MODE CHOICE AND SHOPPING MALL PARKING

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Abstract

In this thesis, I analyze individuals' mode choice decisions and shopping mall's parking space pricing behavior. Individuals have three choices: first they may come to the mall by car in which case they have to park, second they may come by public transportation, or they do not visit the mall and go for their outside option. The mall determines the price of the good and the parking fee after the government sets public transportation fare. I find that the equilibrium parking fees are always less than the marginal cost of providing parking spaces and more importantly they are two of the many parking fees that lead to the social optimum. The mall can be thought as using parking as a loss leader to increase its profits, and implicitly employing mixed bundling in which the good is sold either alone or bundled with parking.

TÜREL DAĞILIM VE ALIŞVERİŞ MERKEZİ PARKLANMASI

Fulya Yüksel Ersoy

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Özet

Bu tezde, tüketicilerin türel dağılım kararları ve alışveriş merkezinin park yeri fiyatlama davranışı analiz edilmektedir. Sunulan modelde, tüketiciler üç seçeneğe sahiptirler: alışveriş merkezine arabalarıyla (ki bu durumda park etmek zorundalar) veya toplu taşıma araçları ile gelebilmektedirler; ya da alışveriş merkezine gelmeyip kendilerine fayda sağlayan diğer seçenekleri değerlendirebilmektedirler. Devlet, toplu taşıma araçlarının ücretlerini belirledikten sonra alışveriş merkezi, sattığı ürünün fiyatını ve park yeri ücretini belirlemektedir. Dengede, park yeri ücretleri her zaman park yeri sağlamanın marjinal maliyetinden daha küçük olmaktadırlar. Daha da önemlisi dengede bulunan park yeri ücretleri toplum açısından en iyi olan park yeri ücretlerindendir. Alışveriş merkezinin park yeri ücreti seçimi, zarar lideri fiyatlandırması, ve ürünün hem park yeri ile birlikte hem de ayrı olarak sunulması, bir karışık paketleme uygulaması olarak düşünülebilmektedir.

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1 Introduction

This paper focuses on shopping mall parking behavior when shoppers have two alternative modes of reaching the mall. They can either come to the mall by car or by public transportation. I analyze the equilibrium and socially optimal parking fees through a set of different variations of the model. The main contribution of the paper is to show that the equilibrium parking fees should be less than the marginal cost of providing parking spaces in all variations I worked out. The products which are sold at prices below their marginal cost to boost other profitable sales are called loss leaders. Thus, the mall uses parking spaces as ”loss leaders”.

Parking is one of the most important intermediate goods in the economy but economists did not pay enough attention on the issue until early 1990s. After then, there have been increasing number of papers on parking, which I discuss in Section 2. One of the most important providers of parking spaces are shopping malls. Shopping malls allocate more space for parking than they allocate for shops (International Council of Shopping Centers and Urban Land Institute, 2003). The same report also states that 94% of the shopping malls provide parking for free.

To my knowledge, Hasker and Inci (2012) are the first to analyze the shopping mall parking behavior. They show why suburban malls provide parking for free even though they allocate vast amount of land for parking and why that may also be socially optimal in a second-best sense. In their model, shoppers can reach the mall only by car and they may end up leaving the mall empty-handed when they do not find their desired good at the mall. To insure the shoppers against this bad outcome to some extent, the mall provides parking for free. Thus, the insurance motive of the risk-neutral mall leads it to embed the costs of parking into the price of the good sold at the mall. They, then, extend the model to a setting in which the mall decides on the parking lot size as well as the parking fee, and again they find zero equilibrium parking fee. They also analyze what might happen in urban malls in which individuals who have no intention of shopping at the mall may want to free ride on the mall's parking lot if it is provided for free. The mall wants to charge for their parking while it wants to provide some ”insurance” to the shoppers. Thus, in this case, if the potential free riders are sufficiently large, the mall may give up providing parking for free.

There are two main differences between my paper and Hasker and Inci (2012). The first main difference is that there is mode choice in my setting. The shoppers may either come by car or public transportation. There are both empirical and theoretical works on mode choice which motivate me to investigate the effects of mode choice on the shopping mall parking behavior. First, there is strong empirical evidence that some shoppers prefer public transportation to reach the mall. For example, a survey conducted at selected sub-urban shopping malls in Singapore reveals that among 1283 shoppers who respond the survey; 63% prefer public transportation, 17% prefer walking and the remaining 20% rely on private transportation (Ooi and Sim, 2007).

Second, there are theoretical papers on mode choice. As far as I know, Voith (1998) is the first to discuss the interplay between parking, transportation, and land use. Its main interest is to analyze the effects of parking taxes and transit subsidies on the equilibrium size of central business district (CBD), the land values of CBD and modal shares. Although the main interest of my paper is quite different, the modelling of individuals and their commuting choices have similarities with Voith (1998). Voith assumes that a worker has two modes to access the CBD: automobile and transit, auto use results in congestion whereas transit use does not. In his base model, the district is assumed to have an unlimited supply of parking spaces, then he discusses an extension of the model in which supply of parking spaces is endogenously determined. He finds a non-monotonic relationship (the Laffer Curve) between parking taxes and transit use. Also, Arnott and Yan (2000) and Kraus (2003) work on ”two-mode problem” which I discuss in details in Section 2.

The second difference of my model from Hasker and Inci (2012) is that shoppers in my model always find the good that they want at the mall. This assumption is mainly due to shut down the insurance motive of the mall in order to highlight the pure effects of introducing public transportation to the model. I find that in the absence of the insurance motive, depending on parameters, the mall either provides parking for free or charges a positive parking fee. However, the equilibrium parking fees are always less than the marginal cost of providing parking spaces. Hence, even in the absence of the insurance motive the mall embeds some of the costs of parking into the price of the good.

The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 sets up the model and derives the equilibrium as well as the social optimum. Section 4 relaxes some of the assumptions of the model and analyzes how the equilibrium and social optimum change. Section 5 relates the paper to the industrial organization literature on bundling and loss leaders. Section 6 concludes. An appendix contains some details and proofs.

2 Literature Review

The parking literature mainly focuses on the pricing of parking. Early work analyzes the relation between parking fees and road pricing. Glazer and Niskanen (1992) construct a model in which two types of drivers exist: parkers, who park downtown, and through-drivers, who drive through downtown. The parking duration and the demand for parking spaces are endogenous. They consider both lump-sum and duration-specific parking fees. They analyze the first-best and find that duration-specific parking fees should be equal to the marginal cost of providing parking spaces and lump-sum parking fees should be zero. They also analyze the second-best in which the road usage fee is suboptimally set and find that there should be a positive lump-sum parking fee in addition to duration specific parking fee which is equal to the marginal cost of providing parking spaces. When there are search costs, increasing parking fees may increase congestion by shortening parking duration.

Verhoef, Nijkamp, and Rietveld (1995) discuss regulatory parking policies as a substitute for road pricing. The authors believe that parking policies can only be effective if the costs of driving do not heavily depend on trip lengths and routes followed. These policies are good at alleviating congestion externalities on urban roads. They compare two forms of parking policies: pricing parking and restricting its supply. They find that time-varying parking fees are preferable to the parking suppy restrictions when parking policies are equivalent to road pricing. Then, they analyze spatially differentiated parking fees and find that under certain assumptions these fees surmount the difficulty that the regulatory parking policies cannot be differentiated according to trip lengths.

Arnott, de Palma, and Lindsey (1991) develop a model that incorporates the travel time (temporal) and location of parking spot (spatial) decisions of commuters. They first analyze the equilibrium with no road tolls and no parking fees. This equilibrium is inefficient in many respects and this inefficiency is due to the uninternalized externalities of commuters. They, then, analyze the cases in which there are time-varying road tolls but not parking fees, there are time-varying road tolls and parking fees, and there are location specific parking fees but not road tolls. The first case eliminates some of the inefficiencies and the second one eliminates all of them. But, since the applicability of road tolls in real life is somehow problematic, the authors also consider the third case. In all these cases, they assume that parking is in the hands of public sector. By relaxing this assumption and assuming private provision of parking, they analyze the equilibrium price of parking in a perfectly competitive environment. They find that without road tolls competitive pricing of parking may be worse than no pricing at all.

There is a growing body of papers studying the effects of congestion externalities on parking fees. Arnott and Rowse (1999) present a stochastic model of parking around a circular road in which drivers cause a parking congestion externality on other drivers since they do not take into account the effect of their parking on the mean density of empty parking spaces. For simplicity, the authors do not incorporate flow congestion into the model. There are two travel modes available to individuals, namely walking and driving. The individuals should decide which travel offers to accept, which mode of transportation to use and the cruising distance. Stochastic nature of finding a parking space complicates the analysis, but it is necessary since there would be no cruising without stochasticity. The authors characterize the equilibrium solution with no parking fees as well as with parking fees, and the social optimum. They find that the optimal parking fee should be equal to the parking congestion externality.

Anderson and de Palma (2004) first analyze a model of unassigned parking in which a parker increases the search time of other parkers in the same area, but does not slow down the traffic in general. They look at the equilibrium with unpriced parking, the social optimum, and the pricing of parking by monopolistically competitive private lot operators. They show that pricing parking in a monopolistically competitive fashion achieves the social optimum. They, then, analyze a model in which both the search externality and the cruising congestion externality exist. They find that market solution can no longer decentralize the social optimum. Their model can be interpreted as a model of on-street parking or off-street parking. Anderson and de Palma (2007) extend the model of Anderson and de Palma (2004) by allowing endogenous land use. They show once again that the social optimum is achieved in a monopolistically competitive parking market.

There are also papers focusing on cruising for parking. Arnott and Inci (2006) analyze the interaction between traffic congestion and curbside parking in a structural model in which cruising for parking is a source of traffic congestion. They are interested in finding the steady states when the parking is saturated (fully occupied). A car cruising for parking in their setting contributes to the traffic at least as much as a car in transit. In equilibrium, there is cruising for parking. They also analyze the first-best steady-state social optimum when the amount of curbside allocated for parking is fixed and variable. They find that cruising for parking creates a pure deadweight loss and the parking fee (or the amount of curbside allocated for parking when the parking fee is fixed) should be set such that the cruising for parking is eliminated without making parking unsaturated.

Shoup (2006) analyzes off-street parking, and its relation with curbside parking and cruising congestion externalities. He explains possible factors that can affect the decision of drivers on choosing between curbside parking and off-street parking. He predicts that drivers will be more likely to cruise if the curbside parking fee is low, the off-street parking fee is high, the parking duration is long, the fuel cost of cruising is low, there are few individuals in the car, and the value of time spent for cruising is low. Shoup presents a simple model and finds that underpriced curbside parking results in cruising for parking and it does not change the utility of drivers, but makes everyone else in the community worse off. The model also predicts that equating the curbside parking fee to the off-street parking fee eliminates cruising for parking.

Arnott and Rowse (2009) analyze the interactions between traffic congestion, curbside parking, and off-street parking (specifically, private garage parking) by extending the model provided in Arnott and Inci (2006). They first analyze the case in which garage parking is provided competitively and curbside parking is underpriced. The equilibrium condition requires that the stock of cars cruising for parking should be such that the full prices of curbside and off-street parking are equal. Then, the authors analyze a more complex case in which parking garages have a market power due to horizontal economies of scale and there is spatial competition among garages, which may lead to inefficient spacing of garage parking. They find that in equilibrium the parking garage fee is above the marginal cost and the parking garages are too close to each other. For tractability reasons, they assume that demand is inelastic. They ignore capacity constraints of garages since these constraints complicate the analysis a lot.

My paper is also related to the two-mode problem. Arnott and Yan (2000) construct a partial equilibrium model of two-mode travel (car and rail) in which two congestible modes are imperfect substitutes in demand. They first discuss the Wheaton and Wilson problem, which analyzes the optimal road width when there is only one mode and the congestion on this mode is underpriced. They, then, discuss the Doi problem, which analyzes the optimal pricing of one mode when the other mode is underpriced. Finally, they try to analyze the (second-best) optimal road width, rail fare, and rail capacity when road congestion is underpriced. To solve this problem, they mostly adopt a local approach in which comparative static properties of the problem are derived; however, the results are too complex and they are interpretable only for some specific cases.

Kraus (2003) constructs a model of the two-mode problem by combining a rail line model with a highway bottleneck model. He solves for the equilibrium, the first-best and the second-best. His paper is similar to Arnott and Yan (2000) in the sense that it analyzes the second-best optimal rail price and rail capacity when the road is underpriced. It differs from Arnott and Yan (2000) in that it focuses on the relationship between the first-best and the second-best levels of rail capacity. Kraus finds that under some elasticity conditions, lowering transit fares and scheduling more trains with higher capacity are desirable when road travel is underpriced.

One of the earliest empirical papers in the literature that analyzes the effects of parking fees on mode choice decision is *Estimation and Specification of the Effects of Parking Costs on Urban Transport Mode Choice* by Gillen (1977). Although the main concern of this paper is to estimate the parameters of specified models of mode choice by using a binary logit estimation, it also offers insights about the relationship between parking and mode choice. Gillen estimates the effects of parking costs and auto running costs separately. Then, he concludes that the impact of the former on the mode choice decision is greater than the impact of the latter, and this is true even when he considers income effects. In his model, a change in parking fees both affects the parking location and mode choice. In my model, parking fees and mode choice endogenously affect each other.

3 The Model

I start off with describing the base model and its assumptions. There is a shopping mall that is owned and operated by a monopolist, risk-neutral shopping mall owner. The mall sells one good, which has no cost, at a non-negative price of P. There are two ways of reaching the mall: by car or by public transportation. The mall provides parking spaces for shoppers coming by car. Providing parking spaces is costly for the mall and this cost is denoted by $c > 0$ per unit of parking space. The parking fee charged by the mall is t. Both P and t are common knowledge and the mall decides on them.

There is also a (local) government which is in charge of the public transportation system. The government decides on the public transportation fare, s, by taking into account the welfare of all parties in society. I also discuss two other possibilities in terms of how the fare for this transportation mode can be determined. In Subsection 4.1, I consider a case in which the mall provides shuttle bus service and in Subsection 4.2, I consider a case in which this mode of transportation is provided privately by a profit-maximizing third party. Providing public transportation is costless for the government in the base model, but I relax this assumption in Subsection 4.3.

There is a mass $\hat{N}$ of risk-neutral individuals who live in a certain neighborhood and they commute to the mall by one of the transportation modes discussed above. Thus, I assume that there is a common origin and destination. Individuals may choose to stay at home rather than coming to the mall, in which case they get $r > 0$ which is the reservation value of not coming to the mall. Each shopper purchases one and only one unit of the good sold by the mall. The value of the good to a shopper is v, which has the cumulative distribution function $F(v)$ with support $[0; \bar{v}]$ and the probability density function $f(v) > 0$. I assume that $F(v)$ has the standard monotone hazard rate property or that $f(v)/[1 - F(v)]$ is increasing.

Since the shoppers are risk-neutral, the utility of a shopper coming by car is $v - P - t - k$, where $k > 0$ represents the fuel cost of shoppers coming to the mall by car, while the utility of a shopper coming by public transportation is $v - P - s - d(N)$, where N represents the number of shoppers coming by public transportation.[1] Here, $d(N)$ represents the disutility of shoppers coming by public transportation and it is assumed that this disutility function has the convex form of $d(N) = aN^2$, where a is a positive constant. I analyze the model with a more general $d(N)$ in Subsection 4.4. There is strong evidence showing that comfort is one of the

[1] Similar to Voith (1998), I abstract from the issues of fixed costs related to car ownership.

main factors affecting commuting preferences of individuals (See, for example, Lisco (1968), Joireman et al. (1997), Steg (2003), and Haywood and Koning (2011)). Hence, I think that incorporating this disutility function into the model is both appropriate and necessary.

Overall, the model can be thought as a sequential game with three players. First, the government determines the public transportation fare s. After observing the behavior of the government, the mall decides on the good price P, and the parking fee t. After observing the choices of the government and the mall, the individuals choose from the following alternatives: staying at home, coming to the mall by car or by public transportation.

3.1 Equilibrium

In this subsection, I determine the equilibrium parking fee, the equilibrium price of the good and the equilibrium transportation fare. Because the model is sequential, I start from the last stage of the game and apply backward induction.

3.1.1 The problem of the individuals

First, I need to solve the problem of the individuals. An individual will come to the shopping mall by car if his utility of coming by car is greater than his utility of coming by public transportation, i.e., $v - P - t - k > v - P - s - d(N)$, and if his utility of coming by car is greater than or equal to his utility of staying at home, i.e., $v - P - t - k \geq r$. Similarly, an individual will come to the mall by public transportation if his utility of coming by public transportation is greater than his utility of coming by car, i.e., $v-P-s-d(N) > v-P-t-k$, and if his utility of coming by public transportation is greater than or equal to his utility of staying at home, i.e., $v - P - s - d(N) \geq r$. An individual will be indifferent between coming by car or by public transportation if his utility of coming by car is equal to his utility of coming by public transportation and this utility is greater than or equal to his utility of staying at home, i.e., $v - P - t - k = v - P - s - d(N) \geq r$.

In an equilibrium; $F(\tilde{v})\hat{N}$ individuals stay at home where $\tilde{v}$ is the value that makes individuals indifferent between staying at home and coming to the mall and $\tilde{v} = P + t + k + r$, N^* individuals come by public transportation where N^* is the number of individuals coming by public trasportation that makes shoppers indifferent between coming by car and coming by public transportation and $N^* = d^{-1}(t + k - s) = \sqrt{(t + k - s)/a}$, and the remaining $(1 - F(\tilde{v}))\hat{N} - N^*$ individuals come by car.[2]

[2]Notice that I do not consider the cases where s and t are determined such that $t + k < s + d(N)$ for all N or $t + k > s + d(N)$ for all N since my aim is to analyze the shopping mall behavior in determining P and t when some of the shoppers come by public transportation and some of them come by car. If there is an interior solution of the problem, there will be an N^* that satisfies $t + k = s + d(N^*)$. For the analysis of the cases in

Notice that the mode choice of shoppers is independent of the value of the good. That is I cannot claim that shoppers with high valuations of the good come by car and those with low valuations by public transportation, or vice versa. Hence, I can only determine how many shoppers choose to come by public transportation and how many of them by car; I cannot determine who chooses to come by public transportation and who by car.

3.1.2 The problem of the mall

After solving the problem of the individuals, I need to deal with the problem of the mall. The objective of the mall is to maximize its profit, $\Pi(P,t)$:

$$\Pi(P,t) = PN^* + (P+t-c)[(1-F(\tilde{v}))\hat{N} - N^*] \, , \tag{1}$$

subject to the rationality constraint $P+t-c \geq 0$ and the non-negativity constraint $P \geq 0$. The first-order conditions with respect to P and t are

$$\begin{aligned} \Pi_P &: (1-F(P+t+k+r))\hat{N} - (P+t-c)f(P+t+k+r)\hat{N} \\ \Pi_t &: (1-F(P+t+k+r))\hat{N} - (P+t-c)f(P+t+k+r)\hat{N} \\ &- \frac{t-c}{2a\sqrt{\frac{t+k-s}{a}}} - \sqrt{\frac{t+k-s}{a}} \, , \end{aligned} \tag{2}$$

respectively. Equating the first-order condition with respect to P, Π_P, to zero implicitly gives the equilibrium price of the good, P_{eq}:

$$P_{eq} = \frac{1-F(\tilde{v})}{f(\tilde{v})} - t + c \, . \tag{3}$$

If both first-order conditions are zero, then I will have

$$\sqrt{\frac{t+k-s}{a}} + \frac{t-c}{2a\sqrt{\frac{t+k-s}{a}}} = 0 \, . \tag{4}$$

Equation (4) implies that $t \leq c$ and leads to the equilibrium parking fee, t_{eq}:

$$t_{eq} = \frac{c+2s-2k}{3} \, . \tag{5}$$

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which an interior solution is not possible (i.e., the cases where $t+k < s+d(N)$ for all N or $t+k > s+d(N)$ for all N), see Appendix A.1.

The problem of the mall has a unique interior maximum.[3]

3.1.3 The problem of the government

Having solved the problem of the individuals and the mall, I can now analyze the problem of the government. The government maximizes the total welfare of all members in society by choosing a public transportation fare s. Thus, its objective function is, $Y(s)$, is given by

$$\begin{aligned} Y(s) &= r\hat{N}F(\tilde{v}) + E[v - P - t - k][\hat{N} - N^* - \hat{N}F(\tilde{v})] \\ &\quad + E[v - P - s - d(N^*)]N^* + PN^* + (P + t - c)[(1 - F(\tilde{v}))\hat{N} - N^*] \\ &\quad + sN^* \, . \end{aligned} \tag{6}$$

When determining s, the government should take into account the effects of its decision on the pricing behavior of the mall. Thus, it should consider two cases: case 1 in which the equilibrium public transportation fare leads to an equilibrium parking fee which is less than or equal to zero and case 2 in which the equilibrium public transportation fare leads to a positive equilibrium parking fee. Now, I analyze these two cases in turn and determine which one occurs in equilibrium.

Case 1: The public transportation fare s determined by the government results in a parking fee of t such that $t \leq 0$. Then, because choosing a negative parking fee is not practically implementable, the mall chooses $t = 0$ and determines the price of the good P accordingly. By using $t = 0$ and equation (3), the mall finds the equilibrium price of the good:

$$P_{eq\_case1} = \frac{1 - F(\tilde{v})}{f(\tilde{v})} + c \, . \tag{7}$$

By substituting equation (7) and $t = 0$ in equation (6), I get

$$\begin{aligned} Y^{case1} &= r\hat{N}F(\tilde{v}) + [(\int_{\tilde{v}}^{\bar{v}} vf(v)dv) - \frac{1 - F(\tilde{v})}{f(\tilde{v})} - c - k][\hat{N} - \hat{N}F(\tilde{v})] \\ &\quad + (\frac{1 - F(\tilde{v})}{f(\tilde{v})} + c)\sqrt{\frac{k - s}{a}} + \frac{1 - F(\tilde{v})}{f(\tilde{v})}[(1 - F(\tilde{v}))\hat{N} - \sqrt{\frac{k - s}{a}}] \\ &\quad + s\sqrt{\frac{k - s}{a}} \, . \end{aligned} \tag{8}$$

Notice that $\tilde{v} = P + t + k + r = [1 - F(\tilde{v})]/f(\tilde{v}) + c + k + r$ does not contain s. The

[3] See Appendix A.2 for the proof.

first-order condition with respect to s is

$$Y_s^{case1} : -\frac{c+s}{2a\sqrt{\frac{k-s}{a}}} + \sqrt{\frac{k-s}{a}} \; . \tag{9}$$

If this first-order condition is equal to zero, then I will have the equilibrium transportation fare, s_{eq_case1}:

$$s_{eq\_case1} = \frac{2k-c}{3} \geq 0 \; , \tag{10}$$

which is the maximizer of the problem of the government for case 1.[4]

Case 2: The public transportation fare s determined by the government results in a parking fee of t such that $t > 0$. Then, the mall chooses $t = (c + 2s - 2k)/3$ and determines the price of the good P accordingly. By using $t = (c + 2s - 2k)/3$ and equation (3), the mall finds the equilibrium price of the good:

$$P_{eq\_case2} = \frac{1 - F(\tilde{v})}{f(\tilde{v})} + \frac{2(c+k-s)}{3} \; . \tag{11}$$

By substituting equation (11) and $t = (c + 2s - 2k)/3$ in equation (6), I get

$$\begin{aligned} Y^{case2} = {} & r\hat{N}F(\tilde{v}) + [(\int_{\tilde{v}}^{\overline{v}} vf(v)dv) - \frac{1-F(\tilde{v})}{f(\tilde{v})} - c - k][\hat{N} - \hat{N}F(\tilde{v})] \\ & + [\frac{1-F(\tilde{v})}{f(\tilde{v})} + \frac{2(c+k-s)}{3}]\sqrt{\frac{c+k-s}{3a}} \\ & + \frac{1-F(\tilde{v})}{f(\tilde{v})}[(1-F(\tilde{v}))\hat{N} - \sqrt{\frac{c+k-s}{3a}}] + s\sqrt{\frac{c+k-s}{3a}} \; . \end{aligned} \tag{12}$$

Notice that $\tilde{v} = P + t + k + r = [1 - F(\tilde{v})]/f(\tilde{v}) + 2(c+k-s)/3 + (c+2s-2k)/3 + k + r = [1 - F(\tilde{v})]/f(\tilde{v}) + c + k + r$ does not contain s. The first-order condition with respect to s is

$$Y_s^{case2} : -\frac{\frac{2c+2k+s}{3}}{6a\sqrt{\frac{c+k-s}{3a}}} + \frac{1}{3}\sqrt{\frac{c+k-s}{3a}} \; . \tag{13}$$

If this first-order condition is equal to zero, then I will have the equilibrium transportation fare, s_{eq_case2}:

$$s_{eq\_case2} = 0 \; , \tag{14}$$

[4]See Appendix A.3 for the proof.

which is the maximizer of the problem of the government for case 2.[4]

Having analyzed the two possible cases, I now compare the value of objective functions Y^{case1} and Y^{case2} so that I can understand which of these cases happens in equilibrium. By substituting the relevant prices, parking fees and transportation fares, I find that

$$Y^{case1} = y + \frac{2c+2k}{3}\sqrt{\frac{c+k}{3a}} \tag{15}$$

$$Y^{case2} = y + \frac{2c+2k}{3}\sqrt{\frac{c+k}{3a}} \, ,$$

where $y = r\hat{N}F(\tilde{v}) + [(\int_{\tilde{v}}^{\bar{v}} vf(v)dv) - (1 - F(\tilde{v}))/f(\tilde{v}) - c - k][\hat{N} - \hat{N}F(\tilde{v})] + [(1 - F(\tilde{v}))^2\hat{N}]/f(\tilde{v})$ and $\tilde{v} = [1-F(\tilde{v})]/f(\tilde{v})+c+k+r$. This comparison shows that the welfare of society in the first case is equal to the welfare of society in the second case. Hence, the government is indifferent between choosing the public transportation fare which leads to case 1 and the public transportation fare which leads to case 2.

3.1.4 The equilibrium solution

If the government chooses public tranportation fare $s_{eq\_case1} = (2k - c)/3 \geq 0$, then the mall in turn will choose the equilibrium parking fee $t_{eq\_case1} = 0$ and the equilibrium good price $P_{eq\_case1} = [1 - F(\tilde{v})]/f(\tilde{v}) + c$. Then $N^* = \sqrt{(c+k)/3a}$ individuals come by public transportation. If the government chooses public tranportation fare $s_{eq\_case2} = 0$, then the mall in turn will choose the equilibrium parking fee $t_{eq\_case2} = (c-2k)/3 > 0$ and the equilibrium good price $P_{eq\_case2} = [1 - F(\tilde{v})]/f(\tilde{v}) + (2c+2k)/3$. Then, once again $N^* = \sqrt{(c+k)/3a}$ individuals come by public transportation.[5] Notice that which of the above equilibria occurs depends on the model parameters. I summarize these findings in the following proposition.

Proposition 1 *If $c \leq 2k$, then the government will choose a non-negative public transportation fare and the mall will prefer free provision of parking. If $c > 2k$, then the government will prefer free provision of public transportation and the mall will choose $(c-2k)/3 > 0$ as the equilibrium parking fee. Furthermore, the equilibrium parking fees are always less than the marginal cost of providing parking spaces.*

The mall faces a trade-off between attracting more individuals and charging higher prices both for the good and parking. In equilibrium, the mall chooses to provide parking at a price that is below the marginal cost of providing it. The reason for this result is that as long as

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[5]Notice that changing the value of a here only changes the modal split but not the total number of individuals who visit the shopping mall.

the parking fee is above the marginal cost of providing parking spaces, decreasing parking fee will result in positive marginal profits for the mall since the mall can earn more profits both by charging higher prices for the good without affecting the total number of individuals buying the good and by collecting parking fees from more shoppers. If the marginal cost of providing parking spaces is small enough, then the mall will prefer free provision of parking since its marginal profit is greater than or equal to zero when parking is provided for free. However, if the marginal cost of providing parking spaces is high, then the mall will choose a positive parking fee since its marginal profit is equal to zero at this parking fee. Thus, the mall embeds at least some of the costs of parking into the price of the good sold at the mall and use parking as a loss leader to increase the price of the good.

I still need to check whether the non-negativity and rationality constraints hold or not. Both for case 1 and case 2, one can simply see that the equilibrium price of the good is always greater than zero and $P_{eq} + t_{eq} - c = [1 - F(\tilde{v})]/f(\tilde{v}) > 0$; so both constraints hold in equilibrium.

3.2 The Socially Optimal Solution

The next question to ask is whether a social planner, who maximizes the welfare of society, agrees with the equilibrium solution or not. The problem of the social planner is to maximize the welfare of society, $Y(P, t, s)$:

$$\begin{aligned} Y(P,t,s) = {} & r\hat{N}F(\tilde{v}) + E[v - P - t - k][\hat{N} - N^* - \hat{N}F(\tilde{v})] \\ & + E[v - P - s - d(N^*)]N^* + PN^* + (P + t - c)[(1 - F(\tilde{v}))\hat{N} - N^*] \\ & + sN^* \,, \end{aligned} \tag{16}$$

by simultaneously choosing P, t and s. The first-order conditions of the problem are

$$\begin{aligned} Y_P &: \hat{N}f(\tilde{v})[r - \int_{\tilde{v}}^{\bar{v}} vf(v)dv + k + c] - [\hat{N} - \hat{N}F(\tilde{v})]f(\tilde{v})\tilde{v} \\ Y_t &: \hat{N}f(\tilde{v})[r - \int_{\tilde{v}}^{\bar{v}} vf(v)dv + k + c] - [\hat{N} - \hat{N}F(\tilde{v})]f(\tilde{v})\tilde{v} \\ & \quad - \frac{t - c - s}{2a\sqrt{\frac{t+k-s}{a}}} - \sqrt{\frac{t + k - s}{a}} \\ Y_s &: \frac{t - c - s}{2a\sqrt{\frac{t+k-s}{a}}} + \sqrt{\frac{t + k - s}{a}} \,. \end{aligned} \tag{17}$$

Equating the first-order conditions with respect to P and t to zero gives the socially optimal parking fee, t_{opt}, for an interior solution:

$$t_{opt} = \frac{3s + c - 2k}{3} \, . \tag{18}$$

Equating the first-order condition with respect to s to zero gives the socially optimal public transportation fare, s_{opt}:

$$s_{opt} = \frac{3t + 2k - c}{3} \, . \tag{19}$$

Equations (18) and (19) are the same, hence there are infinitely many (t, s) pairs that satisfy the planner's solution. This is partly due to the bilateral risk neutrality assumption. Here, the main concern of the social planner is to determine the socially optimal number of shoppers coming by public transportation and to find (t, s) pairs that lead to this optimal number. So, in all socially optimal solutions, $N^* = \sqrt{(c + k)/3a}$ individuals come by public transportation and the social planner chooses one of the (t, s) pairs satisfying equation (18) to achieve this outcome.

If I substitute $t = 0$ in equation (19), I will find that $s = (2k - c)/3$ and if I substitute $s = 0$ in equation (18), I will find that $t = (c - 2k)/3$. Hence, the equilibrium (t, s) pairs, namely $(0, (2k - c)/3)$ when $c \leq 2k$ and $((c - 2k)/3, 0)$ when $c > 2k$, are two of the many socially optimal pairs leading to the outcome that $\sqrt{(c + k)/3a}$ individuals come by public transportation.

Note that the socially optimal number of shoppers coming by public transportation is the same as the equilibrium number of shoppers coming by public transportation. This result does not imply that the social welfare in the equilibrium solution and the social welfare in the socially optimal solution are the same. I am unable to calculate the price of the good either explicitly or implicitly in the socially optimal solution and since I cannot calculate it, I cannot find $\tilde{v}$ and the number of shoppers coming by car. Hence, I cannot make a comparison between the welfare of society in the equilibrium solution and the welfare of society in the socially optimal solution.

4 Extensions

The previous section derives the main results. In this robustness section, I extend the base model in various ways and analyze whether the results change under different specifications of the model. First, I consider a case in which public transportation is not an option to come to the mall and the mall provides shuttle bus service. I find that total number of shoppers coming to the mall, the modal split and the equation determining the equilibrium price of the good are the same as the ones found in Subsection 3.1, and the equations determining the equilibrium parking fee and the equilibrium price of the shuttle bus are the same as the ones found in Subsection 3.2. Then, I analyze a case in which transportation is provided privately by a profit-maximizing transportation agency instead of by the government. In this case, the equilibrium parking fees are still less than the marginal cost of providing parking spaces, the equilibrium transportation fares are always positive, and the equilibrium number of shoppers coming by privately provided transportation is lower than the equilibrium number of shoppers coming by public transportation in Subsection 3.1. I, then, consider a case in which provision of public transportation is costly for the government. I find that the equilibrium parking fees are still less than the marginal cost of providing parking spaces, the equilibrium number of shoppers coming by public transportation in this case is lower than the equilibrium number of shoppers coming by public transportation in Subsection 3.1, and the profit of the mall in this case is less than the profit of it in Subsection 3.1. Lastly, I analyze the base model with a more general set of disutility functions and show that parking fee in an equilibrium is always less than the marginal cost of providing parking spaces for this set of disutility functions.

4.1 The Mall Providing Shuttle Bus Service

Here, I consider a scenario in which there is not public transportation, but the mall provides shuttle bus service. Let q denote the price of the shuttle bus to shoppers and $d(N)$ represent the disutility of shoppers coming by shuttle bus where $d(N) = aN^2$, a is a positive constant. In equilibrium, N^* individuals come by shuttle bus where $N^* = \sqrt{(t+k-q)/a}$. Then, the problem of the mall is to maximize its profit, $\Pi^{shuttle}(P,t,q)$:

$$\Pi^{shuttle}(P,t,q) = (P+q)N^* + (P+t-c)[(1-F(\tilde{v}))\hat{N} - N^*]\,, \tag{20}$$

by choosing P, t and q. The first-order conditions with respect to P, t and q are

$$\begin{aligned}
\Pi_P^{shuttle} &: (1-F(P+t+k+r))\hat{N} - (P+t-c)f(P+t+k+r)\hat{N} \qquad (21)\\
\Pi_t^{shuttle} &: (1-F(P+t+k+r))\hat{N} - (P+t-c)f(P+t+k+r)\hat{N}\\
&\quad - \frac{t-c-q}{2a\sqrt{\frac{t+k-q}{a}}} - \sqrt{\frac{t+k-q}{a}}\\
\Pi_q^{shuttle} &: \frac{t-c-q}{2a\sqrt{\frac{t+k-q}{a}}} + \sqrt{\frac{t+k-q}{a}}\,.
\end{aligned}$$

Equating the first-order condition with respect to P to zero implicitly gives the equilibrium price of the good, $P_{eq}^{shuttle}$:

$$P_{eq}^{shuttle} = \frac{1 - F(\tilde{v})}{f(\tilde{v})} - t + c\,. \tag{22}$$

If I equate the first-order conditions with respect to P and t to zero, then I will have the equilibrium parking fee, $t_{eq}^{shuttle}$:

$$t_{eq}^{shuttle} = \frac{3q + c - 2k}{3}\,. \tag{23}$$

If I equate the first-order condition with respect to q to zero, then I will have the equilibrium price of the shuttle bus, $q_{eq}^{shuttle}$:

$$q_{eq}^{shuttle} = \frac{3t + 2k - c}{3}\,. \tag{24}$$

Equations (23) and (24) imply that there are infinitely many (t, q) pairs from which the mall can choose. In equilibrium, $N^* = \sqrt{(c + k)/3a}$ individuals come by shuttle bus and the mall providing shuttle bus service chooses one of the various (t, q) pairs satisfying equation (23) to reach this equilibrium solution. Notice that this solution is a mixture of the equilibrium solution and the socially optimal solution in Section 3. Equation (22), which is the equation determining the equilibrium price of the good, the number of shoppers coming by car, the number of shoppers coming by the alternative mode, and the number of individuals staying at home are the same as the equilibrium solution in the base model in Subsection 3.1; the (t, q) pairs the mall can choose given by equations (23) or (24) are the same as the (t, s) pairs the social planner can choose in the socially optimal solution in the base model in Subsection 3.2.

4.2 Private Provision of Transportation

I now consider a scenario in which transportation is provided privately by a transportation agency that only cares about its own profits. This service has similar properties with public transportation provided by the government. Let s denote the private transportation fare and $d(N)$ represent the disutility of shoppers coming by transportation provided privately where $d(N) = aN^2$, a is a positive constant. In equilibrium, N^* individuals come by transportation provided privately where $N^* = \sqrt{(t + k - s)/a}$. The problem of the mall in this case is the same as the problem of the mall in Subsubsection 3.1.2 and the parking fee in this case is the same as the parking fee in equation (5), which is $t = (c + 2s - 2k)/3$. Then, the problem of the private transportation agency is to maximize its own profits, X:

$$X = sN^* \,, \tag{25}$$

by choosing s. The first-order condition with respect to s is

$$X_s : \sqrt{\frac{t+k-s}{a}} - \frac{s}{2a\sqrt{\frac{t+k-s}{a}}} \,. \tag{26}$$

By equating this first-order condition to zero, I get

$$s = \frac{2t+2k}{3} \,. \tag{27}$$

If the parking fee chosen by the mall is positive, simultaneously solving equation (5) and equation (27) yields the equilibrium parking fee, $t^{private}_{eq\_case1}$, and the equilibrium transportation fare under private provision of transportation, $s^{private}_{eq\_case1}$:

$$t^{private}_{eq\_case1} = \frac{3c-2k}{5} \,. \tag{28}$$

$$s^{private}_{eq\_case1} = \frac{2c+2k}{5} \,. \tag{29}$$

Notice that $t^{private}_{eq\_case1}$ in equation (28) is positive if and only if $3c > 2k$ and it is always less than the marginal cost of providing parking spaces. If the mall chooses free provision of parking, equation (27) will result in the equilibrium transportation fare, $s^{private}_{eq\_case2}$:

$$s^{private}_{eq\_case2} = \frac{2k}{3} \,, \tag{30}$$

and this case will occur if and only if $3c \leq 2k$. The following propositions summarize these findings.

Proposition 2 *When transportation is provided privately, the equilibrium transportation fares are always positive and they are greater than the equilibrium public transportation fares in the base model. Moreover, the equilibrium parking fees are always less than the marginal cost of providing parking spaces.*

The equilibrium transportation fares should be positive; otherwise profit-maximizing transportation agency will not provide transportation. The agency chooses equilibrium transportation fares such that they are greater than the equilibrium public transportation fares in the base model since it is only interested in its own profits, not the total welfare of all members in society. In equilibrium, the mall still chooses parking fees which are less than the marginal cost of providing parking spaces. Hence, it uses parking as a loss leader in order to charge higher prices for the good sold and maximize its profits.

Proposition 3 *The equilibrium number of shoppers coming by transportation provided privately ($N^* = \sqrt{(c+k)/5a}$ if $t > 0$ and $N^* = \sqrt{k/3a}$ if $t = 0$) is lower than the equilibrium number of shoppers coming by public transportation in the base model. Furthermore, the equilibrium profit of the mall under private provision of transportation is less than the equilibrium profit of the mall in the base model for all positive values of c and k.*[6]

When transportation is provided privately, the equilibrium transportation fares are greater than the ones found in the base model and the equilibrium parking fees are greater than or equal to the ones found in the base model. Moreover, the increase in the equilibrium transportation fares is more than the increase in the equilibrium parking fees and this implies that the equilibrium number of shoppers coming by privately provided transportation will be lower than the equilibrium number of shoppers coming by public transportation in the base model.

The number of shoppers coming to the mall in this case is the same as the number of shoppers coming to the mall in the base model. This information combined with the fact that the equilibrium parking fees in this case are greater than or equal to the equilibrium parking fees in the base model implies that the price of the good in this case will be less than or equal to the one in the base model. This fact and the fact that the equilibrium parking fees are less than the marginal cost of providing parking spaces lead to the following conclusion: The equilibrium profit of the mall under private provision of transportation is less than the equilibrium profit of the mall in the base model.

4.3 Costly Provision of Public Transportation

I now consider a scenario in which provision of public transportation is costly for the government. I show that adding this cost into the base model changes none of my main results qualitatively and hence its exclusion from the model is just a valid simplifying assumption.[7] The problem of the individuals is the same as the problem solved in Subsubsection 3.1.1 and the problem of the mall is the same as the problem solved in Subsubsection 3.1.2. However, the problem of the government is different and I analyze it as two separate cases.

[6] See Appendix A.4 for calculations.

[7] For simplicity, I assume constant public transportation cost per passenger. However, this assumption may not reflect reality since high fixed costs of providing public transportation may lead a decline in per passenger cost as the number of shoppers choosing public transportation increases.

Case 1: s determined by the government results in a t such that $t \leq 0$ and the mall chooses $t = 0$ and $P = [1 - F(\tilde{v})]/f(\tilde{v}) + c$. Then, the problem of the government is to maximize the welfare of society, $Y^{case1\_c_{PT}}$:

$$\begin{aligned} Y^{case1\_c_{PT}} &= r\hat{N}F(\tilde{v}) + [(\int_{\tilde{v}}^{\bar{v}} vf(v)dv) - \frac{1 - F(\tilde{v})}{f(\tilde{v})} - c - k][\hat{N} - \hat{N}F(\tilde{v})] \\ &+ (\frac{1 - F(\tilde{v})}{f(\tilde{v})} + c)\sqrt{\frac{k - s}{a}} + \frac{1 - F(\tilde{v})}{f(\tilde{v})}[(1 - F(\tilde{v}))\hat{N} - \sqrt{\frac{k - s}{a}}] \\ &+ (s - c_{PT})\sqrt{\frac{k - s}{a}} \ , \end{aligned} \tag{31}$$

by choosing s, where $c_{PT} > 0$ denotes the marginal cost of providing public transportation. Equating the first-order condition with respect to s to zero results in the equilibrium transportation fare, $s^{c_{PT}}_{eq\_case1}$:

$$s^{c_{PT}}_{eq\_case1} = \frac{2k - c + c_{PT}}{3} \geq 0 \ . \tag{32}$$

Case 2: s determined by the government results in a t such that $t > 0$ and the mall chooses $t = (c + 2s - 2k)/3$ and $P = [1 - F(\tilde{v})]/f(\tilde{v}) + 2(c + k - s)/3$. Then, the problem of the government is to maximize the welfare of society, $Y^{case2\_c_{PT}}$:

$$\begin{aligned} Y^{case2\_c_{PT}} &= r\hat{N}F(\tilde{v}) + [(\int_{\tilde{v}}^{\bar{v}} vf(v)dv) - \frac{1 - F(\tilde{v})}{f(\tilde{v})} - c - k][\hat{N} - \hat{N}F(\tilde{v})] \\ &+ [\frac{1 - F(\tilde{v})}{f(\tilde{v})} + \frac{2(c + k - s)}{3}]\sqrt{\frac{c + k - s}{3a}} \\ &+ \frac{1 - F(\tilde{v})}{f(\tilde{v})}[(1 - F(\tilde{v}))\hat{N} - \sqrt{\frac{c + k - s}{3a}}] + (s - c_{PT})\sqrt{\frac{c + k - s}{3a}} \ , \end{aligned} \tag{33}$$

by choosing s. Equating the first-order condition with respect to s to zero results in the equilibrium transportation fare, $s^{c_{PT}}_{eq\_case2}$:

$$s^{c_{PT}}_{eq\_case2} = c_{PT} \ . \tag{34}$$

So far, I have analyzed case 1 and case 2. Now, I need to compare $Y^{case1\_c_{PT}}$ and $Y^{case2\_c_{PT}}$ in order to understand which of these cases is superior in terms of welfare. By substituting the relevant prices, parking fees and transportation fares, I find that

$$
\begin{aligned}
Y^{case1\_c_{PT}} &= y + \frac{2c+2k-2c_{PT}}{3}\sqrt{\frac{c+k-c_{PT}}{3a}} \\
Y^{case2\_c_{PT}} &= y + \frac{2c+2k-2c_{PT}}{3}\sqrt{\frac{c+k-c_{PT}}{3a}}\ ,
\end{aligned}
\tag{35}
$$

where $y = r\hat{N}F(\tilde{v}) + [(\int_{\tilde{v}}^{\bar{v}} vf(v)dv) - (1-F(\tilde{v}))/f(\tilde{v}) - c - k][\hat{N} - \hat{N}F(\tilde{v})] + [(1 - F(\tilde{v}))^2\hat{N}]/f(\tilde{v})$ and $\tilde{v} = [1-F(\tilde{v})]/f(\tilde{v}) + c + k + r$. This comparison shows that the welfare of society in the first case is equal to the welfare of society in the second case, hence the government is indifferent between choosing the equilibrium public transportation fare which leads to case 1 and the one which leads to case 2.

Notice that which of the above equilibria occurs depends on the model parameters. If $c + 2c_{PT} \leq 2k$, then the first case will occur, i.e., the government chooses a non-negative transportation fare and the mall prefers free provision of parking. If $c + 2c_{PT} > 2k$, the second case will occur, i.e., the government provides transportation at its marginal cost and the mall chooses $t_{eq\_case2}^{c_{PT}} = (c - 2k + 2c_{PT})/3 > 0$. I summarize these findings in the following proposition.

Proposition 4 *The equilibrium number of shoppers coming by public transportation under costly provision of transportation by the government, $\sqrt{(c+k-c_{PT})/3a}$, is lower than the equilibrium number of shoppers coming by public transportation in the base model. Moreover, as expected, the equilibrium profit of the mall in the costly provision of public transportation by the government case is less than the equilibrium profit of the mall in the base model for all positive values of c and k.*[8]

The equilibrium transportation fares under costly provision of public transportation are greater than the ones found in the base model and the equilibrium parking fees are greater than or equal to the ones found in the base model. Moreover, the increase in the equilibrium transportation fares is more than the increase in the equilibrium parking fees and this implies that the equilibrium number of shoppers coming by public transportation under costly provision of transportation will be lower than the equilibrium number of shoppers coming by public transportation in the base model.

The number of shoppers coming to the mall in this case is the same as the number of shoppers coming to the mall in the base model. This information combined with the fact that the equilibrium parking fees in this case are greater than or equal to the equilibrium parking fees in the base model implies that the price of the good in this case will be less than or equal

[8]See Appendix A.5 for calculations.

to the one in the base model. This fact and the fact that the equilibrium parking fees are less than the marginal cost of providing parking spaces lead to the following conclusion: The equilibrium profit of the mall under costly provision of public transportation is less than the equilibrium profit of the mall in the base model.

The next step is to analyze the social planner's problem when provision of public transportation by the government is costly. The problem of the social planner is to maximize the welfare of society, $Y^{cPT}(P,t,s)$:

$$\begin{aligned} Y^{cPT}(P,t,s) &= r\hat{N}F(\tilde{v}) + [(\int_{\tilde{v}}^{\bar{v}} vf(v)dv) - P - t - k][\hat{N} - \hat{N}F(\tilde{v})] \\ &+ P\sqrt{\frac{t+k-s}{a}} + (P+t-c)[(1-F(\tilde{v}))\hat{N} - \sqrt{\frac{t+k-s}{a}}] \\ &+ (s-c_{PT})\sqrt{\frac{t+k-s}{a}}\ , \end{aligned} \tag{36}$$

by choosing P,t and s. The first-order conditions with respect to P,t and s are

$$\begin{aligned} Y_P^{cPT} &: \hat{N}f(\tilde{v})[r - \int_{\tilde{v}}^{\bar{v}} vf(v)dv + k + c] - [\hat{N} - \hat{N}F(\tilde{v})]f(\tilde{v})\tilde{v} \\ Y_t^{cPT} &: \hat{N}f(\tilde{v})[r - \int_{\tilde{v}}^{\bar{v}} vf(v)dv + k + c] - [\hat{N} - \hat{N}F(\tilde{v})]f(\tilde{v})\tilde{v} \\ &- \frac{t-c-s+c_{PT}}{2a\sqrt{\frac{t+k-s}{a}}} - \sqrt{\frac{t+k-s}{a}} \\ Y_s^{cPT} &: \frac{t-c-s+c_{PT}}{2a\sqrt{\frac{t+k-s}{a}}} + \sqrt{\frac{t+k-s}{a}}\ . \end{aligned} \tag{37}$$

Equating the first-order conditions with respect to P and t to zero yields the socially optimal parking fee, t_{opt}^{cPT}:

$$t_{opt}^{cPT} = \frac{3s+c-2k-c_{PT}}{3}\ . \tag{38}$$

Equating the first-order condition with respect to s to zero yields the socially optimal transportation fare, s_{opt}^{cPT}:

$$s_{opt}^{cPT} = \frac{3t+2k-c+c_{PT}}{3}\ . \tag{39}$$

Equation (38) and equation (39) imply that there are infinitely many (t,s) pairs which lead to the socially optimal solution in which $N^* = \sqrt{(k+c-c_{PT})/3a}$ individuals come by public transportation. Hence, the socially optimal number of shoppers coming by public transportation when provision of transportation by the government is costly is lower than the socially optimal number of shoppers coming by public transportation in Subsection 3.2.

4.4 Generalization of Disutility Function

Up to this point, I have assumed a specific functional form for disutility function, namely $d(N) = aN^2$. In this subsection, I go beyond this assumption and consider a scenario in which $d(N) : (0, \infty) \to (0, \infty)$ is an increasing and differentiable function. I want to investigate if my results are due to the specific functional form of disutility function I have assumed in the base model or if they are generalizable to the set of increasing and differentiable positive functions.

The problem of the individuals is the same as the problem in Subsubsection 3.1.1, and in equilibrium N^* individuals come by public transportation, where $N^* = d^{-1}(t + k - s)$. The problem of the mall is to maximize its profit, $\Pi^{gen}(P, t)$:

$$\Pi^{gen}(P, t) = PN^* + (P + t - c)[(1 - F(\tilde{v}))\hat{N} - N^*] \,, \tag{40}$$

by choosing P and t. The first-order conditions with respect to P and t are

$$\begin{aligned} \Pi^{gen}_P &: (1 - F(P + t + k + r))\hat{N} - (P + t - c)f(P + t + k + r)\hat{N} \\ \Pi^{gen}_t &: (1 - F(P + t + k + r))\hat{N} - (P + t - c)f(P + t + k + r)\hat{N} \\ &- \frac{t - c}{d'(d^{-1}(t + k - s))} - d^{-1}(t + k - s) \,. \end{aligned} \tag{41}$$

If both first-order conditions are zero, then I will have

$$\frac{t - c}{d'(d^{-1}(t + k - s))} + d^{-1}(t + k - s) = 0 \,. \tag{42}$$

Rearranging equation (42) leads to

$$c - t = d'(d^{-1}(t + k - s))d^{-1}(t + k - s) \,. \tag{43}$$

Since the inverse of an increasing function is also increasing, the derivative of an increasing function is non-negative, and the inverse of $d(N)$ is positive; the right-hand side of equation (43) is non-negative, which in turn requires the left-hand side of the equation to be non-negative and implies $t \leq c$. This finding leads to the following proposition.

Proposition 5 *For positive, increasing and differentiable disutility functions, the equilibrium parking fees are always less than the marginal cost of providing parking spaces.*

In this robustness section, I analyze whether the results of the base model change under different specifications. I find that the equilibrium parking fees found in the base model are two of the many equilibrium parking fees when the mall provides shuttle bus service. When transportation is privately provided by a profit-maximizing transportation agency or when provision of public transportation is costly for the government, the equilibrium parking fees are still less than the marginal cost of providing parking spaces. This result also holds for a general set of disutility functions.

5 Related Literature on Bundling and Loss Leaders

The mall can be thought as selling the good alone to the shoppers coming by public transportation and selling the good bundled with parking spaces to the shoppers coming by car. Hence, in this section I analyze the relation between my paper and the bundling literature. *United States v. Loew's, Inc: A Note on Block Booking* (Stigler, 1963) is the first paper that implicitly examines the practice of bundling. Stigler argues that block booking is a price discrimination technique by which one can sort customers into different groups according to their reservation prices and extract consumer surplus. By using the same argument and the same framework as Stigler, Adams and Yellen (1976) show that the commodity bundling can be profitable for a monopolist even in the absence of cost considerations and complementarity of bundle components. They coin the term "mixed bundling" for the practice of selling the same commodities both separately and in packages. Furthermore, they state that "commodity bundles sometimes include goods that cannot be sold separately in the market place", and give the example of an automobile as a package of transportation services and luxury since luxury cannot be sold separately in the market. In line with the definitions and concepts introduced in Adams and Yellen, the mall in my paper can be thought as implicitly employing mixed bundling practices in which the good is sold either alone or bundled with parking. Notice that the mall in my paper does not sell parking spaces separately in the market.

In Adams and Yellen, the two products are independently valued products, i.e., they are neither substitutes nor complements. Lewbel (1985) generalizes the paper of Adams and Yellen to the cases in which customers may perceive commodities as complements or substitutes, including the cases in which commodities are substitutes for some customers and complements for the others. Lewbel analyzes each case graphically and then concludes that bundling does not require complementarity of the goods to be profitable and complementarity of the goods is not a sufficient condition for bundling to be profitable. Telser (1979) in *A theory of monopoly of complementary goods* shows that a monopolist can increase his profits by selling complementary goods in groups to different kinds of shoppers. Venkatesh and Kamakura (2003) in *Optimal bundling and pricing under a monopoly* analyze optimal solutions for complements and substitutes and show how marginal costs play an important role in determining the optimal strategy. In my paper, parking spaces and the good sold by the mall can be thought as complements since when the price of the good increases, ceteris paribus, the demand for parking, $(1 - F(\hat{v}))\hat{N} - N^*$, will decrease and when the parking fee increases, ceteris paribus, the demand for the good, $(1 - F(\hat{v}))\hat{N}$, will decrease. In fact, in my model, parking spaces and the good are perfect complements for the shoppers coming to the mall by car and they are independent goods for the shoppers coming to the mall by public transportation. The value of parking spaces for the shoppers coming by public transportation is zero.

In the base model and also in nearly all modifications of the base model, the mall chooses equilibrium parking fees which are less than the marginal cost of providing parking spaces. This result implies that the mall uses parking as a loss leader. Hence, my paper relates to the loss leader literature. Telser (1979) states that the sale of one or more complementary goods at prices below the marginal costs may be profitable for the seller as long as the seller has the monopoly power over all complementary products and is unable to sell them in different packages to different groups of shoppers. Telser also comments on one of the most common features of loss leader products, namely the commodity sold at a price below its marginal cost is sold earlier than the other commodity, and the desire to explain the success of the loss leader strategy based on this feature. He states that the success of the loss leader strategy does not depend on mistaken customer expectations, but depends on the assumption that the seller cannot perfectly price discriminate. In my model, the mall has a monopoly over the good and parking spaces and it does not charge different prices for different customer groups. It offers the good at the same price to both groups, hence it does not perfectly price discriminate. So, the explanation offered by Telser (1979) totally fits my model.

Hess and Gerstner (1987) interpret the loss leader pricing as a bundling strategy in which retailers bundle impulse goods, the goods on which the retailers have monopoly power, with leader products, the products priced below their costs to attract customers to the retail stores. Lal and Matutes (1989) claim that one reason for which firms employ the loss leader strategy is related to satisfying the needs of heterogeneous customers in a perfect information duopoly setting. Lal and Matutes (1994) consider an economy in which every customer has the same willingness to pay for the products and find that the interplay between rational expectations, multi-product competition and imperfect information of customers about prices may lead to an equilibrium in which retailers offer loss leaders to attract more customers. They also conclude that the goods with lower reservation prices and the goods bought more frequently are offered as loss leaders. DeGraba (2006) takes an approach different from the papers discussed above. He claims that goods should be priced as loss leaders if the group of customers buying these goods is more profitable than the group of customers who does not buy them. DeGraba also opposes the idea of classifying the most frequently bought goods as loss leaders.

The imperfect information explanation offered by Lal and Matutes (1994) and most of the papers in the loss leader literature is not relevant to my paper since the individuals in my model are aware of the price of the good and the parking fee in advance. Moreover, most papers in the loss leader literature analyze competitive environments in which at least two retailers compete for customers whereas the mall in my paper has a monopoly power. However, the individuals' having the outside option of staying at home adds a competitive flavor into the model. Offering parking spaces as loss leaders enables the mall to increase the price of the good without changing the full price and the total number of shoppers, and consequently increase the profits of the mall.

6 Conclusion

In this paper, I construct a model of shopping mall parking, in which shoppers have two alternative modes (car and public transportation) by which they come to the mall. The mall provides parking spaces for those who come by car. I show that in equilibrium, the mall charges parking fees less than the marginal cost of providing parking spaces, so it embeds some proportion of the cost of providing parking spaces into the price of the good. Hence, in a sense the mall employs a kind of mixed bundling practice and uses parking as a loss leader. Moreover, this equilibrium result of charging parking fees less than the marginal cost of providing parking spaces is one of the socially optimal solutions and robust to the extensions of the model.

In my model, parking lot size is exogenous for simplicity. In reality, it is a choice variable and should be endogenously dealt with. Some papers in the parking literature, including Voith (1998), Arnott and Inci (2006), Anderson and dePalma (2007) and Hasker and Inci (2012) discuss endogenous land use. So, an extension of my model may analyze a case in which the demand for the good varies and the mall determines parking lot size as well as the price of the good and the parking fee.

Another extension of my model may deal with the traffic congestion externality. In my model, k represents the fuel cost of shoppers coming by car and it is constant. Modeling k as a function of shoppers coming by car is one way of including traffic congestion into the model. Traffic congestion is an important component of the parking literature and many papers such as Arnott and Yan (2000) and Kraus (2003) incorporate it in their models. So, adding traffic congestion externality to the model would be worthy and the effects of this modification on the equilibrium and socially optimal solutions should be analyzed.

In my model, mode choice of shoppers is independent from the value of the good. In other words, the shoppers may come by car or public transportation regardless of their valuation of the good. An alternative version of this model can be the one in which valuations of the individuals are somehow linked to their mode choice decisions. Analyzing the equilibrium and socially optimal solutions in such a setting may be a fruitful topic for future research.

A further extension of my paper can be to model the shopping mall as a monopoly platform of two-sided markets. Rochet and Tirole (2003) develop a model which mainly deals with usage externalities of two-sided markets. They analyze the model as an illustration of the payment card industry, but it is applicable to more general situations. They give some examples of two-sided markets and the shopping mall case is one of these examples. In this context, a mall can be modeled as a two-sided platform which serves both shoppers by providing parking spaces and shops by providing shopping spaces. Furthermore, Rochet and Tirole (2006) state that the price structure in a two-sided market is very important for many economists, private and public decision makers. For this reason, managers sometimes prefer using one side of the market as a loss leader. Then, the authors analyze under what conditions the price structure really matters.

All in all, this paper is an important step to better understand the shopping mall parking behavior, but further research is needed in order to grasp the big picture.

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A Appendix

Appendix A.1

If s and t are determined such that $t + k < s + d(N)$ for all N, then all shoppers will come by car. In this case, the problem of the mall is to maximize

$$\Pi(P, t) = (P + t - c)(1 - F(\tilde{v}))\hat{N} \;, \tag{A-1}$$

by choosing P and t, where $\tilde{v} = P + t + k + r$. The first-order conditions with respect to P and t are

$$\begin{aligned} \Pi_P &: (1 - F(P + t + k + r))\hat{N} - (P + t - c)f(P + t + k + r)\hat{N} \\ \Pi_t &: (1 - F(P + t + k + r))\hat{N} - (P + t - c)f(P + t + k + r)\hat{N} \;. \end{aligned} \tag{A-2}$$

These two conditions are the same and equating them to zero gives

$$P + t = \frac{1 - F(\tilde{v})}{f(\tilde{v})} + c \;. \tag{A-3}$$

Hence, P and t cannot be uniquely determined in this case. This solution is the same as the solution of Hasker and Inci (2012) model in the absence of insurance motive.

If s and t are determined such that $t + k > s + d(N)$ for all N, then all shoppers will come by public transportation. This result in turn implies that the mall will not choose t since nobody comes by car. Then, the problem of the mall is to maximize

$$\Pi(P) = P(1 - F(\tilde{v}))\hat{N} \;, \tag{A-4}$$

by choosing P, where $\tilde{v} = P + s + d((1 - F(\tilde{v}))\hat{N}) + r$. The first-order condition with respect to P is

$$\Pi_P : (1 - F(\tilde{v}))\hat{N} - Pf(\tilde{v})\hat{N}\frac{\partial \tilde{v}}{\partial P} \;. \tag{A-5}$$

Notice that $H(\tilde{v}, P) = \tilde{v} - P - s - d((1 - F(\tilde{v}))\hat{N}) - r = 0$ defines an implicit function. By using the implicit function theorem, I get

$$\Pi_P : (1 - F(\tilde{v}))\hat{N} - \frac{Pf(\tilde{v})\hat{N}}{1 + d'((1 - F(\tilde{v}))\hat{N})f(\tilde{v})\hat{N}} \;. \tag{A-6}$$

By equating equation (A-6) to zero, the price of the good is implicitly defined as

$$P = \frac{(1 - F(\tilde{v}))[1 + d'((1 - F(\tilde{v}))\hat{N})f(\tilde{v})\hat{N}]}{f(\tilde{v})} \ . \tag{A-7}$$

Appendix A.2

To verify that the equilibrium price of the good determined by equation (3) and the equilibrium parking fee determined by equation (5) are maximizers, I form the Hessian matrix and check for its negative definiteness. To form Hessian, I derive the second-order conditions of the maximization problem of the mall which are

$$\begin{aligned}
\Pi_{PP} &: -2f(P+t+k+r)\hat{N} - f'(P+t+k+r)\hat{N}(P+t-c) \\
\Pi_{Pt} &: -2f(P+t+k+r)\hat{N} - f'(P+t+k+r)\hat{N}(P+t-c) \\
\Pi_{tP} &: -2f(P+t+k+r)\hat{N} - f'(P+t+k+r)\hat{N}(P+t-c) \\
\Pi_{tt} &: -\frac{3t+4k-4s+c}{2\sqrt{a(t+k-s)}(2t+2k-2s)} - 2f(P+t+k+r)\hat{N} \\
&\quad - f'(P+t+k+r)\hat{N}(P+t-c) \ .
\end{aligned} \tag{A-8}$$

First, I need to determine the sign of $-2f(\tilde{v})\hat{N} - f'(\tilde{v})\hat{N}(P+t-c)$. By replacing P and t with their values found in equations (3) and (5) respectively, I get $-2f(\tilde{v})\hat{N} - f'(\tilde{v})\hat{N}(1-F(\tilde{v}))/f(\tilde{v})$. From the monotone hazard rate property discussed in Section 3, I know that $[f^2(\tilde{v}) + f'(\tilde{v})(1-F(\tilde{v}))]/[1-F(\tilde{v})]^2 > 0$. If I multiply this expression by $\hat{N}[1-F(\tilde{v})]^2/f(\tilde{v})$, I will have $f(\tilde{v})\hat{N} + f'(\tilde{v})\hat{N}(1-F(\tilde{v}))/f(\tilde{v}) > 0$ which implies that $2f(\tilde{v})\hat{N} + f'(\tilde{v})\hat{N}(1-F(\tilde{v}))/f(\tilde{v}) > 0$ since the probability density function is always non-negative. Hence, $-2f(\tilde{v})\hat{N} - f'(\tilde{v})\hat{N}(1-F(\tilde{v}))/f(\tilde{v}) < 0$.

Then, I need to determine the sign of $-(3t+4k-4s+c)/[2\sqrt{a(t+k-s)}(2t+2k-2s)]$. By replacing t with its value found in equation (5), I have $-3/(2\sqrt{a(t+k-s)})$, which is negative. The resulting Hessian matrix evaluated at P_{eq} and t_{eq} is

$$\begin{bmatrix}
-2f(\tilde{v})\hat{N} - f'(\tilde{v})\hat{N}\frac{1-F(\tilde{v})}{f(\tilde{v})} & -2f(\tilde{v})\hat{N} - f'(\tilde{v})\hat{N}\frac{1-F(\tilde{v})}{f(\tilde{v})} \\
-2f(\tilde{v})\hat{N} - f'(\tilde{v})\hat{N}\frac{1-F(\tilde{v})}{f(\tilde{v})} & -\frac{3}{2\sqrt{a(t+k-s)}} - 2f(\tilde{v})\hat{N} - f'(\tilde{v})\hat{N}\frac{1-F(\tilde{v})}{f(\tilde{v})}
\end{bmatrix}$$

and it is negative-definite. This negative-definiteness implies that P_{eq} and t_{eq} are local maximizers for the mall.

Notice that there is only one critical (P, t) pair and it is a local maximizer. Hence, either (P_{eq}, t_{eq}) pair is the global maximizer and the problem of the mall has an interior solution or the global maximum lies on the boundaries.

Appendix A.3

To show that the equilibrium transportation fare found in case 1 in Subsubsection 3.1.3 is the global maximum, I need to look at the second derivative of Y^{case1} which is

$$Y_{ss}^{case1} : -\frac{4k-3s+c}{4a(k-s)\sqrt{\frac{k-s}{a}}} . \tag{A-9}$$

By rearranging this derivative, I have $-[(3/(4a\sqrt{(k-s)/a}))+((k+c)/(4a(k-s)\sqrt{(k-s)/a}))]$, which is negative. So, Y_{ss}^{case1} is always negative which implies that $s_{eq\_case1} = (2k-c)/3$ is the only critical point. Since it is the only critical point, it is the global maximum of the problem of the government in case 1 in Subsubsection 3.1.3.

Similarly, to show that the equilibrium transportation fare found in case 2 in Subsubsection 3.1.3 is the global maximum, I need to look at the second derivative of Y^{case2} which is

$$Y_{ss}^{case2} : -\frac{2c+2k-s}{36a(c+k-s)\sqrt{\frac{c+k-s}{3a}}} . \tag{A-10}$$

By rearranging this derivative, I have $-[(1/(36a\sqrt{(c+k-s)/3a})) + ((k+c)/(36a(c+k-s)\sqrt{(c+k-s)/3a}))]$, which is negative. So, Y_{ss}^{case2} is always negative which implies that $s_{eq\_case2} = 0$ is the only critical point. Since it is the only critical point, it is the global maximum of the problem of the government in case 2 in Subsubsection 3.1.3.

Appendix A.4

To prove that the equilibrium profit of the mall under private provision of transportation, $\Pi^{private}$, is less than the equilibrium profit of the mall in the base model, Π, for all positive values of c and k; I analyze three cases.

If $2k \geq 3c$, then the mall will choose free provision of parking spaces in both cases. The resulting profits are

$$\begin{aligned} \Pi &= \frac{[1-F(\tilde{v})]^2\hat{N}}{f(\tilde{v})} + c\sqrt{\frac{c+k}{3a}} \\ \Pi^{private} &= \frac{[1-F(\tilde{v})]^2\hat{N}}{f(\tilde{v})} + c\sqrt{\frac{k}{3a}} , \end{aligned} \tag{A-11}$$

where $\tilde{v} = [1-F(\tilde{v})]/f(\tilde{v})+c+k+r$. So, the equilibrium profit of the mall in the base model is greater than the equilibrium profit of the mall under private provision of transportation.

If $c \leq 2k < 3c$, then the mall in the base model will choose free provision of parking spaces, but the mall in the private provision of transportation case will choose a positive parking fee, $t = (3c - 2k)/5$. The resulting profits are

$$\Pi = \frac{[1 - F(\tilde{v})]^2 \hat{N}}{f(\tilde{v})} + c\sqrt{\frac{c+k}{3a}} \tag{A-12}$$
$$\Pi^{private} = \frac{[1 - F(\tilde{v})]^2 \hat{N}}{f(\tilde{v})} + \frac{2c+2k}{5}\sqrt{\frac{c+k}{5a}},$$

where $\tilde{v} = [1 - F(\tilde{v})]/f(\tilde{v}) + c + k + r$. Since $2k < 3c$ implies $(2c + 2k)/5 < c$, and $\sqrt{(c+k)/5a} < \sqrt{(c+k)/3a}$; the equilibrium profit of the mall in the base model is greater than the equilibrium profit of the mall under private provision of transportation.

If $2k < c$, then the mall will choose positive parking fees in both cases. The resulting profits are

$$\Pi = \frac{[1 - F(\tilde{v})]^2 \hat{N}}{f(\tilde{v})} + \frac{2c+2k}{3}\sqrt{\frac{c+k}{3a}} \tag{A-13}$$
$$\Pi^{private} = \frac{[1 - F(\tilde{v})]^2 \hat{N}}{f(\tilde{v})} + \frac{2c+2k}{5}\sqrt{\frac{c+k}{5a}},$$

where $\tilde{v} = [1 - F(\tilde{v})]/f(\tilde{v}) + c + k + r$. So, the equilibrium profit of the mall in the base model is greater than the equilibrium profit of the mall under private provision of transportation.

Appendix A.5

To prove that the equilibrium profit of the mall in the costly provision of transportation by the government case, $\Pi^{c_{PT}}$, is less than the equilibrium profit of the mall in the base model, Π, for all positive values of c and k; I analyze three cases.

If $2k \geq c + 2c_{PT}$, then the mall will choose free provision of parking spaces in both cases. The resulting profits are

$$\Pi = \frac{[1 - F(\tilde{v})]^2 \hat{N}}{f(\tilde{v})} + c\sqrt{\frac{c+k}{3a}} \tag{A-14}$$
$$\Pi^{c_{PT}} = \frac{[1 - F(\tilde{v})]^2 \hat{N}}{f(\tilde{v})} + c\sqrt{\frac{c+k-c_{PT}}{3a}},$$

where $\tilde{v} = [1 - F(\tilde{v})]/f(\tilde{v}) + c + k + r$. So, the equilibrium profit of the mall in the base model is greater than the equilibrium profit of the mall when provision of public transportation by the government is costly.

If $c \leq 2k < c + 2c_{PT}$, then the mall in the base model will choose free provision of parking spaces, but the mall in the costly provision of public transportation case will choose a positive parking fee, $t = (c - 2k + 2c_{PT})/3$. The resulting profits are

$$\Pi = \frac{[1-F(\tilde{v})]^2 \hat{N}}{f(\tilde{v})} + c\sqrt{\frac{c+k}{3a}} \tag{A-15}$$

$$\Pi^{c_{PT}} = \frac{[1-F(\tilde{v})]^2 \hat{N}}{f(\tilde{v})} + \frac{2c+2k-2c_{PT}}{3}\sqrt{\frac{c+k-c_{PT}}{3a}},$$

where $\tilde{v} = [1-F(\tilde{v})]/f(\tilde{v})+c+k+r$. Since $2k < c+2c_{PT}$ implies $(2c+2k-2c_{PT})/3 < c$, and $\sqrt{(c+k-c_{PT})/3a} < \sqrt{(c+k)/3a}$; the equilibrium profit of the mall in the base model is greater than the equilibrium profit of the mall when provision of public transportation by the government is costly.

If $2k < c$, then the mall will choose positive parking fees in both cases. The resulting profits are

$$\Pi = \frac{[1-F(\tilde{v})]^2 \hat{N}}{f(\tilde{v})} + \frac{2c+2k}{3}\sqrt{\frac{c+k}{3a}} \tag{A-16}$$

$$\Pi^{c_{PT}} = \frac{[1-F(\tilde{v})]^2 \hat{N}}{f(\tilde{v})} + \frac{2c+2k-2c_{PT}}{3}\sqrt{\frac{c+k-c_{PT}}{3a}},$$

where $\tilde{v} = [1-F(\tilde{v})]/f(\tilde{v})+c+k+r$. So, the equilibrium profit of the mall in the base model is greater than the equilibrium profit of the mall when provision of public transportation by the government is costly.