



## Strathprints Institutional Repository

**Galindo-Rosales, Francisco J and Campo-Deaño, Laura and Sousa, P.C. and Ribeiro, Vera M. and Oliveira, Monica and Alves, M.A. and Pinho, F.T. (2014) Viscoelastic instabilities in microscale flows. Experimental Thermal and Fluid Science. pp. 128-139. ISSN 0894-1777 , <http://dx.doi.org/10.1016/j.expthermflusci.2014.03.004>**

This version is available at <http://strathprints.strath.ac.uk/47471/>

**Strathprints** is designed to allow users to access the research output of the University of Strathclyde. Unless otherwise explicitly stated on the manuscript, Copyright © and Moral Rights for the papers on this site are retained by the individual authors and/or other copyright owners. Please check the manuscript for details of any other licences that may have been applied. You may not engage in further distribution of the material for any profitmaking activities or any commercial gain. You may freely distribute both the url (<http://strathprints.strath.ac.uk/>) and the content of this paper for research or private study, educational, or not-for-profit purposes without prior permission or charge.

Any correspondence concerning this service should be sent to Strathprints administrator: [strathprints@strath.ac.uk](mailto:strathprints@strath.ac.uk)

# Viscoelastic instabilities in micro-scale flows

Francisco J. Galindo-Rosales<sup>a</sup>, Laura Campo-Deaño<sup>a</sup>, Patrícia C. Sousa<sup>a</sup>,  
Vera M. Ribeiro<sup>a</sup>, Mónica S. N. Oliveira<sup>b</sup>, Manuel A. Alves<sup>a</sup>, Fernando T.  
Pinho<sup>a,\*</sup>

<sup>a</sup>*CEFT, Faculdade de Engenharia da Universidade do Porto, Rua Dr. Roberto Frias,  
4200-465 Porto, Portugal*

<sup>b</sup>*James Weir Fluids Laboratory, Department of Mechanical and Aerospace Engineering,  
University of Strathclyde, Glasgow G1 1XJ, UK*

---

## Abstract

Many artificial and natural fluids contain macromolecules, particles or droplets that impart complex flow behaviour to the fluid. This complex behaviour results in a non-linear relationship between stress and deformation standing in between Newton's law of viscosity for an ideal viscous liquid and Hooke's law for an ideal elastic material. Such non-linear viscoelastic behaviour breaks down flow reversibility under creeping flow conditions, as encountered at the micro-scale, and can lead to flow instabilities. These instabilities offer an alternative to the development of systems requiring unstable flows under conditions where chaotic advection is unfeasible. Flows of viscoelastic fluids are characterized by the Weissenberg ( $Wi$ ) and Reynolds ( $Re$ ) numbers, and at the micro-scale flow instabilities occur in regions in the  $Wi$ - $Re$  space typically unreachable at the macro-scale, namely high  $Wi$  and low  $Re$ . In this paper, we review recent experimental work by the authors on the topic of elastic instabilities in flows having a strong extensional component, includ-

---

\*Corresponding author. Tel:+ 35 22 508 15 97  
Email address: [fpinho@fe.up.pt](mailto:fpinho@fe.up.pt) (Fernando T. Pinho )

1  
2  
3  
4  
5 ing: flow through a hyperbolic contraction followed by a sudden expansion;  
6 flow in a microfluidic diode and in a flow focusing device; flow around a  
7 confined cylinder; flow through porous media and simplified porous media  
8 analogues. These flows exhibit different types of flow transitions depending  
9 on geometry,  $Wi$  and  $Re$ , including: transition from a steady symmetric to  
10 a steady asymmetric flow, often followed by a second transition to unsteady  
11 flow at high  $Wi$ ; direct transition between steady symmetric and unsteady  
12 flows.  
13  
14  
15  
16  
17  
18

19 *Keywords:* Viscoelasticity, Elastic instabilities, Microfluidics  
20  
21  
22

---

## 23 1. Introduction

24  
25  
26 2 The use of microfluidic devices is growing fast in a variety of applications  
27  
28 3 in biochemistry, drug delivery, medical diagnosis, micro-heat exchangers, se-  
29  
30 4 quencing and synthesis of chemicals, micro-mixing or micro-rheology, among  
31  
32 5 others [1]. Most of the research in this field concerns Newtonian fluids [2]  
33  
34 6 with their flows characterized by linear behaviour typical of low Reynolds  
35  
36 7 number flows, although chaotic advection can also occur in laminar flows  
37  
38 8 at sufficiently high Reynolds numbers [3]. However, a significant number of  
39  
40 9 fluids (natural and mostly synthetic) includes macromolecules, particles or  
41  
42 10 droplets that impart complex properties to the fluids, such as viscoelasticity,  
43  
44 11 thus breaking the flow reversibility typical of inertialess flow of Newtonian  
45  
46 12 fluids [4].

47  
48 13 The Reynolds number ( $Re$ ) is defined as the ratio between inertial and vis-  
49  
50 14 cous forces,  $Re = \rho UL/\eta$ , and often requires the definition of a characteristic  
51  
52 15 rate of deformation  $\dot{\gamma} \approx U/L$  from relevant velocity ( $U$ ) and length scales ( $L$ )  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16 in order to determine the characteristic shear viscosity  $\eta$  for non-Newtonian  
17 fluids. The ratio between the relaxation time of the fluid ( $\lambda$ ) and a charac-  
18 teristic time scale of the flow ( $t_{flow} = L/U$ ) is the so-called Deborah number  
19 ( $De$ ) given by  $De = \lambda U/L$ , which is a dimensionless measure of the rate of  
20 change of flow conditions related to flow unsteadiness in a Lagrangian per-  
21 spective [5]. In shear flows, viscoelastic fluids are also subject to shear-driven  
22 normal stresses and the ratio between the normal and shear stresses quanti-  
23 fies the non-linear response of the viscoelastic fluid and is proportional to the  
24 Weissenberg number ( $Wi = \lambda\dot{\gamma}$ ), so that the final form of the Weissenberg  
25 number in many flows looks like that of the Deborah number, but it has a  
26 clearly different physical meaning (and in some cases may also involve a dif-  
27 ferent length scale when normal stresses and Lagrangian transients co-exist)  
28 [5, 6].

29 The small length scales in microfluidics increases significantly the relevance  
30 of fluid elasticity and allows exploring regions of the  $Wi-Re$  parameter space  
31 typically unreachable at the macro-scale, i.e. low  $Re$  and high  $Wi$ , repre-  
32 sented by large values of the Elasticity number ( $El = Wi/Re = \lambda\eta/\rho L^2$ ) [7].  
33 Therefore, at the micro-scale, flows of viscoelastic fluids can be significantly  
34 different from those of their Newtonian counterparts, because elastic insta-  
35 bilities can be triggered with relative ease. One illustrative example of this  
36 phenomenon can be found in the human circulatory system, as there is re-  
37 cent evidence of viscoelastic behaviour of human blood and plasma, which is  
38 enhanced in the microcirculation [8, 9, 10]. Purely elastic flow instabilities in  
39 shear flow have been extensively studied both at macro [11] and micro-scales  
40 [12] and it is widely accepted that the underlying mechanism is related to the

1  
2  
3  
4  
5 41 elastic normal stresses developing along curved streamlines being unable to  
6  
7 42 sustain minor perturbations appearing on those streamlines [13, 14, 15, 16].  
8  
9 43 Pakdel and McKinley [14, 15, 17] showed that the critical conditions for the  
10  
11 44 onset of purely elastic instabilities can be described for a wide range of flows  
12  
13 45 by a single dimensionless parameter ( $M$ ), which accounts for elastic normal  
14  
15 46 stresses and streamline curvature:

$$M = \sqrt{\frac{\lambda v \tau_{11}}{\mathfrak{R} \tau_{12}}}, \quad (1)$$

16  
17  
18  
19  
20  
21 47 where  $\lambda$  is the relaxation time of the fluid,  $v$  is the local streamwise fluid  
22  
23 48 velocity,  $\tau_{11}$  is the local tensile stress in the flow direction,  $\tau_{12}$  is the shear  
24  
25 49 stress ( $\tau_{12} = \eta\dot{\gamma}$ ) and  $\mathfrak{R}$  is the streamline local radius of curvature. When  
26  
27 50 the flow conditions are such that  $M$  locally exceeds a critical value,  $M_{crit}$ ,  
28  
29 51 elastic instabilities develop. The value of  $M_{crit}$  is slightly dependent on the  
30  
31 52 flow, and for simple flows, where the radius of curvature is known,  $M_{crit}$   
32  
33 53 can be estimated. As discussed by McKinley et al. [15], for Taylor-Couette  
34  
35 54 flow  $M_{crit} \approx 5.9$  and for torsional flow in a cone-and-plate arrangement,  
36  
37 55  $M_{crit} \approx 4.6$ . For more complex flows (see example in Figure 1), the spatial  
38  
39 56 variation of  $M$  needs to be taken into account to identify the critical regions  
40  
41 57 where the largest value of  $M$  occurs. This mechanism for the onset of purely  
42  
43 58 elastic instabilities and the applicability of the  $M$  parameter to identify the  
44  
45 59 critical conditions for the onset of elastic instabilities was confirmed numer-  
46  
47 60 ically by Alves and Poole [18] for creeping flow of upper-convected Maxwell  
48  
49 61 (UCM) fluids in smooth contractions, for a wide range of contraction ratios.

50  
51 62  
52 63 The experimental and numerical studies of purely elastic flow instabilities

1  
2  
3  
4  
5 64 developing in several micro-geometries for extensional-dominated flows have  
6  
7 65 essentially emerged in the last decade (e.g. [12, 19, 20]). Among these ar-  
8  
9 66 rangements, viscoelastic flow in contraction geometries has been the subject  
10  
11 67 of numerous investigations. Although the instability onset can be linked to  
12  
13 68 the ubiquitous presence of large normal stresses, and streamline curvature  
14  
15 69 is also present, in other micro-geometries able to generate extensional domi-  
16  
17 70 nated flows (e.g. stagnation/ flow focusing devices) or mixed kinematic flows  
18  
19 71 (e.g. contraction/expansions), a clear picture of all the observed transitions  
20  
21 72 and their causes has not yet emerged [21]. These are conditions that justify  
22  
23 73 our research program exploring various flows possessing a strong extensional  
24  
25 74 deformation flow field in order to identify common features. These flows ex-  
26  
27 75 hibit a rich variety of unexpected effects, usually anchored on elastic effects.  
28  
29 76 The high sensitivity of viscoelastic fluid flow at the micro-scale to flow insta-  
30  
31 77 bilities under creeping flow conditions offer an alternative to the development  
32  
33 78 of systems requiring unstable flows under conditions where chaotic advection  
34  
35 79 is impossible or difficult to achieve, while they also impose limits of operation  
36  
37 80 for systems where instabilities are to be avoided at all, as in micro-rheology.  
38  
39 81 The next section briefly describes the experimental techniques prior to the  
40  
41 82 presentation and discussion of experimental results obtained in five different  
42  
43 83 geometrical arrangements, where the common thread is the presence of a  
44  
45 84 strong extensional flow component and the onset of elastic instabilities at  
46  
47 85 sufficiently high  $Wi$ .

## 2. Materials and methods

The viscoelastic fluids typically used in our works consist of aqueous polymer solutions with a high molecular weight, namely polyacrylamide (PAA,  $M_w=18\times 10^6$  g mol<sup>-1</sup>, Polysciences) or polyethylene oxide (PEO,  $M_w=8\times 10^6$  g mol<sup>-1</sup>, Sigma-Aldrich), at different weight concentrations (see Table 1 for details).

All these fluids were characterized rheologically under simple shear flow by means of a rotational rheometer (Physica MCR301, Anton Paar) to determine the dependence of the shear viscosity on the shear rate (Figure 2a). In this experiment the first normal stress difference ( $N_1$ ) can also be determined from the measured normal force. However, measuring  $N_1$  in dilute and semi-dilute polymer solutions is rather difficult and obtaining reproducibility in the results is challenging. In these cases, it is more effective to perform the characterization under uniaxial elongational flow as in a capillary-breakup extensional rheometer (Haake CaBER1, Thermo Scientific), which measures the time evolution of the diameter of a stretched fluid filament as it thins, from which the longest relaxation time of the fluids can be determined (Figure 2b) [22]. Despite the significant differences between a steady shear flow (as in a shear rheometer) and an unsteady extensional flow (as in the CaBER device), there is a direct relationship between the elastic properties measured in both devices. Indeed, Zell et al. [23] have found that the normal stress coefficient, measured in shear flow and defined as  $\Psi_{10} = \frac{N_1}{\dot{\gamma}^2}$ , shows a quadratic dependence on the relaxation time determined in the CaBER. Since the normal stress difference coefficient depends on a shear relaxation time, the relation between the shear and extensional relaxation times may

1  
2  
3  
4  
5 111 seem obvious but there are different dependences that remain unexplained  
6  
7 112 as discussed by Zell et al. [23].  
8  
9 113

10  
11 114 The viscoelastic fluids used in the experiments usually exhibit a shear  
12  
13 115 thinning behaviour, i.e. their apparent shear viscosity decreases with the in-  
14  
15 116 crease in the applied shear rate. Moreover, the higher the molecular weight  
16  
17 117 and the concentration of the polymer additive, the higher the shear viscosity,  
18  
19 118 the stronger the shear thinning behaviour and the longer the relaxation time  
20  
21 119 will be [24, 25]. These effects are clearly shown in Figure 2, where the viscos-  
22  
23 120 ity of the various fluids is plotted as a function of the shear rate in Figure 2a  
24  
25 121 and the time evolution of the normalized diameters measured in the CaBER  
26  
27 122 device are shown in Figure 2b. A small percentage of glycerol was added to  
28  
29 123 one of the fluids, in order to increase slightly the viscosity of the solution  
30  
31 124 increasing in turn the relaxation time of the polymer solution. Typically,  
32  
33 125 Boger fluids are prepared by dissolving a flexible polymer with large molec-  
34  
35 126 ular weight in a high viscosity solvent, to minimize the shear thinning due  
36  
37 127 to the addition of the polymer. However, the use of high-viscosity fluids in  
38  
39 128 microfluidics is precluded by the large pressure drops their flows develop with  
40  
41 129 severe consequences to the structural integrity of the chips. The advent of  
42  
43 130 Boger fluids with low viscosity is a useful alternative for microfluidics when  
44  
45 131 it is important to distinguish between shear thinning and elastic effects. By  
46  
47 132 adding salt to aqueous solutions of polyacrylamide, Aitkadi et al. [26] found  
48  
49 133 that the salt has a stabilizing effect on the shear viscosity while maintaining  
50  
51 134 adequate levels of elasticity, thus resulting in a Boger fluid behaviour, a good  
52  
53 135 example being the PAAsg200 fluid shown in Figure 2.  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65



1  
2  
3  
4  
5 136 No storage or loss moduli ( $G'$  and  $G''$ ) are presented because it was not pos-  
6  
7 137 sible to measure  $G'$  accurately in small amplitude oscillatory shear (SAOS)  
8  
9 138 in this rotational rheometer due to instrument limitations and the low elas-  
10  
11 139 ticity of the fluids used. All the rheological experiments, as well as all the  
12  
13 140 microfluidic experiments, were performed at a constant temperature of 20 °C.  
14  
15 141

16 142 The geometries used were planar micro-channels made of polydimethylsilox-  
17  
18 143 ane (PDMS) fabricated from SU-8 photoresist moulds using standard soft-  
19  
20 144 lithography techniques, as described in [27, 28]. The micro-fabrication pro-  
21  
22 145 cess consists of four fundamental steps: 1) drawing of the micro-geometries  
23  
24 146 using a CAD software; 2) manufacture of the mask using the CAD drawings  
25  
26 147 - this type of mask is obtained by sputtering a chrome layer onto a glass or  
27  
28 148 quartz substrate in which the CAD image is etched; 3) manufacture of the  
29  
30 149 SU-8 mould by photolithography using the mask; and 4) fabrication of the  
31  
32 150 PDMS micro-channels using the mould. High-resolution chrome masks were  
33  
34 151 employed to obtain high quality SU-8 moulds with nearly vertical side-walls  
35  
36 152 and well-defined corner features. The SU-8 photoresist was used to create  
37  
38 153 a positive-relief on the mould surface, producing a mould containing the in-  
39  
40 154 verse structure of the micro-channels. The microfluidic devices were then  
41  
42 155 fabricated by casting PDMS on the mould. The PDMS used (Sylgard 184,  
43  
44 156 Dow Corning) is commercially available as a prepolymer kit composed of a  
45  
46 157 PDMS oligomer and a crosslinking agent or curing agent, which are mixed  
47  
48 158 in certain proportions (typically 50:1, 20:1 and 10:1 PDMS:curing agent) to  
49  
50 159 produce the polymer used to fabricate the micro-devices. An overview of the  
51  
52 160 micro-channel fabrication procedure is shown schematically in Figure 3. In

1  
2  
3  
4  
5 161 the next section the different geometrical arrangements and their influence  
6  
7 162 on the fluid flow will be described in detail. Figure 4 shows scanning electron  
8  
9 163 microscopy (SEM) pictures of the micro-channels used and Table 2 lists the  
10  
11 164 corresponding dimensions.

12 165 Pressure measurements were performed using differential pressure transduc-  
13  
14 166 ers (Honeywell 26 PC sensors) operating up to a maximum differential pres-  
15  
16 167 sure of 210 kPa. Pressure transducers with different ranges and sensitivities  
17  
18 168 were used depending on the fluids and flow rates tested. The flow rate was  
19  
20 169 always imposed using a syringe pump, such as the neMESYS (Cetoni GmbH)  
21  
22 170 and PHD 2000 (Harvard Apparatus), equipped with different Hamilton sy-  
23  
24 171 ringes (from 25 $\mu$ l to 1ml, depending on the required flow rate) to ensure  
25  
26 172 pulsation-free dosing.

27  
28 173 Flow visualizations were based on streak photography with the micro-channels  
29  
30 174 placed on the motorized stage of an inverted epi-fluorescence microscope  
31  
32 175 (DMI 5000M, Leica Microsystems GmbH) equipped with a sensitive monochro-  
33  
34 176 matic CCD camera (DFC350 FX, Leica Microsystems GmbH). The fluids  
35  
36 177 were seeded with fluorescent tracer particles (1  $\mu$ m diameter Nile Red par-  
37  
38 178 ticles, Molecular Probes, Invitrogen, Ex/Em: 520/580 nm) and the illu-  
39  
40 179 mination was provided by a 100 W mercury lamp operating together with  
41  
42 180 adequate excitation and emission filters and a dichroic mirror.

43 181 For the velocity measurements, a micro-Particle Image Velocimetry ( $\mu$ PIV)  
44  
45 182 system from Dantec Dynamics was used. Images of the emitted radiation  
46  
47 183 from excited fluorescent tracer particles were captured using a 20 $\times$  (numer-  
48  
49 184 ical aperture, NA=0.4) microscope objective and a camera (Flow Sense 4M,  
50  
51 185 Dantec Dynamics), with a resolution of 2048  $\times$  2048 pixels and running on

1  
2  
3  
4  
5 186 double frame mode. The flow was illuminated by a double-pulsed 532 nm  
6  
7 187 Nd:YAG laser (Dual Power 65-15, Dantec Dynamics). The fluorescent par-  
8  
9 188 ticles used in  $\mu$ PIV were smaller than those used in streak photography and  
10  
11 189 had a diameter of 0.5  $\mu$ m.

### 14 190 **3. Results and Discussion**

15  
16 191 In this section, we review five different experimental investigations in extensional-  
17  
18 192 dominated flows of viscoelastic fluids which explore the onset of elastic insta-  
19  
20 193 bilities. The case studies focus on fundamental issues to understand the flow  
21  
22 194 dynamics itself, but also cover more applied research ranging from fluid char-  
23  
24 195 acterization in a microfluidic rheometer to the development of a microfluidic  
25  
26 196 diode, which can be used as a flow rectifier or in micro-pumps.

#### 27 28 29 197 *3.1. Microfluidic hyperbolic contraction and sudden expansion*

30  
31 198 The flow of Boger fluids in hyperbolic contraction micro-channels was in-  
32  
33 199 vestigated to assess the relation between the observed flow patterns and the  
34  
35 200 dimensionless relaxation time of the fluids. The underlying rationale for us-  
36  
37 201 ing hyperbolic-shaped channels is their ability to generate strong extensional  
38  
39 202 flows with enhanced strain-rate homogeneity near the centreline when com-  
40  
41 203 pared to other contraction flows [29, 30]. This geometry is thus sensitive  
42  
43 204 to effects of elongational viscosity and the use of Boger fluids allows a clear  
44  
45 205 separation between elastic and viscous effects [31].

46 206 In their investigation on the flow through hyperbolic contraction and sudden  
47  
48 207 expansion, Campo-Deaño et al. [32] used aqueous solutions of PAA at differ-  
49  
50 208 ent weight concentrations (50, 125, 250 and 400 ppm) to which 1% of NaCl  
51  
52 209 were added (Table 1). The shear viscosity of the fluids ranges from 1 to 4

1  
2  
3  
4  
5 210 mPa·s (Figure 2).  
6

7 211 The geometry imaged in Figure 4a) includes a hyperbolic contraction fol-  
8  
9 212 lowed by a sudden expansion. The width of the inlet and outlet channels is  
10  
11 213  $D_1=400\ \mu\text{m}$ , the minimum width of the contraction is  $D_2=54\ \mu\text{m}$  and the  
12  
13 214 length of the hyperbolic contraction region is  $L_c=128\ \mu\text{m}$ , resulting in a to-  
14  
15 215 tal Hencky strain of  $\varepsilon_H = \ln(D_1/D_2)=2$ . The depth of the micro-channel is  
16  
17 216 constant,  $h = 45\ \mu\text{m}$ .

18 217 Visualizations of the flow patterns in the hyperbolic contraction/abrupt ex-  
19  
20 218 pansion micro-channel (Figure 5) showed a Newtonian-like behaviour at very  
21  
22 219 low flow rates (low  $Wi$ ) and complex non-Newtonian behaviour at high flow  
23  
24 220 rates (high  $Wi$ ) for all polymer concentrations. This non-Newtonian be-  
25  
26 221 haviour corresponds to the existence of regions of separated flow upstream of  
27  
28 222 the contraction. Thus, for negligible inertia and elasticity no flow separation  
29  
30 223 was observed. For the Boger fluid with 400 ppm of PAA and 1% NaCl the  
31  
32 224 critical flow rate at which the behaviour first changes from Newtonian-like to  
33  
34 225 non-Newtonian was determined as  $Q_{cr} = 0.17\ \text{ml/hr}$ , which corresponds to  
35  
36 226  $Re = 0.7$  and  $Wi = 4$ ; for the PAA solution at 250 ppm the critical value is  
37  
38 227  $Q_{cr} = 0.27\ \text{ml/hr}$  ( $Re = 1.2$ ,  $Wi = 4$ ); for 125 ppm  $Q_{cr} = 0.47\ \text{ml/hr}$  ( $Re =$   
39  
40 228  $2.8$ ,  $Wi = 3.6$ ), and finally for the lowest concentration of 50 ppm the critical  
41  
42 229 flow rate corresponds to  $Q_{cr} = 0.95\ \text{ml/hr}$  ( $Re = 5.8$ ,  $Wi = 2.8$ ). Figure 5  
43  
44 230 shows flow patterns above this critical flow rate for each case. As expected,  
45  
46 231 there is a clear decrease in the critical flow rate when the polymer concentra-  
47  
48 232 tion is increased due to enhanced elasticity with the corresponding increase  
49  
50 233 of the relaxation time. However, the transition was found to correspond to  
51  
52 234 a critical value of  $Wi$  of about 4, which did not decrease significantly except  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

235 when flow inertia became relevant ( $Re > 2$ ). Above the critical flow rate, sym-  
 236 metric vortices develop upstream of the hyperbolic contraction in contrast  
 237 to the behaviour of Newtonian liquids that exhibit vortices downstream as a  
 238 consequence of inertial effects. Further increasing the Weissenberg number  
 239 leads to an increase of the upstream vortex due to the enhancement of elas-  
 240 tic effects which is characterized in terms of the dimensionless vortex length,  
 241  $X_R = L_v / D_1$ , where  $L_v$  is the vortex length. This variation of  $X_R$  with  $Wi$  is  
 242 shown in Figure 6 and it can be used as a simple methodology to estimate  
 243 the value of the relaxation time for solutions of very low polymer concentra-  
 244 tion by doing an extrapolation from the relaxation time obtained using the  
 245 extensional rheometer (CaBER) for the highest concentrations (400 and 250  
 246 ppm) to the lowest concentration (50 ppm) based on the assumption that  
 247  $Wi_{cr}$  is essentially independent of the polymer concentration. This assump-  
 248 tion is expected to hold as long as inertial effects are not important. In this  
 249 case, as the critical Weissenberg number is  $Wi_{cr} \approx 4$ , the relaxation time for  
 250 the 50 ppm solution can be estimated using Equation 2:

$$Wi_{cr} = \lambda \dot{\epsilon} \approx \frac{\lambda (U_2 - U_1)}{L_c} = \lambda \frac{Q_{cr}}{h L_c} \left( \frac{1}{D_2} - \frac{1}{D_1} \right) \quad (2)$$

251 obtaining  $\lambda \approx 5$  ms, which is similar to the value measured in the CaBER  
 252 ( $\lambda = 4 \pm 1$  ms).

253 As already discussed, at high  $Wi$  an increase of the vortex size due to the  
 254 progressive enhancement of elastic effects is observed. Moreover, in the ex-  
 255 periments reported by McKinley et al. [33], and Sousa et al. [34] using a  
 256 different aqueous polymer solution (PEO), the vortex growth upstream of  
 257 the contraction presents either asymmetric and/or time-dependent behaviour

1  
2  
3  
4  
5 258 which increases in magnitude with  $Wi$ , while in the present case the flows  
6  
7 259 are still steady and symmetric at the flow rates studied, as they are below  
8  
9 260 the critical point.

10  
11 261 This vortex growth regime in viscoelastic fluid flows in contractions has been  
12  
13 262 also widely reported at the macro-scale. For instance, Alves et al. [35] used a  
14  
15 263 more viscous Boger fluid to characterize the flow patterns in a square-square  
16  
17 264 contraction, observing the formation of vortices upstream the contraction  
18  
19 265 plane at low  $De$  (or low  $Wi$ ) which increased with  $De$  until the flow be-  
20  
21 266 came chaotic-like at high flow rates. Note that these flows were also char-  
22  
23 267 acterized by small Reynolds numbers,  $Re < 1$ . Rothstein and McKinley [36]  
24  
25 268 studied experimentally the flow of a Boger fluid in an axisymmetric contrac-  
26  
27 269 tion/expansion and they also reported an increase in the normalized pressure  
28  
29 270 drop at high  $De$  which is believed to be the result of an additional resistance  
30  
31 271 due to the extensional flow in the contraction (strain-hardening of the ex-  
32  
33 272 tensional viscosity). Later on, Sousa et al. [37] studied the flow of a Boger  
34  
35 273 fluid through square-square contractions with different contraction ratios,  
36  
37 274 and identified a number of distinct flow type regions with increasing values  
38  
39 275 of  $De$ : at low  $De$  a region in which lip and corner vortices coexist followed at  
40  
41 276 higher  $De$  by two distinct regions of diverging flow which are associated with  
42  
43 277 vortex growth but exhibit different characteristics for low and high contrac-  
44  
45 278 tion ratios; and at large  $De$  the onset of unstable flow in which the vortex  
46  
47 279 size varies periodically in time.

### 48 280 *3.2. Microfluidic diode*

49  
50 281 A microfluidic diode, or rectifier, is a micro-channel with anisotropic flow  
51  
52 282 resistance in both flow directions. The anisotropic behaviour can be due to  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1  
2  
3  
4  
5 283 inertial or elastic up-aft symmetry breaking effects when Newtonian or vis-  
6  
7 284 coelastic fluids flow in these micro-devices, respectively. Microfluidic diodes  
8  
9 285 can be employed for instance in fixed geometry micro-pumps, which are  
10  
11 286 commonly used in micro total analysis systems ( $\mu$ TAS) for pumping fluids  
12  
13 287 [38, 39]. The first fluidic rectifier was patented by Tesla [40] for Newtonian  
14  
15 288 fluids based on inertial non-linear effects at high  $Re$  flows.

16 289 For viscoelastic fluids, Groisman and Quake [41] proposed a microfluidic de-  
17  
18 290 vice consisting of 43 triangular cavities connected in series, which exhibited  
19  
20 291 a maximum diodicity ( $Di$ ) of about 2 ( $Di$  is defined as the ratio between  
21  
22 292 the backward, higher, and forward, lower, flow rates through the device for  
23  
24 293 a given pressure drop). Later, a significant increase in the diodicity was  
25  
26 294 achieved by Sousa et al. [42, 43], using microfluidic diodes made from 42  
27  
28 295 similar hyperbolic shaped cavities connected in series, using different aspect  
29  
30 296 ratios. The aspect ratio ( $AR$ ) is defined as the ratio between the depth of the  
31  
32 297 channel and its smallest width ( $w_c$ ) at the neck of the contraction (cf. Figure  
33  
34 298 7a for the geometry of a single element) and microfluidic diodes with  $AR =$   
35  
36 299 0.73, 1.26 and 1.71 were used in the experiments. The viscoelastic fluid used  
37  
38 300 was an aqueous solution of PEO at a concentration of 0.1 wt% (Table 1). The  
39  
40 301 polymer solution (PEO1000 in Figure 2) has a shear-thinning behaviour with  
41  
42 302 a zero-shear rate viscosity of  $\eta_0 = 7.5$  mPa·s, an infinite shear rate viscosity  
43  
44 303 of  $\eta_\infty = 3$  mPa·s and a relaxation time determined in the CaBER device,  $\lambda$   
45  
46 304  $= 73.9$  ms (Figure 2b). The Reynolds number is defined as  $Re = \rho U_c w_c / \eta_0$ ,  
47  
48 305 where  $U_c$  is the average velocity at the narrow passage with width  $w_c$ , and  
49  
50 306 the Weissenberg number is defined as  $Wi = \lambda U_c / (w_c / 2)$ .

51 307 The flows are characterized by low  $Re$  and under these conditions the flow of  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1  
 2  
 3  
 4  
 5 308 the Newtonian fluid is found to be similar in both flow directions as in the  
 6  
 7 309 limit of  $Re \rightarrow 0$  due to creeping flow reversibility, hence the diodicity is one.  
 8  
 9 310 Recirculations appear in both flow directions and grow within the hyperbolic  
 10  
 11 311 elements as  $Re$  is increased, but the flow remains symmetric relative to the  
 12  
 13 312 centreline, as shown in Figure 7a, with no significant rectification effects.  
 14  
 15 313 For the viscoelastic fluid flow we observed an entirely different dynamic flow  
 16  
 17 314 behaviour, with elasticity-induced recirculations appearing inside the hyper-  
 18  
 19 315 bolic corners, except at low flow rates (low  $Wi$ ) where the flow is Newtonian-  
 20  
 21 316 like. As the elasticity is increased, elastic instabilities appear first in the  
 22  
 23 317 forward direction as shown in Figure 7b and only at higher  $Wi$  do they ap-  
 24  
 25 318 pear in reversed flow. With the onset of the elastic instabilities, the vortices  
 26  
 27 319 appear and disappear along time in some elements of the diode (unsteady  
 28  
 29 320 flow). The critical Weissenberg number ( $Wi_{cr}$ ) for the onset of these elastic  
 30  
 31 321 instabilities in forward flow increases with the depth of the channel ( $h$ ): for  
 32  
 33 322  $AR = 0.73$ ,  $Wi_{cr} \approx 20$ ; for  $AR = 1.26$ ,  $Wi_{cr} \approx 25$ ; and for  $AR = 1.71$ ,  $Wi_{cr} \approx$   
 34  
 35 323 45.  
 36  
 37 324 Profiles of the normalized streamwise velocity measured with  $\mu$ -PIV along  
 38  
 39 325 the centreline of several consecutive elements show a large difference in the  
 40  
 41 326 amplitude of the velocity oscillations in both flow directions, as can be seen in  
 42  
 43 327 Figure 8. This dissimilar behaviour in the two directions is in contrast to the  
 44  
 45 328 results for Newtonian fluids where the amplitudes of the oscillations of the  
 46  
 47 329 velocity profiles in backward and forward flow were the same, as documented  
 48  
 49 330 by Sousa et al. [42] and not shown here for conciseness. In the backward di-  
 50  
 51 331 rection, for which the viscoelastic flow remains steady up to higher flow rates,  
 52  
 53 332 the velocity gradient at the centreline is higher than for the forward direction  
 54  
 55  
 56  
 57  
 58  
 59  
 60  
 61  
 62  
 63  
 64  
 65



1  
2  
3  
4  
5 333 (unsteady flow). Note that in the latter case the velocity field was averaged  
6  
7 334 over a long period of time, in which the flow behaviour was varying in time  
8  
9 335 due to the elastic instability and consequently, the velocity oscillations were  
10  
11 336 smoothed. In those unsteady flow experiments, time-averaged measurements  
12  
13 337 were made over a time scale significantly larger than the fluid relaxation time  
14  
15 338 to establish the overall time-averaged flow field.

16 339 For viscoelastic fluid flow, the pressure drop measured along the micro-  
17  
18 340 channel also depends on the flow direction, leading to enhanced diodicity.  
19  
20 341 As shown in Figure 9a, the pressure drop is higher in the forward direction,  
21  
22 342 where the flow is more sensitive to elastic instabilities leading to unsteady  
23  
24 343 flow at lower  $Wi$  than in the backward flow direction, so that the diodicity  
25  
26 344 increases significantly above this first flow transition. At higher flow rates  
27  
28 345 (or  $Wi$ ), when the flow also becomes unsteady in the backward direction,  
29  
30 346 the corresponding pressure drop ceases to differ so significantly from that in  
31  
32 347 the forward direction leading to a reduction of diodicity at high  $Wi$ . The  
33  
34 348 stronger and more homogeneous extensional flow in the forward direction is  
35  
36 349 responsible for the earlier onset of elastic instabilities and of enhanced diod-  
37  
38 350 icity ( $Di$ ), especially for higher aspect ratios as shown in Figure 9b. Using  
39  
40 351 the hyperbolic-shaped micro-devices a maximum diodicity of about 6.4 was  
41  
42 352 achieved when the effect of the bounding walls decreased, thus minimizing  
43  
44 353 shear effects and enhancing the extensional flow component [42].

### 45 46 354 *3.3. Flow-focusing device*

47  
48 355 The geometry of a flow-focusing device is similar to a cross-slot device, but  
49  
50 356 the flow comprises three inlets and one outlet channel (all of width  $D$ , cf.  
51  
52 357 Figure 4c), thus imposing an extensional flow to the central stream of the  
53

1  
 2  
 3  
 4  
 5 358 device, as shown in the Newtonian fluid flow patterns of Figure 10. This  
 6  
 7 359 geometry has been used at the micro-scale for a number of applications, such  
 8  
 9 360 as rheometry [44], fluid mixing [45], or droplet formation [46].  
 10  
 11 361 We investigated experimentally the flow of a Newtonian fluid, de-ionized wa-  
 12  
 13 362 ter [47, 48], and of a viscoelastic fluid [47], which was an aqueous solution of  
 14  
 15 363 125ppm (w/w) of PAA with 1% of NaCl (Table 1) resulting in a Boger-like  
 16  
 17 364 fluid with a nearly constant viscosity of  $\eta = 1.31$  mPa·s and a relaxation time  
 18  
 19 365 of  $\lambda = 12.4$  ms, measured in the CaBER device (Figure 2). For the Newtonian  
 20  
 21 366 fluid flow, Figure 10 compares the experimental flow patterns with numerical  
 22  
 23 367 predictions obtained at the same flow conditions showing a very close match.  
 24  
 25 368 The Newtonian fluid flow remained symmetric relative to the horizontal cen-  
 26  
 27 369 treline (cf. Figure 10) for a wide range of flow conditions ( $Re \lesssim 113$ ). The  
 28  
 29 370 Reynolds number, based on the exit channel, is defined as  $Re = \rho U_3 D_3 / \eta$ ,  
 30  
 31 371 where  $U_3$  and  $D_3$  are the corresponding average velocity and channel width.  
 32  
 33 372 Furthermore, the flow rate ratio ( $FR = Q_2 / Q_1$ ) or the corresponding velocity  
 34  
 35 373 ratio ( $VR = V_2 / V_1$ ), defined as the ratio of the inlet average velocities in the  
 36  
 37 374 lateral streams to the average velocity in the central inlet stream, controls  
 38  
 39 375 the total Hencky strain experienced by the fluid in the converging region [47]  
 40  
 41 376 (shown by separation streamlines in Figure 10).  
 42  
 43 377 For the viscoelastic fluid, a symmetric flow was also observed but only at  
 44  
 45 378 sufficiently low  $Wi$  as shown in Figure 11a. As  $Wi$  is increased (defined as  
 46  
 47 379  $Wi = \lambda U_3 / D_3$ ), two types of instabilities were observed [48]: a first transition  
 48  
 49 380 in which the steady symmetric flow becomes asymmetric but remains steady  
 50  
 51 381 (Figure 11b) and a second instability at higher  $Wi$  in which the steady asym-  
 52  
 53 382 metric flow becomes unsteady (Figure 11c).  
 54  
 55  
 56  
 57  
 58  
 59  
 60  
 61  
 62  
 63  
 64  
 65

1  
2  
3  
4  
5 383 The first transition is driven by elastic normal stresses that act along the  
6  
7 384 streamlines and is entirely absent from the corresponding Newtonian fluid  
8  
9 385 flow. Compared to the cross-slot flow discussed by Poole et al. [49], the main  
10  
11 386 differences lie in the fact that there is no stagnation point at the centre of the  
12  
13 387 geometry and that the onset of elastic driven instabilities depends not only  
14  
15 388 on  $Wi$ , but also on the velocity ratio as shown in Figure 12. For example,  
16  
17 389 we observed that for very low  $VR$  the flow evolves directly from steady sym-  
18  
19 390 metric to unsteady flow without ever going through the intermediate regime  
20  
21 391 of steady asymmetric flow. This behaviour was also captured in the numeric  
22  
23 392 computations [19] and was attributed to the normal stresses not being suf-  
24  
25 393 ficiently high to trigger that intermediate transition. The transition from  
26  
27 394 steady symmetric to steady asymmetric flows acts therefore as a stress relief  
28  
29 395 mechanism.

### 30 31 32 33 397 *3.4. Confined cylinder*

34  
35 398 The flow past a cylinder is a classic problem in fluid dynamics. The shallow  
36  
37 399 depths typical of microfluidics bring a renewed interest into this topic, in  
38  
39 400 particular on the effects of the aspect ratio. The flow of a Newtonian and  
40  
41 401 a viscoelastic Boger fluid past a confined cylinder of diameter  $D$  placed in  
42  
43 402 the centre of a micro-channel (width  $H = 212 \mu\text{m}$ , cf. Figure 4d) was in-  
44  
45 403 vestigated to assess the effects of the blockage ratio ( $BR$ , ratio between the  
46  
47 404 cylinder diameter and the width of the microfluidic device) and aspect ra-  
48  
49 405 tio ( $AR$ , ratio between the depth of the microfluidic device and the cylinder  
50  
51 406 diameter). To characterize the Boger fluid flow we use the Weissenberg num-  
52  
53 407 ber,  $Wi = \lambda U/(D/2)$  and when necessary the Reynolds number is defined

1  
2  
3  
4  
5 408 as  $Re = \rho UD/2\eta$ . The micro-geometries were designed with different widths  
6  
7 409 and depths to obtain  $AR = 2.0, 1.0$  and  $0.55$  and  $BR = 25\%, 50\%$  and  $75\%$ .  
8  
9 410 De-ionized water was used as Newtonian fluid and the Boger fluid was an  
10  
11 411 85 wt% aqueous solution of glycerol and PAA at a weight concentration of  
12  
13 412 200 ppm with 1% NaCl (Table 1). The Boger fluid showed a nearly constant  
14  
15 413 shear viscosity of 0.152 Pa·s over a wide range of shear rates. The character-  
16  
17 414 istic relaxation time of the Boger fluid measured in the CaBER device was  
18  
19 415  $\lambda = 86.7$  ms at 293.2 K (Figure 2b).  
20  
21 416 For the Newtonian fluid flow at creeping flow conditions, there is no flow  
22  
23 417 separation and the flow patterns are symmetric both upstream and down-  
24  
25 418 stream of the cylinder, due to creeping flow reversibility of Newtonian flows.  
26  
27 419 At non-negligible Reynolds numbers, the measurements confirm the onset of  
28  
29 420 flow separation downstream of the cylinder above a critical value of  $Re$ , in  
30  
31 421 agreement with Ribeiro et al. [50]. This critical  $Re$  depends on  $AR$  and  $BR$   
32  
33 422 and increases with  $BR$  with a more pronounced effect at higher  $BR$ .  
34  
35 423 Over the wide range of  $Wi$  investigated the following flow regimes were iden-  
36  
37 424 tified: Newtonian-like flow; flow with steady divergent streamlines; periodic  
38  
39 425 flow; chaotic-like flow. This sequence of flow transitions takes place as  $Wi$   
40  
41 426 is increased and the critical conditions for the transitions were found to be  
42  
43 427 dependent on  $AR$  and  $BR$ . For low  $Wi$ , regardless of  $AR$  and  $BR$ , the flow is  
44  
45 428 always symmetric upstream and downstream of the cylinder. Increasing  $Wi$ ,  
46  
47 429 diverging streamlines progressively appear upstream of the cylinder. Both  
48  
49 430 the critical conditions for the onset of the diverging streamlines and the in-  
50  
51 431 tensity of this effect depend on the values of  $AR$  and  $BR$ : Figure 13 illustrates  
52  
53 432 that for  $BR=50\%$  and  $Wi \approx 30$  the intensity of the diverging streamlines in-

1  
2  
3  
4  
5 433 creases as  $AR$  decreases. Moreover, we also found that for the same  $Wi$ , the  
6  
7 434 intensity of the divergent streamlines increases as  $BR$  increases. Therefore, in  
8  
9 435 both cases the divergent streamlines become stronger with wall confinement.  
10  
11 436 As  $Wi$  increases further, the divergent streamlines become progressively more  
12  
13 437 pronounced until an elastic instability arises upstream of the cylinder, near  
14  
15 438 the forward stagnation point, leading to time-dependent flow. For the range  
16  
17 439 of flow conditions tested, elastic instabilities were observed only for  $BR=$   
18  
19 440 75%. Figure 14 shows the profiles of the streamwise dimensionless velocity  
20  
21 441 along the centreline for the Boger fluid flow in order to illustrate the effects  
22  
23 442 of: elasticity at constant  $AR$  and  $BR$  (Figure 14a) and of the aspect ratio  
24  
25 443 for constant  $Wi$  and  $BR$  (Figure 14b).  
26  
27 444 The streamwise velocity far upstream of the cylinder is similar to that of  
28  
29 445 the Newtonian fluid flow, since the shear viscosity is constant and the shape  
30  
31 446 of the fully-developed velocity profile is exclusively determined by the shear  
32  
33 447 stress. Upstream of the cylinder the Boger fluid flow is independent of the  
34  
35 448 elasticity of the fluid (cf. Figure 14a), but it naturally depends on the aspect  
36  
37 449 ratio of the inlet channel (cf. Figure 14b) with the maximum velocity of the  
38  
39 450 fully-developed profile occurring for  $AR = 2$  (square channel). Downstream  
40  
41 451 of the cylinder, the length required for flow redevelopment increases progres-  
42  
43 452 sively with  $Wi$  (cf. Figure 14a), regardless of  $AR$  and  $BR$ . Figure 14b shows  
44  
45 453 that as geometric confinement increases the spatial influence of the cylinder  
46  
47 454 grows: the flow deceleration upstream of the cylinder starts slightly earlier  
48  
49 455 and the downstream flow redevelopment length increases.

1  
2  
3  
4  
5 456 *3.5. Microfluidic analogues of a porous medium*

6  
7 457 Over the last decades, the flow of non-Newtonian fluids through porous me-  
8  
9 458 dia has gained significant importance in applications related to the petroleum  
10  
11 459 industry, as in enhanced oil recovery, among others. The combination of the  
12  
13 460 non-linear properties of those fluids with the meandering paths within the  
14  
15 461 media, thus combining shear and extensional flow features, results in a com-  
16  
17 462 plex fluid mechanics problem. Moreover, the porous media are typically  
18  
19 463 opaque thus preventing the use of optical techniques. In order to obtain a  
20  
21 464 better insight about the dynamics of viscoelastic fluid flows through porous  
22  
23 465 media we designed simple micro-channel analogues consisting of a sequence  
24  
25 466 of contractions/expansions arranged in symmetric and asymmetric configu-  
26  
27 467 rations as shown in Figure 4e [51]. Dilute aqueous solutions of PAA at 50  
28  
29 468 and 125ppm (Table 1 and Figure 2) were used as model viscoelastic fluids in  
30  
31 469 the porous media analogues, as well as in the real porous media consisting  
32  
33 470 of unconsolidated packed beds (beach sand).

34  
35 471 Measurements with de-ionized water flowing through symmetric and asym-  
36  
37 472 metric micro-channels showed a linear variation of the streamwise pressure  
38  
39 473 gradient with the flow rate under laminar flow conditions. Using Darcy's law  
40  
41 474 it was possible to relate the modified permeabilities of the micro-channels  
42  
43 475 with the permeability of the porous medium while obtaining the porosity of  
44  
45 476 the porous medium and the Sauter's mean diameter ( $x_{32}$ ) of the particles as  
46  
47 477 described in [51]. It was found that the bed of beach sand with an average  
48  
49 478 particle size of  $x_{32} \approx 400 \mu\text{m}$  and a porosity  $\epsilon = 0.36$  is characterized by  
50  
51 479 a modified permeability of  $k'_{PM}=2.8 \times 10^{-10} \text{ m}^2$ , which is similar to the per-  
52  
53 480 meability of the analogue micro-channel. The results plotted in Figure 15a

1  
2  
3  
4  
5 481 show that both micro-channel configurations mimic well the pressure drop-  
6  
7 482 flow rate variation for Newtonian fluids when the equivalent particle size is  
8  
9 483 selected properly, corroborating in this way and in terms of these flow prop-  
10  
11 484 erties at least, that both simplified microfluidic systems are reasonably good  
12  
13 485 analogues of a real porous medium when a Newtonian fluid is used.  
14  
15 486 For viscoelastic fluids the situation is somehow different (Figure 15b). Here,  
16  
17 487 the Weissenberg number is defined as  $Wi = \lambda \frac{U_i}{d_{pMC}}$ , where  $\lambda$  is the relaxation  
18  
19 488 time of the fluid,  $U_i$  is the velocity in the contraction sections and  $d_{pMC}$  is  
20  
21 489 the equivalent particle size of the analogous porous medium. At low  $Wi$ , the  
22  
23 490 pressure drop measured experimentally across the bed varies linearly with  
24  
25 491 flow rate, but above a critical Weissenberg number ( $Wi_{cr}$ ), a larger slope in  
26  
27 492 the pressure gradient curve is observed due to the onset of elastic instabili-  
28  
29 493 ties in both the asymmetric micro-channel and real porous medium (critical  
30  
31 494 conditions depend on the geometry, see Figure 16). According to the Pakdel-  
32  
33 495 McKinley criterion [14, 15, 17], since the flow in the asymmetric configuration  
34  
35 496 is characterized by more marked streamline curvatures, the onset of elastic  
36  
37 497 instabilities is expected to occur at lower  $Wi$  as found in the experiments.  
38  
39 498 The symmetric configuration was found to reproduce well the flow of vis-  
40  
41 499 coelastic fluids at low  $Wi$ , whereas the asymmetric configuration provides  
42  
43 500 better results at high  $Wi$  (Figure 15b). In this way, both micro-geometries  
44  
45 501 seem to be complementary and could be interesting tools to obtain a better  
46  
47 502 insight about the flow of viscoelastic fluids through a porous medium.  
48  
49 503 Finally, on the pursuit of analysing the elastic instabilities without the influ-  
50  
51 504 ence of shear thinning effects, we have also studied the flow of low-viscosity  
52  
53 505 Boger fluids (aqueous solutions of 50 and 125 ppm of PAA with 1% of NaCl,  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1  
2  
3  
4  
5 506 see Table 1 and Figure 2 cf. [32]), through the same porous medium. Above  
6  
7 507 a critical flow rate, there is a large increase in the slope of the pressure gra-  
8  
9 508 dient curve (cf. Figure 17) due to the onset of elastic effects as discussed  
10  
11 509 previously. Then further increasing the flow rate, a more dramatic rise in  
12  
13 510 the slope of the pressure gradient *vs.* flow rate curve arises, i.e. showing an  
14  
15 511 unexpected very large third slope (Figure 17). This third slope could not  
16  
17 512 be reproduced in any of the microfluidic analogues of a porous medium and  
18  
19 513 was found to be related to the blocking of the porous medium due to the  
20  
21 514 apparent formation of a gel under these very extreme flow conditions [52].  
22

#### 23 515 **4. Conclusions**

24  
25  
26 516 The small size of microfluidic devices implies that often their flows are char-  
27  
28 517 acterized by low Reynolds numbers especially when the fluids contain addi-  
29  
30 518 tives that increase their viscosity. In addition, such additives usually impart  
31  
32 519 non-Newtonian behaviour and in particular elasticity to the fluid, leading  
33  
34 520 to high Weissenberg number flows in micro-channels. These are ultimately  
35  
36 521 responsible for the appearance of elastic instabilities at low or even negligi-  
37  
38 522 ble Reynolds numbers. This work summarizes five experimental investiga-  
39  
40 523 tions on pressure driven flows in micro-geometries that generate flows with  
41  
42 524 a non-negligible component of extensional deformation and where the onset  
43  
44 525 of various elastic instabilities are observed.

45  
46 526 In all the symmetric geometries used the flows are Newtonian-like, steady  
47  
48 527 and symmetric at low Weissenberg numbers. However, upon increasing the  
49  
50 528 Weissenberg number elastic instabilities set in and the flow either becomes  
51  
52 529 steady asymmetric or unsteady. When a transition to steady asymmetric  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65



1  
2  
3  
4  
5 530 flow takes place there is invariably a second elastic transition to unsteady  
6  
7 531 flow at a higher Weissenberg number. The path of transitions also depends  
8  
9 532 on other relevant dimensionless numbers specific to each geometric flow: for  
10  
11 533 flow-focusing it is possible to evolve directly from steady symmetric to un-  
12  
13 534 steady flow, without ever going through the steady asymmetric flow, for some  
14  
15 535 ranges of the velocity ratio and Reynolds number. Presumably, the different  
16  
17 536 transitions are associated with different mechanisms involving elastic normal  
18  
19 537 stresses. In some cases extremely high normal stresses lead to a specific flow  
20  
21 538 feature which is not observed under weaker flow conditions. In other cases  
22  
23 539 the curved streamlines are unable to sustain the normal stresses acting along  
24  
25 540 those streamlines as described by the Pakdel-McKinley criterion [14, 15, 17],  
26  
27 541 and the flow becomes unstable.

28  
29 542 These elastic instabilities have both advantages and drawbacks in practical  
30  
31 543 terms. When the objective is micro-mixing or high rates of heat and mass  
32  
33 544 transfer, these low Reynolds number chaotic-like flows of elastic fluids provide  
34  
35 545 a useful solution which is not available if the fluids are Newtonian (a similar  
36  
37 546 situation here would require higher Reynolds numbers for chaotic advection  
38  
39 547 to emerge). However, if the objective is to minimize or delay the existence of  
40  
41 548 unstable flows, as when operating with micro-rheometers, then ideally there  
42  
43 549 should be no and these transitions impose operational limits.

44  
45 550 As shown in the examples described, new phenomena arise at the micro-scale  
46  
47 551 when fluids are non-Newtonian and these flows have interesting features espe-  
48  
49 552 cially in combination with other unusual conditions not described here, such  
50  
51 553 as in the presence of electrokinetic effects or of gradients in surface tension  
52  
53 554 made possible by new developments and advances in micro-manufacturing

1  
2  
3  
4  
5 555 and surface coating methods. The popularity of micro-flow systems requires  
6  
7 556 the miniaturization of flow forcing methods and the use of electric and mag-  
8  
9 557 netic forcing mechanisms will certainly become more common as an alter-  
10  
11 558 native to pressure driven flow. The combination of these methods with flu-  
12  
13 559 ids made from complex additives, will certainly provide an opportunity to  
14  
15 560 explore new challenging phenomena. Already good examples are the com-  
16  
17 561 binations of microfluidics and optics (optofluidics) and of microfluidics and  
18  
19 562 acoustics (acousto-fluidics), for instance for particle mixing or separation in  
20  
21 563 micro-channels.  
22

## 23 564 **Acknowledgements**

24  
25  
26 565 The authors acknowledge the funding from FCT, COMPETE and FEDER  
27  
28 566 through various projects and scholarships throughout the years: PTDC/EQU-  
29  
30 567 FTT/70727/2006, PTDC/EME-MFE/114322/2009, PTDC/EME-MFE/99109/2008  
31  
32 568 and PTDC/EQU-FTT/118716/2010, SFRH/BPD/69663/2010, SFRH/BPD/69664/2010,  
33  
34 569 SFRH/BPD/75258/2010, SFRH/BD/44737/2008, IF/00148/2013 and IF/00190/2013.  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

Table 1: Formulations of the viscoelastic fluids used.

Acronym <sup>a</sup>	Polymer		Solvent			$\lambda$ [ms]
	PAA	PEO	Water	Glycerol	NaCl	
	[ppm]	[ppm]	[%]	[%]	[%]	
PAA50	50	0	100	0	1	4±1
PAA50	50	0	100	0	0	54±1
PAA125	125	0	100	0	1	10±2
PAA125	125	0	100	0	0	129±1
PAAsg200	200	0	85	15	1	87±1
PAA250	250	0	100	0	1	18±2
PAA400	400	0	100	0	1	29±2
PEO1000	0	1000	100	0	0	74±1

<sup>a</sup>PAA: Polyacrylamide; PAA: PAA with NaCl in water; PAAsg: PAA in water with NaCl and glycerol; PEO: Polyethylene oxide

Table 2: Dimensions of the micro-channels used in the works here reviewed (see Figure 4).

Hyperbolic contraction (Fig.4a)	$h$ [ $\mu\text{m}$ ]	$D_1$ [ $\mu\text{m}$ ]	$D_2$ [ $\mu\text{m}$ ]	$L_c$ [ $\mu\text{m}$ ]		
	45	400	54	128		
Microfluidic diode (Fig.4b)	$AR$ [-]	$h$ [ $\mu\text{m}$ ]	$D$ [ $\mu\text{m}$ ]	$w_c$ [ $\mu\text{m}$ ]	$L$ [ $\mu\text{m}$ ]	
	0.73	46	326	63	128	
	1.26	88	326	70	128	
	1.71	120	326	70	128	
Flow focusing Fig.4c	$h$ [ $\mu\text{m}$ ]	$D$ [ $\mu\text{m}$ ]				
	100	100				
Confined cylinder (Fig.4d)	$AR$ [-]	$BR$ [%]	$h$ [ $\mu\text{m}$ ]	$D$ [ $\mu\text{m}$ ]	$H$ [ $\mu\text{m}$ ]	
	0.37	75	58	158	212	
	0.55	55	58	105	212	
	1.1	25	58	55	212	
	0.66	75	105	158	212	
	1.0	50	105	105	212	
	1.9	25	105	55	212	
	1.3	75	213	158	212	
	2.0	50	213	105	212	
	3.9	25	213	55	212	
Analogue of a porous medium (Fig.4e)	$W_1$ [ $\mu\text{m}$ ]	$W_2$ [ $\mu\text{m}$ ]	$W_c$ [ $\mu\text{m}$ ]	$L_1$ [ $\mu\text{m}$ ]	$L_2$ [ $\mu\text{m}$ ]	$h$ [ $\mu\text{m}$ ]
				Symmetric arrangement		
	108	40	40	106	31	103
				Asymmetric arrangement		
	108	72	52	106	31	103

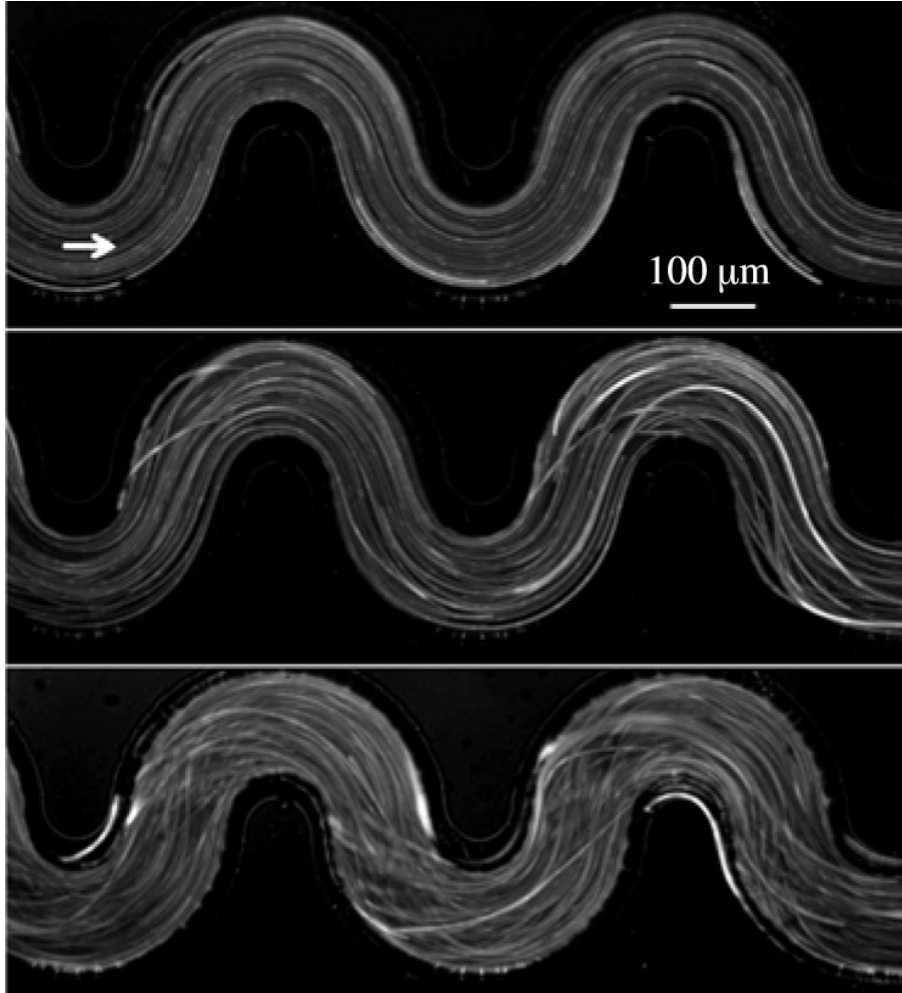
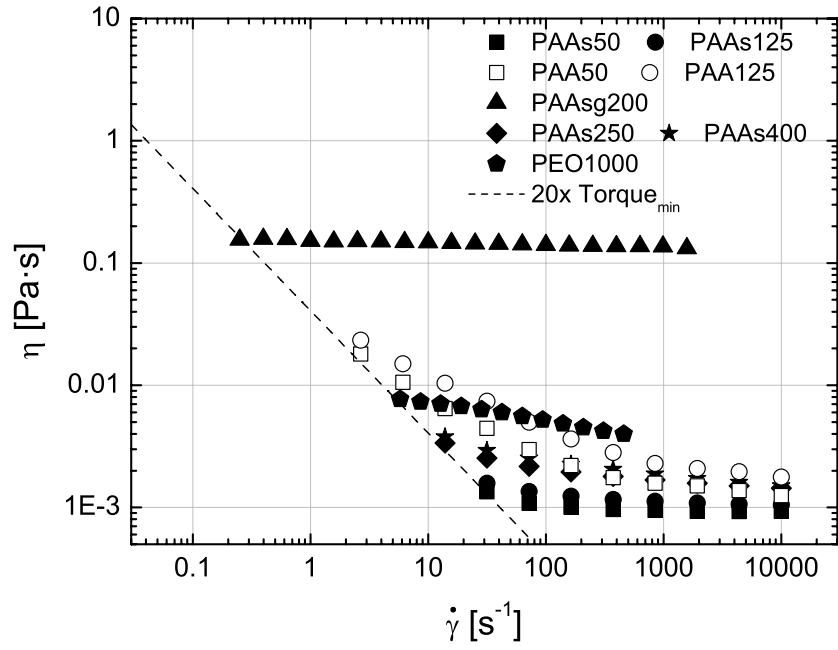
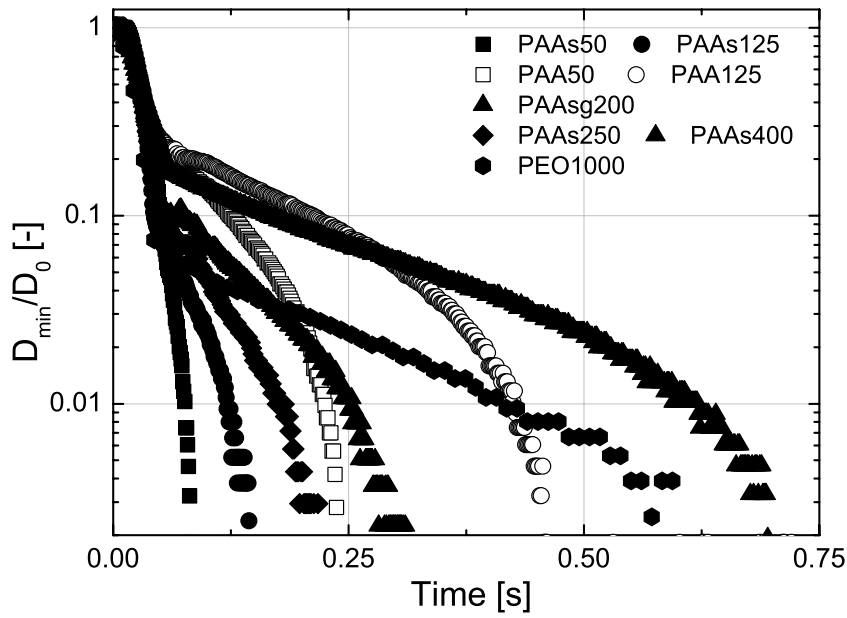


Figure 1: Illustration of the onset of an elastic instability from *instantaneous* flow patterns of a PEO solution in a 50:50 water/glycerol mixture flowing through a serpentine channel ( $M_{crit} \approx 0.68$ ). From top to bottom: stable flow ( $Wi = 0.22$ ); slightly unstable flow, close to the onset of elastic instability ( $Wi = 0.24$ ); and unstable flow ( $Wi = 0.25$ ). Reproduced with permission from Zilz et al. [20].



(a)



29

(b)

Figure 2: Rheological characterization of the viscoelastic fluids at 20 °C: (a) flow curves in steady shear flow and (b) time evolution of the normalized diameter measured in the CaBER device.

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

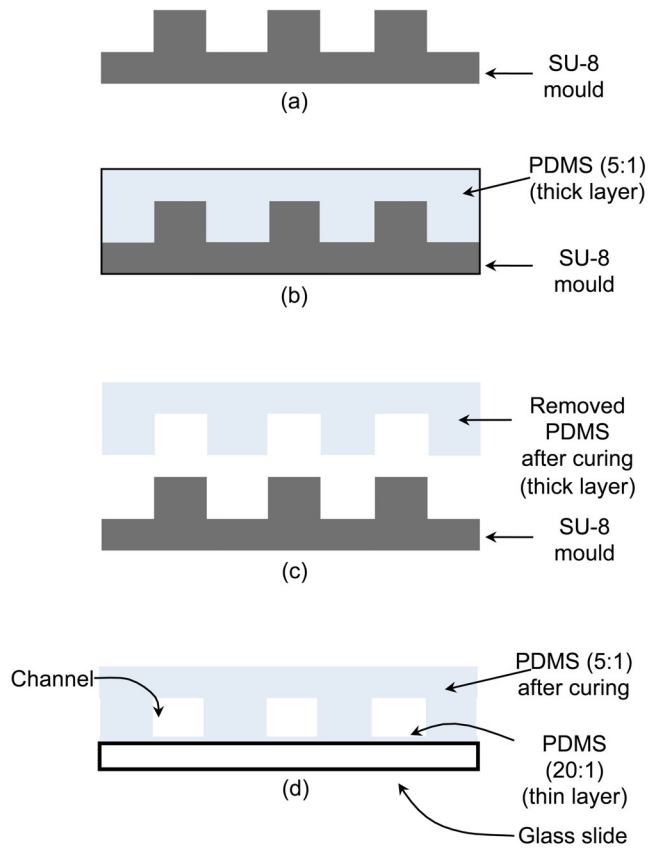


Figure 3: PDMS device fabrication procedure. (a) Cross-section of the SU-8 mould with a positive relief. (b) The mixture of oligomer and curing agent is poured onto the SU-8 mould (thick layer) and placed in the oven to cure for 20 minutes. (c) After curing, the PDMS thick layer is removed from the SU-8 mould and access ports are created. (d) The PDMS layer containing the channel structure is bonded to the glass slide covered with a thin layer of PDMS and placed in the oven to further seal the channel.

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

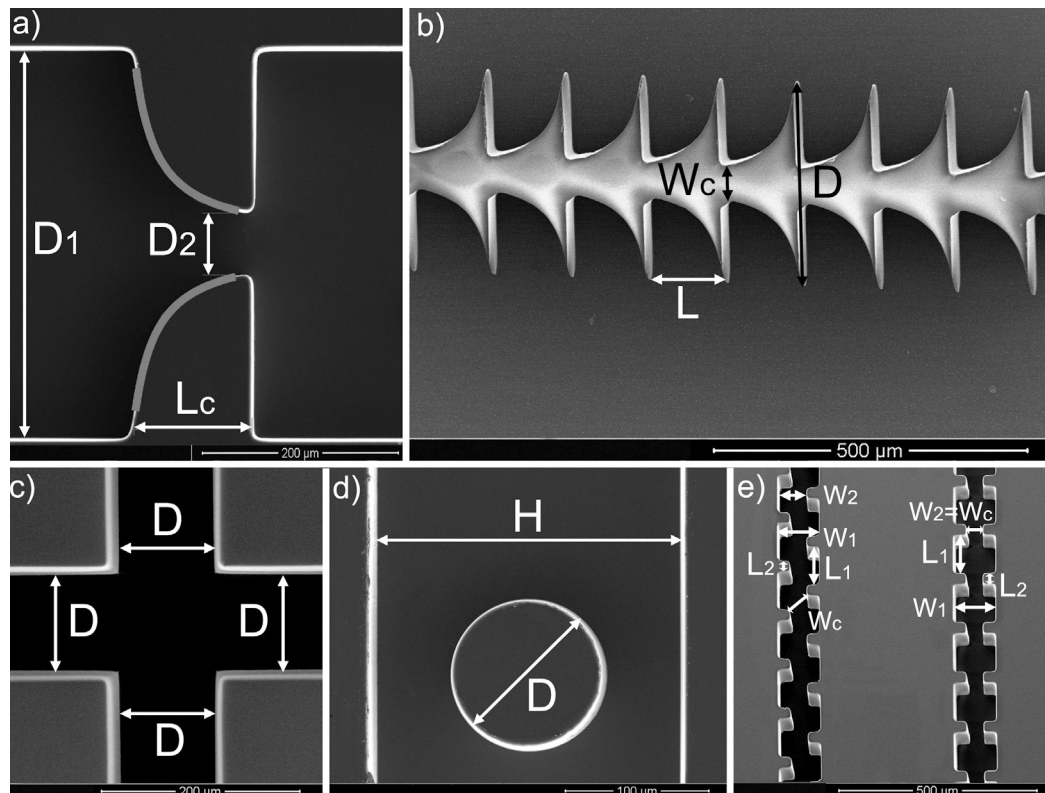


Figure 4: Scanning Electron Microscopy images of the five planar microchannels fabricated from high quality SU-8 photoresist moulds obtained by means of a high-resolution chrome mask using standard soft lithography: (a) hyperbolic contraction and sudden expansion; (b) microfluidic diode; (c) cross-slot device (to be used as a flow focusing micro-device); (d) confined cylinder; and (e) simplified microfluidic analogues of a porous medium. See Table 2 for dimensions.



1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

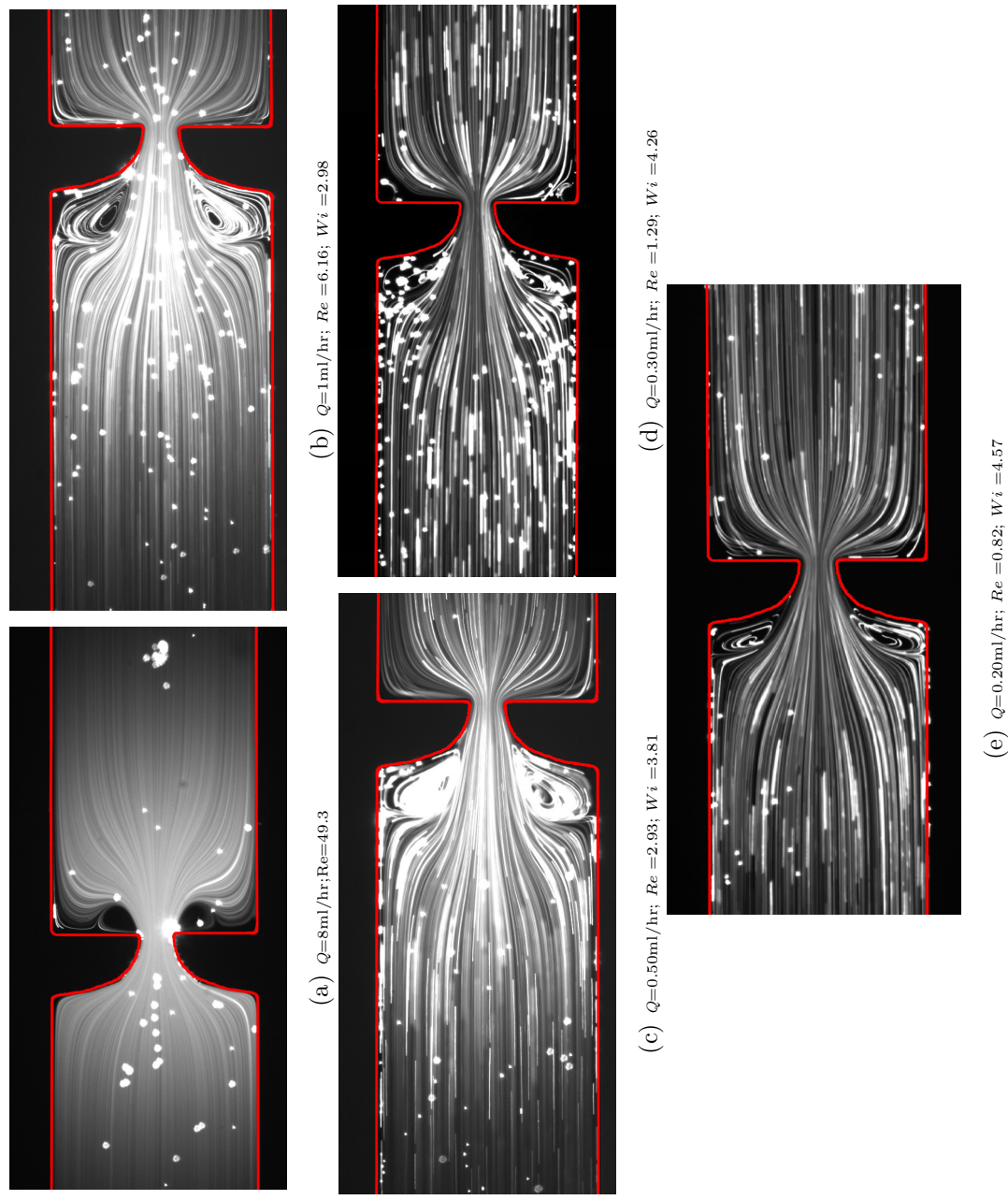


Figure 5: Flow patterns in a hyperbolic-shaped micro-contraction/abrupt expansion: (a) de-ionized water and aqueous PAA solutions at (b) 50 ppm, (c) 125 ppm and (e) 400 ppm, with 1% NaCl, at a flow rate above the critical conditions for the onset of secondary flow. The flow direction is from left to right. Adapted from Campo-Deaño et al. [32].

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

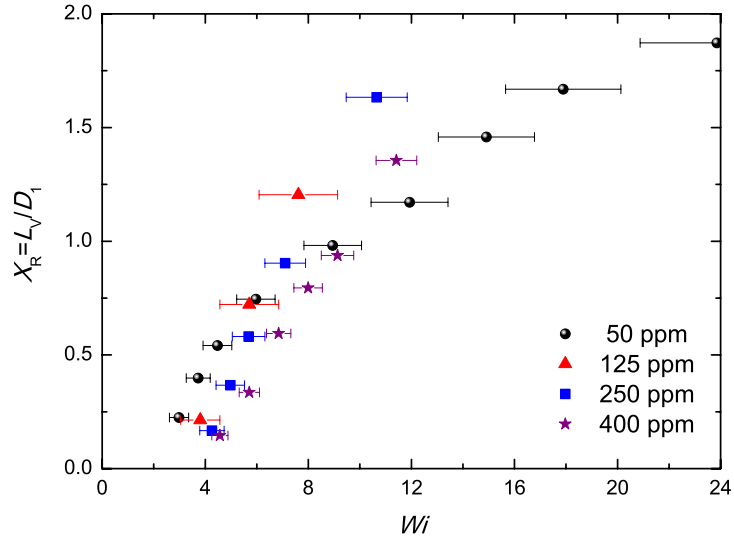


Figure 6: Effect of Weissenberg number on the dimensionless vortex length in the steady symmetric regime for the 50, 125, 250 and 400 ppm PAA aqueous solutions with 1% NaCl flowing through a hyperbolic-shaped micro-contraction. Adapted from Campo-Deaño et al. [32].

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

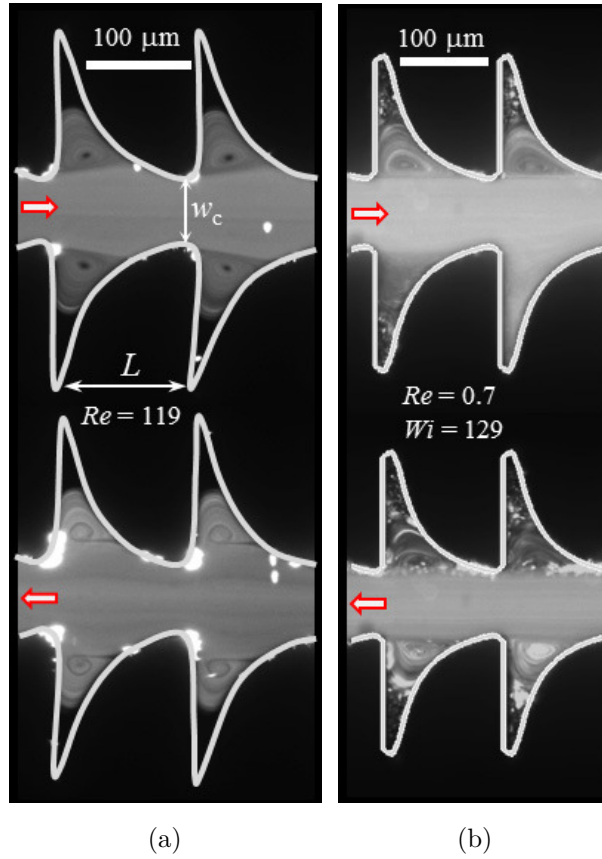


Figure 7: (a) Newtonian fluid flow patterns in a hyperbolic diode with  $AR = 0.73$ . Adapted from Sousa et al. [43]. (b) Viscoelastic fluid flow patterns in a microfluidic diode with  $AR = 1.26$ . Adapted from Sousa et al. [42].

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

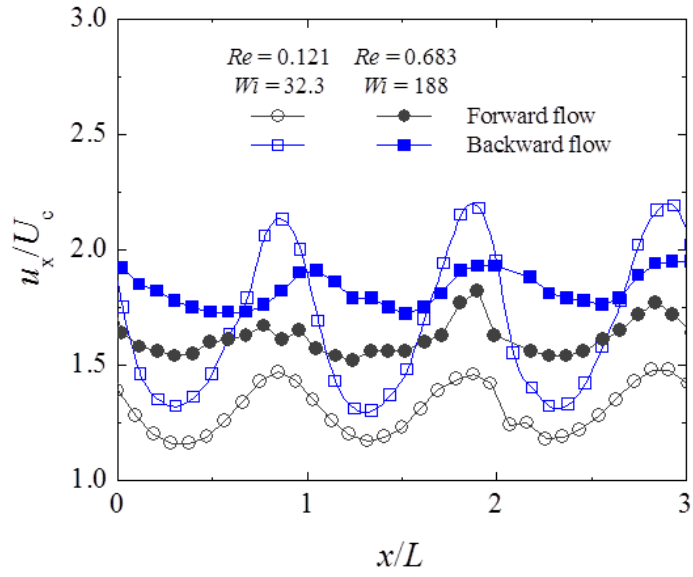


Figure 8: Dimensionless axial velocity profiles along the centreline of the microfluidic diode,  $AR = 0.73$ , for viscoelastic fluid flow under two different flow rates. The streamwise velocity component ( $u_x$ ) is normalised with the bulk velocity at the contraction throat ( $U_c$ ).

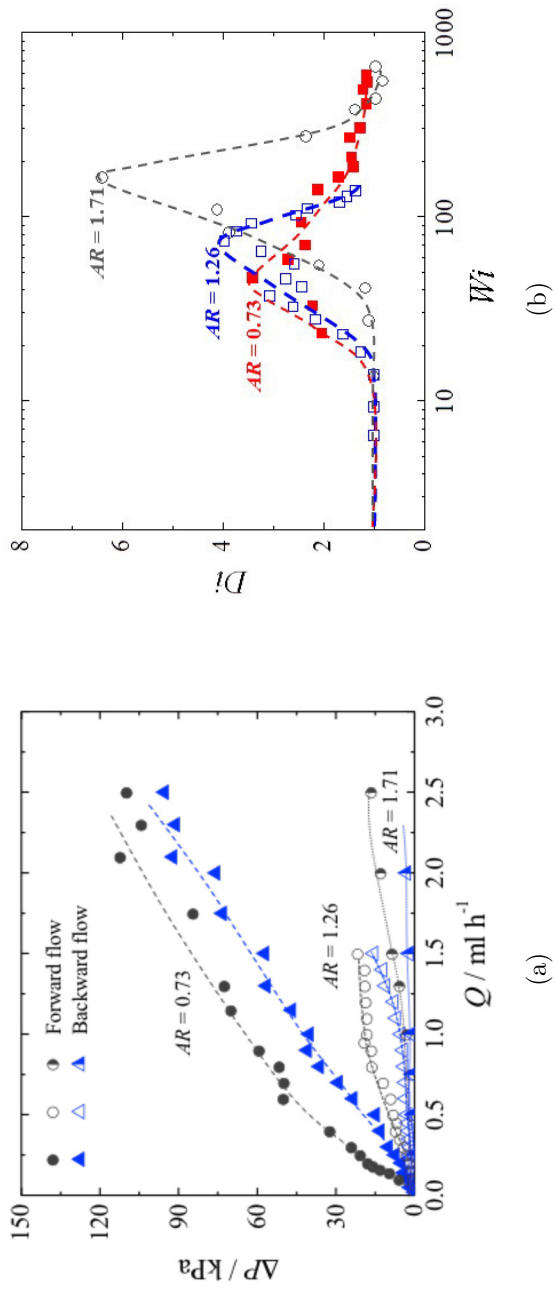


Figure 9: (a) Pressure drop measured in the hyperbolic-shaped diode as a function of the flow rate and channel depth for the viscoelastic fluid used. (b) Diodicity as a function of Weissenberg number for microfluidic diodes with different aspect ratios. Adapted from Sousa et al. [42].

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

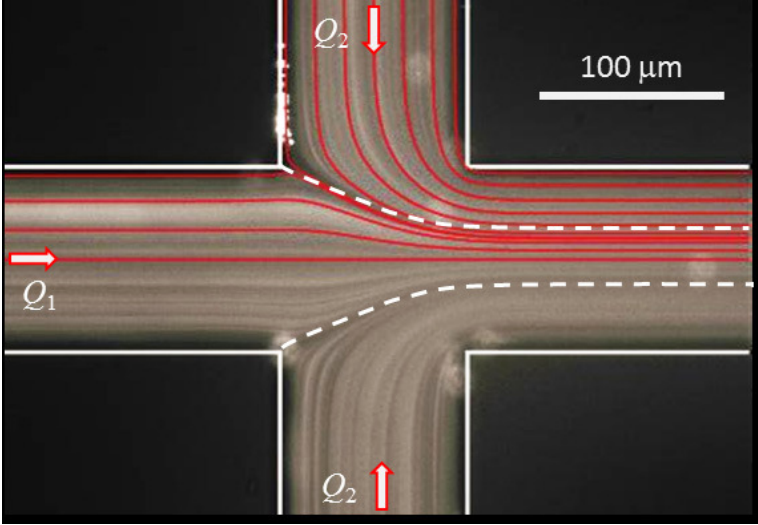


Figure 10: Experimental (micrograph) and numerical (red solid lines) flow patterns obtained for Newtonian fluid flow in a flow-focusing micro-device,  $Q_1 = Q_2 = 0.3 \text{ ml h}^{-1}$ ,  $VR = 1$  and  $Re = 2.8$ . The separation streamlines are highlighted by white dashed lines and arrows indicate the flow direction. Adapted from Oliveira et al. [47].

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

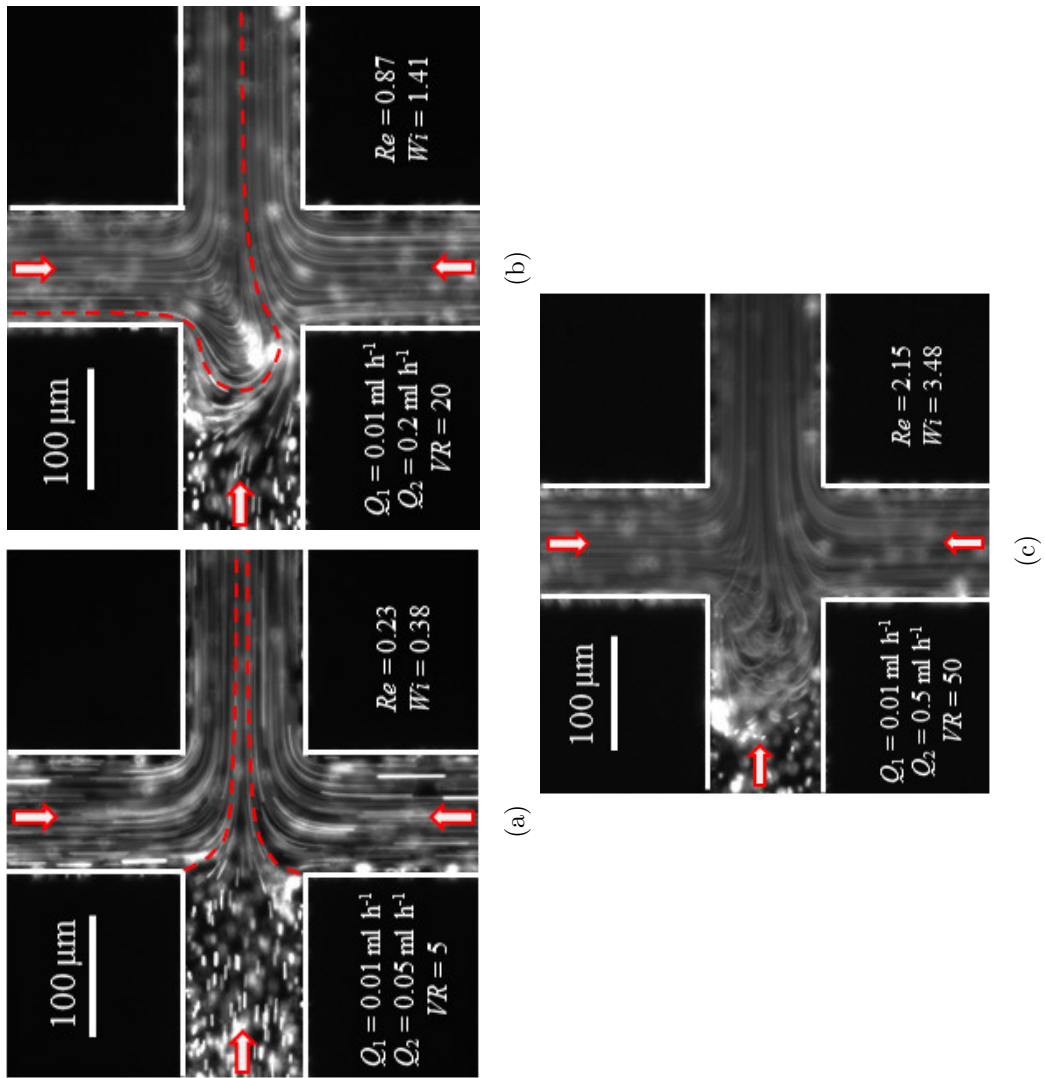


Figure 11: Flow patterns obtained for the viscoelastic fluid in a flow-focusing micro-device: (a) symmetric flow; (b) steady asymmetric flow; (c) unsteady flow (illustrative snapshot). The separation streamlines are highlighted by a red dashed line and arrows indicate the flow direction. Adapted from Oliveira et al. [48].

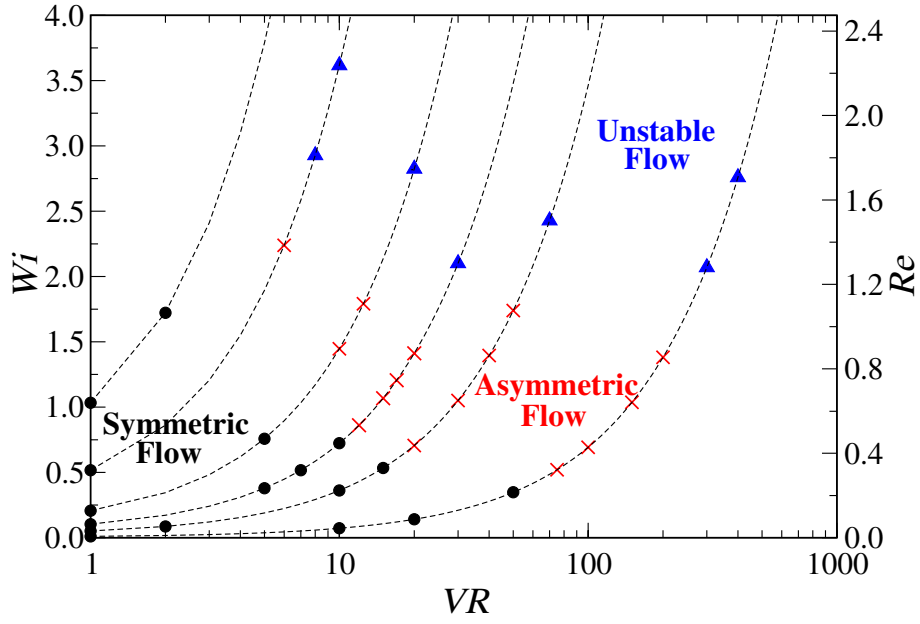


Figure 12: Flow classification map in the  $Wi$ - $VR$  domain for the flow-focusing micro-device. The circles indicate symmetric flow, the crosses steady asymmetric flow and the triangles unsteady flow.



1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

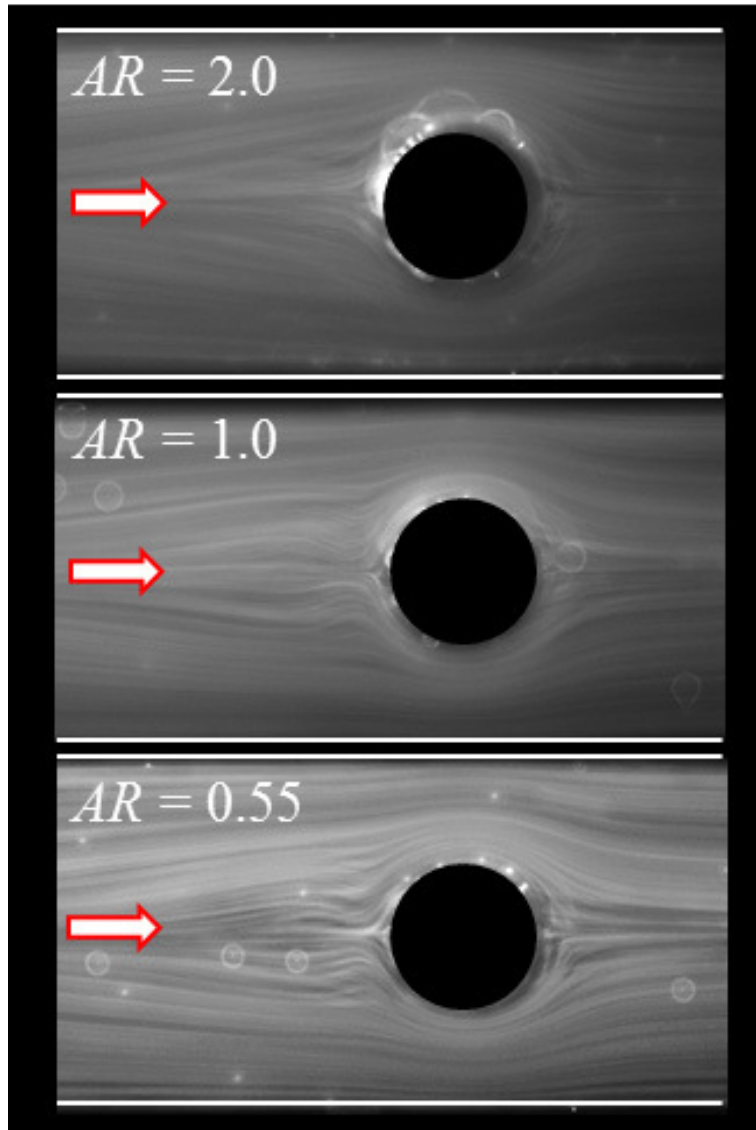


Figure 13: Flow patterns obtained for the Boger fluid flow around a cylinder for  $BR = 50\%$  as a function of  $AR$  under similar flow conditions: a)  $AR = 2.0$ ,  $Wi = 30.5$ ,  $Re = 7.8 \times 10^{-3}$ ; b)  $AR = 1.0$ ,  $Wi = 30.9$ ,  $Re = 7.9 \times 10^{-3}$ ; c)  $AR = 0.55$ ,  $Wi = 29.9$ ,  $Re = 7.6 \times 10^{-3}$ . Arrows indicate the flow direction.

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

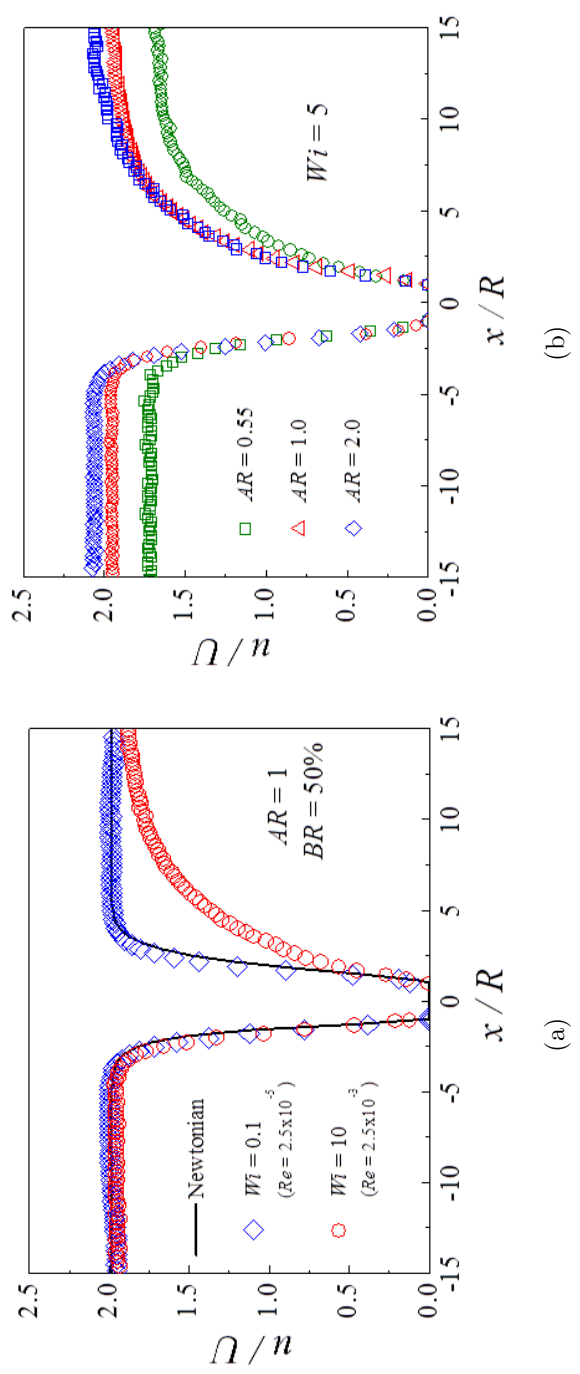
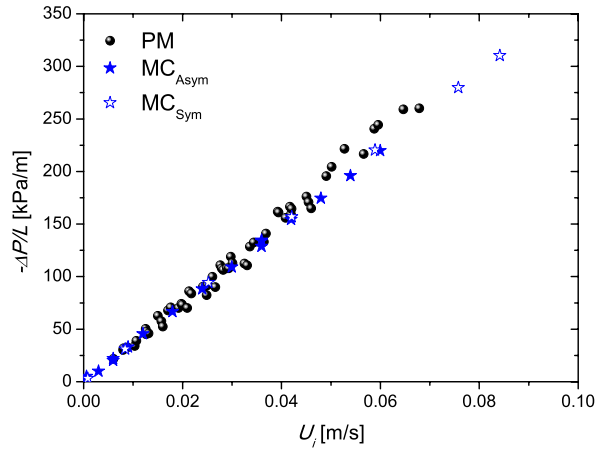
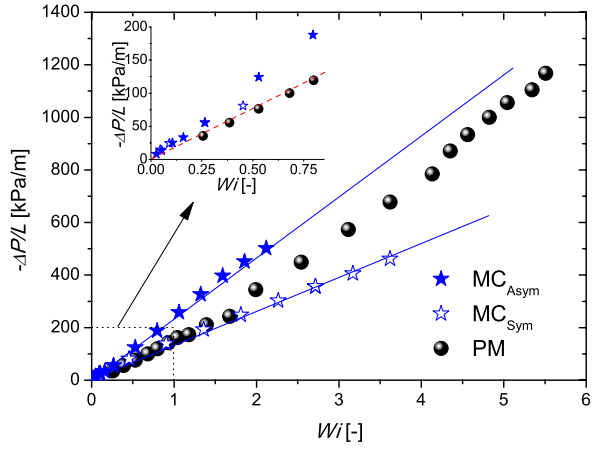


Figure 14: Streamwise dimensionless velocity profiles along the centerline of the flow around a confined cylinder as a function of elasticity for  $AR = 1$  and  $BR = 50\%$  (a) and as a function of  $AR$  for  $Wi = 5$  and  $BR = 50\%$ . (b)

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65



(a)



(b)

Figure 15: Average pressure drop gradient as a function of the interstitial velocity for the flow of (a) de-ionized water through the microfluidic analogues of a porous medium ( $MC_{Asym}$ ,  $MC_{Sym}$ ) and the porous sand bed (PM); (b) an aqueous solution of PAA at 125ppm through the microfluidic analogues of a porous medium ( $MC_{Asym}$ ,  $MC_{Sym}$ ) and the porous sand bed (PM). Adapted from Galindo-Rosales et al. [51].

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

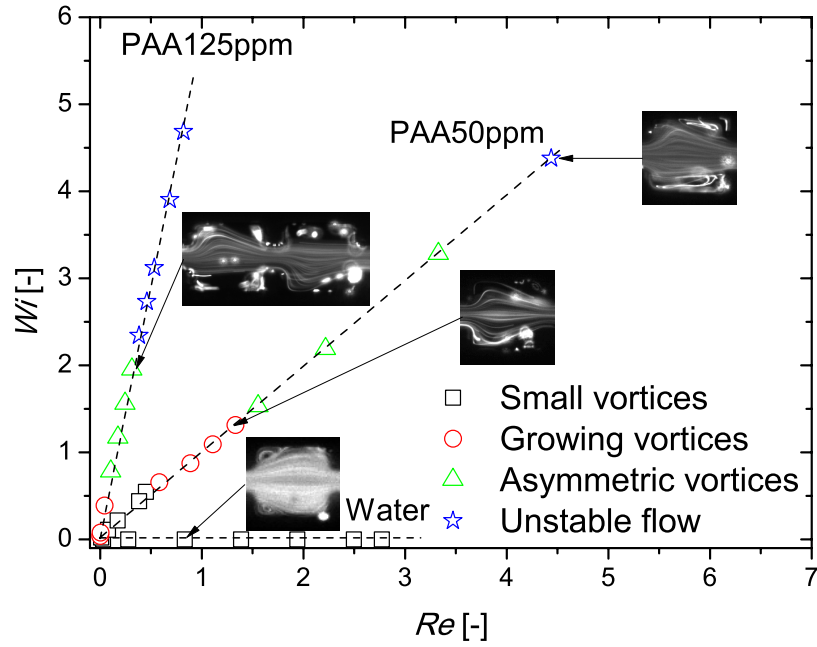


Figure 16:  $Wi-Re$  flow pattern map for the symmetric micro-channel. Adapted from Galindo-Rosales et al. [51].

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

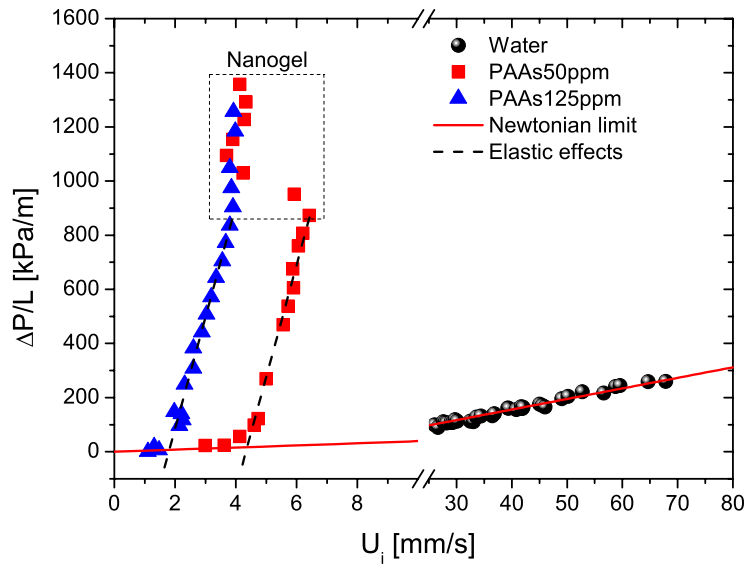


Figure 17: Variation of the pressure gradient with the interstitial velocity for the flow of low viscosity Boger fluids through a porous medium made of sand of 400  $\mu\text{m}$  particles (Sauter mean diameter). Adapted from Campo-Deaño et al. [52].

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

**References**

[1] T. Bayraktar, S. B. Pidugu, Characterization of liquid flows in microfluidic systems, *International Journal of Heat and Mass Transfer* 49 (2006) 815–824.

[2] G. Karniadakis, A. Beskok, N. Aluru, *Microflows and Nanoflows: Fundamentals and Simulation*, Springer, New York, 2005.

[3] A. D. Stroock, S. K. W. Dertinger, A. Adjari, I. Mezic, H. A. Stone, G. M. Whitesides, Chaotic mixer for microchannels, *Science* 295 (2002) 647–651.

[4] R. G. Larson, *The structure and rheology of complex fluids*, Oxford University Press, New York, 1999.

[5] R. J. Poole, The Deborah and Weissenberg numbers, *Rheology Bulletin* 53 (2012) 32–39.

[6] J. M. Dealy, Weissenberg and Deborah numbers  $\dot{\gamma}$  their definition and use, *Rheology Bulletin* 79(2) (2010) 14–18.

[7] T. M. Squires, S. R. Quake, Microfluidics: fluid physics at the nanoliter scale, *Reviews of Modern Physics* 77 (2005) 977–1026.

[8] M. Brust, C. Schaefer, R. Doerr, L. Pan, M. Garcia, P. E. Arratia, C. Wagner, Rheology of human blood plasma: Viscoelastic versus newtonian behavior, *Physical Review Letters* 110 (2013) 078305.

- 1  
2  
3  
4  
5 590 [9] L. Campo-Deaño, R. P. A. Dullens, D. G. L. A. Aarts, F. T. Pinho,  
6  
7 591 M. S. N. Oliveira, Viscoelasticity of blood and viscoelastic blood ana-  
8  
9 592 logues for use in polydimethylsiloxane in vitro models of the circulatory  
10  
11 593 system, *Biomicrofluidics* 7 (2013) 034102.
- 12  
13 594 [10] P. C. Sousa, J. Carneiro, R. Vaz, A. Cerejo, F. T. Pinho, M. A. Alves,  
14  
15 595 M. S. N. Oliveira, Shear viscosity and nonlinear behaviour of whole  
16  
17 596 blood under large amplitude oscillatory shear, *Biorheology* 50 (2013)  
18  
19 597 269–282.
- 20  
21 598 [11] E. S. G. Shaqfeh, Purely elastic instabilities in viscometric flows, *Annual*  
22  
23 599 *Reviews in Fluid Mechanics* 28 (1996) 129–186.
- 24  
25  
26 600 [12] J. A. Pathak, D. Ross, K. B. Migler, Elastic flow instability, curved  
27  
28 601 streamlines, and mixing in microfluidic flows, *Physics of Fluids* 16 (2004)  
29  
30 602 4028–4035.
- 31  
32 603 [13] R. G. Larson, E. S. G. Shaqfeh, S. J. Muller, A purely elastic instability  
33  
34 604 in taylor-couette flow, *Journal of non-Newtonian Fluid Mechanics* 218  
35  
36 605 (1990) 573–600.
- 37  
38  
39 606 [14] P. Pakdel, G. H. McKinley, Elastic instabilities and curved streamlines,  
40  
41 607 *Physical Review Letters* 77 (1996) 2459–2462.
- 42  
43 608 [15] G. H. McKinley, P. Pakdel, A. Oztekin, Rheological and geometric  
44  
45 609 scaling of purely elastic flow instabilities, *Journal of Non-Newtonian*  
46  
47 610 *Fluid Mechanics* 67 (1996) 19–47.
- 48  
49  
50 611 [16] J. Soulages, M. S. N. Oliveira, P. C. Sousa, M. Alves, G. McKinley,  
51  
52 612 Investigating the stability of viscoelastic stagnation flows in t-shaped  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

- 1  
2  
3  
4  
5 613 microchannels, *Journal of non-Newtonian Fluid Mechanics* 163 (2009)  
6 614 9–24.
- 7  
8  
9 615 [17] P. Pakdel, G. McKinley, Cavity flows of elastic liquids: purely elastic  
10 instabilities, *Physics of Fluids* 10(5) (1998) 1058–1070.  
11 616
- 12  
13  
14 617 [18] M. A. Alves, R. J. Poole, Divergent flow in contractions, *Journal of*  
15 Non-Newtonian Fluid Mechanics 144 (2007) 140–148.  
16 618
- 17  
18  
19 619 [19] M. S. N. Oliveira, F. T. Pinho, R. J. Poole, P. J. Oliveira, M. A. Alves,  
20 Purely elastic flow asymmetries in flow-focusing devices, *Journal of non-*  
21 620 *Newtonian Fluid Mechanics* 160 (2009) 31–39.  
22 621
- 23  
24  
25 622 [20] J. Zilz, R. J. Poole, M. A. Alves, D. Bartolo, A. Lindner, Geometric  
26 scaling of a purely elastic flow instability in serpentine channels, *Journal*  
27 623 *of Fluid Mechanics* 712 (2012) 203–218.  
28 624
- 29  
30  
31  
32 625 [21] M. S. N. Oliveira, M. A. Alves, F. T. Pinho, Microfluidic flow of vis-  
33 coelastic fluids, in: R. Grigoriev (Ed.), *Transport and mixing in laminar*  
34 626 *flows: From microfluidics to oceanic currents*, Wiley-VCH Verlag GmbH  
35 & Co. KGaA, Weinheim, Germany, 2011, pp. 131–174.  
36 627  
37 628
- 38  
39  
40 629 [22] C. Tropea, A. L. Yarin, J. F. Foss (Eds.), *Springer Handbook of Exper-*  
41 *imental Fluid Mechanics*, Springer Berlin Heidelberg, Germany, 2007.  
42 630
- 43  
44  
45 631 [23] A. Zell, S. Gier, S. Rafai, C. Wagner, Is there a relation between  
46 the relaxation time measured in caber experiments and the first nor-  
47 632 mal stress coefficient?, *Journal of non-Newtonian Fluid Mechanics* 165  
48 633 (2010) 1265–1274.  
49 634  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65



- 1  
2  
3  
4  
5 635 [24] V. Kantsler, V. Steinberg, Transition to tumbling and two regimes of  
6  
7 636 tumbling motion of a vesicle in shear flow, *Physical Review Letters* 96  
8  
9 637 (2006) 036001.
- 10  
11 638 [25] Y. Liu, Y. Jun, V. Steinberg, Concentration dependence of the longest  
12  
13 639 relaxation times of dilute and semi-dilute polymer solutions, *Journal of*  
14  
15 640 *Rheology* 53(5) (2009) 1069–1085.
- 16  
17  
18 641 [26] A. Aitkadi, P. Carreau, G. Chauveteau, Rheological properties of par-  
19  
20 642 tially hydrolyzed polyacrylamide solutions, *Journal of Rheology* 31  
21  
22 643 (1987) 537–561.
- 23  
24  
25 644 [27] Y. Xia, G. M. Whitesides, Soft lithography, *Annual Review of Materials*  
26  
27 645 *Science* 28 (1998) 153–184.
- 28  
29 646 [28] J. C. McDonald, D. C. Duffy, J. R. Anderson, D. T. Chiu, H. K. Wu,  
30  
31 647 O. J. A. Schueller, G. M. Whitesides, Fabrication of microfluidic systems  
32  
33 648 in poly(dimethylsiloxane), *Electrophoresis* 21 (2000) 27–40.
- 34  
35  
36 649 [29] D. F. James, Flow in a converging channel at moderate reynolds number,  
37  
38 650 *AIChE Journal* 37 (1991) 59–64.
- 39  
40  
41 651 [30] M. S. N. Oliveira, M. A. Alves, F. T. Pinho, G. H. McKinley, Viscous  
42  
43 652 flow through microfabricated hyperbolic contractions, *Experiments in*  
44  
45 653 *Fluids* 43 (2007) 437–451.
- 46  
47 654 [31] D. V. Boger, A highly elastic constant-viscosity fluid, *Journal of Non-*  
48  
49 655 *Newtonian Fluid Mechanics* 3 (1977) 87–91.

- 1  
2  
3  
4  
5 656 [32] L. Campo-Deaño, F. J. Galindo-Rosales, F. T. Pinho, M. A. Alves,  
6  
7 657 M. S. N. Oliveira, Flow of low viscosity Boger fluids through a microflu-  
8  
9 658 idic hyperbolic contraction, *Journal of Non-Newtonian Fluid Mechanics*  
10 659 166 (2011) 1286–1296.
- 11  
12  
13 660 [33] G. H. McKinley, L. E. Rodd, M. S. N. Oliveira, J. J. Cooper-White, Ex-  
14  
15 661 tensional flows of polymer solutions in microfluidic converging/diverging  
16  
17 662 geometries, *Journal of Central South University Technology* 14 (2007)  
18  
19 663 6–9.
- 20  
21  
22 664 [34] P. C. Sousa, F. T. Pinho, M. S. N. Oliveira, M. A. Alves, Extensional  
23  
24 665 flow of blood analogue solutions in microfluidic devices, *Biomicrofluidics*  
25  
26 666 5 (2011) 014108.
- 27  
28  
29 667 [35] M. A. Alves, F. T. Pinho, P. J. Oliveira, Visualizations of Boger fluid  
30  
31 668 flows in a 4:1 square-square contraction, *AIChE Journal* 51 (2005) 2908–  
32  
33 669 2922.
- 34  
35 670 [36] J. P. Rothstein, G. H. McKinley, Extensional flow of a polystyrene Boger  
36  
37 671 fluid through a 4:1:4 axisymmetric contraction/expansion, *Journal of*  
38  
39 672 *Non-Newtonian Fluid Mechanics* 86 (1999) 61–88.
- 40  
41  
42 673 [37] P. C. Sousa, P. M. Coelho, M. S. N. Oliveira, M. A. Alves, Three dimen-  
43  
44 674 sional flow of Newtonian and Boger fluids in square-square contractions,  
45  
46 675 *Journal of Non-Newtonian Fluid Mechanics* 160 (2009) 122–139.
- 47  
48  
49 676 [38] E. Stemme, G. Stemme, A valveless diffuser/nozzle-based fluid pump,  
50  
51 677 *Sensors and Actuators A: Physical* 39 (1993) 159–167.

- 1  
2  
3  
4  
5 678 [39] T. Gerlach, H. Wurmus, Working principle and performance of the  
6  
7 679 dynamic micropump, *Sensors and Actuators A: Physical* 50 (1995) 135–  
8  
9 680 140.
- 10  
11 681 [40] N. Tesla, Valvular conduit, US Patent 1329559., 1920.
- 12  
13  
14 682 [41] A. Groisman, S. R. Quake, A microfluidic rectifier: anisotropic flow  
15  
16 683 resistance at low reynolds numbers, *Physical Review Letters* 92 (2004)  
17  
18 684 094501.
- 19  
20  
21 685 [42] P. C. Sousa, F. T. Pinho, M. S. N. Oliveira, M. A. Alves, High perfor-  
22  
23 686 mance microfluidic rectifiers for viscoelastic fluid flow, *RSC Advances* 2  
24  
25 687 (2012) 920–929.
- 26  
27  
28 688 [43] P. C. Sousa, F. T. Pinho, M. S. N. Oliveira, M. A. Alves, Efficient mi-  
29  
30 689 crofluidic rectifiers for viscoelastic fluid flow, *Journal of Non-newtonian*  
31  
32 690 *Fluid Mechanics* 165 (2010) 652–671.
- 33  
34 691 [44] P. E. Arratia, J. P. Gollub, D. J. Durian, Polymeric filament thinning  
35  
36 692 and breakup in microchannels, *Physical Review E* 77 (2008) 036309.
- 37  
38  
39 693 [45] K. Jensen, Chemical kinetics: smaller, faster chemistry, *Nature* 393  
40  
41 694 (1998) 735–737.
- 42  
43  
44 695 [46] S. L. Anna, H. C. Mayer, Microscale tipstreaming in a microfluidic flow,  
45  
46 696 *Physics of Fluids* 18 (2006) 121512.
- 47  
48 697 [47] M. S. N. Oliveira, F. T. Pinho, M. A. Alves, Divergent streamlines  
49  
50 698 and free vortices in Newtonian fluid flows in microfluidic flow-focusing  
51  
52 699 devices, *Journal of Fluids Mechanics* 711 (2012) 171–191.

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

700 [48] M. S. N. Oliveira, F. T. Pinho, M. A. Alves, Extensional flow of Newtonian and Boger fluids through a flow focusing microdevice, 3rd Micro and  
701 Nano Flows Conference, Thessaloniki, Greece, 22-24th August, 2011.  
702

703 [49] R. J. Poole, M. A. Alves, P. J. Oliveira, Purely-elastic flow asymmetries,  
704 Physical Review Letters 99 (2007) 164503.

705 [50] V. M. Ribeiro, P. M. Coelho, F. T. Pinho, M. A. Alves, Three-  
706 dimensional effects in laminar flow past a confined cylinder, Chemical  
707 Engineering Science 84 (2012) 155–169.

708 [51] F. J. Galindo-Rosales, L. Campo-Deaño, M. S. N. Oliveira, M. A. Alves,  
709 F. T. Pinho, E. v. Bokhorst, P. J. Hamersma, Microfluidic systems  
710 for the analysis of viscoelastic fluid flow phenomena in porous media,  
711 Microfluidics and Nanofluidics 12 (2012) 485–498.

712 [52] L. Campo-Deaño, F. J. Galindo-Rosales, F. T. Pinho, M. A. Alves,  
713 M. S. N. Oliveira, Nanogel formation of polymer solutions flowing  
714 through porous media, Soft Matter 8 (2012) 6445–6453.