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A Branch-and-Cut Algorithm for the Multilevel Generalized Assignment Problem

PASQUALE AVELLA[1], MAURIZIO BOCCIA[1], AND IGOR VASILYEV[2]
[1]Dipartimento di Ingegneria, Università del Sannio, Piazza Roma, Benevento 82100, Italy
[2]Institute for System Dynamics and Control Theory, Siberian Branch of Russian Academy of Sciences, Lermontov 664033, Russia

Corresponding author: P. Avella (avella@unisannio.it)

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ABSTRACT The multilevel generalized assignment problem (MGAP) consists of minimizing the assignment cost of a set of jobs to machines, each having associated therewith a capacity constraint. Each machine can perform a job with different efficiency levels that entail different costs and amount of resources required. The MGAP was introduced in the context of large manufacturing systems as a more general variant of the well-known generalized assignment problem, where a single efficiency level is associated with each machine. In this paper, we propose a branch-and-cut algorithm whose core is an exact separation procedure for the multiple-choice knapsack polytope induced by the capacity constraints and single-level execution constraints. A computational experience on a set of benchmark instances is reported, showing the effectiveness of the proposed approach.

INDEX TERMS Generalized assignement problem, branch-and-cut, exact separation.

I. INTRODUCTION

Let $M = \{1, \ldots, m\}$ be a set of machines and let $N = \{1, \ldots, n\}$ be a set of jobs to be assigned to the machines in M. Each machine $i \in M$ can perform a job $j \in N$ with different *efficiency levels* which entail different costs and different amount of resources required. Let $K = \{1, \ldots, k\}$ be the set of the possible efficiency levels for each machine. Let d_{ijk} be the amount of resource required by the machine i to perform the job j at the efficiency level k and let c_{ijk} be the cost of assigning the job j to the machine i with the efficiency level k. Let u_i be the capacity of the machine $i \in M$.

The MGAP is to find a minimum assignment cost of the jobs N to the machines M satisfying the constraints that the total amount of resources required by each machine $i \in M$ does not exceed its capacity u_i and that an execution level $k \in K$ must be selected.

MGAP is a more general variant of the the well-known Generalized Assignment Problem (GAP). In particular, a GAP instance can be seen as a MGAP instance where $|K| = 1$ (just one possible execution level for each machine).

Because of its computational difficulty, GAP is a challenging integer programming problem, which stimulated a wide interest among researchers. Different heuristic approaches are presented in [2], [10], [20], [21], [23], [30]–[32]. The best known solution for the large-scale instances are found by the parallel ejection chain heuristic [2] and the path relinking approach [31]. The polyhedral structure of the problem is studied in [12], [16], [17]. The most successfull exact approaches are the Stabilized Branch-and-Price algorithm of Pigatti et al. [26], where almost all the test instances with up to 200 tasks were solved, the Cutting Plane algorithm of Avella et al. [5] and the lagrangean-based Branch and Bound algorithm of Posta et al. [28] solving to optimality several previously unsolved instances.

The approach proposed in [5] is based on an exact separation procedure for the knapsack polytopes induced by the capacity constraint defined for each machine, i.e. a separation procedure which either returns a separating hyperplane between a knapsack polytope and a given fractional solution or concludes that the fractional solution is an internal point of the knapsack polytope. In our work we try to generalize this approach by using an exact separation procedure for the multiple choice knapsack polytopes induced by capacity and single-level constraints defined for each machine.

Also the more general MGAP has been widely addressed in literature.

Heuristic approaches for MGAP were proposed by Laguna et al. [20], who reported on results for instances up to 40 jobs, 4 machines and 4 efficiency levels, and French and Wilson [13] who report on a computational experience on larger instances up to 200 jobs, 30 machines and

5 efficiency levels. Both of these heuristics are are developed from solution methods used for the GAP.

A Branch-and-Cut algorithm for the exact solution of the MGAP was proposed by Osorio and Laguna [24] who solved to optimality instances with up to 60 jobs, 30 machines and 2 levels. More recently, these results were outperformed by the Branch-and-Price algorithm of Ceselli and Righini [9] which was able to solve instances up to 400 jobs, 80 machines and 5 levels.

The Ceselli and Righini branch-and-price algorithm is based on a decomposition of the MGAP into a master problem and a pricing subproblem. The master is a set partitioning problem, while the pricing is a multiple-choice knapsack problem. Moreover it uses a branching strategy that is both effective at improving the dual bound and compatible with the combinatorial structure of the pricing subproblem. In this way they solve instances up to 400 jobs, 80 machines and 4 levels. In our computational experience we also compared our results with those returned by the Branch-and-Price algorithm of Ceselli and Righini showing that our approach is higly competitive.

In this paper we propose a reformulation of MGAP based on an exact separation procedure for the multiple-choice knapsack polytopes defined by the capacity and by the single-level execution constraints. The separation procedure is embedded in a Branch-and-Cut scheme and tested on benchmark instances.

The remainder of the paper is organized as follows. In Section II we report two examples of real-life application of the MGAP in the context of large and medium size manufacturing sistems. In Section III MGAP formulations are described. In Section IV the exact knapsack separation procedure is outlined. In Section V we report on computational experiments on benchmark instances.

II. REAL-LIFE APPLICATIONS

In this section we report two examples of real-life application of the MGAP in the context of large and medium size manufacturing sistems. The former was described in [15] where firstly was introduced the *Multilevel Generalized Assignment problem*; the latter is an application of MGAP in the clothing industry described in [18].

Glover et al. [15] introduced MGAP in the context of large manufacturing systems as an *optimal lot sizing and machine loading problem for multiple products*. The *lot sizing and machine loading problem problem* can be described as follows:

- a set of products has to be made in a single-stage process;
- each product can be made in a finite set of possible lot sizes;
- all lots of any single product must be produced on the same machine;
- the machines work in parallel and each machine can produce only one product at a time;
- some machines may be able to product several (ar all) of the products while others may be more specialized;
- each machine has a limited production capacity over the planning horizon;
- the demand of each product is assumed to occur contunuously at a known constant rate.

The objective of the problem is to determine the optimal product lot sizing and the optimal assignment of production to machines with the aim of minimize the production and inventory holding costs.

The second example is described in [18]. It is a task-operator-machine assignment problem in clothing industry where human operators, with a set of available machines, perform tasks consisting of stitching various pieces of a clothing item. Execution times vary from one operator to another. Each task requires a specific machine and must be assigned to one and only one operator. Each machine can perform several tasks although it is allotted at most one operator. An operator is assigned to, at most, one machine. The aim is to find a task-operator-machine assignment that minimizes the total execution time. Secondary objectives, are to minimize the deviation from perfect load balance among the operators, to limit the inter-operator communication cost, the number of machines and the number of operators. The author formulated this problem as a MGAP and solved it by a fuzzy genetic multiobjective optimization algorithm.

For a comprehensive survey on the real-life applications in scheduling, timetabling, telecommunication, transportation, product planning etc. of the GAP and its variants we refer the reader to [22].

III. PROBLEM FORMULATION

Let x_{ijk} be a binary variable which is 1 if the job $j \in N$ is assigned to the machine $i \in M$ at level $k \in K$, 0 otherwise. Let $K' = \{0, 1, \ldots, k\}$ be the extended set of possible levels, where 0 is a dummy level with zero resource consumption. The Multilevel Generalized Assignment problem (MGAP) can be formulated as:

$$\min \sum_{i\in M}\sum_{j\in N}\sum_{k\in K} c_{ijk}x_{ijk} \tag{1}$$

$$\sum_{i\in M}\sum_{k\in K} x_{ijk} = 1, \quad j \in N \tag{2}$$

$$\sum_{j\in N}\sum_{k\in K} d_{ijk}x_{ijk} \leq u_i, \quad i \in M \tag{3}$$

$$\sum_{k\in K'} x_{ijk} = 1 \quad i \in M, j \in N$$

$$x_{ijk} \in \{0, 1\} \quad i \in M, j \in N, k \in K' \tag{4}$$

where the assignment constraints (2) require that each job $j \in N$ is assigned to a machine $i \in M$ with an efficiency level $k \in K$. The capacity constraints (3) enforce the condition that the amount of resources required by the jobs assigned to the machine $i \in M$ does not exceed its capacity u_i. For each pair (i, j) with $i \in M$ and $j \in N$, the single-level constraints (4) imposes that a single execution level can be selected ($x_{ij0} = 1$ if job j is not assigned to the machine i).

The single-level constraints (4) are redundant, as they are implied by the Set Partitioning constraints (2). We use them to tighten the formulation (1)-(4) by generating cutting planes which are valid for the multiple-choice knapsack polytopes $P_{MKN}(i)$ defined by the capacity and by the single-level constraints:

$$P_{MKN}(i) = conv\left(\left\{x \in B^{m\times n\times(k+1)} : \sum_{j\in N}\sum_{k\in K'} d_{ijk}x_{ijk} \leq u_i; \sum_{k\in K'} x_{ijk} = 1 \;\; j \in N\right\}\right)$$

Using the multiple-choice knapsack polytopes $P_{MKN}(i)$, MGAP can be reformulated as:

$$\min \sum_{i\in M}\sum_{j\in N}\sum_{k\in K} c_{ijk}x_{ijk}$$

$$\sum_{i\in M}\sum_{k\in K} x_{ijk} = 1 \quad j \in N \tag{5}$$

$$x \in P_{MKN}(i) \quad i \in M \tag{6}$$

$$x_{ijk} \in \{0, 1\} \quad i \in M, j \in N, k \in K'.$$

IV. EXACT MULTIPLE-CHOICE KNAPSACK SEPARATION

Given a polyhedron $P \subset R^n$ and a point $\bar{y} \in R^n$, an *exact separation procedure* for the polyhedron P is able to find a separating hyperplane between P and $\bar{y}$, or to show that $\bar{y} \in P$.

Following the research stream started by [1] with the "local cuts" for the TSP the rationale of the proposed technique is that spending more computation time in separation can lead to better results for the lower bound with a positive gain in terms of computation time of Branch-and-Cut algorithms. The approach based on "local cuts" has been extended also to other IP problems. In particular in [4] this approach has been successfully used to solve hard Set Covering instances while in [14] and [6] local cuts based on an exact separation procedure for the knapsack set with a single continuous variable are proposed to solve general MIP problems.

The exact separation problem for the knapsack polytope was first studied by Boyd [8] and addressed also in the more recent paper of Kaparis and Letchford [19] on binary knapsack sets. Exact separation procedures for the binary knapsack polytope were effectively used in cutting plane algorithms for the Capacitated p-median problem [7], for the Capacitated Facility Location problem [3] and for the Generalized Assignment problem [5].

In the following we describe an exact separation procedure for the multiple-choice knapsack problem to compute the lower bound given by the tighten formulation (5)-(6). Let $i \in M$ and consider the corresponding *multiple-choice knapsack problem*. Let

$$X_{MKN} = \left\{y \in B^{n\times(k+1)} : \sum_{j\in N}\sum_{k\in K'} a_{jk}y_{jk} \leq u; \sum_{k\in K'} y_{jk} = 1 \;\; j \in N\right\}$$

be the set of its feasible solutions and let $P_{MKN} = conv(X_{MKN})$ be the convex hull of X_{MKN}.

Given a point $\bar{y} \in [0, 1]^{n\times(k+1)}$, the separation problem to find a separating hyperplane between P_{MKN} and $\bar{y}$, or showing that $\bar{y} \in P_{MKN}$, amounts to solve the following LP problem:

$$\theta = \max_{(\alpha,\beta)} \left[\bar{y}^T\alpha - \beta\right]$$

$$v^T\alpha \leq \beta \qquad \forall v \in X_{MKN} \tag{7}$$

$$|\alpha| = 1 \tag{8}$$

$$\alpha \geq 0 \tag{9}$$

where (8)-(9) are normalization constraints inbtroduced to avoid unboundedness. Using the L^1 norm we maximize the ratio between the violation and the size of the support.

The separation LP (7)-(9) includes a huge number of constraints. A way to reduce it is to consider the polytope defined by the fractional support of $\bar{y}$. In other words we consider the reduced polytope

$$P_{MKN}(\bar{y}) = \{y \in P_{MKN} : y_i = 0 \quad if \quad \bar{y}_i = 0, \; y_i = 1 \quad if \quad \bar{y}_i = 1\}$$

It is known that a separation hyperplane for P_{MKN} exists iff it exists for $P_{MKN}(\bar{y})$. Obviously a lifting procedure (e.g. Padberg [25] and Wolsey [29]) has to be used to convert the valid inequalities of $P_{MKN}(\bar{y})$ into valid inequalities of P_{MKN}.

The reduced problem still contains an intractable number of constraints and has to be solved by a row generation approach.

A. ROW GENERATION PROCEDURE

A *row generation procedure* is an iterative procedure where, at each iteration, a relaxed separation problem including only a subset of the constraints (7) (*i.e. the partial separation problem*) is considered. Let $(\bar{\alpha}, \bar{\beta})$ be an optimal solution of the partial separation problem:

- if all solutions $h \in X_{MKN}$ satisfy the inequality $h^T\bar{\alpha} \leq \bar{\beta}$, then $(\bar{\alpha}, \bar{\beta})$ is the optimal solution of the complete separation problem too;
- otherwise a new inequality is added to the partial separation problem and the procedure itarates.

The main steps of the row generation procedure are summarized below.

1) ROW GENERATION PROCEDURE

Step1 Let $U \subset X_{MKN}$ be a subset of the feasible solutions of the multiple-choice knapsack problem.

Step2 Solve the partial separation LP over U:

$$\theta = \max_{(\alpha,\beta)} [\bar{y}T\alpha - \beta]$$
$$v^T\alpha \le \beta \qquad \forall v \in U$$
$$|\alpha| = 1$$
$$\alpha \ge 0$$

Let $(\bar{\alpha}, \bar{\beta})$ be its optimal solution.

Step 3 Solve the multiple-choice knapsack problem:

$$\zeta = \max_v \bar{\alpha}^T v$$
$$v \in X_{MKN}$$

Let $\bar{v}$ be its optimal solution.

Step 4 If $\bar{v}^T\bar{\alpha} > \bar{\beta}$ then set $U = U \cup \{\bar{v}\}$ and go to *Step 2*.

Step 5 If $\bar{v}^T\bar{\alpha} \le \bar{\beta}$ then $(\bar{\alpha}, \bar{\beta})$ is the optimal solution of the separation LP and the inequality $\bar{\alpha}^T y \le \bar{\beta}$ is valid for P_{MKN}.

TABLE 1. Comparison with the Branch and Price.

m	n	k	B&C time (secs)	B&P time (secs)
10	100	3	15.1	63.5
10	100	4	5.2	10.7
10	100	5	10.1	14.4
20	100	3	35.1	53.9
20	100	4	45.2	80.7
20	100	5	43.1	68.6
30	100	3	243.3	598.0
30	100	4	150.5	274.4
30	100	5	98.9	190.9
15	200	4	22.3	975.2
15	200	5	70.1	1037.2
30	200	4	103.1	199.2
30	200	5	95.2	271.3

TABLE 2. Comparison with the EKS lower bound.

m	n	k	$\%CGAP_{EKN}$	$\%CGAP_{EMCKN}$
10	100	3	99.1	99.2
10	100	4	100	100
10	100	5	96.4	97.1
20	100	3	96.4	97.1
20	100	4	96.1	97.1
20	100	5	95.4	96.3
30	100	3	94.2	94.2
30	100	4	95.4	95.7
30	100	5	95.4	96.0
15	200	4	95,1	96,8
15	200	5	95,0	97,5
30	200	4	97,4	98,2
30	200	5	98,4	99,2

The row generation procedure requires to solve a large number of multiple-choice knapsack problems and using an efficient algorithm at each iteration is crucial to the success of the approach. In our computational experience we used a modification of Pisinger's MCKNAP algorithm [27] which combines dynamic programming with bounding and reduction techniques.

V. COMPUTATIONAL EXPERIMENTS

The exact separation procedure has been embedded into the Branch-and-Cut framework provided by ILOG CPLEX Callable Library 12.2 ([11]) to evaluate its effectiveness in reformulating MGAP. All the experiments were carried out on a Personal Computer with a Pentium(R)M processor 1.2 GHz and 504 MB of RAM. We compared our results with those returned by the Branch-and-Price algorithm of Ceselli and Righini [9]. In addition, we implemented for the MGAP also the exact knapsack separation proposed in [5] for the GAP and we compared the lower bounds provided by an exact separation procedure for the binary knapsack with those given by the multiple-choice knapsack exact separation.

The Branch-and-Price algorithm was tested on randomly generated instances of three types (C, D, E). The instances of type C were easily solved by CPLEX in a few seconds. On the other hand the instances of type D turned out to be too hard for our approach, so we focused on the instances of type E.

In Table 1 we report, for each group of instances with the same dimensions (m: number of machines; n: number of jobs; k: number of levels) the average time required by the the Branch-and-Price of Ceselli and Righini and by our Branch-and-Cut algorithm, respectively. The table clearly shows that our approach is highly competitive with Branch-and-Price.

Finally, in Table 2 we report on the percentage of closed gap by exact knapsack separation (column $\%CGAP_{EKN}$) and by exact multiple-choice knapsack separation (column $\%CGAP_{EMCKN}$), respectively. The percentage of closed gap is computed as $\frac{LB-LB_{LP}}{OPT-LB_{LP}} \cdot 100$ where LB_{LP} is the value of the LP relaxation, LB is the lower bound and OPT is the value of the optimal solution. The values reported in Table 2 are the average values for each group of instances with the same dimensions.

It can be easily seen that the lower bounds returned by knapsack separation are very close to those provided by multiple-choice knapsack separation, so replacing multiple-choice knapsack with knapsack separation can be a viable approach, worth to be further investigated.

REFERENCES

[1] D. Applegate, R. Bixby, V. Chvatal, and W. Cook, "Implementing the Dantzig-Fulkerson-Johnson algorithm for large traveling salesman problems," *Math. Program.*, vol. 97, nos. 1–2, pp. 91–153, 2003.

[2] Y. Asahiro, M. Ishibashi, and M. Yamashita, "Independent and cooperative parallel search methods for the generalized assignment problem," *Optim. Methods Softw.*, vol. 18, no. 2, pp. 129–141, 2003.

[3] P. Avella and M. Boccia, "A cutting plane algorithm for the capacitated facility location problem," *Comput. Optim. Appl.*, vol. 43, no. 1, pp. 39–65, 2009.

[4] P. Avella, M. Boccia, and I. Vasilyev, "Computational experience with general cutting planes for the set covering problem," *Operat. Res. Lett.*, vol. 39, no. 1, pp. 16–20, 2009.

[5] P. Avella, M. Boccia, and I. Vasilyev, "A computational study of exact knapsack separation for the generalized assignment problem," *Comput. Optim. Appl.*, vol. 45, no. 3, pp. 543–555, 2010.

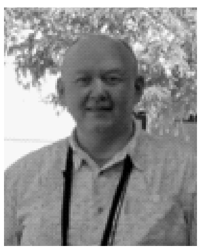
[6] P. Avella, M. Boccia, and I. Vasilyev, "Computational testing of a separation procedure for the knapsack set with a single continuous variable," *INFORMS J. Comput.*, vol. 24, no. 1, pp. 165–171, 2012.

[7] M. Boccia, A. Sforza, C. Sterle, and I. Vasilyev, "A cut and branch approach for the capacitated p-median problem based on fenchel cutting planes," *J. Math. Modell. Algorithms*, vol. 7, no. 1, pp. 43–58, 2008.

[8] E. A. Boyd, "Generating Fenchel cutting planes for knapsack polyhedra," *SIAM J. Optim.*, vol. 3, no. 4, pp. 734–750, 1993.
[9] A. Ceselli and G. Righini, "A branch-and-price algorithm for the multilevel generalized assignment problem," *Operat. Res.*, vol. 54, no. 6, pp. 1172–1184, 2006.
[10] P. C. Chu and J. E. Beasley, "A genetic algorithm for the generalised assignment problem," *Comput. Operat. Res.*, vol. 24, no. 1, pp. 17–23, 1997.
[11] *Using the CPLEX Callable Library*, IBM ILOG AMPL, New York, NY, USA, 2010.
[12] I. R. De Farias and G. L. Nemhauser, "A family of inequalities for the generalized assignment polytope," *Operat. Res. Lett.*, vol. 29, no. 2, pp. 49–51, 2001.
[13] A. P. French and J. M. Wilson, "Heuristic solution methods for the multilevel generalized assignment problem," *J. Heurist.*, vol. 8, no. 2, pp. 143–153, 2002.
[14] R. Fukasawa and M. Goycoolea, "On the exact separation of mixed integer knapsack cuts," *Math. Program.*, vol. 128, nos. 1–2, pp. 19–41, 2011.
[15] F. Glover, J. Hultz, and D. Klingman, "Improved computer-based planning techniques. Part II," *Interfaces*, vol. 9, no. 4, pp. 12–20, 1979.
[16] E. S. Gottlieb and M. R. Rao, "The generalized assignment problem: Valid inequalities and facets," *Math. Program.*, vol. 46, nos. 1–3, pp. 31–52, 1990.
[17] E. S. Gottlieb and M. R. Rao, "$(1, k)$-configuration facets for the generalized assignment problem," *Math. Program.*, vol. 46, nos. 1–3, pp. 53–60, 1990.
[18] S. Hajri-Gabouj, "A fuzzy genetic multiobjective optimization algorithm for a multilevel generalized assignment problem," *IEEE Trans. Syst., Man Cybern. C, Appl. Rev.*, vol. 33, no. 2, pp. 214–224, May 2003.
[19] K. Kaparis and A. N. Letchford, "Separation algorithms for 0-1 knapsack polytopes," *Math. Program.*, vol. 124, nos. 1–2, pp. 69–91, 2010.
[20] M. Laguna, J. P. Kelly, J. L. Gonzales-Velarde, and F. Glover, "Tabu search for the multilevel generalized assignment problem," *Eur. J. Operat. Res.*, vol. 82, no. 1, pp. 176–189, 1995.
[21] R. M. Nauss, "Solving the generalized assignment problem: An optimizing and heuristic approach," *INFORMS J. Comput.*, vol. 15, no. 3, pp. 249–266, 2003.
[22] T. Oncan, "A survey of the generalized assignment problem and its applications," *Inf. Syst. Operat. Res.*, vol. 43, no. 3, pp. 123–141, 2007.
[23] I. H. Osman, "Heuristics for the generalized assignment problem: Simulated annealing and tabu search approaches," *Operat. Res. Spektrum*, vol. 17, no. 4, pp. 211–225, 1995.
[24] M. A. Osorio and M. Laguna, "Logic cuts for multilevel generalized assignment problems," *Eur. J. Operat. Res.*, vol. 151, no. 1, pp. 1238–1246, 2003.
[25] M. W. Padberg, "On the facial structure of set packing polyhedra," *Math. Program.*, vol. 5, no. 1, pp. 199–215, 1973.
[26] A. Pigatti and M. Poggi de Aragao, and E. Uchoa, "Stabilized branch-and-cut-and-price for the generalized assignment problem," *Discrete Math.*, vol. 19, no. 1, pp. 389–395, 2005.
[27] D. Pisinger, "A minimal algorithm for the 0-1 knapsack problem," *Operat. Res.*, vol. 45, no. 5, pp. 758–767, 1995.
[28] M. Posta, J. A. Ferland, and P. Michelon, "An exact method with variable fixing for solving the generalized assignment problem," *Comput. Optim. Appl.*, vol. 52, no. 3, pp. 629–644, 2012.
[29] L. A. Wolsey, "Facets and strong valid inequalities for integer programs," *Operat. Res.*, vol. 24, no. 2, pp. 367–372, 1976.
[30] M. Yagiura, T. Ibaraki, and F. Glover, "An ejection chain approach for the generalized assignment problem," *INFORMS J. Comput.*, vol. 16, no. 2, pp. 133–151, 2004.
[31] M. Yagiura, T. Ibaraki, and F. Glover, "A path relinking approach with ejection chains for the generalized assignment problem," *Eur. J. Operat. Res.*, vol. 169, no. 2, pp. 548–569, 2006.
[32] M. Yagiura, T. Yamaguchi, and T. Ibaraki, "A variable depth search algorithm with branching search for the generalized assignment problem," *Optim. Methods Softw.*, vol. 10, no. 2, pp. 419–441, 1998.

PASQUALE AVELLA is a Full Professor with the University of Sannio, Benevento, Italy. His current research interests include computational mixed-integer programming, in particular he worked on network location, network design, scheduling timetabling, and vehicle routing problems. He received the Ph.D. degree in operations research from the University of Roma "La Sapienza," Rome, Italy, in 1995.

MAURIZIO BOCCIA is an Assistant Professor with the University of Sannio, Benevento, Italy. His current research interests include the study of large scale linear integer programming problems. Recently, he worked on exact separation techniques for binary and integer knapsack problems and for knapsack problems with a single continuous variable. This approach provided good results for the generalized assignment, capacitated P-median and single source capacitated facility Location problems. He received the Ph.D. degree in operations research from the University of Naples "Federico II," Naples, Italy, in 2002.



IGOR VASILYEV received the M.Sc. degree in mathematics from Irkutsk State University, Irkutsk, Russia, in 1995, and the Ph.D. degree in 1998. Since 1999, he has been with the Institute of System Dynamics and Control Theory, Siberian Branch of Russian Academy of Sciences, Lermontov, Russia. He was with Irkutsk State University. His current research interests include all aspects of combinatorial optimization, particularly polyhedral study, cutting planes, parallel, and distributing computing.