INVESTIGATION OF COMPRESSED AIR ENERGY STORAGE EFFICIENCY

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TITLE: Investigation of Compressed Air Energy Storage
                                Efficiency

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ABSTRACT

Investigation of Compressed Air Energy Storage Efficiency

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This study investigates Compressed Air Energy Storage (CAES) application in the electrical power and transportation industries. Information concerning current CAES projects is presented. A thorough thermodynamic analysis of the CAES process is completed; including theoretical efficiency determination for several variants of the compression and expansion processes. Industry claimed efficiencies ranging from 26% to 82% are presented and explained. Isothermal and Isentropic efficiency baselines are developed. Energy density of compressed air on both a mass and volume basis is compared to other energy storage methods. Best expected efficiency of a hypothetical CAES system is determined to be 34% using currently achievable efficiencies and 63% considering 100% efficient compression and expansion. A .5 kW CAES system, built from commercial off the shelf components (COTS) to demonstrate the CAES concept, is documented and discussed. This system includes a LabView data acquisition system which was used to record all test results. LabView was also used to develop a complete test bed program that determined real time thermodynamic state properties, component efficiencies, mass flow rates, power outputs and several other performance characteristics of the demonstration system. The LabView program allowed real time efficiency and power optimization of the demonstration system. Results of demonstration system testing are thoroughly discussed. Total system efficiency was very poor; 3.6% electrical conversion efficiency, .040 refrigeration coefficient of performance (COP) and a 5.0% overall efficiency which considers both cooling and electrical storage properties. Several paths for possible future projects involving the demonstration system and CAES are presented.

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CHAPTER 1

INTRODUCTION

1.1   Global Energy Requirements and Resulting Energy Costs

Our world is changing rapidly. Globalization has introduced the concepts and lifestyles of industrialized societies to a broader segment of the world; increasing energy demand. Energy is commonly obtained from the burning of fossil fuels, which are of limited supply. Increased demand for this limited supply of fuelstock has led to increasing costs; resulting in electricity and transportation fuel pricing that hinders global economic growth. Future generations of our increasing global population will require even more energy than is consumed today, furthering the rise of fuel and energy costs, Figures 1.1-1 and 1.1-2.[1]
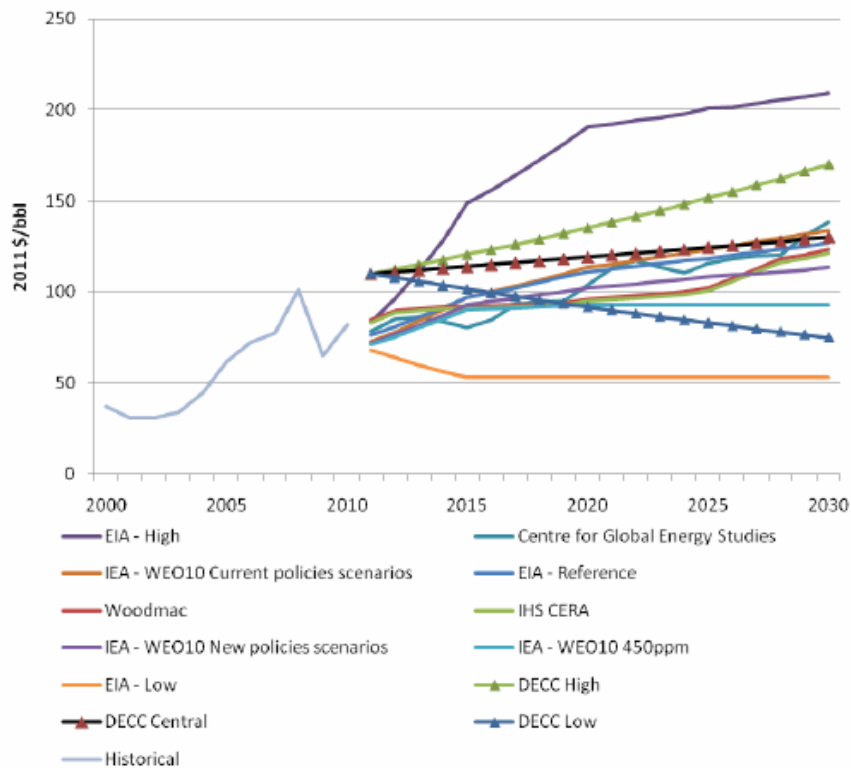
Figure 1.1-1: Comparison of Predictions of Future Oil Prices
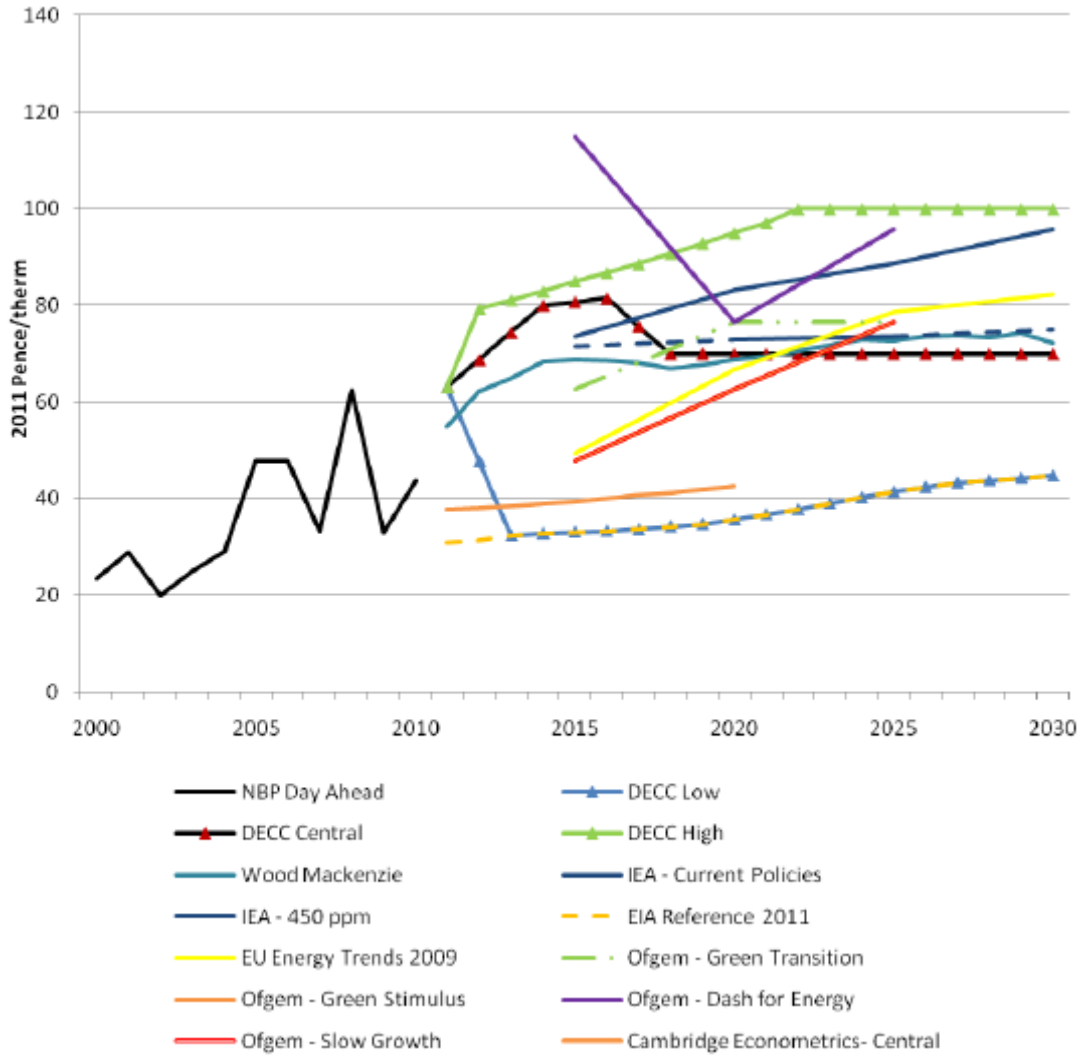
Figure 1.1-2: Comparison of Predictions of Future Natural Gas Prices

The predictions shown in Figures 1.1 and 1.2 are from a variety of respected energy analysis groups. Though the range varies, almost all indicate that future energy prices will be significantly higher than today. These predictions are based on several scenarios of GDP growth, capacity infrastructure growth, level of fuel intensity in emerging economies, supply discovery, supplier behavior, demand elasticity, etc. Worst cases scenarios predict that energy prices will double from current levels.

1.2 Renewable Energy Sources Portion of the Global Energy Portfolio

The effect of increased energy prices is already apparent. Many nations are still recovering from the financial melt-down, hindered, in part, by rising energy costs. Though the economic situation is complex and no source is solely to blame, the effect of rising energy costs does make a return to prosperity more difficult. To reduce consumption of fossil fuels alternate sources of energy are being developed at an astonishing pace. Many of these new energy sources are renewable and

include: biomass, hydro, geothermal, solar and wind.[2] Today these sources make up only 8% of the United States energy portfolio, Figure 1.2-1.[3]
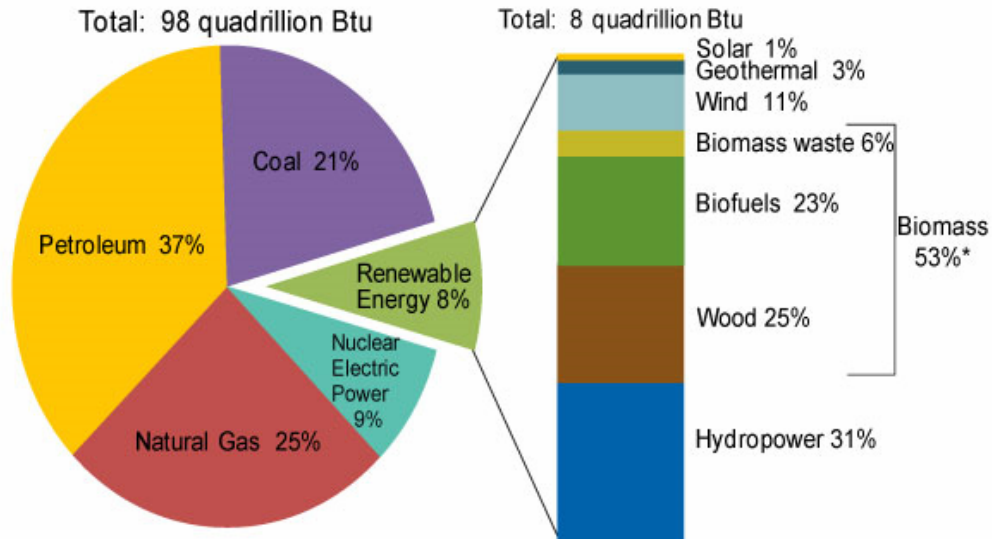
Note: Sum of biomass components does not equal 53% due to independent rounding.
Source: U.S. Energy Information Administration, *Annual Energy Review 2010.*

Figure 1.2-1: United States Primary Energy Sources

As countries industrialize and expend their energy appetite, renewable energy will likely become a larger portion of their energy supply. In the long term this transition is inevitable due to the global increasing energy demand juxtaposed against a fixed, and limited, supply of fossil fuels.

1.3 Renewable Power Generation Profiles and the Need for Energy Storage

Three of the five primary renewable energy sources, solar, hydro (with respect to tidal based systems), and wind, are only available intermittently. Solar energy is only available during the day and not at reliable levels. Though it is true that sunlight is always available somewhere on the planet, the lack of an integrated low loss global electrical power grid requires that solar energy must be stored, locally, to produce a consistent supply of power. Hourly and real time fluctuations in the solar flux at solar panels also create a need for load balancing systems which can react quickly to decreased power output due to clouds, fog, dust storms or other environmental factors.

Though tidal based systems have a more predictable output, it is hardly constant. Many factors affect tidal power systems, including variations associated with seasonal cycles, moon cycles, and storms. Again, storage is needed for tidal based power systems to maintain steady output.

Wind power is extremely variable in most environments. Some parts of the world have consistent trade winds which can provide relatively stable output, but even stable wind sources have much larger output variability than a traditional power plant.

The current exploitation of solar, hydro and wind renewables clearly shows a fundamental need for successful, affordable, and efficient energy storage systems.

1.4 Definition of the CAES Concept

Compressed Air Energy Storage (CAES) uses air as the energy transfer medium. Energy is used to compress air that is then stored under pressure. When needed, that compressed air is expanded, to extract a portion of the energy that was initially used during compression. In its simplest form, air is compressed by a shaft driven machine and stored under pressure. When energy is needed, the compressed air is expanded through a turbine, or other expansion device to drive a generator that produces electrical power or directly drive a process which requires shaft power.

1.5 Variations of the Basic CAES Concept

Air has interesting thermal properties which result in large entropy changes during the compression and expansion processes. In theory this makes it possible to use typical environmental conditions or excess heat from other industrial processes to extract more expansion energy than was required to compress and store the air. Heat exchangers can also be employed to reduce compression energy by creating a near isothermal compression process. During expansion heat exchangers can be used to reduce the cooling effect of expanding air, and increase expansion output. Additional heat can be input to provide even more expansion power. In practice, process efficiencies are well below 100%; even when considering heat input during expansion as free energy.

There are a number of Hybrid CAES concepts in use and/or being developed. The Alabama Electric Cooperative introduced a power plant in 1991 that uses compressed air to assist in the compression stage of an otherwise traditional fossil fuel powered turbine generation facility. PG&E is currently evaluating suitable sites for a similar 300MW facility in California. Other CAES variations use air as the primary storage medium while intermediate substances and mechanisms are used to convert potential energy into shaft energy. An example is Sustain X's patented system that uses hydraulic pumps and motors to convert electricity to stored air energy, and back to electricity when required. It is likely that other novel concepts will be introduced in the future.

1.6   Document Map

The following chapters discuss several facets and applications of CAES technology, including the experiments and results of a demonstration system built specifically for this project. Brief summaries of each chapter are provided here.

Chapter 2 discusses CAES application in the electrical power industry. Operational concepts of the existing Alabama Electric Cooperative plant are provided and energy benefits of this type of system are explained. New, fundamentally different, CAES systems by Sustain X and General compression are also explained. Factors hindering further CAES deployment in the electrical power industry are also discussed.

Chapter 3 discusses CAES application in the transportation and industrial equipment industries. This chapter includes energy density comparisons of air storage, batteries, and traditional fossil fuels utilizing several different efficiency baselines.

Chapter 4 provides insight into the thermodynamics of CAES. Pressure-Volume and Temperature-Entropy diagrams are provided and discussed for both the compression and expansion stages typical in a CAES power cycle. Theoretical process efficiency is evaluated for several different processes.

Chapter 5 introduces the 0.5 kW demonstration system built to display the CAES process and provide experimentally measured efficiencies achieved using commercial off the shelf components.

Chapter 6 provides details of the demonstration system instrumentation, National Instruments data acquisition system, and LabView program which was developed to provide real time efficiency and operating point analysis. It also discusses supplemental calculations which were performed after data acquisition to better define system efficiencies.

Chapter 7 showcases the results of demonstration system testing. Individual components and total system efficiencies are provided.

Chapter 8 provides several paths to continue this initial effort at understanding and evaluating Compressed Air energy Storage.

1.7   Nomenclature

CAES   Compressed Air Energy Storage

COTS   Commercial Off the Shelf

DAQ   Data Acquisition System

DOE   United States Department of Energy

Expansion Efficiency	Percent value indicating ratio of actual energy delivered vs available energy considering a theoretical isentropic expansion from inlet conditions to atmospheric conditions
kW	kilowatt, equivalent to 1000 watts
MW	Mega-Watt (1,000,000 Watts)
NARUC	National Association of Regulatory Commissioners
Renewable Energy	Energy which comes from natural sources such as sunlight, wind, rain, tides and geothermal heat, which are renewable (naturally replenished)[4]
TR	Tons of refrigeration, equivalent to 12,000 BTU/hr

CHAPTER 2

CAES OPERATION IN THE ELECTRICAL POWER INDUSTRY

2.1   Electrical Power Industry Application of CAES

CAES is not a new concept. The electrical power industry first applied CAES technology in 1978 at the Huntorf plant in Germany. Another plant was constructed in McIntosh, Alabama in 1991[5]. Several additional plants are currently in the planning stages, driven by the need to integrate renewables into the power grid. Udi Helman of the National Association of Regulatory Commissioners (NARUC) stated in 2009 "[Increasing the renewable sourced portion of grid power and] evolving sustainable environmental policies will continue to create [additional] demand for CAES or other [energy] storage technologies".[6]   In 2010, the State of California passed into law, state bill AB 2514, setting specific energy storage targets by 2015 and 2020. Driven by AB 2514, on September of 2013, California took the additional step of issuing guidelines to integrate 1.3 Gigawatts of load balancing storage power into the California power grid. These steps are driven by the fact that renewables power sources are typically plagued by inherent intermittency. In the near term traditional power plants can use load balancing turbines to account for the variable power output of the renewable based plants; however as the percentage of grid power sourced from renewable plants increases, so too does the need for large scale grid balancing storage systems.

2.2   Alabama Electric Cooperative CAES Assisted Power Plant

The Power South Energy Cooperative finished the McIntosh Alabama plant in 1991. This plant uses a solution mined salt cavern to store compressed air at pressures up to 1100 psig. It took almost two years to mine the 19 million cubic foot volume cavern.[7] The power generation/air compression system uses a 140 ft long train of compression and expansion equipment which includes a centrally mounted motor/generator. Clutches are used on both sides of the motor/generator to allow either compression of air by electric power or electricity generation by turbine expansion using compressed air to assist natural gas turbines. The compression system uses four-stage compression with intercooling between stages to enhance compression efficiency. Intercooling and multistage compression more closely represent an isothermal process, which has the best theoretical compression efficiency. (CAES compression processes are thoroughly described and evaluated in Chapter 4.) Figure 2.2-1 shows two pictures of the plants primary equipment train.
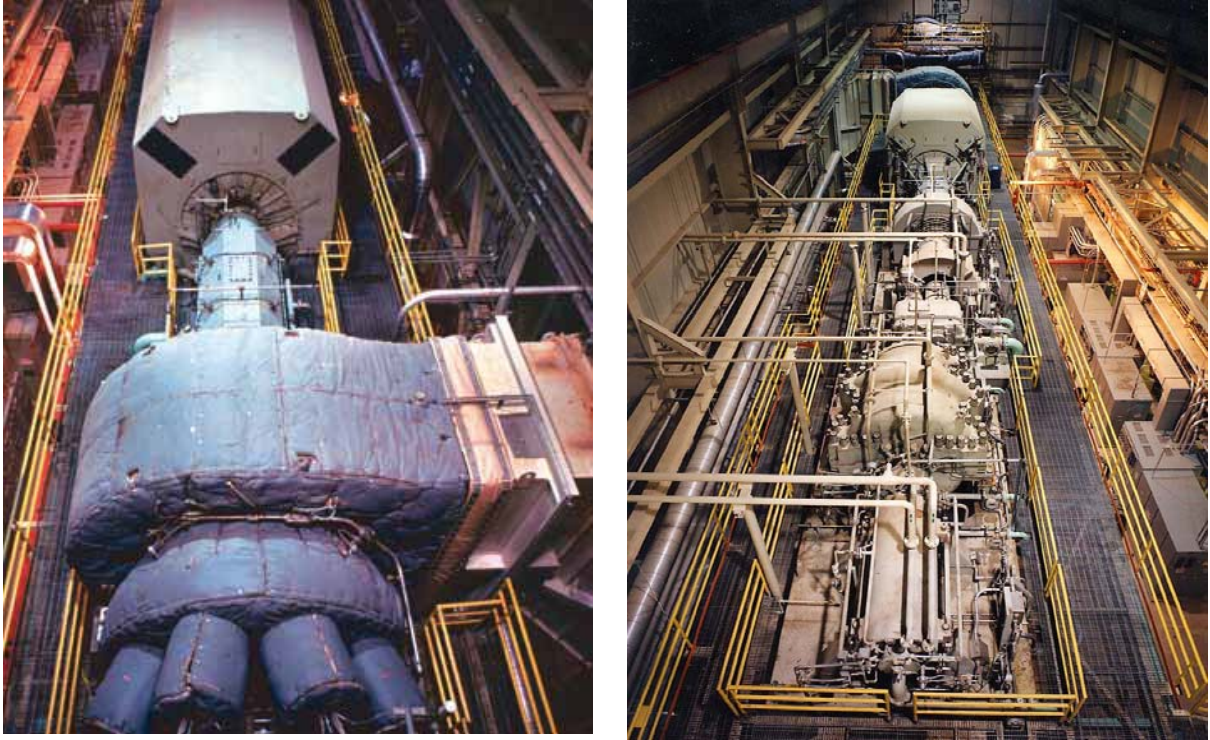
Figure 2.2-1: McIntosh Alabama CAES Plant Power Train

The left photograph shows the low pressure expander and combustors in the foreground, while one of the clutches and the motor/generator are shown in the background. The right photograph is from the opposite field of view. It shows the compression side of the power train, with the motor/generator and low pressure expansion turbine at the back. McIntosh plant power CAES assisted production is completed as follows:

1. During evenings when power grid loading is low the motor/generator is used as a motor to drive the four stage compression process.

2. Compressed air is delivered to the storage cavern.

3. When needed; air is released from the storage cavern and heated in a recuperator that uses expander discharge air to heat the process air.

4. The preheated air is mixed with natural gas and ignited to produce expansion power through a two stage expansion process.

The benefits of this type of system are that less generation capacity is required because plants can operate at full capacity when loads are small, storing the excess energy as compressed air. When demand exceeds capacity the McIntosh plant can quickly meet the load requirements by releasing the stored air energy. The McIntosh plant can reach full output power of 110 MW in 14

minutes, allowing it to act as a grid power balancing system. It also has an impressive reliability record, as shown in Table 2.2.1[8].

Table 2.2-1: McIntosh Plant Reliability

13-YEAR AVERAGE RELIABILITY			
COMPRESSION		GENERATION	
Starting	Running	Starting	Running
92.7%	99.6%	91.6%	96.7%

2.3   SustainX CAES Storage System

The SustainX system is a new patented approach to CAES. This system uses a combination of hydraulic and air compression and expansion processes to convert electrical power to stored compressed air. Figure 2.3-1 shows the general concept of the SustainX approach.
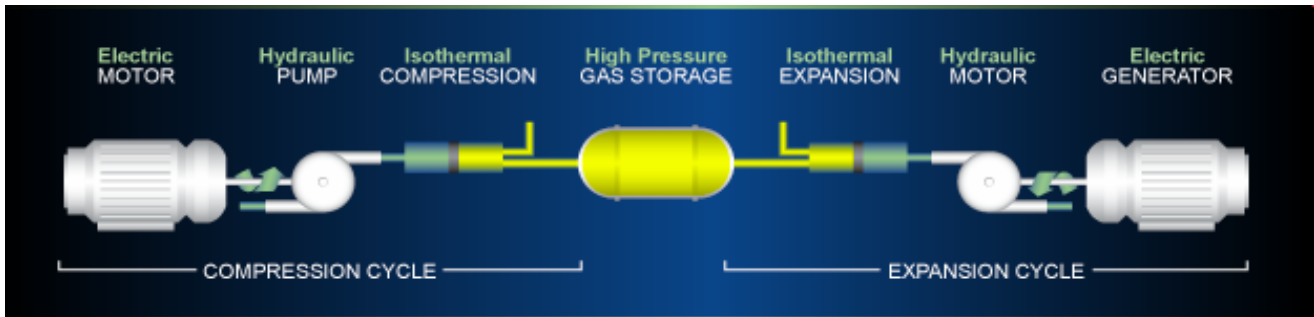
Figure 2.3-1 SustainX Patented CAES Concept

Another unique aspect of the SustainX approach is that the compression process and expansion processes are completed very near the isothermal ideal. Water injection is used along with heat exchangers to maintain stable temperatures throughout the process. Scaled versions of these systems have been documented to achieve 90% expansion efficiency while producing 1 kW out of a 1.5 gallon accumulator.[9] SustainX uses mechanical components proven in other industries for the majority of its innovative energy systems. Theoretically this will allow greater dependability and lower cost. SustainX has built a 40 kilowatt demonstration system and has plans for producing systems from one to four megawatts using off-the-shelf components.

2.4   General Compression Dispatchable Wind Storage Systems

General Compression was founded in 2006 by Eric Ingersol, David Marcus, and Michael Marcus. The company mission is to develop low cost storage for wind farms. The specifics of the method are proprietary, although the general concept has been made public. The system is intended to use electricity produced by wind farms to isothermally compress air into geologic formations

similar to the McIntosh plant. However the expansion process of the General Compression system does not require fuel; apparently the compression system is run in reverse to develop power. General compression has received $54.5 million in DOE grant money[10] and initiated a partnership with ConocoPhillips to build a 2 megawatt demonstration system in Texas.[11] This project is still under development.

2.5   Factors Hindering CAES Systems in the Power Industry

Both the Huntdorf and McIntosh plants prove that CAES is an established technology. Although new CAES methods promise to be more efficient, the existing CAES plants are not as efficient as pumped hydro, the industry standard storage method . In addition, some contemporary natural gas plants have efficiencies approaching the McIntosh plant without using CAES. One example of such is the General Electric 7FA 2x1 combined cycle plant which uses 1.83 MJ (considering 50.0 MJ/kg Lower Heating Value pure methanol as the fuel source) natural gas to provide 1 MJ electrical energy, compared with the McIntosh plant which requires 1.2 MJ (considering 50.0 MJ/kg Lower Heating Value pure methanol as the fuel source) natural gas AND 2.5 MJ electrical energy to produce 1 MJ electrical energy output.[12] This shows that for every MJ of electrical energy produced, the McIntosh plant requires an additional 1.9 MJ of energy, more than twice the total energy input of the newer GE plant. However the benefit of the McIntosh plant is that the 2.5 MJ required for compressing air can be delivered during periods of low grid power demand, thus increasing plant output and minimizing gas consumption when high power is required. This allows equipment with less power generating capacity to meet higher transient demands.

Other factors inhibiting the use of traditional CAES in the electric power industry include restrictive site requirements and the impracticality of turbine assist plants at a small scale. However, new CAES technologies appear to overcome some of these limitations, and may allow CAES to become a significant portion of the power industry energy storage portfolio.

CHAPTER 3

CAES OPERATION IN THE TRANSPORTATION/INDUSTRIAL EQUIPMENT INDUSTRIES

3.1   Transportation and Industrial Equipment Industry Application of CAES Technologies

A significant effort was put forth in the late 1800's to use CAES systems as a power source for trains and vehicles. At the time, CAES was competing with steam to become the primary method to power transportation systems. One of the most notable achievements was a tram system designed by Louis Mekarski in the 1870's. The system was tested in Paris in 1876 and introduced to the tramways of Nantes in 1879. The Mekarski tram was using CAES technology to propel a public transportation system over 125 years ago! In 1900 Nantes had a fleet of 94 trams using the Mekarski design. CAES was off to a seemingly unstoppable start at the beginning of the industrial revolution. However, the Mekarski tramcars were replaced by electric trams in 1917.[13] The CAES technology of that period faced the same technological challenges of today's experimental CAES systems. Section 3.4 discusses the factors that must be overcome for CAES to be successful in the transportation and industrial equipment industries.

As electric and combustion engine technologies improved, air powered systems such as the Mekarski tram became obsolete. By the 1930's air powered systems were only used in mines and other hazardous environments where combustion engines could not be used safely. Today compressed air is used in limited applications such as hand tools, conveyor pneumatics, switching systems, and valve actuators. However, in the experimental arena there is a great deal of research suggesting that a possible reemergence of air powered transportation equipment may occur.

3.2   Motor Development International (MDI); The Primary Air Car Advocate

The primary advocate of air powered transportation is Motor Development International (MDI). This company, founded in 1991 by engine designer Guy Negre ,[14] has successfully produced several models of air and hybrid air/combustion powered vehicles over the last 20 years. MDI is currently testing the latest vehicles on European roadways and has plans to sell them at $5,000-$11,000 USD depending on the model and configuration.[15] Early models were very similar in appearance to traditional vehicles. These cars had severely limited range and it is not clear if they ever actually operated on European roadways. The newer designs have shed lots of weight and do not resemble a traditional vehicle. Figure 3.2-1 compares an early model prototype with a current production model.

Figure 3.2-1:   MDI Concept Cars; Heritage and Today; AIRPod Model Pictured at Right.

Early designs obviously have much more mass than the current models being tested on European roadways. The AIRPod model pictured weighs 573 lb. It is evident that the MDI air power train is not capable of moving a large mass for a reasonable distance. The newer cars have been designed to move passengers in the most efficient manner possible, essentially overcoming significant limitations of engineering factors associated with using CAES and air engines.

MDI has also incorporated a hybrid design that uses low temperature combustion to preheat the compressed air before it enters a piston type engine. This process allows additional power to be obtained while still keeping emissions well below that of traditional combustion engines.

MDI recently released a document that compares several current MDI models with other electric and hybrid vehicles. The document is titled "Comparative Analysis" and is available at the MDI web site, www.MDI.lu. It is clear that for the given test conditions the air cars are meeting or exceeding, the criteria to which they are compared to other vehicles. MDI stresses the importance of initial and lifetime costs when considering a vehicle purchase. The air cars boast the lowest initial cost and have an expected 12,000 cycle life for charge/discharge, compared to 1000 for most batteries. The document does not discuss the comfort or safety of the vehicles being tested. The MDI vehicles are approximately ¼ the weight of the others and do not appear to maintain the same level of comfort or security that comes with a heavier vehicle. It would be interesting to see how an MDI vehicle retrofitted with Lithium Ion batteries and an electric power system would perform in the comparative analysis. This vehicle would likely weigh about ½ as much as the electric vehicles included in the comparison testing, which would substantially increase its' marks in all of the performance criteria of the comparative analysis.

3.3   Engine Air Limited Showcasing Di Petro's Novel Air Motor Design

Other companies are attempting to produce air engines with better efficiencies and power outputs than the MDI designs. Engine Air Pty Ltd of Melbourne Australia is producing motors which use a rotary piston concept invented by Angelo Di Pietro. This motor is reportedly capable of 94.5%

expansion efficiency (as defined in nomenclature) and can rotate with as little as 1 psi.[16] An illustration of the Di Pietro Rotary piston air motor is shown in Figure 3.3-1.
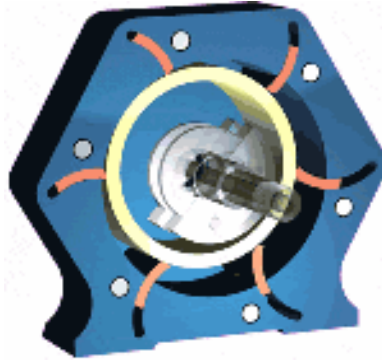
Figure 3.3-1: Di Pietro Rotary Piston Air Motor

This motor shares an important characteristic with electric motors in that it is capable of producing static torque without rotating. This should allow lower operating speeds and reduced friction losses. Motor speed and torque are controlled by throttling the volume or pressure of air into the motor. Higher efficiencies are achieved by using smaller pulses of air, which allows greater expansion efficiencies. Attempts were made to incorporate testing of a Di Pietro motor into this study; however EngineAir was unwilling to provide a test motor.

3.4   Factors Hindering Air Powered Transportation

Several compressed air characteristics must be overcome for CAES technology to establish itself in the transportation industry. MDI is working around these limitations by producing extremely light vehicles with limited load carrying capacities and power outputs. EngineAir is focusing on creating the most efficient engine possible to maximize air motor performance. It is the author's belief that the inherent properties of CAES technology must be overcome if CAES will become part of the emerging alternative fuel vehicle market. The primary characteristics hindering air powered transportation are discussed below.

3.4.1   CAES Has A Low Energy Density On A Volume Basis

Compared to batteries or fossil fuel, compressed air has a very low energy density on a volume basis. Energy density is considered as the amount of energy that can be withdrawn from a source; whether it is a battery, compressed air tank, or energy released during combustion of a liquid fuel. Unfortunately the apparent energy density of compressed air can be skewed because it is dependent on how the air is expanded and at what pressure it is stored. A comparison based on the current accepted limits of CAES and battery technology is presented below. This comparison considers a 4500 psig storage pressure for CAES systems. Note, MDI vehicles are designed to

use a maximum storage pressure of 350 Bar (5075 psig); therefore they would perform slightly better than shown. Figure 3.4.1-1 shows the energy density of air considering all expansion methods, including the actual testing completed for this project. Expansion methods and associated efficiencies will be discussed thoroughly in Chapter 4. It is important to keep in mind that only the as-tested values represent the entire process of converting stored energy into shaft power.

Energy Density of Competing Technologies
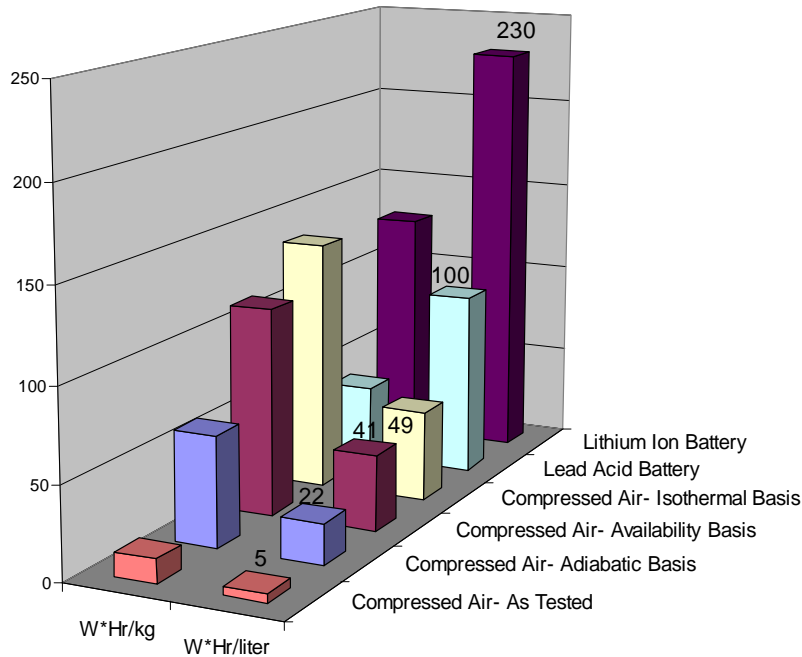
Figure 3.4.1-1: Energy Density of Competing Technologies

It is clear that the volumetric energy density of CAES is not capable of competing with batteries. The situation becomes even worse when CASES is compared with gasoline. Gasoline volumetric energy density is approximately 9440 W*Hr/liter (13,120 W*Hr/kg); 41 times greater than Lithium Ion performance and 192 times greater than the best theoretical CAES. When considering weight and either an availability or isothermal expansion basis, compressed air is theoretically on par with Lithium Ion technology. The problem is that the storage tanks needed to store the energy-equivalent of a battery become difficult to package into a small, efficient CAES powered vehicle.

It is important to note that Figure 3.4.1-1 does not take conversion efficiency into account. Conversion efficiency is a measure of the performance of the mechanism that converts this stored energy into its desired form. For transportation systems it is desired to convert the stored energy into vehicle motion. Therefore energy density is only half the picture; conversion efficiency is also significant. Generally, electric motors have a much better conversion efficiency than air

motors or combustion engines. The Di Pietro motor may be an exception if its' claimed 94.5% efficiency holds true. Generally vehicle sized electric motors can maintain 90-95% conversion efficiencies while typical gasoline engines have 30%- 35%[17] conversion efficiencies. Figure 3.4.1-2 converts the energy density values into expected shaft power outputs considering the best case ISO-thermal expansion model de-rated by MDI published conversion efficiency ranges (43% min 60% Max)[18]. It also includes the Di Pietro claimed efficiency (94.5%) and assumes an electric conversion efficiency of 90% for battery systems. Though it is not shown in Figure 3.4.1-2, gasoline still performs orders of magnitude better than batteries or compressed air.
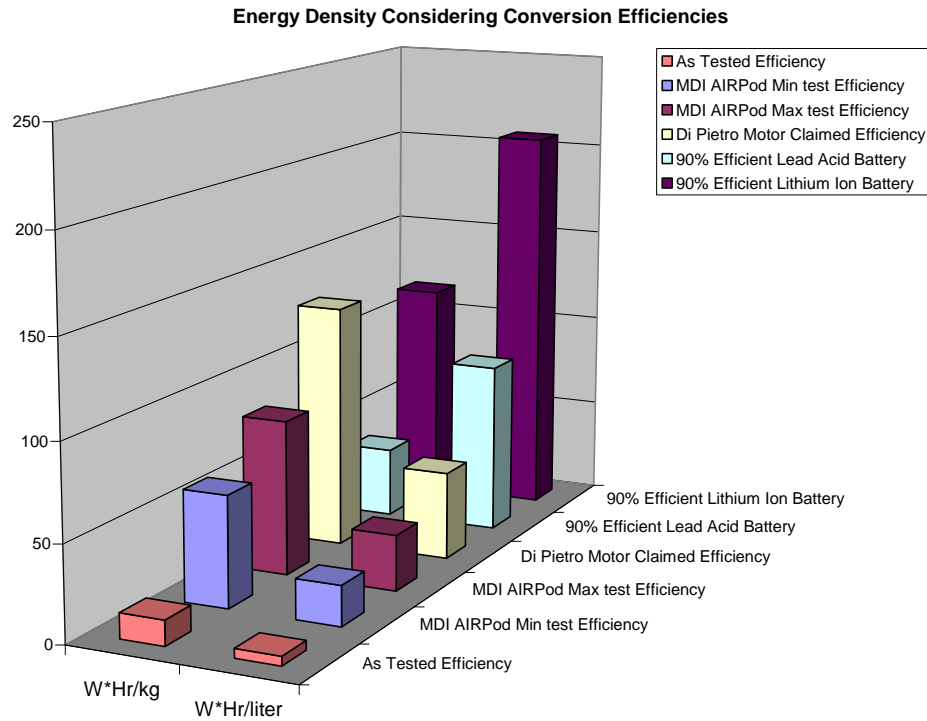
Figure 3.4.1-2 Energy Density Considering Conversion Efficiency

Figure 3.4.1-2 shows lower performance from the CAES systems when using as-tested efficiency data from MDI. MDI reports expansion efficiencies of 43% to 60% depending on operating conditions. Based on MDI's definition of energy density it can be inferred that these efficiencies consider an availablility basis.[19] CAES performance improves when considering the Di Pietro motor claimed peak efficiencies. On a mass basis the Di Pietro motor, theoretically, has better energy density considering conversion of stored energy to shaft power. On a volume basis it is still outmatched by lead-acid batteries and provides less than ¼ of the energy of a lithium ion battery system. For vehicles, volume basis is important because large tanks result in more vehicle mass due to the larger vehicle size needed to carry the fuel tank. Figure 3.4.1-2 shows the overall trend remains unchanged when considering storage to shaft power; batteries can store and convert more energy than current CAES technologies.

3.4.2 Availability Losses Associated with Pressure Regulation of High Pressure Storage

A significant availability loss occurs when regulating a compressed air power source. Most air engines are designed to operate in a narrow pressure range, mandating the use of a pressure regulator. Pressure regulators act as a throttling device; they have a large entropy change which limits the air motors capacity to produce power when expanding the regulated gas. When high pressure air is regulated to a lower pressure it becomes very cold, which is counterproductive to producing expansion in an air motor. The thermodynamics and availability losses associated with pressure regulators are discussed thoroughly in Chapter 4.

3.4.3 Heat Exchangers Are Necessary For Reasonable Efficiency

The thermodynamic properties of air require heat addition during expansion to achieve reasonable efficiencies. In stationary applications heat exchangers can be implemented to recapture waste heat or potentially use ambient conditions to raise process air temperatures. For transportation type systems the size of the required heat exchangers becomes difficult to package. CAES powered vehicles are also very aerodynamic, the airflow through heat exchangers produces additional drag that reduces vehicle performance.

3.4.4 Inefficient to Compress Air to Necessary Storage Pressures

Figures 3.4.1-1 and 3.4.1.-2 show that air has difficulty competing on a volume basis with batteries and fossil fuels. These figures are based on a 4500 psig storage pressure. Compressing air to this pressure requires complex muiltistage compressors which rarely achieve greater than 60% efficiency. MDI's own compressor, which is likely more efficient than most, achieves a published efficiency of 62%-70% while compressing up to 5075 psig.[20] Batteries have much higher charge efficiencies than are possible with current CAES technology; 80% to 90% charge efficiency is typical for a lithium ion battery system.[21]

3.4.5 Lack of Existing Infrastructure to Support Refueling

The equipment necessary to produce and store the compressed air necessary for refueling is not available in our current transportation network. The compressor and necessary storage tanks are more cost prohibitive than a comparable battery charging system. Their installation is also more

involved. Battery charging systems only need electricity, which is widely available. A CAES charging station needs a power source to run the compressor and also requires a storage vessel. Storage vessels are costly and have specific design, installation, inspection, and maintenance requirements. Establishing a refueling network for CAES powered vehicles will require a larger investment than it would to establish a similar network for electric vehicles. One significant advantage that CAES technology seems to offer is a quicker charge time than can be provided with battery technology.

CHAPTER 4

EFFICIENCY AND THERMODYNAMICS OF COMPRESSED AIR ENERGY STORAGE

4.1   CAES Process Efficiency- 26% or 82%?

There are many advocates and opponents of CAES technology. Table 4.1-1 shows an extremely broad range of claimed efficiencies. There are several baselines and individual processes involved with any energy storage method; CAES is no exception. As such, they can not be compared without considering how that efficiency is calculated, and several facets need to be considered before being able to fairly compare efficiencies.

Table 4.1-3: Claimed CAES Conversion Process Efficiencies

Efficiency	Source	Comments
82%	Samir Succar, Princeton Environmental Institute	Round trip basis, theoretical turbine assist efficiency used in Hybrid power plant.
80%-85%	Dresser Rand, Form 85230	Polytropic compression basis, does not include expansion efficiency
68%	Frank S Barnes, Handbook for High Power and High energy Storage	Second law (entropy) basis. Theoretical efficiency with heat recuperator between compression and expansion
37%	Mazza and Hammerschlag, 2005 Wind-toWheel Energy Assessment	46% compression efficiency, unstated basis, 4 stage expansion process
26.70%	Felix Creutzig, Economic and Environmental Evaluation of Compressed Air Cars	Theoretical isothermal compression, 2 stage expansion.

The claimed efficiencies in Table 4.1-1 seem to conflict and cover a very broad range. However , in the broadest terms, it is likely that *all* of these values are correct. The differences are explainable by considering what specific processes are being considered, and how those processes are included in the efficiency calculation. For instance 82% round trip efficiency was claimed by Samir Succar of the Princeton Environmental Institute at his presentation for the InterAcadamy Council (IAC) International Workshop on Energy, October 30, 2005. This is in reference to a hybrid power plant similar to the McIntosh plant discussed in Chapter 2. A closer look at the 82% number reveals that it de-rates the true energy input by the turbine efficiency. The specific slide from the presentation is included as Figure 4.1-1.
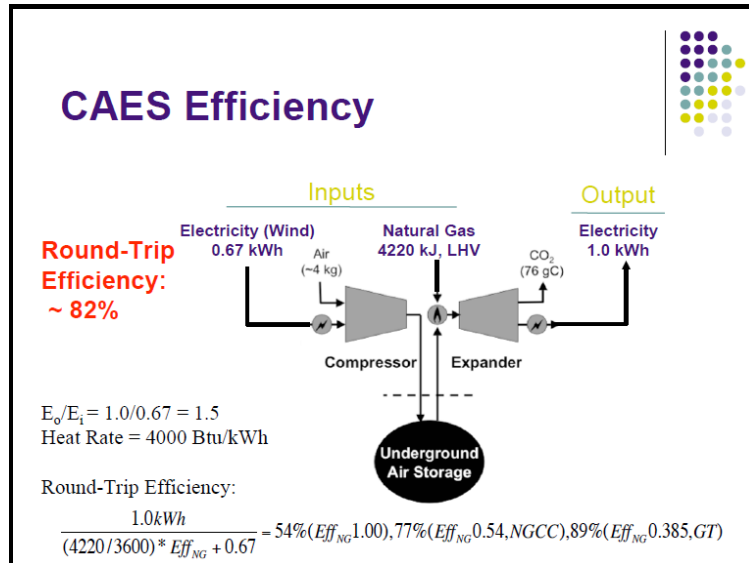
Figure 4.1-1:   Succar "Round Trip Efficiency" Determination, as Discussed at IAC International Workshop on Energy, 10/30/2005

Succars' efficiency considers the actual turbine output energy as the natural gas portion of the input energy for his round trip efficiency. This author believes that an efficiency calculated in this manner skews the true process efficiency in favor of using CAES. For example, using a less efficient turbine in the hybrid plant will actually increase the "round trip efficiency" as calculated by Succar, even though actual energy input will increase for the same 1 kW output. If the Succar analysis is modified to calculate actual (energy out/energy in); the "roundtrip efficiency" decreases to 54% assuming a 100% efficient Gas Turbine. This is much lower than a conventional gas turbine plant. The author believes a more accurate model would determine round trip efficiency as shown in Equation 4.1.

$$\text{Eff}_{\text{Round\_Trip}} := \frac{1.0\_\text{kWh}}{\left[\dfrac{4220\_\text{kJ} \cdot \left(\dfrac{1}{3600}\ \dfrac{\text{kWh}}{\text{kJ}} \right)}{\text{Eff}_{\text{NG}}} + .67\_\text{kWh} \right]}$$

(4.1)

Figure 4.1-2:   Modified "Round Trip Efficiency" Equation Accounting for Actual Natural Gas Energy Input.

Equation 4.1 properly accounts for actual natural gas energy input to get 1 kWh output. However, it still requires the same assumption that .67 kWh is required to compress the amount of air a hybrid plant would use to generate 1 kWh of electrical output from its' gas turbine. In an actual process the compressed air requirement would likely increase as the turbine efficiency drops, resulting in an increase of required electrical input as well. Using the modified round trip efficiency relation as shown in Figure 4.1-2 an 85% efficient gas turbine would result in a 49% round trip efficiency; which is substantially different from the 82% efficiency reported by Succar.

Another example of an apparent conflict in CAES efficiency is the 80% to 85% efficiency claimed by the reputable equipment manufacturer Dressor Rand. This efficiency only considers "flange to flange polytropic efficiency" of the compression process[22]. The method for determining the potential energy of the compressed gas is not discussed. The storage, piping and expansion losses are also not considered. The lower efficiency claims in Table 4.1-1 are referencing transportation applications of CAES. These are theoretical calculations, considering the thermodynamic properties of air. The author believes that all values in Table 4.1-1 are accurate, but they are not all explaining the same process. The focus of this chapter is theoretical efficiencies of the CAES process.

4.2   Chapter 4 Efficiency Calculation Description and Assumptions

This chapter evaluates several compression/expansion models of CAES processes using thermodynamics and determines the associated system efficiencies with those models. Isentropic, Polytropic, and Isothermal compression and expansion processes are considered. A matrix of possible system efficiencies is determined by combining the possible compression and expansion models. Actual calculations are included in the Appendix; reference calculations 15A and 15B for analyses associated with the data presented in this chapter.

An ideal gas model is used to represent air for theoretical processes when state property values are not able to predict final properties. Actual state properties are used whenever possible. There are some pressure/temperature states where air properties deviate as much as 15% from an ideal gas, but for typical CAES processes the ideal gas model predicts CAES system efficiency accurately. The efficiency calculations in this chapter do not consider specialized CAES processes that could be developed and optimized to exploit the unique states where air deviates from ideal gas behavior in a beneficial way. All processes are considered without mechanical and/or friction losses. Heat transfer is assumed perfect within each process; that is nonexistent where insulative properties are desired, and 100% efficient when heat transfer is desired. It is assumed that it is not possible to store the heat of the compression process due to the high temperatures involved and the typical time difference between compression and expansion processes for a typical CAES system (A typical CAES system would not compress air at the same time it was using it). Compression calculations assume 14.7 psia, 70F initial conditions and expansion processes assume cooling to 70F during storage. Actual processes would not be able to achieve the theoretical efficiencies determined in this chapter.

4.3   Compression Process Theoretical Efficiency

Figure 4.3-1 is a Pressure-Volume (P-V) diagram for the possible compression processes.

$$\text{Shaft\_Work} := -\int_{P_1}^{P_2} v \ dp$$

- Isothermal (red) requires least work
- Polytropic (blue)
- Isentropic (green) requires most work
- more work per psi required at lower pressures

P.2 =300

Process direction

P.1=14.7

Pressure (psia)

Specific Vol (ft^3/lbm)
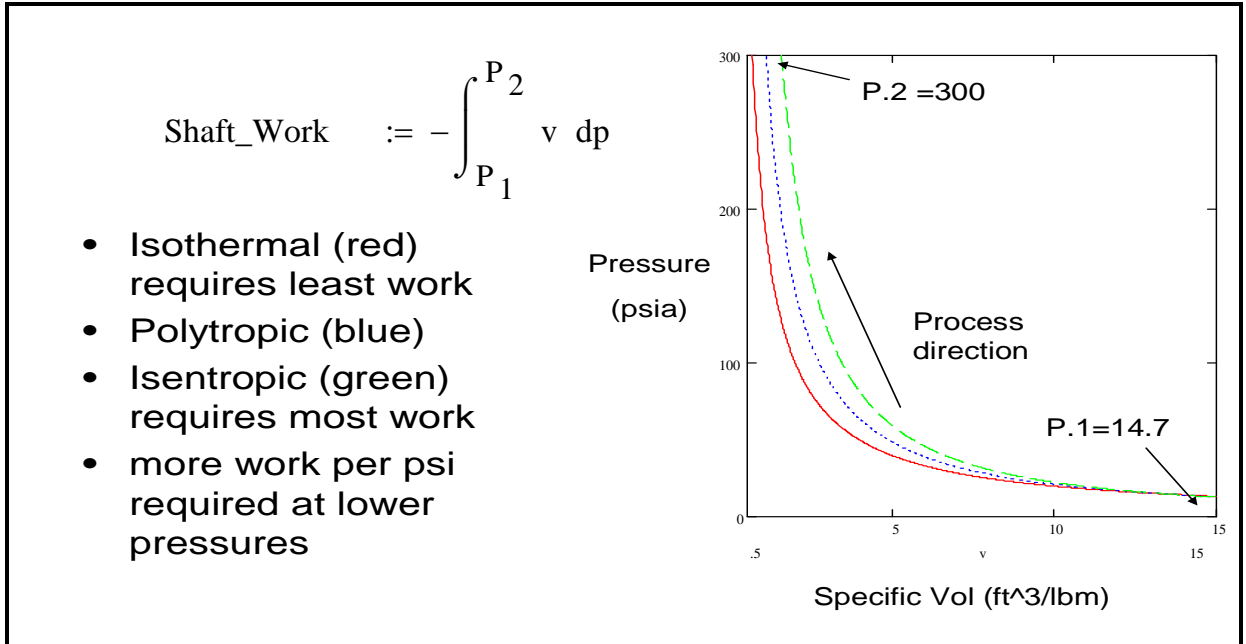
Figure 4.3-1: P-V Diagram for the Compression Process

The actual shaft work required to compress the air is represented by the area under the curve bounded by the starting and ending pressures. Figure 4.3-1 shows that the amount of work required to compress air is significantly greater for an adiabatic process than an isothermal one. The polytropic process represents a typical actual compression process that rejects some heat to the surroundings but also retains some thermal energy in the process air. The P-V diagram shows the importance of achieving near isothermal compression to limit required work to compress air to storage pressure. However, the P-V diagram does not explain the entire thermodynamic process. Figure 4.3-2 is a Temperature- Entropy (T-S) diagram.

- Isothermal (red) requires >> heat removal
- Polytropic (blue)
- Isentropic (green) no heat transfer, metal melting temps at higher pressures

T-S Relations for Compression Process

Q.total Isentropic=0 BTU/lbm (out)

— Isentropic
— PolyTropic, N=1.2
— Isothermal

Q.total polytropic = 60.53 BTU/lbm (out)

Q.total isothermal = 111.75 BTU/lbm (out)

Start

Temperature (R)

Entropy (Btu/lbm*R)
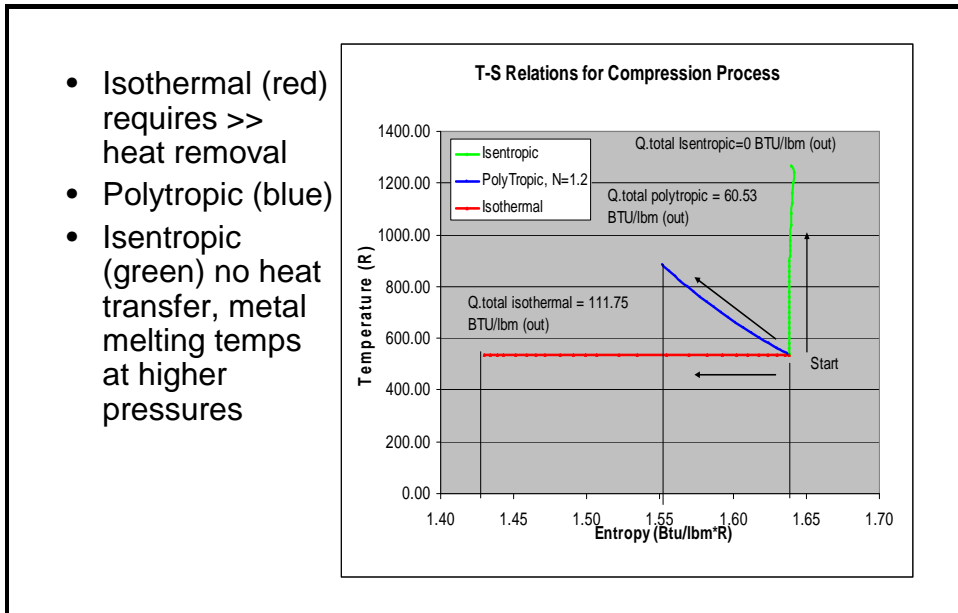
Figure 4.3-2: T-S Diagram for the Compression Process

The T-S diagram shows that when air is compressed it gets hot. To achieve an isothermal compression a significant amount of heat must be rejected during the process. In practice this requires multistage compression and large heat exchangers to remove the heat of compression. The polytropic process retains some heat and transfers some heat to the surroundings; which better represents an actual process. Current work by SustainX uses water as a secondary process material to achieve near isothermal compression. The future will likely bring other novel ways to approach the isothermal ideal. In theory an isentropic compression process would hold the heat of compression within the process by compressing adiabatically. Assuming no heat loss during storage; this approach would seem to conserve energy, resulting in a better overall efficiency. There are a few theoretical analyses supporting the benefits of adiabatic compression, although the author is not aware of any that have successfully produced positive experimental results. This is likely because it is not cost effective to store air without significant heat loss. Also temperatures for an adiabatic compression process (no heat transfer during compression) are capable of destroying all conventional materials as pressure goes above a few thousand psi. With today's technology isothermal compression is the goal because it requires less work per psi of stored air and it doesn't require storing the heat of compression for good efficiency.

4.4 Expansion Process Theoretical Efficiency

Figure 4.4-1 is a Pressure-Volume (P-V) diagram for the possible expansion processes.

$$Shaft\_Work := -\int_{P_1}^{P_2} v \, dp$$

- Isothermal (red) Produces most work
- Polytropic (blue)
- Isentropic (green) produces least work and results in very cold air temperatures
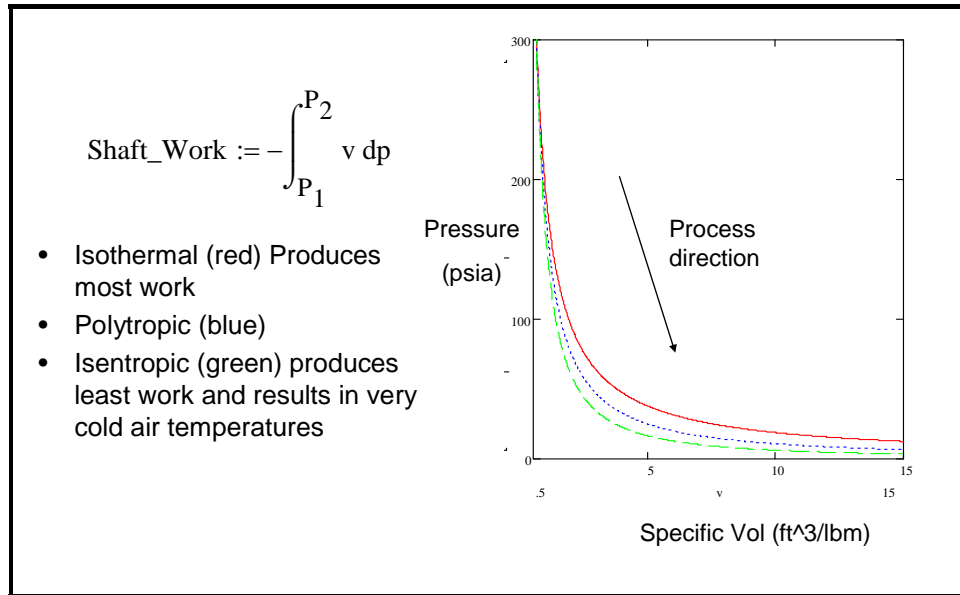
Figure 4.4-1: P-V Diagram for the Expansion Process

The goal of the expansion process is to get work out of the compressed air. As Figure 4.4-1 shows the isothermal process results in the most shaft power. The polytropic process best represents actual expansion where there is not enough heat transfer to maintain a steady temperature. Just as in the compression process it is necessary to look at the Temperature-Entropy diagram to completely understand the thermodynamics for expansion. The T-S diagram is shown in Figure 4.4-2.

- Isothermal (red) requires >> heat addition
- Polytropic (blue)
- Isentropic (green) freezing air temps, results in icing problems including:
  - rotor balance
  - piping blockage
  - stuck valves and controls
  - Incomplete expansion

Figure 4.4-2: T-S Diagram for the Expansion Process

Figure 4.4-2 shows some unique properties for compressed air expansion. The process air will tend to get very cold during expansion. Air will always decrease in temperature during expansion. This highlights another benefit of trying to achieve isothermal compression. If

isothermally compressed air is stored, it is at ambient temperature before expansion; so the ambient environment can then be used to heat the air, maintaining a near ambient temperature throughout the expansion process. Ambient conditions can be used as a heat reservoir to help achieve a near isothermal expansion. In practice this requires multistage expansion along with heat exchangers and/or a large thermal reservoir at ambient temperature. However, properly designed heat exchangers with acceptable time constants and surface areas associated with heating the expanding process air may tend to be cost prohibitive. There are also potential efficiency benefits of compressing cool dense air at night and expanding during the heat of the day; which is when power demand is most significant. These are possibilities for power generation facilities, but have proven difficult to package for transportation applications of CAES. Unlike combustion powered systems, insulating an air powered expansion motor will result in very cold temperatures and decreased power output. When considering an adiabatic expansion cryogenic temperatures are encountered, theoretically resulting in partial liquefaction of the process air. Heat transfer during typical combustion processes reduces efficiency and heats the environment; with CAES expansion it is the opposite. Actual applications require some heat transfer to avoid rotor imbalance, piping blockage, stuck valves and controls, and incomplete expansion resulting from the very cold temperatures associated with the expanding air. Theoretically if an additional heat source is available the CAES expansion process can produce even more power than in the isothermal case. If the heat source is not considered as a source energy input theoretical efficiencies greater than 100% can be obtained. Many industrial processes have significant waste heat that could be used to enhance an air expansion process.

The expansion model shown in Figures 4.4-1 and 4.4-2 does not apply to the hybrid power plant systems such as the McIntosh plant. These types of systems use CAES to save shaft energy typically used to compress the gas before ignition in the turbine. For these systems the author believes it is necessary to consider efficiency at the total system level. This requires looking at the ratio of energy output vs electrical and fuel inputs; as shown in Equation 4.1.

4.5   Pressure Regulation Effects on CAES systems

Most CAES systems will use a pressure regulator to maintain steady state conditions for the expansion process. Pressure regulators act as a regulating valve and typically do not significantly affect the enthalpy state of the process gas. For combustion type process systems maintaining enthalpy is crucial for good pr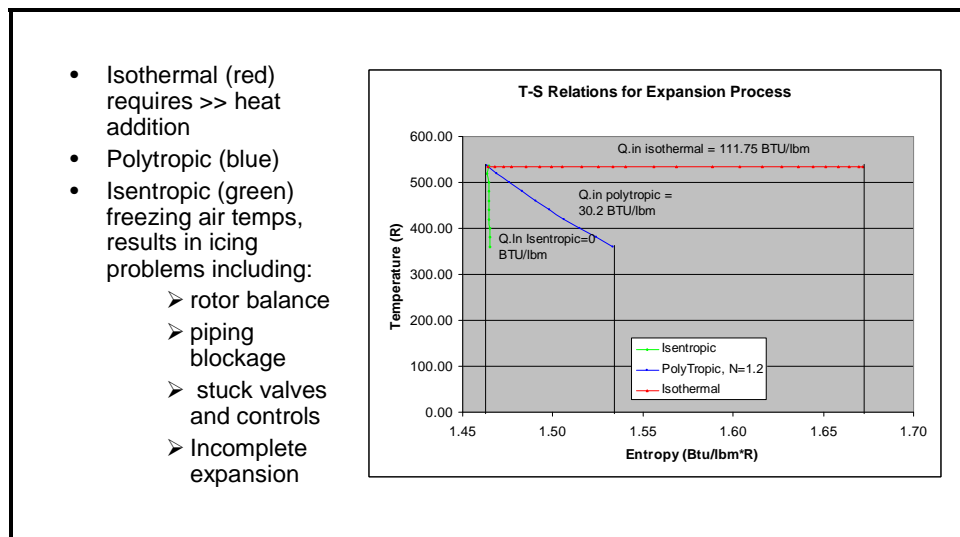ocess efficiency. For CAES systems enthalpy is not near as critical as maintaining entropy. There is a large entropy increase associated with throttling processes. In combustion processes there is significant fuel input; therefore the entropy change associated with the regulated air is insignificant due to the large availability of energy within the fuel. However for a pure air expansion process the air entropy change significantly limits potential shaft power. The ideal gas model does not predict a temperature and entropy change for a pressure regulating valve. However, considering actual state data for air allows calculation of the availability losses associated with pressure regulation. Thermodynamic analysis, as completed in Calculations 15A and 15B, shows that entropy increase associated with a 300 psig regulated input pressure reduces

available expansion power per lb of process air by 21% for a 600 psig storage pressure and 55% for a 4500 psig storage pressure. Theoretically these losses can be overcome, or possibly be turned into availability gains by the use of heat exchangers and/or waste heat sources.

4.6   Ideal Theoretical System Efficiencies

Theoretical system efficiencies considering the compression and expansion methods discussed in sections 4.3 and 4.4 are presented here. Efficiencies are considered for 600 psig and 4500 psig storage pressures, both with and without pressure regulators. These represent transportation and average power generation facility pressures. Other important data such as temperatures and associated heat transfer rates are included. Table 4.6-1 applies to low pressure CAES systems.

Table 4.6-1: Theoretical CAES System Efficiencies for 600 psig Storage Pressure

Theoretical System Efficiencies- Low Pressure (600 psig) CAES				
ALL ENERGY AND HEAT IN BTU/LBM, ALL EFFCIEINCIES ARE % (W_OUT/W_IN)				
COMPRESSION / EXPANSION	ISENTROPIC T.h= 1,921 F Q= -368 btu/lbm	SINGLE STAGE POLYTROPIC T.h= 668 F Q= -207 btu/lbm	THREE STAGE POLYTROPIC T.h= 198 F Q= -156 btu/lbm	ISOTHERMAL T.h= 75 F Q= -141 btu/lbm
CMP ENERGY / EXP ENERGY	-443	-244	-152	-137
ISENTROPIC (600 PSIG) T.c= -276F Q= 0	72	16%	30%	47%	53%
REGULATED (300 PSIG) ISENTROPIC T.c= -240F Q= 0	72	16%	30%	47%	53%
REGULATED (300 PSIG) ISENTROPIC W/REHEAT T.c= -236F Q= 3 btu/lbm	73	16%	30%	48%	53%
POLYTROPIC (600 PSIG) T.c= -206F Q= 52 btu/lbm	115	26%	47%	76%	84%
REGULATED (300 PSIG) POLYTROPIC T.c= -174F Q= 41 btu/lbm	97	22%	40%	64%	71%
REGULATED (300 PSIG) POLYTROPIC W/REHEAT T.c= -167F Q= 46 btu/lbm	100	23%	41%	66%	73%
THREE STAGE POLYTROPIC T.c= -43 F Q= 98 btu/lbm	125	28%	51%	82%	91%
REGULATED (300 PSIG) ISOTHERMAL T.c=62F Q= 109 btu/lbm	108	24%	44%	71%	79%
REGULATED (300 PSIG) ISOTHERMAL W/REHEAT T.c=75F Q=114 btu/lbm	111	25%	45%	73%	81%
ISOTHERMAL (600 PSIG) T.c=75F Q= 141 btu/lbm	137	31%	56%	90%	100%

Table 4.6-1 shows that theoretical CAES system efficiency ranges from 16% to 100% depending on how the process is executed. Perfect Isothermal compression and expansion will result in

100% conversion efficiency. A more realistic, but still not attainable process, uses three stage polytropic compression, incorporates a pressure regulator with reheat to ambient temperature, and uses three stage polytropic expansion to achieve 66% theoretical conversion efficiency. Final temperatures associated with each process are included. Fields shaded in blue represent processes with extremely cold final temperatures, while fields shaded in red indicate processes that have high temperatures that will prove difficult to develop. The heat transfer associated with compression processes includes the losses that are assumed to occur during storage. Isentropic compression has the most heat transfer because it requires the most work to achieve storage pressure and it also includes the losses associated with the stored gas cooling during storage. Actual process input/output energy on a BTU/ per lbm air basis is shown in the second row/column. A quick calculation (-137/-443) shows that isothermal compression can achieve storage pressure with less than 1/3 (31%) of the work required of an isentropic process. Similarly an isothermal expansion theoretically will produce 1.9 times (137/72) the shaft work of an isentropic expansion. Table 4.6-2 is a similar matrix for a high pressure CAES system.

Table 4.6-2: Theoretical CAES System Efficiencies for 4500 psig Storage Pressure

Theoretical System Efficiencies- High Pressure (4500 psig) CAES					
ALL ENERGY AND HEAT IN BTU/LBM, ALL EFFCIEINCIES ARE % (W_OUT/W_IN)					
COMPRESSION / EXPANSION	CMP ENERGY / EXP ENERGY	ISENTROPIC T.h= 4,825 F Q= -1199 btu/lbm	SINGLE STAGE POLYTROPIC T.h= 1221 F Q= -412 btu/lbm	THREE STAGE POLYTROPIC T.h= 275 F Q= -266 btu/lbm	ISOTHERMAL T.h= 75 F Q= -229 btu/lbm
		-1139	-471	-247	-210
ISENTROPIC (4500 PSIG) T.c= -312F Q= 0	92	8%	20%	37%	44%
REGULATED (300 PSIG) ISENTROPIC T.c= -267F Q= 0	63	6%	13%	26%	30%
REGULATED (300 PSIG) ISENTROPIC W/REHEAT T.c= -236F Q= 19 btu/lbm	73	6%	15%	30%	35%
POLYTROPIC (4500 PSIG) T.c= -290F Q= 80 btu/lbm	150	13%	32%	61%	71%
REGULATED (300 PSIG) POLYTROPIC T.c= -210F Q= 36 btu/lbm	85	7%	18%	34%	40%
REGULATED (300 PSIG) POLYTROPIC W/REHEAT T.c= -167F Q= 62 btu/lbm	100	9%	21%	40%	48%
THREE STAGE POLYTROPIC T.c= -95F Q= 149 btu/lbm	178	16%	38%	72%	85%
REGULATED (300 PSIG) ISOTHERMAL T.c= -2F Q= 95 btu/lbm	95	8%	20%	38%	45%
REGULATED (300 PSIG) ISOTHERMAL W/REHEAT T.c=75F Q=148 btu/lbm	111	10%	24%	45%	53%
ISOTHERMAL (4500 PSIG) T.c=75F Q= 229 btu/lbm	210	18%	45%	85%	100%

High pressure CAES generally shows decreased efficiency for an analogous low pressure compression/expansion system; however 100% efficiency is still obtained for an all Isothermal process. Regulated polytropic expansion produces less power per lbm of air (85 BTU/lbm vs 97 BTU/lbm) than the low pressure system due to the larger availability losses associated with pressure regulation across a larger pressure differential. It is interesting that reheating to ambient temperature can restore the losses, as evidenced by both reheat cases producing the same theoretical power regardless of storage pressure. High pressure CAES also results in extremely high temperatures during compression unless an adequate heat removal system is employed. Without heat transfer air temperatures will reach 4,825F, melting most materials.

The efficiencies presented in Tables 4.6-1 and 4.6-2 represent ideal theoretical efficiencies. Actual systems will have many losses and operational variations that will further reduce these

efficiencies. Section 4.7 discusses expected losses for a hypothetical system that uses 100% efficient Isothermal compression and expansion processes.

4.7   Energy Path for Full Cycle CAES System

This section considers losses that will occur in an actual CAES system. This example considers two hypothetical systems each using estimated operating points and standard efficiencies associated with electric motors and generators. The baseline system assumes 100% efficient compression and expansion while the modified system uses best credibly reported compression and expansion efficiencies. The entire CAES process, from electrical input power to electrical output power is considered. Figure 4.7-1 displays the energy path for the process.
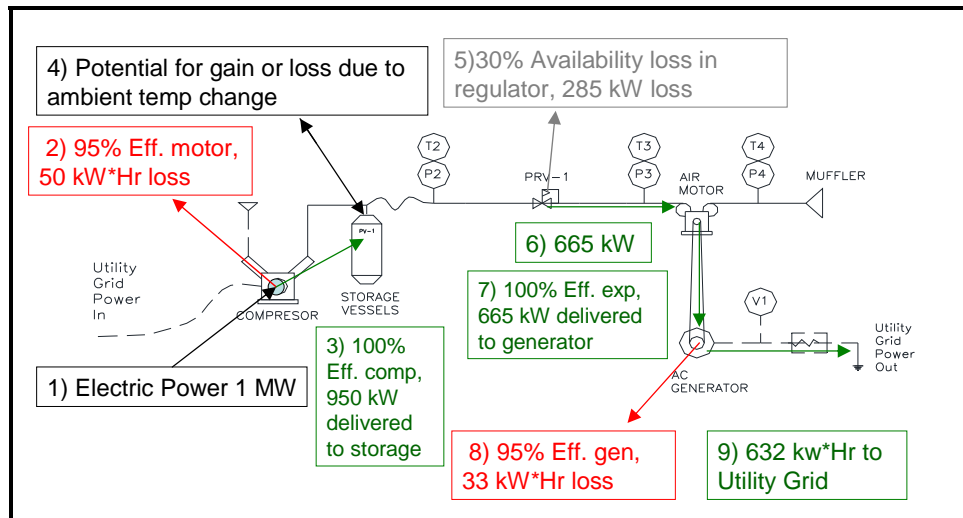
Figure 4.7-1: CAES System Energy Path

1. Electrical Power Input: This is the power required to run the compressor. This example assumes a 1 MW power input for a 1 hr run basis, therefore total energy input is 1MW*hr.

2. Motor Losses: The electric motor that drives the compressor will likely have an efficiency of about 95%, resulting in a 50 kW*hr loss and a potential 950 kW*hr to drive the compression process.

3. Theoretical Isothermal Compression: This example assumes a perfect isothermal compression process for the baseline system which achieves storage pressure with the least amount of work, resulting in a stored potential energy of 950 kW*hr. The modified example assumes 85% compression system efficiency (includes motor loss), resulting in 850 kW*hr stored potential energy.

4. Storage Pressure Gains or Losses: This example assumes there is no thermal loss or gain during storage. An increase in stored gas temperature will increase process output while gas cooling will decrease process output.

5. Pressure Regulator Availability Losses: As discussed in section 4.5 there is a large availability loss across the pressure regulator. The assumed value of 30% was based on an average of data acquired during demonstration system testing. The 30% figure includes operating characteristics of the demonstration system including losses due to friction and gains due to heat transfer from the ambient air. This reduces available expansion energy to 665 kW*hr for the baseline system and 595 kW*hr for the modified system.

6. 665 /595 kW*hr of potential energy is delivered to the air motor for expansion.

7. Theoretical Isothermal expansion: The baseline example assumes perfect isothermal expansion using ambient temperature to keep process air temperatures stable, resulting in 665 kW*hr of shaft energy being delivered to the generator. The modified example assumes 60% efficient expansion efficiency (MDI's best reported efficiency for their 4 kW air motor) which results in 357 kW*hr of shaft energy being delivered to the generator.

8. Generator losses: A 95% efficient generator is assumed resulting in 33kW*hr losses and 632 kW*hr of energy delivered to the electrical grid.for the baseline example and a 18 kW*hr losses and 339 kW*hr being delivered to the electrical grid

This example of a full cycle CAES system shows that even with perfect compression and expansion processes CAES is only capable of 63% energy efficiency. Modifying the example to include best credibly reported efficiencies of 85% compression efficiency[23] and 60% expansion efficiency[24] efficiencies results in a full cycle energy efficiency of 34%. From purely an energy conservation standpoint, these results show it is impossible to justify using a pure CAES system as a power source for the transportation industry or as a load balancer for the electrical power industry. However, when costs and current power pricing structures are considered CAES may be justifiable. 34% efficiency, as defined here, means that a plant would consume 2.94 MW*hr during off peak periods to be able to deliver 1 MW*hr during peak periods.   If peak electricity is priced greater than 3 times the cost of non peak electricity, there will be economic benefits of using a 34% efficient CAES system. It is very important to notice the direct relationship between the viability of CAES and the power pricing structure which is most often set by federal, state, and local regulators.

CHAPTER 5

0.5 KW DEMONSTRATION SYSTEM

5.1   Purpose of the .5 kW Demonstration System

The demonstration system was built to evaluate industry claims concerning the potential for
CAES to power vehicles or be used as an efficient energy storage method for utility grid load
balancing. A pure CAES system that compressed and expanded air without the use of a
supplemental fuel source was chosen because utility scale hybrid systems are already in
operation, and because there are several manufacturers currently developing systems that expand
pure air or use pure air to drive another expansion process.

5.2   Primary System Components

The demonstration system was built entirely of commercial off the shelf (COTS) components to
allow a reasonable build time. It was desired to see what kind of efficiencies could be achieved
using readily available components. A similar approach is being used by CAES pioneering
company Sustain X (reference Chapter 2); as one of their primary justifications for dependability
is that their systems use existing technology in a new application. However, the Sustain X system
does not use the same approach as the demonstration system; and as Chapter 7 shows, the
demonstration system does not achieve the efficiencies claimed by Sustain X. Figure 5.2-1 is a
photograph of the demonstration system.

Figure 5.2-1: CAES Demonstration System Photograph

The demonstration system uses a Nuvair MCH 16/9 compressor (not shown in Figure 5.2-1) originally intended to fill scuba tanks. This compressor was chosen because it was an existing Cal Poly asset and it could be used to demonstrate both low and high pressure CAES storage technologies. The compressor uses 3 stage compression with intercooling between stages. It is rated at 10.8 SCFM with a maximum operating pressure of 5000 psig. The compressor is powered by a 7.5 HP three phase motor.

The storage system consists of a six pack of Norris 8HP508C 4500 psig, 444 scf cylinders connected by a manifold. These cylinders are comparable in size to a standard K bottle. They were chosen because they were an existing Cal Poly asset that allowed a run time of over an hour if completely filled. The piping joining the compressor to the storage system was also existing Cal Poly assets consisting of 1" stainless tubing and ½" stainless jacketed flex hose. A 10 foot long ¼" ID Parker polyflex hose with Kellum grips was used to connect the storage system to the CAES demonstration system.

Two-stage pressure regulation was required to step down from storage pressure to air motor operating pressure. A Harris 8700 roughing regulator and McMaster Carr 1888K34 low pressure regulator were used to keep pressure under control. A McMaster Carr 300 psig relief valve was incorporated to prevent accidental over pressurization.

The air motor is probably the most critical part of the CAES demonstration system. The team wanted a readily available, efficient motor with a reasonable cost. Attempts were made to incorporate a Di Pietro motor into the demonstration system but the vendor was unwilling to

supply one for testing purposes. Building a similar motor based on the Di Petro design was beyond the scope of this phase of the project. Turbine motors were also evaluated but ultimately could not be purchased due to cost constraints. An existing Cal Poly owned aircraft auxiliary propulsion unit (APU) was considered but the idea was abandoned after it was determined that it was not sized appropriately for the available compressed air storage volume. An important benefit of using the APU was that it could allow hybrid and conventional testing, but further investigation showed that it would require a significant amount of work to modify the unit to accomplish dual mode operation. Ultimately a typical vane type air motor was chosen due to cost constraints. The system uses a GAST 2AM-FCC-1, rated at .93 HP with 100 psig operating pressure. The motor has a 3000 RPM speed rating.

A chain drive system couples the air motor to a Leeson 108053 permanent magnet DC motor acting as a generator. Several gear ratios were used during testing by changing out motor and generator sprockets.

The power produced by the generator is dissipated through a resistor bank consisting of a 500W rated 0-1 ohm rheostat and an assortment of .5 and 1 ohm 1000W rated fixed resistors. Several series and parallel resistor configurations were used during testing, with the rheostat and variable engine operating pressure used to control engine speed.

A collage of system components is included as figure 5.2-2.
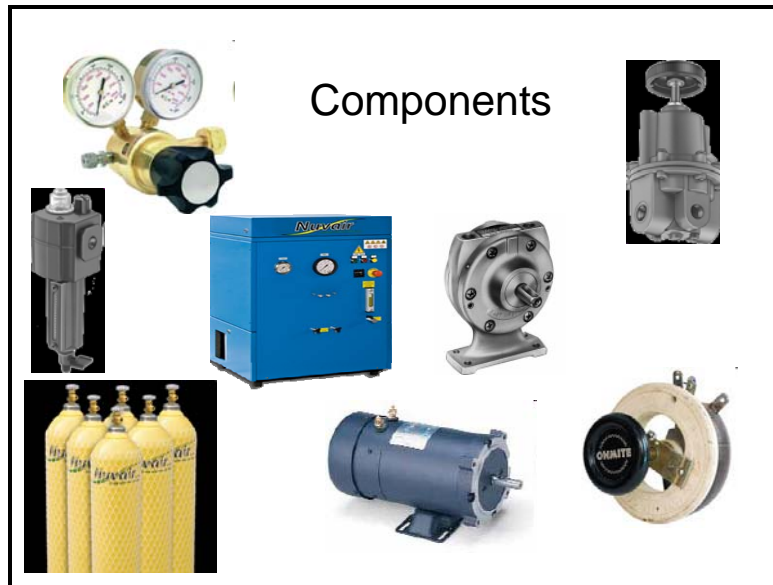
Figure 5.2-2: System Components for the Demonstration System

CHAPTER 6

INSTRUMENTATION, LABVIEW DATA AQUISITION SYSTEM, AND SUPPLEMENTAL CALCULATIONS

6.1   Instrumentation

All system performance calculations are derived from measured parameters of the demonstration system. Pressure and temperature are measured at every location where a significant state change occurs. Figure 6.1-1 is a piping and instrumentation diagram (P&ID) for the demonstration system.

T1 T2 T3 T4
P1 PRV−1 P2 PRV−2 OILER P3 AIR MOTOR P4 MUFFLER
PV−1
PWR−2 V1 RESISTANCE
L15−30 208V 3 ph supply PWR−1 COMPRESOR STORAGE VESSELS
DC GENERATOR RESISTOR BANK
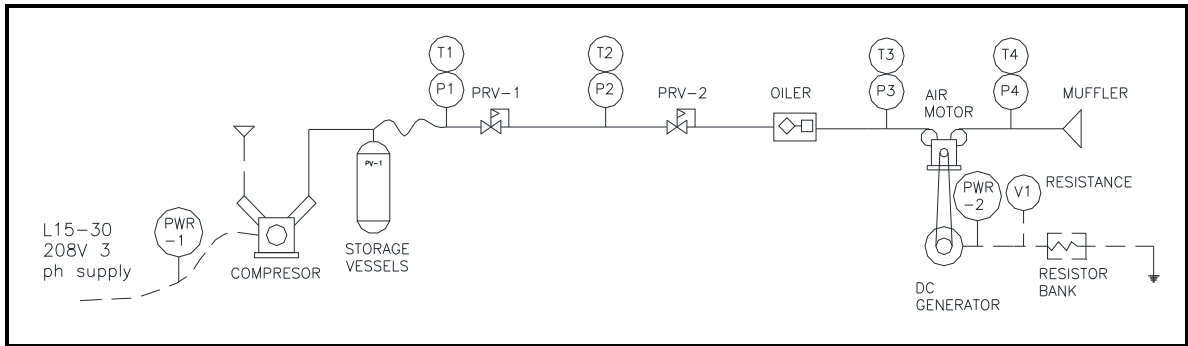
Figure 6.1-1: Demonstration System Piping and Instrumentation Diagram (P&ID)

"T" and "P" indicate temperature and pressure measurement locations. Omega PX319 pressure transducers and Omega TC-K-NPT-U72SMP type K thermocouples are used at each location shown on the P&ID. Compressor input power (PWR-1) is measured using an Ohio Semitronics Inc. (OSI) W-116 three phase AC power meter. Generator output power (PWR-2) is measured with a PC8-001-01D DC power meter from OSI. Generator output voltage is also measured directly by the DAQ.   The DAQ is also used to record absolute and elapsed time. The LabView program uses this acquired data and user inputs to determine real time operating characteristics of the demonstration system. All instrumentation was calibrated at time of purchase; post testing verification calibration was not able to be performed due to budget constraints.

6.2   Data Acquisition System and LabView Program

The National Instruments CDAQ-9174 USB acquisition system acquires and logs all instrumentation signals. The DAQ uses 2 model 9219 4 channel analog input cards and one model 9213 16 channel thermocouple input card.

The LabView program was configured for these experiments to accept 13 measured DAQ inputs and 5 user inputs to determine 36 important system characteristics in real-time. During testing all data is available to the experimentalist. This allows real time adjustments during testing to optimize efficiency, power output, or other system parameters. Important features of the DAQ and details of the critical calculations are described below. All program calculations were verified by hand calculations; and are documented in Calculation 11, Appendix B. The most important calculations are discussed here; reference Calculation 11 for a discussion of all program features.

1. Real Time load resistance is determined using basic power relations expressed by Ohms law. Generator DC voltage and power were measured to determine load resistance. This is used with general test conditions to determine the proper resistor arrangement for future tests. Lowering load resistance will decrease motor speed and increase power output for a given operating pressure.

$$\text{Load\_Res} := \frac{\text{DC\_Volts}^2}{\text{DC\_pwr}}$$

(6.1)

2. Generator Speed was calculated using the manufacturer specified generator constant (13.5 Volt/1000 RPM and the measured generator output voltage.

$$\text{Gen\_speed} := \frac{\text{DC\_Volts}}{\text{Gen\_constant}}$$

(6.2)

3. Motor Speed was calculated using the generator speed and gear ratio; which is a user input entered into the PC during test set up.

$$\text{Mtr\_speed} := \text{Gen\_speed} \cdot \frac{1}{\text{Gear\_ratio}}$$

(6.3)

4. Real time thermodynamic states were determined from the measured pressure and temperature data using state tables originally published by Vargaftik.[25] These tables were converted into data array files that the program referenced based upon the measured pressures and temperatures. Bilinear interpolation functions were used to determine Enthalpy, Entropy, and density of the process air at each location for every DAQ read. The interpolation functions are built into the LabView development package. Real time state data is the foundation for determining real time efficiencies; getting the state tables entered into a readable database and configuring LabView to properly calculate these properties real time was probably the most important and novel feature of the test bed.

5. Real time mass flow rate was determined using storage volume and calculating the rate of change in storage pressure. This calculation requires the user to input the storage volume and step size. The user inputs, pressure data, and elapsed time were used to determine the average mass flow rate during the period. Storage volume is a user input to allow reduced pump up times for high pressure testing. The compressor required over 8 hours to pump up a

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six pack of bottles. Limiting storage volume to 3 bottles allowed full pressure pump up and testing to occur in a single day, with stable ambient conditions. The step size adjusts how many data points are between the two compared pressures, a larger step size results in a greater pressure change and longer elapsed time, which will give a more averaged flow rate; while a small step size will give more of an instantaneous flow rate, although it can be influenced by signal noise or fluctuations in the steady state flow. Testing proved that step sizes between 2 and 5 gave consistent data and adequate response times to changes in test conditions. This means that, for a step size of 2, the Labview program was subtracting the pressure acquired 2 data points previous from the current pressure to achieve a delta pressure (dP). Similarly the absolute time of each measurement was subtracted, giving a delta time (dt). The ratio effectively determines the rate of pressure change in the storage volume (dP/dt). The storage pressure rate change is then converted to a mass flow rate based on density at storage pressure and the system storage volume. The ideal gas law is used with the compressibility factor for air to achieve an accurate flow rate. Compressibility factor is determined using a look up table similar to the state determinations described previously.

$$DP := P_{1\_abs} - P_{1\_past} \qquad\qquad Dt := t - t_{1\_past}$$

$$M_{.dot} := \frac{\dfrac{DP}{Dt} \cdot Storage\_vol}{Z_{.1} \cdot R_{.air} \cdot T_{.1}} \qquad\qquad \textbf{(6.4)}$$

6. Regulator Efficiencies were originally determined considering energy change through the regulator (Eff= Energy in/Energy out); however after initial testing it was obvious that for a compressed air expansion system entropy change had a much more dominate effect on power production than enthalpy and associated gas velocities. In fact enthalpy was actually increasing through the pressure regulator due to heat conduction from the ambient environment, resulting in actual energy efficiencies of over 100%, while power production was 2 orders of magnitude less than input power. The original regulator efficiency calculation is shown below, which turned out invalid for air expansion systems as configured here.

$$\eta\_HP\_reg := \frac{\left(H_{.2} + \dfrac{V_{.2}^{2}}{2} \right)}{\left(H_{.1} + \dfrac{V_{.1}^{2}}{2} \right)} \qquad\qquad \textbf{(6.5)}$$

It was clear that an entropy based model was required to characterize how the regulators were actually limiting the system power output. The regulator efficiency calculation was updated considering an available work basis. This calculation considered a theoretical isentropic expansion to atmospheric pressure from the regulator inlet state and regulator exit state. The ratio of these was considered as the "Isentropic Efficiency". When combined with the system

mass flow rate it is possible to quantify the amount of potential work lost through the regulator. The Isentropic Efficiency and lost potential work calculations are shown below. Typical regulator efficiencies were 70%, indicating significant reduction in system output due to pressure regulator losses.

$$\eta_{.s\_HP\_reg} := \frac{H_{.2} - H_{.0\_2s}}{H_{.1} - H_{.0\_1s}}$$

(6.6)

$$HP\_reg_{exit\_pwr} := -M_{dot} \cdot \left(H_2 - H_{0\_2s} \right)$$

(6.7)

$$HP\_reg_{enter\_pwr} := -M_{dot} \cdot \left(H_1 - H_{0\_1s} \right)$$

(6.8)

$$HP\_reg\_loss := -M_{.dot} \cdot \left[\left(H_{.1} - H_{.0\_1s} \right) - \left(H_{.2} - H_{.0\_2s} \right) \right]$$

(6.9)

7. The LabView Program also calculates Ideal Air Motor Power, Motor Shaft Power, and Thermodynamic Motor Efficiency assuming isentropic expansion from the motor inlet state to ambient pressure. Ideal air motor power was calculated by determining the theoretical enthalpy state that would be achieved during isentropic expansion from the motor inlet condition. The enthalpy change from the inlet condition and this theoretical state are multiplied by the mass flow rate to obtain a theoretical motor power. Actual enthalpy and air velocity changes through the motor are used to calculate the Motor Shaft Power. Motor Shaft Power is a calculated characteristic which does not include shaft friction or air velocity losses; it is not a measured motor power. Thermodynamic motor efficiency is considered as the ratio of Motor Shaft Power and Ideal Air Motor Power. One weakness of the demonstration system is the inability to decouple actual motor power, gearing losses, and measured generator output. It was intended to bench test the generator to allow determination of generator efficiencies. This was not accomplished due to budget restrictions. Characterizing generator performance would allow a more accurate motor shaft power to be calculated using the measured generator output and generator efficiency curve because gearing losses are likely negligible. This was not possible due to the lack of generator characterization testing. The thermodynamic motor efficiency calculation performed here is not a precise calculation but it does offer meaningful data regarding motor performance, especially when comparing demonstration system operating points. Total system efficiencies, which look at actual input and output powers, provide the most accurate efficiency determinations. Total system efficiency calculations are discussed in section 6.3. The LabView motor efficiencies are determined considering an ideal isentropic baseline. As discussed in Chapter 4, the isothermal process, which takes advantage of potential for heat transfer from the environment, produces more power than an isentropic process. Therefore the isentropic based efficiencies presented by the LabView program are considerably higher than if an Isothermal baseline were considered. Ideal Air Motor Power, Motor Shaft Power, and Thermodynamic Motor Efficiency calculations are shown below.

$$\text{Ideal\_mtr\_pwr} := -M_{.dot} \cdot \left(H_{.3} - H_{.4\_3s} \right)$$

(6.10)

$$\text{Mtr\_shaft\_pwr} := -M_{.dot} \cdot \left[\left(H_{.3} + \frac{V_{.3\_mtr}^2}{2} \right) - \left(H_{.4} + \frac{V_{.4\_mtr}^2}{2} \right) \right]$$

(6.11)

$$\text{Mtr\_eff2} := \frac{\text{Mtr\_shaft\_pwr}}{\text{Ideal\_mtr\_pwr}}$$

(6.12)

8. Compressor efficiencies were determined on an isentropic basis. Compressor efficiency is considered as the ratio of Ideal input power and actual power input (EFF= Ideal Input/Actual Input). Power input is a measured parameter while the ideal theoretical power of compression is determined considering an ideal isentropic process from ambient conditions to storage pressure. Mass flow rate is an integral part of the compressor efficiency calculation because measured input power is a system property. Therefore the theoretical Ideal input power requires the use of test data. Using a consistent baseline allows direct comparison of compression system efficiencies for different demonstration system operating pressures. Calculations are shown below.

$$\text{Ideal\_cmp\_pwr} := M_{.dot1} \cdot \left(H_{.1\_0s} - H_{.0} \right) \Big|$$

(6.13)

$$\eta_{.cmp} := \frac{\text{Ideal\_cmp\_pwr}}{\text{AC\_pwr}}$$

(6.14)

9. The LabView program calculates expansion system efficiency considering ideal power as an isentropic expansion from the compressed air pressure and temperature. An isothermal ideal was not used because it relies on additional heat transfer processes not incorporated into the demonstration model. Using an isothermal basis would result in lower reported efficiencies. An actual CAES plant would aspire for an Isothermal expansion, and would likely use an isothermal model in considering expansion efficiency. The calculation is shown below.

$$\text{Ideal\_exp\_pwr} := -M_{.dot} \cdot \left(H_{.1} - H_{.0\_1s} \right)$$

(6.15)

$$\text{Sys\_eff} := \frac{\text{DC\_pwr}}{\text{Ideal\_exp\_pwr}}$$

(6.16)

6.3   Supplemental Calculations Performed After Data Acquisition

The Lab View program provided important real time data and allowed determination of the best operating points of the demonstration system; however additional calculations were required to completely characterize the system and allow comparison with other energy storage methods. These calculations are discussed in this section.

1. Motor/Generator Efficiency. This calculation considered the motor and generator as a system. It used the ratio of measured DC power output and the available isentropic expansion power at the motor inlet. This allows determination of the expansion system efficiency ignoring pressure regulator losses and piping heat gain. This calculation differs from the LabView calculation in that it does not look at the actual air property change through the motor; instead it uses the DC power output. If the generator system efficiency was characterized this calculation would allow determination of actual motor efficiency by dividing the result by the generator efficiency term. This calculation provides a more accurate motor efficiency than the LabView program because it compares a theoretical isentropic expansion with the measured power rather than a measured thermodynamic state. Calculation is shown below.

$$\eta_{.mtr\_gen} := \frac{DC\_pwr}{-M_{.dot} \cdot \left(H_{.3} - H_{.4\_3s}\right)}$$

(6.17)

2. Total Conversion Efficiency. A total conversion efficiency considering both compression and expansion was not calculated real time in Lab View because the linkage between these events is not defined. The demonstration system, and any foreseeable production CAES system, is not designed to compress and expand gas at the same time. If it were it would not be considered a traditional energy storage system. Total conversion efficiency can be presented in a variety of ways because compression and expansion are two different processes occurring at different times and environmental conditions.

One possibility is looking at best compression efficiency and best expansion efficiency. This is not a true efficiency because the compressor must pump from the minimum storage pressure to the maximum storage pressure used. It is not possible to operate at one storage pressure. However, a system could be designed to operate near these conditions and it also serves as a maximum potential efficiency.

Another possibility is looking at total the sum of electrical energy out to total sum electrical energy in. Using this approach maximum efficiency could be defined using the most efficient expansion operating points and a compressor operating pressure range that resulted in the desired volume of compressed gas. Note that the storage volume becomes a factor in this type of calculation because larger storage volumes will result in a smaller storage pressure delta, which allows the compressor to operate closer to its' ideal. pressure. This method would be more applicable to a production CAES plant.

A third possibility would be to define efficiency in terms of daily electrical output and nightly electrical consumption. Most power plant CAES systems will compress through the night and expand during the peak loading requirements of the day. This method would also inherently factor in the storage volume of the system.

The method used in this project is similar to the first method discussed, as the goal was to define the best efficiency that could be obtained with a system using COTS components.

It uses the most efficient steady state condition expansion data and considers the amount of gas used extrapolating this data to a 10 minute operating period. Then it looks at the storage pressure used at this operating point and selects a compression system operating point at or above this pressure. The storage volume is considered infinite to allow a single compressor operating point to be used. This simplification is justified because the compressor efficiency was relatively flat throughout the storage pressure range and also the demonstration system storage volume was much smaller than a production plant would use (the demonstration system showed a much larger storage pressure change for a given operating time than a production plant would use).   This method gives a peak electrical storage efficiency of the demonstration system.

In addition to the electrical storage efficiency two additional calculations were used to evaluate demonstration system performance. The system was evaluated as a stand alone cooling system. CAES systems discharge lots of cold air, this efficiency establishes a cooling coefficient of performance for the CAES system. The other efficiency calculation presents a combined total system efficiency which includes electrical conversion efficiency and cooling capacity. It considers a 1.2 kW/ton baseline efficiency for the cooling system. 1.2 kW/ton is representative of a typical energy efficient commercial cooling system.[26] This efficiency would apply to any system that had electrical and cooling demand. A sample calculation from Calculation 11 is shown below. The best results from demonstration system testing are presented in Chapter 7.

$Test_{time} := 10min$

Compressor operating characteristics:

$$AC\_pwr = \left(4.776 \times 10^3\right) W \qquad M_{dot1} = (0.402) \cdot \frac{lb}{min}$$

Air motor operating characteristics

$$M_{dot} = -0.967 \cdot \frac{lb}{min} \qquad T_4 = (29.628) \cdot °F \quad \text{air motor exit temp}$$

Generator output

$$DC\_pwr = (117.782) W \qquad DC_{energy} := Test_{time} \cdot DC\_pwr$$

$$DC_{energy} = (0.02) \cdot kW \cdot hr$$

Output flow for 10 minutes results in the following total air usage and potential cooling.

$$Air\_usage := Test_{time} \cdot -M_{dot} \qquad\qquad C_{p\_air} := .240 \frac{BTU}{lbm \cdot R}$$

$$Air\_usage = 9.674 lb \qquad\qquad T_{77} := 77°F$$

Cooling Power

$$Cool_{cap} := Air\_usage \cdot C_{p\_air} \cdot \left(T_{77} - T_4\right)$$

$$TR := 12000 \frac{BTU}{hr} \qquad\qquad Cool_{cap} = (0.032) \cdot kW \cdot hr$$

Based on 77 F cooling space.

$$Cool_{pwr} := \frac{Cool_{cap}}{Test_{time}} \qquad Cool_{pwr} = (0.055) \cdot TR \qquad \text{Tons of refrigeration provided by the test bed}$$

Compressor energy to compress that much air:

$$CMP_{energy} := \frac{Air\_usage}{M_{dot1}} \cdot AC\_pwr \qquad \text{Extrapolates compressor energy to match air consumed}$$

$$CMP_{energy} = 1.916 kW \cdot hr$$

Electrical Storage Efficiency
(efficiency method 1)

$$\eta_{elec} := \frac{DC_{energy}}{CMP_{energy}}$$

$\eta_{elec} = (1.025) \cdot \%$ **Efficiency as an electrical storage device**

Total System Efficiency
(efficiency method 3)

Cooling system Efficiency
(efficiency method 2)
This is a refrigeration coefficient of performance (Cooling in/ work to get the cooling)

$$Cool_{COP} := \frac{Cool_{cap}}{CMP_{energy}}$$

$Cool_{COP} = (0.017)$ **Very poor COP, modern systems achieve COP's >3.0**

$$\eta_{total} := \frac{DC_{energy} + Cool_{pwr} \cdot 1.2 \frac{kW}{TR} \cdot Test_{time}}{CMP_{energy}}$$

$\eta_{total} = (1.599) \cdot \%$ **Total system efficiency including cooling effect based on 1.2 kW/TR refrigeration power equivalency.**

CHAPTER 7

DEMONSTRATION SYSTEM TESTING AND RESULTS

7.1   Testing Method

All testing was completed per test procedures with planned operating points. Initial testing captured all possible operating configurations by varying motor inlet pressure, motor/generator gear ratio, motor speed and load resistance. Final testing focused on the best operating points discovered in initial testing. Maximum power output and best compression and expansion system efficiencies were investigated. All test data was reviewed and post processed as described in section 6.3.

7.2   Compressor Performance

Compressor performance was relatively stable throughout the operating pressure range. Two days of testing were dedicated for analyzing compressor performance. Ambient temperatures ranged from 80F to 90F; ambient temperature was recorded at each data point to allow determination of ambient effects on compressor performance, although that is beyond the scope of this initial study. Brief transients of poor efficiency and enhanced efficiency were present due to the operating characteristics of the water removal system and an associated intermediate pressure reservoir utilized in the compressor. Input power increased with increasing storage pressure, but available expansion energy also increased. At lower storage pressures the compressor used less power but also provided less stored energy. The compressor is a fixed volume 3 stage expansion type; therefore the SCFM flow rate was relatively stable regardless of compressor discharge pressure. It did provide approximately 10% higher flow rates at higher discharge pressures. Average flow rate was .64 lbm/min.[27] Figure 7.2-1 shows input power and isentropic efficiency throughout the tested operating range.
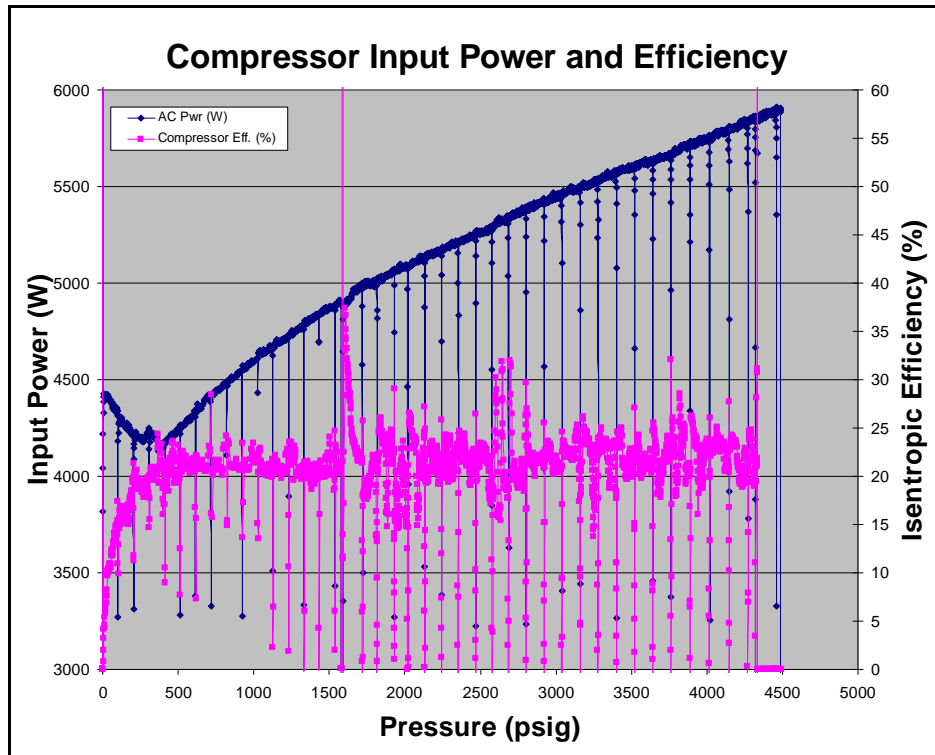
Figure 7.2-1:   Compressor Input Power and Efficiency throughout the Tested Operating Range

It is evident that the compressor power and efficiency repeatedly drop during the compression process. This is an inherent property of the compressor's water removal system. It uses process air to blow out a water trap. During this time compressor power decreases, but process air is discharged into the environment resulting in decreased overall efficiencies. An intermediate pressure reservoir associated with the water removal system also cyclically discharges, resulting in brief periods of increased efficiency. The brief increase in efficiency observed at approximately 1600 psig is not clearly understood. It did not appear in subsequent testing, and is being considered an anomaly that could possibly be investigated in further studies. One possibility is that the cycling of the water trap and intermediate pressure system were operating in a sequential manner that boosted efficiency. The Isentropic efficiency baseline shown in figure 7.2-1 is relative to the demonstration system, but it also suggests poor compressor performance. This efficiency considers the potential expansion power of an isentropic process; consideration of an isothermal expansion would result in a much higher theoretical potential expansion power, which would increase reported efficiency considerably. Compression test data shows that isentropic efficiency is generally between 16% and 25% with an overall average of 19.7%[28]. Table 7.2-1 below groups compressor performance into operating bands and displays the average isentropic based efficiency as well as the isothermal based efficiency at the median pressure of the band. Calculation 21 documents the analysis completed to determine the Isothermal efficiencies, Isentropic efficiencies are calculated real time by the LabView program, and are validated in Calculation 11. Excel was used to average the isentropic efficiencies into the bands shown in the table. Both calculations are included in Appendix B. The LabView data and averaging calculations are documented in file "Full_compression.xls".

Table 7.2-1: Compression Efficiencies Grouped by Discharge Pressure Bands

Discharge Pressure Range (Psig)	Median Pressure (Psig)	Compressor Input Power (W)	Average Isentropic Efficiency*	Isothermal Efficiency at Median Pressure**
0-500	250	4237	16.7	25.5
500-1000	750	4368	20.8	33.0
1000-1500	1250	4724	20.2	36.7
1500-2000	1750	4914	20.4	37.3
2000-2500	2250	5145	20.1	39.5
2500-3000	2750	5330	20.1	40.8
3000-3500	3250	5492	20.1	41.3
3500-4000	3750	5632	19.9	41.3
4000-4330	4165	5755	19.3	45.7

* Defined as average of efficiencies calculated at each data point, Efficiency is calculated for each data point using100*(Potential Isentropic Expansion Power/Compressor AC power in)

**Defined as 100*(Potential Isothermal expansion power at median pressure/Average of measured AC power at each data point in the operating band), Note that this is also equivalent to (Theoretical Isothermal compression input power/Actual average compressor AC power in) because the Ideal Isothermal process is reversible.

Table 7.2-1 shows that, even when an isothermal expansion baseline is considered the COTS compressor used in the CAES system is significantly less efficient than the 80-85% Dressor Rand compression system efficiency discussed in section 4.1. This is expected considering the demonstration system compressor was an existing Cal Poly asset which is designed to fill scuba tanks. It is interesting that the efficiency calculated considering potential isentropic expansion power of stored gas is relatively flat while efficiencies considering a reversible isothermal process get much better with increased storage pressure. This is because, theoretically, there is a significant amount of heat transfer into the expansion system that is assumed when considering the potential power of an isothermal expansion process. This heat transfer increases potential expansion energy. Note that because the isothermal process is reversible the isothermal efficiency can also be considered as the ratio of theoretical input power and actual input power, which is similar to the polytropic basis used by Dressor Rand; discussed in chapter 4.

7.3 Regulator Performance

Section 4.5 describes the availability losses and efficiency degradation associated with pressure regulation in air expansion systems. Test results reinforce the thermodynamics presented in section 4.5. Figure 7.3-1 shows PRV-2 isentropic efficiency vs pressure drop through the regulator.
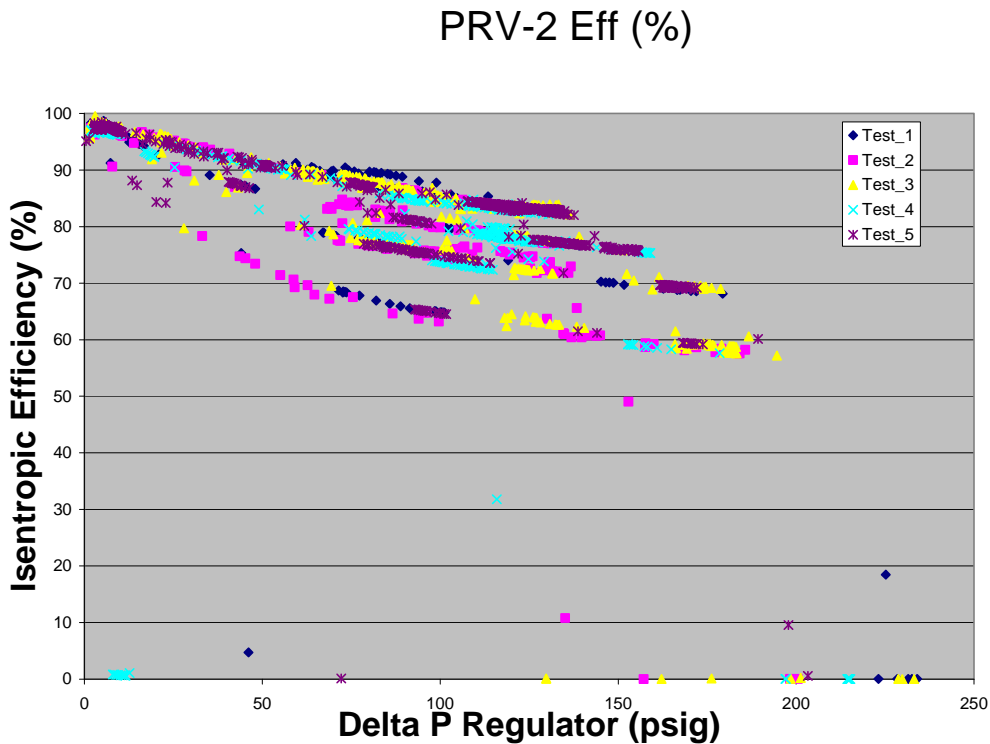
Figure 7.3-1: PR-2 (Low Pressure) Regulator Efficiency as function of Pressure Differential

It is clear that availability losses become significant as pressure differential across the regulator increases. The isentropic efficiency is defined as the ratio of theoretical isentropic expansion power per pound mass of air available after pressure regulation to that before pressure regulation. This is the low pressure regulator of the CAES system. The defined bands prompted further analysis to determine the predominate factor that effects regulator performance. The efficiency data was plotted with respect to several variables to show any correlations between regulator efficiency and the associated variable. Plots depending on Mass flow and motor speed are shown in Figure 7.3-2.
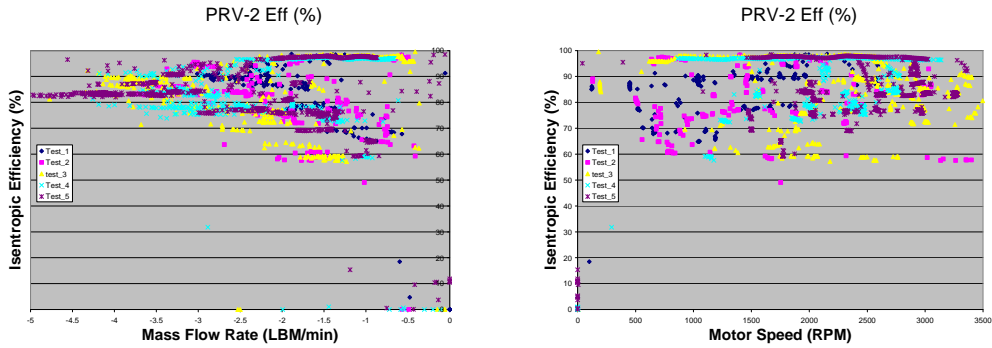
Figure 7.3-2: PR-2 (Low Pressure) Regulator Efficiency as function of Mass Flow rate and Motor Speed

It is evident that the defined bands are not specific to test days, mass flow rates, or motor speed. The mass flow plot shows some correlation in that higher mass flow rates (sign convention results in higher flow rates on the left side of the plot) generally result in higher regulator efficiencies. It was suspected that the pressure ratio of the regulator inlet and exit pressures would have the predominant effect. Figure 7.3-3 shows this to be the case.
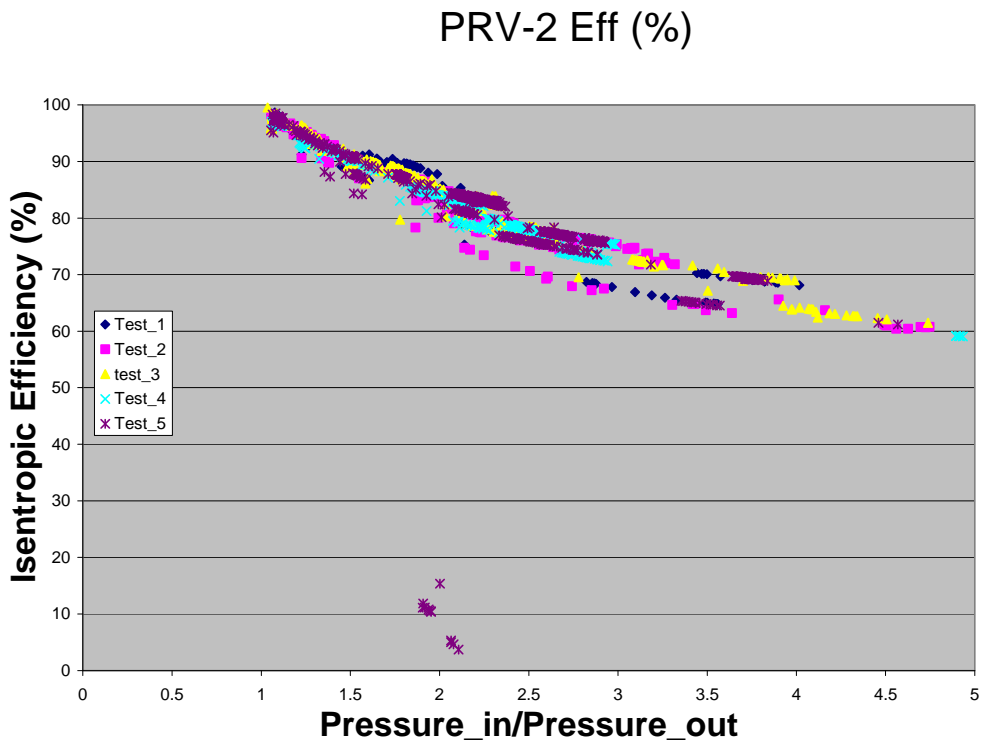
PRV-2 Eff (%)

Figure 7.3-3: PR-2 (Low Pressure) Regulator Efficiency as Function of Pressure Ratio

Figure 7.3-3 clearly shows that pressure ratio is the predominant factor in determining regulator efficiency. This is significant because it means that higher storage pressures will likely result in larger regulator losses and more varied system efficiencies. It also shows the importance of

building a system that can operate efficiently over a wide pressure band. As pressure ratios approach 5:1 regulator efficiency is about 60% regardless of other system operating points.

The High Pressure regulator performed similarly with pressure ratios less than 5:1, as shown in Figure 7.3-4.
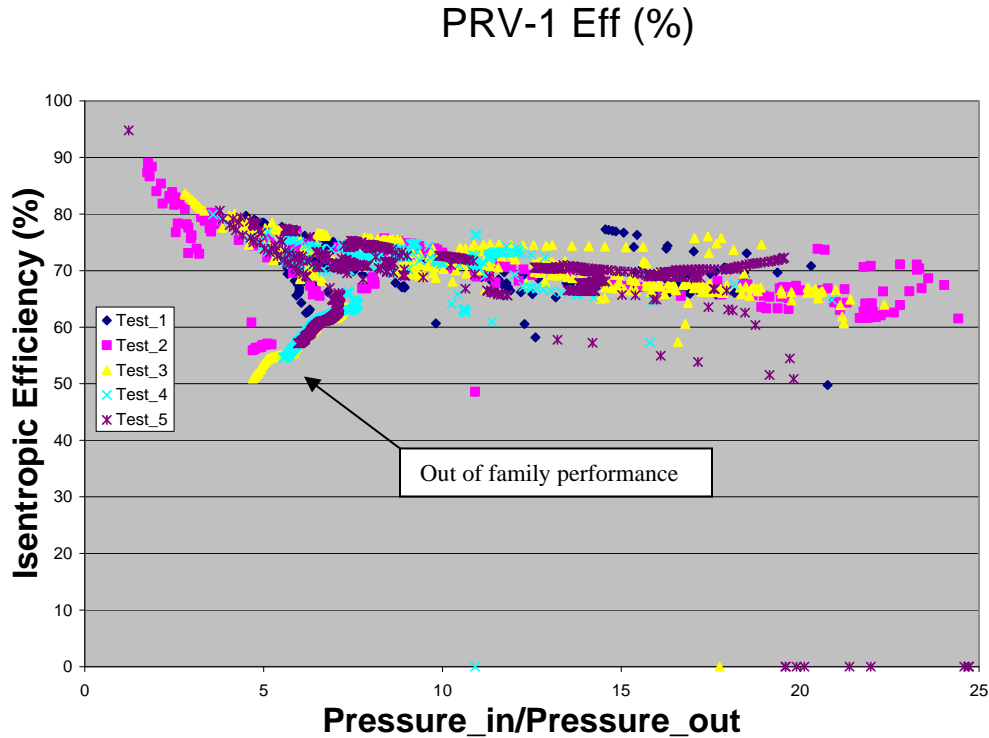
PRV-1 Eff (%)

Figure 7.3-4: PR-1 (High Pressure) Regulator Efficiency as Function of Pressure Ratio

It is interesting that the high pressure plot seems to stabilize at approximately 12:1 pressure ratio. It appears that efficiency is approximately 65-70% for pressure ratios greater than 12:1. The possibility that the 65-70% efficiency plateau is related to regulator design rather than a thermodynamic property of air may justify future research in this area. Possibly design changes could result in a higher efficiency plateau? The ability to use pressure regulators without significant losses would be a great benefit to transportation applications of CAES.

Between pressure ratios of 5:1 and 7:1 there is a significant amount of data, occurring on multiple test days, that does not fit the efficiency trend. This data shows out of family poor efficiency at relatively low pressure ratios. Analysis determined that these data points are all associated with low storage pressures where gas velocities through PRV-1 are very high. It was noted that during these periods PRV-2 Efficiency is near 100%. This is because the entropy change across PRV-1 is very large, resulting in a smaller entropy change across PRV-2 as the air is further reduced in pressure. This data further suggests that regulator design improvements may reduce regulator availability losses.

7.4 Air Motor Expansion Performance

A quick review of the air motor specification sheet was all that was required to determine that air motor performance was going to be less than spectacular. The manufacturer claims .93 shaft HP at 100 psig while using 30 SCFM. 30 SCFM @ 125 psig is about standard performance for a 7.5 HP shop air compressor supply. It was clear that 7.5 HP in and 1 HP out would result in a poor efficiency rating. Note that this motor was chosen because it met budget and schedule; better performing options failed to meet these important criteria. A scaled version of The Di Pietro air motor discussed in section 3.1 would have made an excellent test choice based on its claimed efficiencies, as would an expander based on SustainX's patented isothermal expansion process. The 4 kW MDI engine that powers its AirPod model 45 would also have been suitable if performance was the only consideration. The claimed minimum efficiency of 43% and appropriately sized power output would have allowed a much better performing demonstration system. However all of these choices did not fit the budget or schedule for this project.

Chapter 6 discusses the difficulties encountered in decoupling the motor and generator systems due to the lack of generator system characterization testing. For testing completed thus far; motor performance evaluations without generator system influence are based on air properties at the motor inlet and exit. These results provide a thermodynamic efficiency of the motor but are not directly based on shaft power produced. The efficiency is essentially analyzing the difference in the actual air state exiting the motor versus the theoretical air state exiting with isentropic expansion. As discussed in chapter 4, the isothermal model is the desired expansion process; therefore evaluating motor performance based on comparison with an isentropic ideal has limited value. However, the chosen air motor operates with a thermodynamic process that is much closer to the isentropic model than the isothermal. Section 7.5 shows results considering the motor and generator as one conversion system. Those results are based on actual motor shaft power; rather than the data presented in this section, which is comparing actual thermodynamic change of the process air to that of an ideal Isentropic process. It is the authors opinion that the section 7.5 data provides a better characterization of true motor performance, however motor thermodynamic efficiency is presented here because it measures actual air motor performance against a stable baseline. Figure 7.4-1 and Figure 7.4-2 show thermodynamic efficiency of the air motor calculated as described in section 6.2, #7.
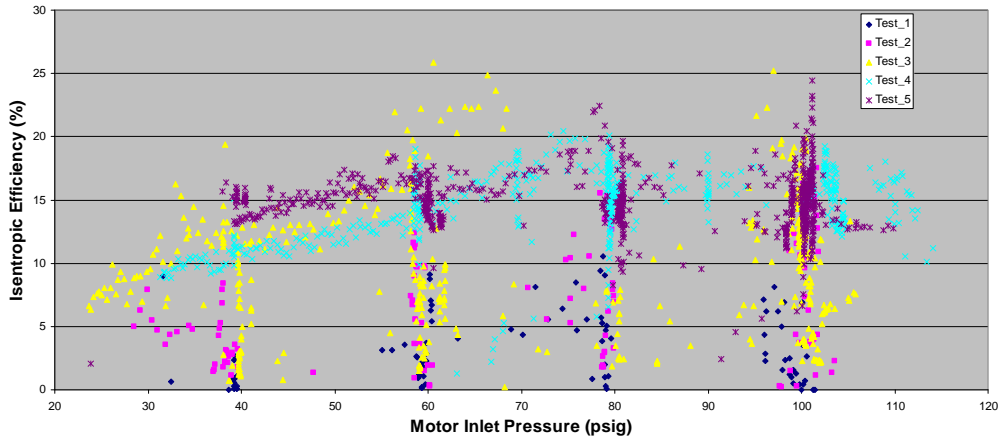
Theoretical Motor Efficiency (%)

Figure 7.4-1: Theoretical Isentropic Motor Efficiency as Function of Motor Inlet Pressure

Figure 7.4-1 shows that air motor efficiency only reaches 25% even when considering an Isentropic baseline. The distinct bands at 40, 60, 80, and 100 psig are a result of the test plan design; these pressures were primary operating points. Efficiencies in test 5 are generally higher than early tests because the system was optimized based on previous test results. Motor break in may have also contributed to the increased efficiency. It is evident that system factors and set points significantly effect motor efficiency. Initial testing was performed with less gearing (1.36:1 and 2:1 motor:generator ratio) than later tests (2.57:1), showing that the air motor performed much better with more gearing. More gearing generally meant faster motor speeds; Figure 7.4-2 shows air motor efficiency as a function of motor speed.
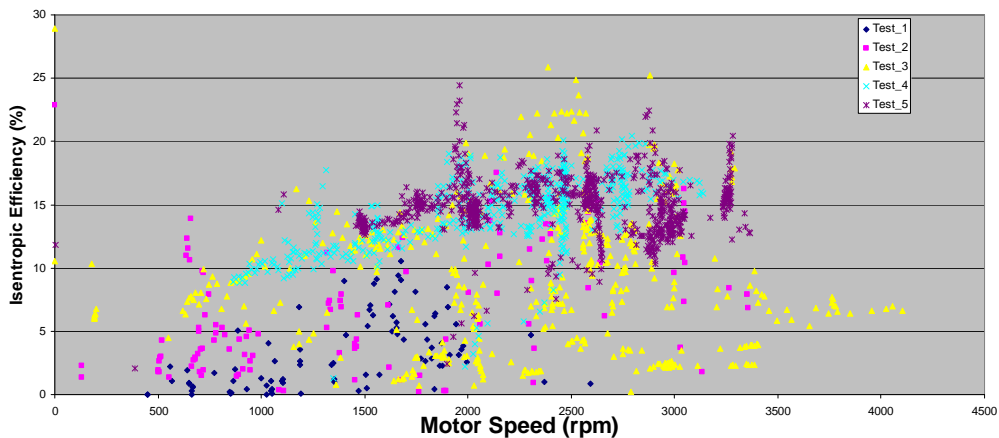
Theoretical Motor Efficiency (%)

Figure 7.4-2: Theoretical Isentropic Motor Efficiency as Function of Motor Speed

It is clear that best efficiencies occurred at or above 2000 RPM. Motor speed was not the only factor effecting motor efficiency; as there are numerous data points from the early tests at speeds above 2000 RPM with low thermodynamic efficiencies.

7.5 Expansion System Performance

The demonstration system measures actual DC power output which is directly related to air motor shaft power. DC power output is also affected by generator and gearing performance. The DC power output is an excellent and accurate measure of the CAES demonstration systems ability to create usable power from compressed air. Figure 7.5-1 shows DC output power versus mass flow rate for all expansion system testing.

Expansion System Output Power (W)
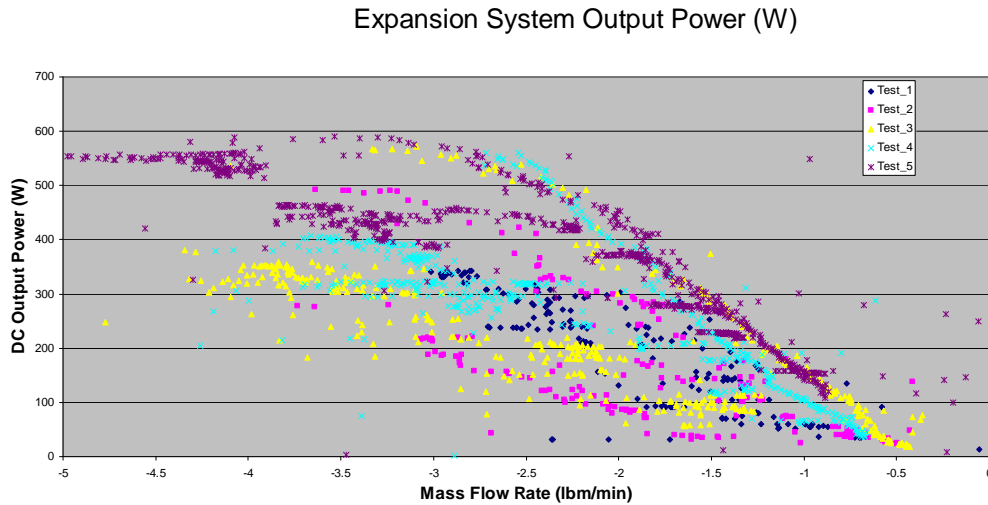
Figure 7.5-1: Expansion System Output Power as Function of Mass Flow Rate

The expected relation between air consumption and power output is clear; consuming more air will result in more power output. There is also a lot of variability in the data; for instance at a flow rate of 2 lbm/min the demonstration system will produce between 90 and 450 watts of output power depending on gearing, load, motor inlet pressure, and ambient conditions. Note the mass flow sign convention is considered positive for air into the system (compression) and negative for airflow out of the storage system; therefore higher flow rates appear on the left side of Figure 7.5-1. DC output power is also plotted versus motor inlet pressure (Figure 7.5-2) and motor speed (Figure 7.5-3) to show how these parameters effect the motor shaft power.

Expansion System Output Power (W)
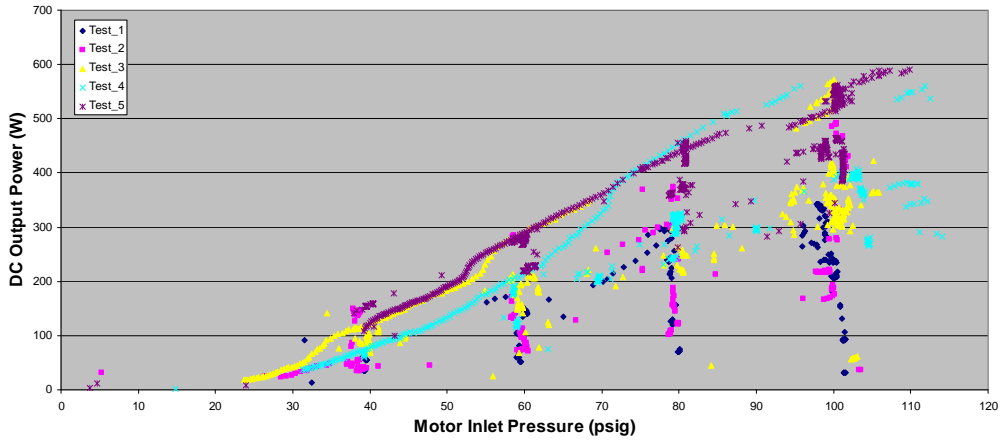
Figure 7.5-2: Expansion System Output Power as Function of Motor Inlet Pressure

It is clear that higher motor inlet pressures enable higher output power. However a high inlet pressure will not result in higher output power unless other system parameters are configured properly. Analysis of the test data at >95 psig inlet pressure shows only 18% of the data points produce greater than 400 W output. Note 400W is 68% of peak power output of 591W.

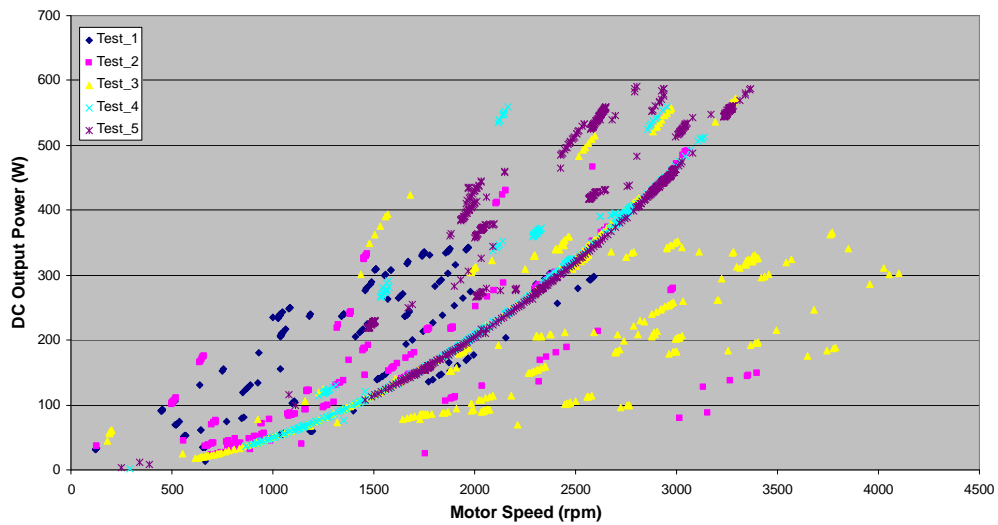## Expansion System Output Power (W)

Figure 7.5-3: Expansion System Output Power as Function of Motor Speed

It is clear that increasing motor speed has a similar effect on output power as increasing motor inlet pressure; it enables higher output power; but only if other system parameters are configured

properly. At less than 1400 RPM the system never produced more than 250W. This performance was anticipated because it is thought that the higher RPM's allow a better match of the inlet air velocity to the motor rotor speed. It was beyond the scope of this study to prove this hypothesis.

Considering the expansion sub system as the power producing part of the CAES demonstration system allows an efficiency determination that is based on actual output power and air input properties. Calculation details are documented in section 6.2 #9. Note the Isentropic model was used as a baseline in this calculation as well because the air motor process is more isentropic than isothermal. It is the author's opinion that an isothermal baseline would only be appropriate if the air motor used multiple stage expansion with heat addition. Figure 7.5-4 shows air expansion system efficiency versus DC output power.

Theoretical Expansion System Efficiency (%)
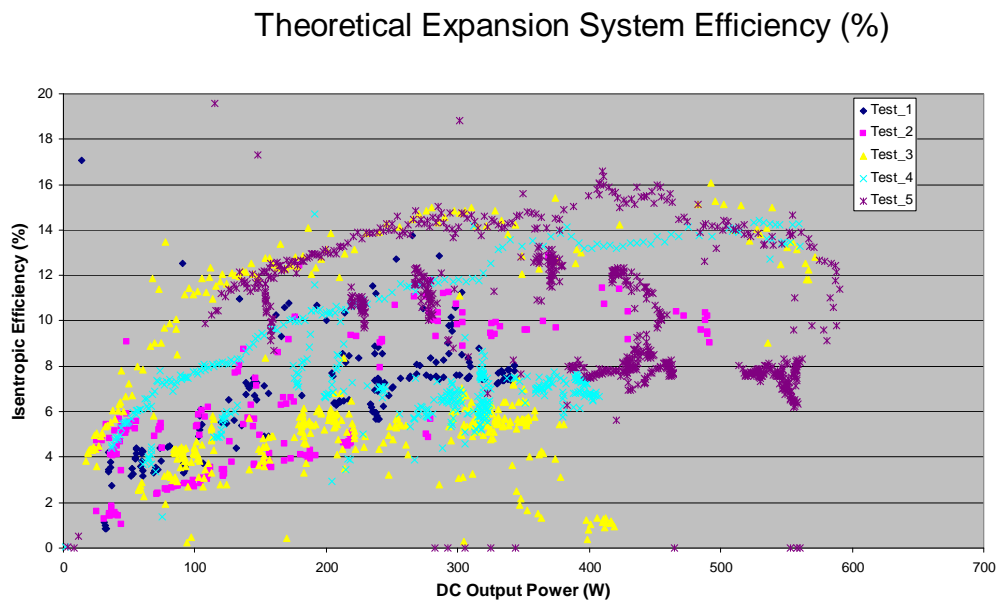
Figure 7.5-4: Expansion System Efficiency as Function of DC Output Power

Figure 7.5-4 shows a couple important facts regarding the CAES demonstration system performance. The first fact is the low expansion system efficiency; which was expected due to motor selection limitations. Excluding a few outlying data points, (which were not steady state conditions) expansion system efficiency is limited to 16%. The second important fact is that the system can produce near peak efficiency in a very broad range of power outputs. Efficiency from 50% to 100% peak output is above 14% (>87.5% of peak efficiency) for the best system set points. Good efficiency over a large range of output power is an important feature for both power generation and transportation applications of CAES technology. It is important to keep in mind that the reported efficiency is considered with an isentropic baseline. If an isothermal baseline was considered the efficiencies would drop considerably. The isentropic baseline efficiencies can be roughly converted to an isothermal basis using data presented in section 4.6 for the low pressure CAES process (isothermal expansion energy of 137 BTU/lbm versus isentropic of 72 BTU/lbm). The calculation is performed below.

$\text{Isentropic\_eff} := 16\%$

$\text{Isentropic\_ideal\_pwr} := 137\dfrac{\text{BTU}}{\text{lbm}}$

$\text{Isothermal\_ideal\_pwr} := 72\dfrac{\text{BTU}}{\text{lbm}}$

$$\text{Isothermal\_eff} := \text{Isentropic\_eff} \cdot \left(\frac{\text{Isothermal\_ideal\_pwr}}{\text{Isentropic\_ideal\_pwr}} \right) \qquad \textbf{(7.1)}$$

$\text{Isothermal\_eff} = 8.409\%$

The simple calculation is not exact but gives some insight into how low the efficiency is when a better performing thermodynamic baseline is used. Best isothermal efficiency of the expansion system is 8%.

7.6 Total System Performance

This section looks at the entire CAES demonstration system as an energy storage device. Efficiencies are looked at in three different ways; as discussed in Section 6.3, Total Conversion Efficiency Calculations. The values reported here consider the properties of the most efficient steady state expansion efficiency observed during all testing. Similarly, the properties of the most efficient steady state compression efficiency are used to determine energy required to store the air. This calculation and test data are documented in an abbreviated version of calculation 11 titled "Calculation_011-Total_sys_eff_only".

Table 7.6-1: Total System Efficiency Considering Electrical and Cooling Properties

Electrical Storage Efficiency	3.6%
Cooling COP	0.040
Total System Efficiency*	5.0%
*Based on electrical and cooling system outputs, cooling effect considers 1.2 kW/TR refrigeration power equivalency	

Best overall system efficiency data is consistent with the first look data originally reported in the October 2011 progress report. The demonstration system was completely characterized with the available hardware before the October progress report was submitted. Extensive data review showed some slightly better operating points, which are presented above. Table 7.6-2 contains all three sets of total efficiency data submitted for the project.

Table 7.6-2: Total System Efficiency Improvements During System Optimization Testing

Date Reported	Electrical Storage Efficiency (%)	Cooling COP	Total System Efficiency* (%)
7/23/2011	1.0	0.019	1.7
10/28/2011	3.3	0.034	4.5
6/15/2012	3.6	0.040	5.0
*Based on electrical and cooling system outputs, cooling effect considers 1.2 kW/TR refrigeration power equivalency			

While all the efficiencies are very poor, it is clear that the LabView program allowed large relative improvements with a limited amount of test time. Electrical efficiency improved by a factor of 3.6 by choosing better operating points. The LabView program data was essential for efficiently characterizing the demonstration system.

CHAPTER 8

POTENTIAL FUTURE STUDIES WITH CAL POLY'S CAES PROJECT

8.1 Generator Characterization to Allow Decoupling of Motor/Generator Efficiencies

The inability to accurately separate the generator and air motor performance left some uncertainty in the component level efficiencies of these parts of the demonstration system. In many areas the demonstration system had testing redundancy, which allowed verification of test data and calculation results. This is apparent in the pressure regulator performance where individual regulator losses could be added to verify that the total loss from the bottle to the motor inlet was comparable. If the generator efficiency was mapped, probably using speed and power output as the test variables, precise claims of air motor and generator performance could be claimed. This would also aid in gearing selection for future testing. Currently it is assumed that demonstration system efficiency could be improved by using more gearing. Best efficiency was obtained using the lowest gearing combinations that were purchased.

8.2 Additional Testing Using Cal Poly's 75 HP Screw Compressor Air Supply

The expansion system could only be run for approximately 1 hour using the available storage volume. While this allowed "steady state" operation of individual operating points it did not test the demonstration system's ability to operate over longer time periods. The 75 HP screw compressor would allow unlimited testing time and extensive evaluation at individual operating points.

8.3 Incorporation of a Heat Exchanger or Heat System to the Demonstration System.

The analysis presented in Chapter 4 shows that the isothermal expansion process produces much better power output and efficiency than an Isentropic process. Adding a heat exchanger, or controlled heat source to simulate a heat exchanger, would add an entire new optimization input to the demonstration system. The effect of adding heat between one or both pressure regulator expansions, or possibly reheating beyond ambient temperature could be investigated. This would probably require complete re-characterization of the system with regards to operating points for best power and efficiency. Most, if not all, practical CAES plants will use some form of reheat to improve efficiency; the demonstration system should also include this aspect of CAES.

8.4   Modify the Demonstration System to include Multi-Stage Expansion

The demonstration system could be modified to use several air motors in series. This would allow an excellent exercise in system modeling and provide some useful data for expected performance of multistage expansion systems without significant cost.

8.5   Fabrication of a Larger Scale System

At .5 kW, the demonstration system has no practical use other than as an educational demonstration tool. The Cal Poly Engines Lab owns a small aircraft auxiliary Propulsion unit (APU) and large air storage tank that could likely be used to fabricate an experimental micro sized CAES Hybrid Power plant. A project of this scale would require significant funding, but would be more likely to produce usable efficiency data that could be extrapolated and applied to larger CAES systems.

8.6   Economic Analysis of a Hybrid CAES Power Plant

This study focused on researching, analyzing, and testing the efficiencies of CAES in both the transportation and power industries. While overall theoretical efficiencies are quite high it is clear that actual process efficiencies achieved thus far (both by industry and Cal Poly) are not great. Yet industry is planning, investing, and building several new CAES plants, some of which are using essentially using the same technology as the McIntosh Alabama plant, which was built in 1991. Current power pricing structure and significant government funding encourages such plant development. An economic model of the CAES application would likely explain industry's interest in the CAES arena. A profit driven model could be developed to optimize plant characteristics based on electrical pricing and historic demand. These plants would likely be able to supply high amounts of power for brief periods of time. Compression capability could be downsized based on operating periods justified by power pricing schedules, reducing plant construction costs. While a more efficient energy storage plant would always have better performance and use less energy overall, an analysis which considered lifecycle costs may show CAES as a viable or even superior storage method.

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[27] Calculation 22, derived from test data documented and analyzed in "Full compression.xls"

[28] Calculation 22, derived from test data documented and analyzed in "Full compression.xls"

Appendix A

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Appendix B

CALCULATIONS

Table of Contents
Calculation'11'- LabView Program Verification

Calculation_011-Verification of LabVIEW real time calculations.

This analysis verifies state determination and real time calculations performed in LabVIEW. Data points were collected during test_001 performed on 8/22/11. Results between LabVIEW and Mathcad are compared. Mathcad calcuation method is same as used in Calc 010; which this file was derived from.

REV_ A: This analysis is updated to reflect other potential methods for calculating efficiency and calculates other parameters that may be of interest. It is intended to have Dr Patrick Lemieux review these calculations for accuracy and to help determine how to best present test bed attributes in the LabVIEW Program as well as final reports.

Background and Assumptions:

This analysis considers dry air. Moisture is removed by the compressor and this effect is an inherent part of the compressor efficiency determination. The compression efficiency is compared against a baseline process consisting of isentropic compression of dry air. Actual ambient air contains moisture which is condensed, adding additional energy to the process gas. It is thought that this may create a slightly conservative theoretical isentropic efficiency calculation for the compression process. After compression; the actual process gas is essentially moisture free and the assumption of dry air does not produce further error.

Efficiencies are determined using a single stage isentropic process as the baseline. The actual compressor is a three stage unit with intercooling between stages. Theoretically the compressor can have better efficiency than a single stage isentropic process. Intermediate pressures are not measured and the single stage isentropic baseline is used throughout this analysis to provide relative results for the different storage pressures tested. On the expansion side there are two pressure regulators which are analyzed considering both Isentropic and Isenthalpic efficiencies. Initial test data shows that although there is enthalpy gain through each pressure regulator there is also a large entropy gain, which limits available expansion power. The air motor is a single stage expansion process and efficiency is accurately determined considering single stage isentropic expansion.

Isentropic efficiencies are determined considering expansion from or compression to 1 bar pressure, which is slightly less than atmospheric pressure. This assumption greatly simplifies the automated LabVIEW calculations by allowing linear interpolation of pressure/entropy data rather than a bilinear interpolation of custom built entropy tables. Error analysis is performed to determine error associated with this assumption.

Ambient conditions significantly effect isentropic compression power and inherently effect the expansion process. The compression process requires more energy during high ambient or high humidity conditions. Regarding expansion; Increased ambient temperatures produce more enthalpy gain through the piping during the expansion process and may allow slightly higher motor power output. Ambient temperature is considered for the compression process but it's effect on expansion is only evident through increased heat transfer to the process air, likely slightly effecting results. At this time the LabVIEW program will not be updated to consider the effects of ambient conditions on expansion power. A separate post processing analysis may be performed after test data is acquired.

Calculation Method

All test data is presented in this analysis. Flow rate is determined using the change in source pressure during a measured time period. State tables are used to determine density, enthalpy, and entropy at each measurement location. Then efficiencies and losses are determined for each component based on standard thermodynamic analysis.

Test Point A Data- is row 95 of Test_001_8/22/11; pertaining to 1084 seconds. LabVIEW export file has 54 columns containing log and calculated Data. Index to data is shown below.

$\text{Test\_001}_{data} :=$

	0	1
0	"Absolute Time (s)"	"Elapsed Time (s)"
1	$3.397 \cdot 10^9$	...

$$A_{data} := \text{submatrix}\left(\text{Test\_001}_{data}, 94, 94, 0, 53\right)$$

All point A data is displayed on the following Page. Note the step size is 2 indicating that the DP/Dt calculation is using Time and P1 data from 2 rows previous. These are parsed from the test data below. Pressure is converted to absolute pressure

$$P_{1\_past} := \left(14.7 + \text{submatrix}\left(\text{Test\_001}_{data}, 92, 92, 8, 8\right)\right)\frac{\text{lbf}}{\text{in}^2}$$

$$P_{1\_past} = \left(1.288 \times 10^3\right) \cdot \frac{\text{lbf}}{\text{in}^2}$$

$$t_{1\_past} := \text{submatrix}\left(\text{Test\_001}_{data}, 92, 92, 0, 0\right) \cdot s$$

$$t_{1\_past} = (3396906188.419)\, s$$

Elapsed time verification also requires knowledge of the initial time stamp (before the while loop in the program) which is not available, however the first row of data is taken within milliseconds of the initial time stamp, it is used to verify the elapsed time calculation later in this analysis.

$$t_{initial} := submatrix\left(Test\_001_{data}, 1, 1, 0, 0\right) \cdot s$$

$$t_{initial} = (\ 3396905127.128\ )\ s$$

	Point A, 1084 sec Test_001_8_22_11, Used for LabView Program Verification				
0	Absolute Time (s)	3396906211.976	27	S1 (Btu/lbm*F)	1.322
1	Elapsed Time (s)	1084.848	28	S2 (Btu/lbm*F)	1.473
2	Temp_1 (F)	78.006	29	S3 (Btu/lbm*F)	1.549
3	Temp_2 (F)	59.977	30	S4 (Btu/lbm*F)	1.616
4	Temp_3 (F)	71.123	31	Rho1 (lbm/ft^3)	6.448
5	Temp_4 (F)	29.628	32	Rho2 (lbm/ft^3)	0.773
6	DC PWR (W)	117.782	33	Rho3 (lbm/ft^3)	0.272
7	DC Volts (V)	11.390	34	Rho4 (lbm/ft^3)	0.086
8	Press_1 (psi)	1257.520	35	PRV-1 Eff (%)	102.089
9	Press_2 (psi)	133.521	36	PRV-2 Eff (%)	102.762
10	Press_3 (psi)	38.711	37	Total Piping Loss (W)	-5.734
11	Press_4 (psi)	0.965	38	Ideal Mtr Pwr (W)	668.207
12	AC Pwr (W)	-3.483	39	Ideal Expansion Pwr (W)	1486.004
13	AC WH Not Used	0.000	40	Compressor Eff. (%)	-42669.630
14	Load Res. (ohms)	1.101	41	Air Compress Factor	0.990
15	Gen Speed (rpm)	843.685	42	DP/DT (psi/sec)	-0.658
16	Mtr Speed (rpm)	1687.370	43	M dot (lbm/min)	-0.968
17	mtr gear (# teeth)	18.000	44	H_exp_idl (Btu/lbm)	33.378
18	gen gear (# teeth)	36.000	45	H4 ideal (Btu/lbm)	87.358
19	Gen Constant (V/rpm)	0.014	46	Mtr Vel Loss (W)	2.385
20	Gear Ratio	0.500	47	Mtr Shaft Pwr (W)	162.734
21	Stor Vol (ft^3)	4.830	48	Gen/Gear Eff (%)	72.377
22	Avail Air (%)	28.397	49	Mtr Eff (%)	24.354
23	H1 (Btu/lbm)	120.723	50	System Conversion Eff. (%)	7.926
24	H2 (Btu/lbm)	123.244	51	step ind.	91.000
25	H3 (Btu/lbm)	126.634	52	loop ind.	93.000
26	H4 (Btu/lbm)	116.928	53	step size	2.000

Verification of LabVIEW Calculations. Verification is completed using order of appearance of log data when possible. For some calculations intermediate values are required; necessitating some calculations to appear earlier. The LabVIEW/MathCad column index (shown previously) is used to identify the calculation being verified. In some cases additional calculations are performed in anticipation of Dr Lemieux's review. These calculations may be added to the LabVIEW program if it's proven that they provide meaningful information. These calculations are identified as supplemental calculations and are assigned an "S number" for identification. For example S1 is supplemental calculation 1.

#2 Elapsed Time

The LabVIEW program uses two absolute time stamps, one is inside the while loop (current time) and the other runs once at the beginning of the test (initial time) . The elapsed time is determined by subtracting the initial time from the current time.

$$t := submatrix\left(Test\_001_{data}, 94, 94, 0, 0\right) \cdot s$$

$$t_{elapsed} := t - t_{initial}$$

$$t_{elapsed} = (1084.848) \, s \quad \text{This matches log data}$$

#14 Load Resistance

Load resistance is determined using basic power relations derived from ohms law. Power= Volts^2/Load_res, thus Load_res = Volts^2/Power . Both DC voltage and power are measured, this allows determination of the load resistance.

$$DC\_pwr := submatrix\left(A_{data}, 0, 0, 6, 6\right) \cdot W \qquad DC\_Volts := submatrix\left(A_{data}, 0, 0, 7, 7\right) \cdot V$$

$$DC\_pwr = (117.782) \, W$$

$$DC\_Volts = (11.39) \, V$$

$$Load\_Res := \frac{DC\_Volts^2}{DC\_pwr}$$

$$Load\_Res = 1.101 \, \Omega$$

This matches log data.

#15 Generator Speed

Generator speed is calculated by dividing the generator constant (user input #19),
.0135 V/RPM, by the generator voltage (DC Volts, log data #7)

$$\text{Gen\_constant} := \text{submatrix}\left(A_{\text{data}}, 0, 0, 19, 19\right) \frac{V}{\text{rpm}}$$

$$\text{Gen\_constant} = (0.0135) \cdot \frac{V}{\text{rpm}}$$

$$\text{Gen\_speed} := \frac{\text{DC\_Volts}}{\text{Gen\_constant}}$$

$$\text{Gen\_speed} = 843.685 \cdot \text{rpm}$$

This matches log data.

#16 Motor Speed and #20 Gear Ratio

Air Motor speed is proportional to generator speed with 1/gear ratio being the proportionality
constant between Generator speed and motor speed. Gear ratio is calculated by dividing
the Motor Gear Teeth (#17) and Generator Gear teeth (#18) user inputs

$$\text{Mtr\_gear} := \text{submatrix}\left(A_{\text{data}}, 0, 0, 17, 17\right)$$

$$\text{Mtr\_gear} = (18)$$

$$\text{Gen\_gear} := \text{submatrix}\left(A_{\text{data}}, 0, 0, 18, 18\right)$$

$$\text{Gen\_gear} = (36)$$

$$\text{Gear\_ratio} := \frac{\text{Mtr\_gear}}{\text{Gen\_gear}}$$

$$\text{Gear\_ratio} = 0.5$$

This matches log data.

$$\text{Mtr\_speed} := \text{Gen\_speed} \cdot \frac{1}{\text{Gear\_ratio}}$$

$$\text{Mtr\_speed} = 1687.37 \cdot \text{rpm}$$

This matches log data.

#22 Available Air and #41 Compressibility Factor

Available air is determined by reading the bottle pressure and temperature and looking up the compressibility factor for air, LabVIEW uses a look up table and interpolation to determine the appropriate compressibility factor (#41). The storage volume (#21), current bottle pressure (P1, #8), temperature (#2), and 4500 psig maximum bottle pressure are used to determine % air remaining. In this verification analysis table density values are used to determine the amount of air at max pressure and the amount of air at the given pressure. Standard conditions of 4500 psig and 70F are used to determine maximum air capacity of the storage array. The LabVIEW calculation currently assumes the current temperature is the basis for the maximum air capacity and also assumes a unity compression factor for maximum capacity. In actuality the max capacity Z factor will vary with temperature between 1.07 and 1.14. The amount of air in the bottle also depends on the initial conditions when the bottle was filled. This introduces some measurable error between the LabVIEW calculation and actual air remaining. The available air parameter is not used in any of the efficiency calculations and is intended as a gage to facilitate test planning and is therefore considered valid. The following verification includes an error analysis that presents a typical error associated with this calculation

Maximum air at MOP (4500 psig), assume standard 70F temp during bottle fill.

$$\text{Storage\_vol} := \text{submatrix}\left(A_{\text{data}}, 0, 0, 21, 21\right) \text{ft}^3$$

$$\text{Storage\_vol} = (4.83) \cdot \text{ft}^3$$

The bottle manufacturer has provided CAD drawings which show bottle capacity is 2844 in^3. Using this number the storage volume per cylinder that should be used in LabVIEW is calculated below.

$$2844 \text{in}^3 = 1.646 \cdot \text{ft}^3$$ LabVIEW program should be updated to use 1.646 constant in the bottle volume calculation.

interpolation using 4500 psig and 70F for the initial compressibility factor A standard bilinear interpolation formula is used, shown below

$$
\begin{aligned}
f(x, y, x1, y1, x2, y2, f11, f12, f21, f22) := \ & \frac{f11}{(x2 - x1)\cdot(y2 - y1)} \cdot (x2 - x)\cdot(y2 - y) \ \ldots \\
& + \frac{f21}{(x2 - x1)\cdot(y2 - y1)} \cdot (x - x1)\cdot(y2 - y) \ \ldots \\
& + \frac{f12}{(x2 - x1)\cdot(y2 - y1)} \cdot (x2 - x)\cdot(y - y1) \ \ldots \\
& + \frac{f22}{(x2 - x1)\cdot(y2 - y1)} \cdot (x - x1)\cdot(y - y1)
\end{aligned}
$$

Initial compressibility factor, Note not used in LabVIEW

$$Z_0 := f(70, 4500, -9.67, 4336.2, 80.33, 5786.5, 1.0702, 1.199, 1.1089, 1.2073)$$

$$Z_0 = 1.116$$

Z factor for 4500 psig and 70 F

$$R_{air} := 53.33 \frac{ft \cdot lbf}{lbm \cdot R}$$ Ideal gas constant for air

$$M_0 := \frac{4500 \frac{lbf}{in^2}(Storage\_vol)}{Z_0 \cdot R_{air} \cdot 529.67R}$$ $$M_0 = (99.287) \cdot lbm$$

$$P_{1\_abs} := \left[\left(submatrix(A_{data}, 0, 0, 8, 8) + 14.7\right)\frac{lbf}{in^2}\right]$$ $$P_{1\_abs} = (1.272 \times 10^3) \cdot \frac{lbf}{in^2}$$

Need to use abs pressure for Z value

$$T_1 := submatrix(A_{data}, 0, 0, 2, 2) \, °F$$

$$T_1 = (537.676) \cdot R$$

$$T_1 / °F = (78.006)$$

$$Z_1 := f(78.006, 1272, -9.67, 1145.54, 80.33, 1435.6, .9463, .9411, .9903, .993)$$

$$Z_1 = 0.99$$ This matches log data

$$M_1 := \frac{P_{1\_abs}(Storage\_vol)}{Z_1 \cdot R_{air} \cdot T_1}$$ $$M_1 = (31.163) \cdot lbm$$

$$Avail\_air := \frac{M_1}{M_0}$$ $$Avail\_air = 31.386 \cdot \%$$

This value is different from LabVIEW because the LabVIEW calculation does not use the correct compressibility factor for air at maximum pressure, causing a slightly conservative reading. Percent difference is calculated below.

$$Avail\_air_{lb\_vw} := submatrix(A_{data}, 0, 0, 22, 22) \, \%$$

$$Avail\_air_{lb\_vw} = (28.397) \cdot \%$$

$$Avail\_air_{\%\_dif} := Avail\_air_{lb\_vw} - Avail\_air$$

$\text{Avail\_air}_{\%\_dif} = (-2.989) \cdot \%$ **LabVIEW data reports almost 3% low at this pressure, This calculation does not effect any efficiency calculations but should be corrected in LabVIEW by including the Z.0 value of 1.116 in the numerator of the LabVIEW calculation.**

#23 thru #34

#23 thru #34 are State properties that are determined using linear interpolation from table data presented in Vargaftik's Handbook of Physical Properties of Liquids and Gases, 2nd edition, ISBN 3-540-16972-5. Table data was entered into spreadsheet files which the LabVIEW program loads into memory and references each time a read is performed. LabVIEW has a built-in linear interpolation program. Spreadsheet tables are of slightly lower resolution than the complete Vargaftik tables, therefore insignificant deviations are expected. Verification is performed by using the same standard Bilinear interpolation function used in the available air calculation.

#23 H1, #27 S1, and #31 RHO_1,

$\text{Bar} := 14.50377 \dfrac{\text{lbf}}{\text{in}^2}$ $\text{kJ} := 1000\text{J}$

$P_{1\_abs} = (87.717) \cdot \text{Bar}$

$T_1 = (298.709) \text{ K}$

$H_1 := f(87.717, 298.709, 80, 290, 100, 300, 272.3, 283.7, 268.3, 279.9) \dfrac{\text{kJ}}{\text{kg}}$

$H_1 = 120.702 \cdot \dfrac{\text{BTU}}{\text{lbm}}$ LabVIEW shows 120.723, nearly identical

$S_1 := f(87.717, 298.709, 80, 290, 100, 300, 5.524, 5.562, 5.447, 5.486) \dfrac{\text{kJ}}{\text{kg} \cdot \text{K}}$

$S_1 = 1.3203 \cdot \dfrac{\text{BTU}}{\text{lb} \cdot \text{R}}$ LabVIEW shows 1.322, nearly identical

$\rho_1 := \dfrac{1}{f(87.717, 298.709, 80, 290, 100, 300, 10.24, 10.66, 8.204, 8.551) \cdot \dfrac{1}{\text{kg}}}$

$\rho_1 = 6.373 \cdot \dfrac{\text{lbm}}{\text{ft}^3}$ Density Calculated in LabVIEW is 6.448, % difference is calculated below

$$\rho_{1.lb\_vw} := 6.448 \frac{lbm}{ft^3}$$

$$\rho_{1\_\%\_dif} := \frac{\rho_{1.lb\_vw} - \rho_1}{\rho_{1.lb\_vw}}$$

$\rho_{1\_\%\_dif} = 1.163 \cdot \%$ **There is 1.2% difference in density calculations, this will effect efficiency calculations and is investigated further to verify that this is solely a result of differences in LabVIEW table resolution.**

$1145 \frac{lbf}{in^2} = 78.945 \cdot Bar$ $1435.6 \frac{lbf}{in^2} = 98.981 \cdot Bar$

$290K = 62.33 \cdot °F$ $300K = 80.33 \cdot °F$

LabVIEW table values include pressures and temperatures at same resolution as Vargaftik at this pressure-data should be identical if interpolation methods are identical.

Review of actual table data is below.

$P1\_T1 := 6.096484 \frac{lbm}{ft^3}$ $\frac{1}{P1\_T1} = 10.24 \cdot \frac{1}{kg}$ LabVIEW Table data matches Vargaftik Table Data.

$P1\_T2 := 5.856285 \cdot \frac{lbm}{ft^3}$ $\frac{1}{P1\_T2} = 10.66 \cdot \frac{1}{kg}$

$P2\_T1 := 7.609459 \cdot \frac{lbm}{ft^3}$ $\frac{1}{P2\_T1} = 8.204 \cdot \frac{1}{kg}$

$P2\_T2 := 7.300667 \cdot \frac{lbm}{ft^3}$ $\frac{1}{P2\_T2} = 8.551 \cdot \frac{1}{kg}$

The rows surrounding the data point being verified are calculated to see if the program is analyzing the correct data point.

$P_{1\_previous} := 1273.026 \frac{lb}{in^2}$ $P_{1\_next} := 1257.52 \frac{lb}{in^2}$

$P_{1\_previous} = 8.95 \frac{s^2}{m} \cdot Bar$ $P_{1\_next} = 8.841 \frac{s^2}{m} \cdot Bar$

$T_{1\_previous} := 78.06 °F$ $T_{1\_next} := 78.01 °F$

$T_{1\_previous} = 298.739 K$ $T_{1\_next} = 298.711 K$

$$\rho_{1\_previous} := \frac{1}{f(87.772, 298.739, 80, 290, 100, 300, 10.24, 10.66, 8.204, 8.551) \cdot \frac{1}{kg}}$$

$$\rho_{1\_previous} = 6.376 \cdot \frac{lbm}{ft^3} \qquad \text{Does not match LabVIEW Row 95 calculation}$$

$$\rho_{1\_next} := \frac{1}{f(86.703, 298.711, 80, 290, 100, 300, 10.24, 10.66, 8.204, 8.551) \cdot \frac{1}{kg}}$$

$$\rho_{1\_next} = 6.304 \cdot \frac{lbm}{ft^3} \qquad \text{Does not match LabVIEW Row 95 calculation}$$

Review of Enthalpy and Entropy calculations shows that LabVIEW data is always higher than the value calculated using the standard interpolation relation found on Wikipedia. It is more likely that the Wikipedia relation was copied incorrectly or contains an error than the LabVIEW calculation is incorrect.

A second source was used to confirm the Wikipedia Interpolation relation. Helmuth Spath's "Two dimensional Spline Interpolation Algorithms" 1995 ISBN 1-56881-017-2, pg 16 formula 2.3 was used.

$$g(x, y, x_1, y_1, x_2, y_2, z_{11}, z_{12}, z_{21}, z_{22}) := z_{11} \cdot \frac{(x - x_2) \cdot (y - y_2)}{(x_1 - x_2) \cdot (y_1 - y_2)} + z_{12} \cdot \frac{(x - x_2) \cdot (y - y_1)}{(x_1 - x_2) \cdot (y_2 - y_1)} \dots$$
$$+ \left[z_{21} \cdot \frac{(x - x_1) \cdot (y - y_2)}{(x_2 - x_1) \cdot (y_1 - y_2)} \right] + z_{22} \cdot \frac{(x - x_1) \cdot (y - y_1)}{(x_2 - x_1) \cdot (y_2 - y_1)}$$

$$\rho_{1\_verify} := \frac{1}{g(86.703, 298.711, 80, 290, 100, 300, 10.24, 10.66, 8.204, 8.551) \cdot \frac{1}{kg}}$$

$$\rho_{1\_verify} = 6.304 \cdot \frac{lbm}{ft^3}$$

Results are identical to the Wikipedia relation. The discrepancy resides in the LabVIEW VI. It will be monitored for significant variations and corrected if necessary.

#24 H2, #28 S2, and #32 RHO_2,

$$P_{2\_abs} := \left[\left(\text{submatrix}\left(A_{data}, 0, 0, 9, 9\right) + 14.7 \right) \frac{lbf}{in^2} \right]$$

$$P_{2\_abs} = (10.219) \cdot Bar$$

$$T_2 := \text{submatrix}\left(A_{data}, 0, 0, 3, 3\right) °F$$

$$T_2 / °F = (59.977) \qquad\qquad T_2 = (288.693)\, K$$

$$H_2 := g\left(\frac{P_{2\_abs}}{Bar}, \frac{T_2}{K}, 10, 280, 15, 290, 277.8, 288, 276.5, 286.8 \right) \frac{kJ}{kg}$$

$$H_2 = 123.222 \cdot \frac{BTU}{lbm} \qquad$$ LabVIEW shows 123.244, nearly identical

$$S_2 := g\left(\frac{P_{2\_abs}}{Bar}, \frac{T_2}{K}, 10, 280, 15, 290, 6.134, 6.169, 6.019, 6.05 \right) \frac{kJ}{kg \cdot K}$$

$$S_2 = 1.4711 \cdot \frac{BTU}{lb \cdot R} \qquad$$ LabVIEW shows 1.473, nearly identical

$$\rho_2 := \frac{1}{g\left(\frac{P_{2\_abs}}{Bar}, \frac{T_2}{K}, 10, 280, 15, 290, 80, 82.95, 53.45, 55.45 \right) \cdot \frac{1}{kg}}$$

$$\rho_2 = 0.767 \cdot \frac{lbm}{ft^3} \qquad$$ Density Calculated in LabVIEW is .773, % difference is calculated below

$$\rho_{2lb\_vw} := .773 \frac{lbm}{ft^3}$$

$$\rho_{2\_\%\_dif} := \frac{\rho_{2lb\_vw} - \rho_2}{\rho_{2lb\_vw}}$$

$\rho_{2\_\%\_dif} = 0.74 \cdot \%$ **There is <1% difference in density calculations, will continue to verify LabVIEW interpolation method does not show significant difference than standard linear interpolation.**

#25 H3, #29 S3, and #33 RHO_3,

$$P_{3\_abs} := \left[\left(\text{submatrix}\left(A_{data}, 0, 0, 10, 10\right) + 14.7\right)\frac{lbf}{in^2}\right]$$

$$P_{3\_abs} = (3.683) \cdot Bar$$

$$T_3 := \text{submatrix}\left(A_{data}, 0, 0, 4, 4\right) {}^\circ F$$

$$T_3 / {}^\circ F = (71.123) \qquad T_3 = (294.885) K$$

$$H_3 := g\left(\frac{P_{3\_abs}}{Bar}, \frac{T_3}{K}, 3, 290, 4, 300, 289.7, 299.8, 289.5, 299.6\right)\frac{kJ}{kg}$$

$$H_3 = 126.611 \cdot \frac{BTU}{lbm} \qquad \text{LabVIEW shows 126.634, nearly identical}$$

$$S_3 := g\left(\frac{P_{3\_abs}}{Bar}, \frac{T_3}{K}, 3, 290, 4, 300, 6.520, 6.554, 6.437, 6.471\right)\frac{kJ}{kg \cdot K}$$

$$S_3 = 1.5477 \cdot \frac{BTU}{lb \cdot R} \qquad \text{LabVIEW shows 1.549, nearly identical}$$

$$\rho_3 := \frac{1}{g\left(\dfrac{P_{3\_abs}}{Bar}, \dfrac{T_3}{K}, 3, 290, 4, 300, 277.2, 286.8, 207.8, 215.1\right) \cdot \dfrac{1}{kg}}$$

$$\rho_3 = 0.267 \cdot \frac{lbm}{ft^3} \qquad \text{Density Calculated in LabVIEW is .272, \% difference is calculated below}$$

$$\rho_{3lb\_vw} := .272 \frac{lbm}{ft^3}$$

$$\rho_{3\_\%\_dif} := \frac{\rho_{3lb\_vw} - \rho_3}{\rho_{3lb\_vw}}$$

There is 1.8% difference in density calculations, will continue to verify LabVIEW interpolation method does not show increased significant difference than standard linear interpolation.

$\rho_{3\_\%\_dif} = 1.813 \cdot \%$

#26 H4, #30 S4, and #34 RHO_4,

$$P_{4\_abs} := \left[\left(\text{submatrix}(A_{data}, 0, 0, 11, 11) + 14.7 \right) \frac{lbf}{in^2} \right]$$

$$P_{4\_abs} = (1.08) \cdot Bar$$

$$T_4 := \text{submatrix}(A_{data}, 0, 0, 5, 5) \, °F$$

$$T_4 / °F = (29.628) \qquad T_4 = (271.832) \, K$$

$$H_4 := g\left(\frac{P_{4\_abs}}{Bar}, \frac{T_4}{K}, 1, 270, 2, 280, 270.1, 280.2, 269.8, 279.9 \right) \frac{kJ}{kg}$$

$$H_4 = 116.907 \cdot \frac{BTU}{lbm}$$ LabVIEW shows 116.928, nearly identical

$$S_4 := g\left(\frac{P_{4\_abs}}{Bar}, \frac{T_4}{K}, 1, 270, 2, 280, 6.765, 6.802, 6.565, 6.602 \right) \frac{kJ}{kg \cdot K}$$

$$S_4 = 1.6136 \cdot \frac{BTU}{lb \cdot R}$$ LabVIEW shows 1.616, nearly identical

$$\rho_4 := \frac{1}{g\left(\frac{P_{4\_abs}}{Bar}, \frac{T_4}{K}, 1, 270, 2, 280, 774.6, 803.4, 387.1, 401.5 \right) \cdot \frac{1}{kg}}$$

$$\rho_4 = 0.083 \cdot \frac{lbm}{ft^3}$$ Density Calculated in LabVIEW is .086, % difference is calculated below

$$\rho_{4lb\_vw} := .086 \frac{lbm}{ft^3}$$

$$\rho_{4\_\%\_dif} := \frac{\rho_{4lb\_vw} - \rho_4}{\rho_{4lb\_vw}}$$

$\rho_{4\_\%\_dif} = 3.035 \cdot \%$ **There is 3% difference in density calculations, will continue to verify LabVIEW interpolation method does not show increased significant difference than standard linear interpolation.**

Mollier Diagram Test_001 1084 sec
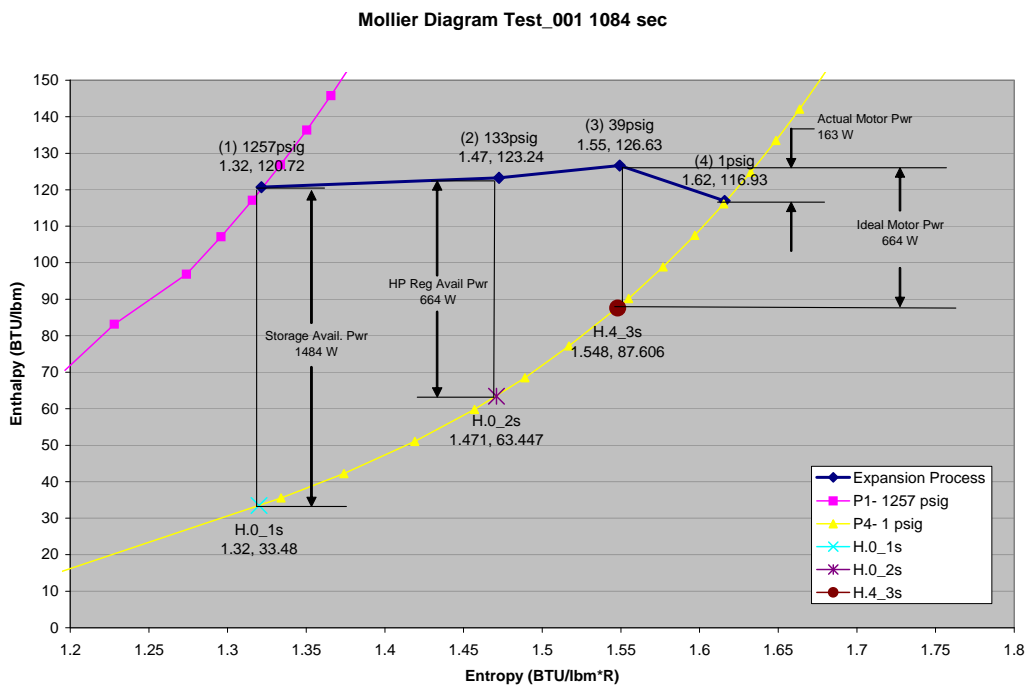
Mollier Diagram shows the results of the following calculations, including Isentropic power that is available at each location. It is evident that large losses occur due to the pressure regulators and that the motor expansion is not close to Isentropic. Temperature decreases rapidly as Entropy is decreased along a constant pressure line. It is evident that ideal isentropic expansions will be difficult to achieve due to the extremely low temperatures involved. H.4_3S (isentropic expansion through the air motor) results in exit temperature of approximately -100 F. Isentropic expansion from location 1 results in air approaching the liquid state and bordering on deomposition. The temperature is approximately -300 F. Note: a full page copy of this diagram is inclued in the Appendix as Figure 1. A genralized Mollier diagram is included as Figure 3 to give an idea of overall air properties.

#35 PRV-1 Efficiency, #42 DP/Dt, and #43 Mass Flow rate.

High pressure Regulator efficiency is determined assuming an ideal case of constant enthalpy through the regulator. Pressure and temperature data before and after the regulator are used to determine actual states. Velocity changes are included in the energy balance for increased accuracy; which requires determination of mass flow rate.

DP/Dt is based on the step size input which is 2 for this data row. The pressure and temperature data at state 1 was separated from the test data matrix earlier in this analysis.

$$P_{1\_past} = (1287.726) \cdot \frac{lbf}{in^2}$$

$$t_{1\_past} = (3396906188.419) \, s$$

$$P_{1\_abs} = (1272.22) \cdot \frac{lbf}{in^2}$$

$$t = (3396906211.976) \, s$$

$$DP := P_{1\_abs} - P_{1\_past}$$

$$Dt := t - t_{1\_past}$$

$$\frac{DP}{Dt} = -0.658 \cdot \frac{lbf}{in^2 \cdot s}$$

This value matches
LabVIEW

$$M_{dot} := \frac{\frac{DP}{Dt} \cdot Storage\_vol}{Z_1 \cdot R_{air} \cdot T_1}$$

$$M_{dot} = -0.967 \cdot \frac{lbm}{min}$$
LabVIEW shows -.968, likely roundoff error

In SCFM this equates as:

$$M_{dot\_scfm} := \frac{M_{dot}}{\left(.076 \frac{lbm}{ft^3}\right)}$$

$$M_{dot\_scfm} = -12.729 \cdot \frac{ft^3}{min}$$

Note the sign convention for mass flow is positive into system (compression); resulting in negative flow rates for motor operations

Piping diameter is different on each side of the regulator, therefore air velocities are different; they are determined below.

Tube dimensions

$$HP\_tube_{ID} := .364 \, in$$
1/4" Sch40 Pipe.

$$LP\_tube_{ID} := .622 \, in$$
1/2" Sch40 Pipe.

$$V_1 := \frac{M_{dot}}{\rho_1 \cdot \frac{\pi \cdot HP\_tube_{ID}^2}{4}}$$

$$V_1 = -3.501 \cdot \frac{ft}{s}$$

$$V_2 := \frac{M_{dot}}{\rho_2 \cdot \frac{\pi \cdot LP\_tube_{ID}^2}{4}}$$

$$V_2 = -9.959 \cdot \frac{ft}{s}$$

$$\eta\_HP\_reg := \frac{\left(H_2 + \frac{V_2^2}{2} \right)}{\left(H_1 + \frac{V_1^2}{2} \right)}$$

$$\eta\_HP\_reg = 102.089 \cdot \%$$

This matches LabVIEW Data

Supplemental Calculation #S1: PRV-1 Isentropic Efficiency

S1 looks at expansion through the high pressure regulator considering changes in available expansion power through the regulator. Even though there is an enthalpy gain through the regulator there is also an entropy gain which reduces capability of the process air to perform expansion work. An Isentropic efficiency calculation characterizes losses with regard to the process air's potential to perform expansion work. Isentropic efficiency is being defined as the ratio of isentropic expansion power leaving and entering the pressure regulator.

$$P_{1\_abs} = \left(1.272 \times 10^3 \right) \cdot psi$$

$$P_{2\_abs} = (10.219) \cdot Bar$$

$$H_1 = 120.702 \cdot \frac{BTU}{lbm}$$

$$H_2 = 123.222 \cdot \frac{BTU}{lbm}$$

$$S_1 = 5.528 \cdot \frac{kJ}{kg \cdot K}$$

$$S_2 = 6.159 \cdot \frac{kJ}{kg \cdot K}$$

$$T_1 = (298.709) \, K$$

$$T_2 = (288.693) \, K$$

Interpolation with data From Vargaftik Table, requires using three linear interpolations because entropy change is not equal between data points. LabVIEW program performs linear interpolation at 1 Bar because atmospheric pressure is so close to 1 bar. This calculation will verify that the difference is insignificant. Note that entropy is at the lower limit of the 1 bar data (approaching absolute zero) where property behavior is erratic. Therefore this is a good data point to verify that using 1 bar data is acceptable instead of performing multiple interpolations as is completed here.

Linear interpolation relation (1D)

$$h(x, x_0, f_0, x_1, f_1) := \frac{(x - x_0)f_1 - (x - x_0)f_0}{(x_1 - x_0)} + f_0$$

First, Interpolation performed using entropy and enthalpy at each pressure that has table data (1 bar and 2 bar)

$H_{0c} := h(5.528, 2.918, -131.7, 5.591, 82.7)$ $H_{0c} = 77.6468$

$H_{0d} := h(5.528, 5.494, 91.1, 5.549, 96.5)$ $H_{0d} = 94.43818$

Then interpolation performed using ambient pressure and enthalpies associated with isentropic expansion to table pressures.

The nomenclature used for the isentropic derived states used in this analysis is as follows:

H.A_Bs, where A is the location of the property and B is the location where the isentropic process is considered from. For example for enthalpy considering isentropic expansion across the motor (from location 3 to 4) the enthalpy would be expressed as H.4_3s.

$$H_{0\_1s} := h\left(\frac{14.7}{14.504}, 1, H_{0c}, 2, H_{0d}\right) \frac{kJ}{kg}$$

$$H_{0\_1s} = 33.48 \cdot \frac{BTU}{lbm}$$

$$H_{0\_1s\_lb\_vw} := 33.378 \frac{BTU}{lbm}$$

LabVIEW value is 33.378 % difference is calculated below

$$H_{0\_1s\_\%\_dif} := \frac{H_{0\_1s\_lb\_vw} - H_{0\_1s}}{H_{0\_1s\_lb\_vw}}$$

$$H_{0\_1s\_\%\_dif} = -0.305 \cdot \%$$

Less than 1% error, LabVIEW method is adequate.

This suggests the simplified LabVIEW single interpolation method adequately predicts entropy state at ambient conditions.

The process is repeated for the regulator exit state

First, Interpolation performed using entropy and enthalpy at each pressure that has table data (1 bar and 2 bar)

$$H_{2a} := h(6.159, 6.139, 144.2, 6.173, 149.2)$$

$$H_{2a} = 147.14118$$

$$H_{2b} := h(6.159, 6.156, 178.9, 6.211, 189.1)$$

$$H_{2b} = 179.45636$$

Then interpolation performed using pressure and enthalpy.

$$H_{0\_2s} := h\left(\frac{14.7}{14.504}, 1, H_{2a}, 2, H_{2b}\right) \frac{kJ}{kg}$$

$$H_{0\_2s} = 63.447 \cdot \frac{BTU}{lbm}$$

$$\eta_{s\_HP\_reg} := \frac{H_2 - H_{0\_2s}}{H_1 - H_{0\_1s}}$$

$$\eta_{s\_HP\_reg} = 68.531 \cdot \%$$

This shows that significant loss of potential to perform expansion work occurs across the high pressure regulator. This appears to be an important parameter that should be calculated real time in the LabVIEW program.

Available power before and after the HP reg is calculated below

$$HP\_reg_{exit\_pwr} := -M_{dot} \cdot (H_2 - H_{0\_2s})$$

$$HP\_reg_{enter\_pwr} := -M_{dot} \cdot (H_1 - H_{0\_1s})$$

$$HP\_reg_{exit\_pwr} = 1.017 \times 10^3 \, W$$

$$HP\_reg_{enter\_pwr} = 1.484 \times 10^3 \, W$$

$$HP\_reg\_loss := -M_{dot} \cdot \left[(H_1 - H_{0\_1s}) - (H_2 - H_{0\_2s})\right]$$

$$HP\_reg\_loss = 466.913 \, W$$

#36 PRV-2

The low pressure regulator is calculated similarly, although velocities are different due to pressure differences

$$V_3 := \frac{M_{dot}}{\rho_3 \cdot \dfrac{\pi \cdot LP\_tube_{ID}^2}{4}}$$

$$V_3 = -28.611 \cdot \frac{ft}{s}$$

$$\eta\_LP\_reg := \frac{\left(H_3 + \dfrac{V_3^2}{2} \right)}{\left(H_2 + \dfrac{V_2^2}{2} \right)}$$

$$1\frac{ft^2}{s^2} = 3.994 \times 10^{-5} \cdot \frac{BTU}{lbm}$$

$$1\frac{BTU}{min} = 17.584 \, W$$

$$\eta\_LP\_reg = 102.762 \cdot \%$$ This matches LabVIEW Data

Supplemental Calculation #S2: PRV-2 Isentropic Efficiency

S2 looks at expansion through the low pressure regulator and is analogous to calculation S1. Only the exit state requires analysis, entrance state analyzed previously.

$$P_{2\_abs} = (10.219) \cdot Bar$$

$$P_{3\_abs} = (3.683) \cdot Bar$$

$$H_2 = 123.222 \cdot \frac{BTU}{lbm}$$

$$H_3 = 126.611 \cdot \frac{BTU}{lbm}$$

$$S_2 = 6.159 \cdot \frac{kJ}{kg \cdot K}$$

$$S_3 = 6.48 \cdot \frac{kJ}{kg \cdot K}$$

$$T_2 = (288.693) \, K$$

$$T_3 = (294.885) \, K$$

First, Interpolation performed using entropy and enthalpy at each pressure that has table data (1 bar and 2 bar)

$$H_{3a} := h(6.48, 6.463, 199.7, 6.512, 209.7)$$

$$H_{3a} = 203.16939$$

$$H_{3b} := h(6.48, 6.447, 239.6, 6.488, 249.7)$$

$$H_{3b} = 247.72927$$

Then interpolation performed using pressure and enthalpy.

$$1\,\mathrm{Bar} = 14.504 \cdot \frac{\mathrm{lbf}}{\mathrm{in}^2}$$

$$H_{0\_3s} := h\left(\frac{14.7}{14.504}, 1, H_{3a}, 2, H_{3b}\right)\frac{\mathrm{kJ}}{\mathrm{kg}} \qquad H_{0\_3s} = 87.606 \cdot \frac{\mathrm{BTU}}{\mathrm{lbm}}$$

$$\eta_{s\_LP\_reg} := \frac{H_3 - H_{0\_3s}}{H_2 - H_{0\_2s}} \qquad \eta_{s\_LP\_reg} = 65.254 \cdot \%$$

This shows that significant loss of potential to perform expansion work also occurs across the low pressure regulator. This appears to be an important parameter that should be calculated real time in the LabVIEW program.

Available power after the LP reg is calculated below; note this is approximately equivalant to ideal motor power, only varying by the muffler back pressure

$$LP\_reg_{exit\_pwr} := -M_{dot} \cdot (H_3 - H_{0\_3s}) \qquad LP\_reg\_loss := -M_{dot} \cdot \left[(H_2 - H_{0\_2s}) - (H_3 - H_{0\_3s})\right]$$

$$LP\_reg_{exit\_pwr} = 663.521\,\mathrm{W} \qquad\qquad LP\_reg\_loss = 353.312\,\mathrm{W}$$

$$LP\_reg_{enter\_pwr} := -M_{dot} \cdot (H_2 - H_{0\_2s})$$

$$LP\_reg_{enter\_pwr} = 1.017 \times 10^3\,\mathrm{W}$$

#37 Total Piping Loss

Total piping loss is determined by looking at the difference in enthalpy states upstream of the high pressure regulator (location 1) and the motor inlet (location 3). This is an energy balance that assumes losses due to friction and imperfect throttling. test data shows that heat gain from the atmosphere into the process air actually results in a net gain in energy. This apparent gain is analyzed further, determining and verifying the required heat input to produce these gains.

$$Pipe\_loss := -M_{dot} \cdot \left[\left(H_1 + \frac{V_1^2}{2}\right) - \left(H_3 + \frac{V_3^2}{2}\right)\right]$$

negative sign to create a positive mass flow, current system configuration is that flow out of system is negative.

Pipe_loss $= -100.8$ W

This value does not match the LabVIEW value of -5.734W. Further investigation is necessary Hand calculations have been performed and the desired relation with proper unit corrections has been determined. Need to update the LabVIEW file.

100 watts of energy gain through the tubing rather than losses seems incorrect. There is some expected heat transfer into the process air through the tubing but a value greater than the total system output is not considered accurate. Estimated heat gain through tubing using conduction heat transfer based on piping area and wall thickness. The calculation below is a rough approximation that may be refined in future revisions of this analysis.

A check to see the influence of velocity change is performed below:

$$-M_{dot} \cdot \left[\left(\frac{V_1^2}{2} \right) - \left(\frac{V_3^2}{2} \right) \right] = -0.274 \text{ W}$$ Difference due to velocity change is negligible

Review of Entropy value shows that entropy is actually higher at state 3 than state 1, verifying that the energy gain is real.

$$H_1 = 120.702 \cdot \frac{BTU}{lbm} \qquad H_3 = 126.611 \cdot \frac{BTU}{lbm}$$

Regulator specific gains are determined below.

$$HP_{reg\_ht\_gain} := -M_{dot} \cdot \left[\left(H_2 + \frac{V_2^2}{2} \right) - \left(H_1 + \frac{V_1^2}{2} \right) \right]$$

$$HP_{reg\_ht\_gain} = 42.896 \text{ W}$$

$$LP_{reg\_ht\_gain} := -M_{dot} \cdot \left[\left(H_3 + \frac{V_3^2}{2} \right) - \left(H_2 + \frac{V_2^2}{2} \right) \right]$$

$$LP_{reg\_ht\_gain} = 57.903\,W$$

Expected heat gain considering a resistance model including free convective heat transfer from the air to the pipe wall, conduction through the pipe wall, and forced convection to the process air is evaluated below.

Initially assume all pipe is 1/2" schedule 40, use 42" total length.

$Tot\_half\_pipe := 42\,in$ $half\_pipe\_od := .840\,in$ $1\,\dfrac{W}{m^2 \cdot K} = 0.176 \cdot \dfrac{BTU}{ft^2 \cdot hr \cdot R}$

$Tot\_qtr\_pipe := 12\,in$ $qtr\_pipe\_od := .540\,in$

$r_o := \dfrac{half\_pipe\_od}{2}$ $h_o := 20\,\dfrac{W}{m^2 \cdot K}$ $h_i := 150\,\dfrac{W}{m^2 \cdot K}$ $K_{sst} := 16\,\dfrac{W}{m \cdot K}$

$r_i := .311\,in$ $delta\_T := 25\,R$

$$U_{0\_half} := \frac{1}{r_o} \left(\frac{1}{r_o \cdot h_o} + \frac{\ln\left(\dfrac{r_o}{r_i}\right)}{K_{sst}} + \frac{1}{r_i \cdot h_i} \right)^{-1} \qquad U_{0\_half} = 2.975 \cdot \frac{BTU}{ft^2 \cdot hr \cdot R}$$

$$Q_{estimate} := U_{0\_half} \cdot \pi \, half\_pipe\_od \cdot Tot\_half\_pipe \cdot delta\_T \qquad Q_{estimate} = 16.775\,W$$

The estimated heat flow is 5 times less than actual measured gain. Joule Thompson effect may actually cause air temp to drop much more than the delta T shown. This would explain the large increase in entropy through the pressure regulator. A Joule Thomson analysis will be performed to predict the air temperature after expansion without heat transfer. In addition entropy calculations can be used to determine the theoretical temperature without heat transfer. This temp can then be used in the above resistance model to determine if the 100 W is valid. The larger Delta T may increase the expected heat transfer to 100 W of energy gain.

The important concept of entropy contribution to available air power is evident here. During compression air entropy is reduced (entropy of surroundings increases) This allows considerable energy to be stored due to entropy reduction of the air. When an expansion occurs entropy increases significantly, if this is expansion in the motor there is potential for work to be produced, in the regulator this is a loss of availability of the process air,

#38 Ideal Motor Power and #45 H_4_Ideal

This calculation assumes isentropic expansion from the motor inlet state to atmospheric pressure.

$$S_3 = 1.548 \cdot \frac{BTU}{lbm \cdot R} \qquad S_3 = 6.48 \cdot \frac{kJ}{kg \cdot K}$$

Interpolation with data From Vargaftik Table, requires using three linear interpolations because entropy change is not equal between data points. LabVIEW program performs linear interpolation at 1 Bar because atmospheric pressure is so close to 1 bar. This calculation will verify that the difference is insignificant. Note that entropy is at the lower limit of the 1 bar data (approaching absolute zero) where property behavior is erratic. Therefore this is a good data point to verify that using 1 bar data is acceptable instead of performing multiple interpolations as is completed here.

First, Interpolation performed using entropy and enthalpy at each pressure that has table data (1 bar and 2 bar)

$$H_{4a} := h(6.48, 6.463, 199.7, 6.512, 209.7) \qquad H_{4a} = 203.16939$$

$$H_{4b} := h(6.48, 6.447, 239.6, 6.488, 249.7) \qquad H_{4b} = 247.72927$$

Then interpolation performed using pressure and enthalpy.

$$H_{4\_3s} := h\left(\frac{14.7}{14.504}, 1, H_{4a}, 2, H_{4b}\right) \frac{kJ}{kg} \qquad H_{4\_3s} = 87.606 \cdot \frac{BTU}{lbm}$$

LabVIEW value is 33.378. % difference is calculated below

$$H_{4\_3s\_lb\_vw} := 87.358 \frac{BTU}{lbm} \qquad 1 Bar = 14.504 \cdot \frac{lbf}{in^2}$$

$$H_{4\_3s\_\%\_dif} := \frac{H_{4\_3s\_lb\_vw} - H_{4\_3s}}{H_{4\_3s\_lb\_vw}}$$

$$H_{4\_3s\_\%\_dif} = -0.284 \cdot \%$$

Less than 1% error, LabVIEW method is adequate.

$$\text{Ideal\_mtr\_pwr} := -M_{dot} \cdot \left(H_3 - H_{4\_3s}\right)$$

$$\text{Ideal\_mtr\_pwr} = 663.521 \, W$$

LabVIEW data is 668.207 W, nearly identical; difference due to interpolation method.

#39 Ideal Expansion Power, and #44 H_exp_Ideal

This calculation assumes isentropic expansion from the compressed state at location 1 to atmospheric pressure It is completed in same manner as #38 and #44.

$$S_1 = 1.32 \cdot \frac{BTU}{lbm \cdot R} \qquad S_1 = 5.528 \cdot \frac{kJ}{kg \cdot K}$$

Interpolation to determine theoretical enthalpy resulting from Isentropic expansion to ambient conditions performed previously in supplemental calculation S1.

$$\text{Ideal\_exp\_pwr} := -M_{dot} \cdot \left(H_1 - H_{0\_1s}\right)$$

$$\text{Ideal\_exp\_pwr} = 1.484 \times 10^3 \, W$$

LabVIEW data is 1486.004 W, nearly identical; difference due to interpolation method.

#40 Isentropic Compressor Efficiency

The compressor efficiency calculation is only valid when the compressor is running and the air motor is stopped. A different data point must be used to verify this calculation. Row 207 is used, details below.

Test Point A1 Data- is row 207 of Test_002_8/22/11; pertaining to 2540 seconds.
LabVIEW export file has 54 columns containing log and calculated Data.

Efficiency is determined considering an ideal case of Isentropic compression from ambient conditions to the measured pressure at location 1

$$A1_{data} := \text{submatrix}\left(\text{Test\_001}_{data}, 207, 207, 0, 53\right)$$

$$M_{dot1} := \text{submatrix}\left(A1_{data}, 0, 0, 43, 43\right) \frac{lbm}{min} \quad M_{dot1} = (0.402) \cdot \frac{lbm}{min}$$

$$M_{dot1\_scfm} := \frac{M_{dot1}}{\left(.076 \frac{lbm}{ft^3}\right)} \qquad M_{dot1\_scfm} = (5.289) \cdot \frac{ft^3}{min}$$

$$P_{1a} := submatrix\left(A1_{data}, 0, 0, 8, 8\right) \frac{lbf}{in^2} \qquad P_{1a} = (78.308) \cdot Bar$$

$$Cmp\_eff\_lbvw := submatrix\left(A1_{data}, 0, 0, 40, 40\right) \qquad P_{1a} = \left(1.136 \times 10^3\right) \cdot \frac{lbf}{in^2}$$

$$Cmp\_eff\_lbvw = (-12.948) \qquad \text{Negative sign is known problem and needs correction}$$

$$AC\_pwr := submatrix\left(A1_{data}, 0, 0, 12, 12\right) W \qquad AC\_pwr = \left(4.776 \times 10^3\right) W$$

Assume general ambient condition of 14.7 psia and 70 F. It is assumed that ambient temperature will have negligible influence and that water will be condensed out early in the compression process, meaning negligible effect on work to compress.

Ambient temperature will be added to datalogger and LabVIEW program will be updated to calculate compressor efficiency based on ambient conditions.

Vargaftik tables to determine ambient enthalpy and entropy at standard condition

$$P_{amb} := 14.7 \frac{lbf}{in^2} \qquad T_{amb} := 70\,°F \qquad T_{amb} = 294.261\,K$$

Using standard conditions for ambient, not possible to know temp when gas will be expanded.

$$H_0 := g\left(\frac{P_{amb}}{Bar}, \frac{T_{amb}}{K}, 1, 290, 2, 300, 290.2, 300.3, 290.0, 300.1\right) \frac{kJ}{kg}$$

$$H_0 = 126.613 \cdot \frac{BTU}{lbm}$$

$$S_0 := g\left(\frac{P_{amb}}{Bar}, \frac{T_{amb}}{K}, 1, 290, 2, 300, 6.837, 6.871, 6.637, 6.671\right) \frac{kJ}{kg \cdot K}$$

$$S_0 = 6.849 \cdot \frac{kJ}{kg \cdot K}$$

Assume Ideal isentropic compression from ambient state to State 1s, use 3 linear interpolations rather than single bilinear.

First, Interpolation performed using entropy and enthalpy at each pressure that has table data (60 bar and 80 bar)

$H_{1a} := h(6.849, 6.792, 823.8, 6.860, 879.4)$

$$H_{1a} = 870.40588$$

$H_{1b} := h(6.849, 6.829, 879.6, 6.893, 935.7)$

$$H_{1b} = 897.13125$$

Then interpolation performed using pressure and enthalpy.

$$H_{1\_0s} := h\left(\frac{P_{1a}}{Bar}, 60, H_{1a}, 80, H_{1b}\right)\frac{kJ}{kg}$$

$$H_{1\_0s} = (384.725)\cdot\frac{BTU}{lbm}$$

$$Ideal\_cmp\_pwr := M_{dot1}\cdot\left(H_{1\_0s} - H_0\right)$$

$$Ideal\_cmp\_pwr = 1.825 \times 10^3 \, W$$

$$\eta_{cmp} := \frac{Ideal\_cmp\_pwr}{AC\_pwr}$$

$$\eta_{cmp} = (38.2)\cdot\%$$

This number is significantly different due to difference in the LabVIEW calculation method. LabVIEW program should be updated and calculation should be re verified.

Assume a hot day (310K,98F) to verify ambient influence is negligible.

$$(310K)/°F = 98.33$$

$$H_{0\_hot} := h\left(\frac{14.7}{14.503}, 1, 310.4, 2, 320.4\right)\frac{kJ}{kg} \qquad H_{0\_hot} = 133.506\cdot\frac{BTU}{lbm}$$

$$S_{0\_hot} := h\left(\frac{14.7}{14.503}, 1, 6.904, 2, 6.705\right) \qquad S_{0\_hot} = 6.901$$

60 bar interpolation

$H_{1a\_hot} := 992.8$ (Is on table value)

80 bar interpolation

$H_{1b\_hot} := h(6.901, 6.877, 1051.1, 6.933, 1108.8)$ $H_{1b\_hot} = 1075.82857$

Then interpolation performed using pressure and enthalpy.

$H_{1\_0s\_hot} := h\left(\dfrac{P_{1a}}{Bar}, 60, H_{1a\_hot}, 80, H_{1b\_hot}\right)\dfrac{kJ}{kg}$ $H_{1\_0s\_hot} = (459.503)\cdot\dfrac{BTU}{lbm}$

$Ideal\_cmp\_pwr\_hot := M_{dot1}\cdot\left(H_{1\_0s\_hot} - H_{0\_hot}\right)$

$Ideal\_cmp\_pwr\_hot = 2.304 \times 10^3 \, W$

$\eta_{cmp\_hot} := \dfrac{Ideal\_cmp\_pwr\_hot}{AC\_pwr}$ $\eta_{cmp\_hot} = (0.482)$

Percent difference on a hot day

$\%\_dif\_cmp\_pwr := \dfrac{Ideal\_cmp\_pwr\_hot - Ideal\_cmp\_pwr}{Ideal\_cmp\_pwr}$

$\%\_dif\_cmp\_pwr = 26.3\cdot\%$ Ambient temperature makes a significant difference in ideal compressor efficiency. LabVIEW program should be modified to include ambient pressure for the compressor efficiency calculation

Note this process assumes single stage isentropic compression as best case compression power. Ideal compression power would be much lower if a multiple stage isentropic compression process was considered. A multistage polytropic process (which would account for some inevitable heat loss to surroundings) would result in an even lower ideal compression power. Thus the reported compression efficiency is higher than it would be if other compression methods were considered as a baseline. The reported efficiencies allow direct comparison of compression efficiencies for different system storage pressures.

Supplemental Calculation #S3: Compression Efficiency; standard condition baseline

S3 looks at reporting compression efficiency based on available power to isentropically expand from storage pressure at standard temperature to standard atmospheric pressure. It is thought that this will help better allow evaluation of compressor performance with regard to temperature and storage pressure; while also providing a more reasonable efficiency number which compares energy to compress with energy available for expansion under standard conditions. Using standard conditions will allow determination of ambient effects on compressor performance (AC pwr consumption will change dependant on ambient conditions and storage pressure, while Ideal available isentropic expansion power will depend only on pressure due to the standard 1 bar, 70F condition

A mollier diagram showing the possible ways to calculate compression efficiency is located in the Appendix, Figure 2.

Note that to accomplish calculation #40 a new lookup table will be required in the LabVIEW program. Each entry in this table is the result of an interpolation of published table data. It is estimated that it will take approximately 6 hours to create this table and implement the new lookup function in LabVIEW. This is another reason to use the following method in the LabVIEW program instead of the efficiency calculated in #40.

Vargaftik tables to determine ambient enthalpy and entropy at standard condition

$$P_{std} := 14.7 \frac{lbf}{in^2} \qquad T_{std} := 70\,°F$$

$$T_{std} = 294.261\ K \qquad P_{1a} = (78.308)\cdot bar$$

$$H_{std} := g\left(\frac{P_{std}}{Bar}, \frac{T_{std}}{K}, 1, 290, 2, 300, 290.2, 300.3, 290.0, 300.1\right)\frac{kJ}{kg}$$

$$H_{std} = 126.613\cdot\frac{BTU}{lbm}$$

$$S_{1\_P1\_std\_T} := g\left(\frac{P_{1a}}{Bar}, \frac{T_{std}}{K}, 60, 290, 80, 300, 5.620, 5.657, 5.524, 5.562\right) \frac{kJ}{kg \cdot K}$$

$$S_{1\_P1\_std\_T} = (5.548) \cdot \frac{kJ}{kg \cdot K} \qquad \text{Entropy at std compressed state, assume isentropic expansion to Pstd for this efficiency calculation}$$

$$H_{1\_P1\_std\_T} := g\left(\frac{P_{1a}}{Bar}, \frac{T_{std}}{K}, 60, 290, 80, 300, 274.4, 285.6, 272.3, 283.7\right) \frac{kJ}{kg}$$

$$H_{1\_P1\_std\_T} = (277.328) \cdot \frac{kJ}{kg} \qquad \text{Enthalpy at std compressed state, assume isentropic expansion to Pstd for this efficiency calculation}$$

Assume Ideal isentropic expansion from P1 to ambient using 3 interpolations rather than single bilinear.

First, Interpolation performed using entropy and enthalpy at each pressure that has table data (1 bar and 2 bar)

$$H_{0e} := h(5.548, 2.918, -131.7, 5.591, 82.7) \qquad H_{0e} = 79.251$$

$$H_{0f} := h(5.548, 5.494, 91.1, 5.549, 96.5) \qquad H_{0f} = 96.402$$

Then interpolation performed using pressure and enthalpy.

$$H_{0\_1s\_std} := h\left(\frac{P_{std}}{Bar}, 1, H_{0e}, 2, H_{0f}\right) \frac{kJ}{kg} \qquad H_{0\_1s\_std} = 34.172 \cdot \frac{BTU}{lbm}$$

$$Ideal\_cmp\_exp\_pwr := M_{dot1} \cdot \left(H_{1\_P1\_std\_T} - H_{0\_1s\_std}\right)$$

$$Ideal\_cmp\_exp\_pwr = 601.257 \, W$$

$$\eta_{cmp\_avail\_pwr} := \left[\frac{M_{dot1} \cdot \left(H_{1\_P1\_std\_T} - H_{0\_1s\_std}\right)}{AC\_pwr}\right]$$

$$\eta_{cmp\_avail\_pwr} = (12.588)\cdot\%$$

This is similar to the method currently used in the LabVIEW program, with exception that the LabVIEW program uses the temperature at T1, rather than a standard temperature.

#46 Motor Velocity Loss

Motor velocity loss determines power associated with accelerating low density high speed flow leaving the motor exhaust port. It was thought that this could be significantly reducing efficiency although initial testing shows negligible power consumption. Note that the velocity change can not simply be looked at as a loss because enthalpy state is changed by the changes in velocity which effects motor output.

Energy associated with velocity changes is the kinetic energy change in the process air.

$$V_{3\_mtr} := \frac{M_{dot}}{\rho_3 \cdot \dfrac{\pi \cdot LP\_tube_{ID}^2}{4}} \qquad\qquad V_{4\_mtr} := \frac{M_{dot}}{\rho_4 \cdot \dfrac{\pi \cdot HP\_tube_{ID}^2}{4}}$$

$$Mtr\_vel\_loss := -M_{dot} \cdot \frac{V_{4\_mtr}^2 - V_{3\_mtr}^2}{2}$$

$$Mtr\_vel\_loss = 24.042 \, W$$

This value does not match Lab View value of 2 Watts, review of the LabVIEW calculation shows that the incorrect pipe diameter is used for the exit measurement location. Location 4 uses 1/4"NPT, not 1/2" as is assumed in LabVIEW. Correction factor for LabVIEW Location 4 term is determined below

The ratio of the Pipe areas is:

$$Pipe\_ratio := \frac{\dfrac{\pi \cdot HP\_tube_{ID}^2}{4}}{\dfrac{\pi \cdot LP\_tube_{ID}^2}{4}}$$

$$Pipe\_ratio = 0.342$$

Multiply the LabVIEW state 4 area term by .342 correction factor to correct existing error

#47 Motor Shaft Power

$$\text{Mtr\_shaft\_pwr} := -M_{dot} \cdot \left[\left(H_3 + \frac{V_{3\_mtr}^2}{2} \right) - \left(H_4 + \frac{V_{4\_mtr}^2}{2} \right) \right]$$

$\text{Mtr\_shaft\_pwr} = 141.029\,W$ 　　　This differs from LabVIEW calculation by the difference in velocity loss. Review of LabVIEW calculation shows that the velocity loss calculation is used to determine shaft power.

$\text{Mtr\_shaft\_pwr} + \text{Mtr\_vel\_loss} - 2.385\,W = 162.685\,W$ 　　LabVIEW value is 162.734, when velocity loss is corrected this calculation will execute correctly

$\text{Mtr\_shaft\_pwr\_lbvw} := 162.734\,W$

Note the motor shaft power is a calculated value based on measured air properties, it does not include motor shaft friction or motor thermal losses. System Conversion efficiency (#50) considers the entire system; coupling motor/generator/gearing and regulators. Ideally the generator performance would be characterized by separate testing allowing back calculating of Motor performance.

#48 Gen/Gear Efficiency

Generator and gear efficiency is determined by the ratio of measured DC power output and calculated motor power.

$$\text{Gen\_gear\_eff} := \frac{DC\_pwr}{Mtr\_shaft\_pwr\_lbvw}$$ 　　　$\text{Gen\_gear\_eff} = (72.377)\cdot\%$ 　Matches LabVIEW

With correction of velocity loss the efficiency is

$$\text{Gen\_gear\_eff\_2} := \frac{DC\_pwr}{Mtr\_shaft\_pwr}$$ 　　　$\text{Gen\_gear\_eff\_2} = (83.516)\cdot\%$ 　Seems high for a 1 hp generator

#49 Motor Efficiency

Motor efficiency is determined by looking at calculated power from state data and considering ideal isentropic expansion from state 3 (verified previously #38 Ideal motor power).

$$\text{Mtr\_eff} := \frac{\text{Mtr\_shaft\_pwr\_lbvw}}{\text{Ideal\_mtr\_pwr}}$$

$$\text{Mtr\_eff} = 24.526 \cdot \%$$ Matches LabVIEW

$$\text{Mtr\_eff2} := \frac{\text{Mtr\_shaft\_pwr}}{\text{Ideal\_mtr\_pwr}}$$

$$\text{Mtr\_eff2} = 21.255 \cdot \%$$ Actual Efficiency when shaft power is corrected

#50 System Conversion Efficiency

This calculation determines the total efficiency by the ratio of measured power out and power that is available considering an isentropic expansion from state 1 (verified previously in #39 Ideal Expansion Power.

$$\text{Sys\_eff} := \frac{\text{DC\_pwr}}{\text{Ideal\_exp\_pwr}}$$

$$\text{Sys\_eff} = (7.938) \cdot \%$$ LabVIEW shows 7.926, difference likely associated with interpolation differences.

Supplemental Calculation #S4: Motor/Generator efficiency

This calculation considers actual DC power output and measured available air power at the motor inlet (location 3). Available air power is determined considering isentropic expansion from state 3 to ambient conditions. It is performed because the motor/generator system is coupled with regard to experimental measurements. The motor power determined previously is based on measured state data and does not include additional losses such as motor friction and internal inefficiencies. Although this supplemental calculation does not allow individual component efficiencies to be determined it does provide an accurate number of efficiency of the motor/generator system. In the future if generator efficiency is characterized the actual motor efficiency will be able to be determined.

$$\eta_{\text{mtr\_gen}} := \frac{\text{DC\_pwr}}{-M_{\text{dot}} \cdot \left(H_3 - H_{4\_3s} \right)}$$

Motor generator system efficiency, should be part of LabVIEW program because it's more accurate than the decoupled component efficiencies

$$\eta_{\text{mtr\_gen}} = (17.751) \cdot \%$$

Supplemental Calculation #S5 Total Conversion Efficiency (Not calculated in LabVIEW)

A total conversion efficiency considering both compression and expansion is not calculated in LabVIEW because the linkage between these events is not defined. Total conversion efficiency can be presented in a variety of ways.

One possibility is looking at best compression efficiency and best expansion efficiency. This is not a true efficiency because the compressor must pump from the minimum storage pressure to the maximum storage pressure used. It is not possible to operate at one storage pressure. However a system could be designed to operate near these conditions, and it also serves as a maximum potential efficiency.

Another possibility is looking at total the ration of electrical energy out to total electrical energy in. This approach would be best performed after the most efficient expansion operating points were defined. This method will give a more obtainable overall efficiiency number regarding an actual operating system and would allow definition of optimum storage pressure ranges based on test data.

The CAES system also offers potential cooling capability due to the low temperature air discharge. Efficiencies can also be determined considering this system attribute. Two potential ways, along with the electrical energy method are calculated below.

Three total system efficiencies are determined:
1. Ratio of energy out to energy in. This represents the quality of storing electrical power only and ignores the potential cooling effect of the air leaving the air motor.
2. The coefficient of performance for the air stream as a cooling source is determined using the difference between energy in and energy out as the energy required to perform the cooling operation. This is a measure of the systems performance as a refrigeration or cooling system.
3. A multi-purpose consideration of efficiency that includes cooling and electrical storage efficiency. Cooling quantity is translated to typical electrical demand associated with a modern ammonia refrigeration system. This calculation assumes 1.2 kW/ton as a standard cooling plant efficiency.

This sample calculation extrapolates data from the two operating points used in this analysis. Actual calculations will be performed in Excel using an actual operating period for both compression and expansion. Compression flow rate is less than the expansion flow rate; therefore either a longer time period will be used or the data will be scaled to match the amount of air used for expansion.

$\text{Test}_{time} := 10\,\text{min}$

Compressor operating characteristics:

$$\text{AC\_pwr} = \left(4.776 \times 10^3\right)\text{W} \qquad \text{M}_{dot1} = (0.402)\cdot\frac{\text{lb}}{\text{min}}$$

Air motor operating characteristics

$$\text{M}_{dot} = -0.967\cdot\frac{\text{lb}}{\text{min}} \qquad \text{T}_4 = (29.628)\cdot{}^\circ\text{F} \text{ air motor exit temp}$$

Generator output

$$\text{DC\_pwr} = (117.782)\,\text{W} \qquad \text{DC}_{energy} := \text{Test}_{time}\cdot\text{DC\_pwr}$$

$$\text{DC}_{energy} = (0.02)\cdot\text{kW}\cdot\text{hr}$$

Output flow for 10 minutes results in the following total air usage and potential cooling.

$$\text{Air\_usage} := \text{Test}_{time}\cdot-\text{M}_{dot} \qquad\qquad \text{C}_{p\_air} := .240\,\frac{\text{BTU}}{\text{lbm}\cdot\text{R}}$$

$$\text{Air\_usage} = 9.674\cdot\text{lb} \qquad\qquad \text{T}_{77} := 77\,{}^\circ\text{F}$$

Cooling Power

$$\text{Cool}_{cap} := \text{Air\_usage}\cdot\text{C}_{p\_air}\cdot\left(\text{T}_{77} - \text{T}_4\right)$$

$$\text{TR} := 12000\,\frac{\text{BTU}}{\text{hr}} \qquad\qquad \text{Cool}_{cap} = (0.032)\cdot\text{kW}\cdot\text{hr}$$

Based on 77 F cooling space.

$$\text{Cool}_{pwr} := \frac{\text{Cool}_{cap}}{\text{Test}_{time}} \qquad \text{Cool}_{pwr} = (0.055)\cdot\text{TR} \qquad \text{Tons of refrigeration provided by the test bed}$$

Compressor energy to compress that much air:

$$\text{CMP}_{energy} := \frac{\text{Air\_usage}}{\text{M}_{dot1}}\cdot\text{AC\_pwr} \qquad \text{Extrapolates compressor energy to match air consumed}$$

$$\text{CMP}_{energy} = 1.916\cdot\text{kW}\cdot\text{hr}$$

Electrical Storage Efficiency
(efficiency method 1)

$$\eta_{elec} := \frac{DC_{energy}}{CMP_{energy}}$$

$\eta_{elec} = (1.025) \cdot \%$ **Efficiency as an electrical storage device**

Cooling system Efficiency
(efficiency method 2)
This is a refrigeration coefficient of performance (Cooling in/ work to get the cooling)

$$Cool_{COP} := \frac{Cool_{cap}}{CMP_{energy}}$$

$Cool_{COP} = (0.017)$ **Very poor COP, modern systems achieve COP's >3.0**

Total System Efficiency
(efficiency method 3)

$$\eta_{total} := \frac{DC_{energy} + Cool_{pwr} \cdot 1.2 \frac{kW}{TR} \cdot Test_{time}}{CMP_{energy}}$$

$\eta_{total} = (1.599) \cdot \%$ **Total system efficiency including cooling effect based on 1.2 kW/TR refrigeration power equivalency.**

These calculations will be performed by post processing using the Excel data template that formats the LabVIEW data file.

$$1.708 \frac{BTU}{lb \cdot R} = 7.151 \cdot \frac{kJ}{kg \cdot K}$$

$$306.9 \frac{BTU}{lbm} = 713.849 \cdot \frac{kJ}{kg}$$

$$1700 \,^{\circ}F = 1.2 \times 10^{3} \, K$$

$$7.688 \frac{kJ}{kg \cdot K} = 1.836 \cdot \frac{BTU}{lbm \cdot R}$$

$$1279 \frac{kJ}{kg} = 549.871 \cdot \frac{BTU}{lbm}$$

$$1100K = 1.52 \times 10^{3} \cdot \,^{\circ}F$$

Calculation'11'-'LabView Program Verification
Calculation 11 Appendix

Mollier Diagram Test_001 1084 sec (Expansion Process)
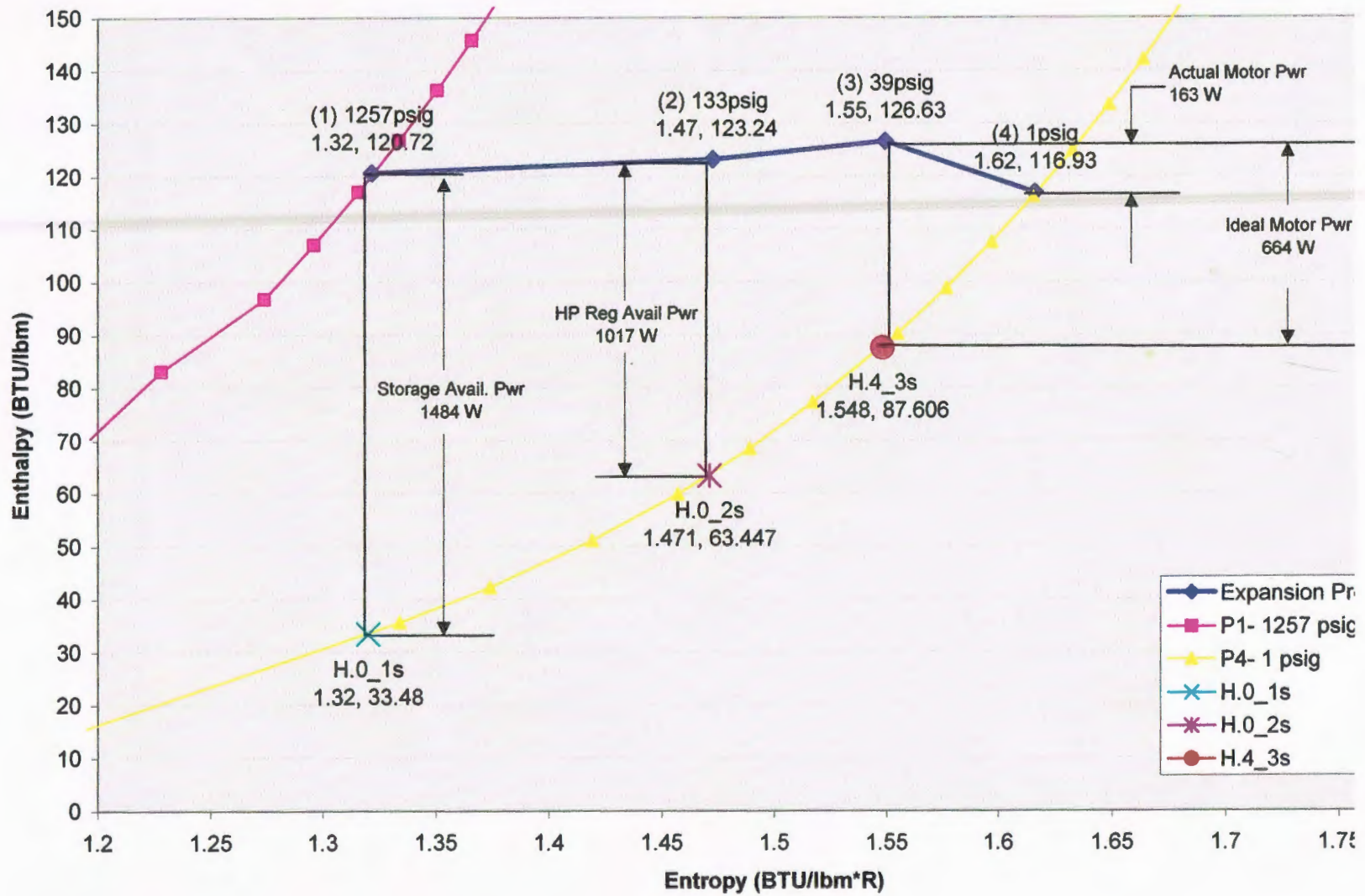
Figure 1

A 1

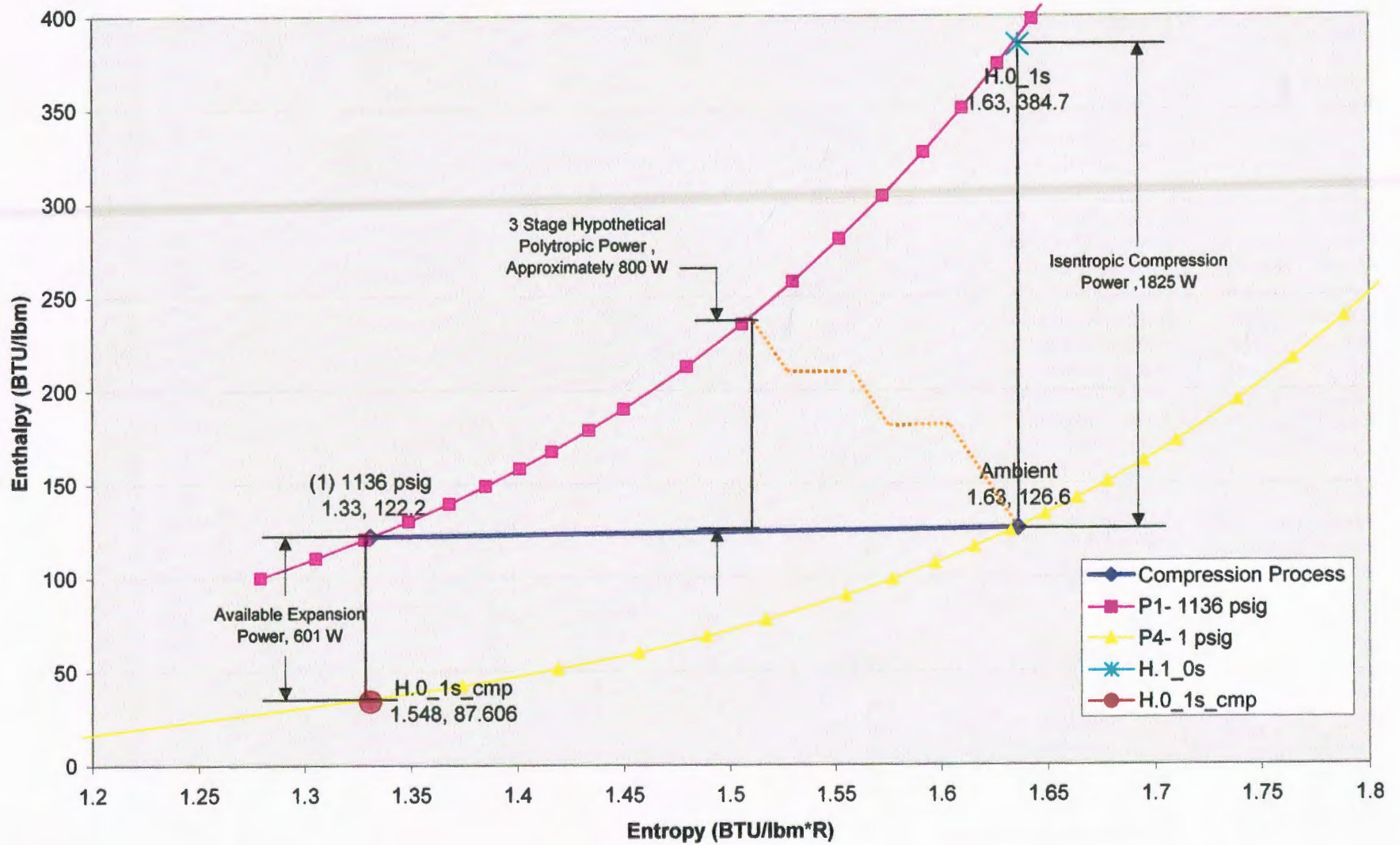**Mollier Diagram Test_001 2540 sec (Compression Process)**

Figure 2

A2

General Mollier Daigram
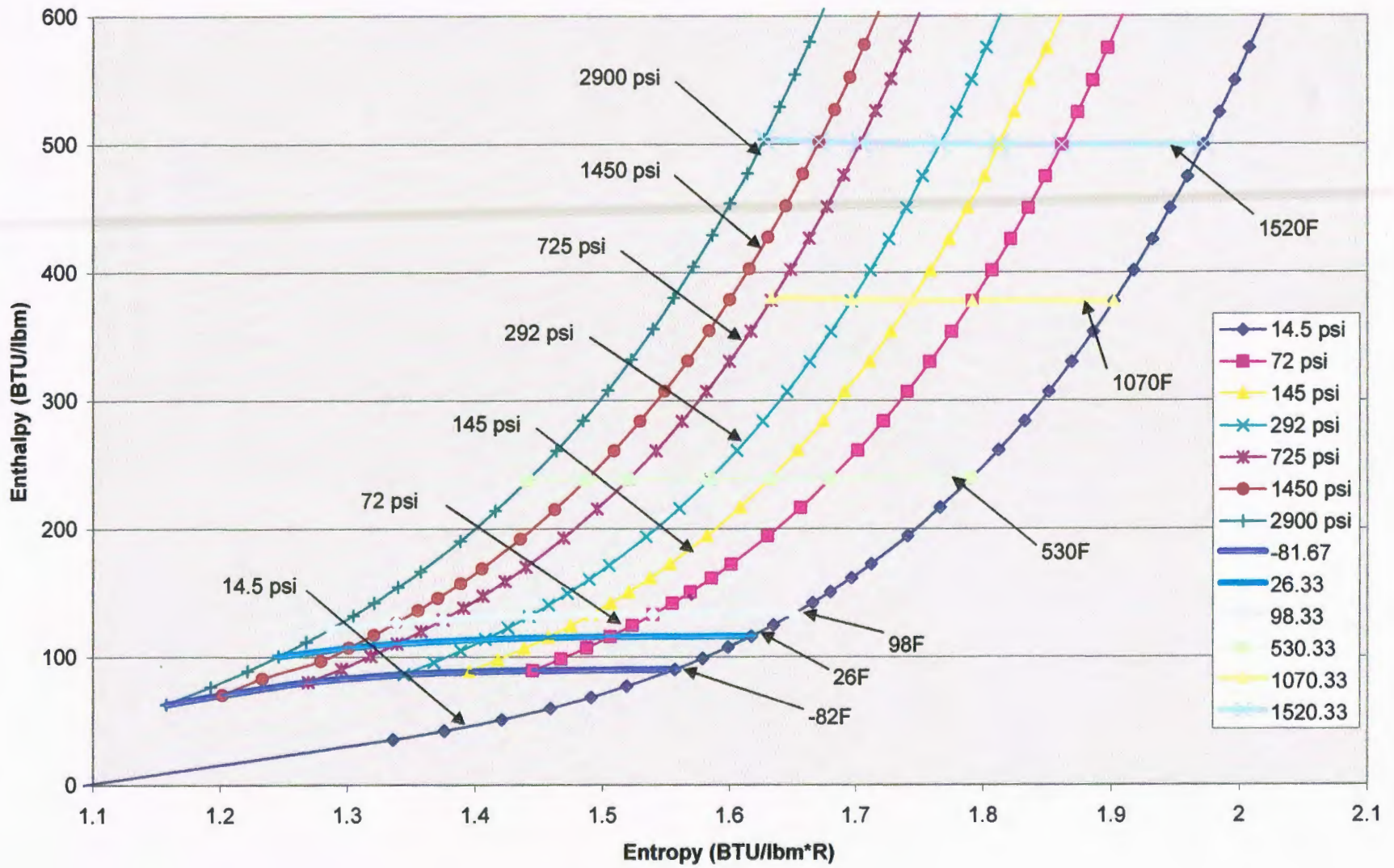
Figure 3

A3

MASS FLOW RATE DETERMINATION

$P_1 = 2328.2$ PSI   ROWS 3-5   6/6/11 TEST RUN 2

$P_2 = 2329.85$ PSI   ROWS 71-73   "

ELAPSED TIME $\Delta t = 13.6$

ρ - VATGAFTIK DENSITY VALUE @ 77F, 2329 psig

$\dot{m}_{IN}$ 1 BAR = 14.5038 psi
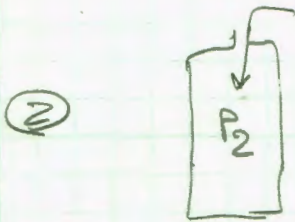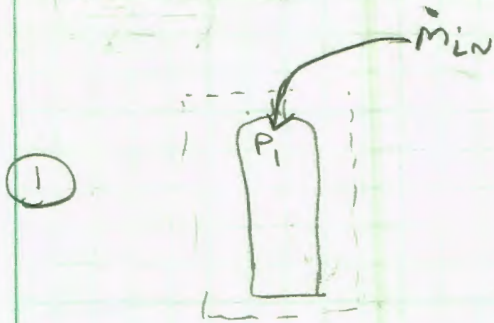
$\dot{m}_{IN} \cdot \Delta t =$ CHANGE OF MASS IN STORAGE

IDEAL GAS LAW

$P = \rho R T$

$P V = Z M R T$ Z @ 160 BAR $< 1.015>$

$V = \dfrac{V}{M}$

$\dfrac{d}{dt}(P V) = \dfrac{d}{dt}(Z M R T)$, CONSTANT Z, R, T, V DURING THIS BRIEF PERIOD

$\dfrac{dP}{dt}V + P\dfrac{dV}{dt}^{0} = \dfrac{dm}{dt} Z R T$

$\dfrac{\dfrac{dP}{dt}V}{Z R T} = \dot{m}$ ← EVALUATED IN MATHCAD

COMPRESSOR RATED IN STD FT^3/MIN,

$\dot{m} = \rho_{STD} Q_{STD} \Rightarrow Q_{STD} = \dfrac{\dot{m}}{\rho_{STD}}$

CALC 11   A4
CALC 10   A4

CALC 010

Air motor Efficiency
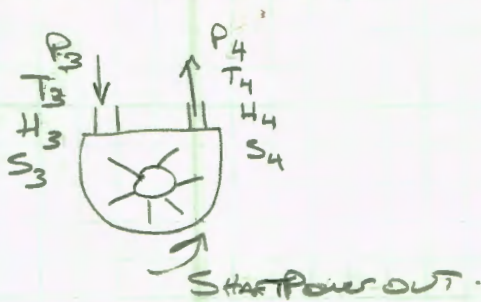
SHAFT Power OUT.

Ideal case would be isentropic expansion

$$S_3 = S_{4s}$$

$$\text{Air PWR IN} = \text{Shaft Power OUT} + \text{Air PWR OUT}$$

$$\dot{m}\left(h_3 + \frac{V_3^2}{2}\right) = \Pi_s + \dot{m}\left(h_4 + \frac{V_4^2}{2}\right)$$

Solving for shaft power

$$\Pi_s = \dot{m}\left(h_3 - h_4 + \frac{V_3^2 - V_4^2}{2}\right)$$

Efficiency is actual case / isentropic case

$$\eta_{motor} = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

CALC 11 A5
CALC 10 A9

TOTAL SYSTEM EFFICIENCY

TOTAL SYSTEM EFFICIENCY IS BASED ON POWER INPUT
& POWER OUTPUT. FOR INITIAL CALCULATIONS POWER INPUT
IS BASED ON COMPRESSOR MANUFACTURER SPECS &
MEASURED POWER CONSUMPTION. THE COMPRESSOR DOES NOT
PRODUCE ENOUGH AIR TO RUN THE SYSTEM IN AN ENERGY
NEUTRAL WAY, THEREFORE EFFICIENCY IS DETERMINED USING
ENERGY METHODS BASED ON A 1 HR AIR MOTOR RUN TIME.
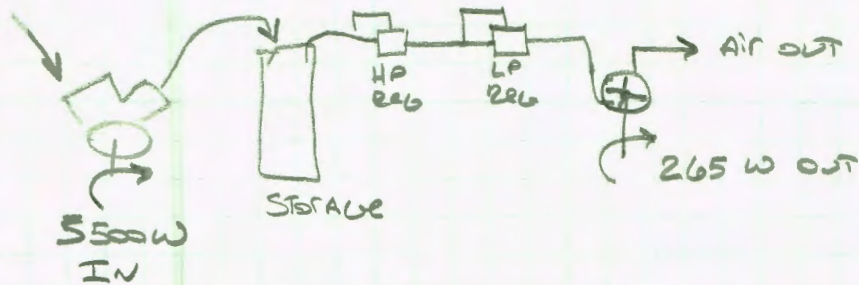
$$\eta_{SYSTEM} = \frac{ENERGY\ OUT}{ENERGY\ IN}$$

DETERMINE
1) AMOUNT OF AIR REQUIRED FOR 1 HR TEST
2) COMPRESSOR ENERGY TO PRODUCE THAT AIR
3) CALCULATE EFFICIENCY

CALCULATIONS PERFORMED WITH MATHCAD

CALC 11 A6
CALC 10 A10

PIPING POWER LOSS CALCULATION CORRECTION

CALCULATION IS AN ENERGY BALANCE BETWEEN MEASUREMENT
LOCATIONS 1 & 3

$$\Sigma_{in} = \Sigma_{out}$$

$$-\dot{m}\left(h_1 + \frac{v_1^2}{2}\right) = -\dot{m}\left(h_3 + \frac{v_3^2}{2}\right) + LOSSES$$

NEGATIVE B/C
$\dot{m}$ IS NEG FOR
FLOW OUT

① $\qquad LOSSES = -\dot{m}\left[\left(h_1 + \frac{v_1^2}{2}\right) - \left(h_3 + \frac{v_3^2}{2}\right)\right]$

⟵ [EVALUATED IN MATHCAD
SHOWS 100.8 W LOSS
LABVIEW CALC SHOWS
5.734 W GAIN]

LABVIEW ANALYSIS REQUIRES SEVERAL UNIT CORRECTIONS
AND REQUIRES EXPRESSING VELOCITIES IN TERMS OF
$\dot{m}$, TUBE $\varnothing$'s, AND DENSITIES. THE LABVIEW RELATION IS
DETERMINED BELOW AND COMPARED TO THE PREVIOUS RELATION

LABVIEW UNITS

$$\dot{m} = LBM/MIN \qquad \rho = LBM/ft^3$$

$$h_x = \frac{BTU}{LBM} \qquad TUBE \; \varnothing \rightarrow PART \; OF \; CALC, \; USE \; ft$$

VELOCITY DETERMINATION

$$\dot{m} = \rho V A \implies V = \frac{\dot{m}}{\rho A}$$

LOCATION 1 USES ¼" NPT, LOCATION 3 USES ½" NPT. AREAS ARE
DETERMINED BELOW

LOCATION 1

$HP.ID = .364 \, IN = .03033 \, ft \qquad HP AREA = \frac{\pi(.03033 \, ft)^2}{4} = 7.2265 \times 10^{-4} \, ft^2$

$LP.ID = .622 \, IN = .05183 \, ft \qquad LP AREA = \frac{\pi(.05183 \, ft)^2}{4} = 2.1101 \times 10^{-3} \, ft^2$

$$V_1 = \frac{-\dot{m}(LBM/MIN) \frac{1 \, MIN}{60 \, SEC}}{\rho\left(\frac{LBM}{ft^3}\right) 7.2265 \times 10^{-4} (ft^2)} = \frac{(-\dot{m})(ft)}{\rho(60 \, SEC)(7.2265 \times 10^{-4})} = \boxed{\frac{-23.063 \, \dot{m}}{\rho} \, ft/sec} = V_1$$

$$V_3 = \frac{-\dot{m}(LBM/MIN) \frac{1 \, MIN}{60 \, SEC}}{\rho \frac{LBM}{ft^3} \, 2.1101 \times 10^{-3} \, ft^2} = \boxed{\frac{-7.898 \cdot \dot{m}}{\rho} \, ft/sec} = V_2$$

CALC 11

THESE ARE SQUARED AND CONVERTED TO $\frac{BTU}{LBm}$ BELOW

$$\frac{V_1^2}{2} = -\frac{531.91\,\dot{m}^2}{2\,\rho^2}\left(\frac{ft^2}{sec^2}\right)\frac{3.994\times10^{-5}\frac{BTU}{LBm}}{\left(\frac{ft^2}{sec^2}\right)} = \boxed{\frac{.01062\,\dot{m}^2}{\rho_1^2}\ \frac{BTU}{LBm} = \frac{V_1^2}{2}}$$

$$\frac{V_3^2}{2} = \frac{.62.385\,\dot{m}^2}{2\,\rho^2}\left(3.994\times10^{-5}\right) = \boxed{\frac{.0012458\,\dot{m}^2}{\rho_3^2} = \frac{V_3^2}{2}}$$

SUBSTITUTING INTO RELATION ① AND CONVERTING TO WATTS

$$Losses = -\dot{m}\left(\frac{LBm}{MIN}\right)\left[\left(h_1 + \frac{.01062\,\dot{m}^2}{\rho_1^2}\right) - \left(h_3 + \frac{.0012458\,\dot{m}^2}{\rho_3^2}\right)\right]\left(\frac{BTU}{LBm}\right)\frac{17.584\,W}{\left(\frac{BTU}{MIN}\right)}$$

$$\boxed{Losses = -17.584\,\dot{m}\left[\left(h_1 + \frac{.01062\,\dot{m}^2}{\rho_1^2}\right) - \left(h_3 + \frac{.0012458\,\dot{m}^2}{\rho_3^2}\right)\right]\ (W)}$$

⎡ LABVIEW RELATION FOR PIPING POWER LOSS W/ UNIT ⎤
CORRECTION

→ THIS DOES NOT MATCH THE EQUATION CURRENTLY USED, LABVIEW
FILE NEEDS TO BE UPDATED.

→ UPDATED CAES_MAIN_20.T RIO ON MEM STICK, NEEDS TO UPDATE
LAB COMPUTER 9/5/11 BL

/

ESTIMATED HEAT TRANSFER THROUGH UNINSULATED PIPING.

T_1, T_2, T_3

Q

h_1

K

$T_{AMB} = 70F$

h_2

$t = \dfrac{OD - ID}{2}$
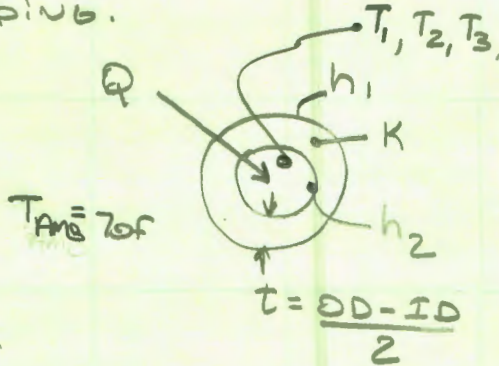
	1/2" PIPE	1/4" PIPE
EXT. AREA =	.554	.229
INT. AREA =	.304	.104
r_o =	.420"	
r_i =	.311"	

1) FREE CONVECTION FROM AIR TO OUTER PIPE WALL
2) CONDUCTION THROUGH PIPE
3) FORCED CONVECTION FROM PIPE ID TO PROCESS AIR

STAINLESS $K = 16 \dfrac{W}{M \cdot K}$ CONDUCTION

$1 \; W/_{M \cdot K} = .5779 \; \dfrac{BTU}{FT \; hr \; f}$

HEAT TRANSFER COEFFICIENT FROM ENGINEERING TOOLBOX

OVERALL HEAT TRANSFER COEFFICIENT, USING RESISTANCE MODEL

$$U_o = \frac{1}{r_o}\left[\frac{1}{r_o h_o} + \frac{LN \; r_o/r_i}{K} + \frac{1}{r_i h_i}\right]^{-1}$$

    ↑ OUTER CONVECTION   ↑ CONDUCTION THROUGH PIPE   ↑ INNER CONVECTION

ASSUME $h_o = 15 \;\; W/m^2 K$ ⎱ ESTIMATES

$h_i = 150$

$K = 16 \dfrac{W}{M \cdot K}$

$Q = U_o A \Delta T$

FOR A USE AREA OF OUTER WALL
T_2 IS AMBIENT → 75°F, $\Delta T_2 = 0$
T_1 IS AIR TEMP (ACTUALLY A FUNCTION OF LOCATION ASSUME T3 TEMP IN THIS CASE →

THIS RELATION IS EVALUATED IN MATHCAD CALCULATION CALC-011.

CALC 11 A9

#46 MOTOR VELOCITY LOSSES

MOTOR ENTRANCE IS LOCATION 3, EXIT IS LOC. 4
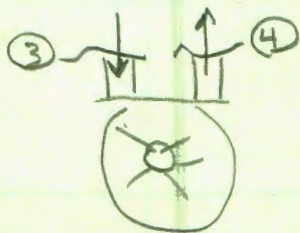
POWER ASSOCIATED W/ VELOCITY CHANGE IS EMBEDDED
IN MOTOR ENERGY BALANCE

$$\cancel{P_{IN}}^{\;0} + E_{IN} = E_{OUT} + WORK$$

$$\dot{m}\left(h_3 + \frac{V_3^2}{2}\right) = \dot{m}\left(h_4 + \frac{V_4^2}{2}\right) + WORK$$

THIS CALCULATION SHOWS HOW MOTOR PRESSURE DIFFERENTIAL
AND DENSITY CHANGE EFFECT KINETIC ENERGY LOSSES
LOW DENSITY EXIT FLOW MAY REQUIRE SIGNIFICANT ACCELERATION
TO MAINTAIN MASS FLOW RATE. IT IS IMPORTANT TO NOTE
THAT THE VELOCITY CHANGE IS NOT NECESSARILY A LOSS
AS THE TEMPERATURE, DENSITY, AND PRESSURE EFFECTS ARE
NOT CONSIDERED.

IGNORING ENTHALPY STATE CHANGE EFFECTS, THE
KINETIC ENERGY CHANGE IS

$$\dot{m}\left(\frac{V_4^2}{2} - \frac{V_3^2}{2}\right)$$

THIS IS CONSIDERED
A LOSS BECAUSE THE
ENERGY IS BEING USED TO
ACCELERATE THE FLOW
RATHER THAN TURN THE
MOTOR SHAFT.

THE PREDICTED MOTOR SHAFT POWER IS ALSO CALCULATED
USING THIS ENERGY RELATION. (#47)

Compressor Efficiency

From Moran & Shapiro Fundamentals of Thermodynamics 3rd Edition ISBN 0-471-07681-3, pg 244

$$\eta_c = \frac{\dot{W}_{cv/\dot{m}})_s}{\dot{W}_{cv/\dot{m}}}$$

Evaluated at same inlet state & same exit pressure

where $-\dot{W}_{cv/\dot{m}} = h_1 - h_0 =$ work input

For CAES project actual inlet power is known. Ideal work input is determined assuming isentropic compression from AMB condition to pressure measured at location 1 (storage pressure)

⌐ Currently LABView program uses negative of isentropic expansion from actual enthalpy at location 1. This is incorrect. ⌐

Assume variability in ambient condition & water content are negligible (water condensed out early). Will verify AMB temp later in this analysis

Assume - VARGAFTIK tables use for state determination, ref. MATHCAD
$P_0 = 14.7$ psig
$T_0 = 70 F$
$S_0 = 6.849$ KJ/Kg·k
$h_0 = 126.613$

$h_{1s} = 384 \frac{BTU}{Lbm}$ determined using isentropic compression from S_0

IDEAL comp power $= \dot{m}(h_{1s} - h_0) = 1825 W$

$$\eta_{cmp} = \frac{IDEAL\_cmp\_pwr}{Ac\_pwr} = 38.2 \%$$

↑ measured compressor power

The ideal compressor power results in a high temperature 1s state. Actual temp. of air leaving the comp is likely higher than this. Future analysis should be performed to quantify this waste heat.

CALC 11 A11

INTERPOLATION RELATION USED TO DETERMINE OTHER STATES
WHEN ONE STATE IS KNOWN - FOR EXAMPLE DETERMINING ENTHALPY
AT AN EXPANDED STATE AFTER AN ISENTROPIC EXPANSION.

$$S_4 = 5.528 \; \frac{kJ}{kgK} \leftarrow \text{[TREAT PROPERTY AS WOULD TREAT TEMP]}$$

PROBLEM IS NO LONGER WORKING WITH
RECTANGULAR GRID AT 'ENTROPY BELOW
5.5.

VARGAFTIK DATA

P = 1 BAR	S	h
75K (S₁)	2.918	-131.7
95K	5.591	82.7

P = 2 BAR	S	h
95	5.494	91.1
100	5.549	96.5
105	5.601	101.8

INTERPOLATION RELATION CAN BE USED TO GET TEMP OR ENTHALPY

SHOULD BE SQUARE   ACTUAL

$$[\Delta S @ P_1 = \Delta S @ P_2]$$

$$[\Delta S @ P_1 \neq \Delta S @ P_2]$$
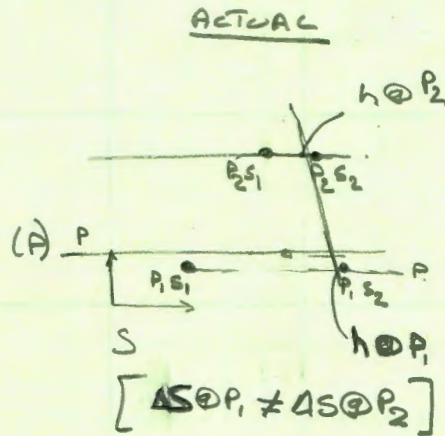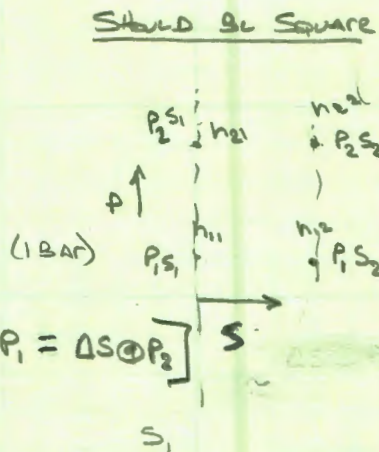
BECAUSE ΔS IS SO MUCH DIFFERENT NEED TO INTERPOLATE
LINEARLY AT EACH PRESSURE AND THEN AGAIN USING
ENTHALPY @ P_1 & P_2

LINEAR INTERPOLATION RELATION

$$f(x, x_0, f_0, x_1, f_1) = f_0 + \frac{(x - x_0) f_1 - (x - x_0) f_0}{(x - x_0)}$$

1 BAR

$$h_{4A}(\underset{S}{5.528}, \underset{S_0}{2.918}, \underset{h_0}{-131.7}, \underset{S_1}{5.591}, \underset{h_1}{82.7}) = 77.6468$$

$$h_{4B}(5.528, 5.494, 91.1, 5.549, 96.5) = 94.438$$

$$h_4(\underset{P}{\tfrac{14.7}{14.503}}, \underset{P_0}{1}, \underset{h_{4A}}{77.65}, \underset{P_1}{2}, \underset{h_{4B}}{94.44}) = 77.874$$

See
MATHCAD
for
EVALUATION

CALC 11 A12

ISENTROPIC EFFIC. THROUGH PRESSURE REGULATORS

$P_1 =$ $P_2 =$

$h_1 =$ $h_2 =$

$S_1 =$ $S_2 =$

IDEAL EXPANSION $S_1 = S_2$

THEN H_{2-IS} is DETERMINED USING STATE TABLES, CONSIDERING ISENTROPIC EXPANSION FROM THE INLET & OUTLET STATES.

CONSIDERING ISENTROPIC EXPANSION TO AMBIENT CONDITIONS AS A BASELINE FOR AVAILABLE AIR POWER WE CAN DETERMINE AN EFFICIENCY MEASURE FOR THE REGULATOR BASED ON THE CHANGE IN ABILITY OF THE PROCESS GAS TO PERFORM WORK.

$$\eta_{ISENTROPIC} = \frac{\text{IDEAL AVAIL EXPANSION WORK OUT}}{\text{IDEAL AVAIL EXPANSION WORK IN}} = \frac{\dot{M}(h_2 - h_{0-2S})}{\dot{m}(h_1 - h_{0-1S})}$$

$$\eta_{S-XX} = \frac{(h_{OUT} - h_{0-OUTS})}{(h_{IN} - h_{0-INS})} \quad \leftarrow \text{GENERAL FORM}$$

SIMILARILY COMPRESSION EFFICIENCY CAN BE EVALUATED BASED ON POWER AVAILABLE TO EXPAND ISENTROPICALLY FROM THE COMPRESSED STATE. THE TEST SET UP CAN NOT MEASURE THE TEMPERATURE OF THE COMPRESSED GAS, A STANDARD 70°F TEMP IS USED TO DETERMINE AVAILABLE IDEAL EXPANSION POWER. THIS WILL ALSO ALLOW EVALUATION OF TEMPERATURE INFLUENCE ON COMPRESSOR EFFICIENCY BY ELIMINATING THE TEMPERATURE DEPENDANCE OF THE IDEAL CASE.

$$\eta_{CMP} = \frac{\dot{m}\left(h(P_1, 70F) - h(P_{AMB}, S(P_1, 70F))\right)}{AC\_PWR\_IN}$$

AVAL PWR

(#49)   IDEAL COMP PWR

ACTUAL COMPRESSIO   S

IDEAL EXPANSION FROM $(P_1, 70F)$ (AVAILABLE WORK METHOD) #S3
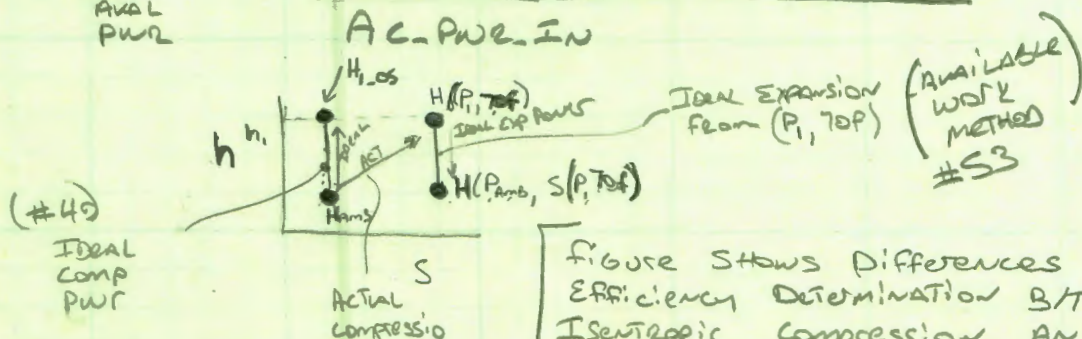
FIGURE SHOWS DIFFERENCES IN EFFICIENCY DETERMINATION B/T ISENTROPIC COMPRESSION AND AVAILABLE WORK METHODS

MOTOR/GENERATOR SYSTEM EFFICIENCY

MEASURING DC POWER OUT, KNOW AVAILABLE AIR POWER AT
MOTOR INLET (ISENTROPIC EXPANSION ASSUMPTION)
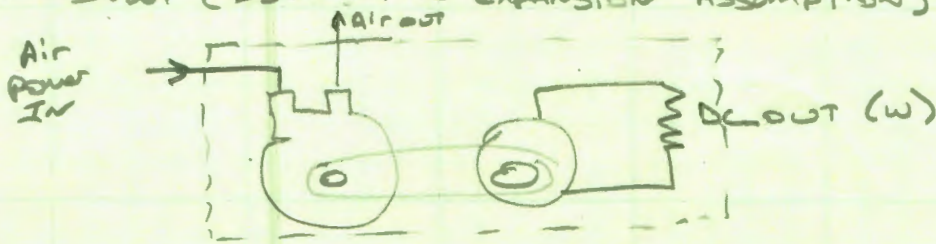
IDEAL CASE ALL AIR POWER IS EXTRACTED THROUGH
ISENTROPIC EXPANSION IN THE AIR MOTOR, TRANSFERED
WITHOUT LOSS TO THE GENERATOR & WITHOUT LOSS
TO THE LOAD RESISTOR WHERE POWER IS MEASURED

$$\eta_{\substack{MOTOR \\ GEN}} = \frac{DC\ PWR}{\dot{m}(h_3 - h_{4,3S})} \qquad \left(\frac{MEASURED\ POWER\ OUT}{IDEAL\ MOTOR\ POWER} \right)$$

Compressibility Factor of Air

Temperature range 250 - 1000 °K

Source: Perry's Chemical Eng Handbook (1984) 6th ed. (table 3-162)

Pressure, bars

Legend:
- 250 °K
- 300 °K
- 350 °K
- 400 °K
- 450 °K
- 500 °K
- 600 °K
- 800 °K
- 1000 °K

CALC 11 A15
CALCO All
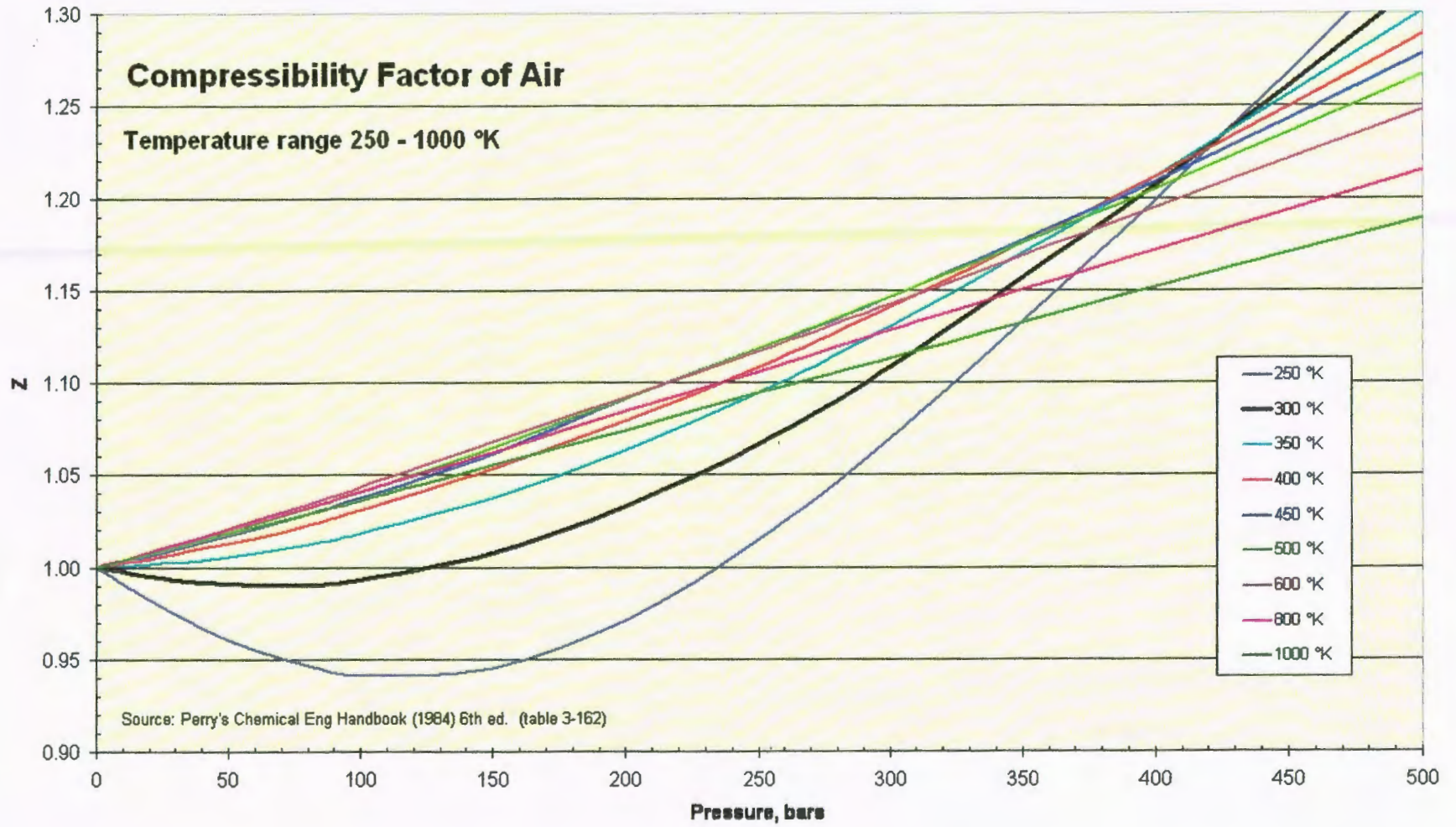
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Calculation '12'- Energy Density of Air

10/25/11

  Calculation 012- Energy density of air

This calculation determines the energy density of air considering isothermal and adiabatic expansion processes. Data will be compared to that published for gasoline and batteries. Gasoline data is 12,700 watt*hr/kg, 8.76kW/liter (Source:Zittel, Werner & Reinhold Wurster (Ludwig-Blkow-Systemtechnik). Hydrogen in the Energy Sector. . HyWeb). Lithium Ion battery data is 128 watt*hr/kg, 230 wh/liter. Lead Acid battery data is 41 watt*hr/kg, 100 wh/liter. Source: http://www.allaboutbatteries.com/Battery-Energy.html

Air data varies widely which is why this calculation is performed. Xtronics.com states 17 w*hr/l and 34 w*hr/kg, while zebu.uroegon.edu states 2000w*h/l

Isothermal Energy Density.

This model considers Isothermal expansion from the maximum expected storage pressure of 4500 psia to atmospheric pressure. In reality an air engine will not be able to operate over this large of a pressure range, therefore this is an optimistic and theoretical number. It also assumes a perfect heat exchanger and expansion engine.

  Available work for an ideal gas expansion is calculated using a frictionless cylinder expansion system starting at 4500 psia and expanding to 14.7 psia under constant temperature of 300K Environment is assumed as 300K and 1 Bar. This requires heat addition and can be considered as the best case air motor performance, (what would be approached with multi-stage expansion and perfect heat exchangers between stages).

$$R_{air} := 53.33 \frac{ft \cdot lbf}{lbm \cdot R} \qquad M := 1 lbm \quad T_0 := 300K \qquad T_0 / °F = 80.33$$

$$Bar := 14.5038 \frac{lbf}{in^2} \qquad 300 Bar = 4.351 \times 10^3 \cdot \frac{lbf}{in^2}$$

$$P_i := 4500 \frac{lbf}{in^2} \qquad P_f := 14.7 \frac{lbf}{in^2} \qquad P(V) := \frac{M \cdot R_{air} \cdot T_0}{V}$$

$$V_i := \frac{M \cdot R_{air} \cdot T_0}{P_i} \qquad V_f := \frac{M \cdot R_{air} \cdot T_0}{P_f}$$

$$V_i = 1.258 L \qquad V_f = 385.239 L \qquad \text{Initial and final volumes}$$

$$W_{avail\_isoT} := \int_{V_i}^{V_f} P(V) \, dV \qquad \frac{W_{avail\_isoT}}{M} = 136.866 \cdot \frac{W \cdot hr}{kg}$$

Isothermal energy density, mass basis

$$\rho_{4500} := \frac{P_i}{R_{air} \cdot T_0} \qquad \rho_{4500} = 22.501 \cdot \frac{lbm}{ft^3} \qquad \text{Density at 4500 psia}$$

$$\frac{W_{avail\_isoT} \cdot \rho_{4500}}{M} = 49.332 \cdot W \cdot \frac{hr}{l}$$

Isothermal energy density, volume basis

An availability analysis as outlined in Shaprio and Moran's Fundamentals of Engineering Thermodynamics, 3rd Edition, ISBN 0-471-07681-3 is also performed to verify the assumed ideal expansion.

$$T_0 = 540 \cdot R \qquad p_0 := 14.7 \frac{lbf}{in^2} \qquad \text{Atmospheric Temperature and Pressure}$$

$$U := 92.04 \frac{BTU}{lbm} \qquad \text{Internal energy of compressed air} \qquad \text{Shaprio ideal gas air table values}$$

$$U_0 := 92.04 \frac{BTU}{lbm} \qquad \text{Internal energy of ambient air}$$

Note internal energy and Absolute entropy are functions of temp only for an ideal gas, therefore U=U.0, specific entropy needs to be calculated with the Delta S relation shown.

$$S_{oi} := .60078 \frac{BTU}{lbm \cdot R}$$

$$S_{o0} := .60078 \frac{BTU}{lbm \cdot R}$$

$$\text{Delta\_S} := S_{oi} - S_{o0} - R_{air} \cdot \ln\left(\frac{P_i}{p_0}\right) \qquad \text{Delta\_S} = -0.392 \cdot \frac{BTU}{lbm \cdot R}$$

$$V_0 := V_f \qquad \text{Volume after expansion, need to use per lbm basis to work with units of other properties}$$

Ideal gas law assumption used to

$$\text{Availability} := (U - U_0) + p_0 \cdot \left(\frac{V_i}{M} - \frac{V_f}{M}\right) - T_0 \cdot (\text{Delta\_S})$$

$$\text{Availability} = 113.034 \cdot \frac{W \cdot hr}{kg}$$

Availability is lower than that when considering Isothermal expansion. This is due to the availability considering a closed system which includes the environment, Availability considers the heat transfer from the environment during expansion, but also negates the PV work because it includes the expansion of the system and the contraction of the environment.

$$\text{Availability} \cdot \rho_{4500} = 40.742 \cdot W \cdot \frac{hr}{l}$$

Closed system availability volume basis

Adiabatic Model

A third way to express available energy density is the adiabatic model which assumes no heat transfer during a single stage reversible expansion process. This is an isentropic expansion and represents the best case expansion of a closed system (No heat transfer). This is the method used in the Lab view program for determining efficiencies of each component. The ideal gas model is continued here. LabVIEW uses air table data for better accuracy.

$p_{r1} := 1.3860$ Ideal gas table A22E Shaprio

$$p_{r2} := \frac{p_{r1} \cdot P_f}{P_i} \qquad p_{r2} = 4.528 \times 10^{-3}$$

$T_f := 360R$

$h_f := 85.97 \dfrac{BTU}{lb}$

$h_i := 129.06 \dfrac{BTU}{lb}$

p.r2 used to determine expected final temperature.p.r2 value is lower than the limit of the ideal gas table. Review of state tables indicates that air is approaching the liquid state at these low temperatures and no longer behaves as an ideal gas. Lowest p.r2 in the table is used here. See below for state table analysis of same expansion.

$$W_{avail\_adiabatic} := M \cdot (h_i - h_f)$$

$$\frac{W_{avail\_adiabatic}}{M} = 27.841 \cdot \frac{W \cdot hr}{kg}$$

This is likely a more realistic expansion value as the exit temperature is within the operating range of some materials. However it does not represent an adiabatic reversible process.

$$\frac{W_{avail\_adiabatic} \cdot \rho_{4500}}{M} = 10.035 \cdot W \cdot \frac{hr}{l} \qquad \text{Volume Basis}$$

State table analysis, using 300 bar (4350 psia) pressure 300 K temp, same isentropic expansion to 1 Bar pressure.Vargaftik Handbook of Physical properties of Liquids and Gases, 2nd edition is used for table values.

$kJ := 1000J \qquad H_i := 257.7 \dfrac{kJ}{kg} \qquad S_i := 5.088 \dfrac{kJ}{kg \cdot K}$

$$h\left(x, x_0, f_0, x_1, f_1\right) := \frac{\left(x - x_0\right) f_1 - \left(x - x_0\right) f_0}{\left(x_1 - x_0\right)} + f_0$$ 1D interpolation relation used to determine enthalpy at 1 bar pressure

$$h(5.088, 2.918, -131.7, 5.591, 82.7) = 42.355$$

$$H_f := h(5.088, 2.918, -131.7, 5.591, 82.7) \frac{kJ}{kg}$$

$$H_f = 18.209 \cdot \frac{BTU}{lbm}$$

$$W_{avail\_adiabatic2} := M \cdot \left(H_i - H_f\right)$$

$$\frac{W_{avail\_adiabatic2}}{M} = 59.818 \cdot \frac{W \cdot hr}{kg}$$ This value is not likely representative of a real system due to the extremely cold exit temperatures (approximately 85K)

$$\frac{W_{avail\_adiabatic2} \cdot \rho_{4500}}{M} = 21.561 \cdot W \cdot \frac{hr}{l}$$ Volume basis

The adiabatic values are lower than both the availability and isothermal cases. It is lower than the isothermal case because it does not receive the additional heat from the environment required to maintain constant temperature during expansion. It is lower than the calculated availability because the availability relation determines the potential to perform work of a combined system that includes the system and the environment, only looking at the difference between the initial state and the ambient environment.

It is interesting to note that unlike a combustion expansion process the actual air motor expansion process is more likely to move toward Isothermal as heat transfer effects are considered. This is because the process air is decreasing in temperature, actually gaining energy from the environment. Typical combustion processes lose heat and expansion potential from heat transfer. Therefore while an isothermal expansion assumption is not a valid assumption for a combustion process it would represent an ideal multiple stage expansion process for an air motor. This indicates that the efficiency basis used in the LabVIEW program (single stage isentropic expansion) results in higher stated efficiencies.

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Calculation'13'-'Total System Efficiency

Calculation 013

This calculation uses the total system efficiency calcualtion method first presented in Calculation 010. It has been updated with the highest observed total expansion system efficiency data from all testing and the highest valid compression testing efficiency at the pressure at which the expansion data is aquired. Compression data is row 4320 of Test_data_all file corresponding to 9625 sec, Test_008 on 10/17/11. Expansion data is row 5150 of Test_data_all file corresponding to 3384 sec, Test_009 on 10/17/11

Characteristic	Compression	Expansion
Absolute Time (s)	3401736645.05	3401741150.09
Elapsed Time (s)	9625.66	3384.93
Temp_1 (F)	92.37	64.37
Temp_2 (F)	83.57	54.26
Temp_3 (F)	83.50	58.42
Temp_4 (F)	81.11	18.38
DC PWR (W)	-3.51	410.56
DC Volts (V)	0.00	14.83
Press_1 (psi)	4291.77	574.57
Press_2 (psi)	-0.43	82.93
Press_3 (psi)	-0.19	75.25
Press_4 (psi)	-0.25	6.77
AC Pwr (W)	5839.67	3.85
AC WH Not Used	-0.01	0.00
Ambient Temp (F)	81.41	72.69
Load Res. (ohms)	0.00	0.54
Gen Speed (rpm)	0.00	1098.65
Mtr Speed (rpm)	0.00	2825.09
mtr gear (# teeth)	14.00	14.00
gen gear (# teeth)	36.00	36.00
Gen Constant (V/rpm)	0.01	0.01
Gear Ratio	0.39	0.39
Stor Vol (ft^3)	8.23	8.23
Avail Air (%)	95.92	14.52
H1 (Btu/lbm)	114.65	121.29
H2 (Btu/lbm)	129.91	122.22
H3 (Btu/lbm)	129.89	123.29
H4 (Btu/lbm)	129.32	114.16
S1 (Btu/lbm*F)	1.22	1.38
S2 (Btu/lbm*F)	1.64	1.50
S3 (Btu/lbm*F)	1.64	1.51
S4 (Btu/lbm*F)	1.64	1.60
Rho1 (lbm/ft^3)	18.96	3.07
Rho2 (lbm/ft^3)	0.07	0.51
Rho3 (lbm/ft^3)	0.07	0.47
Rho4 (lbm/ft^3)	0.07	0.12
PRV-1 Eff (%)	-0.35	64.93
PRV-2 Eff (%)	-2.56	97.48

Characteristic	Compression	Expansion
Piping Enthalpy Gain (W)	-234.00	64.26
Ideal Mtr Pwr (W)	-0.13	1586.69
Available Expansion Pwr (W)	-1445.82	2506.95
Compressor Eff. (%)	24.76	-65123.84
Air Compress Factor	1.11	0.99
DP/DT (psi/sec)	0.40	-0.70
M dot (lbm/min)	0.86	-1.82
H0_1S (Btu/lbm)	19.36	42.82
H4_3s (Btu/lbm)	129.88	73.62
Mtr Vel Loss (W)	-20.42	75.63
Mtr Shaft Pwr (W)	-8.73	291.52
Gen/Gear Eff (%)	40.17	140.84
TheoreticalMtr Eff (%)	6631.93	18.37
System Conversion Eff. (%)	0.24	16.38
step ind.	1919.00	672.00
loop ind.	1924.00	677.00
step size	5.00	5.00
H0_2s (BTU/lbm)	130.25	71.27
Theoretical HP Reg Entropic Loss (W)	-82.52	50.00
Theoretical LP Reg Entropic Loss (W)	0.30	2.33
Motor/Gen Eff. (%)	26.64	0.26
H1_polytropic (BTU/lbm)		
Motor/Gen Eff. (%)Corrected		
TEST NUMBER	Test_008_10_17_11	Test_009_10_17_11
DeltaP_1_2	4292.20	491.63
DeltaP_2_3	-0.24	7.68

Three total system efficiencies are determined:
1. Ratio of energy out to energy in. This represents the quality of storing electrical power only and ignores the potential cooling effect of the air leaving the air motor.
2. The coefficient of performance for the air stream as a cooling source is determined using the difference between energy in and energy out as the energy required to perform the cooling operation. This is a measure of the systems performance as a refrigeration or cooling system.
3. A multi-purpose consideration of efficiency that includes cooling and electrical storage efficiency. Cooling quantity is translated to typical electrical demand associated with a modern ammonia refrigeration system. This calculation assumes 1.2 kW/ton as a standard cooling plant efficiency.

$$Test_{time} := 10 min$$ Arbitrary expansion test time used to balance compressor and motor flow rates

Compressor operating characteristics:

$$AC_{pwr} := 5839.67 W \qquad CMP_{flow} := .86 \frac{lbm}{min}$$

Air motor operating characteristics

$$Test_{flow\_A} := 1.82 \frac{lbm}{min} \qquad T_{4A} := 18.38 °F \quad \text{air motor exit temp} \qquad T_{amb} := 72.69 °F$$

Generator output

$$DC_{pwrA} := 410.56 W \qquad DC_{energy} := Test_{time} \cdot DC_{pwrA}$$

$$DC_{energy} = 0.068 \cdot kW \cdot hr$$

Output flow for 10 minutes results in the following total air usage and potential cooling.

$$Air\_usage := Test_{time} \cdot Test_{flow\_A} \qquad C_{p\_air} := .240 \frac{BTU}{lbm \cdot R}$$

$$Air\_usage = 18.2 \cdot lbm \qquad Cool_{cap} := Air\_usage \cdot C_{p\_air} \cdot (T_{amb} - T_{4A})$$

Cooling Power $$Cool_{cap} = 0.07 \cdot kW \cdot hr$$

$$TR := 12000 \frac{BTU}{hr}$$

$$Cool_{pwr} := \frac{Cool_{cap}}{Test_{time}} \qquad Cool_{pwr} = 0.119 \cdot TR \qquad \text{Tons of refrigeration provided by the test bed}$$

Compressor energy to compress that much air:

$$CMP_{energy} := \frac{Air\_usage}{CMP_{flow}} \cdot AC_{pwr}$$

$$CMP_{energy} = 2.06 \cdot kW \cdot hr$$

Electrical Storage Efficiency
(efficiency method 1)

Cooling system Efficiency
(efficiency method 2)
This is a refrigeration coefficient of
performance (Cooling in/ work to get the
cooling)

$$\eta_{elec} := \frac{DC_{energy}}{CMP_{energy}}$$

$$\eta_{elec} = 3.322 \cdot \% \qquad \textbf{Efficiency as an electrical storage device}$$

$$Cool_{COP} := \frac{Cool_{cap}}{CMP_{energy}}$$

$$Cool_{COP} = 0.034 \qquad \textbf{Very poor COP, modern systems achieve COP's >3.0}$$

Total System Efficiency
(efficiency method 3)

$$\eta_{total} := \frac{DC_{energy} + Cool_{pwr} \cdot 1.2 \frac{kW}{TR} \cdot Test_{time}}{CMP_{energy}}$$

$$\eta_{total} = 4.474 \cdot \% \qquad \textbf{Total system efficiency including cooling effect based on 1.2 kW/TR refrigeration power equivalency.}$$

Remainder of this analysis concerns the test bed energy flow diagram presented in the final repc and presentation.

1) AC input power. Need to scale to match flow rate with expansion process.

$$M_{dot\_cmp} := .86 \frac{lbm}{min} \qquad M_{dot\_mtr} := -1.82 \frac{lbm}{min}$$

$$AC\_pwr := 5839.67 \, W$$

$$M_{dot\_ratio} := \frac{-M_{dot\_mtr}}{M_{dot\_cmp}} \qquad M_{dot\_ratio} = 2.116$$

$$AC\_pwr\_scaled := M_{dot\_ratio} \cdot AC\_pwr$$

$$AC\_pwr\_scaled = 1.236 \times 10^4 \, W$$

2) Isentropic Expansion Power of Compressed Air (labview calculated from Calc 11, #039 for method.

interpolation using a standard bilinear interpolation formula;shown below

$$f(x,y,x1,y1,x2,y2,f11,f12,f21,f22) := \frac{f11}{(x2-x1)\cdot(y2-y1)}\cdot(x2-x)\cdot(y2-y) \ldots$$
$$+ \frac{f21}{(x2-x1)\cdot(y2-y1)}\cdot(x-x1)\cdot(y2-y) \ldots$$
$$+ \frac{f12}{(x2-x1)\cdot(y2-y1)}\cdot(x2-x)\cdot(y-y1) \ldots$$
$$+ \frac{f22}{(x2-x1)\cdot(y2-y1)}\cdot(x-x1)\cdot(y-y1)$$

Standard condition temperature of 70F used for Available air power calculation

$$4291.77 \frac{\text{lbf}}{\text{in}^2} = 295.907 \cdot \text{bar} \qquad\qquad 92\,^\circ\text{F} = 306.483 \cdot \text{K} \qquad \text{kJ} := 1000\text{J}$$

$$H_{p1\_t\_amb} := f(295.9, 294.3, 200, 290, 300, 300, 252.7, 265.5, 244.4, 257.7) \frac{\text{kJ}}{\text{kg}}$$

$$H_{p1\_t\_amb} = 107.674 \cdot \frac{\text{BTU}}{\text{lbm}} \qquad H_{p1\_t\_amb} = 250.45 \cdot \frac{\text{kJ}}{\text{kg}}$$

1 bar table data is used to determine H(p_atm,S(p.1,T.amb))

   Linear interpolation relation (1D)

$$h(x, x_0, f_0, x_1, f_1) := \frac{(x - x_0)f_1 - (x - x_0)f_0}{(x_1 - x_0)} + f_0$$

$$H_{isoS\_exp} := h(5.069, 2.918, -131.7, 5.591, 82.7) \frac{\text{kJ}}{\text{kg}}$$

$$H_{isoS\_exp} = 17.554 \cdot \frac{\text{BTU}}{\text{lbm}}$$

$$\text{Avail\_exp\_pwr} := -M_{dot\_mtr} \cdot \left(H_{p1\_t\_amb} - H_{isoS\_exp}\right)$$

$$\text{Avail\_exp\_pwr} = 2.884 \times 10^3\,\text{W} \quad \text{scaled to reflect motor flow rate}$$

$$\frac{\text{Avail\_exp\_pwr}}{M_{dot\_ratio}} = 1.363 \times 10^3\,\text{W} \quad \text{Actual available power delivered by compressor}$$

3) Compression thermodynamic Losses

Compression process rejects a lot of heat to ambient. This lowers necessary compression power but because this heat is not recovered it is considered a thermodynamic loss.

Reference hand calculations in the appendix for development of the following relation

$$H_{amb} := h(294.3, 290, 290.2, 300, 300.3) \frac{\text{kJ}}{\text{kg}} \qquad\qquad S_{amb} := h(294.3, 290, 6.837, 300, 6.871) \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$H_1 := 114.65 \frac{BTU}{lbm} \qquad H_{amb} = 126.631 \cdot \frac{BTU}{lbm} \qquad S_{amb} = 1.636 \cdot \frac{BTU}{lbm \cdot R}$$

$$S_1 := 1.22 \frac{BTU}{lbm \cdot R} \qquad AC\_pwr = 5.84 \times 10^3 \, W$$

$$Q_{out\_total} := AC\_pwr + M_{dot\_cmp} \cdot \left[\left(H_{amb} - H_1 \right) - T_{amb} \cdot \left(S_{amb} - S_1 \right) \right]$$

$$Q_{out\_total} = 2.668 \times 10^3 \, W \qquad \text{Total unrecovered heat rate during compression process}$$

$$Q_{out\_total} \cdot M_{dot\_ratio} = 5.646 \times 10^3 \, W \qquad \text{scaled to reflect motor flow rate}$$

4) Compressor Mechanical and electric motor losses

$$Cmp_{mech\_loss} := AC\_pwr - \left(\frac{Avail\_exp\_pwr}{M_{dot\_ratio}} + Q_{out\_total} \right)$$

$$Cmp_{mech\_loss} = 1.809 \times 10^3 \, W$$

$$Cmp_{mech\_loss} \cdot M_{dot\_ratio} = 3.828 \times 10^3 \, W \qquad \text{scaled to reflect motor rate}$$

5) Heat Gain at HP regulator

Calculated by looking at enthalpy change, states determined in LabView program

$$H_2 := 122.22 \frac{BTU}{lbm} \qquad H_{1\_mtr} := 121.29 \frac{BTU}{lbm}$$

$$HP_{reg\_ht\_gain} := -M_{dot\_mtr} \cdot \left(H_2 - H_{1\_mtr} \right)$$

$$HP_{reg\_ht\_gain} = 29.763 \, W$$

6) Available Air Power at HP reg exit (Isentropic Expansion model)

$$P_2 := 82.93 \, \frac{lbf}{in^2} \quad T_2 := 54.26 \, °F \quad T_{1mtr} := 64.37 \, °F \quad P_{1mtr} := 574.57 \, \frac{lbf}{in^2}$$

Assuming isentropic expansion from regulated pressure to 1 Bar pressure

$$S_2 := 1.5 \, \frac{BTU}{lbm \cdot R} \quad S_2 = 6.28 \cdot \frac{kJ}{kg \cdot K}$$

$$H_{2\_0\_isos} := h(6.28, 6.238, 159.3, 6.299, 169.4) \, \frac{kJ}{kg}$$

$$H_{2\_0\_isos} = 71.476 \cdot \frac{BTU}{lbm}$$

$$Hp_{exit\_air\_pwr} := -M_{dot\_mtr} \cdot \left(H_2 - H_{2\_0\_isos} \right)$$

$$Hp_{exit\_air\_pwr} = 1.624 \times 10^3 \, W \qquad \text{available power leaving HP reg}$$

7) Availability losses at HP regulator

This is a first law analysis, heat gain and avail power in and losses and avail power out

$$HP_{avail\_loss} := Avail\_exp\_pwr + HP_{reg\_ht\_gain} - Hp_{exit\_air\_pwr}$$

$$HP_{avail\_loss} = 1.29 \times 10^3 \, W \qquad \text{Availability losses in the HP reg}$$

8) Heat gain at LP regulator

Calculated by looking at enthalpy change, states determined in LabView program

$$H_3 := 123.29 \, \frac{BTU}{lbm} \quad H_2 := 122.22 \, \frac{BTU}{lbm}$$

$$LP_{reg\_ht\_gain} := -M_{dot\_mtr} \cdot \left(H_3 - H_2 \right)$$

$LP_{reg\_ht\_gain} = 34.244\,W$

9) Available Air Power at LP reg exit (Isentropic Expansion model)

$$P_3 := 75.25\,\frac{lbf}{in^2} \qquad T_3 := 58.42\,°F$$

Assuming isentropic expansion from regulated pressure to 1 Bar pressure

$$S_3 := 1.51\,\frac{BTU}{lbm \cdot R} \qquad S_3 = 6.322 \cdot \frac{kJ}{kg \cdot K}$$

$$H_{3\_0\_isos} := h(6.32, 6.299, 169.4, 6.357, 179.5)\,\frac{kJ}{kg}$$

$$H_{3\_0\_isos} = 74.401 \cdot \frac{BTU}{lbm}$$

$$Lp_{exit\_air\_pwr} := -M_{dot\_mtr} \cdot \left(H_3 - H_{3\_0\_isos}\right)$$

$$Lp_{exit\_air\_pwr} = 1.565 \times 10^3\,W \qquad \text{available power leaving HP reg}$$

10) Availability losses at LP regulator

This is a first law analysis, heat gain and avail power in and losses and avail power out

$$LP_{avail\_loss} := Hp_{exit\_air\_pwr} + LP_{reg\_ht\_gain} - Lp_{exit\_air\_pwr}$$

$$LP_{avail\_loss} = 93.599\,W \qquad \text{Availability losses in the HP reg}$$

11) Measured Generator Output

$$DC\_pwr := 410.56\,W$$

12) using 75% generator efficiency calcuate estimated motor shaft power

$\eta_{gen} := .75$

$Mtr\_shaft\_pwr := \dfrac{DC\_pwr}{\eta_{gen}}$

$Mtr\_shaft\_pwr = 547.413 \, W$

13) Generator Losses

$Gen_{losses} := Mtr\_shaft\_pwr - DC\_pwr$

$Gen_{losses} = 136.853 \, W$

14) Availability loss in the Motor (isentropic basis)

$Mtr_{losses} := Lp_{exit\_air\_pwr} - Mtr\_shaft\_pwr$

$Mtr_{losses} = 1.017 \times 10^3 \, W$

Expansion system efficiency reported in presentation. This is the ratio of output power to availabl air power (isentropic basis) after compression.

$Exp_{eff} := \dfrac{DC\_pwr}{Avail\_exp\_pwr}$

$Exp_{eff} = 14.235 \cdot \%$

Calculation'13'-'Votal System Efficiency

Calculation 13 Appendix

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HAND CALCULATIONS TO ESTABLISH A RELATIONSHIP B/T
MEASURED PROPERTIES DURING COMPRESSION AND HEAT
REJECTED BY THE COMPRESSION PROCESS.

AIR COMPRESSION RESULTS IN STORED ENERGY BY
LOWERING THE ENTROPY OF THE AIR. ENTHALPY IS
ACTUALLY REDUCED WHEN CONSIDERING ISOTHERMAL
COMPRESSION.
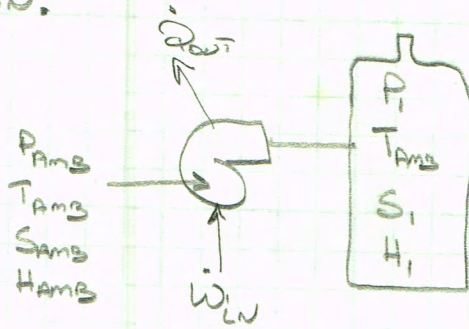
1ST LAW ANALYSIS, IGNORE KINETIC ENERGY & GRAVITY POTENTIAL

$$\dot{E}_{IN} = \dot{E}_{OUT} + \dot{E}_{STORED}$$

$$\dot{W}_{IN} + \dot{m}(h_{AMB}) = \dot{Q}_{OUT} + \left[ENTROPY + \dot{m}(h_1) \right]$$
$$\qquad\qquad\qquad\qquad\qquad\qquad POTENTIAL$$

known   known   UNKNOWN S_{AMB} KNOWN
                      (WASTE H.) $\dot{m}\int_{S_1} T dS$

$$= \dot{m} T_{AMB}(S_{AMB} - S_1)$$

$$\dot{Q}_{OUT} = \dot{W}_{IN} + \dot{m}(h_{AMB} - h_1) - \dot{m}\int_{S_{AMB}}^{S_1} T_{AMB} dS$$

$\dot{Q}_{OUT}$ IS THE AMOUNT OF UNUTILIZED HEAT DURING THE
COMPRESSION PROCESS. REFERENCE MATHCAD FOR
SPECIFIC ANALYSIS OF TEST DATA.

SIMPLIFIED EQN FOR USE IN MATHCAD

$$\dot{Q}_{OUT} = \dot{W}_{IN} + \dot{m}_{CMP} \left((h_{AMB} - h_1) - T_{AMB}(S_{AMB} - S_1) \right)$$
$$\qquad\uparrow$$
$$\quad (AC-PWR)$$

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Calculation '14'- Compressor Efficiency

This Calculation determines compressor efficiency for row 4320 of "Test_data_all" file corresponding to 9625 sec, Test_008 on 10/17/11. Several efficiency baselines are used; including Isentropic available air power basis, Isothermal available air power basis, Isentropic compression basis and Isothermal compression basis). Justification and explanantion of each method is discussed.

Unit Conversion Data

$$Bar := 14.5038 \frac{lbf}{in^2} \qquad kJ := 1000J$$

LabView Data:

$$M_{dot1} := .86 \frac{lbm}{min}$$ Mass flow rate, LabView calculated

$$P_{1a} := 4291.77 \frac{lbf}{in^2}$$ Storage Pressure, Measured

$$AC\_pwr := 5839.67W$$ Compressor power, measured

$$H_1 := 114.65 \frac{BTU}{lbm}$$ Enthalpy at storage pressure, LabView calcualted

$$S_1 := 1.22 \frac{BTU}{lbm \cdot R}$$ Entropy at storage pressure, LabView calcualted

$$T_1 := 92.37 \,°F$$ Measured temp for Location 1

$$P_{amb} := 14.7 \frac{lbf}{in^2}$$ Atmospheric Pressure, assumed

$$T_{amb} := 81.41 \,°F$$

$$T_{amb} = 300.6 \,K$$ Ambeint temp, measured

$$H_{0\_1s} := 19.36 \frac{BTU}{lbm}$$ Enthalpy at ambient assuming isentropic expansion from storage pressure, LabView calcualted

$$T_{std} := 300K$$ Assumed standard temp for available air power

Interpolation Methods

Interpolation relation for 2D interpolations. Helmuth Spath's "Two dimensional Spline Interpolation Algorithms" 1995 ISBN 1-56881-017-2, pg 16 formula 2.3.

$$g\left(x, y, x_1, y_1, x_2, y_2, z_{11}, z_{12}, z_{21}, z_{22}\right) := z_{11} \cdot \frac{\left(x - x_2\right) \cdot \left(y - y_2\right)}{\left(x_1 - x_2\right) \cdot \left(y_1 - y_2\right)} + z_{12} \cdot \frac{\left(x - x_2\right) \cdot \left(y - y_1\right)}{\left(x_1 - x_2\right) \cdot \left(y_2 - y_1\right)} \, ...$$
$$+ \left[z_{21} \cdot \frac{\left(x - x_1\right) \cdot \left(y - y_2\right)}{\left(x_2 - x_1\right) \cdot \left(y_1 - y_2\right)} \right] + z_{22} \cdot \frac{\left(x - x_1\right) \cdot \left(y - y_1\right)}{\left(x_2 - x_1\right) \cdot \left(y_2 - y_1\right)}$$

Interpolation with data From Vargaftik Table, requires using three linear interpolations for entropy based data because entropy change is not equal between data points. This calculation performs single linear interpolation at 1 Bar for ambient pressures because atmospheric pressure is so close to 1 bar. Note that, for expansion based efficiencies, theoretical entropy is at the lower limit of the 1 bar data (approaching absolute zero) where property behavior is erratic. Actual expansion process will absorb heat from ambient; not adhering to the isentropic model.

Linear interpolation relation (1D)

$$h\left(x, x_0, f_0, x_1, f_1\right) := \frac{\left(x - x_0\right)f_1 - \left(x - x_0\right)f_0}{\left(x_1 - x_0\right)} + f_0$$

The nomenclature used for the isentropic derived states used in this analysis is as follows:

H.A_Bs, where A is the location of the property and B is the location where the isentropic process is considered from. For example for enthalpy considering isentropic expansion across the motor (from location 3 to 4) the enthalpy would be expressed as H.4_3s.

Compressor Efficiency-Isentropic Compression Basis

Efficiency is determined considering an ideal case of single stage Isentropic compression from ambient conditions to the measured pressure. Actual compression process uses three stage compression with intercooling between stages; therefore the compressor is theoretically capable of producing efficiencies greater than 100% using this baseline. The current test set up does not measure intermediate pressures or temperatures in the compressor, therefore the actual theoretical power is bounded between isentropic and isothermal cases. Isothermal efficiency will be calculated next.

$$M_{dot1\_scfm} := \frac{M_{dot1}}{\left(.076 \frac{lbm}{ft^3}\right)}$$

$$M_{dot1\_scfm} = 11.316 \cdot \frac{ft^3}{min}$$

Compressor flow in SCFM, note rated flow is 10.8

Vargaftik tables to determine ambient enthalpy and entropy at standard condition

$$H_0 := g\left(\frac{P_{amb}}{Bar}, \frac{T_{amb}}{K}, 1, 290, 2, 300, 290.2, 300.3, 290.0, 300.1\right)\frac{kJ}{kg}$$

$$H_0 = 129.365 \cdot \frac{BTU}{lbm}$$

$$S_0 := g\left(\frac{P_{amb}}{Bar}, \frac{T_{amb}}{K}, 1, 290, 2, 300, 6.837, 6.871, 6.637, 6.671\right)\frac{kJ}{kg \cdot K}$$

$$S_0 = 6.87 \cdot \frac{kJ}{kg \cdot K}$$

Theoretical state 1 (storage pressure) based on isentropic compression to storage pressure

$$P_{1a} = 295.907 \cdot Bar$$

First, Interpolation performed using entropy and enthalpy at each pressure that has table data (200 bar and 300 bar)

$H_{0c} := h(6.87, 6.822, 1293.5, 6.871, 1353)$ $H_{0c} = 1351.78571$

$H_{0d} := h(6.87, 6.752, 1361.3, 6.799, 1421.4)$

$H_{0d} = 1512.18936$

Note: Small extrapolation necessary, 6.799 max table value, very high temperatures associated with isentropic compression

Then interpolation performed using ambient pressure and enthalpies associated with isentropic expansion to table pressures.

$$H_{1\_0s} := h\left(\frac{P_{1a}}{Bar}, 200, H_{0c}, 300, H_{0d}\right)\frac{kJ}{kg}$$

$$H_{1\_0s} = 647.302 \cdot \frac{BTU}{lbm} \quad \text{Theoretical enthalpy after isentropic compression}$$

$$\text{Ideal\_cmp\_pwr} := M_{dot1} \cdot \left(H_{1\_0s} - H_0\right)$$

$$\text{Ideal\_cmp\_pwr} = 7.832 \times 10^3 \, W$$

$$\eta_{cmp\_isoS\_cmp} := \frac{Ideal\_cmp\_pwr}{AC\_pwr}$$

Compressor efficiency is greater than 100% considering an isentropic compression model. Note that temperatures would be greater than 1300K in an isentropic process to 300 Bar.

$$\eta_{cmp\_isoS\_cmp} = 134.125 \cdot \%$$

Compressor Efficiency-Isothermal Compression Basis

Theoretical power required for Isothermal compression can be calculated by assuming an ideal gas model relation and using a integral to calculate the PV work.

$$R_{air} := 53.33 \frac{ft \cdot lbf}{lbm \cdot R} \qquad P(V) := \frac{R_{air} \cdot T_{amb}}{V}$$

$$P_i := P_{amb} \qquad\qquad P_f := P_{1a}$$

$$V_i := \frac{R_{air} \cdot T_{amb}}{P_i} \qquad\qquad V_f := \frac{R_{air} \cdot T_{amb}}{P_f}$$

$$V_i = 0.851 \frac{m^3}{kg} \qquad V_f = 2.915 \times 10^{-3} \frac{m^3}{kg} \qquad \text{Initial and final volumes}$$

$$IsoT_{pwr} := \int_{V_f}^{V_i} M_{dot1} \cdot P(V) \, dV \qquad IsoT_{pwr} = 3.183 \times 10^3 \cdot W \qquad \textbf{Isothermal power, ideal gas compression}$$

$$\eta_{cmp\_isoT\_cmp} := \frac{IsoT_{pwr}}{AC\_pwr}$$

Compressor efficiency considering an Isothermal compression model. Note work required for Isothermal compression is significantly less than that for the Isentropic model. The actual compression process (three stage compression with intercooling) more closely resembles an isothermal process. This efficiency considers compressing the air only and does not include availability losses associated further processing of compressed air.

$$\eta_{cmp\_isoT\_cmp} = 54.511 \cdot \%$$

Heat transfer out associated with Isothermal compression

$$S_{1\_isot} := g\left(\frac{P_{1a}}{Bar}, \frac{T_{amb}}{K}, 200, 300, 300, 310, 5.238, 5.279, 5.088, 5.131\right) \frac{kJ}{kg \cdot K} \qquad T_{amb} = 300.6 \, K$$

$$Q_{out} := -M_{dot1} \cdot T_{amb} \cdot \left(S_{1\_isot} - S_0 \right)$$

$$Q_{out} = 3.466 \times 10^3 \, W$$

$IsoT_{pwr} + Q_{out} = 6.65 \times 10^3 \, W$ Total energy is less than that for isentropic compression.

Compressor Efficiency- Available air power considering Isentropic expansion basis

This method determines compression efficiency based on available power to isentropically expand from storage pressure at standard temperature to standard atmospheric pressure. This basis includes the availability losses associated with using air as an energy transfer medium. It is applicable for the compression process of a system that will expand air to produce electricity or shaft power. This efficiency assumes a reversible expansion process (no heat transfer from the environment) as the baseline. Unlike typical combustion processes; air expansion systems tend to benefit from heat interaction with the environement because the process air temperature becomes much lower than ambient as it expands. This results in energy input into the system. The isentropic expansion basis efficiency is applicable to systems that use quick single stage expansions that occur without significant heat transfer. The experimental test system uses a single stage expansion motor with residence times of approximately .5 sec (based on 1/2" pipe velocity, 12" path) and it has proven to have significant heat transfer based on state properties of entrance and exit air. However; this model is still used in the labview calculations because it provides a stable baseline to compare all components. Most components could theoretically exceed 100% efficiency using this baseline due to heat transfer from the surroundings.

Note: this efficiency method is not relevent to turbine assist compressed air systems because these systems use compressed air directly. The Isothermal compression efficiency previously calculated is a more appropriate efficiency to consider for turbine assist systems.

$$S_{1\_P1\_std\_T} := g\left(\frac{P_{1a}}{Bar}, \frac{T_{std}}{K}, 200, 290, 300, 300, 5.195, 5.238, 5.043, 5.088 \right) \frac{kJ}{kg \cdot K}$$

$S_{1\_P1\_std\_T} = 5.094 \cdot \dfrac{kJ}{kg \cdot K}$ Entropy at std compressed state, assume isentropic expansion to Pstd for this efficiency calculation

$$H_{1\_P1\_std\_T} := g\left(\frac{P_{1a}}{Bar}, \frac{T_{std}}{K}, 200, 290, 300, 300, 252.7, 265.5, 244.4, 257.7 \right) \frac{kJ}{kg}$$

$H_{1\_P1\_std\_T} = 258.019 \cdot \dfrac{kJ}{kg}$ Enthalpy at std temp compressed state,

$H_{0\_1s} = 19.36 \cdot \dfrac{BTU}{lbm}$ Theoretical enthalpy at atmosheric pressure after isentropic expansior (labview determined)

$\text{Ideal\_cmp\_exp\_pwr} := M_{dot1} \cdot \left(H_{1\_P1\_std\_T} - H_{0\_1s} \right)$

$\text{Ideal\_cmp\_exp\_pwr} = 1.385 \times 10^3 \, W$

$\eta_{cmp\_avail\_pwr} := \left[\dfrac{M_{dot1} \cdot \left(H_{1\_P1\_std\_T} - H_{0\_1s} \right)}{\text{AC\_pwr}} \right]$

$\eta_{cmp\_avail\_pwr} = 23.713 \cdot \%$ This is compressor efficiency based on available expansion power of the stored air assuming a single stage isentropic expansion process is used.

Compressor Efficiency- Available air power considering Isothermal expansion basis

$P_i := P_{1a}$ $P_f := P_{amb}$ $P_{exp}(V) := \dfrac{R_{air} \cdot T_{std}}{V}$

$V_i := \dfrac{R_{air} \cdot T_{amb}}{P_i}$ $V_f := \dfrac{R_{air} \cdot T_{amb}}{P_f}$ specific volume of air

$V_i = 2.915 \times 10^{-3} \dfrac{m^3}{kg}$ $V_f = 0.851 \dfrac{m^3}{kg}$ Initial and final specific volumes

$\text{IsoT}_{cmp} := \displaystyle\int_{V_i}^{V_f} M_{dot1} \cdot P_{exp}(V) \, dV$ **Isothermal power, ideal gas compression**

$\text{IsoT}_{pwr} = 3.183 \times 10^3 \cdot W$

$$\eta_{cmp\_isoT\_cmp} := \frac{IsoT_{cmp}}{AC\_pwr}$$

Compressor efficiency considering an Isothermal expansion model. Note this is the same efficiency as determined considering an isothermal compression basis. These efficiencies are identical because the isothermal process is reversible and compression occurs at near the same temperature used for the theoretical constant temperature expansion process.

$$\eta_{cmp\_isoT\_cmp} = 54.402 \cdot \%$$

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Calculation'15A'/'Low Pressure (600 psig)
Theoretical Efficiencies

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This Calculation determines theoretical efficiencies of compression and expansion processes associated with compressed air. It analyzes an open system with closed storage vessel using an Ideal gas model and constant specific heats. Both a high pressure CAES system and a low pressure CAES system are considered under different file names. Both systems are considered with and without a pressure regulator. All energy expressions are on a per mass basis, titled as work terms, which they would become when multiplied by an air mass.

Unit Conversion, Constants, and Thermodynamic State Determination

$$\text{Bar} := 14.5038 \; \frac{\text{lbf}}{\text{in}^2} \qquad \text{kJ} := 1000\text{J} \qquad \gamma_{air} := 1.4 \qquad \text{Heat capacity ratio for air}$$

2) Interpolation Methods

Interpolation relation for 2D interpolations. Helmuth Spath's "Two dimensional Spline Interpolation Algorithms" 1995 ISBN 1-56881-017-2, pg 16 formula 2.3.

$$g\left(x,y,x_1,y_1,x_2,y_2,z_{11},z_{12},z_{21},z_{22}\right) := z_{11} \cdot \frac{\left(x - x_2\right)\cdot\left(y - y_2\right)}{\left(x_1 - x_2\right)\cdot\left(y_1 - y_2\right)} + z_{12}\cdot\frac{\left(x - x_2\right)\cdot\left(y - y_1\right)}{\left(x_1 - x_2\right)\cdot\left(y_2 - y_1\right)} \; ...$$
$$+ \left[z_{21}\cdot\frac{\left(x - x_1\right)\cdot\left(y - y_2\right)}{\left(x_2 - x_1\right)\cdot\left(y_1 - y_2\right)}\right] + z_{22}\cdot\frac{\left(x - x_1\right)\cdot\left(y - y_1\right)}{\left(x_2 - x_1\right)\cdot\left(y_2 - y_1\right)}$$

Interpolation with data From Vargaftik Table, requires using three linear interpolations for entropy based data because entropy change is not equal between data points. This calculation performs single linear interpolation at 1 Bar for ambient pressures because atmospheric pressure is so close to 1 bar. Note that, for expansion based efficiencies, theoretical entropy is at the lower limit of the 1 bar data (approaching absolute zero) where property behavior is erratic. Actual expansion process will absorb heat from ambient; not adhering to the isentropic model.

Linear interpolation relation (1D)

$$h\left(x,x_0,f_0,x_1,f_1\right) := \frac{\left(x - x_0\right)f_1 - \left(x - x_0\right)f_0}{\left(x_1 - x_0\right)} + f_0$$

The nomenclature used for the isentropic derived states used in this analysis is as follows:

H.A_Bs, where A is the location of the property and B is the location where the isentropic process is considered from. For example for enthalpy considering isentropic expansion across the motor (from location 3 to 4) the enthalpy would be expressed as H.4_3s.

Low Pressure CAES (600 psig, McIntosh Plant Min. Operating pressure)

Theoretical Compression efficiency

$$P_{amb} := 14.7 \frac{lbf}{in^2} \qquad T_{amb} := 75\,°F \qquad P_3 := 614.7 \frac{lbf}{in^2} \qquad T_3 := T_{amb}$$

$$R_{air} := 53.33 \frac{ft \cdot lbf}{lbm \cdot R} \qquad T_{amb} = 297.039 \cdot K$$

3) Isothermal Compression

The Isothermal case is used to determine state 3, the compressed state at ambient temperature. this state will be reached in the Isentropic case after a constant volume heat removal process.

$$v(P,T) := \frac{R_{air} \cdot T}{P} \qquad \text{Ideal gas law} \qquad v_3 := v(P_3, T_3)$$

$$v_3 = 0.322 \cdot \frac{ft^3}{lbm}$$

$$W_{1\_3\_isoT} := \int_{P_{amb}}^{P_3} -v(P, T_{amb})\, dP$$

$$W_{1\_3\_isoT} = -136.796 \cdot \frac{BTU}{lbm} \qquad \text{Note T is constant in this case, work is in, noted by the neg.}$$

Heat removal requirement for isothermal compression

$$h_{amb} := g\left(\frac{P_{amb}}{Bar}, \frac{T_{amb}}{K}, 1, 290, 2, 300, 290.2, 300.3, 290.0, 300.1\right)\frac{kJ}{kg} \quad P_{amb} = 1.014 \cdot bar$$

$$h_3 := g\left(\frac{P_3}{Bar}, \frac{T_{amb}}{K}, 40, 290, 45, 300, 281, 291.7, 279.9, 290.7\right)\frac{kJ}{kg} \quad \begin{array}{l} P_3 = 42.382 \cdot bar \\ T_{amb} = 297.039\,K \end{array}$$

$$h_{amb} = 127.819 \cdot \frac{BTU}{lb} \qquad h_3 = 123.835 \cdot \frac{BTU}{lb}$$

This relation below is deduced from a conservation of energy balance.

$$Q_{1\_3\_isoT} := \left[W_{1\_3\_isoT} + (h_3 - h_{amb})\right]$$

$$Q_{1\_3\_isoT} = -140.78 \cdot \frac{BTU}{lb} \qquad \text{Heat flow is out during isothermal compression}$$

4) Isentropic Compression

specific volume at ambient:

$v_2 := v_3$ same specific volume during the constant volume cooling while in storage

$v_1 := v(P_{amb}, T_{amb})$ $v_1 = 13.47 \cdot \dfrac{ft^3}{lbm}$

Polytropic process for an ideal gas (n=k=1.4) is used to determine T.2., formula 3.56 in Shapiro's Fundamentals of Engineering thermodynamics

$$T_2 := T_{amb} \cdot \left(\frac{v_1}{v_2}\right)^{\gamma_{air}-1}$$ $T_2 = 1.921 \times 10^3 \cdot °F$ $T_2 = 1.322 \times 10^3 \, K$

Note this temperature is higher than many materials can withstand at high pressure.

$$P_2 := \frac{P_3 \cdot T_2}{T_{amb}}$$ $P_2 = 2.737 \times 10^3 \cdot \dfrac{lbf}{in^2}$ Ideal gas law to determine corresponding pressure

A very high pressure is required to allow for depressurization during cooling to ambient. Work relation is for a polytropic process using eqn 6.66a in Shapiro's Fundamentals of Engineering thermodynamics

$$W_{1\_2\_isoS} := \frac{-1 \gamma_{air} \cdot R_{air}}{\gamma_{air} - 1} \cdot (T_2 - T_{amb})$$ $W_{1\_2\_isoS} = -442.691 \cdot \dfrac{BTU}{lbm}$ Isentropic compression work

This is considerably more work than required for the isothermal process. This process also produces significant heat which likely transfers from the storage vessel as waste after compression. Lost heat is calculated below; assuming a closed constant volume with heat removal. No work is completed, velocity and gravity effects are not present, therefore a first law analysis simplifies to the following relation

$h_3 = 123.835 \cdot \dfrac{BTU}{lbm}$ $P_3 = 42.382 \cdot Bar$ $T_3 = 297.039 \, K$

$$h_2 := g\left(\frac{P_2}{Bar}, \frac{T_2}{K}, 150, 1250, 200, 1300, 1348.9, 1408.6, 1353.0, 1412.8\right)\frac{kJ}{kg}$$

$P_2 = 188.677 \cdot Bar$

$T_2 = 1.322 \times 10^3 \, K$

$$h_2 = 618.482 \cdot \frac{BTU}{lbm}$$

Note: small extrapolation necessary because of high temperatures.

$$u_3 := h_3 - P_3 \cdot v_3 \qquad u_3 = 87.193 \cdot \frac{BTU}{lbm}$$

$$u_2 := h_2 - P_2 \cdot v_2 \qquad u_2 = 455.356 \cdot \frac{BTU}{lbm}$$

$$Q_{2\_3\_iso\_v} := u_3 - u_2 \qquad Q_{2\_3\_iso\_v} = -368.163 \cdot \frac{BTU}{lbm}$$

heat out in an isentropic compression process during cooling in the storage vessel

Using constant volume specific heat at the approximate average temp to confirm this value.

$$c_v := .195 \, \frac{BTU}{lbm \cdot R} \qquad Q_{2\_3\_cv} := c_v \cdot \left(T_3 - T_2\right)$$

$$Q_{2\_3\_cv} = -359.89 \cdot \frac{BTU}{lbm} \qquad \text{Similar result, using constant specific heat.}$$

5) A throttling process is evaluated considering lost availability

300 psig is the assumed throttling pressure. This analysis is similar to example 7.7 in Shaprio's fundamentals of Engineering Thermodynamics, third edition pg 296.

$$P_4 := 300 \, \frac{lbf}{in^2} \qquad P_4 = 20.684 \cdot Bar$$

$$h_{amb} := g\left(\frac{P_{amb}}{Bar}, \frac{T_{amb}}{K}, 1, 290, 2, 300, 290.2, 300.3, 290.0, 300.1\right)\frac{kJ}{kg}$$

$$h_{amb} = 127.819 \cdot \frac{BTU}{lbm}$$

$$s_{amb} := g\left(\frac{P_{amb}}{Bar}, \frac{T_{amb}}{K}, 1, 290, 2, 300, 6.837, 6.871, 6.637, 6.671\right)\frac{kJ}{kg \cdot K}$$

$$s_{amb} = 1.638 \cdot \frac{BTU}{lbm \cdot R}$$

$$s_3 := g\left(\frac{P_3}{Bar}, \frac{T_{amb}}{K}, 40, 290, 45, 300, 5.75, 5.821, 5.713, 5.749\right)\frac{kJ}{kg \cdot K}$$

$$s_3 = 1.378 \cdot \frac{BTU}{lbm \cdot R}$$

$$h_4 := h_3 \qquad\qquad h_3 = 288.041 \cdot \frac{kJ}{kg}$$

Constant enthalpy through throttling device

Entropy on low side of regulator is determined by three linear interpolations of table data

First, Interpolation performed using enthalpy and entropy at each pressure that has table data with (20 bar and 25 bar)

$$s_{4c} := h\left(\frac{h_3}{\frac{kJ}{kg}}, 285.6, 5.963, 296, 5.998\right) \cdot \frac{kJ}{kg \cdot K} \qquad\qquad s_{4c} = 1.4262 \cdot \frac{BTU}{lbm \cdot R}$$

$$s_{4d} := h\left(\frac{h_3}{\frac{kJ}{kg}}, 284.5, 5.896, 294.9, 5.931\right) \cdot \frac{kJ}{kg \cdot K} \qquad s_{4d} = 1.41108 \cdot \frac{BTU}{lbm \cdot R}$$

Then interpolation performed using regulated pressure and entropy's associated associated with isenthalpic expansion to table pressures.

$$s_4 := h\left(\frac{P_4}{Bar}, 20, s_{4c}, 25, s_{4d}\right)$$

T.4 is approx 293K--Note, the ideal gas model can not predict temperature change during throttling behavior

$$s_4 = 1.424 \cdot \frac{BTU}{lbm \cdot R} \qquad\qquad s_4 = 5.963 \cdot \frac{kJ}{kg \cdot K}$$

$$a_{in} := h_3 - h_{amb} - (s_3 - s_{amb}) \cdot T_{amb} \qquad\qquad a_{in} = 134.909 \cdot \frac{BTU}{lbm}$$

$$a_{out} := h_4 - h_{amb} - (s_4 - s_{amb}) \cdot T_{amb}$$

$$\eta_{throttle} := \frac{a_{out}}{a_{in}}$$

$$a_{out} = 110.397 \cdot \frac{BTU}{lbm}$$

$$\eta_{throttle} = 81.831 \cdot \%$$

Isentropic Expansion, performed from both storage pressure and regulated pressure.

This will confirm that throttle efficiency above can be used as any other component efficiency to determine the overall system efficiency

6) 600 psig Isentropic expansion work

It is assumed that expansion occurs to 1 Bar pressure (14.5 psig), this allows 1d interpolation with table data. Previous analysis in Calculation 11 has determined negligible error is introduced by this assumption

$$h_{5\_isos} := h\left(\frac{s_3}{\frac{kJ}{kg \cdot K}}, 5.741, 117.5, 5.783, 122.7\right) \cdot \frac{kJ}{kg}$$

$$s_3 = 5.771 \cdot \frac{kJ}{kg \cdot K}$$

$$h_{5\_isos} = 52.092 \cdot \frac{BTU}{lbm}$$

Note temperature is between 100 and 105 K, approaching the liquid state.

$$102K = -276.07 \cdot °F$$

$$W_{3\_5\_isoS} := h_3 - h_{5\_isos}$$

$$W_{3\_5\_isoS} = 71.743 \cdot \frac{BTU}{lbm}$$

No heat transfer during isentropic expansion

7) 300 psig Isentropic expansion work

$$h_{5\_reg\_isos} := h\left(\frac{s_4}{\frac{kJ}{kg \cdot K}}, 5.946, 118.8, 5.988, 123.9\right) \cdot \frac{kJ}{kg} \qquad s_4 = 5.963 \cdot \frac{kJ}{kg \cdot K}$$

$$h_{5\_reg\_isos} = 51.939 \cdot \frac{BTU}{lbm}$$

Note temperature is between 120 and 125 K, approaching the liquid state.

$$122K = -240.07 \cdot °F$$

$$W_{4\_5\_isoS} := h_4 - h_{5\_reg\_isos}$$

$$W_{4\_5\_isoS} = 71.896 \cdot \frac{BTU}{lbm}$$

No heat transfer during isentropic expansion

This shows that even though the regulator reduces availability it does allow approximately same work on a per mass basis when an isentropic expansion is considered. Note that both of these processes result in air exit temperatures that are approaching the liquid state; building such a system would be difficult.

8) 300 psig Isentropic Expansion Work with Reheat

This calculation assumes that after the pressure regulator the gas is reheated to ambient temperature through a heat exchanger, fuel, or any other heat transfer process.

$$h_{4\_reheat} := g\left(\frac{P_4}{Bar}, \frac{T_{amb}}{K}, 20, 290, 25, 300, 285.6, 296, 284.5, 294.9\right)\frac{kJ}{kg} \qquad P_4 = 20.684 \cdot bar$$

$$T_{amb} = 297.039 \ K$$

$$h_{4\_reheat} = 125.868 \cdot \frac{BTU}{lbm}$$

$$s_{4\_reheat} := g\left(\frac{P_4}{Bar}, \frac{T_{amb}}{K}, 20, 290, 25, 300, 5.963, 5.998, 5.896, 5.931\right)\frac{kJ}{kg \cdot K}$$

$$s_{4\_reheat} = 5.978 \cdot \frac{kJ}{kg \cdot K} \qquad 124K = -236.47 \cdot °F$$

Assume expansion to 1 bar with constant entropy to determine h and T for exit state

$$h_{5\_reg\_isos\_reheat} := h\left(\frac{s_{4\_reheat}}{\frac{kJ}{kg \cdot K}}, 5.946, 118.8, 5.988, 123.9\right) \cdot \frac{kJ}{kg}$$

$$W_{4\_5\_isoS\_reheat} := h_{4\_reheat} - h_{5\_reg\_isos\_reheat}$$

$$W_{4\_5\_isoS\_reheat} = 73.099 \cdot \frac{BTU}{lbm}$$ Isentropic work if air is reheated after pressure regulation

9) Reheat Heating Requirements

The amount of heat addition required to achieve reheat to ambient is calculated below. This is constant pressure heat addition in which temp is raised from T.4 to ambient. The pressure regulator maintains system pressure, therefore there is expansion work of the process gas during this heat addition. A constant specific heat at 40F is used (.240 BTU/lbm*R) to approximate the required heat addition.

$$C_{p\_40F} := .240 \frac{BTU}{lbm \cdot R}$$

20 bar data is used with linear interpolation

$$T_4 := h\left(\frac{s_4}{\frac{kJ}{kg \cdot K}}, 5.963, 290, 5.998, 300\right) \cdot K$$

$$P_4 = 20.684 \cdot bar \qquad s_4 = 5.963 \cdot \frac{kJ}{kg \cdot K}$$

$$T_4 = 62.1 \cdot °F \qquad T_4 = 521.77 \cdot R$$

$$T_{amb} = 75 \cdot °F$$

$$Q_{3\_4\_reheat} := C_{p\_40F} \cdot (T_{amb} - T_4) \qquad Q_{3\_4\_reheat} = 3.096 \cdot \frac{BTU}{lbm}$$

Heat add requirement to bring pressure regulated gas to ambient temp.

Not much heat addition is required to achieve ambient pressure after pressure regulation. As larger storage pressures are used this value will go up. Note that this is a below ambient temperature process that could likely be achieved with a heat exchanger.

10) 600 psig Isothermal Expansion

Isothermal expansion from the storage pressure is the reverse of the isothermal compression previously analyzed. In an ideal system, this is a reversible process, resulting in 100% efficiency (work out = work in). The pressure regulated case is canalized below.

$$W_{3\_5\_isoT} := -W_{1\_3\_isoT}$$

$$W_{3\_5\_isoT} = 136.796 \cdot \frac{BTU}{lbm}$$ full pressure isothermal expansion

Heat addition required for isothermal expansion

This relation below is deduced from a conservation of energy balance. Sign convention is adjusted to give a positive heat out term.

$$h_{5\_isoT} := h_{amb}$$

$$Q_{3\_5\_isoT} := \left[W_{3\_5\_isoT} + \left(h_{5\_isoT} - h_3 \right) \right]$$

$$Q_{3\_5\_isoT} = 140.78 \cdot \frac{BTU}{lb}$$ Heat in required for isothermal expansion

11) Regulated Pressure (300 psig) Isothermal Expansion Work

$$v(P,T) := \frac{R_{air} \cdot T}{P}$$ Ideal gas law $$v_4 := v\left(P_4, T_4 \right)$$ $$v_4 = 0.644 \cdot \frac{ft^3}{lbm}$$

$$W_{4\_5\_isoT} := \int_{P_4}^{P_{amb}} -v\left(P, T_4 \right) dP$$ Note T is constant in this case.

$$W_{4\_5\_isoT} = 107.845 \cdot \frac{BTU}{lbm}$$

With Isothermal expansion a pressure regulator significantly lowers available work.

Heat addition required for isothermal expansion from regulated pressure

This relation below is deduced from a conservation of energy balance. Sign convention is

adjusted to give a positive heat out term.

$$h_{5\_isoT\_reg} := g\left(\frac{P_{amb}}{Bar}, \frac{T_4}{K}, 1, 280, 2, 290, 280.2, 290.2, 279.9, 290\right)\frac{kJ}{kg}$$

$$T_4 = 62.1 \cdot °F$$

$$Q_{4\_5\_isoT} := \left[W_{4\_5\_isoT} + \left(h_{5\_isoT\_reg} - h_4\right)\right]$$

$$T_4 = 289.872 \ K$$

$$P_{amb} = 1.014 \cdot bar$$

$$Q_{4\_5\_isoT} = 108.717 \cdot \frac{BTU}{lb}$$
Heat in required for isothermal expansion from regulated pressure

12) Regulated Pressure (300 psig) Isothermal Expansion Work with Reheat

Note that the temperature resulting from an ideal throttling process is very low. It is likely that significant heat transfer from the environment would occur even without a heat exchanger. This is what occurred with the test set up where actual exit gas temps were about 20F cooler than input gas temps. The energy output with reheat to ambient before expansion is calculated below.

$$W_{4\_5\_isoT\_reheat} := \int_{P_4}^{P_{amb}} -v\left(P, T_{amb}\right) dP$$
Note T is constant in this case.

$$W_{4\_5\_isoT\_reheat} = 110.511 \cdot \frac{BTU}{lbm}$$

Energy output is much better with reheat after pressure regulation, although not as high as the unregulated case

Heat addition is also required for the isothermal expansion

Heat addition required for isothermal expansion from regulated pressure after reheat

This relation below is deduced from a conservation of energy balance. Sign convention is adjusted to give a positive heat out term.

$$h_{5\_isoT\_reg\_reheat} := h_{amb}$$
Isothermal expansion exit state is amb temp and press

$$Q_{4\_5\_isoT\_reheat} := \left[W_{4\_5\_isoT\_reheat} + \left(h_{5\_isoT\_reg\_reheat} - h_4 \right) \right]$$

$$Q_{4\_5\_isoT\_reheat} = 114.494 \cdot \frac{BTU}{lb}$$ Heat in required for isothermal expansion from regulated pressure after reheat

Total heat required for isothermal expansion from regulated pressure with reheat after pressure regulation

$$Q_{isoT\_reg\_reheat} := Q_{3\_4\_reheat} + Q_{4\_5\_isoT\_reheat}$$

$$Q_{isoT\_reg\_reheat} = 117.59 \cdot \frac{BTU}{lbm}$$ Total heat requirement for isothermal expansion from regulate pressure with reheat after regulation

13) Evaluation of Throttling Efficiency to Predict Isothermal Process Expansion efficiency

$$\eta_{throttle} \cdot W_{3\_5\_isoT} = 111.942 \cdot \frac{BTU}{lbm}$$ Prediction of isothermal expansion from regulated pressure by using Availability basis efficiency of throttling process.

% difference is calculated below

$$\%\_dif\_1 := \frac{W_{4\_5\_isoT\_reheat} - \eta_{throttle} \cdot W_{3\_5\_isoT}}{W_{4\_5\_isoT\_reheat}}$$

$$\%\_dif\_1 = -1.295 \cdot \%$$

The regulator efficiency correlates well with the Isothermal case which accounts for heat transfer from the surroundings during the process.

This shows that the availability based throttling efficiency likely can be applied to the reheated isothermal expansion case without significant error. This correlation will be demonstrated at other pressures.

Polytropic Processes

A theoretical polytropic process using a coefficient of 1.2 probably more closely resembles an actual compression or expansion process. This process will be modeled in compression and expansion for a single stage and three stage process.

14) Single Stage Polytropic Compression

specific volume at ambient:

same specific volume during the constant volume cooling while in storage

$$v_1 := v(P_{amb}, T_{amb}) \qquad v_1 = 13.47 \cdot \frac{ft^3}{lbm}$$

Polytropic process for an ideal gas (n=1.2) is used to determine T.2.5, formula 3.56 in Shapiro's Fundamentals of Engineering thermodynamics. State 2.5 is the title given to the peak pressure during polytropic compression. Like the Isentropic compression there is cooling to the final storage pressure.

$$T_{2.5} := T_{amb} \cdot \left(\frac{v_1}{v_{2.5}} \right)^{1.2-1} \qquad T_{2.5} = 668.449 \cdot °F \qquad T_{2.5} = 626.733\ K \qquad v_{2.5} := v_3$$

$$P_{2.5} := \frac{P_3 \cdot T_{2.5}}{T_{amb}} \qquad P_{2.5} = 1.297 \times 10^3 \cdot \frac{lbf}{in^2}$$

Ideal gas law to determine corresponding pressure

A very high pressure is required to allow for depressurization during cooling to ambient. Work relation is for a polytropic process using eqn 6.66a in Shapiro's Fundamentals of Engineering thermodynamics

$$W_{1\_2.5\_poly} := \frac{-1 \cdot 1.2 \cdot R_{air}}{1.2 - 1} \cdot (T_{2.5} - T_{amb}) \qquad W_{1\_2.5\_poly} = -244.024 \cdot \frac{BTU}{lbm}$$

Work for polytrop compression

This is still considerably more work than required for the isothermal process. This process also produces significant heat which likely transfers from the storage vessel as waste after compression. Lost heat is calculated below; assuming a closed constant volume with heat removal. No work is completed, velocity and gravity effects are not present, therefore a first law analysis simplifies to the following relation

$$h_3 = 123.835 \cdot \frac{BTU}{lbm} \qquad P_3 = 42.382 \cdot Bar \qquad T_3 = 297.039 \ K \qquad u_3 = 87.193 \cdot \frac{BTU}{lbm}$$

$$h_{2.5} := g\left(\frac{P_{2.5}}{Bar}, \frac{T_{2.5}}{K}, 80, 600, 100, 650, 606.7, 660.6, 606.6, 660.8\right)\frac{kJ}{kg}$$

$$P_{2.5} = 89.423 \cdot Bar$$

$$T_{2.5} = 626.733 \ K$$

$$h_{2.5} = 273.236 \cdot \frac{BTU}{lbm}$$

$$u_{2.5} := h_{2.5} - P_{2.5} \cdot v_2 \qquad\qquad u_{2.5} = 195.923 \cdot \frac{BTU}{lbm}$$

$$Q_{2.5\_3\_iso\_v} := \left(u_3 - u_{2.5}\right) \qquad Q_{2.5\_3\_iso\_v} = -108.73 \cdot \frac{BTU}{lbm}$$

Lost heat in polytropic compression process after storage pressure is obtained.

Heat is also removed during the polytropic compression process

$$Q_{1\_2.5\_poly} := \left[W_{1\_2.5\_poly} + \left(h_{2.5} - h_{amb}\right)\right]$$

$$Q_{1\_2.5\_poly} = -98.607 \cdot \frac{BTU}{lbm} \qquad \text{This heat out of the system during polytropic compression}$$

Both heat losses are summed to achieve the total heat loss for polytropic compression

$$Q_{1\_3\_poly} := Q_{2.5\_3\_iso\_v} + Q_{1\_2.5\_poly} \qquad Q_{1\_3\_poly} = -207.337 \cdot \frac{BTU}{lbm} \quad \begin{array}{l}\text{Total heat loss} \\ \text{polytropic} \\ \text{compression}\end{array}$$

15) 600 psig Single Stage Polytropic Expansion

polytropic expansion could be used from the storage state (state3). A coefficient of 1.2 is used.

Polytropic process for an ideal gas (n=1.2) is used to determine T.5_poly., formula 3.56 in Shapiro's Fundamentals of Engineering thermodynamics

$$T_3 = 75 \cdot °F \qquad\qquad T_3 = 297.039 \ K$$

$$P_{5\_poly} := P_{amb}$$

$$T_{5\_poly} := T_3 \cdot \left(\frac{P_{5\_poly}}{P_3} \right)^{1.2-1} \qquad T_{5\_poly} = -206.264 \cdot °F$$

$$T_{5\_poly} = 140.781 \text{ K}$$

$$W_{3\_5\_poly} := \left[\frac{1.2 \cdot R_{air}}{1.2 - 1} \cdot \left(T_{5\_poly} - T_3 \right) \right] \cdot -1$$

$$W_{3\_5\_poly} = 115.655 \cdot \frac{BTU}{lbm} \qquad \text{Power out for polytropic process from storage pressure}$$

This process also requires significant heat from the environment or another heat source. Requirement is calculated below.

$$h_3 = 123.835 \cdot \frac{BTU}{lbm} \qquad P_3 = 42.382 \cdot Bar \qquad T_3 = 297.039 \text{ K} \qquad u_3 = 87.193 \cdot \frac{BTU}{lbm}$$

$$P_{5\_poly} = 1.014 \cdot Bar \qquad T_{5\_poly} = 140.781 \text{ K}$$

$$h_{5\_poly} := g\left(\frac{P_{5\_poly}}{Bar}, \frac{T_{5\_poly}}{K}, 1, 140, 2, 145, 139.1, 144.2, 138.2, 143.3 \right) \frac{kJ}{kg}$$

$$h_{5\_poly} = 60.139 \cdot \frac{BTU}{lbm}$$

$$Q_{3\_5\_poly} := \left[W_{3\_5\_poly} + \left(h_{5\_poly} - h_3 \right) \right]$$

$$Q_{3\_5\_poly} = 51.959 \cdot \frac{BTU}{lbm} \qquad \begin{array}{l} \text{Heat required (heat in)} \\ \text{for polytropic expansion} \end{array}$$

16) 300 Psig Regulated Single Stage Polytropic Expansion

polytropic expansion could be used from after the pressure regulator. A coefficient of 1.2 is used.

Polytropic process for an ideal gas (n=1.2) is used to determine T.5_poly., formula 3.56 in Shapiro's Fundamentals of Engineering thermodynamics

$$T_4 = 289.872 \text{ K} \qquad P_{5\_poly} := P_{amb} \qquad P_4 = 300 \cdot \frac{lbf}{in^2}$$

$$T_{5\_poly\_regulated} := T_4 \cdot \left(\frac{P_{5\_poly}}{P_4}\right)^{1.2-1} \qquad T_{5\_poly\_regulated} = -174.228 \cdot °F$$

$$T_{5\_poly\_regulated} = 158.579 \text{ K}$$

$$W_{4\_5\_poly} := \left[\frac{1.2 \cdot R_{air}}{1.2 - 1} \cdot \left(T_{5\_poly\_regulated} - T_4\right)\right] \cdot -1$$

$$W_{4\_5\_poly} = 97.177 \cdot \frac{BTU}{lbm}$$ Power out for polytropic process from regulated pressure, significantly less than non pressure regulated power, note this is an unobtainable process because the air would liquefy before expansion was complete.

This process also requires significant heat from the environment or another heat source. Requirement is calculated below.

$$h_4 = 123.835 \cdot \frac{BTU}{lbm} \qquad P_4 = 20.684 \cdot Bar \qquad T_4 = 289.872 \text{ K}$$

$$P_{5\_poly} = 1.014 \cdot Bar \qquad T_{5\_poly\_regulated} = 158.579 \text{ K}$$

$$h_{5\_poly\_regulated} := g\left(\frac{P_{5\_poly}}{Bar}, \frac{T_{5\_poly\_regulated}}{K}, 1, 150, 2, 160, 149.2, 159.3, 148.4, 158.6\right) \frac{kJ}{kg}$$

$$h_{5\_poly\_regulated} = 67.866 \cdot \frac{BTU}{lbm}$$ Note: Extrapolation is necessary, results in liquid air, not a true state

$$Q_{4\_5\_poly} := \left[W_{4\_5\_poly} + \left(h_{5\_poly\_regulated} - h_4 \right) \right]$$

$$Q_{4\_5\_poly} = 41.207 \cdot \frac{BTU}{lbm}$$

Heat required for polytropic expansion with pressure regulator, not a valid analysis due to phase change to liquid during expansion

17) Single Stage Polytropic Expansion with regulator and reheat

$$T_{4\_reheat} := T_{amb} \quad T_{4\_reheat} = 297.039 \text{ K} \qquad P_{5\_poly} := P_{amb}$$

$$T_{5\_poly\_reheat} := T_{4\_reheat} \cdot \left(\frac{P_{5\_poly}}{P_4} \right)^{1.2-1} \qquad P_4 = 300 \cdot \frac{lbf}{in^2}$$

$$T_{5\_poly\_reheat} = -167.171 \cdot °F$$

$$W_{4\_5\_poly\_reheat} := \left[\frac{1.2 \cdot R_{air}}{1.2 - 1} \cdot \left(T_{5\_poly\_reheat} - T_{4\_reheat} \right) \right] \cdot -1 \qquad T_{5\_poly\_reheat} = 162.5 \text{ K}$$

$$W_{4\_5\_poly\_reheat} = 99.58 \cdot \frac{BTU}{lbm}$$

Power out for polytropic process from regulated pressure with reheat to ambient

Heat addition required for the reheated polytropic expansion. This process also requires significant heat from the environment or another heat source. Requirement is calculated below.

$$P_{5\_poly} = 14.7 \cdot \frac{lbf}{in^2} \qquad T_{5\_poly\_reheat} = 162.5 \text{ K}$$

$$h_{5\_poly\_reheat} := g \left(\frac{P_{5\_poly}}{Bar}, \frac{T_{5\_poly\_reheat}}{K}, 1, 160, 2, 170, 159.3, 169.4, 158.6, 168.8 \right) \frac{kJ}{kg}$$

$$h_{5\_poly\_reheat} = 69.568 \cdot \frac{BTU}{lbm}$$

$$h_{5\_poly\_reheat} := g\left(\frac{P_{5\_poly}}{Bar}, \frac{T_{5\_poly\_reheat}}{K}, 1, 160, 2, 170, 159.3, 169.4, 158.6, 168.8\right)\frac{kJ}{kg}$$

$$h_{5\_poly\_reheat} = 69.568 \cdot \frac{BTU}{lbm}$$

$$T_{4\_poly\_reheat} := T_{amb} \qquad P_4 = 20.684 \cdot bar \qquad T_{4\_poly\_reheat} = 297.039 \ K$$

$$h_{4\_poly\_reheat} := g\left(\frac{P_4}{Bar}, \frac{T_{4\_poly\_reheat}}{K}, 20, 290, 25, 300, 285.6, 296.0, 284.5, 294.9\right)\frac{kJ}{kg}$$

$$Q_{4\_5\_poly\_reheat} := \left[W_{4\_5\_poly\_reheat} + \left(h_{5\_poly\_reheat} - h_{4\_poly\_reheat}\right)\right]$$

$$Q_{4\_5\_poly\_reheat} = 43.279 \cdot \frac{BTU}{lbm}$$ Heat required for polytropic expansion with regulator after reheat

$$Q_{total\_poly\_reg\_reheat} := Q_{4\_5\_poly\_reheat} + Q_{3\_4\_reheat}$$

$$Q_{total\_poly\_reg\_reheat} = 46.375 \cdot \frac{BTU}{lb}$$ Total heat addition requirement for regulated, reheated, polytropic expansion

Efficiency Determinations.

All efficiencies are determined using Excel, a sample calculation is performed below. Heat

removal and addition requirements are shown in the Excel table; but are not influential in the theoretical efficiency number. The heating /cooling requirements help one understand what it would take to achieve these theoretical efficiencies. Efficiency is determined by the ratio of Power out to Power in.

Example: For Isothermal Compression and Regulated polytropic Expansion the maximum theoretical system efficiency is:

$$W_{1\_3\_isoT} = -136.796 \cdot \frac{BTU}{lbm}$$

$$W_{4\_5\_poly} = 97.177 \cdot \frac{BTU}{lbm}$$

$$\eta_{IsoT\_Reg\_poly} := \frac{W_{4\_5\_poly}}{W_{1\_3\_isoT}}$$

$$\eta_{IsoT\_Reg\_poly} = -71.038 \cdot \%$$

Maximum theoretical efficiency for Isothermal compression and regulated polytropic expansion.

Other combinations are calculated in Excel. The resulting spreadsheet is shown below.

Theoretical System Efficiencies- Low Pressure (600 psig) CAES					
ALL ENERGY AND HEAT IN BTU/LBM, ALL EFFCIEINCIES ARE % (W_OUT/W_IN)					
COMPRESSION / EXPANSION		ISENTROPIC T.h= 1,921 F Q= -368 btu/lbm	SINGLE STAGE POLYTROPIC T.h= 668 F Q= -207 btu/lbm	THREE STAGE POLYTROPIC T.h= 198 F Q= -156 btu/lbm	ISOTHERMAL T.h= 75 F Q= -141 btu/lbm
	CMP ENERGY / EXP ENERGY	-443	-244	-152	-137
ISENTROPIC (600 PSIG) T.c= -276F Q= 0	72	16%	30%	47%	53%
REGULATED (300 PSIG) ISENTROPIC T.c= -240F Q= 0	72	16%	30%	47%	53%
REGULATED (300 PSIG) ISENTROPIC W/REHEAT T.c= -236F Q= 3 btu/lbm	73	16%	30%	48%	53%
POLYTROPIC (600 PSIG) T.c= -206F Q= 52 btu/lbm	115	26%	47%	76%	84%
REGULATED (300 PSIG) POLYTROPIC T.c= -174F Q= 41 btu/lbm	97	22%	40%	64%	71%
REGULATED (300 PSIG) POLYTROPIC W/REHEAT T.c= -167F Q= 46 btu/lbm	100	23%	41%	66%	73%
THREE STAGE POLYTROPIC T.c= -43 F Q= 98 btu/lbm	125	28%	51%	82%	91%
REGULATED (300 PSIG) ISOTHERMAL T.c=62F Q= 109 btu/lbm	108	24%	44%	71%	79%
REGULATED (300 PSIG) ISOTHERMAL W/REHEAT T.c=75F Q=114 btu/lbm	111	25%	45%	73%	81%
ISOTHERMAL (600 PSIG) T.c=75F Q= 141 btu/lbm	137	31%	56%	90%	100%

THEORETICAL EFFICIENCY OF ISOTHERMAL & ADIABATIC CAES

THIS CALCULATION LOOKS AT IDEAL EFFICIENCIES
CONSIDERING AN IDEAL GAS MODEL W/ CONSTANT SPECIFIC
HEATS. BOTH HIGH PRESSURE (3,500 psig) AND LOW PRESSURE
(800 psig) SYSTEMS ARE INVESTIGATED

ISENTROPIC COMPRESSION - NO HEAT TRANSFER ASSUME NEGLIBLE HEIGHT, VELOCITY CHANGES

IDEAL GAS LAW

$$PV = MRT$$

$$\dot{Q}_{in} - \dot{W}_{out} = \Delta \dot{E} \quad \leftarrow \text{1ST LAW}$$

$$\gamma = 1.4 = \frac{C_p}{C_V}$$

$$W = E_1 - E_2$$

$$\dot{E}_1 = \dot{m}\left(u_1 + \frac{P}{\rho} + g\cancel{z}^0\right) = \dot{m}h_1$$

$$\dot{E}_2 = \dot{m}\left(u_2 + \frac{P_2}{\rho_2} + g\cancel{z}^0\right) = \dot{m}h_2$$

$$\left(U + \frac{P}{\rho} = h \leftarrow \text{ENTHALPY STATE}\right)$$

$$\frac{\dot{W}_{out}}{\dot{m}} = h_1 - h_2 = C_p(T_1 - T_2)$$

IF HEAT WAS NOT LOST AN ISENTROPIC EXPANSION COULD
BE PERFORMED AND EFFICIENCY WOULD BE 100%. BUT THE
HEAT OF COMPRESSION IS DIFFICULT TO STORE. THE AMOUNT
OF HEAT LOST AND FINAL PRESSURE AS GAS COOLS TO
AMBIENT IS CALCULATED BELOW.

THE FINAL STATE AFTER COOLING IS AT A LOWER PRESSURE
AND AMBIENT TEMPERATURE.

ISENTROPIC COMPRESSION

CONSTANT VOLUME COOLING TO AMB

$$T_3 = T_1$$

$$\frac{\dot{Q}_{23\,ISOS}}{\dot{m}} = C_V(T_2 - T_3) = \frac{C_p}{1.4}(T_2 - T_3)$$

[HEAT LOST DURING STORAGE]

THE DIFFERENT EFFICIENCIES ARE COMPARED AT STORAGE PRESSURE, THEREFORE WE NEED TO FIX STATE 3 AT THE STORAGE PRESSURE AND DETERMINE THE CORRESPONDING PRESSURE AT STATE 2.

<u>KNOWN</u>

$S_1 = S_2$

$T_1 = T_3$

$P_3 = KNOWN$

$h_1 = KNOWN$

$V_2 = V_3 = \dfrac{P_1 V_1}{P_3}$

$\left(P_1 V_1 = P_3 V_3 \right)$

$P_1 = 14.7 \; PSia$

<u>UNKNOWN</u>

P_2

T_2

h_2

AT LOST EXPANSION WORK DURING COOLING

EVEN IF HEAT WAS CAPTURED THIS IS WASTED ENERGY?

iSENTROPIC

ISOTHERMAL

$P \left(\dfrac{LBF}{iN^2} \right)$

$V \left(\dfrac{ft^3}{LBm} \right)$

THE IDEAL GAS RELATION ADDS THE FOLLOWING B/T STATES 2 & 3

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_2}{T_3} \implies P_2 = \frac{P_3 T_2}{T_3}$$

IDEAL GAS RELATION FOR A POLYTROPIC PROCESS $\left(\begin{array}{l} N = 1.4 \\ iSENTROPIC \end{array} \right)$

$(N = 1.0 \; ISOTHERMAL)$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{1.4 - 1/N} = \left(\frac{V_1}{V_2} \right)^{N-1}$$

ISENTROPIC COMPRESSION WORK AND HEAT ARE EVALUATED IN MATHCAD.

ISOTHERMAL COMPRESSION

$\dot{m} \longrightarrow$

$\longrightarrow \dot{Q}_{OUT}$

$\longrightarrow \dot{m}_{OUT}$

$V(P) = \dfrac{RT}{P}$

$RT \left[LN \dfrac{V_3}{V_1} \right]$

$\dot{Q}_{IN} - \dot{W}_{OUT} = \Delta \dot{\varepsilon}$

$\dfrac{\dot{W}_{OUT}}{\dot{m}} = \displaystyle\int_{P_1}^{P_2} V(P) \, dP$

WHERE $P(V) = \dfrac{R_{Air} T}{P}$

δV is SPECIFIC VOLUME OF AIR

$\frac{12.8}{P}$

LIGHT PRESSURE GOES UP DENSITY is SAME

THIS IS VERIFIED BY LOOKING @ THE PRESSURE VOLUME
DIAGRAM FOR THE PROCESS, FOLLOWING THE T_{AMB} ISOTHERM

FLOW-
WORK =
$\int v\,dp$

NOTE STATE 2 IS OMITTED B/C THE PROCESS IS ISOTHERMAL
THERE ARE NO THERMAL LOSSES AFTER COMPRESSION. HOWEVER
HEAT MUST BE REMOVED CONTINUOUSLY DURING THE PROCESS.
AN ACTUAL PROCESS APPROACHING THE IDEAL ISOTHERMAL
CASE WOULD HAVE MULTIPLE STAGE COMPRESSION WITH
INTERCOOLING B/T STAGES. ISOTHERMAL COMPRESSION IS
STILL REVERSIBLE BUT NOT ISENTROPIC. THE AMOUNT
OF HEAT REMOVED IS DETERMINED BY LOOKING AT
ENTROPY CHANGE

$$\frac{Q_{OUT}}{\underset{\text{ISOT}}{\underset{(1-2)}{\dot{m}}}} = \int_{S_1}^{S_2} T\,ds = T_{AMB}(S_2 - S_1)$$

B/C PROCESS STARTS AT AMBIENT TEMP

EXPANSION SIDE AVAILABILITY & WORK ARE
CALCULATED IN CALC 12, WHICH WAS CONCERNED
WITH HOW ENERGY DENSITY WAS DEFINED. THIS
CALCULATION IS ADAPTED HERE.

EXPANSION METHODS

1) ISENTROPIC EXPANSION - ISENTROPIC EXPANSION RESULTS
   IN AIR EXITING AT CLOSE TO LIQUID STATE AS PRESSURE
   DIFFERENTIAL INCREASES ABOVE 3,000 PSIg. UNLIKE
   COMBUSTION EXPANSION PROCESSES, AIR EXPANSION USES A
   PROCESS GAS THAT IS WELL BELOW AMBIENT AS IT
   EXPANDS. THIS MEANS THAT THERE WILL LIKELY BE SOME
   HEAT TRANSFER TO THE PROCESS GAS, WHICH WILL HELP
   INCREASE EFFICIENCY. WITH FREE HEAT ADDITION FROM
   AMBIENT AIR, IT IS THERMODYNAMICALLY POSSIBLE TO
   ACHIEVE >100% ISENTROPIC EFFICIENCY.

$$\frac{W_{OUT}}{\dot{M}}\bigg|_{ISO-s} = h_4 - h_2 \quad \leftarrow \text{TYPICAL TURBINE IDEAL WORK RELATION}$$

CONSTANT S, NOTE EXPANSION IS NOT TO ORIGINAL TEMPERATURE, INDICATING THIS IS NOT A CLOSED CYCLE. THE EXIT AIR IS VERY COLD.

ISOTHERMAL EXPANSION.

TO ACHIEVE ISOTHERMAL EXPANSION SIGNIFICANT HEAT ADDITION IS REQUIRED. THIS MAY BE POSSIBLE IF WASTE HEAT FROM AN EXISTING TURBINE PLANT IS AVAILABLE

THE WORK OUTPUT FROM ISOTHERMAL EXPANSION IS THE SAME AS THE WORK INPUT. THE HEAT ADDITION REQUIRED IS ALSO THE SAME AS THES PREVIOUS HEAT OUTPUT DURING COMPRESSION. THIS IS EVIDENT BECAUSE THESE ARE REVERSIBLE PROCESSES.

LOSSES IN A REGULATOR

THERE IS A LARGE AVAILABILITY LOSS THROUGH A PRESSURE REGULATOR, THIS IS A THEORETICAL LOSS, ACTUAL LOSSES WILL BE GREATER. A REGULATOR IS NOT DESIRED BUT MAY BE REQUIRED IN AUTOMOTIVE APPLICATIONS WHERE STORAGE PRESSURES MAY BE HIGH, AND IT MAY BE DIFFICULT TO BUILD A MOTOR THAT OPERATES THROUGHOUT THE STORAGE PRESSURE RANGE.

AVAILABILITY OF AIR, PER SECTION 7.5 OF MORAN'S FUNDAMENTALS OF ENGINEERING THERMODYNAMICS.

$$a_{IN} = h_{STORAGE} - h_{AMB} - T_{AMB}(S_{STORAGE} - S_{AMB})$$

$$a_{OUT} = (h_{REG} - h_{AMB} - T_{AMB}(S_{REG} - S_{AMB})$$

NOTE $h_{STORAGE} = h_{REG}$ (CONSTANT ENTHALPY FOR THROTTLING)

THE AVAILABILITY RATIO CAN BE CONSIDERED AS AN EFFICIENCY LOSS IN THE PROCESS BECAUSE IT DIRECTLY REDUCES THE POTENTIAL TO PERFORM EXPANSION WORK.

$$\frac{a_{OUT}}{a_{IN}} = \text{REGULATOR EFFICIENCY}$$

ALL OF THE ABOVE RELATIONS ARE EVALUATED
IN MATHCAD

$$\frac{a_{OUT}}{a_{IN}} = \text{REGULATOR EFFICIENCY}$$

Table of Contents
Calculation '15B' High Pressure (4500 psig)
Theoretical Efficiencies

This Calculation determines theoretical efficiencies of compression and expansion processes associated with compressed air. It analyzes an open system with closed storage vessel using an Ideal gas model and constant specific heats. Both a high pressure CAES system and a low pressure CAES system are considered under different file names. Both systems are considered with and without a pressure regulator. All energy expressions are on a per mass basis, titled as work terms, which they would become when multiplied by an air mass.

High Pressure CAES (4500 psig)

Unit Conversion, Constants, and Thermodynamic State Determination

$$\text{Bar} := 14.5038 \, \frac{\text{lbf}}{\text{in}^2} \qquad \text{kJ} := 1000\text{J} \qquad \gamma_{air} := 1.4 \qquad \text{Heat capacity ratio for air}$$

2) Interpolation Methods

Interpolation relation for 2D interpolations. Helmuth Spath's "Two dimensional Spline Interpolation Algorithms" 1995 ISBN 1-56881-017-2, pg 16 formula 2.3.

$$g\left(x,y,x_1,y_1,x_2,y_2,z_{11},z_{12},z_{21},z_{22}\right) := z_{11} \cdot \frac{\left(x - x_2\right) \cdot \left(y - y_2\right)}{\left(x_1 - x_2\right) \cdot \left(y_1 - y_2\right)} + z_{12} \cdot \frac{\left(x - x_2\right) \cdot \left(y - y_1\right)}{\left(x_1 - x_2\right) \cdot \left(y_2 - y_1\right)} \dots$$
$$+ \left[z_{21} \cdot \frac{\left(x - x_1\right) \cdot \left(y - y_2\right)}{\left(x_2 - x_1\right) \cdot \left(y_1 - y_2\right)} \right] + z_{22} \cdot \frac{\left(x - x_1\right) \cdot \left(y - y_1\right)}{\left(x_2 - x_1\right) \cdot \left(y_2 - y_1\right)}$$

Interpolation with data From Vargaftik Table, requires using three linear interpolations for entropy based data because entropy change is not equal between data points. This calculation performs single linear interpolation at 1 Bar for ambient pressures because atmospheric pressure is so close to 1 bar. Note that, for expansion based efficiencies, theoretical entropy is at the lower limit of the 1 bar data (approaching absolute zero) where property behavior is erratic. Actual expansion process will absorb heat from ambient; not adhering to the isentropic model.

Linear interpolation relation (1D)

$$h\left(x,x_0,f_0,x_1,f_1\right) := \frac{\left(x - x_0\right)f_1 - \left(x - x_0\right)f_0}{\left(x_1 - x_0\right)} + f_0$$

The nomenclature used for the isentropic derived states used in this analysis is as follows:

H.A_Bs, where A is the location of the property and B is the location where the isentropic process is considered from. For example for enthalpy considering isentropic expansion across the motor (from location 3 to 4) the enthalpy would be expressed as H.4_3s.

High Pressure CAES (4500 psig); transportation industry pressure

Theoretical Compression efficiency

$$P_{amb} := 14.7 \frac{lbf}{in^2} \quad T_{amb} := 75\,°F \quad P_3 := 4514.7 \frac{lbf}{in^2} \quad T_3 := T_{amb}$$

$$R_{air} := 53.33 \frac{ft \cdot lbf}{lbm \cdot R} \quad T_{amb} = 297.039 \cdot K$$

3) Isothermal Compression

The Isothermal case is used to determine state 3, the compressed state at ambient temperature. this state will be reached in the Isentropic case after a constant volume heat removal process.

$$v(P,T) := \frac{R_{air} \cdot T}{P} \quad \text{Ideal gas law} \quad v_3 := v(P_3, T_3)$$

$$v_3 = 0.044 \cdot \frac{ft^3}{lbm}$$

$$W_{1\_3\_isoT} := \int_{P_{amb}}^{P_3} -v(P, T_{amb})\, dP$$

$$W_{1\_3\_isoT} = -209.86 \cdot \frac{BTU}{lbm} \quad \begin{array}{l}\text{Note T is constant in}\\ \text{this case, work is in,}\\ \text{noted by the neg.}\end{array}$$

Heat removal requirement for isothermal compression

$$h_{amb} := g\left(\frac{P_{amb}}{Bar}, \frac{T_{amb}}{K}, 1, 290, 2, 300, 290.2, 300.3, 290.0, 300.1\right) \frac{kJ}{kg} \quad P_{amb} = 1.014 \cdot bar$$

$$h_3 := g\left(\frac{P_3}{Bar}, \frac{T_{amb}}{K}, 300, 290, 400, 300, 244.4, 257.7, 241.4, 254.8\right) \frac{kJ}{kg} \quad \begin{array}{l} P_3 = 311.278 \cdot bar \\ T_{amb} = 297.039\,K \end{array}$$

$$T_{amb} = 75 \cdot °F$$

$$h_{amb} = 127.819 \cdot \frac{BTU}{lb} \quad h_3 = 108.956 \cdot \frac{BTU}{lb}$$

This relation below is deduced from a conservation of energy balance.

$$Q_{1\_3\_isoT} := \left[W_{1\_3\_isoT} + (h_3 - h_{amb}) \right]$$

$$Q_{1\_3\_isoT} = -228.723 \cdot \frac{BTU}{lb} \quad \text{Heat flow is out during isothermal compression}$$

4) 4500 psi Isentropic Compression   specific volume at ambient:

$v_2 := v_3$ same specific volume during the constant volume cooling while in storage $v_1 := v\left(P_{amb}, T_{amb}\right)$ $v_1 = 13.47 \cdot \dfrac{ft^3}{lbm}$

Polytropic process for an ideal gas (n=k=1.4) is used to determine T.2., formula 3.56 in Shapiro's Fundamentals of Engineering thermodynamics

$T_2 := T_{amb} \cdot \left(\dfrac{v_1}{v_2}\right)^{\gamma_{air}-1}$ $T_2 = 4.825 \times 10^3 \cdot {}^\circ F$ $T_2 = 2.936 \times 10^3 \, K$

Note this temperature is higher than many materials can withstand at high pressure.

$P_2 := \dfrac{P_3 \cdot T_2}{T_{amb}}$ $P_2 = 4.462 \times 10^4 \cdot \dfrac{lbf}{in^2}$ Ideal gas law to determine corresponding pressure

A very high pressure is required to allow for depressurization during cooling to ambient. Work relation is for a polytropic process using eqn 6.66a in Shapiro's Fundamentals of Engineering thermodynamics

$W_{1\_2\_isoS} := \dfrac{-1\gamma_{air} \cdot R_{air}}{\gamma_{air} - 1} \cdot \left(T_2 - T_{amb}\right)$ $W_{1\_2\_isoS} = -1.139 \times 10^3 \cdot \dfrac{BTU}{lbm}$ Isentropic compression work

This is considerably more work than required for the isothermal process. This process also produces significant heat which likely transfers from the storage vessel as waste after compression. Lost heat is calculated below; assuming a closed constant volume with heat removal. No work is completed, velocity and gravity effects are not present, therefore a first law analysis simplifies to the following relation

$h_3 = 108.956 \cdot \dfrac{BTU}{lbm}$ $P_3 = 311.277 \cdot Bar$ $T_3 = 297.039 \, K$

$h_2 := g\left(\dfrac{P_2}{Bar}, \dfrac{T_2}{K}, 150, 1250, 200, 1300, 1348.9, 1408.6, 1353.0, 1412.8\right) \dfrac{kJ}{kg}$ $P_2 = 3.077 \times 10^3 \cdot Bar$

$T_2 = 2.936 \times 10^3 \, K$

$h_2 = 1.633 \times 10^3 \cdot \dfrac{BTU}{lbm}$

$u_3 := h_3 - P_3 \cdot v_3$ $u_3 = 72.314 \cdot \dfrac{BTU}{lbm}$ Note: large extrapolation necessary because of high temperatures.

$u_2 := h_2 - P_2 \cdot v_2$ $u_2 = 1.271 \times 10^3 \cdot \dfrac{BTU}{lbm}$

$$Q_{2\_3\_iso\_v} := u_3 - u_2 \qquad Q_{2\_3\_iso\_v} = -1.199 \times 10^3 \cdot \frac{BTU}{lbm}$$

heat out in an isentropic compression process during cooling in the storage vessel

Using constant volume specific heat at the approximate average temp to confirm this value.

$$c_v := .195 \frac{BTU}{lbm \cdot R} \qquad Q_{2\_3\_cv} := c_v \cdot (T_3 - T_2)$$

$$Q_{2\_3\_cv} = -926.232 \cdot \frac{BTU}{lbm} \qquad \text{Similar result, using constant specific heat.}$$

5) A throttling process is evaluated considering lost availability

300 psig is the assumed throttling pressure. This analysis is similar to example 7.7 in Shaprio's fundamentals of Engineering Thermodynamics, third edition pg 296.

$$P_4 := 300 \frac{lbf}{in^2} \qquad P_4 = 20.684 \cdot Bar$$

$$h_{amb} := g\left(\frac{P_{amb}}{Bar}, \frac{T_{amb}}{K}, 1, 290, 2, 300, 290.2, 300.3, 290.0, 300.1\right) \frac{kJ}{kg}$$

$$h_{amb} = 127.819 \cdot \frac{BTU}{lbm}$$

$$s_{amb} := g\left(\frac{P_{amb}}{Bar}, \frac{T_{amb}}{K}, 1, 290, 2, 300, 6.837, 6.871, 6.637, 6.671\right) \frac{kJ}{kg \cdot K}$$

$$s_{amb} = 1.638 \cdot \frac{BTU}{lbm \cdot R}$$

$$s_3 := g\left(\frac{P_3}{Bar}, \frac{T_{amb}}{K}, 300, 290, 400, 300, 5.043, 5.088, 4.938, 4.983\right) \frac{kJ}{kg \cdot K} \qquad \begin{array}{l} P_3 = 311.278 \cdot bar \\ T_{amb} = 297.039 \ K \end{array}$$

$$s_3 = 1.209 \cdot \frac{BTU}{lbm \cdot R}$$

$$h_4 := h_3 \qquad h_3 = 253.431 \cdot \frac{kJ}{kg}$$

Constant enthalpy through throttling device

Entropy on low side of regulator is determined by three linear interpolations of table data

First, Interpolation performed using enthalpy and entropy at each pressure that has table data with (20 bar and 25 bar)

$$s_{4c} := h\left(\frac{h_3}{\frac{kJ}{kg}}, 243.8, 5.808, 254.3, 5.849\right) \cdot \frac{kJ}{kg \cdot K} \qquad s_{4c} = 1.3962 \cdot \frac{BTU}{lbm \cdot R}$$

$$s_{4d} := h\left(\frac{h_3}{\frac{kJ}{kg}}, 252.8, 5.781, 263.4, 5.821\right) \cdot \frac{kJ}{kg \cdot K} \qquad s_{4d} = 1.38134 \cdot \frac{BTU}{lbm \cdot R}$$

Then interpolation performed using regulated pressure and entropy's associated associated with isenthalpic expansion to table pressures.

$$s_4 := h\left(\frac{P_4}{Bar}, 20, s_{4c}, 25, s_{4d}\right)$$

T.4 is approx 260K--Note, the ideal gas model can not predict temperature change during throttling behavior

$$s_4 = 1.394 \cdot \frac{BTU}{lbm \cdot R} \qquad s_4 = 5.837 \cdot \frac{kJ}{kg \cdot K} \qquad 260K = 8.33 \cdot {}^\circ F$$

$$a_{in} := h_3 - h_{amb} - (s_3 - s_{amb}) \cdot T_{amb} \qquad a_{in} = 210.415 \cdot \frac{BTU}{lbm}$$

$$a_{out} := h_4 - h_{amb} - (s_4 - s_{amb}) \cdot T_{amb}$$

$$a_{out} = 111.54 \cdot \frac{BTU}{lbm}$$

$$\eta_{throttle} := \frac{a_{out}}{a_{in}}$$

$$\eta_{throttle} = 53.009 \cdot \%$$

With a large pressure drop availability losses are much greater than seen in 600 psig throttling.

Isentropic Expansion, performed from both storage pressure and regulated pressure.

This will confirm that throttle efficiency above can be used as any other component efficiency to determine the overall system efficiency

6) 4500 psig Isentropic expansion work

It is assumed that expansion occurs to 1 Bar pressure (14.5 psig), this allows 1d interpolation with table data. Previous analysis in Calculation 11 has determined negligible error is introduced by this assumption

$$P_3 = 4.515 \times 10^3 \cdot \text{psi}$$

$$h_{5\_isos} := h\left(\frac{s_3}{\frac{kJ}{kg \cdot K}}, 2.918, -131.7, 5.591, 82.7\right) \cdot \frac{kJ}{kg}$$

$$s_3 = 5.063 \cdot \frac{kJ}{kg \cdot K}$$

$$h_{5\_isos} = 17.341 \cdot \frac{BTU}{lbm}$$

Note temperature is between 100 and 105 K, approaching the liquid state.

$$82K = -312.07 \cdot °F$$

$$W_{3\_5\_isoS} := h_3 - h_{5\_isos}$$

$$W_{3\_5\_isoS} = 91.614 \cdot \frac{BTU}{lbm}$$

No heat transfer during isentropic expansion

7) 300 psig Isentropic expansion work

$$h_{5\_reg\_isos} := h\left(\frac{s_4}{\frac{kJ}{kg \cdot K}}, 5.810, 103.4, 5.858, 108.6\right) \cdot \frac{kJ}{kg}$$

$$s_4 = 5.837 \cdot \frac{kJ}{kg \cdot K}$$

Note temperature is between 120 and 125 K, approaching the liquid state.

$$h_{5\_reg\_isos} = 45.716 \cdot \frac{BTU}{lbm}$$

$$107K = -267.07 \cdot °F$$

$$W_{4\_5\_isoS} := h_4 - h_{5\_reg\_isos}$$

$$W_{4\_5\_isoS} = 63.24 \cdot \frac{BTU}{lbm}$$

No heat transfer during isentropic expansion

This shows that the regulator reduces availability and work on a per mass basis when an isentropic expansion with high storage pressure is considered. Note that both of these processes result in air exit temperatures that are approaching the liquid state; building such a system would be difficult.

8) 300 psig Isentropic Expansion Work with Reheat

This calculation assumes that after the pressure regulator the gas is reheated to ambient temperature through a heat exchanger, fuel, or any other heat transfer process.

$$h_{4\_reheat} := g\left(\frac{P_4}{Bar}, \frac{T_{amb}}{K}, 20, 290, 25, 300, 285.6, 296, 284.5, 294.9\right)\frac{kJ}{kg} \quad P_4 = 20.684 \cdot bar$$

$$T_{amb} = 297.039 \ K$$

$$h_{4\_reheat} = 125.868 \cdot \frac{BTU}{lbm}$$

$$s_{4\_reheat} := g\left(\frac{P_4}{Bar}, \frac{T_{amb}}{K}, 20, 290, 25, 300, 5.963, 5.998, 5.896, 5.931\right)\frac{kJ}{kg \cdot K}$$

$$s_{4\_reheat} = 5.978 \cdot \frac{kJ}{kg \cdot K} \qquad 124K = -236.47 \cdot °F$$

Assume expansion to 1 bar with constant entropy to determine h and T for exit state

$$h_{5\_reg\_isos\_reheat} := h\left(\frac{s_{4\_reheat}}{\frac{kJ}{kg \cdot K}}, 5.946, 118.8, 5.988, 123.9\right) \cdot \frac{kJ}{kg}$$

$$W_{4\_5\_isoS\_reheat} := h_{4\_reheat} - h_{5\_reg\_isos\_reheat}$$

$$W_{4\_5\_isoS\_reheat} = 73.099 \cdot \frac{BTU}{lbm} \qquad \text{Isentropic work if air is reheated after pressure regulation}$$

9) Reheat Heating Requirements

The amount of heat addition required to achieve reheat to ambient is calculated below. This is constant pressure heat addition in which temp is raised from T.4 to ambient. The pressure regulator maintains system pressure, therefore there is expansion work of the process gas during this heat addition. A constant specific heat at 40F is used (.240 BTU/lbm*R) to approximate the required heat addition.

$$C_{p\_40F} := .240 \frac{BTU}{lbm \cdot R}$$

$$T_4 := h\left(\frac{s_4}{\frac{kJ}{kg \cdot K}}, 5.963, 290, 5.998, 300\right) \cdot K$$

20 bar data is used with linear interpolation

$P_4 = 20.684 \cdot bar$ $s_4 = 5.837 \cdot \frac{kJ}{kg \cdot K}$

$T_4 = -2.422 \cdot °F$ $T_4 = 457.248 \cdot R$

$T_{amb} = 75 \cdot °F$

$$Q_{3\_4\_reheat} := C_{p\_40F} \cdot (T_{amb} - T_4)$$ $Q_{3\_4\_reheat} = 18.581 \cdot \frac{BTU}{lbm}$

Heat add requirement to bring pressure regulated gas to ambient temp.

Not much heat addition is required to achieve ambient temperature after pressure regulation. As larger storage pressures are used this value will go up. Note that this is a below ambient temperature process that could likely be achieved with a heat exchanger.

10) 4500 psig Isothermal Expansion

Isothermal expansion from the storage pressure is the reverse of the isothermal compression previously analyzed. In an ideal system, this is a reversible process, resulting in 100% efficiency (work out = work in). The pressure regulated case is analized below.

$$W_{3\_5\_isoT} := -W_{1\_3\_isoT}$$

$$W_{3\_5\_isoT} = 209.86 \cdot \frac{BTU}{lbm}$$ full pressure isothermal expansion

Heat addition required for isothermal expansion

This relation below is deduced from a conservation of energy balance. Sign convention is adjusted to give a positive heat out term.

$$h_{5\_isoT} := h_{amb}$$

$$Q_{3\_5\_isoT} := \left[W_{3\_5\_isoT} + \left(h_{5\_isoT} - h_3\right)\right]$$

$$Q_{3\_5\_isoT} = 228.723 \cdot \frac{BTU}{lb}$$ Heat in required for isothermal expansion

11) Regulated Pressure (300 psig) Isothermal Expansion Work

$$v(P,T) := \frac{R_{air} \cdot T}{P} \qquad \text{Ideal gas law} \qquad v_4 := v(P_4, T_4) \qquad v_4 = 0.564 \cdot \frac{ft^3}{lbm}$$

$$W_{4\_5\_isoT} := \int_{P_4}^{P_{amb}} -v(P, T_4)\, dP \qquad \text{Note T is constant in this case.}$$

$$W_{4\_5\_isoT} = 94.509 \cdot \frac{BTU}{lbm}$$

With Isothermal expansion a pressure regulator significantly lowers available work.

Heat addition required for isothermal expansion from regulated pressure

This relation below is deduced from a conservation of energy balance. Sign convention is adjusted to give a positive heat out term.

$$h_{5\_isoT\_reg} := g\left(\frac{P_{amb}}{Bar}, \frac{T_4}{K}, 1, 250, 2, 260, 250, 260, 249.7, 259.7\right) \frac{kJ}{kg} \qquad T_4 = -2.422 \cdot {}^\circ F$$

$$Q_{4\_5\_isoT} := \left[W_{4\_5\_isoT} + \left(h_{5\_isoT\_reg} - h_4\right)\right] \qquad \begin{aligned} T_4 &= 254.026\ K \\ P_{amb} &= 1.014 \cdot bar \end{aligned}$$

$$Q_{4\_5\_isoT} = 94.763 \cdot \frac{BTU}{lb} \qquad \text{Heat in required for isothermal expansion from regulated pressure}$$

12) Regulated Pressure (300 psig) Isothermal Expansion Work with Reheat

Note that the temperature resulting from an ideal throttling process is very low. It is likely that significant heat transfer from the environment would occur even without a heat exchanger. This is what occurred with the test set up where actual exit gas temps were about 20F cooler than input gas temps. The energy output with reheat to ambient before expansion is calculated below.

$$W_{4\_5\_isoT\_reheat} := \int_{P_4}^{P_{amb}} -v(P, T_{amb}) \, dP$$

Note T is constant in this case.

$$W_{4\_5\_isoT\_reheat} = 110.511 \cdot \frac{BTU}{lbm}$$

Energy output is much better with reheat after pressure regulation, although not as high as the unregulated case

Heat addition is also required for the isothermal expansion

Heat addition required for isothermal expansion from regulated pressure after reheat

This relation below is deduced from a conservation of energy balance. Sign convention is adjusted to give a positive heat out term.

$$h_{5\_isoT\_reg\_reheat} := h_{amb}$$ Isothermal expansion exit state is amb temp and press

$$Q_{4\_5\_isoT\_reheat} := \left[W_{4\_5\_isoT\_reheat} + \left(h_{5\_isoT\_reg\_reheat} - h_4 \right) \right]$$

$$Q_{4\_5\_isoT\_reheat} = 129.374 \cdot \frac{BTU}{lb}$$ Heat in required for isothermal expansion from regulated pressure after reheat

Total heat required for isothermal expansion from regulated pressure with reheat after pressure regulation

$$Q_{isoT\_reg\_reheat} := Q_{3\_4\_reheat} + Q_{4\_5\_isoT\_reheat}$$

$$Q_{isoT\_reg\_reheat} = 147.955 \cdot \frac{BTU}{lbm}$$ Total heat requirement for isothermal expansion from regulated pressure with reheat after regulation

13) Evaluation of Throttling Efficiency to Predict Isothermal Process Expansion efficiency

$$\eta_{throttle} \cdot W_{3\_5\_isoT} = 111.245 \cdot \frac{BTU}{lbm}$$

Prediction of isothermal expansion from regulated pressure by using Availability basis efficiency of throttling process.

% difference is calculated below

$$\%\_dif\_1 := \frac{W_{4\_5\_isoT\_reheat} - \eta_{throttle} \cdot W_{3\_5\_isoT}}{W_{4\_5\_isoT\_reheat}}$$

$$\%\_dif\_1 = -0.664 \cdot \%$$

The regulator efficiency correlates well with the Isothermal case which accounts for heat transfer from the surroundings during the process.

This shows that the availability based throttling efficiency likely can be applied to the reheated isothermal expansion case without significant error. This correlation will be demonstrated at other pressures.

Polytropic Processes

A theoretical polytropic process using a coefficient of 1.2 probably more closely resembles an actual compression or expansion process. This process will be modeled in compression and expansion for a single stage and three stage process.

14) Single Stage Polytropic Compression

specific volume at ambient:

same specific volume during the constant volume cooling while in storage

$$v_1 := v(P_{amb}, T_{amb})$$

Polytropic process for an ideal gas (n=1.2) is used to determine T.2.5, formula 3.56 in Shapiro's Fundamentals of Engineering thermodynamics. State 2.5 is the title given to the peak pressure during polytropic compression. Like the Isentropic compression there is cooling to the final storage pressure.

$$v_1 = 13.47 \cdot \frac{ft^3}{lbm}$$

$$T_{2.5} := T_{amb} \cdot \left(\frac{v_1}{v_{2.5}}\right)^{1.2-1} \qquad T_{2.5} = 1.221 \times 10^3 \cdot °F \quad T_{2.5} = 933.847 \ K \qquad v_{2.5} := v_3$$

$$P_{2.5} := \frac{P_3 \cdot T_{2.5}}{T_{amb}} \qquad P_{2.5} = 1.419 \times 10^4 \cdot \frac{lbf}{in^2}$$

Ideal gas law to determine corresponding pressure

A very high pressure is required to allow for depressurization during cooling to ambient. Work relation is for a polytropic process using eqn 6.66a in Shapiro's Fundamentals of Engineering thermodynamics

$$W_{1\_2.5\_poly} := \frac{-1 \cdot 1.2 \cdot R_{air}}{1.2 - 1} \cdot \left(T_{2.5} - T_{amb}\right) \qquad W_{1\_2.5\_poly} = -471.335 \cdot \frac{BTU}{lbm} \qquad$$ Work for polytrop compression

This is still considerably more work than required for the isothermal process. This process also produces significant heat which likely transfers from the storage vessel as waste after compression. Lost heat is calculated below; assuming a closed constant volume with heat removal. No work is completed, velocity and gravity effects are not present, therefore a first law analysis simplifies to the following relation

$$h_3 = 108.956 \cdot \frac{BTU}{lbm} \qquad P_3 = 311.277 \cdot Bar \qquad T_3 = 297.039\ K \qquad u_3 = 72.314 \cdot \frac{BTU}{lbm}$$

$$h_{2.5} := g\left(\frac{P_{2.5}}{Bar}, \frac{T_{2.5}}{K}, 900, 900, 1000, 950, 986.7, 1046.3, 993.6, 1053.3\right)\frac{kJ}{kg} \qquad P_{2.5} = 978.609 \cdot Bar$$

$$T_{2.5} = 933.847\ K$$

$$h_{2.5} = 443.905 \cdot \frac{BTU}{lbm}$$

$$u_{2.5} := h_{2.5} - P_{2.5} \cdot v_2 \qquad\qquad u_{2.5} = 328.707 \cdot \frac{BTU}{lbm}$$

$$Q_{2.5\_3\_iso\_v} := \left(u_3 - u_{2.5}\right) \qquad Q_{2.5\_3\_iso\_v} = -256.393 \cdot \frac{BTU}{lbm}$$
Lost heat in polytropic compression process after storage pressure is obtained.

Heat is also removed during the polytropic compression process

$$Q_{1\_2.5\_poly} := \left[W_{1\_2.5\_poly} + \left(h_{2.5} - h_{amb}\right)\right]$$

$$Q_{1\_2.5\_poly} = -155.249 \cdot \frac{BTU}{lbm} \qquad$$ This heat out of the system during polytropic compression

Both heat losses are summed to achieve the total heat loss for polytropic compression

$$Q_{1\_3\_poly} := Q_{2.5\_3\_iso\_v} + Q_{1\_2.5\_poly} \qquad Q_{1\_3\_poly} = -411.642 \cdot \frac{BTU}{lbm}$$
Total heat loss polytropic compression

15) 4500 psig Single Stage Polytropic Expansion

polytropic expansion could be used from the storage state (state3). A coefficient of 1.2 is used.

Polytropic process for an ideal gas (n=1.2) is used to determine T.5_poly., formula 3.56 in Shapiro's Fundamentals of Engineering thermodynamics

$$T_3 = 75 \cdot °F \qquad T_3 = 297.039 \, K$$

$$P_{5\_poly} := P_{amb}$$

$$T_{5\_poly} := T_3 \cdot \left(\frac{P_{5\_poly}}{P_3} \right)^{1.2-1} \qquad T_{5\_poly} = -289.602 \cdot °F$$

$$T_{5\_poly} = 94.482 \, K$$

$$P_3 = 311.278 \cdot bar$$

$$W_{3\_5\_poly} := \left[\frac{1.2 \cdot R_{air}}{1.2 - 1} \cdot \left(T_{5\_poly} - T_3 \right) \right] \cdot -1$$

$$W_{3\_5\_poly} = 149.923 \cdot \frac{BTU}{lbm} \qquad \text{Power out for polytropic process from storage pressure}$$

This process also requires significant heat from the environment or another heat source. Requirement is calculated below.

$$h_3 = 108.956 \cdot \frac{BTU}{lbm} \qquad P_3 = 311.277 \cdot Bar \qquad T_3 = 297.039 \, K \qquad u_3 = 72.314 \cdot \frac{BTU}{lbm}$$

$$P_{5\_poly} = 1.014 \cdot Bar \qquad T_{5\_poly} = 94.482 \, K \qquad T_{5\_poly} = -289.602 \cdot °F$$

$$h_{5\_poly} := g \left(\frac{P_{5\_poly}}{Bar}, \frac{T_{5\_poly}}{K}, 1, 90, 2, 95, 87.9, 91.3, 85.7, 91.1 \right) \frac{kJ}{kg}$$

$$h_{5\_poly} = 39.098 \cdot \frac{BTU}{lbm}$$

$$Q_{3\_5\_poly} := \left[W_{3\_5\_poly} + \left(h_{5\_poly} - h_3 \right) \right]$$

$$Q_{3\_5\_poly} = 80.065 \cdot \frac{BTU}{lbm}$$ Heat required (heat in)
for polytropic expansion

16) 300 Psig Regulated Single Stage Polytropic Expansion

polytropic expansion could be used from after the pressure regulator. A coefficient of 1.2 is used.

Polytropic process for an ideal gas (n=1.2) is used to determine T.5_poly., formula 3.56 in Shapiro's Fundamentals of Engineering thermodynamics

$$T_4 = 254.026\ K \qquad P_{5\_poly} := P_{amb} \qquad P_4 = 300 \cdot \frac{lbf}{in^2}$$

$$T_{5\_poly\_regulated} := T_4 \cdot \left(\frac{P_{5\_poly}}{P_4} \right)^{1.2-1} \qquad T_{5\_poly\_regulated} = -209.526 \cdot °F$$

$$T_{5\_poly\_regulated} = 138.969\ K$$

$$W_{4\_5\_poly} := \left[\frac{1.2 \cdot R_{air}}{1.2 - 1} \cdot \left(T_{5\_poly\_regulated} - T_4 \right) \right] \cdot -1$$

$$W_{4\_5\_poly} = 85.16 \cdot \frac{BTU}{lbm}$$ Power out for polytropic process from regulated pressure, significantly less than non pressure regulated power, note this is an unobtainable process because the air would liquefy before expansion was complete.

This process also requires significant heat from the environment or another heat source. Requirement is calculated below.

$$h_4 = 108.956 \cdot \frac{BTU}{lbm} \qquad P_4 = 20.684 \cdot Bar \qquad T_4 = 254.026 \ K$$

$$P_{5\_poly} = 1.014 \cdot Bar \qquad T_{5\_poly\_regulated} = 138.969 \ K$$

$$T_{5\_poly\_regulated} = -209.526 \cdot °F$$

$$h_{5\_poly\_regulated} := g\left(\frac{P_{5\_poly}}{Bar}, \frac{T_{5\_poly\_regulated}}{K}, 1, 135, 2, 140, 134, 139.1, 133.1, 138.2\right) \frac{kJ}{kg}$$

$$h_{5\_poly\_regulated} = 59.345 \cdot \frac{BTU}{lbm}$$ Note: Extrapolation is necessary, results in liquid air, not a true state

$$Q_{4\_5\_poly} := \left[W_{4\_5\_poly} + \left(h_{5\_poly\_regulated} - h_4 \right) \right]$$

$$Q_{4\_5\_poly} = 35.549 \cdot \frac{BTU}{lbm}$$ Heat required for polytropic expansion with pressure regulator, not a valid analysis due to phase change to liquid during expansion

17) Single Stage Polytropic Expansion with regulator and reheat

$$T_{4\_reheat} := T_{amb} \qquad T_{4\_reheat} = 297.039 \ K \qquad\qquad P_{5\_poly} := P_{amb}$$

$$T_{5\_poly\_reheat} := T_{4\_reheat} \cdot \left(\frac{P_{5\_poly}}{P_4}\right)^{1.2-1} \qquad\qquad P_4 = 300 \cdot \frac{lbf}{in^2}$$

$$T_{5\_poly\_reheat} = -167.171 \cdot °F$$

$$W_{4\_5\_poly\_reheat} := \left[\frac{1.2 \cdot R_{air}}{1.2 - 1} \cdot \left(T_{5\_poly\_reheat} - T_{4\_reheat} \right) \right] \cdot -1 \qquad T_{5\_poly\_reheat} = 162.5 \ K$$

$$W_{4\_5\_poly\_reheat} = 99.58 \cdot \frac{BTU}{lbm}$$ Power out for polytropic process from regulated pressure with reheat to ambient

Heat addition required for the reheated polytropic expansion. This process also requires significant heat from the environment or another heat source. Requirement is calculated below.

$$P_{5\_poly} = 14.7 \cdot \frac{lbf}{in^2} \qquad T_{5\_poly\_reheat} = 162.5 \, K$$

$$h_{5\_poly\_reheat} := g\left(\frac{P_{5\_poly}}{Bar}, \frac{T_{5\_poly\_reheat}}{K}, 1, 160, 2, 170, 159.3, 169.4, 158.6, 168.8\right) \frac{kJ}{kg}$$

$$h_{5\_poly\_reheat} = 69.568 \cdot \frac{BTU}{lbm}$$

$$h_{5\_poly\_reheat} := g\left(\frac{P_{5\_poly}}{Bar}, \frac{T_{5\_poly\_reheat}}{K}, 1, 160, 2, 170, 159.3, 169.4, 158.6, 168.8\right) \frac{kJ}{kg}$$

$$h_{5\_poly\_reheat} = 69.568 \cdot \frac{BTU}{lbm}$$

$$T_{4\_poly\_reheat} := T_{amb} \qquad P_4 = 20.684 \cdot bar \qquad T_{4\_poly\_reheat} = 297.039 \, K$$

$$h_{4\_poly\_reheat} := g\left(\frac{P_4}{Bar}, \frac{T_{4\_poly\_reheat}}{K}, 20, 290, 25, 300, 285.6, 296.0, 284.5, 294.9\right) \frac{kJ}{kg}$$

$$Q_{4\_5\_poly\_reheat} := \left[W_{4\_5\_poly\_reheat} + \left(h_{5\_poly\_reheat} - h_{4\_poly\_reheat}\right)\right]$$

$$Q_{4\_5\_poly\_reheat} = 43.279 \cdot \frac{BTU}{lbm} \qquad \text{Heat required for polytropic expansion with regulator after reheat}$$

$$Q_{total\_poly\_reg\_reheat} := Q_{4\_5\_poly\_reheat} + Q_{3\_4\_reheat}$$

$$Q_{total\_poly\_reg\_reheat} = 61.861 \cdot \frac{BTU}{lb} \qquad \text{Total heat addition requirement for regulated, reheated, polytropic expansion}$$

Efficiency Determinations.

All efficiencies are determined using Excel, a sample calculation is performed below. Heat removal and addition requirements are shown in the Excel table; but are not influential in the theoretical efficiency number. The heating /cooling requirements help one understand what it would take to achieve these theoretical efficiencies. Efficiency is determined by the ratio of Power out to Power in.

Example: For Isothermal Compression and Regulated polytropic Expansion the maximum theoretical system efficiency is:

$$W_{1\_3\_isoT} = -209.86 \cdot \frac{BTU}{lbm}$$

$$W_{4\_5\_poly} = 85.16 \cdot \frac{BTU}{lbm}$$

$$\eta_{IsoT\_Reg\_poly} := \frac{W_{4\_5\_poly}}{-W_{1\_3\_isoT}}$$

$$\eta_{IsoT\_Reg\_poly} = 40.579 \cdot \%$$

Maximum theoretical efficiency for Isothermal compression and regulated polytropic expansion.

Other combinations are calculated in Excel. The resulting spreadsheet is shown below.

Theoretical System Efficiencies- High Pressure (4500 psig) CAES					
ALL ENERGY AND HEAT IN BTU/LBM, ALL EFFCIEINCIES ARE % (W_OUT/W_IN)					
COMPRESSION EXPANSION		ISENTROPIC T.h= 4,825 F Q= -1199 btu/lbm	SINGLE STAGE POLYTROPIC T.h= 1221 F Q= -412 btu/lbm	THREE STAGE POLYTROPIC T.h= 275 F Q= -266 btu/lbm	ISOTHERMAL T.h= 75 F Q= -229 btu/lbm
	CMP ENERGY EXP ENERGY	-1139	-471	-247	-210
ISENTROPIC (4500 PSIG) T.c= -312F Q= 0	92	8%	20%	37%	44%
REGULATED (300 PSIG) ISENTROPIC T.c= -267F Q= 0	63	6%	13%	26%	30%
REGULATED (300 PSIG) ISENTROPIC W/REHEAT T.c= -236F Q= 19 btu/lbm	73	6%	15%	30%	35%
POLYTROPIC (4500 PSIG) T.c= -290F Q= 80 btu/lbm	150	13%	32%	61%	71%
REGULATED (300 PSIG) POLYTROPIC T.c= -210F Q= 36 btu/lbm	85	7%	18%	34%	40%
REGULATED (300 PSIG) POLYTROPIC W/REHEAT T.c= -167F Q= 62 btu/lbm	100	9%	21%	40%	48%
THREE STAGE POLYTROPIC T.c= -95F Q= 149 btu/lbm	178	16%	38%	72%	85%
REGULATED (300 PSIG) ISOTHERMAL T.c= -2F Q= 95 btu/lbm	95	8%	20%	38%	45%
REGULATED (300 PSIG) ISOTHERMAL W/REHEAT T.c=75F Q=148 btu/lbm	111	10%	24%	45%	53%
ISOTHERMAL (4500 PSIG) T.c=75F Q= 229 btu/lbm	210	18%	45%	85%	100%

THEORETICAL EFFICIENCY OF ISOTHERMAL & ADIABATIC CAES

THIS CALCULATION LOOKS AT IDEAL EFFICIENCIES
CONSIDERING AN IDEAL GAS MODEL W/ CONSTANT SPECIFIC
HEATS. BOTH HIGH PRESSURE (3,500 psig) AND LOW PRESSURE
(800 psig) SYSTEMS ARE INVESTIGATED

ISENTROPIC COMPRESSION - NO HEAT TRANSFER ASSUME NEGLIGIBLE HEIGHT, VELOCITY CHANGES

IDEAL GAS LAW

$$P \Psi = MRT$$

$$\dot{Q}_{IN} - \dot{W}_{OUT} = \Delta \dot{E} \quad \leftarrow \text{1ST LAW} \qquad \gamma = 1.4 = \frac{C_p}{C_v}$$

$$W = E_1 - E_2$$

$$\dot{E}_1 = \dot{m}\left(u_1 + \frac{P}{\rho} + g z^0\right) = \dot{m} h_1$$

$$\dot{E}_2 = \dot{m}\left(u_2 + \frac{P_2}{\rho_2} + g z^0\right) = \dot{m} h_2$$

$$\left(u + \frac{P}{\rho} = h \leftarrow \text{ENTHALPY STATE}\right)$$

$$\frac{\dot{W}_{OUT}}{\dot{m}} = h_1 - h_2 = C_p (T_1 - T_2)$$

IF HEAT WAS NOT LOST AN ISENTROPIC EXPANSION COULD
BE PERFORMED AND EFFICIENCY WOULD BE 100%. BUT THE
HEAT OF COMPRESSION IS DIFFICULT TO STORE. THE AMOUNT
OF HEAT LOST AND FINAL PRESSURE AS GAS COOLS TO
AMBIENT IS CALCULATED BELOW.

THE FINAL STATE AFTER COOLING IS AT A LOWER PRESSURE
AND AMBIENT TEMPERATURE.

CONSTANT VOLUME COOLING TO AMB

$$\overset{T_3 = T_1}{\dot{Q}_{23_{ISOS}}} = C_v (T_2 - T_3) = \frac{C_p}{1.4}(T_2 - T_3)$$

[HEAT LOST DURING STORAGE]

THE DIFFERENT EFFICIENCIES ARE COMPARED AT STORAGE PRESSURE, THEREFORE WE NEED TO FIX STATE 3 AT THE STORAGE PRESSURE AND DETERMINE THE CORRESPONDING PRESSURE AT STATE 2.

KNOWN

$$S_1 = S_2$$
$$T_1 = T_3$$
$$P_3 = KNOWN$$
$$h_1 = KNOWN$$
$$V_2 = V_3 = \frac{P_1 V_1}{P_3}$$
$$\left(P_1 V_1 = P_3 V_3 \right)$$
$$P_1 = 14.7 \; PSia$$

UNKNOWN

$$P_2$$
$$T_2$$
$$h_2$$

AT LOST EXPANSION WORK DURING COOLING

EVEN IF HEAT WAS CAPTURED THIS IS WASTED ENERGY?

ISENTROPIC

ISOTHERMAL

$P \; \left(\frac{LBF}{LN^2} \right)$

$V \; \left(\frac{ft^3}{LBM} \right)$

(2) (3) (1)

L2B

LOST PRESSURE GOES UP DENSITY IS SAME

THE IDEAL GAS RELATION ADDS THE FOLLOWING B/T STATES 2 & 3

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_2}{T_3} \implies P_2 = \frac{P_3 T_2}{T_3}$$

IDEAL GAS RELATION FOR A POLYTROPIC PROCESS $\left(\begin{array}{c} N=1.4 \\ ISENTROPIC \end{array} \right)$ $(N=1.0 \; ISOTHERMAL)$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{1.4 - 1/N} = \left(\frac{V_1}{V_2} \right)^{N-1}$$

ISENTROPIC COMPRESSION WORK AND HEAT ARE EVALUATED IN MATHCAD.

ISOTHERMAL COMPRESSION

$\dot{m}$ →

$\dot{Q}_{OUT}$

$\dot{m}_{OUT}$

$$V(P) = \frac{RT}{P}$$

$$RT \left[LN \frac{V_3}{V_1} \right]$$

$$\dot{Q}_{IN} - \dot{W}_{OUT} = \Delta \dot{\varepsilon}$$

$$\frac{\dot{W}_{OUT}}{\dot{m}} = \int_{P_1}^{P_2} V(P) \, dP$$

WHERE $P(V) = \frac{R_{Air} T}{P}$

∂V IS SPECIFIC VOLUME OF AIR

THIS is VERIFIED BY LOOKING @ THE Pressure VOLUME Diagram for THE process, following THE T_{AMB} isotherm

FLOW-
WORK=
$\int v dP$

NOTE STATE 2 is OMITTED B/c THE process is ISOTHERMAL THERE ARE NO THERMAL LOSSES AFTER COMPRESSION. However HEAT MUST BE REMOVED CONTINUOUSLY DURING THE process. AN ACTUAL process APPROACHING THE IDEAL ISOTHERMAL CASE WOULD HAVE MULTIPLE STAGE COMPRESSION WITH INTERCOOLING B/T STAGES. ISOTHERMAL COMPRESSION is STILL REVERSIBLE BUT NOT ISENTROPIC. THE AMOUNT OF HEAT REMOVED IS DETERMINED BY LOOKING AT ENTROPY CHANGE

$$\frac{Q_{OUT}}{\substack{ISOT \\ (1-2)}} = \int_{S_1}^{S_2} T \, ds = T_{AMB}(S_2 - S_1)$$

$\underset{\dot{m}}{}$

B/c Process STARTS AT AMBIENT TEMP

EXPANSION SIDE AVAILABILITY & WORK Are CALCULATED IN CALC 12, WHICH WAS CONCERNED WITH HOW ENERGY DENSITY WAS DEFINED. THIS CALCULATION is ADAPTED HERE.

EXPANSION METHODS

1) ISENTROPIC EXPANSION - ISENTROPIC EXPANSION RESULTS IN AIR EXITING AT CLOSE TO LIQUID STATE AS pressure DIFFERENTIAL INCREASES ABOVE 3,000 PSig. UNLIKE COMBUSTION EXPANSION processes, AIR EXPANSION USES A PROCESS GAS THAT is WELL BELOW AMBIENT AS IT EXPANDS. THIS MEANS THAT THERE WILL LIKELY BE SOME HEAT TRANSFER TO THE process GAS, WHICH WILL HELP INCREASE EFFICIENCY. WITH FREE HEAT ADDITION FROM AMBIENT AIR, IT is THERMODYNAMICALLY POSSIBLE TO ACHIEVE >100% ISENTROPIC EFFICIENCY.

$$\frac{W_{OUT}}{ISO\text{-}s} = h_4 - h_2 \quad \leftarrow \text{ TYPICAL TURBINE}$$
$$\frac{}{\dot{M}} \qquad\qquad\qquad \text{IDEAL WORK}$$
$$\qquad\qquad\qquad\qquad \text{RELATION}$$

CONSTANT S, NOTE EXPANSION IS NOT
TO ORIGINAL TEMPERATURE, INDICATING THIS IS
NOT A CLOSED CYCLE. THE EXIT AIR IS VERY
COLD.

ISOTHERMAL EXPANSION.

TO ACHIEVE ISOTHERMAL EXPANSION SIGNIFICANT HEAT ADDITION
IS REQUIRED. THIS MAY BE POSSIBLE IF WASTE HEAT
FROM AN EXISTING TURBINE PLANT IS AVAILABLE

THE WORK OUTPUT FROM ISOTHERMAL EXPANSION IS THE
SAME AS THE WORK INPUT. THE HEAT ADDITION REQUIRED
IS ALSO THE SAME AS THES PREVIOUS HEAT OUTPUT
DURING COMPRESSION. THIS IS EVIDENT BECAUSE
THESE ARE REVERSIBLE PROCESSES.

LOSSES IN A REGULATOR

THERE IS A LARGE AVAILABILITY LOSS THROUGH A PRESSURE
REGULATOR, THIS IS A THEORETICAL LOSS, ACTUAL LOSSES
WILL BE GREATER. A REGULATOR IS NOT DESIRED BUT
MAY BE REQUIRED IN AUTOMOTIVE APPLICATIONS WHERE
STORAGE PRESSURES MAY BE HIGH AND IT MAY BE
DIFFICULT TO BUILD A MOTOR THAT OPERATES THROUGHOUT THE
STORAGE PRESSURE RANGE.

AVAILABILITY OF AIR, PER SECTION 7.5 OF MORAN'S FUNDAMENTALS
OF ENGINEERING THERMODYNAMICS.

$$\cdot \; a_{IN} = h_{STORAGE} - h_{AMB} - T_{AMB}\left(S_{STORAGE} - S_{AMB} \right)$$

$$a_{OUT} = \left(h_{REG} - h_{AMB} - T_{AMB}\left(S_{REG} - S_{AMB} \right) \right.$$

NOTE $h_{STORAGE} = h_{REG}$ (CONSTANT ENTHALPY FOR THROTTLING)

THE AVAILABILITY RATIO CAN BE CONSIDERED AS AN
EFFICIENCY LOSS IN THE PROCESS BECAUSE IT
DIRECTLY REDUCES THE POTENTIAL TO PERFORM EXPANSION
WORK.

$$\frac{a_{OUT}}{a_{IN}} = \text{REGULATOR EFFICIENCY}$$

ALL OF THE ABOVE RELATIONS ARE EVALUATED
IN MATHCAD

$$\frac{a_{OUT}}{a_{IN}} = \text{REGULATOR EFFICIENCY}$$

Table of Contents
Calculation '16' Motor/Generator Efficiency

This Calculation determines air motor/generator system efficiency for row 5150 of "Test_data_all" file corresponding to 3384 sec, Test_009 on 10/17/11. Isentropic available air power basis, Isothermal available air power basis, Isentropic compression basis and Isothermal compression basis). Justification and explanantion of each method is discussed.

Unit Conversion Data

$$Bar := 14.5038 \frac{lbf}{in^2} \qquad kJ := 1000J$$

LabView Data:

$$M_{dot1} := 1.82 \frac{lbm}{min}$$ Mass flow rate, LabView calculated

$$P_3 := 75.25 \frac{lbf}{in^2}$$ Motor Inlet pressure

$$T_3 := 58.42 \,°F$$

$$DC\_pwr := 410.56W$$ Output power, measured

$$H_3 := 123.29 \frac{BTU}{lbm}$$ Enthalpy at storage pressure, LabView calcualted

$$S_3 := 1.51 \frac{BTU}{lbm \cdot R}$$ Entropy at storage pressure, LabView calcualted

Motor exit pressure

$$P_4 := 6.77 \frac{lbf}{in^2}$$

$$S_4 := 1.60 \frac{BTU}{lbm \cdot R}$$

$$U_3 := H_3 - \frac{P_3}{\rho_3} \qquad U_3 = 93.662 \cdot \frac{BTU}{lbm}$$

$$U_4 := H_4 - \frac{P_4}{\rho_4} \qquad U_4 = 103.72 \cdot \frac{BTU}{lbm}$$

$$P_{amb} := 14.7 \frac{lbf}{in^2}$$ Atmospheric Pressure, assumed

$$T_{amb} := 72.69 \,°F$$ Ambient temp, measured

$$T_{amb} = 295.756 \,K$$

$$H_4 := 114.16 \frac{BTU}{lbm}$$

$$H_{4\_3s} := 73.62 \frac{BTU}{lbm}$$ Enthalpy at motor exit assuming isentropic expansion from motor inlet pressure LabView calculated

$$T_{std} := 300K$$ Assumed standard temp for available air power

$$\rho_3 := .47 \frac{lbm}{ft^3} \qquad \rho_4 := .12 \frac{lbm}{ft^3}$$

Interpolation Methods

Interpolation relation for 2D interpolations. Helmuth Spath's "Two dimensional Spline Interpolation Algorithms" 1995 ISBN 1-56881-017-2, pg 16 formula 2.3.

$$g(x,y,x_1,y_1,x_2,y_2,z_{11},z_{12},z_{21},z_{22}) := z_{11} \cdot \frac{(x-x_2)\cdot(y-y_2)}{(x_1-x_2)\cdot(y_1-y_2)} + z_{12} \cdot \frac{(x-x_2)\cdot(y-y_1)}{(x_1-x_2)\cdot(y_2-y_1)} \ldots$$
$$+ \left[z_{21} \cdot \frac{(x-x_1)\cdot(y-y_2)}{(x_2-x_1)\cdot(y_1-y_2)} \right] + z_{22} \cdot \frac{(x-x_1)\cdot(y-y_1)}{(x_2-x_1)\cdot(y_2-y_1)}$$

Interpolation with data From Vargaftik Table, requires using three linear interpolations for entropy based data because entropy change is not equal between data points. This calculation performs single linear interpolation at 1 Bar for ambient pressures because atmospheric pressure is so close to 1 bar. Note that, for expansion based efficiencies, theoretical entropy is at the lower limit of the 1 bar data (approaching absolute zero) where property behavior is erratic. Actual expansion process will absorb heat from ambient; not adhering to the isentropic model.

Linear interpolation relation (1D)

$$h(x,x_0,f_0,x_1,f_1) := \frac{(x-x_0)f_1 - (x-x_0)f_0}{(x_1-x_0)} + f_0$$

The nomenclature used for the isentropic derived states used in this analysis is as follows:

H.A_Bs, where A is the location of the property and B is the location where the isentropic process is considered from. For example for enthalpy considering isentropic expansion across the motor (from location 3 to 4) the enthalpy would be expressed as H.4_3s.

Motor/gen Efficiency-Isentropic expansion Basis

Efficiency is determined considering an ideal case of single stage Isentropic expansion from motor inlet pressure to motor outlet pressure. Isothermal efficiency will be calculated next.

$$M_{dot1\_scfm} := \frac{M_{dot1}}{\left(.076 \frac{lbm}{ft^3}\right)}$$

$M_{dot1\_scfm} = 23.947 \cdot \frac{ft^3}{min}$ Motor flow in SCFM,

$$Ideal\_mtr\_pwr := M_{dot1} \cdot \left(H_3 - H_{4\_3s}\right)$$

$$Ideal\_mtr\_pwr = 1.59 \times 10^3 \, W$$

$$\eta_{motor\_gen\_isoS} := \frac{DC\_pwr}{Ideal\_mtr\_pwr}$$

$$\eta_{motor\_gen\_isoS} = 25.828 \cdot \%$$

Motor/generator Efficiency-Isothermal Basis

Theoretical power required for Isothermal compression can be calculated by assuming an ideal gas model relation and using a integral to calculate the PV work. This calculation is performed using motor inlet temperature, average temperature and motor outlet temperature to show how temperature will effect theorretical performance.

$$R_{air} := 53.33 \frac{ft \cdot lbf}{lbm \cdot R}$$ $$P(V,T) := \frac{R_{air} \cdot T}{V}$$ $$T_{avg} := \frac{T_3 + T_4}{2}$$

$$P_i := P_3$$ $$P_f := P_4$$

$$V_i(T) := \frac{R_{air} \cdot T}{P_i}$$ $$V_f(T) := \frac{R_{air} \cdot T}{P_f}$$

Initial and final volumes for the temperatures evaluted

$$T_3/°F = 58.42$$

$$T_4/°F = 18.38$$

$$V_i(T_3) = 2.55 \cdot \frac{ft^3}{lbm} \qquad V_i(T_{avg}) = 2.451 \cdot \frac{ft^3}{lbm} \qquad V_i(T_4) = 2.353 \cdot \frac{ft^3}{lbm}$$

$$V_f(T_3) = 28.342 \cdot \frac{ft^3}{lbm} \qquad V_f(T_{avg}) = 27.247 \cdot \frac{ft^3}{lbm} \qquad V_f(T_4) = 26.151 \cdot \frac{ft^3}{lbm}$$

Note that specifi volume is not significantly effected by the significant 40F temperature difference between T3 and T4.

$$IsoT_{pwr\_t3} := \int_{2.55}^{28.342} M_{dot1} \cdot P(V, T_3) \, dV \qquad IsoT_{pwr\_t3} = 2.737 \times 10^3 \cdot W$$

$$\eta_{mtr\_gen\_isoT3} := \frac{DC\_pwr}{IsoT_{pwr\_t3}}$$

$$IsoT_{pwr\_tavg} := \int_{2.451}^{27.247} M_{dot1} \cdot P(V, T_{avg}) \, dV \qquad IsoT_{pwr\_tavg} = 2.631 \times 10^3 \cdot W$$

$$\eta_{mtr\_gen\_isoTavg} := \frac{DC\_pwr}{IsoT_{pwr\_tavg}}$$

$$IsoT_{pwr\_t4} := \int_{2.55}^{28.342} M_{dot1} \cdot P(V, T_4) \, dV \qquad IsoT_{pwr\_t4} = 2.525 \times 10^3 \cdot W$$

$$\eta_{mtr\_gen\_isoT4} := \frac{DC\_pwr}{IsoT_{pwr\_t4}}$$

$$\eta_{mtr\_gen\_isoT3} = 15.003 \cdot \%$$

$$\eta_{mtr\_gen\_isoTavg} = 15.605 \cdot \%$$

$$\eta_{mtr\_gen\_isoT4} = 16.26 \cdot \%$$

Motor efficiency considering an Isothermal expansion model. The isothermal model can be approached by a multistage expansion which includes interheating between expansion stages or by a slow process that is always in near equilibrium with the environment. Transportation industry air motors attempt to achieve Isothermal expansion by using heat exhangers or fuel driven heat sources. It is evidenced that the test bed motor is not achieving good isothermal expansion efficiency

The amount of heat required for isothermal expansion is calculated below. Future work may include adding a heat source to the test bed to determine how effective and efficient heat addition is for achieving an isothermal expansion. Reference hand calcualtion in appendix for development of the following relations.

$$S_{4\_theo} := \frac{IsoT_{pwr\_tavg}}{M_{dot1} \cdot T_{amb}} + S_3$$

$$S_{4\_theo} = 1.664 \cdot \frac{BTU}{lbm \cdot R}$$

Additional heat to obtain an isothermal process is:

$$Q_{add\_isoT} := M_{dot1} \cdot (S_{4\_theo} - S_4) \cdot T_{amb}$$

$$Q_{add\_isoT} = 1.098 \times 10^3 \, W$$

Hand calcualtion in the appendix also attempts to predict actual motor shaft power considering known entropy and temperature of motor inlet and outlet. This may allow decoupling of generator and motor efficiencies.

$$Pwr_{mtr\_shft} := M_{dot1} \left[T_{amb} \cdot (S_4 - S_3) - (U_4 - U_3) \right]$$

$$Pwr_{mtr\_shft} = 1.211 \times 10^3 \, W$$

$$\eta_{mtr} := \frac{Pwr_{mtr\_shft}}{IsoT_{pwr\_t3}}$$

$$\eta_{mtr} = 44.271 \cdot \%$$

Motor Efficiency based on exit properties of air, considering Isothermal expansion from motor inlet temperature. This number represents best case expansion power based on actual motor properties.

These results show that generator efficiency may be significantly less than 80%; although the motor shaft power predicted by the air properties analysis is significantly higher than the rated output power of the motor (.694 kW). Motor internal friction and other losses likely effect true motor shaft power.

Motor/gen efficiency based on motor inlet/outlet conditions. This calculation considers mechanical losses in the motor/gen system. It considers a 100% efficient baseline of actual process air expansion properties.

$$\eta_{mtr\_gen\_mech} := \frac{DC\_pwr}{Pwr_{mtr\_shft}}$$

$$\eta_{gen} := \frac{DC\_pwr}{Pwr_{mtr\_shft}}$$

$\eta_{mtr\_gen\_mech} = 33.889 \cdot \%$ Motor/Generator efficiency based on motor air properties (assumes 100% mechanical motor efficiency.

$\eta_{mtr} \cdot \eta_{mtr\_gen\_mech} = 15.003 \cdot \%$ Verification that this matches coupled motor/gen efficiency previously calculated.

Calculation_016_rev- Motor/Generator Efficiency

Calculation 16 Appendix

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ENERGY BALANCE TO DETERMINE REQUIRED HEAT FOR
AN ISOTHERMAL EXPANSION & TO ESTIMATE ACTUAL
HEAT TRANSFER THROUGH THE AIR MOTOR.
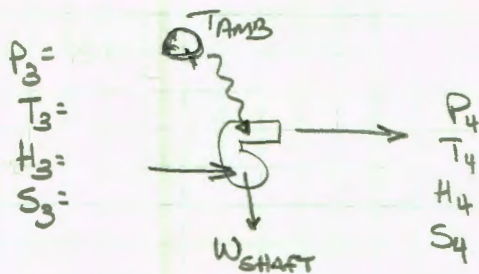
ASSUMPTIONS.

1) 80% GENERATOR EFFICIENCY, GENERATOR POWER IS MEASURED
   BUT GENERATOR EFFICIENCY HAS NOT BEEN CHARACTERIZED
   THE 80% ASSUMPTION IS BASED ON A HIGH EFFICIENCY 1 HP
   MOTOR.

2) VELOCITY POWER IS NEGLIBLE

3) FRICTION & GEAR TRAIN LOSSES ARE NEGLIBLE

4) POTENTIAL ENERGY CHANGES DUE TO HEIGHT ARE NEGLIBLE

$$E_{IN} = E_{OUT}$$

$$\dot{m}(H_3) + \dot{Q}_{IN} = \dot{W}_{OUT} + \dot{m}(H_4) \quad (1)$$

$\dot{m}, H_3, H_4$ ARE CALCULATED IN THE LABVIEW PROGRAM

MOTOR POWER IS DETERMINED FROM MEASURED POWER OUT &
ASSUMED 80% EFF

$$(2) \quad \eta_{GEN} = \frac{DC \; PWR}{MTR \; PWR} \implies \begin{array}{l} \text{MATHCAD FOR EVALUATION} \\ \text{TO DETERMINE MTR SHAFT} \\ \text{POWER} \end{array}$$

HEAT IS CALCULATED CONSIDERING ENTROPY CHANGE
OF THE PROCESS AIR CONSIDERING AMBIENT ENVIRONMENT
AS A HEAT RESEVOIR AT AMBIENT TEMPERATURE.

FROM MORAN & SHAPIRO FUNDAMENTALS OF ENGINEERING
THERMODYNAMICS, THIRD EDITION. PGS 201-209 DEFINITION OF
ENTROPY, CLAUSIUS INEQUALITY, AN TdS RELATIONS

$$S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} \bigg)_{INT \; REV} \qquad EQN \; 6.7 \qquad \begin{array}{ll} TdS = dH - Vdp & 6.16 \\ TdS = dU + pdV & 6.15 \end{array}$$

THE HEAT TRANSFER TO THE AIR OCCURS THROUGH THE
AIR MOTOR AND THEN BACK TO THE AMBIENT
RESERVOIR WHEN THE AIR IS DISCHARGED. THIS
IS NOT A REVERSIBLE PROCESS BECAUSE WORK IS
PRODUCED IN THE FORM OF SHAFT POWER.

EQN 6.15 CAN BE USED TO DETERMINE THE THEORETICAL
HEAT TRANSFER RATE REQUIRED FOR ISOTHERMAL
EXPANSION

$$T dS = dU + p dV$$

↑ AMBIENT TEMP (CONSTANT)

↑ CHANGE IN INTERNAL ENERGY

↖ THEORETICAL CONSTANT TEMP EXPANSION WORK
(CALCULATED PREVIOUSLY IN CALC0115)

$$T \int dS = \int du + \int p dV \quad (3)$$

$$T_{AMB} \Delta S = \Delta u_{Air} + \text{THEORETICAL EXP. WORK}$$

↑ FOR CONSTANT TEMP $\Delta u = 0$

$$T_{AMB} \Delta S_{THEORETICAL} = \text{THEORETICAL EXP. WORK.}$$

THIS CAN BE USED TO DETERMINE EXPECTED ENTROPY
STATE FOR AN IDEAL ISOTHERMAL PROCESS

$$S_{4_{THEO}} = \frac{\text{THEO. EXP WORK}}{T_{AMB}} + S_3$$

← USE ON (PER FLOW RATE BASIS)

THE ADDITIONAL HEAT REQUIRED TO OBTAIN AN ISOTHERMAL
PROCESS CAN BE PREDICTED BY LOOKING AT ACTUAL S_4

$$Q_{ADD} = \left(S_{4_{THEO}} - S_{4_{ACT}} \right) T_{AMB}$$

USING TEST DATA IN EQUATION (3) WE CAN PREDICT
MOTOR EXPANSION WORK, POSSIBLY SEPERATING THE CURRENT
PROBLEM OF COUPLED MOTOR/GENERATOR EFFICIENCIES.

$$T_{AMB} \left(S_4 - S_3 \right) = \left(U_4 - U_3 \right) + \text{MTR SHAFT POWER}$$

$$\begin{matrix} \text{PREDICTED} \\ \text{MOTOR SHAFT POWER} \end{matrix} = T_{AMB} \left(S_4 - S_3 \right) - \left(U_4 - U_3 \right)$$

THESE RELATIONS ARE EVALUATED IN MATHCAD

Table of Contents
Calculation '18A'- Low Pressure CAES 3 Stage Polytropic Efficiency

A theoretical polytropic process using a coefficient of 1.2 probably more closely resembles an actual compression or expansion process than the isothermal or isentropic processes analyzed previously. The polytropic process will be modeled in compression and expansion considering a three stage process with an even pressure ratio between stages.This Calculation determines theoretical efficiencies of 3 stage polytropic compression and expansion processes associated with compressed air. It analyzes an open system with closed storage vessel using an Ideal gas model. Both a high pressure CAES system and a low pressure CAES system are considered under different file names. All energy expressions are on a per mass basis, titled as work terms, which they would become when multiplied by an air mass.

Low Pressure CAES (600 psig)

Unit Conversion, Constants, and Thermodynamic State Determination

$$Bar := 14.5038 \frac{lbf}{in^2} \qquad kJ := 1000J \qquad \gamma_{air} := 1.4 \qquad \text{Heat capacity ratio for air}$$

2) Interpolation Methods

Interpolation relation for 2D interpolations. Helmuth Spath's "Two dimensional Spline Interpolation Algorithms" 1995 ISBN 1-56881-017-2, pg 16 formula 2.3.

$$g(x,y,x_1,y_1,x_2,y_2,z_{11},z_{12},z_{21},z_{22}) := z_{11} \cdot \frac{(x-x_2)\cdot(y-y_2)}{(x_1-x_2)\cdot(y_1-y_2)} + z_{12} \cdot \frac{(x-x_2)\cdot(y-y_1)}{(x_1-x_2)\cdot(y_2-y_1)} \ldots$$
$$+ \left[z_{21} \cdot \frac{(x-x_1)\cdot(y-y_2)}{(x_2-x_1)\cdot(y_1-y_2)} \right] + z_{22} \cdot \frac{(x-x_1)\cdot(y-y_1)}{(x_2-x_1)\cdot(y_2-y_1)}$$

Interpolation with data From Vargaftik Table, requires using three linear interpolations for entropy based data because entropy change is not equal between data points. This calculation performs single linear interpolation at 1 Bar for ambient pressures because atmospheric pressure is so close to 1 bar. Note that, for expansion based efficiencies, theoretical entropy is at the lower limit of the 1 bar data (approaching absolute zero) where property behavior is erratic. Actual expansion process will absorb heat from ambient; not adhering to the isentropic model.

Linear interpolation relation (1D)

$$h(x,x_0,f_0,x_1,f_1) := \frac{(x-x_0)f_1 - (x-x_0)f_0}{(x_1-x_0)} + f_0$$

The nomenclature used for the isentropic derived states used in this analysis is as follows:

H.A_Bs, where A is the location of the property and B is the location where the isentropic process is considered from. For example for enthalpy considering isentropic expansion across the motor (from location 3 to 4) the enthalpy would be expressed as H.4_3s.

Low Pressure CAES (600 psig); Energy Industry minimum pressure

Ambient Conditions:

$$P_{amb} := 14.7 \frac{lbf}{in^2} \qquad T_{amb} := 75\,°F \qquad\qquad P_3 := 614.7 \frac{lbf}{in^2} \qquad T_3 := T_{amb}$$

$$R_{air} := 53.33 \frac{ft \cdot lbf}{lbm \cdot R} \qquad T_{amb} = 297.039 \cdot K$$

$$v(P,T) := \frac{R_{air} \cdot T}{P} \qquad \text{Ideal gas law used throughout this analysis}$$

specific volume at ambient:

$$v_1 := v(P_{amb}, T_{amb}) \qquad v_1 = 13.47 \cdot \frac{ft^3}{lbm}$$

$$h_{amb} := g\left(\frac{P_{amb}}{Bar}, \frac{T_{amb}}{K}, 1, 290, 2, 300, 290.2, 300.3, 290.0, 300.1 \right) \frac{kJ}{kg} \qquad P_{amb} = 1.014 \cdot bar$$

$$T_{amb} = 297.039 \cdot K$$

$$h_{amb} = 127.819 \cdot \frac{BTU}{lb}$$

Pressure ratio for 600 psig final pressure.

Given

$$(14.7) \cdot PR^3 = 614$$

$$\text{Find}(PR) \rightarrow (3.4696341106673474488 \quad -1.7348170553336737244 + 3.0047912816749512598i \quad -1.734817$$

$$Press_{ratio} := 3.470$$

3) Three Stage Polytropic Compression

The process is considered using a coefficent of 1.2 for all three compressions. It is assumed that there is constant pressure heat removal to ambient temperature between compression stages and a final constant pressure heat removal to ambient before the air enters storage. A diagram of the process is included in the Appendix.

 Polytropic process for an ideal gas (n=1.2) is used to determine T.2, formula 3.56 in Shapiro's Fundamentals of Engineering thermodynamics. State 2 is the title given to the 1st stage exit pressure during polytropic compression.

Stage 1

$$P_2 := Press_{ratio} \cdot P_{amb} \qquad P_2 = 51.009 \cdot \frac{lbf}{in^2} \qquad \text{Pressure ratio used to determine P2}$$

$$T_2 := T_{amb} \cdot \left(\frac{P_2}{P_{amb}} \right)^{\frac{1.2-1}{1.2}} \qquad T_2 = 198.201 \cdot °F \qquad T_2 = 365.484 \ K$$

Work relation is for a polytropic process using eqn 6.66a in Shapiro's Fundamentals of Engineering thermodynamics

$$W_{1\_2} := \frac{-1 \cdot 1.2 \cdot R_{air}}{1.2 - 1} \cdot \left(T_2 - T_{amb} \right) \qquad W_{1\_2} = -50.66 \cdot \frac{BTU}{lbm}$$

Work for stage 1 polytropic compression

Heat removed during polytropic compression is calculated below

$$h_2 := g\left(\frac{P_2}{Bar}, \frac{T_2}{K}, 3, 350, 4, 375, 350.3, 375.6, 350.2, 375.5 \right) \frac{kJ}{kg} \qquad \begin{matrix} P_2 = 3.517 \cdot Bar \\[4pt] T_2 = 365.484 \ K \end{matrix}$$

$$h_2 = 157.316 \cdot \frac{BTU}{lbm} \qquad h_{amb} = 127.819 \cdot \frac{BTU}{lbm}$$

$$Q_{1\_2} := \left[W_{1\_2} + \left(h_2 - h_{amb} \right) \right]$$

$$Q_{1\_2} = -21.162 \cdot \frac{BTU}{lbm} \qquad \text{This is heat out of the system during polytropic compression}$$

Heat removal after stage 1 compression is calculated below; assuming a constant pressure. No work is completed, velocity and gravity effects are negligible, therefore a first law analysis simplifies to the following relation

$$P_3 := P_2 \qquad T_3 := T_{amb}$$

$$h_3 := g\left(\frac{P_3}{Bar}, \frac{T_3}{K}, 3, 290, 4, 300, 289.7, 299.8, 289.5, 299.6\right)\frac{kJ}{kg}$$

$$P_3 = 3.517 \cdot bar$$

$$T_3 = 297.039 \ K$$

$$Q_{2\_3} := h_3 - h_2$$

$$h_3 = 127.561 \cdot \frac{BTU}{lbm}$$

$$Q_{2\_3} = -29.756 \cdot \frac{BTU}{lbm}$$ This is intercooling heat transfer (out of process air)

Both heat losses are summed to achieve the total heat loss for polytropic compression

$$Q_{1\_3\_poly} := Q_{1\_2} + Q_{2\_3}$$

$$Q_{1\_3\_poly} = -50.918 \cdot \frac{BTU}{lbm}$$ Total heat loss
Stage 1 polytropic compression

Stage 2

$$P_4 := Press_{ratio} \cdot P_2 \qquad P_4 = 177.001 \cdot \frac{lbf}{in^2}$$ Pressure ratio used to determine P4

$$T_4 := T_3 \cdot \left(\frac{P_4}{P_3}\right)^{\frac{1.2-1}{1.2}}$$

$$T_4 = 198.201 \cdot °F \quad T_4 = 365.484 \ K$$

Note because pressure ratio is the same, and the ideal gas law is assumed; the temperatures leaving the compressor are the same as for the first stage

Work relation is for a polytropic process using eqn 6.66a in Shapiro's Fundamentals of Engineering thermodynamics

$$W_{3\_4} := \frac{-1 \cdot 1.2 \cdot R_{air}}{1.2 - 1} \cdot \left(T_4 - T_3 \right) \qquad\qquad W_{3\_4} = -50.66 \cdot \frac{BTU}{lbm}$$

Work for stage 2 polytropic compression, same as stage 1 due to same pressure ratio and the ideal gas assumption

Heat removed during stage 2 polytropic compression is calculated below

$$h_4 := g\left(\frac{P_4}{Bar}, \frac{T_4}{K}, 10, 350, 15, 375, 349.2, 374.7, 348.4, 374.0 \right) \frac{kJ}{kg} \quad P_4 = 12.204 \cdot Bar$$

$$T_4 = 365.484 \text{ K}$$

$$h_4 = 156.779 \cdot \frac{BTU}{lbm}$$

$$Q_{3\_4} := \left[W_{3\_4} + \left(h_4 - h_3 \right) \right]$$

$$Q_{3\_4} = -21.441 \cdot \frac{BTU}{lbm}$$ This heat out of the system during stage 2 polytropic compression

Heat removal after stage 2 compression is calculated below; assuming a constant pressure. No work is completed, velocity and gravity effects are negligible, therefore a first law analysis simplifies to the following relation

$$P_5 := P_4 \qquad T_5 := T_{amb}$$

$$h_5 := g\left(\frac{P_5}{Bar}, \frac{T_5}{K}, 10, 290, 15, 300, 288, 298.3, 286.8, 297.1 \right) \frac{kJ}{kg} \quad P_5 = 12.204 \cdot bar$$

$$T_5 = 297.039 \text{ K}$$

$$Q_{4\_5} := h_5 - h_4$$

$$Q_{4\_5} = -30.072 \cdot \frac{BTU}{lbm}$$

Both heat losses are summed to achieve the total heat loss for polytropic compression

$$Q_{3\_5} := Q_{3\_4} + Q_{4\_5} \qquad\qquad Q_{3\_5} = -51.513 \cdot \frac{BTU}{lbm} \qquad$$ Total heat loss
Stage 2 polytropic compression

Stage 3

$$P_6 := Press_{ratio} \cdot P_4 \qquad P_6 = 614.194 \cdot \frac{lbf}{in^2} \qquad \text{Pressure ratio used to determine P6}$$

$$T_6 := T_5 \cdot \left(\frac{P_6}{P_5}\right)^{\frac{1.2-1}{1.2}} \qquad T_6 = 198.201 \cdot °F \quad T_6 = 365.484 \text{ K}$$

Note because pressure ratio is the same, and the ideal gas law is assumed; the temperatures leaving the compressor are the same as for the other stages

Work relation is for a polytropic process using eqn 6.66a in Shapiro's Fundamentals of Engineering thermodynamics

$$W_{5\_6} := \frac{-1 \cdot 1.2 \cdot R_{air}}{1.2 - 1} \cdot \left(T_6 - T_5\right) \qquad W_{5\_6} = -50.66 \cdot \frac{BTU}{lbm}$$

Work for stage 3 polytropic compression

Heat removed during stage 3 polytropic compression is calculated below

$$h_6 := g\left(\frac{P_6}{Bar}, \frac{T_6}{K}, 40, 350, 45, 375, 344.6, 370.8, 343.9, 370.2\right) \frac{kJ}{kg}$$

$$P_6 = 42.347 \cdot Bar$$

$$T_6 = 365.484 \text{ K}$$

$$h_6 = 154.999 \cdot \frac{BTU}{lbm}$$

$$Q_{5\_6} := \left[W_{5\_6} + \left(h_6 - h_5\right)\right]$$

$$Q_{5\_6} = -22.368 \cdot \frac{BTU}{lbm} \quad \text{This is heat out of the system during stage 3 polytropic compression}$$

Heat removal after stage 3 compression is calculated below; assuming a constant pressure. No work is completed, velocity and gravity effects are negligible, therefore a first law analysis simplifies to the following relation

$$P_{storage} := P_6 \qquad T_{storage} := T_{amb}$$

$$P_{storage} = 42.347 \cdot bar \qquad T_{storage} = 297.039 \ K$$

$$h_{storage} := g\left(\frac{P_{storage}}{Bar}, \frac{T_{storage}}{K}, 40, 290, 45, 300, 281, 291.7, 279.9, 290.7\right)\frac{kJ}{kg}$$

$$Q_{6\_storage} := h_{storage} - h_6$$

$$Q_{6\_storage} = -31.16 \cdot \frac{BTU}{lbm}$$

Both heat losses are summed to achieve the total heat loss for polytropic compression

$$Q_{5\_storage} := Q_{6\_storage} + Q_{5\_6} \qquad Q_{5\_storage} = -53.529 \cdot \frac{BTU}{lbm}$$

Total heat loss
Stage 3 polytropic
compression

Total Work Input for three stage Polytropic compression

$$W_{3\_stage\_cmp} := W_{1\_2} + W_{3\_4} + W_{5\_6}$$

$$W_{3\_stage\_cmp} = -151.979 \cdot \frac{BTU}{lbm}$$

Total heat removal during compressions

$$Q_{cmp\_process} := Q_{1\_2} + Q_{3\_4} + Q_{5\_6}$$

$$Q_{cmp\_process} = -64.971 \cdot \frac{BTU}{lbm}$$

Total heat removed by Heat Exchangers

$$Q_{cmp\_heatX} := Q_{2\_3} + Q_{4\_5} + Q_{6\_storage}$$

$$Q_{cmp\_heatX} = -90.988 \cdot \frac{BTU}{lbm}$$

Total heat removed for three stage polytropic compression

$$Q_{3\_stage\_cmp} := Q_{cmp\_process} + Q_{cmp\_heatX}$$

$$Q_{3\_stage\_cmp} = -155.96 \cdot \frac{BTU}{lbm}$$

Three Stage Polytropic Expansion

The process is considered using a coeffcient of 1.2 for all three expansions. It is assumed that there is constant pressure heat addition to ambient temperature between expansion stages. A diagram of the process is included in the Appendix.

Polytropic process for an ideal gas (n=1.2) is used to determine T.5_poly., formula 3.56 in Shapiro's Fundamentals of Engineering thermodynamics

$$T_{storage} = 75 \cdot °F \qquad T_{storage} = 297.039\ K \qquad P_{storage} = 42.347 \cdot bar$$

$$T_8 := T_{storage} \cdot \left(\frac{1}{Press_{ratio}} \right)^{\frac{1.2-1}{1.2}}$$

$$T_8 = -25.129 \cdot °F$$

$$T_{10} := T_8$$

$$T_8 = 241.412\ K$$

$$T_{12} := T_8$$

Temperatures for other expansions are the same because there is heat addition to ambient b/t stages.

Because the pressure ratio is the same for all stages and the air is heated to ambient between stages the work is the same for all stages

$$W_{storage\_8} := \left[\frac{1.2 \cdot R_{air}}{1.2 - 1} \cdot \left(T_8 - T_{storage} \right) \right] \cdot -1$$

$$W_{storage\_8} = 41.173 \cdot \frac{BTU}{lbm} \qquad \text{Power out for each stage of polytropic process}$$

$$W_{9\_10} := W_{storage\_8} \qquad W_{11\_12} := W_{storage\_8}$$

This process also requires significant heat from the environment or another heat source. Requirement is calculated below.

$$h_{storage} = 123.838 \cdot \frac{BTU}{lbm} \qquad\qquad P_8 := \frac{P_{storage}}{Press_{ratio}}$$

$$T_8 = 241.412\ K \qquad\qquad T_8 = -25.129 \cdot °F \qquad P_8 = 12.204 \cdot bar$$

$$h_8 := g\left(\frac{P_8}{Bar}, \frac{T_8}{K}, 10, 240, 15, 250, 236.7, 247.0, 235, 245.4\right)\frac{kJ}{kg}$$

$$h_8 = 102.068 \cdot \frac{BTU}{lbm}$$

$$Q_{storage\_8} := \left[W_{storage\_8} + \left(h_8 - h_{storage}\right)\right]$$

$$Q_{storage\_8} = 19.402 \cdot \frac{BTU}{lbm} \qquad$$ Heat required (heat in) for first polytropic expansion stage (during process)

Heat addition requirement after stage 1 expansion is calculated below; assuming a constant pressure. No work is completed, velocity and gravity effects are negligible, therefore a first law analysis simplifies to the following relation

$$P_9 := P_8 \qquad T_9 := T_{amb}$$

$$h_9 := g\left(\frac{P_9}{Bar}, \frac{T_9}{K}, 10, 290, 15, 300, 288, 298.3, 286.8, 297.1\right)\frac{kJ}{kg} \qquad\qquad P_9 = 12.204 \cdot bar$$

$$T_9 = 297.039\ K$$

$$Q_{8\_9} := h_9 - h_8 \qquad\qquad h_9 = 126.707 \cdot \frac{BTU}{lbm}$$

$$Q_{8\_9} = 24.639 \cdot \frac{BTU}{lbm} \qquad$$ Interstage heating requirement

Both heat requirements are summed to achieve the total heat addition required for stage1 polytropic expansion

$$Q_{storage\_9} := Q_{storage\_8} + Q_{8\_9}$$

$$Q_{storage\_9} = 44.041 \cdot \frac{BTU}{lbm} \qquad$$ Total heat requirement Stage 1 polytropic expansion

Stage 2

$$W_{9\_10} = 41.173 \cdot \frac{BTU}{lbm} \qquad T_{10} = 241.412 \text{ K}$$

This process also requires significant heat from the environment or another heat source. Requirement is calculated below.

$$h_9 = 126.707 \cdot \frac{BTU}{lbm} \qquad\qquad P_{10} := \frac{P_9}{Press_{ratio}}$$

$$T_{10} = 241.412 \text{ K} \qquad\qquad T_{10} = -25.129 \cdot °F \qquad P_{10} = 3.517 \cdot bar$$

$$h_{10} := g\left(\frac{P_{10}}{Bar}, \frac{T_{10}}{K}, 3, 240, 4, 250, 239.2, 249.3, 238.9, 249.0\right) \frac{kJ}{kg}$$

$$h_{10} = 103.384 \cdot \frac{BTU}{lbm}$$

$$Q_{9\_10} := \left[W_{9\_10} + \left(h_{10} - h_9\right)\right]$$

$$Q_{9\_10} = 17.849 \cdot \frac{BTU}{lbm}$$ Heat required (heat in) for second polytropic expansion stage

Heat addition requirement after stage 2 expansion is calculated below; assuming a constant pressure. No work is completed, velocity and gravity effects are negligible, therefore a first law analysis simplifies to the following relation

$$P_{11} := P_{10} \qquad T_{11} := T_{amb}$$

$$h_{11} := g\left(\frac{P_{11}}{Bar}, \frac{T_{11}}{K}, 3, 290, 4, 300, 289.7, 299.8, 289.5, 299.6\right) \frac{kJ}{kg} \qquad P_{11} = 3.517 \cdot bar$$

$$T_{11} = 297.039 \text{ K}$$

$$Q_{10\_11} := h_{11} - h_{10} \qquad\qquad h_{11} = 127.561 \cdot \frac{BTU}{lbm}$$

$$Q_{10\_11} = 24.177 \cdot \frac{BTU}{lbm}$$

Both heat requirements are summed to achieve the total heat addition required for stage 2 polytropic expansion

$$Q_{9\_11} := Q_{9\_10} + Q_{10\_11}$$

$$Q_{9\_11} = 42.026 \cdot \frac{BTU}{lbm}$$ Total heat addition requirement Stage 2 polytropic expansion

Stage 3

$$W_{11\_12} = 41.173 \cdot \frac{BTU}{lbm} \quad T_{12} = 241.412\,K$$

This process also requires significant heat from the environment or another heat source. Requirement is calculated below.

$$h_{11} = 127.561 \cdot \frac{BTU}{lbm} \qquad\qquad P_{12} := \frac{P_{11}}{Press_{ratio}}$$

$$T_{12} = 241.412\,K \qquad\qquad T_{12} = -25.129 \cdot °F \qquad P_{12} = 1.014 \cdot bar$$

$$h_{12} := g\left(\frac{P_{12}}{Bar}, \frac{T_{12}}{K}, 1, 240, 2, 250, 239.9, 250.0, 239.6, 249.7\right)\frac{kJ}{kg}$$

$$h_{12} = 103.75 \cdot \frac{BTU}{lbm}$$

$$Q_{11\_12} := \left[W_{11\_12} + \left(h_{12} - h_{11} \right) \right]$$

$$Q_{11\_12} = 17.362 \cdot \frac{BTU}{lbm}$$ Heat required (heat in) for first polytropic expansion stage

Heat addition is not required after the last expansion. This air could be used for a cooling process, if there was a need.

Total Work for 3 stage expansion is calculated below

$$W_{3\_stage\_exp} := W_{storage\_8} + W_{9\_10} + W_{11\_12}$$

$$W_{3\_stage\_exp} = 123.518 \cdot \frac{BTU}{lbm}$$

Total Heat addition required for 3 stage expansion (during process):

$$Q_{3\_stage\_exp} := Q_{storage\_8} + Q_{9\_10} + Q_{11\_12}$$

$$Q_{3\_stage\_exp} = 54.613 \cdot \frac{BTU}{lbm}$$

Total Heat addition through the heat exchangers is:

$$Q_{3\_stage\_exp\_heatX} := Q_{8\_9} + Q_{10\_11}$$

$$Q_{3\_stage\_exp\_heatX} = 48.816 \cdot \frac{BTU}{lbm}$$

$$Q_{total\_3\_stage\_exp} := Q_{3\_stage\_exp\_heatX} + Q_{3\_stage\_exp\_heatX}$$

$$Q_{total\_3\_stage\_exp} = 97.631 \cdot \frac{BTU}{lbm}$$

Efficiency Determinations.

Efficiencies are determined using Excel, using different combinations of compression and expansion processes. Heat removal and addition requirements are shown in the Excel table; but are not influential in the theoretical efficiency number. The heating /cooling requirements help one understand what it would take to achieve these theoretical efficiencies. Efficiency is determined by the ratio of Power out to Power in.

Example: For 3 stage polytropic compression and expansion the maximum theoretical system efficiency is:

$$W_{3\_stage\_cmp} = -151.979 \cdot \frac{BTU}{lbm} \qquad W_{3\_stage\_exp} = 123.518 \cdot \frac{BTU}{lbm}$$

$$T_{12} = -25.129 \cdot °F$$

$$\eta_{IsoT\_Reg\_poly} := \frac{W_{3\_stage\_exp}}{-W_{3\_stage\_cmp}}$$

$$\eta_{IsoT\_Reg\_poly} = 81.273 \cdot \% \qquad \text{Maximum theoretical efficiency for 3 stage polytropic expansion and compression}$$

Other combinations are calculated in Excel with data from calculation 15.

Theoretical System Efficiencies- Low Pressure (600 psig) CAES					
ALL ENERGY AND HEAT IN BTU/LBM, ALL EFFCIEINCIES ARE % (W_OUT/W_IN)					
COMPRESSION EXPANSION		ISENTROPIC T.h= 1,921 F Q= -368 btu/lbm	SINGLE STAGE POLYTROPIC T.h= 668 F Q= -207 btu/lbm	THREE STAGE POLYTROPIC T.h= 198 F Q= -156 btu/lbm	ISOTHERMAL T.h= 75 F Q= -141 btu/lbm
	CMP ENERGY EXP ENERGY	-443	-244	-152	-137
ISENTROPIC (600 PSIG) T.c= -276F Q= 0	72	16%	30%	47%	53%
REGULATED (300 PSIG) ISENTROPIC T.c= -240F Q= 0	72	16%	30%	47%	53%
REGULATED (300 PSIG) ISENTROPIC W/REHEAT T.c= -236F Q= 3 btu/lbm	73	16%	30%	48%	53%
POLYTROPIC (600 PSIG) T.c= -206F Q= 52 btu/lbm	115	26%	47%	76%	84%
REGULATED (300 PSIG) POLYTROPIC T.c= -174F Q= 41 btu/lbm	97	22%	40%	64%	71%
REGULATED (300 PSIG) POLYTROPIC W/REHEAT T.c= -167F Q= 46 btu/lbm	100	23%	41%	66%	73%
THREE STAGE POLYTROPIC T.c= -43 F Q= 98 btu/lbm	125	28%	51%	82%	91%
REGULATED (300 PSIG) ISOTHERMAL T.c=62F Q= 109 btu/lbm	108	24%	44%	71%	79%
REGULATED (300 PSIG) ISOTHERMAL W/REHEAT T.c=75F Q=114 btu/lbm	111	25%	45%	73%	81%
ISOTHERMAL (600 PSIG) T.c=75F Q= 141 btu/lbm	137	31%	56%	90%	100%

Calculation 18A - Low Pressure CAES 3 Stage
Polytropic Efficiency
Calculation 18A Appendix

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CALC 18

3 STAGE POLYTROPIC COMPRESSION ANALYSIS

W_{1-2} W_{3-4} W_{5-6}

P_1
T_1
(AMBIENT)

Q_{12} Q_{23} Q_{34} Q_{45} Q_{56}

P_6
T_6

$P_{STORAGE}$
$T_{STORAGE}$
(AMBIENT)

$Q_{6\_STORAGE}$

3 STAGE EXPANSION

P_5
T_5 Q_{58} Q_{8-9} Q_{9-10} Q_{10-11} Q_{11-12}

W_{5-8} W_{9-10} W_{11-12}

$\Rightarrow$ EXIT AIR
$T_{12} = -42.8F$

FIRST LAW ANALYSIS for REHEAT/INTERCOOL CALCULATIONS

P_1
T_1
h_1

P_2
T_2
h_2
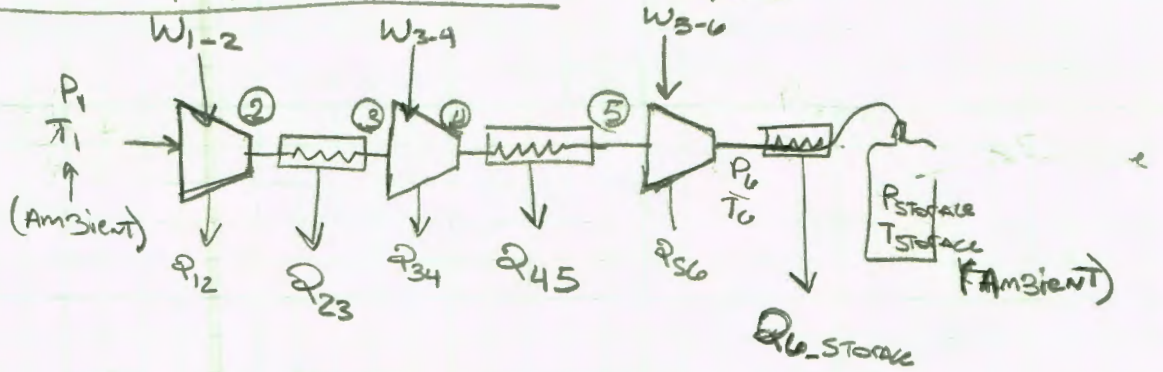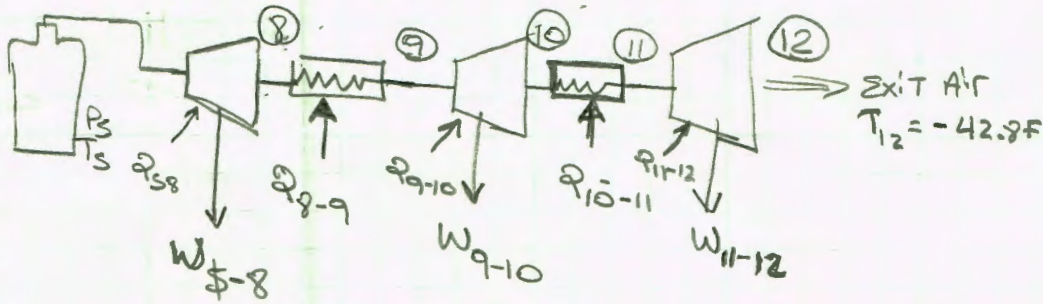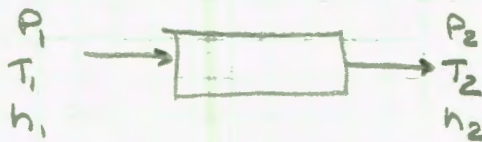
$$\varepsilon_{IN} = \varepsilon_{OUT} \qquad (P_1 = P_2)$$

$$Q_{IN} + \dot{m}(h_1) = \cancel{W_{OUT}}^{0} + \dot{m}(h_2)$$

$$Q_{IN} = h_2 - h_1$$

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Calculation 18B High Pressure CAES 3 Stage Polytropic Efficiency

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A theoretical polytropic process using a coefficient of 1.2 probably more closely resembles an actual compression or expansion process than the isothermal or isentropic processes analyzied previously. The polytropic process will be modeled in compression and expansion considering a three stage process with an even pressure ratio between stages.This Calculation determines theoretical efficiencies of 3 stage polytropic compression and expansion processes associated with compressed air. It analyzes an open system with closed storage vessel using an Ideal gas model. Both a high pressure CAES system and a low pressure CAES system are considered under different file names. All energy expressions are on a per mass basis, titled as work terms, which they would become when multiplied by an air mass.

High Pressure CAES (4500 psig)

Unit Conversion, Constants, and Thermodynamic State Determination

$$\text{Bar} := 14.5038 \frac{\text{lbf}}{\text{in}^2} \qquad \text{kJ} := 1000\text{J} \qquad \gamma_{air} := 1.4 \qquad \text{Heat capacity ratio for air}$$

2) Interpolation Methods

Interpolation relation for 2D interpolations. Helmuth Spath's "Two dimensional Spline Interpolation Algorithms" 1995 ISBN 1-56881-017-2, pg 16 formula 2.3.

$$g(x,y,x_1,y_1,x_2,y_2,z_{11},z_{12},z_{21},z_{22}) := z_{11} \cdot \frac{(x - x_2) \cdot (y - y_2)}{(x_1 - x_2) \cdot (y_1 - y_2)} + z_{12} \cdot \frac{(x - x_2) \cdot (y - y_1)}{(x_1 - x_2) \cdot (y_2 - y_1)} \dots$$
$$+ \left[z_{21} \cdot \frac{(x - x_1) \cdot (y - y_2)}{(x_2 - x_1) \cdot (y_1 - y_2)} \right] + z_{22} \cdot \frac{(x - x_1) \cdot (y - y_1)}{(x_2 - x_1) \cdot (y_2 - y_1)}$$

Interpolation with data From Vargaftik Table, requires using three linear interpolations for entropy based data because entropy change is not equal between data points. This calculation performs single linear interpolation at 1 Bar for ambient pressures because atmospheric pressure is so close to 1 bar. Note that, for expansion based efficiencies, theoretical entropy is at the lower limit of the 1 bar data (approaching absolute zero) where property behavior is erratic. Actual expansion process will absorb heat from ambient; not adhering to the isentropic model.

Linear interpolation relation (1D)

$$h(x,x_0,f_0,x_1,f_1) := \frac{(x - x_0)f_1 - (x - x_0)f_0}{(x_1 - x_0)} + f_0$$

The nomenclature used for the isentropic derived states used in this analysis is as follows:

H.A_Bs, where A is the location of the property and B is the location where the isentropic process is considered from. For example for enthalpy considering isentropic expansion across the motor (from location 3 to 4) the enthalpy would be expressed as H.4_3s.

High Pressure CAES (4500 psig); transportation industry pressure

Ambient Conditions:

$$P_{amb} := 14.7 \frac{lbf}{in^2} \qquad T_{amb} := 75\,°F \qquad\qquad P_3 := 4514.7 \frac{lbf}{in^2} \qquad T_3 := T_{amb}$$

$$R_{air} := 53.33 \frac{ft \cdot lbf}{lbm \cdot R} \qquad T_{amb} = 297.039 \cdot K$$

$$v(P,T) := \frac{R_{air} \cdot T}{P} \qquad \text{Ideal gas law used throughout this analysis}$$

specific volume at ambient:

$$v_1 := v\left(P_{amb}, T_{amb}\right) \qquad v_1 = 13.47 \cdot \frac{ft^3}{lbm}$$

$$h_{amb} := g\left(\frac{P_{amb}}{Bar}, \frac{T_{amb}}{K}, 1, 290, 2, 300, 290.2, 300.3, 290.0, 300.1\right)\frac{kJ}{kg} \qquad P_{amb} = 1.014 \cdot bar$$

$$T_{amb} = 297.039 \cdot K$$

$$h_{amb} = 127.819 \cdot \frac{BTU}{lb}$$

Pressure ratio for 4500 psig final pressure.

Given

$$(14.7) \cdot PR^3 = 4514$$

$$\text{Find}(PR) \rightarrow (6.7465447697576508201 \quad -3.37327238487882541 + 5.8426791583791622133i \quad -3.3732723\text{8}$$

$$\text{Press}_{ratio} := 6.747$$

3) Three Stage Polytropic Compression

The process is considered using a coeffcent of 1.2 for all three compressions. It is assumed that there is constant pressure heat removal to ambient temperature between compression stages and a final constant pressure heat removal to ambient before the air enters storage. A diagram of the process is included in the Appendix.

Polytropic process for an ideal gas (n=1.2) is used to determine T.2, formula 3.56 in Shapiro's Fundamentals of Engineering thermodynamics. State 2 is the title given to the 1st stage exit pressure during polytropic compression.

Stage 1

$$P_2 := Press_{ratio} \cdot P_{amb} \qquad P_2 = 99.181 \cdot \frac{lbf}{in^2}$$ Pressure ratio used to determine P2

$$T_2 := T_{amb} \cdot \left(\frac{P_2}{P_{amb}} \right)^{\frac{1.2-1}{1.2}} \qquad T_2 = 275.302 \cdot °F \qquad T_2 = 408.318 \; K$$

Work relation is for a polytropic process using eqn 6.66a in Shapiro's Fundamentals of Engineering thermodynamics

$$W_{1\_2} := \frac{-1 \cdot 1.2 \cdot R_{air}}{1.2 - 1} \cdot \left(T_2 - T_{amb} \right) \qquad W_{1\_2} = -82.363 \cdot \frac{BTU}{lbm}$$

Work for stage 1 polytropic compression

Heat removed during polytropic compression is calculated below

$$h_2 := g\left(\frac{P_2}{Bar}, \frac{T_2}{K}, 6, 400, 8, 425, 400.7, 426.2, 400.5, 426.0 \right) \frac{kJ}{kg} \qquad \begin{array}{l} P_2 = 6.838 \cdot Bar \\ \\ T_2 = 408.318 \; K \end{array}$$

$$h_2 = 175.882 \cdot \frac{BTU}{lbm}$$

$$Q_{1\_2} := \left[W_{1\_2} + \left(h_2 - h_{amb} \right) \right]$$

$$Q_{1\_2} = -34.301 \cdot \frac{BTU}{lbm}$$ This heat out of the system during polytropic compression

Heat removal after stage 1 compression is calculated below; assuming a constant pressure. No work is completed, velocity and gravity effects are negligible, therefore a first law analysis simplifies to the following relation

$$P_3 := P_2 \qquad T_3 := T_{amb}$$

$$h_3 := g\left(\frac{P_3}{Bar}, \frac{T_3}{K}, 6, 290, 8, 300, 289, 299.2, 288.5, 298.7\right) \frac{kJ}{kg} \qquad \begin{array}{l} P_3 = 6.838 \cdot bar \\[6pt] T_3 = 297.039 \ K \end{array}$$

$$Q_{2\_3} := h_3 - h_2$$

$$Q_{2\_3} = -48.637 \cdot \frac{BTU}{lbm}$$

Both heat losses are summed to achieve the total heat loss for polytropic compression

$$Q_{1\_3\_poly} := Q_{1\_2} + Q_{2\_3}$$

$$Q_{1\_3\_poly} = -82.938 \cdot \frac{BTU}{lbm}$$

Total heat loss Stage 1 polytropic compression

Stage 2

$$P_4 := Press_{ratio} \cdot P_2 \qquad P_4 = 669.174 \cdot \frac{lbf}{in^2}$$

Pressure ratio used to determine P4

$$T_4 := T_3 \cdot \left(\frac{P_4}{P_3}\right)^{\frac{1.2-1}{1.2}} \qquad T_4 = 275.302 \cdot °F \quad T_4 = 408.318 \ K$$

Note because pressure ratio is the same, and the ideal gas law is assumed; the temperatures leaving the compressor are the same as for the first stage

Work relation is for a polytropic process using eqn 6.66a in Shapiro's Fundamentals of Engineering thermodynamics

$$W_{3\_4} := \frac{-1 \cdot 1.2 \cdot R_{air}}{1.2 - 1} \cdot (T_4 - T_3) \qquad W_{3\_4} = -82.363 \cdot \frac{BTU}{lbm}$$

Work for stage 2 polytropic compression, same as stage 1 due to same pressure ratio and the ideal gas assumption

Heat removed during stage 2 polytropic compression is calculated below

$$h_4 := g\left(\frac{P_4}{Bar}, \frac{T_4}{K}, 45, 400, 50, 425, 396.5, 422.7, 395.9, 422.2\right) \frac{kJ}{k}$$

$P_4 = 46.138 \cdot Bar$

$T_4 = 408.318\ K$

$$h_4 = 174.157 \cdot \frac{BTU}{lbm}$$

$$Q_{3\_4} := \left[W_{3\_4} + \left(h_4 - h_3\right)\right]$$

$$Q_{3\_4} = -35.451 \cdot \frac{BTU}{lbm}$$ This heat out of the system during stage 2 polytropic compression

Heat removal after stage 2 compression is calculated below; assuming a constant pressure. No work is completed, velocity and gravity effects are negligible, therefore a first law analysis simplifies to the following relation

$$P_5 := P_4 \qquad T_5 := T_{amb}$$

$$h_5 := g\left(\frac{P_5}{Bar}, \frac{T_5}{K}, 45, 290, 50, 300, 279.9, 290.7, 278.7, 289.6\right) \cdot \frac{kJ}{}$$

$P_5 = 46.138 \cdot bar$

$T_5 = 297.039\ K$

$$Q_{4\_5} := h_5 - h_4$$

$$Q_{4\_5} = -50.663 \cdot \frac{BTU}{lbm}$$

Both heat losses are summed to achieve the total heat loss for polytropic compression

$$Q_{3\_5} := Q_{3\_4} + Q_{4\_5} \qquad\qquad Q_{3\_5} = -86.115 \cdot \frac{BTU}{lbm}$$ Total heat loss Stage 2 polytropic compression

Stage 3

$$P_6 := Press_{ratio} \cdot P_4 \qquad P_6 = 4.515 \times 10^3 \cdot \frac{lbf}{in^2} \qquad \text{Pressure ratio used to determine P6}$$

$$T_6 := T_5 \cdot \left(\frac{P_6}{P_5}\right)^{\frac{1.2-1}{1.2}} \qquad T_6 = 275.302 \cdot °F \quad T_6 = 408.318 \text{ K}$$

Note because pressure ratio is the same, and the ideal gas law is assumed; the temperatures leaving the compressor are the same as for the other stages

Work relation is for a polytropic process using eqn 6.66a in Shapiro's Fundamentals of Engineering thermodynamics

$$W_{5\_6} := \frac{-1 \cdot 1.2 \cdot R_{air}}{1.2 - 1} \cdot (T_6 - T_5) \qquad W_{5\_6} = -82.363 \cdot \frac{BTU}{lbm}$$

Work for stage 3 polytropic compression

Heat removed during stage 3 polytropic compression is calculated below

$$h_6 := g\left(\frac{P_6}{Bar}, \frac{T_6}{K}, 300, 400, 400, 425, 380.9, 410.1, 380.0, 409.7\right) \frac{kJ}{}$$

$$\begin{array}{l} = 311.292 \cdot Bar \\ T_6 = 408.318 \text{ K} \end{array}$$

$$h_6 = 167.899 \cdot \frac{BTU}{lbm}$$

$$Q_{5\_6} := \left[W_{5\_6} + (h_6 - h_5)\right]$$

$$Q_{5\_6} = -37.958 \cdot \frac{BTU}{lbm}$$ This heat out of the system during stage 2polytropic compression

Heat removal after stage 3 compression is calculated below; assuming a constant pressure. No work is completed, velocity and gravity effects are negligible, therefore a first law analysis simplifies to the following relation

$$P_{storage} := P_6 \qquad T_{storage} := T_{amb}$$

$$P_{storage} = 311.292 \cdot bar \qquad T_{storage} = 297.039 \text{ K}$$

$$h_{storage} := g\left(\frac{P_{storage}}{Bar}, \frac{T_{storage}}{K}, 300, 290, 400, 300, 244.4, 257.7, 241.4, 254.8 \right) \frac{kJ}{kg}$$

$$Q_{6\_storage} := h_{storage} - h_6$$

$$Q_{6\_storage} = -58.943 \cdot \frac{BTU}{lbm}$$

Both heat losses are summed to achieve the total heat loss for polytropic compression

$$Q_{5\_storage} := Q_{6\_storage} + Q_{5\_6} \qquad Q_{5\_storage} = -96.901 \cdot \frac{BTU}{lbm}$$

Total heat loss
Stage 3 polytropic
compression

Total Work Input for three stage Polytropic compression

$$W_{3\_stage\_cmp} := W_{1\_2} + W_{3\_4} + W_{5\_6}$$

$$W_{3\_stage\_cmp} = -247.09 \cdot \frac{BTU}{lbm}$$

Total heat removal during compressions

$$Q_{cmp\_process} := Q_{1\_2} + Q_{3\_4} + Q_{5\_6}$$

$$Q_{cmp\_process} = -107.71 \cdot \frac{BTU}{lbm}$$

Total heat removed by Heat Exchangers

$$Q_{cmp\_heatX} := Q_{2\_3} + Q_{4\_5} + Q_{6\_storage}$$

$$Q_{cmp\_heatX} = -158.244 \cdot \frac{BTU}{lbm}$$

Total heat removed for three stage polytropic compression

$$Q_{3\_stage\_cmp} := Q_{cmp\_process} + Q_{cmp\_heatX}$$

$$Q_{3\_stage\_cmp} = -265.954 \cdot \frac{BTU}{lbm}$$

Three Stage Polytropic Expansion

The process is considered using a coeffcient of 1.2 for all three expansions. It is assumed that there is constant pressure heat addition to ambient temperature between expansion stages. A diagram of the process is included in the Appendix.

 Polytropic process for an ideal gas (n=1.2) is used to determine T.5_poly., formula 3.56 in Shapiro's Fundamentals of Engineering thermodynamics

$$T_{storage} = 75 \cdot °F \qquad T_{storage} = 297.039\ K \qquad P_{storage} = 311.292 \cdot bar$$

$$T_8 := T_{storage} \cdot \left(\frac{1}{Press_{ratio}} \right)^{\frac{1.2-1}{1.2}}$$

$$T_8 = -70.714 \cdot °F$$

$$T_{10} := T_8$$

Temperatures for other expansions are the same because there is heat addition to ambient b/t stages.

$$T_8 = 216.087\ K$$

$$T_{12} := T_8$$

Because the pressure ratio is the same for all stages and the air is heated to ambient between stages the work is the same for all stages

$$W_{storage\_8} := \left[\frac{1.2 \cdot R_{air}}{1.2 - 1} \cdot \left(T_8 - T_{storage} \right) \right] \cdot -1$$

$$W_{storage\_8} = 59.917 \cdot \frac{BTU}{lbm} \qquad \text{Power out for each stage of polytropic process}$$

$$W_{9\_10} := W_{storage\_8} \qquad W_{11\_12} := W_{storage\_8}$$

This process also requires significant heat from the environment or another heat source. Requirement is calculated below.

$$h_{storage} = 108.956 \cdot \frac{BTU}{lbm} \qquad\qquad P_8 := \frac{P_{storage}}{Press_{ratio}}$$

$$T_8 = 216.087 \, K \qquad\qquad T_8 = -70.714 \cdot {}^\circ F \qquad\qquad P_8 = 46.138 \cdot bar$$

$$h_8 := g\left(\frac{P_8}{Bar}, \frac{T_8}{K}, 45, 210, 50, 220, 189.4, 201.4, 187.1, 199.3\right) \frac{kJ}{kg}$$

$$h_8 = 84.354 \cdot \frac{BTU}{lbm}$$

$$Q_{storage\_8} := \left[W_{storage\_8} + \left(h_8 - h_{storage} \right) \right]$$

$$Q_{storage\_8} = 35.316 \cdot \frac{BTU}{lbm} \qquad$$ Heat required (heat in)
for first polytropic
expansion stage

Heat addition requirement after stage 1 expansion is calculated below; assuming a constant pressure. No work is completed, velocity and gravity effects are negligible, therefore a first law analysis simplifies to the following relation

$$P_9 := P_8 \qquad T_9 := T_{amb}$$

$$h_9 := g\left(\frac{P_9}{Bar}, \frac{T_9}{K}, 45, 290, 50, 300, 279.9, 290.7, 278.7, 289.6\right) \frac{kJ}{kg} \qquad P_9 = 46.138 \cdot bar$$

$$T_9 = 297.039 \, K$$

$$Q_{8\_9} := h_9 - h_8 \qquad\qquad h_9 = 123.493 \cdot \frac{BTU}{lbm}$$

$$Q_{8\_9} = 39.139 \cdot \frac{BTU}{lbm}$$

Both heat losses are summed to achieve the total heat loss for stage1 polytropic expansion

$$Q_{storage\_9} := Q_{storage\_8} + Q_{8\_9}$$

$$Q_{storage\_9} = 74.454 \cdot \frac{BTU}{lbm} \qquad$$ Total heat requirement
Stage 1 polytropic
expansion

Stage 2

$$W_{9\_10} = 59.917 \cdot \frac{BTU}{lbm} \qquad T_{10} = 216.087 \, K$$

This process also requires significant heat from the environment or another heat source. Requirement is calculated below.

$$h_9 = 123.493 \cdot \frac{BTU}{lbm} \qquad\qquad P_{10} := \frac{P_9}{Press_{ratio}}$$

$$T_{10} = 216.087 \, K \qquad\qquad T_{10} = -70.714 \cdot °F \qquad P_{10} = 6.838 \cdot bar$$

$$h_{10} := g\left(\frac{P_{10}}{Bar}, \frac{T_{10}}{K}, 6, 210, 8, 220, 207.5, 218.1, 206.6, 217.7\right) \frac{kJ}{kg}$$

$$h_{10} = 91.875 \cdot \frac{BTU}{lbm}$$

$$Q_{9\_10} := \left[W_{9\_10} + \left(h_{10} - h_9\right)\right]$$

$$Q_{9\_10} = 28.299 \cdot \frac{BTU}{lbm}$$ Heat required (heat in) for first polytropic expansion stage

Heat addition requirement after stage 1 expansion is calculated below; assuming a constant pressure. No work is completed, velocity and gravity effects are negligible, therefore a first law analysis simplifies to the following relation

$$P_{11} := P_{10} \qquad T_{11} := T_{amb}$$

$$h_{11} := g\left(\frac{P_{11}}{Bar}, \frac{T_{11}}{K}, 6, 290, 8, 300, 289.0, 299.2, 288.5, 298.7\right) \frac{kJ}{kg} \qquad\qquad P_{11} = 6.838 \cdot bar$$

$$T_{11} = 297.039 \, K$$

$$Q_{10\_11} := h_{11} - h_{10} \qquad\qquad h_{11} = 127.244 \cdot \frac{BTU}{lbm}$$

$$Q_{10\_11} = 35.369 \cdot \frac{BTU}{lbm}$$

Both heat losses are summed to achieve the total heat loss for stage1 polytropic expansion

$$Q_{9\_11} := Q_{9\_10} + Q_{10\_11}$$

$$Q_{9\_11} = 63.668 \cdot \frac{BTU}{lbm}$$ Total heat addition requirement Stage 2 polytropic expansion

Stage 3

$$W_{11\_12} = 59.917 \cdot \frac{BTU}{lbm} \quad T_{12} = 216.087 \, K$$

This process also requires significant heat from the environment or another heat source. Requirement is calculated below.

$$h_{11} = 127.244 \cdot \frac{BTU}{lbm} \qquad\qquad P_{12} := \frac{P_{11}}{Press_{ratio}}$$

$$T_{12} = 216.087 \, K \qquad\qquad T_{12} = -70.714 \cdot °F \qquad P_{12} = 1.014 \cdot bar$$

$$h_{12} := g\left(\frac{P_{12}}{Bar}, \frac{T_{12}}{K}, 1, 210, 2, 220, 209.7, 219.8, 209.3, 219.4\right) \frac{kJ}{kg}$$

$$h_{12} = 92.795 \cdot \frac{BTU}{lbm}$$

$$Q_{11\_12} := \left[W_{11\_12} + \left(h_{12} - h_{11}\right)\right]$$

$$Q_{11\_12} = 25.468 \cdot \frac{BTU}{lbm}$$ Heat required (heat in) for first polytropic expansion stage

Heat addition is not required after the last expansion. This air could be used for a cooling process, if there was a need.

Total Work for 3 stage expansion is calculated below

$$W_{3\_stage\_exp} := W_{storage\_8} + W_{9\_10} + W_{11\_12}$$

$$W_{3\_stage\_exp} = 179.751 \cdot \frac{BTU}{lbm}$$

Total Heat addition required for 3 stage expansion (during process):

$$Q_{3\_stage\_exp} := Q_{storage\_8} + Q_{9\_10} + Q_{11\_12}$$

$$Q_{3\_stage\_exp} = 89.083 \cdot \frac{BTU}{lbm}$$

Total Heat addition through the heat exchangers is:

$$Q_{3\_stage\_exp\_heatX} := Q_{8\_9} + Q_{10\_11}$$

$$Q_{3\_stage\_exp\_heatX} = 74.507 \cdot \frac{BTU}{lbm}$$

$$Q_{total\_3\_stage\_exp} := Q_{3\_stage\_exp\_heatX} + Q_{3\_stage\_exp\_heatX}$$

$$Q_{total\_3\_stage\_exp} = 149.015 \cdot \frac{BTU}{lbm}$$

Efficiency Determinations.

Efficiencies are determined using Excel, using different combinations of compression and expansion processes. Heat removal and addition requirements are shown in the Excel table; but are not influential in the theoretical efficiency number. The heating /cooling requirements help one understand what it would take to achieve these theoretical efficiencies. Efficiency is determined by the ratio of Power out to Power in.

Example: For 3 stage polytropic compression and expansion the maximum theoretical system efficiency is:

$$W_{3\_stage\_cmp} = -247.09 \cdot \frac{BTU}{lbm} \qquad W_{3\_stage\_exp} = 179.751 \cdot \frac{BTU}{lbm}$$

$T_{12} = -70.714 \cdot °F$

$$\eta_{IsoT\_Reg\_poly} := \frac{W_{3\_stage\_exp}}{-W_{3\_stage\_cmp}}$$

$\eta_{IsoT\_Reg\_poly} = 72.747 \cdot \%$ Maximum theoretical efficiency for 3 stage polytropic expansion and compression

Other combinations are calculated in Excel with data from calculation 15.

Theoretical System Efficiencies- High Pressure (4500 psig) CAES				
ALL ENERGY AND HEAT IN BTU/LBM, ALL EFFCIEINCIES ARE % (W_OUT/W_IN)				
COMPRESSION / EXPANSION	ISENTROPIC T.h= 4,825 F Q= -1199 btu/lbm	SINGLE STAGE POLYTROPIC T.h= 1221 F Q= -412 btu/lbm	THREE STAGE POLYTROPIC T.h= 275 F Q= -266 btu/lbm	ISOTHERMAL T.h= 75 F Q= -229 btu/lbm
CMP ENERGY / EXP ENERGY	-1139	-471	-247	-210
ISENTROPIC (4500 PSIG) T.c= -312F Q= 0	92	8%	20%	37%	44%
REGULATED (300 PSIG) ISENTROPIC T.c= -267F Q= 0	63	6%	13%	26%	30%
REGULATED (300 PSIG) ISENTROPIC W/REHEAT T.c= -236F Q= 19 btu/lbm	73	6%	15%	30%	35%
POLYTROPIC (4500 PSIG) T.c= -290F Q= 80 btu/lbm	150	13%	32%	61%	71%
REGULATED (300 PSIG) POLYTROPIC T.c= -210F Q= 36 btu/lbm	85	7%	18%	34%	40%
REGULATED (300 PSIG) POLYTROPIC W/REHEAT T.c= -167F Q= 62 btu/lbm	100	9%	21%	40%	48%
THREE STAGE POLYTROPIC T.c= -95F Q= 149 btu/lbm	178	16%	38%	72%	85%
REGULATED (300 PSIG) ISOTHERMAL T.c= -2F Q= 95 btu/lbm	95	8%	20%	38%	45%
REGULATED (300 PSIG) ISOTHERMAL W/REHEAT T.c=75F Q=148 btu/lbm	111	10%	24%	45%	53%
ISOTHERMAL (4500 PSIG) T.c=75F Q= 229 btu/lbm	210	18%	45%	85%	100%

Calculation 18B - Low Pressure CAES 3 Stage Polytropic Efficiency

"
"
"
"

Calculation 18B Appendix

#	**Contents**	**Page**

CALC 18

3 STAGE POLYTROPIC COMPRESSION ANALYSIS

3 STAGE EXPANSION

$$\Rightarrow \text{EXIT AIR}$$
$$T_{12} = -42.8F$$
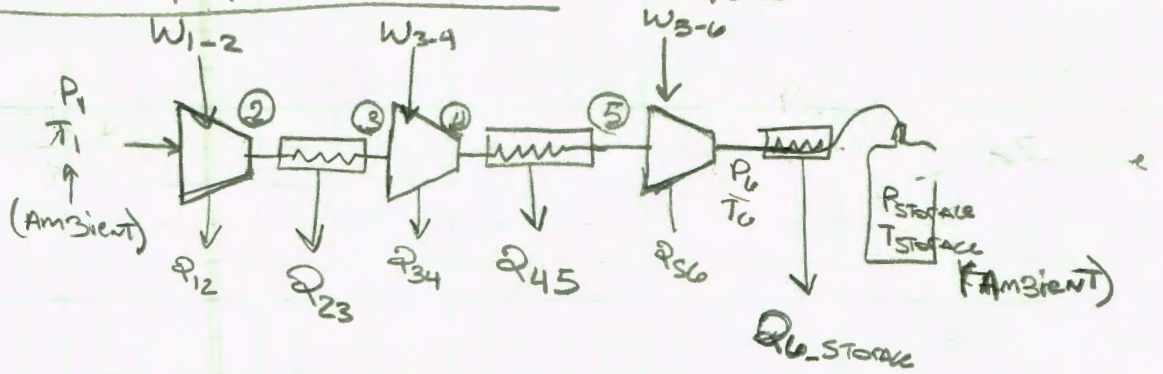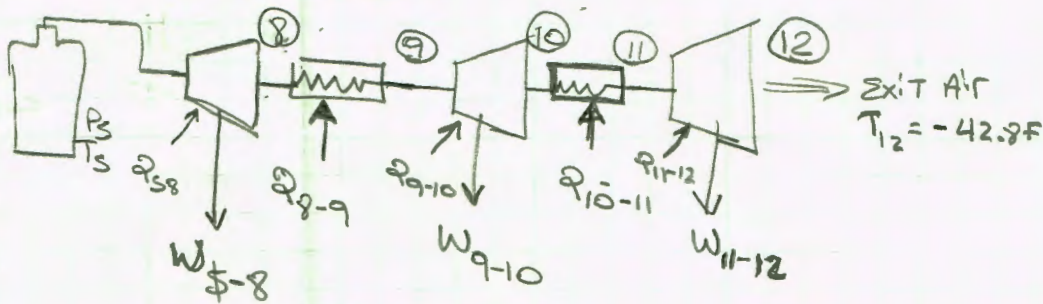
FIRST LAW ANALYSIS for REHEAT/INTERCOOL CALCULATIONS

$$\varepsilon_{IN} = \varepsilon_{OUT} \quad (P_1 = P_2)$$

$$Q_{IN} + \dot{m}(h_1) = \cancel{W_{OUT}}^0 + \dot{m}(h_2)$$
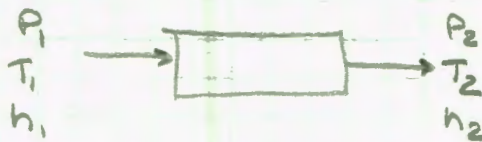
$$Q_{IN} = h_2 - h_1$$

Table of Contents
Calculation'20'- Process Plots

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This calculation establishes curves used to explain compression and expansion processes for the CAES presentation. It plots isothermal, isentropic, and polytropic (n=1.2) processes for compression and expansion based on ideal gas law.

Unit Conversion, Constants, and Thermodynamic State Determination

$$Bar := 14.5038 \frac{lbf}{in^2} \qquad kJ := 1000J \qquad \gamma_{air} := 1.4 \qquad \text{Heat capacity ratio for air}$$

$$P_{amb} := 14.7 \frac{lbf}{in^2} \qquad T_{amb} := 75\,°F \qquad P_3 := 614.7 \frac{lbf}{in^2} \qquad T_3 := T_{amb}$$

$$R_{air} := 53.33 \frac{ft \cdot lbf}{lbm \cdot R} \qquad T_{amb} = 297.039 \cdot K$$

$$53.33 ft \cdot \frac{lbf}{lbm \cdot R} = 0.069 \cdot \frac{BTU}{lbm \cdot R} \qquad v_{amb} := \frac{R_{air} \cdot T_{amb}}{P_{amb}}$$

Compression Process:

$$v_{amb} = 13.47 \cdot \frac{ft^3}{lbm}$$

$$Cmp_{isoT}(v, T) := \frac{R_{air} \cdot T_{amb}}{v} \qquad \text{Isothermal plot of pressure vs. specific volume}$$

The relation for a polytropic plot is derived in the appendix. It results in the following equation for pressure as a function of specific volume, initial state, and polytopic coefficient. n=1 isothermal, 1.2 polytropic, and 1.4 for isentropic. the equation is tested for equivalency with the constant temp ideal gas law equation above.

$$P_{poly}(v, P_1, v_1, n) := \frac{P_1 \cdot v_1^n}{v^n} \cdot \frac{lbf}{in^2} \qquad \begin{array}{l} \text{For this equation values must be unitless, v must} \\ \text{be in ft\textasciicircum3/lbm and P must be in lbf/in\textasciicircum2} \end{array}$$

Test Values:

$$P_{poly}\left(13.47, 14.7, \frac{v_{amb}}{\frac{ft^3}{lbm}}, 1\right) = 14.7 \cdot \frac{lbf}{in^2} \qquad v_{100} := \frac{R_{air} \cdot T_{amb}}{114.7 \frac{lbf}{in^2}} \qquad v_{100} = 1.726 \cdot \frac{ft^3}{lbm}$$

$$P_{poly}\left(1.5624, 14.7, \frac{v_{amb}}{\frac{ft^3}{lbm}}, 1.4\right) = 300.011 \cdot \frac{lbf}{in^2} \quad P_{poly}\left(\frac{v_{100}}{\frac{ft^3}{lbm}}, 14.7, \frac{v_{amb}}{\frac{ft^3}{lbm}}, 1.0\right) = 114.7 \cdot \frac{lbf}{in^2}$$

Isentropic plot of pressure vs. specific volume

$$\dfrac{P_{poly}\left(v, \dfrac{P_{amb}}{\dfrac{lbf}{in^2}}, \dfrac{v_{amb}}{\dfrac{ft^3}{lbm}}, 1\right)}{\dfrac{lbf}{in^2}}$$

———

$$\dfrac{P_{poly}\left(v, \dfrac{P_{amb}}{\dfrac{lbf}{in^2}}, \dfrac{v_{amb}}{\dfrac{ft^3}{lbm}}, 1.2\right)}{\dfrac{lbf}{in^2}}$$

$$\dfrac{P_{poly}\left(v, \dfrac{P_{amb}}{\dfrac{lbf}{in^2}}, \dfrac{v_{amb}}{\dfrac{ft^3}{lbm}}, 1.4\right)}{\dfrac{lbf}{in^2}}$$

— ·

v
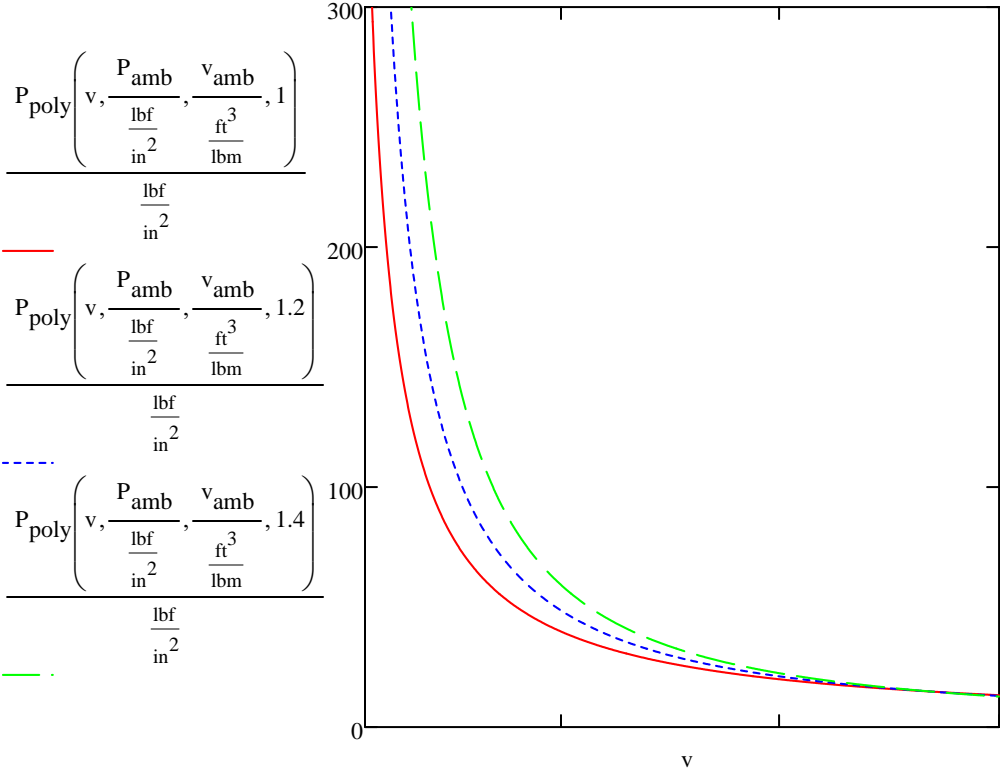
Temperature entropy plot shows heat removal requirements for Isentropic and polytropic processes.

Relation below is derived in the Appendix

$$T\left(v, P_1, v_1, n\right) := \left[\frac{P_1 \cdot v_1^{\ n} \cdot v^{(1-n)}}{53.33} \cdot 144R\right] / °F$$

Values must be entered unitless, pressure in psia, specific volume in ft^3/lbm.

Test Values:

$$T\left(13.47, \frac{P_{amb}}{\frac{lbf}{in^2}}, \frac{v_{amb}}{\frac{ft^3}{lbm}}, 1.0\right) = 75 \qquad T\left(5, \frac{P_{amb}}{\frac{lbf}{in^2}}, \frac{v_{amb}}{\frac{ft^3}{lbm}}, 1.0\right) = 75$$

$$T\left(13.47, \frac{P_{amb}}{\frac{lbf}{in^2}}, \frac{v_{amb}}{\frac{ft^3}{lbm}}, 1.2\right) = 75.002 \qquad T\left(5, \frac{P_{amb}}{\frac{lbf}{in^2}}, \frac{v_{amb}}{\frac{ft^3}{lbm}}, 1.2\right) = 192.21$$

$$T\left(13.47, \frac{P_{amb}}{\frac{lbf}{in^2}}, \frac{v_{amb}}{\frac{ft^3}{lbm}}, 1.4\right) = 75.005 \qquad T\left(5, \frac{P_{amb}}{\frac{lbf}{in^2}}, \frac{v_{amb}}{\frac{ft^3}{lbm}}, 1.4\right) = 335.114$$

Temperature/entropy plot is produced using ideal gas tables. the plot is performed in Excel and is included separately in this analysis.

The expansion process is considered from 300 psia and 70F (cooling to ambient is assumed during storage)

$$P_{300} := 300\frac{lbf}{in^2}$$

$$v_{300} := \frac{R_{air} \cdot T_{amb}}{314.7\frac{lbf}{in^2}}$$
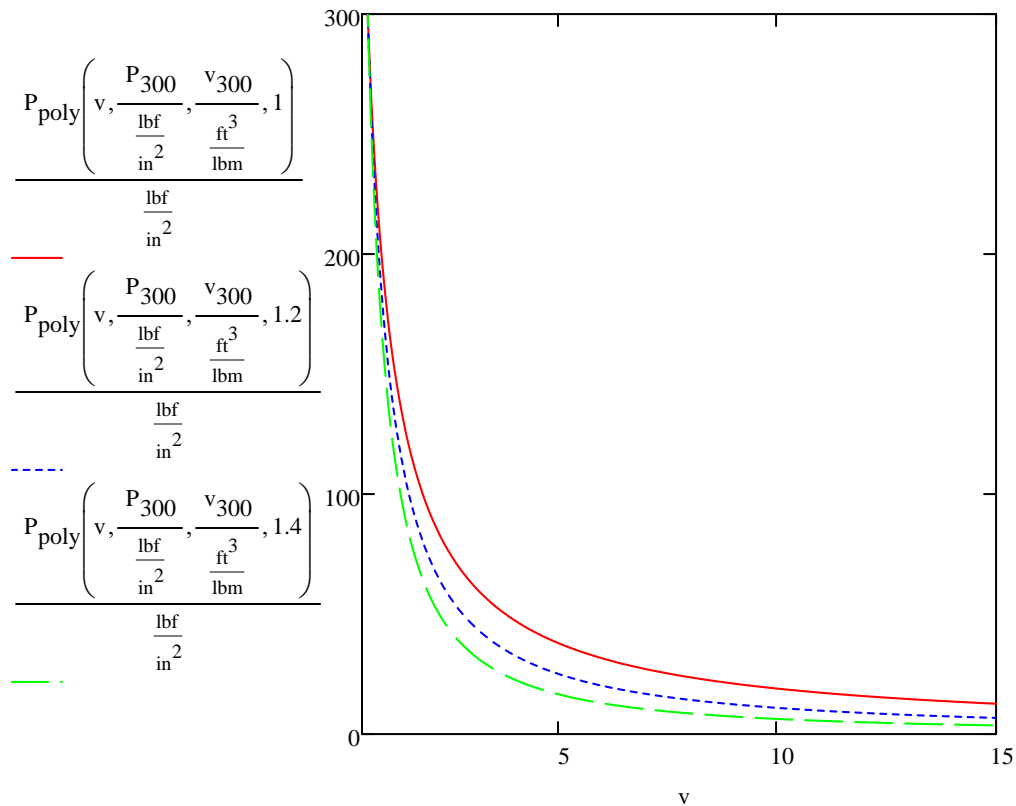
$$6.13\frac{kJ}{kg \cdot K} = 1.464 \cdot \frac{BTU}{lbm \cdot R}$$

Isentropic expansion initial entropy after cooling to ambeint (128 psia)

$$6.03\frac{kJ}{kg \cdot K} = 1.44 \cdot \frac{BTU}{lbm \cdot R}$$

polytropic expansion initial entropy after cooling to amient (190 psia)

Calculation'20'/'Process Plots

Calculation 20 Appendix

"

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PLOTS OF COMPRESSION & EXPANSION FOR PRESENTATION

IDEAL GAS LAW

$$PV = RT \implies P(V) = \frac{RT}{V} \leftarrow \text{FOR ISOTHERMAL PLOT THIS}$$

LOOKING AT POLYTROPIC & ISENTROPIC $PV^N = \text{CONSTANT}$

FOR POLYTROPIC $N = 1.2$ (AS USED IN PREVIOUS ANALYSIS
FOR ISENTROPIC $N = 1.4 = \gamma = \frac{Cp}{Cv}$. DERIVED IN SHAPIRO THERMO TEXT.

① $P(V) = \frac{RT}{V}$

② $PV^N = K \leftarrow$ NEED A STARTING POINT WHICH
                                is $T_{AMBIENT}$, $P_{AMBIENT}$ FOR COMPRESSION

$$K = \left(P_{AMB}\right)\left(V_{AMB}\right)^N \qquad \text{WHERE } N = 1.4 \text{ FOR ISENTROPIC}$$
$$1.2 \text{ FOR POLYTROPIC}$$

THEN CAN SOLVE FOR $T(V)$ USING IDEAL GAS LAW

$$\left(\frac{RT}{V}\right)\left(V^N\right) = P_{AMB} V_{AMB}^N$$

$$T = \frac{\left(P_{AMB}\right)\left(V_{AMB}\right)^N V}{R \, V^N}$$

③ $T(V, P_{AMB}, V_{AMB}, N) = \dfrac{P_{AMB}\left(V_{AMB}\right)^N V^{(1-N)}}{R} \leftarrow$ RELATION FOR TEMPERATURE
                                                                                    AS A FUNCTION OF SPECIFIC
                                                                                    VOLUME

THEN SUBBING ③ INTO ①:

④ $P(V, P_{AMB}, V_{AMB}, N) = \dfrac{\cancel{R} \left(P_{AMB}\right)\left(V_{AMB}\right)^N V^{(-N)}}{R \, \cancel{V}} = \dfrac{P_{AMB}\left(V_{AMB}\right)^N}{V^N}$

└── MAKE N A VARIABLE

└── P_{AMB} & T_{AMB} CAN BE CONSIDERED THE STARTING
       POINTS, WHICH WILL ALLOW THIS RELATION
       TO WORK FOR EXPANSION FROM AN INITIAL
       PRESSURE AS WELL.

THIS RELATION SHOULD MATCH THE IDEAL GAS LAW LAW WHEN N=1 (ISOTHERMAL). IT CHECKS IN THAT WY N=1, $P(V_{Amb}, T_{Amb}, 1) = P_{Amb}$, WHICH IT SHOULD

NOTE: THE RELATION $PV^N = K = P_1(V_1^N)$ IS ENOUGH TO PLOT THE $P(V)$ CURVE. THE TEMPERATURE TERM DERIVED PREVIOUSLY CANCELS OTHER TERMS REDUCING THE EQUATION TO THE STARTING POINT. HOWEVER THE TEMPERATURE RELATION IS USEFUL FOR PLOTTING TEMPERATURE - ENTROPY CURVES. THESE WILL ALSO BE DISCUSSED IN THE PRESENTATION. TEMPERATURE RELATION IS REITERATED BELOW

$$T(V, P_{Amb}, V_{Amb}) = \frac{P_{Amb}(V_{Amb})^N V^{(1-N)}}{R}$$

N=1.4 ISENTROPIC
N=1.2 POLYTROPIC
N=1.0 ISOTHERMAL

UNITS CHECK

$$\frac{\left(\frac{LBF}{IN^2}\right)\left(\frac{Ft^3}{LBm}\right)^N \left(\frac{Ft^3}{LBm}\right)^2}{\frac{Ft \cdot LBF}{LBm \cdot R}\left(\frac{Ft^3}{LBm}\right)^N} = \frac{Ft^2 \cdot R}{IN^2}\frac{(144 IN^2)}{Ft^2}$$

NEED TO ADD 144 TO EQN FOR PROPER UNITS WHEN USING $V = \frac{Ft^3}{LBm}$ & $P = PSia$ FOR RESULT IN RANKEEN.

$$(5) \quad T(V, P_1, V_1, N) = \frac{(P_1)(V_1)^N V_2^{(1-N)}(144)(R)}{R_{air}} \Rightarrow \binom{OUTPUT}{TEMP\ IN\ R}$$

(USE P_{Amb}, V_{Amb} MOST) CALCS

R_{air} (USE R UNITLESS 53.33)

THE ABOVE RELATION ALLOWS DETERMINATION OF RESULTING TEMPERATURES FROM THE COMPRESSION/EXPANSION PROCESSES. ENTROPY PROPERTIES ARE DETERMINED USING TABLES.
IDEAL GAS TABLES ARE USED TO DETERMINE ENTROPY ASSOCIATED WITH SELECTED POINTS TS-CURVES ARE PRODUCED IN EXCEL.
REFERENCE EXCEL FILE CALC_020.XLS (ATTACHED).

PSia

$\frac{Ft^3}{LBm}$

AMPAD

$H = \Delta Q + W$ $\int VdP$ OPEN SYSTEM

$\int_{V_1}^{V_2} PdV =$

$\int_P VdP$ OPEN SYSTEM SHAFT WORK

$Q = \int Tds$

Determining Specific Volume @ 300 psia

$$P(.66, 14.7, V_{AMB}, 1.0) = 300,021$$

$V = .66 ft^3/LB,$ isothermal

Similarly for $N=1.2$ (polytropic), & $N=1.4$ (isentropic)

$V = 1.091$ polytropic $V = 1.562$

To avoid interpolation it is convenient to obtain a relation for V & P as a function of temperature for the same pressure range.

Solving EQN ⑤ for $V(T, \ \_\_\_\_)$

(5) $$T(V, P, V_1, N) = \frac{P_1 V_1^N V_2^{(1-N)} (144)}{R}$$

$$V^{1-N} = \frac{T \cdot R}{P_1 V_1^N (144)}$$

$$(1-N) \log(V) = \log\left(\frac{T R}{P_1 V_1^N \cdot 144}\right)$$

$$V = 10^{\left[\frac{\log\left(\frac{TR}{P_1 V_1^N \cdot 144}\right)}{(1-N)}\right]}$$

$\log(10) = 1$

$(100) = 2$ 10^2

Similarily EQN (4): for $P(V, \ \_\_\_\_\_)$

$$P_2 V_2^N = P_1 V_1^N$$

$$P_2 = \frac{P_1 V_1^N}{V_2^N}$$

for isothermal case the PV^N relation can not be used it results in dividing by 0

An alternate relation is as follows

$S_{O2} = S_{O1}$ for all P_2 b/c temp is constant therefore entropy state is determined directly from the pressure ratio.

ISOTHERMAL CASE CON'T

ENTROPY RELATION IS AS FOLLOWS

$$S_2 = S_1 + \cancel{S^{\circ}_{2}} - \cancel{S^{\circ}_{1}} - R\,LN\left(\frac{P_2}{P_1}\right)$$

$$S_2 = S_1 - R\,LN\left(\frac{P_2}{P_1}\right)$$

EXPANSION PROCESS

ISENTROPIC STARTING POINT

ASSUME AIR COOLS TO AMBIENT DURING STORAGE. DURING THIS TIME SPECIFIC VOLUME IS UNCHANGED BUT PRESSURE DROPS SIGNIFICANTLY

ISENTROPIC → 127 PSia @ AMBIENT ← DETERMINED BY ITERATING P_{o_1} PRESSURE UNTIL EXCEL SPREADSHEET SHOWS EQUIVALENT DATA FOR P_2. $P_2(V, P_1, V_1, N)$

SAME AS, V_1 AT START OF EXPANSION

DROPS AS STORED GAS COOLS

FIXED AT END OF COMPRESSION
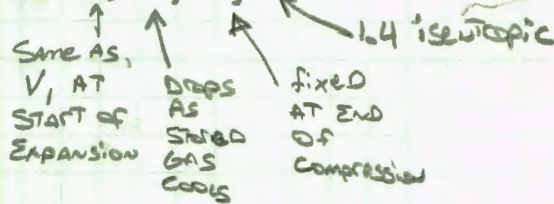
1.4 ISENTROPIC

PRESSURE SEEMS LOW. VERIFYING BY ANALYSIS.
ISENTROPIC COMPRESSION FINAL STATE   P = 314.7 PSia
                                        T = 1265 R

CONSTANT VOLUME COOLING TO 534 R

IDEAL GAS LAW

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \implies P_2 = \frac{P_1 V_1 T_2}{T_1 V_2} \qquad V_1 = V_2 \left(\begin{array}{c}\text{SAME GAS IN}\\ \text{SAME VOLUME}\end{array}\right)$$

$$P_2 = \frac{P_1 T_2}{T_1}$$

$$P_2 = \frac{(314.7\,Psia)\,534\,R}{1265\,R} = 132 \quad PSia$$

FOR POLYTROPIC $P_1 = 314.7$
$T = 884\,R$

$$P_2 = \frac{(314.7\,Psia)\,534\,R}{884\,R}$$

$$P_2 = 190\,PSia$$

USING THE REDUCED PRESSURE ASSUMES EXPANSION METHOD IS
SAME AS COMPRESSION, WHICH IS NOT NECESSARILY THE
CASE. REWORKED EXCEL SPREAD SHEET ASSUMING ALL
EXPANSIONS COMPUTED FROM 300 PSIA AND 534°2.
THIS GIVES BETTER COMPARISON AND ISOLATES EXPANSION
FROM THE COMPRESSION PROCESS. EXCEL SPREADSHEET
ATTACHED

Calculation 020 temperature Entropy Plot Data

This calcualtion uses entropy values determined from Shaprio's air Ideal gas table (Table A22E pg 816)
specific volume is chosen at points that allow good curve fitting based on the PV diagram.
v,P are calculated as a function of temperature throughout the established temp range to allow table values to be used without
interpolation. Interpolation is still used to capture the last data point corresponding to 300 psia.
Isothermal,Poly tropic and Isentropic processes are considered.
Pressure and temperature relations developed in hand calcs
So.2 is from ideal gas table
S2 calculated from S.2=S.1+So.2-So.1-R*ln(P.2/P.1)

Starting state for Isentropic compression								
T (R)	P.1 (psia)	v.1 (ft^3/lbm)	S.1 (BTU/lbm*R)	R (ft-lbf/lbm*R)	n	S.01 (BTU/lbm*R)		
534.67	14.7	13.47	1.638	53.33	1.4	0.598		
				R (BTU/lbm*R)				
				0.069				

Isentropic Compression- note S2 is constant, S2 completed to verify calculation only.								
arbitrary	P(v,P.1,v.1,n)	T(v,P.1,v.1,n)	Gas Table value	v(T,P.1,v.1,n)*	P(v,P.1,v.1,n)*	From Gas Table	See Above	Q.incremental
v (ft^3/lbm)	P (psia)	Temperature (R)	Temperature (R)	v (ft^3/lbm)	P (psia)	So.2 (Btu/lbm*R)	S2 (Btu/lbm*R)	BTU/lbm
13.47	14.70	534.66	537.00	13.32	14.93	0.60	1.64	No heat transfer
13.00	15.45	542.31	540.00	13.14	15.22	0.60	1.64	
12.00	17.28	559.95	560.00	12.00	17.29	0.61	1.64	
11.00	19.52	579.78	580.00	10.99	19.55	0.62	1.64	
10.00	22.31	602.31	600.00	10.10	22.01	0.63	1.64	
8.50	28.01	642.77	620.00	9.30	24.68	0.63	1.64	
8.00	30.49	658.55	640.00	8.59	27.59	0.64	1.64	
7.00	36.75	694.68	660.00	7.96	30.72	0.65	1.64	
6.50	40.77	715.58	680.00	7.38	34.11	0.66	1.64	
6.00	45.61	738.86	700.00	6.87	37.75	0.66	1.64	
5.50	51.51	765.03	720.00	6.40	41.66	0.67	1.64	
5.00	58.87	794.76	740.00	5.98	45.85	0.68	1.64	
4.60	66.16	821.71	760.00	5.59	50.34	0.68	1.64	
4.40	70.40	836.45	780.00	5.24	55.13	0.69	1.64	
4.20	75.14	852.16	800.00	4.92	60.24	0.70	1.64	
3.80	86.44	886.97	820.00	4.62	65.68	0.70	1.64	
3.60	93.24	906.36	840.00	4.35	71.45	0 71	1.64	
3.40	101.01	927.32	860.00	4.10	77.59	0.71	1.64	
3.20	109.96	950.09	880.00	3.88	84.09	0.72	1.64	
2.90	126.20	988.24	900.00	3.66	90.97	0.72	1.64	
2.8	132.6	1002.21	920.00	3.47	98.24	0.73	1.64	
2.7	139.5	1016.90	940.00	3.29	105.93	0.74	1.64	
2.6	147.1	1032.37	960.00	3.12	114.03	0.74	1.64	
2.5	155.4	1048.69	980.00	2.96	122.56	0.75	1.64	
2.4	164.5	1065.95	1000.00	2.82	131.54	0.75	1.64	
2.3	174.6	1084.26	1040.00	2.55	150.89	0.76	1.64	
2.2	185.8	1103.71	1080.00	2.32	172.20	0.77	1.64	
2.1	198.3	1124.44	1120.00	2.12	195.58	0.78	1.64	
2	212.3	1146.60	1160.00	1.94	221.13	0.79	1.64	
1.9	228.1	1170.37	1200.00	1.78	248.99	0.80	1.64	
1.8	246.1	1195.95	1240.00	1.64	279.27	0.80	1.64	
1.7	266.6	1223.61	1265.00	1.56	299.48	0.81	1.64	
1.6	290.2	1253.65						
1.5625	300.0	1265.60						

Starting state for polytropic compression								
T (R)	P.1 (psia)	v.1 (ft^3/lbm)	S.1 (BTU/lbm*R)	R (ft-lbf/lbm*R)	n	S.01 (BTU/lbm*R)		
534.67	14.7	13.47	1.638	53.33	1.2	0.598		
				R (BTU/lbm*R)				
				0.069				

Polytropic Compression- note S2 is constant, S2 completed to verify calculation only.								
arbitrary	P(v,P.1,v.1,n)	T(v,P.1,v.1,n)	Gas Table value	v(T,P.1,v.1,n)*	P(v,P.1,v.1,n)*	From Gas Table	See Above	Q.incremental
v (ft^3/lbm)	P (psia)	Temperature (R)	Temperature (R)	v (ft^3/lbm)	P (psia)	So.2 (Btu/lbm*R)	S2 (Btu/lbm*R)	BTU/lbm
13.47	14.70	534.66	537.00	13.18	15.09	0.60	1.64	n/a
13.00	15.34	538.47	540.00	12.82	15.60	0.60	1.64	-0.53
12.00	16.89	547.16	560.00	10.69	19.41	0.61	1.63	-3.55
11.00	18.75	556.76	580.00	8.97	23.96	0.62	1.62	-3.54
10.00	21.02	567.48	600.00	7.57	29.36	0.63	1.62	-3.54
9.00	23.85	579.56	620.00	6.42	35.74	0.63	1.61	-3.53
8.00	27.47	593.38	640.00	5.48	43.25	0.64	1.61	-3.52
7.00	32.24	609.44	660.00	4.70	52.01	0.65	1.60	-3.50
6.00	38.80	628.52	680.00	4.05	62.22	0.66	1.60	-3.52
5.00	48.28	651.86	700.00	3.50	74.04	0.66	1.59	-3.50
4.00	63.11	681.61	720.00	3.04	87.67	0.67	1.59	-3.49
3.00	89.13	721.98	740.00	2.65	103.34	0.68	1.58	-3.49
2.50	110.93	748.79	760.00	2.32	121.27	0.68	1.58	-3.47
2.00	144.98	782.97	780.00	2.04	141.72	0.69	1.57	-3.47
1.80	164.52	799.64	800.00	1.80	164.97	0.70	1.57	-3.46
1.60	189.50	818.70	820.00	1.59	191.31	0.70	1.56	-3.45
1.40	222.44	840.86	840.00	1.41	221.07	0.71	1.56	-3.45
1.20	267.63	867.19	860.00	1.25	254.60	0.71	1.56	-3.42
1.10	297.09	882.41	880.00	1.12	292.25	0.72	1.55	-3.42
1.09	300.03	883.86	884.00	1.09	300.31	0.72	1.55	-0.68
						Total Heat transfer (Btu/lbm)		-60.53

Starting state for Isothermal compression					
T (R)	P.1 (psia)	v.1 (ft^3/lbm)	S.1 (BTU/lbm*R)	R (ft-lbf/lbm*R)	n
534.67	14.7	13.47	1.638	53.33	1
				R (BTU/lbm*R)	
				0.069	

Isothermal Compression- note S2 is constant, S2 completed to verify calculation only.					
arbitrary	P(v,P.1,v.1,n)	T(v,P.1,v.1,n)	Gas Table value	=S.1-R*ln(P.2/P.1)	Q.incremental
v (ft^3/lbm)	P (psia)	Temperature (R)	Temperature (R)	S2 (Btu/lbm*R)	BTU/lbm
13.47	14.70	534.66	537.00	1.64	n/a
13.00	15.23	534.66	537.00	1.64	-1.32
12.00	16.50	534.66	537.00	1.63	-2.97
11.00	18.00	534.66	537.00	1.62	-3.22
10.00	19.80	534.66	537.00	1.62	-3.53
9.00	22.00	534.66	537.00	1.61	-3.90
8.00	24.75	534.66	537.00	1.60	-4.36
7.00	28.29	534.66	537.00	1.59	-4.95
6.00	33.00	534.66	537.00	1.58	-5.71
5.00	39.60	534.66	537.00	1.57	-6.76
4.00	49.50	534.66	537.00	1.55	-8.27
3.00	66.00	534.66	537.00	1.53	-10.66
2.50	79.20	534.66	537.00	1.52	-6.76
2.00	99.00	534.66	537.00	1.51	-8.27
1.80	110.01	534.66	537.00	1.50	-3.90
1.60	123.76	534.66	537.00	1.49	-4.36
1.40	141.44	534.66	537.00	1.48	-4.95
1.20	165.01	534.66	537.00	1.47	-5.71
1.10	180.01	534.66	537.00	1.47	-3.22
1.00	198.01	534.66	537.00	1.46	-3.53
0.9	220.01	534.66	537.00	1.45	-3.90
0.80	247.51	534.66	537.00	1.44	-4.36
0.75	264.01	534.66	537.00	1.44	-2.39
0.7	282.87	534.66	537.00	1.43	-2.56
0.66	300.01	534.66	537.00	1.43	-2.18
			Total Heat transfer (Btu/lbm)		-111.75

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Starting state for Isentropic expansion, assume standard condition								
T (R)	P.1 (psia)	v.1 (ft^3/lbm)	S.1 (BTU/lbm*R)	R (ft-lbf/lbm*R)	n	S.01 (BTU/lbm*R)		
534.67	300.0	0.66	1.464	53.33	1.4	0.598		
				R (BTU/lbm*R)				
				0.069				

Isentropic expansion- note S2 is constant, S2 completed to verify calculation only.								
arbitrary	P(v,P.1,v.1,n)	T(v,P.1,v.1,n)	Gas Table value	v(T,P.1,v.1,n)*	P(v,P.1,v.1,n)*	From Gas Table	See Above	Q.incremental
v (ft^3/lbm)	P (psia)	Temperature (R)	Temperature (R)	v (ft^3/lbm)	P (psia)	So.2 (Btu/lbm*R)	S2 (Btu/lbm*R)	BTU/lbm
0.66	300.00	534.63	537.00	0.65	304.67	0.60	1.46	No heat transfer
0.80	229.17	495.04	520.00	0.71	272.23	0.59	1.46	
1.00	167.68	452.77	500.00	0.78	237.31	0.58	1.46	
1.50	95.05	384.98	480.00	0.86	205.72	0.57	1.46	
2.00	63.54	343.13	460.00	0.96	177.25	0.56	1.46	
2.50	46.49	313.83	440.00	1.07	151.71	0.55	1.46	
3.00	36.02	291.76	420.00	1.21	128.91	0.54	1.46	
4.00	24.08	260.05	400.00	1.36	108.68	0.53	1.46	
5.00	17.62	237.84	380.00	1.55	90.82	0.52	1.47	
5.69	14.70	225.85	360.00	1.77	75.16	0.50	1.47	

Starting state for polytropic expansion								
T (R)	P.1 (psia)	v.1 (ft^3/lbm)	S.1 (BTU/lbm*R)	R (ft-lbf/lbm*R)	n	S.01 (BTU/lbm*R)		
534.67	300.0	0.66	1.464	53.33	1.2	0.598		
				R (BTU/lbm*R)	31			
				0.069				

Isentropic expansion- note S2 is constant, S2 completed to verify calculation only.								
arbitrary	P(v,P.1,v.1,n)	T(v,P.1,v.1,n)	Gas Table value	v(T,P.1,v.1,n)*	P(v,P.1,v.1,n)*	From Gas Table	See Above	Q.incremental
v (ft^3/lbm)	P (psia)	Temperature (R)	Temperature (R)	v (ft^3/lbm)	P (psia)	So.2 (Btu/lbm*R)	S2 (Btu/lbm*R)	BTU/lbm
0.66	300.00	534.63	537.00	0.65	308.06	0.60	1.46	n/a
0.80	238.16	514.45	520.00	0.76	253.98	0.59	1.47	2.91
1.00	182.21	492.00	500.00	0.92	200.73	0.58	1.48	3.42
2.00	79.31	428.31	480.00	1.13	157.12	0.57	1.48	3.42
3.00	48.76	394.95	460.00	1.40	121.71	0.56	1.49	3.41
4.00	34.52	372.87	440.00	1.75	93.22	0.55	1.50	3.42
5.00	26.41	356.59	420.00	2.21	70.51	0.54	1.51	3.41
5.50	23.56	349.86	400.00	2.82	52.62	0.53	1.52	3.41
6.00	21.22	343.82	380.00	3.64	38.68	0.52	1.52	3.41
6.50	19.28	338.36	360.00	4.77	27.96	0.50	1.53	3.40
7.00	17.64	333.38						
8.00	15.03	324.60						
8.15	14.70	323.40				Total Heat transfer (Btu/lbm)		30.20

Starting state for Isothermal Expansion					
T (R)	P.1 (psia)	v.1 (ft^3/lbm)	S.1 (BTU/lbm*R)	R (ft-lbf/lbm*R)	n
534.67	300.0	0.66	1.464	53.33	1
				R (BTU/lbm*R)	
				0.069	

Isothermal Compression- note S2 is constant, S2 completed to verify calculation only.					
arbitrary	P(v,P.1,v.1,n)	T(v,P.1,v.1,n)	Gas Table value	=S.1-R*ln(P.2/P.1)	Q.incremental
v (ft^3/lbm)	P (psia)	Temperature (R)	Temperature (R)	S2 (Btu/lbm*R)	BTU/lbm
0.66	300.00	534.63	537.00	1.46	n/a
0.7	282.86	534.63	537.00	1.47	2.18
0.75	264.00	534.63	537.00	1.47	2.56
0.80	247.50	534.63	537.00	1.48	2.39
0.9	220.00	534.63	537.00	1.49	4.36
1.00	198.00	534.63	537.00	1.49	3.90
1.10	180.00	534.63	537.00	1.50	3.53
1.20	165.00	534.63	537.00	1.51	3.22
1.40	141.43	534.63	537.00	1.52	5.71
1.60	123.75	534.63	537.00	1.53	4.95
1.80	110.00	534.63	537.00	1.53	4.36
2.00	99.00	534.63	537.00	1.54	3.90
2.50	79.20	534.63	537.00	1.56	8.27
3.00	66.00	534.63	537.00	1.57	6.76
4.00	49.50	534.63	537.00	1.59	10.66
5.00	39.60	534.63	537.00	1.60	8.27
6.00	33.00	534.63	537.00	1.62	6.76
7.00	28.29	534.63	537.00	1.63	5.71
8.00	24.75	534.63	537.00	1.64	4.95
9.00	22.00	534.63	537.00	1.64	4.36
10.00	19.80	534.63	537.00	1.65	3.90
11.00	18.00	534.63	537.00	1.66	3.53
12.00	16.50	534.63	537.00	1.66	3.22
13.00	15.23	534.63	537.00	1.67	2.97
13.47	14.70	534.63	537.00	1.67	1.32
			Total Heat transfer (Btu/lbm)		111.75

T-S Relations for Expansion Process

T-S Relations for Compression Process

Q.total Isentropic=0 BTU/lbm (out)

Q.total polytropic = 60.53 BTU/lbm (out)

Q.total isothermal = 111.75 BTU/lbm (out)

Legend:
- Isentropic
- PolyTropic, N=1.2
- Isothermal

Temperature (R) — vertical axis: 0.00, 200.00, 400.00, 600.00, 800.00, 1000.00, 1200.00, 1400.00

Entropy (Btu/lbm*R) — horizontal axis: 1.40, 1.45, 1.50, 1.55, 1.60, 1.65, 1.70
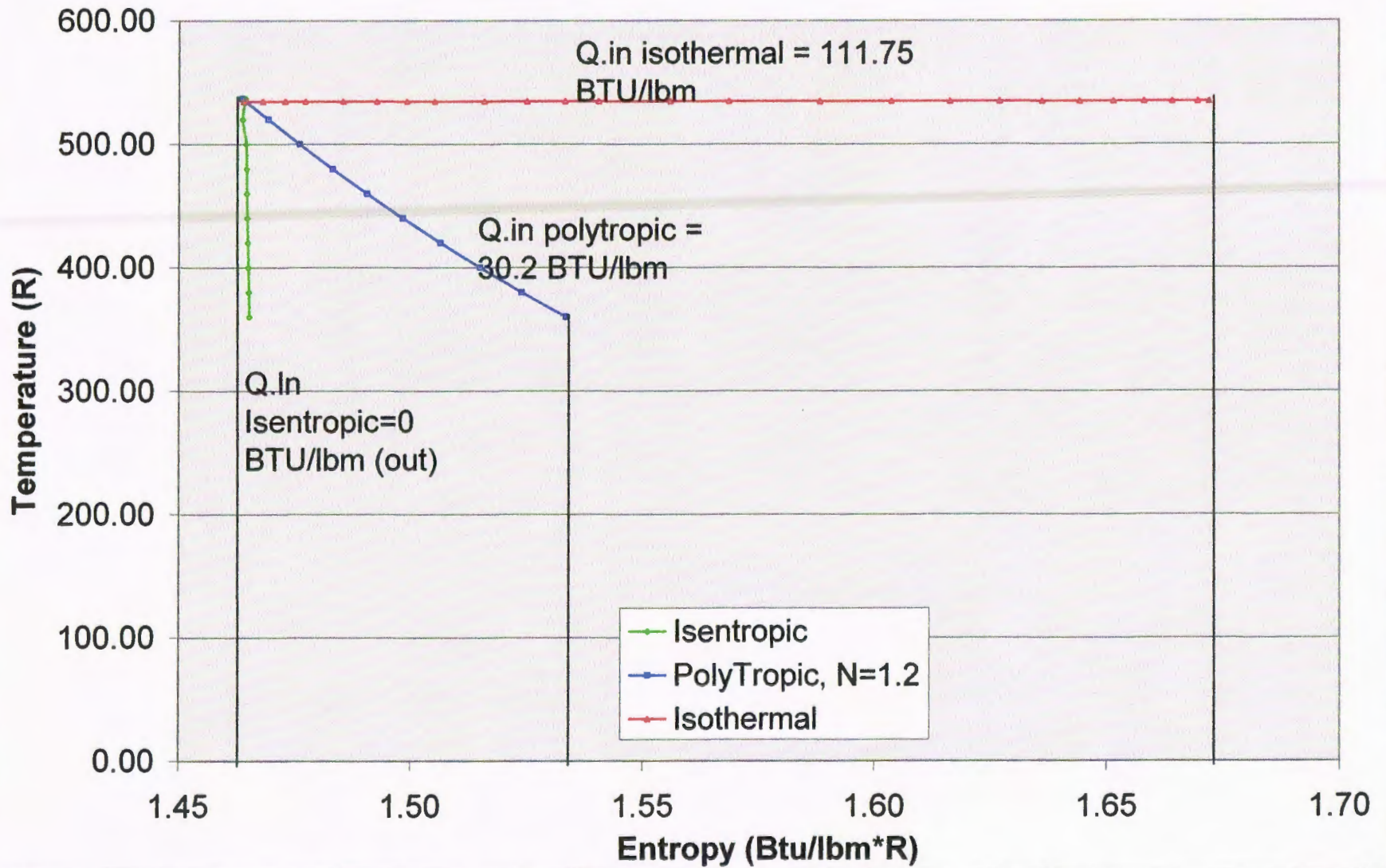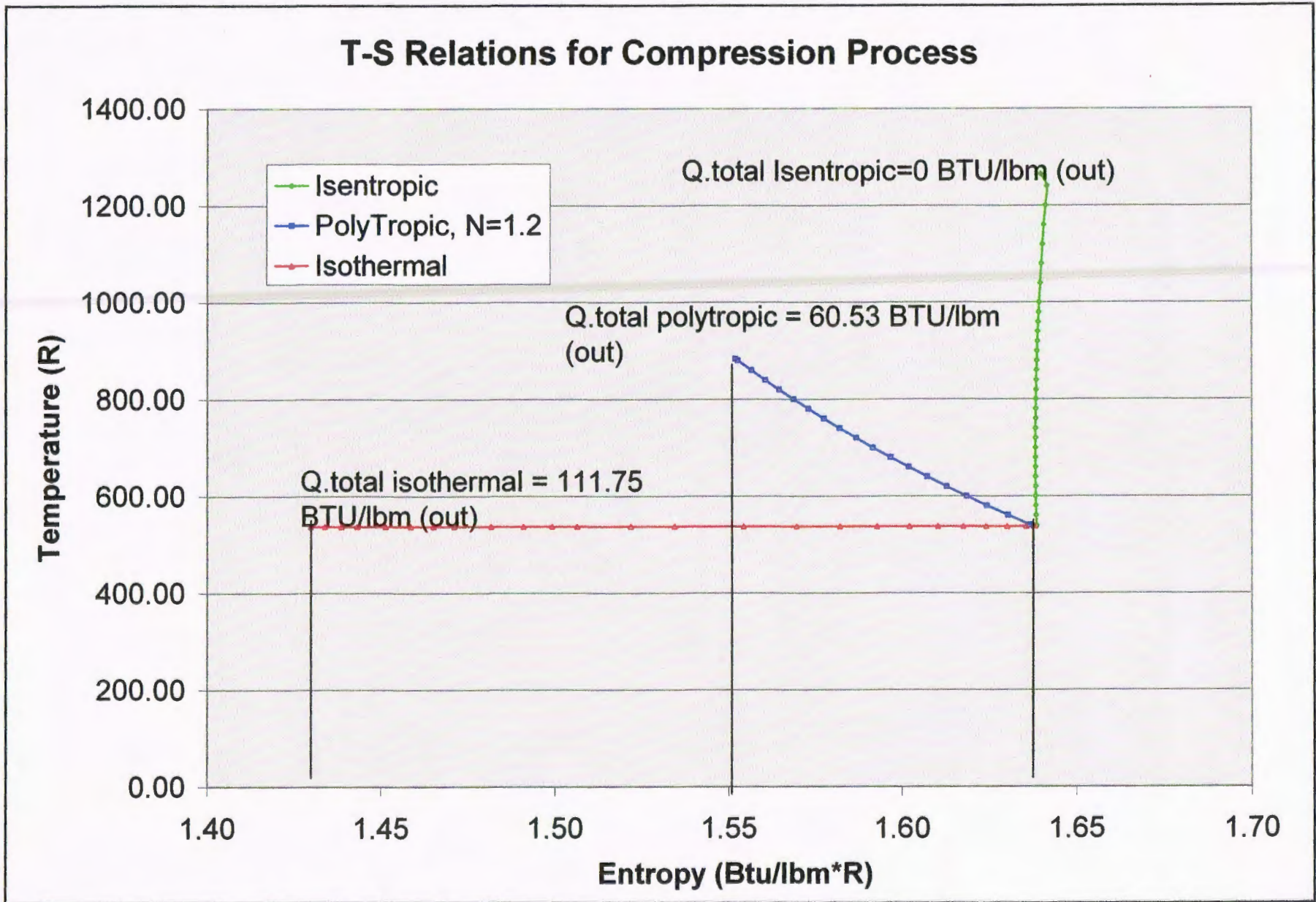
Calculation 11.1- Total System Efficiency

Calculation_011-Total_sys_eff_only

This analysis is based on the original calulation 11, which was developed to verify the Labview program. This analysis is used to determine the total system efficiency using the best results from all testing completed.

Background and Assumptions:

This analysis considers dry air. Moisture is removed by the compressor and this effect is an inherent part of the compressor efficiency determination. The compression efficiency is compared against a baseline process consisting of isentropic compression of dry air. Actual ambient air contains moisture which is condensed, adding additional energy to the process gas. It is thought that this may create a slightly conservative theoretical isentropic efficiency calculation for the compression process. After compression; the actual process gas is essentially moisture free and the assumption of dry air does not produce further error.

Efficiencies are determined using a single stage isentropic process as the baseline. The actual compressor is a three stage unit with intercooling between stages. Theoretically the compressor can have better efficiency than a single stage isentropic process. Intermediate pressures are not measured and the single stage isentropic baseline is used throughout this analysis to provide relative results for the different storage pressures tested. On the expansion side there are two pressure regulators which are analyzed considering both Isentropic and Isenthalpic efficiencies. Initial test data shows that although there is enthalpy gain through each pressure regulator there is also
a large entropy gain, which
limits available expansion power. The air motor is a single stage expansion process and efficiency is accurately determined considering single stage isentropic expansion.

Isentropic efficiencies are determined considering expansion from or compression to 1 bar pressure, which is slightly less than atmospheric pressure. This assumption greatly simplifies the automated LabVIEW calculations by allowing linear interpolation of pressure/entropy data rather than a bilinear interpolation of custom built entropy tables. Error analysis is performed to determine error associated with this assumption.

Ambient conditions significantly effect isentropic compression power and inherently effect the expansion process. The compression process requires more energy during high ambient or high humidity conditions. Regarding expansion; Increased ambient temperatures produce more enthalpy gain through the piping during the expansion process and may allow slightly higher motor power output. Ambient temperature is considered for the compression process but it's effect on expansion is only evident through increased heat transfer to the process air, likely slightly effecting results. At this time the LabVIEW program will not be updated to consider the effects of ambient conditions on expansion power. A separate post processing analysis may be performed after test data is acquired.

Calculation Method

All test data is presented in this analysis. Flow rate is determined using the change in source pressure during a measured time period. State tables are used to determine density, enthalpy, and entropy at each measurement location. Then efficiencies and losses are determined for each component based on standard thermodynamic analysis.

Test Point A Data- is row 5146 of Test_All; pertaining to best expansion efficiency on 10/17/11. LabVIEW export file has 54 columns containing log and calculated Data. Index to data is shown below.

$\text{Test\_001}_{\text{data}} :=$

	0	1
1	"Absolute Time (s)"	"Elapsed Time (s)"
2	$3.401 \cdot 10^9$	...

$$A_{\text{data}} := \text{submatrix}\left(\text{Test\_001}_{\text{data}}, 5145, 5145, 0, 53\right)$$

All point A data is displayed on the following Page. Note the step size is 2 indicating that the DP/Dt calculation is using Time and P1 data from 2 rows previous. These are parsed from the test data below. Pressure is converted to absolute pressure

$$P_{1\_\text{past}} := \left(14.7 + \text{submatrix}\left(\text{Test\_001}_{\text{data}}, 5143, 5143, 8, 8\right)\right)\frac{\text{lbf}}{\text{in}^2}$$

$$P_{1\_\text{past}} = (610.721)\cdot\frac{\text{lbf}}{\text{in}^2}$$

$$t_{1\_\text{past}} := \text{submatrix}\left(\text{Test\_001}_{\text{data}}, 5143, 5143, 0, 0\right)\cdot s$$

$$t_{1\_\text{past}} = (3401741120.096)\, s$$

	Point A, 5146 sec of Test_all, used to determine best total energy conversion efficiency			
0	Absolute Time (s)	3401741130	34	Rho3 (lbm/ft^3)	0.472709811
1	Elapsed Time (s)	3364.928993	35	Rho4 (lbm/ft^3)	0.122124126
2	Temp_1 (F)	64.37071266	36	PRV-1 Eff (%)	64.88937277
3	Temp_2 (F)	54.02046454	37	PRV-2 Eff (%)	97.35852213
4	Temp_3 (F)	58.23634607	38	Piping Enthalpy Gain (W)	64.71908948
5	Temp_4 (F)	17.12495058	39	Ideal Mtr Pwr (W)	1563.070046
6	DC PWR (W)	410.3265947	40	Available Expansion Pwr (W)	2474.177719
7	DC Volts (V)	14.82145875	41	Compressor Eff. (%)	-63493.96037
8	Press_1 (psi)	589.0065822	42	Air Compress Factor	0.987054843
9	Press_2 (psi)	83.81542441	43	DP/DT (psi/sec)	-0.692899421
10	Press_3 (psi)	75.77101989	44	M dot (lbm/min)	-1.786100452
11	Press_4 (psi)	6.836119819	45	H0_1S (Btu/lbm)	42.41529909
12	AC Pwr (W)	3.896713489	46	H4_3s (Btu/lbm)	73.47149647
13	AC WH Not Used	0	47	Mtr Vel Loss (W)	71.01026947
14	Ambient Temp (F)	72.98838664	48	Mtr Shaft Pwr (W)	294.613474
15	Load Res. (ohms)	0.535367783	49	Gen/Gear Eff (%)	139.2762487
16	Gen Speed (rpm)	1097.885833	50	TheoreticalMtr Eff (%)	18.84838589
17	Mtr Speed (rpm)	2823.134999	51	System Conversion Eff. (%)	16.58436222
18	mtr gear (# teeth)	14	52	step ind.	668
19	gen gear (# teeth)	36	53	loop ind.	673
20	Gen Constant (V/rpm)	0.0135	54	step size	5
21	Gear Ratio	0.388888889	55	H0_2s (BTU/lbm)	71.04052671
22	Stor Vol (ft^3)	8.23	56	Theoretical HP Reg Entropic Loss (W)	49.40282734
23	Avail Air (%)	14.87945486	57	Theoretical LP Reg Entropic Loss (W)	2.411758232
24	H1 (Btu/lbm)	121.1937318	58	Motor/Gen Eff. (%)	0.262513248
25	H2 (Btu/lbm)	122.1593575	59	H1_polytropic (BTU/lbm)	
26	H3 (Btu/lbm)	123.2400347	60	Motor/Gen Eff. (%)Corrected	
27	H4 (Btu/lbm)	113.8594686	61	TEST NUMBER	Test_009_10_17_11
28	S1 (Btu/lbm*F)	1.377675143	62	DeltaP_1_2	505.1911578
29	S2 (Btu/lbm*F)	1.499915537	63	DeltaP_2_3	8.044404515
30	S3 (Btu/lbm*F)	1.507974821	64	PRV2 Press ratio	1.106167299
31	S4 (Btu/lbm*F)	1.594353691	65	PRV1 Press Ratio	7.027424682
32	Rho1 (lbm/ft^3)	3.14891537	66	Stat_1*	0
33	Rho2 (lbm/ft^3)	0.519214505	67	Stat_2*	1

Supplemental Calculation #S5 Total Conversion Efficiency (Not calculated in LabVIEW)

A total conversion efficiency considering both compression and expansion is not calculated in LabVIEW because the linkage between these events is not defined. Total conversion efficiency can be presented in a variety of ways.

One possibility is looking at best compression efficiency and best expansion efficiency. This is not a true efficiency because the compressor must pump from the minimum storage pressure to the maximum storage pressure used. It is not possible to operate at one storage pressure. However a system could be designed to operate near these conditions, and it also serves as a maximum potential efficiency.

Another possibility is looking at total the ration of electrical energy out to total electrical energy in. This approach would be best performed after the most efficient expansion operating points were defined. This method will give a more obtainable overall efficiiency number regarding an actual operating system and would allow definition of optimum storage pressure ranges based on test data.

The CAES system also offers potential cooling capability due to the low temperature air discharge. Efficiencies can also be determined considering this system attribute. Two potential ways, along with the electrical energy method are calculated below.

Three total system efficiencies are determined:
1. Ratio of energy out to energy in. This represents the quality of storing electrical power only and ignores the potential cooling effect of the air leaving the air motor.
2. The coefficient of performance for the air stream as a cooling source is determined using the difference between energy in and energy out as the energy required to perform the cooling operation. This is a measure of the systems performance as a refrigeration or cooling system.
3. A multi-purpose consideration of efficiency that includes cooling and electrical storage efficiency. Cooling quantity is translated to typical electrical demand associated with a modern ammonia refrigeration system. This calculation assumes 1.2 kW/ton as a standard cooling plant efficiency.

This sample calculation extrapolates data from the two operating points used in this analysis. Actual calculations will be performed in Excel using an actual operating period for both compression and expansion. Compression flow rate is less than the expansion flow rate; therefore either a longer time period will be used or the data will be scaled to match the amount of air used for expansion.

$Test_{time} := 10min$

Compressor operating characteristics:

Compressor operating points are the representative of peak steady state compression efficiency observed during testing. The particular data point being used is row 5925 of Full_Compression file. The storage pressure is well above that of the storage pressure where best expansion efficiency was observed; therefore it is theoretically possible to operate the CAES demonstration system with these calculated efficiencies. Labview Data for the compression characteristics used is shown below.

	Point B, 5925 sec of Full_compression, used to determine best total energy conversion efficiency, Compression side			
0	Absolute Time (s)	3401732945	30	S3 (Btu/lbm*F)	1.65181864
1	Elapsed Time (s)	5925.635242	31	S4 (Btu/lbm*F)	1.649055854
2	Temp_1 (F)	95.0918743	32	Rho1 (lbm/ft^3)	14.70033177
3	Temp_2 (F)	91.57311955	33	Rho2 (lbm/ft^3)	0.069951632
4	Temp_3 (F)	101.2672443	34	Rho3 (lbm/ft^3)	0.069846506
5	Temp_4 (F)	94.70030451	35	Rho4 (lbm/ft^3)	0.070560871
6	DC PWR (W)	-3.434121813	36	PRV-1 Eff (%)	-0.335958813
7	DC Volts (V)	5.00679E-05	37	PRV-2 Eff (%)	-7.839877219
8	Press_1 (psi)	3183.321597	38	Piping Enthalpy Gain (W)	-250.3499023
9	Press_2 (psi)	-0.419142121	39	Ideal Mtr Pwr (W)	-0.37856616
10	Press_3 (psi)	-0.189861372	40	Available Expansion Pwr (W)	-1437.296905
11	Press_4 (psi)	-0.211763666	41	Compressor Eff. (%)	26.13803105
12	AC Pwr (W)	5498.872131	42	Air Compress Factor	1.051527904
13	AC WH Not Used	-0.004928112	43	DP/DT (psi/sec)	0.378727634
14	Ambient Temp (F)	86.42452306	44	M dot (lbm/min)	0.865648499
15	Load Res. (ohms)	-7E-10	45	H0_1S (Btu/lbm)	23.49634202
16	Gen Speed (rpm)	0.003708734	46	H4_3s (Btu/lbm)	134.152261
17	Mtr Speed (rpm)	0.009536744	47	Mtr Vel Loss (W)	-21.49320125
18	mtr gear (# teeth)	14	48	Mtr Shaft Pwr (W)	-24.0620459
19	gen gear (# teeth)	36	49	Gen/Gear Eff (%)	14.2719444
20	Gen Constant (V/rpm)	0.0135	50	TheoreticalMtr Eff (%)	6356.100578
21	Gear Ratio	0.388888889	51	System Conversion Eff. (%)	0.238929187
22	Stor Vol (ft^3)	8.23	52	step ind.	1179
23	Avail Air (%)	75.04080949	53	loop ind.	1184
24	H1 (Btu/lbm)	117.9213945	54	step size	5
25	H2 (Btu/lbm)	131.8433615	55	H0_2s (BTU/lbm)	132.1605908
26	H3 (Btu/lbm)	134.1771314	56	Theoretical HP Reg Entropic Loss (W)	-82.01351403
27	H4 (Btu/lbm)	132.5963446	57	Theoretical LP Reg Entropic Loss (W)	0.296138068
28	S1 (Btu/lbm*F)	1.253065917	58	Motor/Gen Eff. (%)	9.071391407
29	S2 (Btu/lbm*F)	1.64822597	59	H1_polytropic (BTU/lbm)	

$$AC\_pwr := 5498.87\,W \qquad M_{dot1} := .8656\,\frac{lbm}{min}$$

Air motor operating characteristics

$$M_{dot} := -1.7861\,\frac{lbm}{min} \qquad T_4 := 17.125\,°F \quad \text{air motor exit temp}$$

Generator output

$$DC\_pwr := 410.327\,W \qquad DC_{energy} := Test_{time} \cdot DC\_pwr$$

$$DC_{energy} = 0.068 \cdot kW \cdot hr$$

Output flow for 10 minutes results in the following total air usage and potential cooling.

$$Air\_usage := Test_{time} \cdot -M_{dot} \qquad\qquad C_{p\_air} := .240\,\frac{BTU}{lbm \cdot R}$$

$$Air\_usage = 17.861 \cdot lb \qquad\qquad T_{77} := 77\,°F$$

Cooling Power

$$Cool_{cap} := Air\_usage \cdot C_{p\_air} \cdot (T_{77} - T_4)$$

$$TR := 12000 \frac{BTU}{hr}$$

$$Cool_{cap} = 0.075 \cdot kW \cdot hr$$

Based on 77 F cooling space.

$$Cool_{pwr} := \frac{Cool_{cap}}{Test_{time}} \qquad Cool_{pwr} = 0.128 \cdot TR$$

Tons of refrigeration provided by the test bed

Compressor energy to compress that much air:

$$CMP_{energy} := \frac{Air\_usage}{M_{dot1}} \cdot AC\_pwr$$

Extrapolates compressor energy to match air consumed

$$CMP_{energy} = 1.891 \cdot kW \cdot hr$$

Electrical Storage Efficiency
(efficiency method 1)

$$\eta_{elec} := \frac{DC_{energy}}{CMP_{energy}}$$

$$\eta_{elec} = 3.616 \cdot \%$$ **Efficiency as an electrical storage device**

Cooling system Efficiency
(efficiency method 2)
This is a refrigeration coefficient of performance (Cooling in/ work to get the cooling)

$$Cool_{COP} := \frac{Cool_{cap}}{CMP_{energy}}$$

$$Cool_{COP} = 0.04$$ **Very poor COP, modern systems achieve COP's >3.0**

Total System Efficiency
(efficiency method 3)

$$\eta_{total} := \frac{DC_{energy} + Cool_{pwr} \cdot 1.2 \frac{kW}{TR} \cdot Test_{time}}{CMP_{energy}}$$

$$\eta_{total} = 4.974 \cdot \%$$ **Total system efficiency including cooling effect based on 1.2 kW/TR refrigeration power equivalency.**

Calculation 21 - Theoretical Isothermal Compression Energy Requirements

This Calculation determines theoretical Isothernal Compression energy requirements. It is derived from Calcualtion 15, which develops theoretical efficiency relations for the compression process. Like calcualtion 15 It analyzes an open system with closed storage vessel using an Ideal gas model and constant specific heats. All energy expressions are on a per mass basis, titled as work terms, which they would become when multiplied by an air mass.

Unit Conversion, Constants, and Thermodynamic State Determination

$$\text{Bar} := 14.5038 \; \frac{\text{lbf}}{\text{in}^2} \qquad \text{kJ} := 1000\text{J} \qquad \gamma_{air} := 1.4 \qquad \text{Heat capacity ratio for air}$$

Median Pressures of bands used in Table 7.2-1 of final report

Theoretical Isothernal Compression efficiency

$$P_{amb} := 14.7 \frac{\text{lbf}}{\text{in}^2} \qquad T_{amb} := 75 \,°\text{F} \qquad\qquad T_3 := T_{amb}$$

$$R_{air} := 53.33 \frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot\text{R}} \qquad T_{amb} = 297.039\cdot\text{K}$$

$$v(P, T) := \frac{R_{air}\cdot T}{P} \qquad \text{Ideal gas law}$$

$$W_{1\_3\_isoT}(P) := \int_{P_{amb}}^{P} -v\left(P, T_{amb}\right) dP$$

Relation for work required to compress on a per mass basis. Note T is constant in this case, work is in, noted by the neg. Median pressures from bands in table 7.2-1 are determined below

$$W_{1\_3\_isoT}\left(250 \frac{\text{lbf}}{\text{in}^2}\right) = -103.83\cdot\frac{\text{BTU}}{\text{lbm}} \qquad\qquad W_{1\_3\_isoT}\left(2750 \frac{\text{lbf}}{\text{in}^2}\right) = -191.695\cdot\frac{\text{BTU}}{\text{lbm}}$$

$$W_{1\_3\_isoT}\left(750 \frac{\text{lbf}}{\text{in}^2}\right) = -144.086\cdot\frac{\text{BTU}}{\text{lbm}} \qquad\qquad W_{1\_3\_isoT}\left(3250 \frac{\text{lbf}}{\text{in}^2}\right) = -197.816\cdot\frac{\text{BTU}}{\text{lbm}}$$

$$W_{1\_3\_isoT}\left(1250\,\frac{lbf}{in^2}\right) = -162.804 \cdot \frac{BTU}{lbm}$$

$$W_{1\_3\_isoT}\left(3750\,\frac{lbf}{in^2}\right) = -203.06 \cdot \frac{BTU}{lbm}$$

$$W_{1\_3\_isoT}\left(1750\,\frac{lbf}{in^2}\right) = -175.133 \cdot \frac{BTU}{lbm}$$

$$W_{1\_3\_isoT}\left(4165\,\frac{lbf}{in^2}\right) = -206.906 \cdot \frac{BTU}{lbm}$$

$$W_{1\_3\_isoT}\left(2250\,\frac{lbf}{in^2}\right) = -184.342 \cdot \frac{BTU}{lbm}$$

Efficiency Determinations.

All efficiencies are determined below using measured power inputs and flow rates during compression testing. Efficiency is determined by the ratio of Power out to Power in.

Input_pwr :=

	0	1
0	0.609	$4.368 \cdot 10^3$
1	0.614	$4.724 \cdot 10^3$
2	0.63	$4.914 \cdot 10^3$
3	0.623	$5.145 \cdot 10^3$
4	0.649	$5.33 \cdot 10^3$
5	0.665	$5.492 \cdot 10^3$
6	0.668	$5.632 \cdot 10^3$
7	0.665	$5.755 \cdot 10^3$
8	0.636	$5.066 \cdot 10^3$

Flow rate and input power Data imported from "Full Compression.xls"

Considering an Isothermal Compression basis the Demonstration CAES system efficiency for each pressure band is (Theoretical Pwr in/ Actual Pwr in):

$$\eta_{IsoT\_250} := \frac{-W_{1\_3\_isoT}\left(250\,\frac{lbf}{in^2}\right) \cdot Input\_pwr_{0,0}\,\frac{lbm}{min}}{Input\_pwr_{0,1}\,W}$$

$$\eta_{IsoT\_250} = 25.475\,\%$$

This relation is put into function form and evaluated for all median pressure bands below

$$\eta_{IsoT}(P,x) := \frac{-W_{1\_3\_isoT}\left(P\frac{lbf}{in^2}\right)\cdot Input\_pwr_{x,0}\frac{lbm}{min}}{Input\_pwr_{x,1}\,W}$$

$\eta_{IsoT}(250,0) = 25.475\,\%$ Matches original Calcualtion, other median values follow

$\eta_{IsoT}(250,0) = 25.475\,\%$ $\eta_{IsoT}(2250,4) = 39.493\,\%$

$\eta_{IsoT}(750,1) = 32.953\,\%$ $\eta_{IsoT}(2750,5) = 40.836\,\%$

$\eta_{IsoT}(1250,2) = 36.689\,\%$ $\eta_{IsoT}(3250,6) = 41.279\,\%$

$\eta_{IsoT}(1750,3) = 37.292\,\%$ $\eta_{IsoT}(3750,7) = 41.254\,\%$

$\eta_{IsoT}(4165,8) = 45.693\,\%$

Isothermal Efficiency baselines show improved performance at the higher pressures while the Isentropic efficiency (reference Excel file "Full Compression.xls") is relatively flat. The Labview reported efficiency considers the available Isentropic expansion power as the baseline. Meaning that the actual input energy and the available energy to perform work by isentropic expansion are being compared. Isentropic expansion power does not increase with increasing storage pressure as much as the isothermal case becasue it does not assume the benefit of heat transfer with the environment during expansion. At the higher storage pressures significant heat addition is required to maintain an Isothermal expansion. The Isothermal case is a reversible process; that is compression will require the same quantity of power to compress to storage pressure that expansion from stroage pressure will produce. This means that the Isothermal efficiency can be looked at either as the ratio of avaialble isothermal expansion power/ actual input power or as the ratio of ideal isothernal compression power/actual input power.