

## Fine tuning in general gauge mediation

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**ABSTRACT:** We study the fine-tuning problem in the context of general gauge mediation. Numerical analyses toward for relaxing fine-tuning are presented. We analyse the problem in typical three cases of the messenger scale, that is, GUT ( $2 \times 10^{16}$  GeV), intermediate ( $10^{10}$  GeV), and relatively low energy ( $10^6$  GeV) scales. In each messenger scale, the parameter space reducing the degree of tuning as around 10% is found. Certain ratios among gluino mass, wino mass and soft scalar masses are favorable. It is shown that the favorable region becomes narrow as the messenger scale becomes lower, and tachyonic initial conditions of stop masses at the messenger scale are favored to relax the fine-tuning problem for the relatively low energy messenger scale. Our spectra would also be important from the viewpoint of the  $\mu - B$  problem.

**KEYWORDS:** Supersymmetry Breaking, Beyond Standard Model, Supersymmetric Standard Model

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**1 Introduction**

Low-energy supersymmetric extension of the standard model is one of promising candidates for a new physics at a TeV scale. The supersymmetry (SUSY) can stabilize the huge hierarchy between the weak scale and the Planck scale. That is a motivation for the low-energy SUSY. In addition, the three gauge couplings are unified at the grand unified theory (GUT) scale,  $2 \times 10^{16}$  GeV, in the minimal supersymmetric standard model (MSSM). Also, supersymmetric standard models have candidates for the dark matter.

Although low-energy SUSY solves the (huge) hierarchy problem between the weak scale and Planck/GUT scale, a few percent of fine-tuning is required in the MSSM as follows. The lightest CP-even Higgs mass  $m_h$  is predicted as  $m_h \lesssim M_Z$  at the tree level in the MSSM, but that is smaller than the experimental bound  $m_h \gtrsim 114.4$  GeV. However, the Higgs mass receives a large radiative correction depending on the averaged stop mass  $m_{\bar{t}}$  [1–5]. The experimental bound  $m_h \gtrsim 114.4$  GeV requires  $m_{\bar{t}} \gtrsim 1$  TeV when  $|A_t|/m_{\bar{t}} \lesssim 1.0$ , where  $A_t$  is the so-called A-term corresponding to the top Yukawa coupling. On the other hand, the stop mass also has a renormalization group (RG) effect on the soft scalar mass  $m_{H_u}$  of the up-sector Higgs field as [6–12]

$$\Delta m_{H_u}^2 \sim -\frac{3y_t^2}{4\pi^2} m_{\bar{t}}^2 \ln \frac{\Lambda}{m_{\bar{t}}}, \tag{1.1}$$

where  $y_t$  is the top Yukawa coupling and  $\Lambda$  denotes a cut-off scale of the MSSM such as the Planck scale or GUT scale. This RG effect  $|\Delta m_{H_u}^2|$  would be comparable to the stop mass with a negative sign. Furthermore, the successful electroweak (EW) symmetry breaking requires

$$\frac{1}{2} M_Z^2 \sim -\mu^2 - m_{H_u}^2, \tag{1.2}$$

where  $\mu$  denotes the supersymmetric mass of the up-sector Higgs field  $H_u$  and the down-sector Higgs field  $H_d$ . If  $m_{H_u}^2 \sim -m_{\tilde{t}}^2$  and  $m_{\tilde{t}} = \mathcal{O}(1)$  TeV, one needs a few percent of fine-tuning between  $\mu^2$  and  $m_{H_u}^2$  in order to derive the correct value of  $M_Z$ . That is the so-called little hierarchy problem [13–18]. Several works have been done to address this issue [19]–[45]. Some of them include extensions of the MSSM.

In the bottom-up approach [46], it is found that non-universal gaugino masses with a certain ratio are favorable to improve fine-tuning in the MSSM when the messenger scale of SUSY breaking is the Planck/GUT scale. Such a favorable ratio of gaugino masses can be realized in the TeV scale mirage mediation [33, 34, 47, 49] and gravity mediation, e.g. moduli mediation [43, 51–53] and the SUSY breaking scenario, where F-components of gauge non-singlets are sizable [42, 54, 59].<sup>1</sup> On the other hand, the spectrum of the constrained MSSM with the universal gaugino mass would be unfavorable. It is also pointed out that a negative value of the stop mass squared at the Planck/GUT scale would also be favorable [39, 40].

Since the minimal gauge mediation [64] leads to the universal gaugino mass, that would be unfavorable from the viewpoint of fine-tuning [29, 63]. Recently, Meade, Seiberg and Shih have extended the gauge mediation to general gauge mediation (GGM) [65]. (See also [66]–[77].) That leads to non-universal gaugino and soft scalar masses. Thus, it is important to study fine-tuning in the GGM. That is our purpose.<sup>2</sup> The important difference of the gauge mediation (including GGM) from other mediation scenarios such as gravity mediation is that the messenger scale can vary from the GUT scale to a TeV scale and predicted A-terms are very small in most of models. These would also lead to an important difference in the fine-tuning behavior.

This paper is organized as follows. In section 2, we briefly review on the fine-tuning problem in the MSSM. Section 3 is also a brief review on the GGM. In section 4, we analyse numerically on fine-tuning in the GGM. In section 5, we give a comment on the  $\mu - B$  problem. Section 6 is devoted to conclusion.

## 2 Fine tuning in the MSSM

Here, we briefly review the fine-tuning problem in the MSSM by showing explicitly equations. In our analysis, we neglect the Yukawa couplings except the top Yukawa coupling  $y_t$ . Then, the Higgs sector in the MSSM is described as the following superpotential,

$$W_{\text{Higgs}} = \mu H_u H_d + y_t Q_3 U_3 H_u, \tag{2.1}$$

where  $Q_3$ , and  $U_3$  are the chiral superfields corresponding to the left- and right-handed top quarks, respectively. The Higgs fields and top-stop multiplets as well as the gaugino fields play an important role in the fine-tuning problem. Thus, we concentrate on these fields. Their soft SUSY breaking terms are given by

$$V_{\text{soft}} = m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_{Q_3}^2 |Q_3|^2 + m_{U_3}^2 |U_3|^2 + (\mu B H_u H_d + y_t A_t Q_3 U_3 H_u + \text{h.c.}), \tag{2.2}$$

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<sup>1</sup> Those spectra with less fine-tuning also have interesting aspects on the dark matter physics [60–62].

<sup>2</sup> See also [63, 72].

where  $m_X$  ( $X = H_{u,d}, Q_3, U_3$ ) are the soft scalar masses for  $X$ , respectively,  $\mu B$  is the SUSY breaking mass, i.e. the so-called  $B$ -term. Note that we utilize the same notation for denoting a chiral superfield and its lowest scalar component.

The soft SUSY breaking mass for the up-type Higgs  $m_{H_u}$  is subject to relatively large logarithmic radiative correction (1.1) from mainly stop loops. The radiative correction  $\Delta m_{H_u}^2$  is comparable to the stop mass with the negative sign, i.e.  $\Delta m_{H_u}^2 \sim -m_t^2$ . Such a large and negative correction leads to the EW symmetry breaking at the weak scale. Here, we define the averaged top squark mass  $m_{\tilde{t}}$  as

$$m_{\tilde{t}}^2 \equiv \sqrt{m_{Q_3}^2(M_Z)m_{U_3}^2(M_Z)}. \quad (2.3)$$

A stationary condition of the Higgs potential gives the relation among the  $Z$  boson mass  $M_Z$ , the  $\mu$  parameter and soft scalar masses,  $m_{H_u}^2$  and  $m_{H_d}^2$ , as

$$\frac{M_Z^2}{2} = -\mu^2(M_Z) - \frac{m_{H_u}^2(M_Z) \tan^2 \beta - m_{H_d}^2(M_Z)}{\tan^2 \beta - 1}, \quad (2.4)$$

where  $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle$ . The lightest Higgs boson mass is constrained by

$$\begin{aligned} m_h^2 \leq & M_Z^2 \cos^2 2\beta \left( 1 - \frac{3m_t^2}{8\pi^2 v^2} \ln \frac{m_{\tilde{t}}^2}{m_t^2} \right) \\ & + \frac{3m_t^4}{4\pi^2 v^2} \left[ \ln \frac{m_{\tilde{t}}^2}{m_t^2} + \frac{\tilde{A}_t^2}{m_{\tilde{t}}^2} \left( 1 - \frac{\tilde{A}_t^2}{12m_{\tilde{t}}^2} \right) \right. \\ & \left. + \frac{1}{16\pi^2} \left( \frac{3m_t^2}{2v^2} - 32\pi\alpha_3 \right) \left\{ \frac{2\tilde{A}_t^2}{m_{\tilde{t}}^2} \left( 1 - \frac{\tilde{A}_t^2}{12m_{\tilde{t}}^2} \right) \ln \frac{m_{\tilde{t}}^2}{m_t^2} + \left( \ln \frac{m_{\tilde{t}}^2}{m_t^2} \right)^2 \right\} \right], \quad (2.5) \end{aligned}$$

within the 2-loop approximation [4, 5], where  $v = 174$  GeV,  $\tilde{A}_t \equiv A_t(M_Z) - \mu \cot \beta$  and  $m_t$  is the running top squark mass at  $M_Z$ .

The current experimental lower bound for the Higgs mass is given by the LEP experiment as  $m_h \geq 114.4$  GeV. In order to realize  $m_h \geq 114.4$  GeV, a large top squark mass is required as  $m_{\tilde{t}} \gtrsim 1$  TeV when  $|A_t(M_Z)/m_{\tilde{t}}| \lesssim 1.0$ . The soft scalar mass of the up-sector Higgs field,  $m_{H_u}$  suffers from a large radiative correction according to such a large top squark mass through (1.1). Therefore, a few percent of fine-tuning between  $m_{H_u}^2$  and  $\mu^2$  is required in (2.4) in order to realize the EW symmetry breaking with the experimentally observed  $Z$  boson mass,  $M_Z \simeq 91.2$  GeV. That is the so-called little hierarchy problem. We investigate this fine-tuning problem in the context of the GGM. Furthermore, when  $|A_t(M_Z)/m_{\tilde{t}}| \lesssim 1.5$ , the condition  $m_h \geq 114.4$  GeV requires  $m_{\tilde{t}} \gtrsim 500$  GeV. Hence, the stop mixing  $A_t/m_{\tilde{t}}$  is important [4, 5, 78].

### 3 General gauge mediation

Before considering the fine-tuning problem in the GGM, we also give a brief review on the GGM. Recently, Meade, Seiberg and Shih have presented the most general spectrum which can be obtained in gauge mediated SUSY breaking model [65]. A careful definition

of gauge mediation mechanism has been given in the work, that is, in the limit that the MSSM gauge couplings  $\alpha_i \rightarrow 0$ , the theory decouples into the MSSM and a separate hidden sector which breaks SUSY. Following the convention, we label the gauge groups, SU(3), SU(2) and U(1) of the MSSM by  $a = 3, 2, 1$ , respectively. Within the framework of the GGM, the three gaugino masses  $M_a$  ( $a = 1, 2, 3$ ) of the MSSM are given at the messenger scale  $M$  as,

$$M_a = 2g_a^2 B_a. \tag{3.1}$$

In general,  $B_a$  ( $a = 1, 2, 3$ ) are three independent complex parameters. If CP phases of  $B_a$  are not aligned each other, that would lead to a serious CP problem. Thus, we use  $B_a$  as three real parameters. The soft scalar masses squared are also given in the GGM as

$$m_f^2 = g_1^2 Y_f \zeta + \sum_{a=1}^3 g_a^4 c_2(f; a) A_a, \tag{3.2}$$

at  $M$ , where  $c_2(f; a)$  is the quadratic Casimir of the representation of fermion  $f$  under the gauge group corresponding to the label  $a$ . Here,  $A_a$  ( $a = 1, 2, 3$ ) are three independent real parameters. Hereafter, we concentrate on the models with  $\zeta = 0$ .<sup>3</sup> In this case, there are the mass relations at  $M$

$$m_{Q_f}^2 + m_{D_f}^2 + m_{E_f}^2 - m_{L_f}^2 - 2m_{U_f}^2 = 0, \quad m_{H_u} = m_{H_d}, \tag{3.3}$$

where  $m_{Q_f}$ ,  $m_{U_f}$ ,  $m_{D_f}$ ,  $m_{L_f}$ , and  $m_{E_f}$  denote soft scalar masses for the  $f$ -th generation of the left-handed squarks, up-sector right-handed squarks, down-sector right-handed squarks, left-handed sleptons and right-handed sleptons. Thus, the U(1)<sub>Y</sub>  $D$ -term  $S$ , i.e.,

$$S = m_{H_u}^2 - m_{H_d}^2 + \sum_{f=1}^3 (m_{Q_f}^2 + m_{D_f}^2 + m_{E_f}^2 - m_{L_f}^2 - 2m_{U_f}^2), \tag{3.4}$$

vanishes at the messenger scale  $M$ . Furthermore, its RG equation is given as

$$(4\pi)^2 \frac{dS}{dt} = -b_1 g_1^2(t) S(t), \tag{3.5}$$

where  $t \equiv 2 \log(M_Z/\bar{\mu})$ ,  $\bar{\mu}$  is an arbitrary energy scale, and  $b_1 = 33/5$  (and  $b_2 = 1$ ,  $b_3 = -3$  for references). Thus, when  $S$  is vanishing at  $M$ , it vanishes at any scale. For concreteness, we show explicitly the initial conditions of the soft scalar masses,  $m_{Q_3}$ ,  $m_{U_3}$ ,  $m_{H_u}$  and  $m_{H_d}$  as

$$\begin{aligned} m_{Q_3}^2(M) &= \frac{4}{3} g_3^4(M) A_3 + \frac{3}{4} g_2^4(M) A_2 + \frac{3}{5} \left(\frac{1}{6}\right)^2 g_1^4(M) A_1 \\ &= (4\pi)^4 B_3^2 \left[ \frac{4}{3} \tilde{\alpha}_3^2(M) a_3 + \frac{3}{4} \tilde{\alpha}_2^2(M) a_2 + \frac{1}{60} \tilde{\alpha}_1^2(M) a_1 \right], \\ m_{U_3}^2(M) &= \frac{4}{3} g_3^4(M) A_3 + \frac{3}{5} \left(-\frac{2}{3}\right)^2 g_1^4(M) A_1 \\ &= (4\pi)^4 B_3^2 \left[ \frac{4}{3} \tilde{\alpha}_3^2(M) a_3 + \frac{4}{15} \tilde{\alpha}_1^2(M) a_1 \right], \end{aligned} \tag{3.6}$$

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<sup>3</sup> This situation,  $\zeta = 0$ , can be realized by invoking messenger parity.

$$\begin{aligned}
m_{H_u}^2(M) &= m_{H_d}^2(M) \\
&= \frac{3}{4}g_2^4(M)A_2 + \frac{3}{5}\left(\pm\frac{1}{2}\right)^2g_1^4(M)A_1 \\
&= (4\pi)^4B_3^2\left[\frac{3}{4}\tilde{\alpha}_2^2(M)a_2 + \frac{3}{20}\tilde{\alpha}_1^2(M)a_1\right],
\end{aligned}$$

where  $\tilde{\alpha}_a \equiv \alpha_a/(4\pi) \equiv g_a^2/(4\pi)^2$ . Here, we have defined the ratios

$$a_a \equiv \frac{A_a}{B_3^2}, \tag{3.7}$$

for convenience. Similarly, we define the ratios of gaugino masses to the gluino mass,

$$b_a \equiv \frac{B_a}{B_3}. \tag{3.8}$$

The initial condition of the  $A$ -term in the GGM is given as

$$A_t = 0, \tag{3.9}$$

at  $M$ . Thus, the  $A$ -term  $A_t$  at the weak scale is given only by the RG effect between the weak scale and the messenger scale  $M$ . This initial condition is important because the stop mixing  $A_t/m_{\tilde{t}}$  at the weak scale has a significant effect on the Higgs mass (2.5).

By utilizing these gaugino and sfermion masses given in the GGM, we numerically analyze the fine-tuning problem in the next section.

## 4 Numerical analyses

We study the fine-tuning problem in the GGM and present numerical analyses. In gauge mediated SUSY breaking models, phenomenological consequences at the EW scale generally depend on the messenger scale  $M$ . We present our analyses for three typical messenger scales, that is (i) GUT scale  $M = \Lambda_{\text{GUT}} \equiv 2 \times 10^{16}$  GeV, (ii) intermediate scale  $M = 10^{10}$  GeV, and (iii) relatively low energy scale  $M = 10^6$  GeV.

Firstly, we give the soft parameters at the EW scale by integrating the 1-loop RG equations [6–10]. The gaugino mass at the EW scale are

$$M_1(M_Z) \simeq 0.428B_1, \tag{4.1}$$

$$M_2(M_Z) \simeq 0.859B_2, \tag{4.2}$$

$$M_3(M_Z) \simeq 3.00B_3. \tag{4.3}$$

In this analysis, we use the values of gauge couplings at the EW scale as  $\tilde{\alpha}_1(M_Z) \simeq 1.36 \times 10^{-3}$ ,  $\tilde{\alpha}_2(M_Z) \simeq 2.72 \times 10^{-3}$ , and  $\tilde{\alpha}_3(M_Z) \simeq 9.50 \times 10^{-3}$ . These couplings in the MSSM would be unified at the GUT scale within a good accuracy. In addition, we use the running top mass  $m_t = 164.5$  GeV at  $M_Z$  and  $\tan\beta = 10$  for numerical analysis.

The scalar masses such as  $m_{Q_3}$ ,  $m_{U_3}$ ,  $m_{H_{u,d}}$ , and  $A_t$ , which are important to discuss the fine-tuning problem, are given for each typical messenger scale as

(i)  $M = \Lambda_{\text{GUT}}$ ,

$$\begin{aligned}
 m_{Q_3}^2(M_Z) &\simeq 6.07B_3^2 - 0.0120B_1B_3 - 0.00754B_1^2 - 0.0834B_2B_3 \\
 &\quad - 0.00245B_1B_2 + 0.437B_2^2 \\
 &\quad - 0.116m_{H_u}^2(M) + 0.884m_{Q_3}^2(M) - 0.116m_{U_3}^2(M), \quad (4.4)
 \end{aligned}$$

$$\begin{aligned}
 m_{U_3}^2(M_Z) &\simeq 5.11B_3^2 - 0.0240B_1B_3 + 0.0495B_1^2 - 0.167B_2B_3 \\
 &\quad - 0.00490B_1B_2 - 0.202B_2^2 \\
 &\quad - 0.232m_{H_u}^2(M) - 0.232m_{Q_3}^2(M) + 0.768m_{U_3}^2(M), \quad (4.5)
 \end{aligned}$$

$$\begin{aligned}
 m_{H_u}^2(M_Z) &\simeq -2.90B_3^2 - 0.0361B_1B_3 + 0.00505B_1^2 - 0.250B_2B_3 \\
 &\quad - 0.00735B_1B_2 + 0.235B_2^2 \\
 &\quad + 0.652m_{H_u}^2(M) - 0.348m_{Q_3}^2(M) - 0.348m_{U_3}^2(M), \quad (4.6)
 \end{aligned}$$

$$m_{H_d}^2(M_Z) \simeq 0.538B_2^2 + 0.0415B_1^2 + m_{H_d}^2(M), \quad (4.7)$$

$$A_t(M_Z) \simeq 2.20B_3 + 0.278B_2 + 0.0352B_1, \quad (4.8)$$

(ii)  $M = 10^{10}$  GeV,

$$\begin{aligned}
 m_{Q_3}^2(M_Z) &\simeq 5.43B_3^2 - 0.00327B_1B_3 - 0.000940B_1^2 - 0.0331B_2B_3 \\
 &\quad - 0.000404B_1B_2 + 0.227B_2^2 \\
 &\quad - 0.0958m_{H_u}^2(M) + 0.904m_{Q_3}^2(M) - 0.0958m_{U_3}^2(M), \quad (4.9)
 \end{aligned}$$

$$\begin{aligned}
 m_{U_3}^2(M_Z) &\simeq 4.76B_3^2 - 0.00654B_1B_3 + 0.0142B_1^2 - 0.0661B_2B_3 \\
 &\quad - 0.000807B_1B_2 - 0.0701B_2^2 \\
 &\quad - 0.192m_{H_u}^2(M) - 0.192m_{Q_3}^2(M) + 0.808m_{U_3}^2(M), \quad (4.10)
 \end{aligned}$$

$$\begin{aligned}
 m_{H_u}^2(M_Z) &\simeq -2.03B_3^2 - 0.00981B_1B_3 + 0.00405B_1^2 - 0.0992B_2B_3 \\
 &\quad - 0.00121B_1B_2 + 0.157B_2^2 \\
 &\quad + 0.712m_{H_u}^2(M) - 0.288m_{Q_3}^2(M) - 0.288m_{U_3}^2(M), \quad (4.11)
 \end{aligned}$$

$$m_{H_d}^2(M_Z) \simeq 0.262B_2^2 + 0.0103B_1^2 + m_{H_d}^2(M), \quad (4.12)$$

$$A_t(M_Z) \simeq 1.93B_3 + 0.181B_2 + 0.0167B_1, \quad (4.13)$$

(iii)  $M = 10^6$  GeV,

$$\begin{aligned}
 m_{Q_3}^2(M_Z) &\simeq 4.24B_3^2 - 0.000733B_1B_3 - 0.0000353B_1^2 - 0.00879B_2B_3 \\
 &\quad - 0.0000603B_1B_2 + 0.111B_2^2 \\
 &\quad - 0.0669m_{H_u}^2(M) + 0.933m_{Q_3}^2(M) - 0.0669m_{U_3}^2(M), \quad (4.14)
 \end{aligned}$$

$$\begin{aligned}
 m_{U_3}^2(M_Z) &\simeq 3.90B_3^2 - 0.00147B_1B_3 + 0.00563B_1^2 - 0.0176B_2B_3 \\
 &\quad - 0.000121B_1B_2 - 0.0198B_2^2 \\
 &\quad - 0.134m_{H_u}^2(M) - 0.134m_{Q_3}^2(M) + 0.866m_{U_3}^2(M), \quad (4.15)
 \end{aligned}$$

$$\begin{aligned}
 m_{H_u}^2(M_Z) &\simeq -1.03B_3^2 - 0.00220B_1B_3 + 0.00234B_1^2 - 0.0264B_2B_3 \\
 &\quad - 0.000181B_1B_2 + 0.0914B_2^2 \\
 &\quad + 0.799m_{H_u}^2(M) - 0.201m_{Q_3}^2(M) - 0.201m_{U_3}^2(M), \quad (4.16)
 \end{aligned}$$

$$m_{H_d}^2(M_Z) \simeq 0.121B_2^2 + 0.00366B_1^2 + m_{H_d}^2(M), \quad (4.17)$$

$$A_t(M_Z) \simeq 1.47B_3 + 0.105B_2 + 0.00850B_1. \quad (4.18)$$

Here, we have used the initial conditions,  $A_t(M) = S(M) = 0$ . The change of RG effects between the cases (ii) and (iii) is rather drastic compared with one between (i) and (ii).

If all soft parameters are taken as the same order,  $B_a \sim m_X(M)$ , the averaged top squark mass is approximated for each messenger scale as

$$m_{\tilde{t}}^2 \sim \begin{cases} 6.0B_3^2 & \text{in the case (i)} \\ 5.7B_3^2 & \text{in the case (ii)} \\ 4.8B_3^2 & \text{in the case (iii)} \end{cases}. \quad (4.19)$$

For a fixed value of  $|A_t(M_Z)/m_{\tilde{t}}|$ , a large value of  $m_{\tilde{t}}^2$  would be favorable to realize the Higgs mass  $m_h \geq 114.4$  GeV. That implies that a higher messenger scale would be favorable for a fixed value of the gluino mass, i.e.  $B_3$ . In order to satisfy the experimental bound for the Higgs mass, the lower bound for  $B_3$  is roughly estimated as

$$B_3 \gtrsim \begin{cases} 200 \text{ (410) GeV for } |A_t(M_Z)/m_{\tilde{t}}| \lesssim 1.5 \text{ (1.0) in the case (i)} \\ 210 \text{ (420) GeV for } |A_t(M_Z)/m_{\tilde{t}}| \lesssim 1.5 \text{ (1.0) in the case (ii)} \\ 230 \text{ (460) GeV for } |A_t(M_Z)/m_{\tilde{t}}| \lesssim 1.5 \text{ (1.0) in the case (iii)} \end{cases}. \quad (4.20)$$

Furthermore, we can estimate the stop mixing  $|A_t(M_Z)/m_{\tilde{t}}|$ . For example, for  $B_a \sim m_X(M)$  we estimate

$$|A_t(M_Z)/m_{\tilde{t}}| \sim \begin{cases} 1.0 & \text{in the case (i)} \\ 0.89 & \text{in the case (ii)} \\ 0.72 & \text{in the case (iii)} \end{cases}. \quad (4.21)$$

A large value of  $|A_t(M_Z)/m_{\tilde{t}}|$  would be favorable to realize the Higgs mass  $m_h \geq 114.4$  GeV. That implies that a higher messenger scale would be favorable.

On the other hand, the dominant part of the RG effects in  $m_{H_d}^2$  (4.6), (4.11) and (4.16) is due to the gluino mass, i.e.  $B_3^2$ . If  $B_3 \sim 500$  GeV, we need fine-tuning between  $m_{H_u}^2$  and  $\mu^2$  to realize  $M_Z$ . The absolute value of coefficient of  $B_3^2$  in  $m_{H_d}^2(M_Z)$  decreases as the messenger scale  $M$  decreases. Thus, for a fixed value of  $B_3$ , the degree of fine-tuning is reduced as the messenger scale becomes lower.

Thus, the tension between the fine-tuning and the lower bound of the Higgs mass  $m_h \geq 114.4$  GeV depends non-trivially on the messenger scale  $M$ . Also that would depend on ratios among gaugino masses and scalar masses, although we have used  $B_a \sim m_X(M)$  in the above estimation.

Toward the numerical analyses of the fine-tuning problem, we introduce fine-tuning parameters [13–18],

$$\Delta_Y \equiv \frac{1}{2} \frac{Y}{M_Z^2} \frac{\partial M_Z^2}{\partial Y}, \quad (4.22)$$

which indicates that we need  $100/\Delta_Y$  percent of fine-tuning for  $Y$  to derive  $M_Z$ . A larger value of  $\Delta_Y$  means more severe fine-tuning to be required.



If  $B_a$  and  $A_a$  are independent of each other, fine-tuning for  $B_3$  would be most severe, because  $m_{H_d}^2(M_Z)$  depends dominantly on  $B_3$ . For example, we can calculate

$$(i) \quad M = \Lambda_{\text{GUT}}$$

$$\Delta_{B_3} = 5.85\hat{M}_3^2 + (0.0364\hat{M}_1 + 0.253\hat{M}_2)\hat{M}_3, \quad (4.23)$$

$$(ii) \quad M = 10^{10} \text{ GeV}$$

$$\Delta_{B_3} = 4.10\hat{M}_3^2 + (0.00990\hat{M}_1 + 0.100\hat{M}_2)\hat{M}_3, \quad (4.24)$$

$$(iii) \quad M = 10^6 \text{ GeV}$$

$$\Delta_{B_3} = 2.08\hat{M}_3^2 + (0.00222\hat{M}_1 + 0.0266\hat{M}_2)\hat{M}_3, \quad (4.25)$$

where  $\hat{M}_a \equiv B_a/M_Z$ . It is found that the coefficients of the terms become small as the messenger scale becomes lower. If  $\Delta_{B_3} \leq 10$  is required under the condition  $B_1 = B_2 = B_3$ , the allowed value of  $B_3$  are (i)  $B_3 \leq 110 \text{ GeV}$ , (ii)  $B_3 \leq 140 \text{ GeV}$ , and (iii)  $B_3 \leq 190 \text{ GeV}$ . They could not satisfy the bounds on the Higgs mass (4.20). On the other hand, when we take  $B_3 \simeq 500 \text{ GeV}$ , we find that severe fine-tunings such as (i)  $\Delta_{B_3} \simeq 200$ , (ii)  $\Delta_{B_3} \simeq 140$ , (iii)  $\Delta_{B_3} \simeq 70$  are needed.

We have assumed that  $B_a$  and  $A_a$  are independent of each other. However, in a definite theory, they are not independent, but certain ratios are predicted in each theory. That is, in a definite theory there is one parameter, which determines the overall size of soft SUSY breaking terms. We choose  $B_3$  as such a parameter and the ratios  $a_a$  and  $b_a$  are fixed in a theory. Then, we consider the fine-tuning only for  $B_3$ , i.e.  $\Delta_{B_3}$  under fixed ratios of  $a_a$  and  $b_a$ . Varying  $a_a$  and  $b_a$  means that we compare different theories in the theory space of the GGM. Then, the fine-tuning parameter can be rewritten as

$$(i) \quad M = \Lambda_{\text{GUT}}$$

$$\begin{aligned} \Delta_{B_3} = & \hat{M}_3^2(5.85 + 0.506b_2 - 0.465b_2^2 + 0.508a_3 - 0.122a_2 + 0.0728b_1 + 0.0148b_1b_2 \\ & - 0.00936b_1^2 + 0.00132a_1), \end{aligned} \quad (4.26)$$

$$(ii) \quad M = 10^{10} \text{ GeV}$$

$$\begin{aligned} \Delta_{B_3} = & \hat{M}_3^2(4.10 + 0.200b_2 - 0.311b_2^2 + 0.825a_3 - 0.143a_2 + 0.0198b_1 + 0.00245b_1b_2 \\ & - 0.00798b_1^2 - 0.00495a_1), \end{aligned} \quad (4.27)$$

$$(iii) \quad M = 10^6 \text{ GeV}$$

$$\begin{aligned} \Delta_{B_3} = & \hat{M}_3^2(2.08 + 0.0533b_2 - 0.182b_2^2 + 1.04a_3 - 0.183a_2 + 0.00444b_1 + 0.000365b_1b_2 \\ & - 0.00465b_1^2 - 0.00821a_1). \end{aligned} \quad (4.28)$$

Coefficients of  $b_1$  and  $a_1$  in the above equations are very small. Thus, those terms would not be important unless  $b_1 = \mathcal{O}(10)$  or  $a_1 = \mathcal{O}(100)$ . Therefore, we concentrate on others and throughout our numerical analyses we take  $b_1 = a_1 = 1$  as a typical value. It is found that the coefficients of  $a_2$  and  $b_2^2$ , which determines the wino mass, are negative. Hence, it would be favorable to cancel the dominant term by relatively large  $b_2$  and/or  $a_2$ . That is, models satisfying

(i)  $M = \Lambda_{\text{GUT}}$

$$|5.85 + 0.506b_2 - 0.465b_2^2 + 0.508a_3 - 0.122a_2| \ll 1, \quad (4.29)$$

(ii)  $M = 10^{10} \text{ GeV}$

$$|4.10 + 0.200b_2 - 0.311b_2^2 + 0.825a_3 - 0.143a_2| \ll 1, \quad (4.30)$$

(iii)  $M = 10^6 \text{ GeV}$

$$|2.08 + 0.0533b_2 - 0.182b_2^2 + 1.04a_3 - 0.183a_2| \ll 1, \quad (4.31)$$

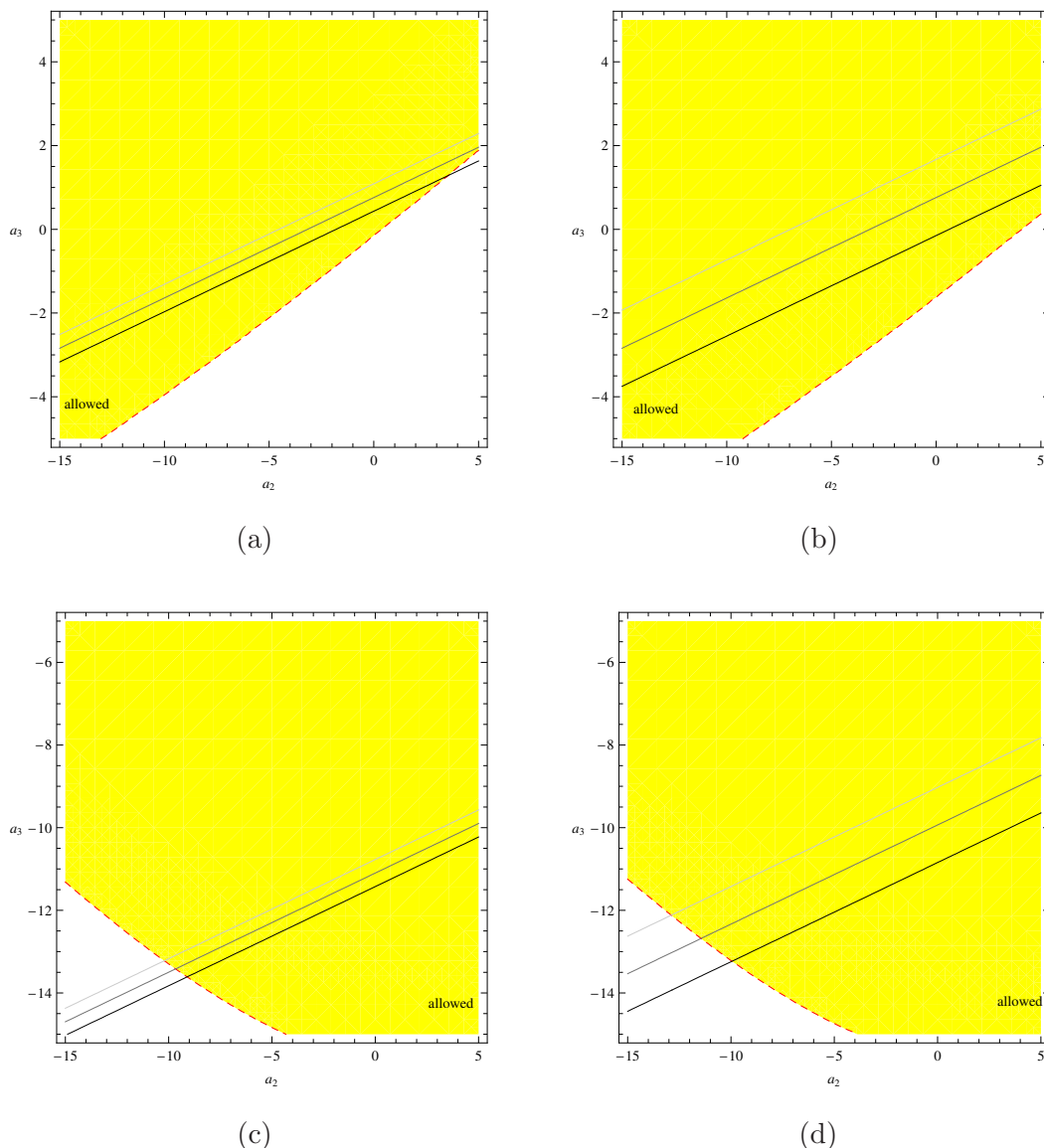
would be interesting in the theory space. For fixed values of  $a_2$  and  $a_3$ , a favorable value of  $b_2$  is determined. That means a favorable ratio between the gluino and wino masses such as ref. [46]. For a fixed value of  $b_2$ , a linear correlation between  $a_2$  and  $a_3$  is required. On the other hand, for a fixed value of  $a_2$  ( $a_3$ ) a quadratic relation between  $b_2$  and  $a_3$  ( $a_2$ ) is required.

The results of numerical analyses are shown in figures 1–4. Figure 1 (a) and (b) show three curves corresponding to  $\Delta_{B_3} = 5, 10, 15$  for  $B_3 = 500$  and  $300 \text{ GeV}$  in the case (i), respectively. The darkest (darker) solid lines correspond  $\Delta_{B_3} = 5$  (10). The dotted (red) curve is  $m_h = 114.4 \text{ GeV}$  and the shaded (yellow) region corresponds to the region with  $m_h \geq 114.4 \text{ GeV}$ . In these figures, we fix  $b_1 = a_1 = 1$  and  $b_2 \simeq 4.19$  for (a) and  $b_2 \simeq 4.01$  for (b). These values of  $b_2$  lead to  $\Delta_{B_3} = 10$  when  $b_1 = a_1 = a_2 = a_3 = 1$ .<sup>4</sup> These figures mean how much stable the region with  $\Delta_{B_3} = 10$  is in the  $(a_2, a_3)$  plane, when  $b_2$  is fixed such that  $\Delta_{B_3} = 10$  is realized for  $b_1 = a_1 = a_2 = a_3 = 1$ . We find from figure 1 (a) that  $a_2 \lesssim 5$  and  $a_3 \lesssim 2$  are required to realize  $\Delta_{B_3} \sim 10$ . Figure 1 (b) shows that these upper bounds of both  $a_2$  and  $a_3$  are raised for  $B_3 = 300 \text{ GeV}$ . It is seen from (4.26) that the widths among three lines become wider as the  $B_3$  becomes lower. The lower bound for  $a_2$  and  $a_3$  are evaluated as  $(a_2, a_3) \gtrsim (-45, -10)$  for  $B_3 = 500 \text{ GeV}$  and  $(-40, -9)$  for  $300 \text{ GeV}$ . These results are insensitive to a value of  $a_1$ , even if  $a_1$  is larger such as  $a_1 \sim \mathcal{O}(10)$ . Figure 1 (c) and (d) correspond to the case of  $(b_2, B_3) = (1, 500 \text{ GeV})$  and  $(b_2, B_3) = (1, 300 \text{ GeV})$ , respectively. The lower bounds for  $a_2$  are raised to  $a_2 \gtrsim -10$ . It can be also found that the favorable region is  $a_3 \gtrsim -15$ .

Figures 2 and 3 show the results of the same analyses as case (i) for the cases (ii) and (iii), respectively, but (a) and (c) for  $B_3 = 1 \text{ TeV}$  and (b) and (d) for  $B_3 = 500 \text{ GeV}$

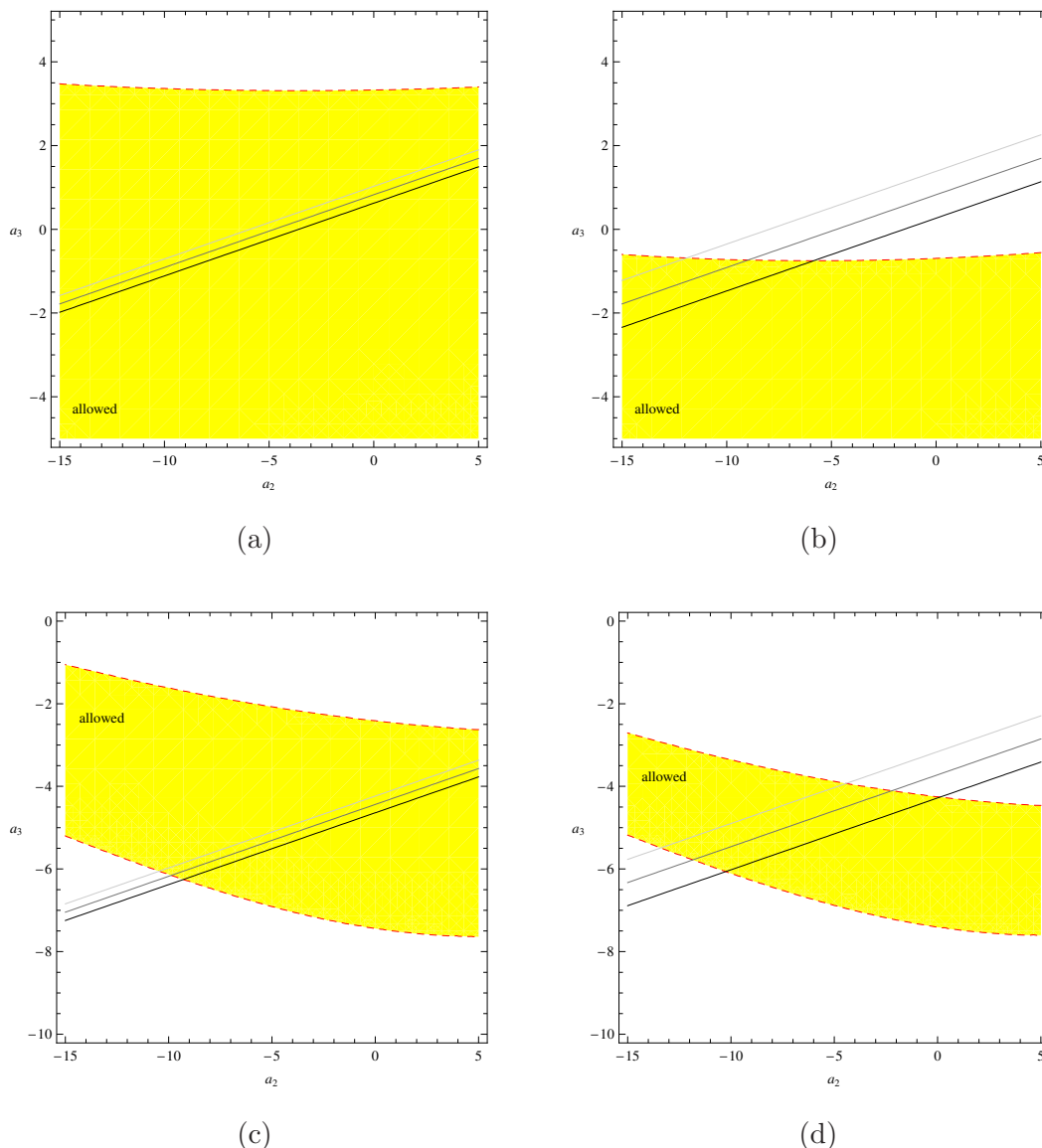
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<sup>4</sup>There is another value of  $b_2$ , which is negative and its absolute value is similar for positive one, to lead to  $\Delta_{B_3} = 10$  when  $b_1 = a_1 = a_2 = a_3 = 1$ . In this work, we focus on only a positive value of solution but our results are not modified for a negative one.



**Figure 1.** Lines and curve for the case (i) determined by constraints from  $\Delta_{B_3} = 5, 10, 15$  (solid lines),  $m_h = 114.4 \text{ GeV}$  (dashed (red) curve), and experimentally allowed region  $m_h \geq 114.4 \text{ GeV}$  (shaded (yellow) region). The darker and darkest solid lines correspond  $\Delta_{B_3} = 10$  and  $5$ , respectively. We take as  $b_1 = a_1 = 1$  in all figures. (a) for  $B_3 = 500 \text{ GeV}$  and  $b_2 \simeq 4.19$ . (b) for  $B_3 = 300 \text{ GeV}$  and  $b_2 \simeq 4.01$ . These values of  $b_2$  lead to  $\Delta_{B_3} = 10$  when  $b_1 = a_1 = a_2 = a_3 = 1$  in each value of  $B_3$ . (c) for  $B_3 = 500 \text{ GeV}$  and  $b_2 = 1$ . (d) for  $B_3 = 300 \text{ GeV}$  and  $b_2 = 1$ .

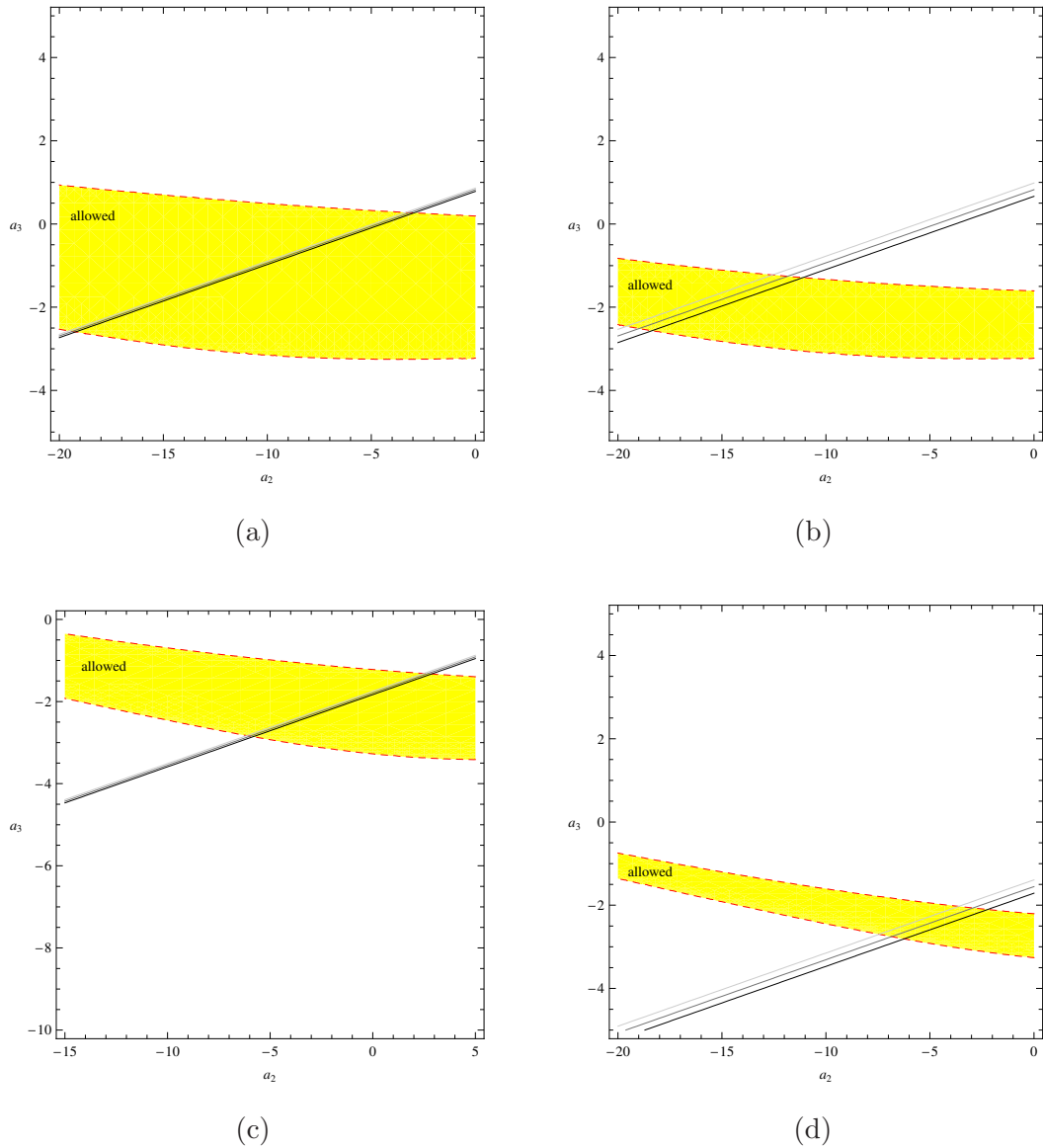
in figure 3. For the messenger scale of  $M = 10^6 \text{ GeV}$ , there is no region corresponding to  $\Delta_{B_3} \sim 10$  and  $m_h \geq 114.4 \text{ GeV}$  when the gluino mass is relatively light such as  $B_3 \sim 300 \text{ GeV}$ . Figure 4 corresponds to the enlargement of figure 3. All favorable regions shown in figures also satisfy the experimental bound of the top squark mass,  $m_{\tilde{t}_1} \geq 95.7 \text{ GeV}$ . The allowed regions become generally narrow as the messenger scale becomes lower. Especially, the values of  $a_2$  and  $a_3$  are constrained to only negative values



**Figure 2.** The same lines and curve as figure 1 but in the case (ii)

for  $(M, B_3, b_2) = (10^6 \text{ GeV}, 500 \text{ GeV}, 1)$  shown in figure 3 (d). This means that tachyonic scalar masses are required at the messenger scale to reduce the fine-tuning in the context of the GGM.

Toward for future model building of the GGM to relax the fine-tuning problem, we present a summary of a typical parameter space in tables 1, 2, 3. When we fix as  $a_3 = 1$ , which is always allowed in all cases of the messenger scale, the favorable regions are obtained as



**Figure 3.** The same lines and curve as figure 1 but (a) and (c) for  $B_3 = 1$  TeV and (b) and (d) for  $B_3 = 500$  GeV in the case (iii).

(i)  $M = \Lambda_{\text{GUT}}$

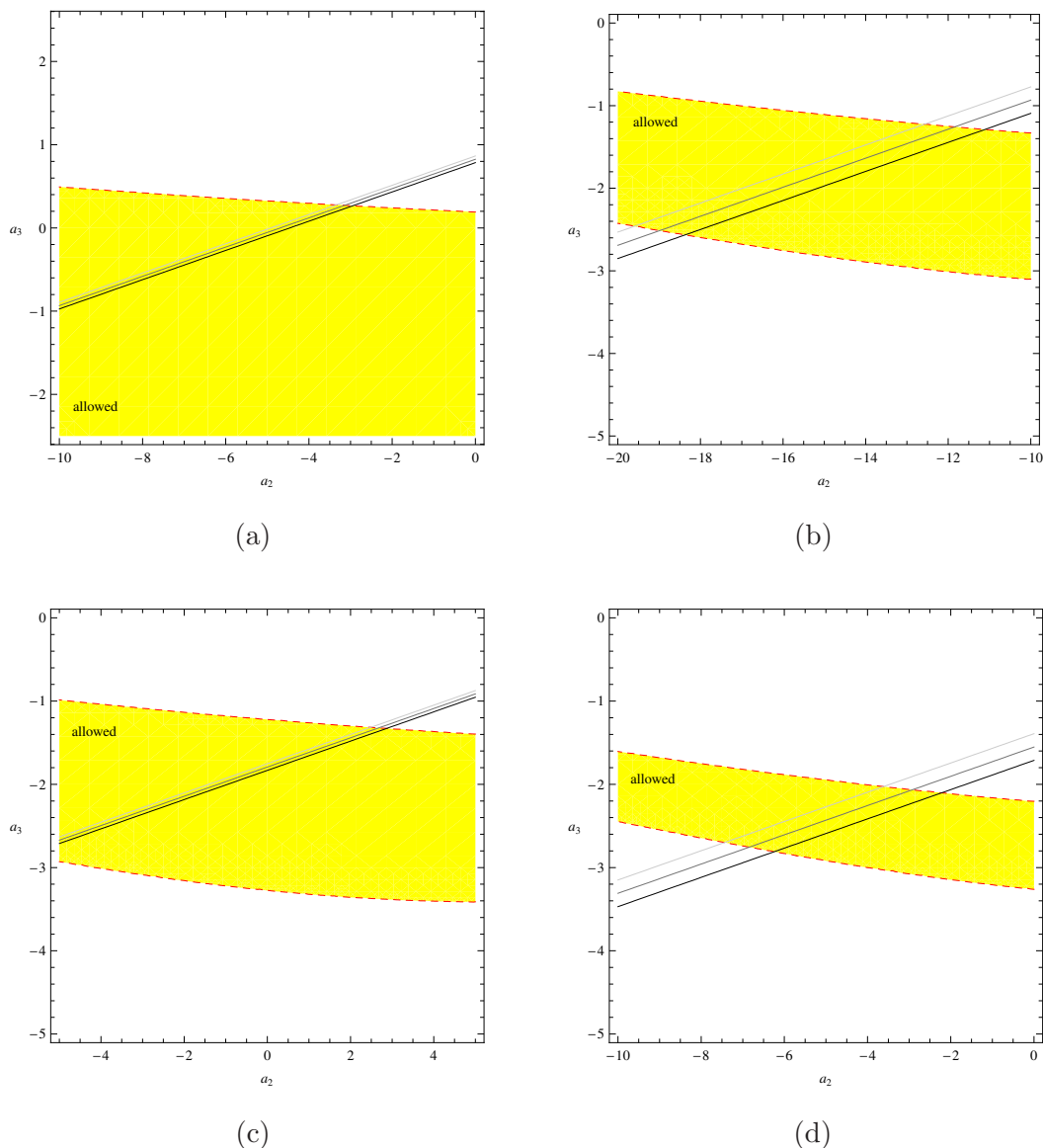
$$0 \lesssim b_2 \lesssim 7 \text{ for } -100 \lesssim a_2 \lesssim 40 \text{ and } B_3 = 500 \text{ GeV}, \quad (4.32)$$

$$0 \lesssim b_2 \lesssim 6.5 \text{ for } -100 \lesssim a_2 \lesssim 40 \text{ and } B_3 = 300 \text{ GeV}, \quad (4.33)$$

(ii)  $M = 10^{10}$  GeV

$$2 \lesssim b_2 \lesssim 8 \text{ for } a_3 = 1, \quad -100 \lesssim a_2 \lesssim 20, \text{ and } B_3 = 500 \text{ GeV}, \quad (4.34)$$

$$0 \lesssim b_2 \lesssim 6.5 \text{ for } a_3 = 1, \quad -100 \lesssim a_2 \lesssim -10, \text{ and } B_3 = 300 \text{ GeV}, \quad (4.35)$$



**Figure 4.** The enlargements of figure 3.

(iii)  $M = 10^6$  GeV

$$5 \lesssim b_2 \lesssim 11 \text{ for } a_3 = 1, \quad -100 \lesssim a_2 \lesssim 5, \text{ and } B_3 = 1000 \text{ GeV}, \quad (4.36)$$

$$5 \lesssim b_2 \lesssim 10 \text{ for } a_3 = -1, \quad -100 \lesssim a_2 \lesssim -10, \text{ and } B_3 = 500 \text{ GeV}. \quad (4.37)$$

Our results show that a certain ratio between the gluino mass and wino mass is favorable. Also, the tachyonic initial condition for stop masses at the messenger scale would be favorable, in particular in the low messenger scale scenario. For  $M < 10^6$  GeV, the favorable region corresponds to only negative values of both  $a_2$  and  $a_3$ . The  $A$ -term  $A_t$  plays a role in this result. Its initial value vanishes at  $M$ , i.e.  $A_t(M) = 0$ , and its value at  $M_Z$  is generated by RG effect as eqs. (4.8),(4.13),(4.18), which are determined mainly by

$B_3$ [GeV]	$b_2$	$a_2$	$a_3$	Figure
500	4.19	$-45 \lesssim a_2 \lesssim 5$	$-10 \lesssim a_3 \lesssim 2$	2 (a)
300	4.01	$-40 \lesssim a_2 \lesssim 10$	$-9 \lesssim a_3 \lesssim 5$	2 (b)
500	1	$-10 \lesssim a_2 \lesssim 50$	$-15 \lesssim a_3 \lesssim 0$	2 (c)
300	1	$-10 \lesssim a_2 \lesssim 50$	$-12 \lesssim a_3 \lesssim 3$	2 (d)

**Table 1.** Favorable parameter regions for (i)  $M = \Lambda_{\text{GUT}}$ .

$B_3$ [GeV]	$b_2$	$a_2$	$a_3$	Figure
500	4.12	$-35 \lesssim a_2 \lesssim 15$	$-5 \lesssim a_3 \lesssim 0$	3 (a)
300	3.86	$-35 \lesssim a_2 \lesssim -10$	$-5 \lesssim a_3 \lesssim -1$	3 (b)
500	1	$-10 \lesssim a_2 \lesssim 10$	$-6 \lesssim a_3 \lesssim -3$	3 (c)
300	1	$-12 \lesssim a_2 \lesssim -2$	$-5 \lesssim a_3 \lesssim -4$	3 (d)

**Table 2.** Favorable parameter regions for (ii)  $M = 10^{10}$  GeV.

$B_3$ [GeV]	$b_2$	$a_2$	$a_3$	Figure
1000	4.10	$-20 \lesssim a_2 \lesssim -5$	$-2 \lesssim a_3 \lesssim 0$	4 (a)
500	3.93	$-20 \lesssim a_2 \lesssim -10$	$-2 \lesssim a_3 \lesssim -1$	4 (b)
1000	1	$-5 \lesssim a_2 \lesssim 5$	$-2 \lesssim a_3 \lesssim -1$	4 (c)
500	1	$-5 \lesssim a_2 \lesssim -3$	$-3 \lesssim a_3 \lesssim -2$	4 (d)

**Table 3.** Favorable parameter regions for (ii)  $M = 10^6$  GeV.

$B_3$  and  $B_2$ . However, a value of  $|A_t(M_Z)|$  at  $M_Z$  is smaller as the messenger scale becomes lower, because the RG effects become smaller. On the other hand, a large value of the stop mixing  $|A_t/m_{\tilde{t}}|$  is favorable to increase the Higgs mass,  $m_h$ . Thus, if a value of  $|A_t(M_Z)|$  is small, we have to decrease a value  $m_{\tilde{t}}$  to obtain a large stop mixing  $|A_t/m_{\tilde{t}}|$ . That can be realized by imposing the tachyonic initial condition of the stop mass at  $M$ .

We also give the mass spectra of gluino, wino, and stop for typical parameters of the favorable regions in table 4. We find that the smallest masses of wino and stop are realized in the case (i) with  $B_3 = 300$  GeV,  $a_3 = -1$ , and  $a_2 = 30$  as  $M_2 \simeq 517$  GeV and  $m_{\tilde{t}} \simeq 555$  GeV. On the other hand, the largest masses of wino and stop are given in the case (iii) with  $B_3 = 10^3$  GeV,  $a_3 = 1$  and  $a_2 = -50$  as  $M_2 \simeq 7150$  GeV and  $m_{\tilde{t}} \simeq 2420$  GeV.

## 5 $\mu - B$ problem

Here, we comment on the  $\mu$ -term and  $B$ -term. How to generate the  $\mu$ -term and  $B$ -term is another important issue. Within the framework of the gauge mediation, a simple mechanism to generate the  $\mu$ -term would lead to

$$\frac{\mu B}{\mu^2} = \mathcal{O}(16\pi^2). \quad (5.1)$$

$M$ [GeV]	$B_3$ [GeV]	$a_3$	$a_2$	$b_2$	$M_3$ [GeV]	$M_2$ [GeV]	$m_{\tilde{t}}$ [GeV]
$2 \times 10^{16}$	500	1	-50	5.71	1500	2450	1180
$2 \times 10^{16}$	500	-1	-50	4.22	1500	1810	863
$2 \times 10^{16}$	500	-1	-1	1.31	1500	563	865
$2 \times 10^{16}$	500	-1	40	1.42	1500	609	803
$2 \times 10^{16}$	300	1	-50	5.59	900	1440	722
$2 \times 10^{16}$	300	1	1	4.01	900	1030	616
$2 \times 10^{16}$	300	1	30	2.63	900	677	522
$2 \times 10^{16}$	300	-1	-50	5.36	900	1380	704
$2 \times 10^{16}$	300	-1	1	3.67	900	947	615
$2 \times 10^{16}$	300	-1	30	2.01	900	517	555
$10^{10}$	500	1	-50	6.48	1500	2790	1280
$10^{10}$	500	1	1	4.12	1500	1770	1230
$10^{10}$	500	1	10	3.58	1500	1520	1210
$10^{10}$	500	-1	-50	6.04	1500	2590	1150
$10^{10}$	500	-1	1	3.34	1500	1440	1110
$10^{10}$	500	-1	10	2.55	1500	1100	1100
$10^{10}$	300	1	-30	5.50	900	1420	755
$10^{10}$	300	-1	-50	5.89	900	1510	682
$10^{10}$	300	-1	-10	3.83	900	987	669
$10^6$	1000	1	-50	8.32	3000	7150	2420
$10^6$	1000	-1	-50	7.59	3000	6520	1800
$10^6$	1000	-1	1	2.21	3000	1900	1800
$10^6$	500	-1	-30	5.97	1500	2570	897

**Table 4.** Mass spectra of gluino, wino, and stop in typical parameter space.

This ratio would cause a problem if

$$\mu^2 \sim m_{H_u}^2(M_Z), m_{H_d}^2(M_Z). \tag{5.2}$$

When both (5.1) and (5.2) hold, we could not realize the successful EW symmetry breaking. That is often called the  $\mu - B$  problem of the gauge mediation.

However, in the previous section, we have studied models with spectra different from eq. (5.2). From the viewpoint of fine-tuning between  $\mu^2$  and  $m_{H_u}^2(M_Z)$ , the favorable spectrum is that  $\mu, |m_{H_u}(M_Z)| = \mathcal{O}(100)\text{GeV}$  and other SUSY breaking masses are of order of a few TeV. Indeed, if we can obtain the following hierarchy,

$$\mu^2 \sim m_{H_u}^2 \ll \mu B \ll m_{H_d}^2, \tag{5.3}$$

we can realize the successful EW symmetry breaking. It has been already pointed out in [79] that the above hierarchy would be favorable in the gauge mediation. Also, such a pattern has been studied within the framework of the TeV scale mirage scenario [34], i.e. the mass pattern II.



This pattern of hierarchy can be realized in our analyses. A relatively large  $B_2$  is favorable to obtain a large  $m_{H_d}$  seen as in (4.7), (4.12), and (4.17). For example, if we take  $M = 10^6$  GeV,  $B_3 = 1$  TeV,  $a_3 = 1$ ,  $a_2 = -50$ , and  $b_2 \simeq 8.32$ , which lead to  $\Delta_{B_3} = 10$ ,  $M_3 \simeq 3$  TeV, and  $M_2 \simeq 7.15$  TeV, and  $m_{\tilde{t}} = 2.42$  TeV, we obtain

$$m_{H_d}^2(M_Z) \simeq 2.89^2 \text{ TeV}^2. \tag{5.4}$$

By using

$$\sin 2\beta = \frac{2\mu B}{2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2}, \tag{5.5}$$

with  $\tan \beta = 10$ , the above value of  $m_{H_d}^2(M_Z) \simeq 2.89^2 \text{ TeV}^2$  determines the value of  $\mu B$  as

$$\mu B \simeq 911^2 \text{ GeV}^2. \tag{5.6}$$

That is, we have  $\mu B/\mu^2 = \mathcal{O}(100)$  for  $\mu \sim 100$  GeV. Such a ratio  $\mu B/\mu^2$  could be realized by a simple mechanism to generate the  $\mu$ -term and  $B$ -term (5.1).<sup>5</sup> Therefore, this parameter set, which relaxes the fine-tuning problem, would also be favorable from the viewpoint of the  $\mu - B$  problem.

## 6 Summary

We have studied the fine-tuning problem in the context of general gauge mediation. Numerical analyses toward for relaxing the fine-tuning in the problem have been presented. We analysed the problem in typical three cases of the messenger scale, that is, GUT ( $2 \times 10^{16}$  GeV), intermediate ( $10^{10}$  GeV), and relatively low energy ( $10^6$  GeV) scales. In each case, the parameter space with less fine-tuning such as 10% has been found. It has also been shown that the favorable region becomes narrow as the messenger scale becomes lower, especially,  $-10 \lesssim a_2 \lesssim 50$  and  $-15 \lesssim a_3 \lesssim 0$  are allowed for  $B_3 = 500$  GeV and  $b_1 = b_2 = a_1 = 1$  in the case (i),  $-10 \lesssim a_2 \lesssim 10$  and  $-6 \lesssim a_3 \lesssim -3$  for  $B_3 = 500$  GeV and  $b_1 = b_2 = a_1 = 1$  in the case (ii), and  $-5 \lesssim a_2 \lesssim -3$  and  $-3 \lesssim a_3 \lesssim -2$  for  $B_3 = 500$  GeV and  $b_1 = b_2 = a_1 = 1$  in the case (iii). Our results imply that certain ratios between the gluino and wino masses as well as scalar masses are favorable to relax the fine-tuning problem. Also, tachyonic initial conditions of scalar masses are favored, in particular in the relatively low messenger scale scenario. Furthermore, the type of spectra with  $\mu \approx 100$  GeV and a few TeV of other SUSY breaking masses is also favorable from the viewpoint of the  $\mu - B$  problem. Thus, it would be important to construct explicit models, which realize certain ratios among gaugino and scalar masses.

**Note to be added.** While this paper was being completed, ref. [80] appeared, where also fine tuning in the GGM was studied.

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<sup>5</sup> In this example, we use the large ratio of  $|a_2/a_3|$ , i.e.  $a_3 = 1$  and  $a_2 = -50$ , but realization of such a ratio may not be straightforward in explicit model building. As another example, we take  $M = 10^{10}$  GeV,  $B_3 = 500$  GeV,  $a_3 = 1$ ,  $a_2 = 1$ , and  $b_2 \simeq 4.12$ . This example leads to  $m_{H_d}(M_Z) \simeq 1.06$  TeV and  $\mu B/\mu^2 = \mathcal{O}(10)$ . That would lead to the above hierarchy (5.3) although the gap of hierarchy would be smaller than the first example.

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