contrary, if the merger creates a market participant whose size is such that the inequality $D(0)/(n - m) > k \geq D(0)/(n - 1)$ holds, then the coalition and the outsiders play a
mixed strategy whose bounds are strictly positive. Thus, merging allows firms to play a strictly greater price than the pre-merger and earn positive profits. Horizontal merger is profitable for both the survival firm and the outsiders. An important distinction between the basic triopoly model and the oligopoly one is the profile of the mixed strategies post-merger. In triopoly a merger that turns the pure strategy equilibrium à la Bertrand into a mixed strategy equilibrium necessarily creates a firm whose size allows to satisfy all upcoming demand at the infimum of the mixed strategy support.

On the other hand, if firms capacity constraints are binding the premerger equilibrium is in pure strategies and firms charge the price that markets total production and earn competitive profits on their capacities. If horizontal mergers create a firm whose size is below its Cournot best-response quantity, then equilibrium will be in pure strategies (positive competitive price) with the same characteristics both pre and post merger. As we have seen, if firm’s capacity is such that $k \in (\hat{k}^*/m, \hat{k}^*)$, then horizontal mergers lead to a mixed strategy equilibrium. If firms play a mixed strategy then horizontal mergers increase industry profitability as well as firms’ profitability. Both merged firm and outsiders gain from horizontal merging and increase their profits from the competitive ones.

If $\hat{k}^* \geq k > \hat{k}^*/m$, postmerger equilibrium is in mixed strategies and merged firm increases its profits over its premerger joint profit level. Postmerger mixed strategy supports shift rightwards since both extrema increase. The reduction in competitors’ number allows merged firm to monopolize a higher residual demand and increase profits over the premerger value of its constituent firms. This in turn allows firms randomizing on a higher price range, and undercut a certain price level with lower probability than premerger. Horizontal mergers relax competitive constraints leading to unilateral price increase by both the coalition and the outsiders if mixed strategies are played postmerger.

6. Conclusions

We have shown that horizontal mergers among equally-sized competitors in Bertrand-Edgeworth games are always non-detrimental for merging parties. Contrary to symmetric treatment in Cournot competition, in which a multiplant Cournot competitor forms post-merger (with the same dimension of the competitors prior to the merger), this essay tackles horizontal mergers as exogenous market structure changes that create a new firm (survival firm) whose size is a proper multiple of the merging firms’ capacity. Horizontal combinations of price-setters in a homogenous product market, under efficient rationing, are studied first in a symmetric triopoly and then in a symmetric oligopoly. Oligopolists’ strategic profiles hinge on their dimensional characteristics. Equilibrium prior to the merger arises either in pure or mixed strategies. If firms’ capacity is unlimited then a pure strategy equilibrium exists and each firm is capable of supplying all forthcoming demand at marginal cost. Thus, price drives down to marginal cost and firms earn zero profits. In $n$-firm oligopolies the same
is true if the aggregate capacity but the largest one is enough to supply market demand at marginal cost. On the other hand, if firms supply (at most) their Cournot best-response function then a pure strategy equilibrium exists, as well. With limited capacities, firms exhaust their productive capabilities and charge the price that allows selling the aggregate capacity on the market, earning competitive profits. If capacities are indeterminate, then equilibrium arises in mixed strategies. Mixed equilibria are characterized by firms randomizing over a price range, whose lower bound is strictly greater than the price that markets the aggregate production.

Firms’ takeovers then characterize equilibrium features. If firms are either large or small enough, the survival firm represents either a sufficiently large or small competitor, whose size does not alter equilibrium strategy profile. That is, if the merger creates a firm that is sufficiently large to accommodate forthcoming demand at marginal cost then equilibrium post-merger will rest à la Bertrand. Both the merged firm and the outsiders will charge marginal cost pricing and earn zero profits. Thus, post-merger equilibrium is the same as pre-merger and horizontal takeovers do not affect price, profits, consumer surplus and total welfare. Similarly, if the merger creates a firm that is sufficiently small, that is merged firm’s capacity is no greater than its Cournot best-response, then equilibrium will arise in pure strategies with the same characteristics as pre-merger. Firms have no incentive to charge either a higher or a lower price than the pre-merger equilibrium one. Pure strategy equilibria post-merger do not alter market equilibrium characteristics and exert no real effect on the market.

If the merged firm gains enough capacity after the merger, then different pricing incentive arise between the merged firm and the symmetric outsiders. That is, post-merger capacity constraints might not be necessarily binding, unlike pure strategy equilibria à la Cournot in which capacities represent a quantity commitment for firms. In mixed strategies firm randomize their price offers, exhausting full capacity only with some positive probability. If mixed strategy equilibria arise post-merger all market firms have an incentive to raise price (and profits) with respect to the pre-merger equilibrium. Horizontal merges are then benefiting both insiders and outsiders preventing outsiders from free-riding the merged firm’s output restriction (by increasing their sales). In particular, if firms play mixed strategies pre-merger then post-merger equilibrium (necessarily mixed) witness an increase in price due to higher mixed strategy supports’ extrema than the ones before the merger. Reducing the number of competitors increases residual demand and shifts up and rightwards both the infimum and supremum of equilibrium mixed strategies support. Horizontal merging effect, relaxing competitive externality among oligopolists is clearer in this case: post-merger price distributions are (first-order) stochastically dominated by the pre-merger ones. That is, post-merger firms’ probability to charge a lower price than any given price level is inferior than the one prior to the merger. Consequently, profitability for both insiders and outsiders increase.
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References


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