# A HUMAN ACHIEVEMENT: MATHEMATICS WITHOUT BOUNDARIES

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After suffering a series of defeats in battles against Napoleonic armies, Prussia decided to reform, amongst other things, its system of education. This work was entrusted to Wilhelm von Humboldt, who was appointed as the head of department of culture and education of the kingdom. During the eighteen months at this post, von Humboldt completely reorganized the school system of Prussia and wrote the charter of a new university. This university was called Berlin University: It enrolled its first students in 1810 and operated in a royal mansion donated by the King of Prussia.

The charter of Berlin University was revolutionary and it was based on three fundamental principles dictated by von Humboldt. The first was the inseparable unity of education and research. According to von Humboldt, research activity was what distinguished a university from other institutions of education. In Berlin University all subjects were present from philosophy to natural sciences, from medicine and engineering to arts and religious studies. University professors and students were constantly engaged in research, accepting no theory or idea as given, without subjecting it to critical reasoning.

The second fundamental principle concerned academic freedom. Berlin University was to be an arena of intellectual freedom. Activities of the university were to be conducted without any influence or interference of external sources of authority. This principle was summarized in the German motto “Lehrfreiheit und Lernfreiheit”.

Students in Berlin University were obliged to have a fundamental education in natural sciences, philosophy and humanities in their first years before specializing in their degree areas. This was the third fundamental principle of von Humboldt. Wilhelm von Humboldt himself was a philosopher and a linguist. He knew thirty languages. It was von Humboldt’s hope that the graduates of Berlin University would be first and foremost universal intellectuals and propagators of enlightenment. This was in direct contrast to the new French institutions of higher education whose mission was to educate expert professionals who were also good citizens of France.

The new model of Berlin University was received enthusiastically by other German Universities. Although universities in other European countries did not altogether take Berlin University as a blueprint, the fundamental principles set forth by von Humboldt were acclaimed by many. It was in the United States that von Humboldt’s principles were widely adapted as the basic philosophy of higher education. Von Humboldt had hoped that in the new universities

modelled on his fundamental principles, a unified grand theory of knowledge would develop by time, transcending all national or geographical borders. With a common culture based on similar general courses taken in the first years of their universities, the age of enlightenment would produce a new generation of professionals, who would also be intellectuals equipped with all tools necessary for critical thinking, refuting all dogma and bigotry.

However this did not happen - history took a different turn: Europe entered a phase of rapid industrialization and formation of strong nation states, creating new rivalries. The industry needed workers willing to do the same simple manual work for long hours at low wages. The state needed loyal and obedient citizens, who would heed a call to arms without hesitation whenever this was considered necessary by the government.

There was definitely a need for experts -engineers, doctors, and so on- but there was almost no "*Lebensraum"* for independently minded intellectuals who would not automatically hate the designated enemy of the state.

At the beginning of the new millenium, we are somewhat caught between two main currents of historical events. Or rather, there is one main current, that of globalization and some strong reactions to globalization which can form a strong coalition of opposition. There are also those who think that mankind cannot do without the devil, which has to be invented if there is none readily available.

Yet the alteration of geographic borders, fear of clash of civilizations, globalization, anti-globalization may well be temporary trends here today and gone tomorrow, belonging to the world that we see on the surface, the world where ideas are limited by boundaries of the widest variety. To the erring person who imagines the true world to be just a reflection of what he sees, everything is bound to appear like a seemingly endless, unproductive tug-of-war.

Yet below this surface is another world, the world of the infinite, where progress is always in a steady forward direction. In this world there can be no notion of "the shortness of the human life span" or even "time"; definitely no notion of material gains, for each idea is a drop that will expand within the never-ending flow that endures beyond centuries and milleniums. This may be why we mathematicians are perhaps among those people who can sense the true meaning of the word "infinite" in the most acute way.

Mathematics is a precious human achievement. It transcends boundaries of all kinds - geographical, historical, national, philosophical or linguistic. Mathematics is accumulative and ageless. Whenever I give the proof of Euclides that there are infinitely many primes, I ask my students to conduct a survey of the physics or astronomy of that period in history, and to compare it with what we know now. The proof attributed to Euclides is still valid today. Furthermore, I hope that it gives at least to some of my students as it does to me, a sense of aesthetic pleasure, whereas the model of the universe by Ptolemy, although at its time of formulation considered a masterpiece, is actually not only false but also quite naive.

Mathematics is full of true masterpieces. It is through the use of accumulation of the masterpieces of mathematics that scientists understand nature much better today than even just a century ago. We have developed means of harnessing the forces of nature for the benefit of humanity. What we describe as "high technology" has its roots in some field of mathematics. Today we use mathematics more widely than ever. Mathematics is and has always been a part of our common heritage, a part of the common wealth we share. We mathematicians do not patent our theorems, but publish them so that everyone can use them, criticize them or even prove them false.

To teach mathematics in the general context of humanities, I propose a course or a series of courses highlighting some concepts of mathematics, interplay between the concrete and the abstract and between heuristic arguments and formal proofs. Let me try to illustrate by means of some sketchy examples: Assuming that our students know basic arithmetic, one could define prime numbers and prove the prime factorization theorem that there are an infinite number of primes. We can then continue to discuss the twin prime problem and the Goldbach conjecture. For a more advanced class one can describe some of the futile attempts to obtain a formula giving all primes, and even include a discussion of some of the heuristic arguments making the conjecture that there are an infinite number of twin primes plausible. A discussion of the use of big primes in cryptography would bring us to today, from our starting point which was around 300 BC.

Another line of advance could start from utilitarian geometry and how it was formalized in the *Elements*. This masterpiece deserves certainly some attention, especially as the first example of the axiomatic approach and rigorous proofs. The fifth postulate could be discussed at some depth. One could also deal with the systematic approach of Appolonius to the conic sections and jump to Kepler's laws and maybe mention Newton's discovery of the gravitational force. Another path could take us to different geometries motivated certainly by the fifth postulate. In this discussion of geometry one could display how the *Elements* survived until the modern times, transmitted from one civilization to another through translations from one language to another, written on papyrus, parchment, "in palimpsest" and on paper.

A more ambitious project would be to take up the abstract notion of a group and illustrate the wide range of applications that is hidden in this simple algebraic structure. Even if briefly, one could touch upon Galois groups and how one can prove the impossibility of the trisection of an angle using compass and ruler only. Symmetry and ornaments can also be discussed in this context. A short discussion of Klein's Erlangen program would demonstrate the link between algebra and geometry.

These are just the initial thoughts that spring to my mind within the framework of what I know as a $20^{th}$ century mathematician – within time and the fertility of the human imagination, naturally new projects will be produced, existing projects will change form. However, we know from the history of mankind that in the land of the infinite, no idea or project –however incomplete it may be- goes wasted, if it is of any value: Sooner or later, it is bound to sparkle someone else's imagination -be it in another geography or another century- and in the end, turn into a sturdy brick contributing to the beauty of the magnificent joint product of mankind of all ages -immortal and transcending all worldly matters.

If education is to make a significant contribution to our future, I believe it must stress much more the achievements of humanity, not only in technology, health or natural sciences, but also in humanities in general.

We should strive to increase the awareness of our young people, that throughout history we have created a tremendous amount of human wealth – in music, literature, architecture, philosophy and in mathematics. These human values, when taught properly, will infuse a new sense of pride and confidence in ourselves, a new hope for a better and peaceful life on our planet. We should revise the unfulfilled dream of von Humboldt and try to make it come true.