Authors manuscript, published in ELECTRONICS LETTERS, 47(7) pp. 442-444. (2011)

Optimal stopping condition for iterative image deconvolution by a new orthogonality criterion

Dániel Szolgay[1] and Tamás Szirányi[2]

[1]Pázmány Péter Catholic University, Budapest, Hungary

[2]Comuter and Automation Research Center, MTA SZTAKI, Hungary

The stopping condition is a common problem for non-regularized deconvolution methods. We introduce an automatic procedure for estimating the ideal stopping point based on a new measure of independence, checking an orthogonality criterion of the estimated signal and its gradient at a given iteration. We provide an effective lower bound estimate than the conventional ad-hoc methods, proving its superiority to the others at a wide range of different noise models.

Introduction: Blurring is a common issue in almost all image acquisition processes. The distortion is often modeled as convolution: the original unknown image is convolved with a Point Spread Function (PSF):

$$Y = H * U + N \quad (1)$$

where Y is the measured blurry image, U is the unknown original image, H is the PSF and N is additional noise. Y, U and N are (n, m) sized 2D images and H is a (k, l) sized kernel $(k \le n, l \le m)$ with some boundary constraints.

In this letter we will focus on a common issue of non-regularized iterative methods, the stopping condition. Although nowadays regularization is the main trend, non-regularized methods also produce results comparable to the state of the art [1]. We introduce a new error metric based on the independence of the estimated signal and the estimation noise. We show that the ideal (but unknown) criterion can be well estimated by our theoretically new calculus, without any regularization or other constraints.

In the experiments an iterative, non-blind deconvolution algorithm, the Richardson-Lucy (RL) method was used: [2, 3].

The Necessity of Stopping Condition: Since we do not know the original image (U), only the blurry measured one (Y) can be used to guide us toward U. If $X(t)$ is the output of a deconvolution process after t iterations (starting with $X(t=0)=Y$), then its cost function is usually ([2]) based on minimizing the Mean Square Error (MSE):

$$MSE\big(Y, H*X(t)\big) = \frac{1}{n \cdot m}|Y - H*X(t)|^2 \qquad (2)$$

where $|.|$ is the Euclidean norm.

The above MSE measures similarity between two images. In the ideal case the goal is to minimize $MSE(U, X(t))$ (or $|U - X(t)|$) by stopping the iterations at the minimum. However, we can only access the smoothed and thus vague $MSE(Y, H*X(t))$. Let $X(t)$ be an iterated estimation, while another one is $X'(t) = X(t) + N(t)$, where they differ in an additional $N(t)$ noise and residual error with zero mean. In this case $H*N(t) \approx 0$ can be considered, and $H*X'(t) = H*X(t) + H*N(t) \approx H*X(t)$. Since the iterations of $X(t)$ are controlled by $H*X(t)$, this allows possible cases for $t_n \neq t_m$ where $|X'(t_n) - U| >> |X(t_m) - U|$ is true, while $|H*X'(t_n) - Y| \leq |H*X(t_m) - Y|$ (see Fig.1.), and this is why the problem is ill-posed. As stated in [3, 4] this problem affects the quality of the solution of the iterative algorithms highly.

One way to stop this corruption is to make additional assumptions about the target image through regularization [5, 6]. This can enhance the image quality, but the output depends on the chosen regularization parameters.

Without regularization one has to estimate the number of iterations needed to reach the image quality considered best, and stop the process before the image gets corrupted. A straightforward idea is to stop the process after a constant number of iterations or after the change between two consecutive estimations of the image has become lower than a certain threshold [6]. In the following we will refer to this latter as Differential Based Stopping Condition (DBSC):

$$DBSC\colon \frac{|X(t)-X(t-1)|}{|X(t)|} < th \qquad (3)$$

where th is a heuristic choice for threshold, usually between 10^{-3} and 10^{-6}.

We have also tested a modified version of DBSC (in the following: MDBSC), where the re-blurred estimated images ($H * X(t)$) were considered instead of $X(t)$ in Eq.(3). Other similar solutions are summarized in [4].

Orthogonality Based Stopping Condition: In recent years, the concept of a new estimation error has been introduced for similar purposes. Its efficiency has been proven for focus measurement in blind deconvolution problems, see [7]. This error definition, termed as Angle Deviation Error (ADE), is based on the orthogonality principle [8], considering the independence of noise and the estimated signal, using the scalar product:

$$ADE(Q,P) = \left| arcsin \frac{\langle Q,P \rangle}{|Q|\cdot|P|} \right| \qquad (4)$$

where Q and P are n*m sized vectors on $\mathbb{R}$, and $\langle Q,P \rangle$ is their scalar product.

We will show that conventional MSE measures cannot help us to find optimal stopping criteria; while ADE has an optimum, close to the minimum of the practically unknown $MSE(U,X(t))$.

When estimating the $X(t)$ image, there is a point where further iterations do not enhance the image anymore and the difference between two consecutive estimated images ($X_e(t) = X(t) - X(t-1)$) contains minimal information about the residual error of the estimated image vs. the original, ($X_{re}(t) = X(t) - U$), that we want to minimize. We can assume that at this point the independence of $X_e(t)$ and $X_{re}(t)$ is maximal. Once this independence has been reached, the process must be stopped, since any steps after this point may add false information - which is not part of U - to the reconstructed image. In other words, we have to stop the iterative process when the $ADE(X_e(t), X_{re}(t))$ function reaches its minimum.

This theory has been confirmed by checking the statistical dependency between $ADE(X_e(t), X_{re}(t))$ and $MSE(U, X(t))$: the $argmin_t ADE(X_e(t), X_{re}(t))$ correlates well with the $argmin_t MSE(U, X(t))$, the correlation coefficient is 0.9986 for our image database (see *Results* section for database details).

$ADE(X_e(t), X_{re}(t))$ still refers to the unknown image U, and $(Y - H * X(t))$ cannot be used instead of the unknown $X_{re}(t)$ because the deviation error is blurred by the function H in $H * X(t)$. We found that the best estimation for the independence of the signal and the noise is using the difference between two consecutive estimated images $X_e(t)$ and the unblurred estimation $X(t)$:

$$ADE(X_e(t), X(t)) \qquad (5)$$

Function in (5) only contains measurable images and provides a promising solution for the stopping problem, assuming that at the minimum of fn. (5) the change between two consecutive iterations $X_e(t)$ contains mostly independent noise and not structural information about the image $X(t)$, resulting in the highest possible independence of the actual reconstructed image and the iterative modification; hence further iterations will not enhance the image quality, but may add more noise.

The hypothesis that $argmin_t ADE\left(X_e(t), X(t)\right)$ gives the t closest to the optimum has been proved by comparing it to the $argmin_t\, E\left(MSE\left(U, X(t)\right)\right)$, where $E(.)$ is the expected value: the correlation was 95.56% for our database. This high correlation shows that fn. (5) stops the iterations near to the theoretically best but unknown minimum of $MSE\big(U,X(t)\big)$. In the following we present the test conditions and the results achieved with the proposed algorithm and other competing methods.

Results: To test the proposed method and to compare it to other algorithms, we used 25 images as database, which contain landscapes, images of buildings, animals, textures, black and white drawings. The PSF is a Gaussian kernel defined by different blur radii between 1 and 5. We tested a wide range of noise levels and different models: Poisson noise or white Gaussian noise with $SNR = 20, 25, 30, 35, 40dB$ was added to the blurred images.

To compare the proposed method to other existing stopping conditions, we calculated the ratio between the MSE value at the real minimum location of $MSE(U, X(t))$ and at the point where the stopping condition would stop the iteration.

We compared the proposed method to fixed iteration count (the best results were obtained when this constant was 7), to DBSC (eq. (3)) and as a baseline to the blurred image, Y. The experiments were taken using all the 25 images with different blur radii and noise levels. The results can be seen on Fig. 2.

Our tests prove that the commonly used DBSC is outperformed by the one using the blurred comparison (MDBSC), and both of them are outperformed by the proposed ADE based function. DBSC and MDBSC work well occasionally, but their quality is not stable.

We have also tested the stability of the proposed and compared methods at different noise levels between 20 and 40 dB separately and against an inaccurate estimation of the radius of H Gaussian deconvolution kernel. The results show that fn. (5) based stopping criterion gives the best SNR estimation of U at each noise level and it is robust as long as the deconvolution kernel's radius deviates from the blurring kernel's size with less than 10%, which is a reasonable assumption.

Conclusions: We presented a novel method for calculating the optimal stopping point and bounding criterion for iterative deconvolution processes. The proposed method is capable of estimating a theoretically reasonable stopping point of iterations, indicating it when an aimless section of the iterations is starting. It outperforms the generally used ad-hoc methods, while we did not apply any constraints about dimensionality or regularization issues.

References

[1] L. Zou, H. Zhou, S. Cheng, C. He, “Dual Range Deringing for Non-blind Image Deconvolution,” ICIP, (2010).

[2] W. H. Richardson, “Bayesian-based iterative method of image restoration,” Journal of the Optical Society of America, vol. 62, no. 1, pp. 55–59, (1972).

[3] D.S.C. Biggs, M. Andrews, “Acceleration of iterative image restoration algorithms,” Appl. Opt, 36, 1766-1775, (1997).

[4] J. Verbeeck, G. Bertoni, “Deconvolution of core electron energy loss spectra,” Ultramicroscopy 109, 1343-1352, (2009).

[5] A. N. Tikhonov, V. Y. Arsenin, “Solutions of ill-posed problems,” ser. Scripta series in mathematics. Washington: Winston, (1977).

[6] You-Wei Wen, A. M. Yip. “Adaptive parameter selection for total variation image deconvolution,” Numer. Math. Theor. Meth. Appl. 2, 427-438, (2009).

[7] L. Kovács, T. Szirányi, “Focus area extraction by blind deconvolution for defining regions of interest,” IEEE Tr. PAMI 29, 1080-1085, (2007).

[8] A. Papoulis, Probability, “Random Variables ad Stochastic Processes.” New York: McGraw-Hills, (1984).

Authors manuscript, published in ELECTRONICS LETTERS, 47(7) pp. 442-444. (2011)

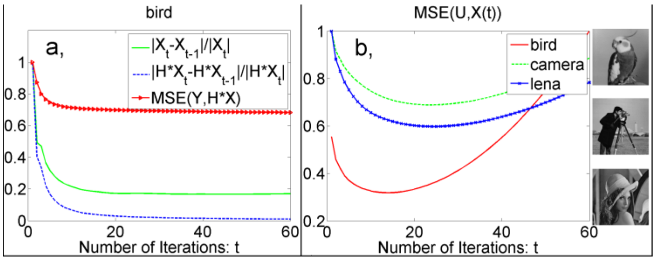
List of Captions:

Fig. 1: The measurable function $MSE(Y, H * X(t))$ and other investigated methods (on the left, relative values) do not follow the immeasurable function $MSE(U, X(t))$ (on the right).

Fig. 2: The relative MSE quality error of the deconvolved image using different methods with Gaussian (a) and Poisson (b) noise; The proposed ADE based stopping condition gives a lower bound than any other methods.

Authors manuscript, published in ELECTRONICS LETTERS, 47(7) pp. 442-444. (2011)

Figure 1:



Authors manuscript, published in ELECTRONICS LETTERS, 47(7) pp. 442-444. (2011)

Figure 2: