CP and *T* violation tests in neutrino oscillation

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We examine how large violation effects of *CP* and *T* are allowed in long baseline neutrino experiments. When we attribute only the atmospheric neutrino anomaly to neutrino oscillation we may have large *CP*-violation effects. When we attribute both the atmospheric neutrino anomaly and the solar neutrino deficit to neutrino oscillation we may have sizable *T* violation effects proportional to the ratio of the two mass differences; it is difficult to see *CP* violation since we cannot ignore the matter effect. We give a simple expression for *T* violation in the presence of matter. [S0556-2821(97)00903-X]

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I. INTRODUCTION

CP or *T* violation is a fundamental and important problem of particle physics and cosmology. The *CP* study of the lepton sector, though it has been less examined than that of the quark sector, is indispensable, since the neutrinos are allowed to have masses and complex mixing angles in the electroweak theory.

The neutrino oscillation search is a powerful experiment which can examine masses and mixing angles of the neutrinos. In fact, several underground experiments have shown a lack of the solar neutrinos [1–4] and anomaly in the atmospheric neutrinos [5–9], implying that neutrino oscillations may occur. The atmospheric neutrino anomaly suggests a mass difference around 10^{-3}–10^{-2} eV^2 [10–12], which encourages us to perform long baseline neutrino experiments. Recently such experiments have been planned and will be in operation in the near future [13,14]. It seems necessary for us to examine whether there is a chance to observe not only neutrino oscillations but also *CP* or *T* violation by long baseline experiments. In this paper we study such possibilities taking into account the atmospheric neutrino experiments and also considering the solar neutrino experiments and others.

II. FORMULATION OF *CP* AND *T* VIOLATION IN NEUTRINO OSCILLATION

A. Brief review

We briefly review *CP* and *T* violation in vacuum oscillation [15–17] to clarify our notation.

Let us denote the mass eigenstates of three generations of neutrinos by $\nu_m=(\nu_1,\nu_2,\nu_3)$ with mass eigenvalues[1] (m_1,m_2,m_3) and the weak eigenstates by $\nu_w=(\nu_e,\nu_\mu,\nu_\tau)$ corresponding to electron, μ and τ, respectively. They are connected by a unitary transformation

$$\nu_w=U\nu_m, \tag{1}$$

where U is a unitary (3×3) matrix similar to the Cabibbo-Kobayashi-Maskawa (CKM) matrix for quarks. We will use the parametrization for U by Chau and Keung [18–20]:

$$U=\begin{pmatrix}1&0&0\\0&c_\psi&s_\psi\\0&-s_\psi&c_\psi\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&0\\0&0&e^{i\delta}\end{pmatrix}\begin{pmatrix}c_\phi&0&s_\phi\\0&1&0\\-s_\phi&0&c_\phi\end{pmatrix}\times\begin{pmatrix}c_\omega&s_\omega&0\\-s_\omega&c_\omega&0\\0&0&1\end{pmatrix} \tag{2}$$

$$=\exp(i\psi\lambda_7)\Gamma\,\exp(i\phi\lambda_5)\exp(i\omega\lambda_2), \tag{3}$$

where the λ's are the Gell-Mann matrices.

The evolution equation for the weak eigenstate is given by

$$\begin{aligned}i\frac{d}{dx}\nu_w&=-U\,\mathrm{diag}(p_1,p_2,p_3)U^\dagger\nu_w\\&\simeq\left(-p_1+\frac{1}{2E}U\,\mathrm{diag}(0,\delta m_{21}^2,\delta m_{31}^2)U^\dagger\right)\nu_w\\&\sim\frac{1}{2E}U\,\mathrm{diag}(0,\delta m_{21}^2,\delta m_{31}^2)U^\dagger\nu_w,\end{aligned} \tag{4}$$

where p_i's are the momenta, E is the energy, and $\delta m_{ij}^2=m_i^2-m_j^2$. A term proportional to a unit matrix like p_1 in Eq. (4) is dropped because it is irrelevant to the transition probability.

The solution for the equation is

$$\nu_w(x)=U\,\exp\left(-i\frac{x}{2E}\mathrm{diag}(0,\delta m_{21}^2,\delta m_{31}^2)\right)U^\dagger\nu_w(0). \tag{5}$$

The transition probability of $\nu_\alpha\to\nu_\beta$ $(\alpha,\beta=e,\mu,\tau)$ at a distance L is given by

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[1]We assume $m_1<m_2<m_3$ in vacuum.

$$P(\nu_\alpha \to \nu_\beta ;E,L)$$
$$= \left| \sum_{i,j} U_{\beta i} \left[\exp\left(-i \frac{L}{2E} \operatorname{diag}(0,\delta m_{21}^2, \delta m_{31}^2) \right) \right]_{ij} U_{\alpha j}^* \right|^2 \tag{6}$$

$$= \sum_{i,j} U_{\beta i} U_{\beta j}^* U_{\alpha i}^* U_{\alpha j} \exp\{-i\,\delta m_{ij}^2 (L/2E)\}. \tag{7}$$

T violation gives the difference between the transition probability of $\nu_\alpha \to \nu_\beta$ and that of $\nu_\beta \to \nu_\alpha$ [21]:

$$P(\nu_\alpha \to \nu_\beta ;E,L) - P(\nu_\beta \to \nu_\alpha ;E,L)$$
$$= -4(\operatorname{Im} U_{\beta 1} U_{\beta 2}^* U_{\alpha 1}^* U_{\alpha 2})$$
$$\times(\sin \Delta_{21} + \sin \Delta_{32} + \sin \Delta_{13}) \tag{8}$$
$$\equiv 4Jf, \tag{9}$$

where

$$\Delta_{ij} \equiv \delta m_{ij}^2 \frac{L}{2E} = 2.54 \frac{(\delta m_{ij}^2/10^{-2} \ \text{eV}^2)}{(E/\text{GeV})} (L/100 \ \text{km}), \tag{10}$$

$$J \equiv -\operatorname{Im} U_{\beta 1} U_{\beta 2}^* U_{\alpha 1}^* U_{\alpha 2}, \tag{11}$$

$$f \equiv (\sin \Delta_{21} + \sin \Delta_{32} + \sin \Delta_{13}) \tag{12}$$

$$= -4 \sin \frac{\Delta_{21}}{2} \sin \frac{\Delta_{32}}{2} \sin \frac{\Delta_{13}}{2}. \tag{13}$$

The unitarity of U gives

$$J = \pm \sin\omega \cos\omega \sin\psi \cos\psi \sin\phi \cos^2\phi \sin\delta \tag{14}$$

with the sign $+$ $(-)$ for α,β in cyclic (anticyclic) order [$+$ for $(\alpha,\beta)=(e,\mu)$, (μ,τ), or (τ,e)]. In the following we assume the cyclic order for (α,β) for simplicity.

There are bounds for J and f,

$$|J| \leqslant \frac{1}{6\sqrt{3}}, \tag{15}$$

where the equality holds for $|\sin\omega| = 1/\sqrt{2}$, $|\sin\psi| = 1/\sqrt{2}$, $|\sin\phi| = 1/\sqrt{3}$, and $|\sin\delta| = 1$, and [22]

$$|f| \leqslant \frac{3\sqrt{3}}{2}, \tag{16}$$

where the equality holds for $\Delta_{21} \equiv \Delta_{32} \equiv 2\pi/3 \pmod{2\pi}$.

In the vacuum the CPT theorem gives the relation between the transition probability of an antineutrino and that of a neutrino,

$$P(\overline{\nu_\alpha} \to \overline{\nu_\beta} ;E,L) = P(\nu_\beta \to \nu_\alpha ;E,L), \tag{17}$$

which relates CP violation to T violation:

$$P(\nu_\alpha \to \nu_\beta ;E,L) - P(\overline{\nu_\alpha} \to \overline{\nu_\beta} ;E,L)$$
$$= P(\nu_\alpha \to \nu_\beta ;E,L) - P(\nu_\beta \to \nu_\alpha ;E,L). \tag{18}$$

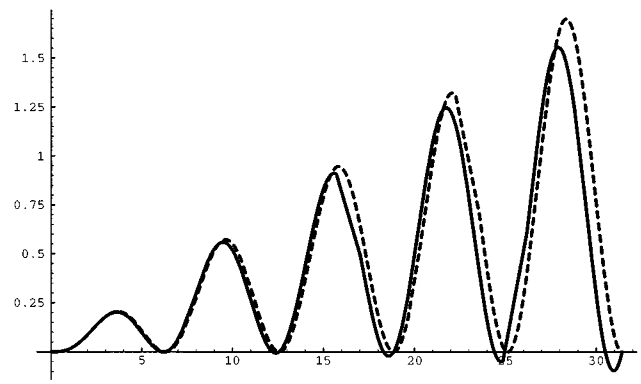

FIG. 1. Graph of $f(\Delta_{31},\epsilon)$ for $\epsilon = 0.03$. The solid line and dashed line represent the exact expression, Eq. (19), and the approximated one, Eq. (20), respectively. The approximated f has peaks at $\Delta_{31} = 3.67$, 9.63, 15.8, ... irrespectively of ϵ.

B. CP and T violation with disparate mass differences

Let us consider how large the $T(CP)$ violation can be in the ''disparate'' mass difference case,[2] say $\epsilon \equiv \delta m_{21}^2 / \delta m_{31}^2 \ll 1$. In this case the following two situations are interesting [21], since in the case $\Delta_{31} \ll 1$ we have too small $f[\approx O(\epsilon \Delta_{31}^3)$ due to Eq. (13)] to observe the $T(CP)$ violation effect.

Situation 1. $\Delta_{31} \sim 1$. Because $|\epsilon \Delta_{31}| \ll 1$ in this case, the oscillatory part f becomes $O(\epsilon)$:

$$f(\Delta_{31},\epsilon) = \sin \Delta_{21} + \sin \Delta_{32} + \sin \Delta_{13}$$
$$= \sin(\epsilon \Delta_{31}) + \sin\{(1-\epsilon)\Delta_{31}\} - \sin \Delta_{31} \tag{19}$$
$$= \epsilon \Delta_{31}(1 - \cos \Delta_{31}) + O(\epsilon^2 \Delta_{31}^2). \tag{20}$$

Figure 1 shows the graph of $f(\Delta_{31}, \epsilon = 0.03)$. The approximation Eq. (20) works very well up to $|\epsilon \Delta_{31}| \sim 1$. In the following we will use Eq. (20) instead of Eq. (19). We see many peaks of $f(\Delta_{31},\epsilon)$ in Fig. 1. In practice, however, we do not see such sharp peaks but observe the value averaged around there, for Δ_{31} has a spread due to the energy spread of neutrino beam ($|\delta\Delta_{31}/\Delta_{31}| = |\delta E/E|$). In the following we will assume $|\delta\Delta_{31}/\Delta_{31}| = |\delta E/E| = 20\%$ [23] as a typical value.

Table I gives values of $f(\Delta_{31},\epsilon)/\epsilon$ at the first several peaks and the averaged values around there.

We see the T-violation effect,

$$\langle P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha) \rangle_{20\%}$$
$$= 4J\langle f \rangle_{20\%} = J\epsilon \times \begin{cases} 25.9, \\ 56.0, \\ 62.4, \\ \vdots \end{cases} \text{for } \Delta_{31} = \begin{cases} 3.67, \\ 9.63, \\ 15.8, \\ \vdots \end{cases} \tag{21}$$

at peaks for neutrino beams with a 20% energy spread. Note that the averaged peak values decrease with the spread of neutrino energy.

[2]Hereafter we denote the larger mass difference by δm_{31}^2 and the smaller one by δm_{21}^2 in the case that the mass differences have a large ratio.

TABLE I. The peak values of $f(\Delta_{31},\epsilon)/\epsilon$ and the corresponding averaged values. Here $\langle f/\epsilon\rangle_{20\%(10\%)}$ is a value of $f(\Delta_{31},\epsilon)/\epsilon = \Delta_{31}(1-\cos\Delta_{31})$ [see Eq. (20)] averaged over the range $0.8\Delta_{31}$–$1.2\Delta_{31}$ ($0.9\Delta_{31}$–$1.1\Delta_{31}$).

Δ_{31}	f/ϵ	$\langle f/\epsilon\rangle_{10\%}$	$\langle f/\epsilon\rangle_{20\%}$
3.67	6.84	6.75	6.48
9.63	19.1	17.6	14.0
15.8	31.5	25.7	15.6
⋮	⋮	⋮	⋮

Which peak we can reach depends on δm^2_{31}, L, and E. The first peak $\Delta_{31}=3.67$ is reached, for example, by $\delta m^2_{31}=10^{-2}$ eV^2, $L=250$ km (for the KEK-Kamiokande long baseline experiment) and neutrino energy $E=1.73$ GeV. In this case we see the $T(CP)$-violation effect best at $|25.9J\epsilon|\leqslant 2.50\epsilon$ since we have a bound on J as Eq. (15).

Situation 2. $\Delta_{31}\geqslant 1$. Because $\sin\Delta_{32}$ and $\sin\Delta_{13}$ oscillate rapidly and vanish after being averaged over the energy spread in this case, the oscillatory part f is dominated by $\sin\Delta_{21}$. Since f now has a bound $|f|\leqslant 1$ instead of Eq. (16), the T-violation effect $4Jf$ is bounded as $|4Jf|\leqslant|4J|$. (For an energy spread of 10–20% of the neutrino beam [23], $\Delta_{31}>30$ is enough for $\sin\Delta_{32}$ and $\sin\Delta_{13}$ to oscillate rapidly and vanish after being averaged.)

III. CP VIOLATION

There are a variety of possible combinations of the parameters, three mixing angles, two mass differences, and a CP-violating phase. When we consider only the atmospheric neutrino anomaly to be attributed to the neutrino oscillation, we can take the mass differences, δm^2_{21} and δm^2_{31} (and hence δm^2_{32}), to be comparable, while when we consider both the solar and the atmospheric neutrino anomalies to be attributed to the neutrino oscillation, we expect δm^2_{21} and δm^2_{31} to be ''disparate,'' $\delta m^2_{21}/\delta m^2_{31}\ll 1$.

We investigate how large the CP-violation effect can be in the neutrino oscillation for the above two cases.

A. Comparable mass difference case

Let us examine the case of mass differences to be the same order of magnitude.

We use a parameter set that $(\delta m^2_{21},\delta m^2_{31})=(3.8, 1.4)\times 10^{-2}$ eV^2, $(\omega,\phi,\psi)=(19°, 43°, 41°)$, and δ is arbitrary, derived by Yasuda [12] through the analysis of the atmospheric neutrino anomaly. Here the matter effect [24,25] is negligibly small and Eq. (18) is available.

With the use of Eqs. (9), (14), and (18) this parameter set gives the CP-violation effect

$$P(\nu_\alpha\to\nu_\beta)-P(\overline{\nu_\alpha}\to\overline{\nu_\beta})=0.22\,\sin\delta f(x), \tag{22}$$

where

$$f(x)=(\sin 3.8x+\sin 2.4x-\sin 1.4x) \tag{23}$$

and

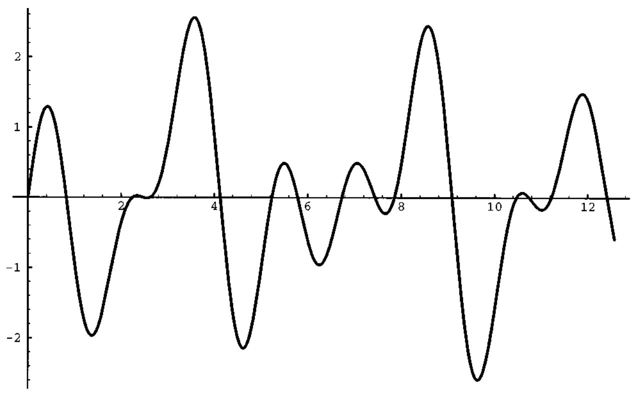

FIG. 2. Graph of $f(x)$ of Eq. (23). There are high peaks (positive or negative) at $x=0.42$, 1.4, 3.6, 4.6, ... Values of $f(x)$ at peaks averaged over an energy spread of 10–20% are $\langle f(0.42)\rangle =1.3$–1.3, $\langle f(1.4)\rangle=-1.9$ to -1.8, $\langle f(3.6)\rangle=2.2$–1.4, $\langle f(4.6)\rangle= -1.5$ to -0.40, ...

$$x=2.5\,\frac{(L/100\ \text{km})}{(E/\text{GeV})}. \tag{24}$$

Figure 2 shows the oscillatory part $f(x)$. There are many peaks $f(x)$ showing the possibility to observe the large CP-violation effect. For example, we may see a very large difference between the transition probabilities, $\langle P(\nu_\alpha\to\nu_\beta)-P(\overline{\nu_\alpha}\to\overline{\nu_\beta})\rangle_{20\%}\sim 0.4\sin\delta$ for $L=250$ km (for the KEK-Kamiokande experiment) and $E\sim 4.5$ GeV corresponding to $x\sim 1.4$, if we have a large $\sin\delta$.

Incidentally we may remark that the survival probability of solar neutrino is calculated to be 0.45 for those mixing angles. This value is consistent with both Gallium experiments [1,2] and the Kamiokande experiment [3], but it is inconsistent with the Homestake result [4] if all of the solar neutrino anomaly should be attributed to the neutrino oscillation [26].

In conclusion we may see a large CP-violation effect when we have comparable mass differences. In this respect we note that the long baseline experiments are urgently desirable.

B. Disparate mass difference case

Next we consider the ''disparate'' mass difference case $\delta m^2_{21}/\delta m^2_{31}\ll 1$.

The case $\delta m^2_{31}\sim 1$ eV^2 and $\delta m^2_{21}\sim 10^{-2}$ eV^2 is favored by the hot dark matter scenario [27] and the atmospheric neutrino anomaly. This case is already analyzed by Tanimoto [28] and we will not discuss it here.

The case $\delta m^2_{31}\sim 10^{-2}$ eV^2 and $\delta m^2_{21}\sim 10^{-4}$ eV^2 could typically explain the anomalies of the atmospheric and the solar neutrinos [11]. In this case we cannot neglect the matter effect [24,25]

$$2\sqrt{2}G_F n_e E\sim 2\times 10^{-4}\ \text{eV}^2\left(\frac{E}{\text{GeV}}\right)\left(\frac{n}{3\ \text{g/c.c.}}\right), \tag{25}$$

where n_e is the electron number density of the earth and n is the matter density of the surface of the earth, since it is greater than δm^2_{21}. It requires one to subtract such an effect in order to deduce the pure CP-violation effect [29]. In principle, it is possible because the matter effect is proportional to E while δm_{21}^2 is constant.

IV. T VIOLATION

In the matter with constant density,[3] we have a pure T-violation effect $P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$, though we do not observe a pure CP-violation effect because of an apparent CP violation due to matter.

A. T violation in matter

When a neutrino is in matter, its matrix of the effective mass squared M_m^2 of weak eigenstates is [19,20]

$$M_m^2 = U \begin{pmatrix} 0 & & \\ & \delta m_{21}^2 & \\ & & \delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix}, \quad (26)$$

where $a = 2\sqrt{2} G_F n_e E$ and U is given by Eq. (2). This is diagonalized by a mixing matrix U_m as $M_m^2 = U_m \, \mathrm{diag}(\tilde{m}_1^2, \tilde{m}_2^2, \tilde{m}_3^2) U_m^\dagger$. It is written with a real unitary (orthogonal) matrix $\tilde{U}$ as

$$U_m = \exp(i\psi\lambda_7)\Gamma\tilde{U}. \quad (27)$$

With arguments analogous to Sec. II A we have the T-violation effect

$$P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha) = 4 J_m f_m, \quad (28)$$

where

$$J_m = -\mathrm{Im} U_{m\beta 1} U_{m\beta 2}^* U_{m\alpha 1}^* U_{m\alpha 2}$$
$$= \sin\psi \cos\psi \tilde{U}_{11}\tilde{U}_{12}\tilde{U}_{13}\sin\delta, \quad (29)$$

$$f_m = \sin\frac{\tilde{m}_2^2 - \tilde{m}_1^2}{2E} L + \sin\frac{\tilde{m}_3^2 - \tilde{m}_2^2}{2E} L + \sin\frac{\tilde{m}_1^2 - \tilde{m}_3^2}{2E} L. \quad (30)$$

We get

$$D_m \equiv \mathrm{diag}(\tilde{m}_1^2, \tilde{m}_2^2, \tilde{m}_3^2) = U_m^\dagger M_m^2 U_m = \tilde{U}^\dagger \left\{ U_\phi U_\omega \begin{pmatrix} 0 & & \\ & \delta m_{21}^2 & \\ & & \delta m_{31}^2 \end{pmatrix} U_\omega^T U_\phi^T + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right\} \tilde{U}$$
$$= \tilde{U}^\dagger \left\{ \begin{pmatrix} a + \delta m_{31}^2 \sin^2\phi & 0 & \delta m_{31}^2 \cos\phi \sin\phi \\ 0 & 0 & 0 \\ \delta m_{31}^2 \cos\phi \sin\phi & 0 & \delta m_{31}^2 \cos^2\phi \end{pmatrix} + \delta m_{21}^2 U_\phi U_\omega \begin{pmatrix} 0 & & \\ & 1 & \\ & & 0 \end{pmatrix} U_\omega^T U_\phi^T \right\} \tilde{U}, \quad (31)$$

where $U_\phi = \exp(i\phi\lambda_5)$ and $U_\omega = \exp(i\omega\lambda_2)$.

An exact result for U_m and D_m is given in [30], though their result is rather complicated. Here we show a simple expression for U_m and D_m in the case $\delta m_{21}^2 \ll a, \delta m_{31}^2$. We derive U_m and D_m in this case using perturbation with respect to small δm_{21}^2.

First we decompose $\tilde{U} = U_0 V$ and diagonalize by U_0 the first term of the parenthesis {} of Eq. (31), the eigenvalues of which we denote by Λ_i s. We find

$$U_0 = \exp(i\phi'\lambda_5) \quad \text{with} \quad \tan 2\phi' = \frac{\delta m_{31}^2 \sin 2\phi}{\delta m_{31}^2 \cos 2\phi - a}, \quad (32)$$

and

$$\Lambda_1 = \frac{(a + \delta m_{31}^2) - \sqrt{(a + \delta m_{31}^2)^2 - 4a\,\delta m_{31}^2 \cos^2\phi}}{2},$$

$$\Lambda_2 = 0,$$

$$\Lambda_3 = \frac{(a + \delta m_{31}^2) + \sqrt{(a + \delta m_{31}^2)^2 - 4a\,\delta m_{31}^2 \cos^2\phi}}{2}. \quad (33)$$

We have

[3]Note that the time reversal of $\nu_\alpha \to \nu_\beta$ requires the exchange of the production point and the detection point and the time reversal of $P(\nu_\alpha \to \nu_\beta)$ in matter is in general different from $P(\nu_\beta \to \nu_\alpha)$ [19].

$$D_m = V^\dagger\left\{\begin{pmatrix}\Lambda_1 & & \\ & \Lambda_2 & \\ & & \Lambda_3\end{pmatrix} + \delta m_{21}^2 U_{\phi-\phi'} U_\omega \begin{pmatrix}0 & & \\ & 1 & \\ & & 0\end{pmatrix} U_\omega^T U_{\phi-\phi'}^T\right\} V \equiv V^\dagger\{\mathrm{diag}(\Lambda_1,\Lambda_2,\Lambda_3) + \delta m_{21}^2 H\} V. \tag{34}$$

Next we diagonalize the whole M_m^2 by V with perturbation with respect to small δm_{21}^2.

At the zeroth order of δm_{21}^2 we have $\widetilde{m}_i^2 = \Lambda_i$, $V_{ij} = \delta_{ij}$, and $\widetilde{U} = U_0$ which gives $\widetilde{U}_{12} = (U_0)_{12} = 0$ and hence $J_m = 0$ [see Eq. (29)].

At the first order of perturbation, we have

$$\widetilde{m}_i^2 = \Lambda_i + \delta m_{21}^2 H_{ii}, \tag{35}$$

$$V_{ij} = \begin{cases} 1 & \text{for} \quad i = j, \\ \delta m_{21}^2 \dfrac{H_{ij}}{\Lambda_j - \Lambda_i} & \text{for} \quad i \neq j, \end{cases} \tag{36}$$

and with Eq. (29)

$$J_m = -\frac{\delta m_{21}^2}{a} \frac{\delta m_{31}^2}{\{(\delta m_{31}^2 + a)^2 - 4\,\delta m_{31}^2 a \cos^2\phi\}^{1/2}} \times \sin\omega \cos\omega \sin\psi \cos\psi \sin\phi \sin\delta. \tag{37}$$

B. Most likely case: $\delta m_{21}^2 \ll a \ll \delta m_{31}^2$

It seems most likely to be realized that $\delta m_{21}^2 \ll a \ll \delta m_{31}^2$ as is discussed in Sec. III B. Here we study this case in detail. Since J_m is $O(\delta m_{21}^2)$ we neglect $O(\delta m_{21}^2)$ in estimating f_m. We also neglect $O(a^2)$ since $a/\delta m_{31}^2 \ll 1$.

Then we have the effective masses

$$\widetilde{m}_1^2 \simeq \Lambda_1 \simeq a \cos^2\phi,$$

$$\widetilde{m}_2^2 \simeq \Lambda_2 \simeq 0, \tag{38}$$

$$\widetilde{m}_3^2 \simeq \Lambda_3 \simeq \delta m_{31}^2 + a \sin^2\phi$$

and "mass difference ratio"

$$\epsilon_m = \frac{\widetilde{m}_2^2 - \widetilde{m}_1^2}{\widetilde{m}_3^2 - \widetilde{m}_2^2} \simeq -\frac{a \cos^2\phi}{\delta m_{31}^2}. \tag{39}$$

Note that $|\epsilon_m| \ll 1$.

We find

$$J_m \sim -\frac{\delta m_{21}^2}{a} \sin\omega \cos\omega \sin\psi \cos\psi \sin\phi \sin\delta \tag{40}$$

and

$$J_m \epsilon_m = J\epsilon. \tag{41}$$

Using the argument similar to that used to derive Eq. (21), we obtain the T-violation effect

$$\langle P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)\rangle_{20\%} = J_m \epsilon_m \times \begin{cases} 25.9 \\ 56.0 \\ 62.4 \\ \vdots \end{cases} = J\epsilon \times \begin{cases} 25.9, \\ 56.0, \\ 62.4, \\ \vdots, \end{cases} \tag{42}$$

at peaks, where we choose the mean neutrino energy E to satisfy (see Table I)

$$\Delta_{31} = \delta m_{31}^2 \frac{L}{2E} = 3.67, \;\; 9.63, \;\; 15.8, \ldots . \tag{43}$$

According to the analysis by Fogli *et al.* [11], $J/\sin\delta \sim 0.06$ and $\epsilon \sim 10^{-2}$ are allowed,[4] for example. Then

$$\langle P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)\rangle_{20\%} = \left(\frac{J/\sin\delta}{0.06}\right)\left(\frac{\epsilon}{10^{-2}}\right)\sin\delta \times \begin{cases} 0.015, \\ 0.033, \\ 0.037, \\ \vdots. \end{cases} \tag{44}$$

V. SUMMARY

We have examined the CP and T violation in the neutrino oscillation and analyzed how large the violation can be by taking account of the constraints of the neutrino experiments.

In the case of the comparable mass differences of δm_{21}^2, δm_{31}^2, and δm_{32}^2 in the range 10^{-3}–10^{-2} eV^2, which is consistent with the analysis of the atmospheric neutrino anomalies, it is found that there is a possibility that the CP-violation effect is large enough to be observed by 100–1000 km baseline experiments if the CP-violating parameter $\sin\delta$ is sufficiently large.

In case that δm_{21}^2 is much smaller than the matter parameter "a" and δm_{31}^2, which is favored both by the solar and atmospheric neutrino anomalies, we have derived a simple formula for the T-violation effect. We note that the probability of a CP- or T-violation effect should vanish for $\delta m_{21}^2 \to 0$, and therefore be proportional to $\delta m_{21}^2/\delta m_{31}^2$, $\delta m_{21}^2/(E/L)$ or $\delta m_{21}^2/a$ by the dimensional analysis. Our calculation confirms this expectation. If the solar and atmospheric neutrino anomalies are both attributed to the neutrino oscillation, the CP-violation test is found difficult since the matter effect is

[4]Here $\sin\omega \sim 1/2$ $\sin\psi \sim 1/\sqrt{2}$, and $\sin\phi = \sqrt{0.1}$.

larger than the pure CP-violation effect. How to extract the matter effect in such a case will be discussed in a separate paper [29].

In conclusion the long baseline neutrino oscillation experiments are very important and desirable to study not only neutrino masses and mixings but the CP or T violation in the lepton sector and there is some possibility to find such an effect explicitly.

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