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Duality of a Supersymmetric Standard Model without R Parity

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Recently one of the authors proposed a dual theory of a Supersymmetric Standard Model (SSM) in which it is naturally understood that at least one quark (the top quark) should be heavy, i.e., almost the same order as the weak scale, and the supersymmetric Higgs mass parameter μ can naturally be expected to be small. Unfortunately, the model cannot possess Yukawa couplings of the lepton sector. In this paper, we examine a dual theory of a Supersymmetric Standard Model without R parity. In this scenario, we can introduce Yukawa couplings of the lepton sector. In order to induce sufficiently large Yukawa couplings of leptons, we must introduce fairly large R parity breaking terms, which may be observed in the near future.

Recently, it has become clear that certain aspects of four dimensional supersymmetric field theories can be analyzed exactly.[1)~4)] By using this innovation, it has been tried to build models in order to solve some phenomenological problems.[4)~7)] One of the most interesting aspects is "duality".[1),3)] By using "duality", we can infer the low energy effective theory of a strong coupling gauge theory. One of the authors suggested that nature may use this "duality". He discussed a duality of a Supersymmetric Standard Model (SSM). Unfortunately, his model does not possess Yukawa couplings of the lepton sector. One possibility to introduce these is to unify quarks and leptons by considering the Pati-Salam gauge group.[8)] In this paper, we discuss the model with R-parity breaking terms in order to obtain the Yukawa couplings of the lepton sector.

First we recapitulate the previous model. Then we discuss the extension of the previous model and see how leptons acquire their mass. Then we give a summary and discussion.

To get the point of the previous idea, first we review Seiberg's duality. Following his discussion,[1)] we examine $SU(N_C)$ Supersymmetric (SUSY) QCD with N_F flavors of chiral superfields,

	$SU(N_C)$	$SU(N_F)_L$	$SU(N_F)_R$	$U(1)_B$	$U(1)_R$
Q^i	N_C	N_F	1	1	$(N_F-N_C)/N_F$
$\bar{Q}_j$	$\bar{N}_C$	1	$\bar{N}_F$	-1	$(N_F-N_C)/N_F$

which has the global symmetry $SU(N_F)_L\times SU(N_F)_R\times U(1)_B\times U_R$. In the case $N_F \geq N_C+2$, Seiberg suggested[1)] that at the low energy scale the above theory is equivalent to the following $SU(\tilde{N}_C)$ SUSY QCD theory ($\tilde{N}_C=N_F-N_C$) with N_F flavors

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of chiral superfields q_i and $\bar{q}^j$ and meson fields T_j^i:

	$SU(\tilde{N}_C)$	$SU(N_F)_L$	$SU(N_F)_R$	$U(1)_B$	$U(1)_R$
q_i	$\tilde{N}_C$	$\bar{N}_F$	1	$N_C/(N_F-N_C)$	N_C/N_F
$\bar{q}^j$	$\bar{\tilde{N}}_C$	1	N_F	$-N_C/(N_F-N_C)$	N_C/N_F
T_j^i	1	N_F	$\bar{N}_F$	0	$2(N_F-N_C)/N_F$

and with a superpotential

$$W=q_i T_j^i \bar{q}^j \,. \tag{1}$$

Then the idea of the duality of a SSM is the following.[5]

We introduce ordinary matter superfields except Higgs doublets:

$$Q_L{}^i=(U_L{}^i, D_L{}^i):(3,2)_{1/6}\,,\quad U_{Ri}^c:(\bar{3},1)_{-2/3}\,,\quad D_{iR}^c:(\bar{3},1)_{1/3}\,,$$
$$L^i=(N_L{}^i, E_L{}^i):(1,2)_{-1/2}\,,\quad E_{Ri}^c:(1,1)_1\,,\quad i=1,2,3 \tag{2}$$

which transform under the gauge group $SU(3)_{\tilde{C}}\times SU(2)_L\times U(1)_Y$.

Let us examine the dual theory of this theory with respect to the gauge group $SU(3)_{\tilde{C}}$. In this case the number of the flavor is 6 and the global symmetry in the Seiberg's sense is $SU(6)_{QL}\times SU(6)_{QR}\times U(1)_B\times U(1)_R$. We can assign $Q=(U_L{}^1, D_L{}^1, U_L{}^2, D_L{}^2, U_L{}^3, D_L{}^3)$ and $\bar{Q}=(U_R{}^{c1}, D_R{}^{c1}, U_R{}^{c2}, D_R{}^{c2}, U_R{}^{c3}, D_R{}^{c3})$ in the table. Since $N_F=6$, the dual gauge group is also $SU(3)_C$ $(\tilde{N}_C=N_F-N_C)$, which we will assign to the ordinary QCD gauge group. A subgroup, $SU(2)_L\times U(1)_Y$, of the global symmetry group $SU(6)_{QL}\times SU(6)_{QR}\times U(1)_B\times U(1)_R\times SU(6)_L\times SU(3)_{ER}\times U(1)_L\times U(1)_{ER}$ is gauged. $SU(6)_L\times SU(3)_{ER}\times U(1)_L\times U(1)_{ER}$ is the global symmetry of the lepton sector. For example, $SU(6)_L\times U(1)_L$ acts on the multiplet $L\equiv(N_L{}^1, E_L{}^1, N_L{}^2, E_L{}^2, N_L{}^3, E_L{}^3)$. $SU(2)_L$ generators are given by

$$I_L{}^a=I_{QL1}^a+I_{QL2}^a+I_{QL3}^a+I_{L1}^a+I_{L2}^a+I_{L3}^a\,,\quad a=1,2,3\,, \tag{3}$$

where $I_{QLi}^a[I_{Li}^a]$ are generators of $SU(2)_{QLi}$ $[SU(2)_{Li}]$ symmetries which rotate $(U_L{}^i, D_L{}^i)$ $[(N_L{}^i, E_L{}^i)]$, and the generator of hypercharge Y is given by

$$Y=\frac{1}{6}B-(I_{R1}^3+I_{R2}^3+I_{R3}^3)-\frac{1}{2}L+ER\,, \tag{4}$$

where I_{QRi}^a are generators of $SU(2)_{Ri}$ symmetries which rotate (U_{Ri}^c, D_{Ri}^c). In this theory, the global symmetry group is $SU(3)_{QL}\times SU(3)_{UR}\times SU(3)_{DR}\times U(1)_B\times U(1)_R\times SU(3)_L\times SU(3)_{ER}\times U(1)_L\times U(1)_{ER}$.*) Then we can write down the quantum numbers of dual fields,

$$q_{Li}=(d_{Li}, -u_{Li}):(3,\bar{2})_{1/6}\,,\quad u_R{}^{ci}:(3,1)_{-3/2}\,,\quad d_R{}^{ci}:(\bar{3},1)_{1/3}\,,$$
$$M_j{}^i:(1,2)_{-1/2}\,,\quad N_j{}^i:(1,2)_{1/2}\,,$$
$$L^i=(N_L{}^i, E_L{}^i):(1,2)_{-1/2}\,,\quad E_{Ri}^c:(1,1)_1\,,\quad i=1,2,3 \tag{5}$$

*) Strictly speaking two of four global $U(1)$ are broken by $SU(2)_L$ and $U(1)_Y$ anomalies.

under the standard gauge group $SU(3)_C\times SU(2)_L\times U(1)_Y$. Here $M_j^i\sim Q_L{}^iU_{Rj}^c$ and $N_j^i\sim Q_L{}^iD_{Rj}^c$ are the meson fields, and we assign $q=(d_L{}^1, -u_L{}^1, d_L{}^2, -u_L{}^2, d_L{}^3, -u_L{}^3)$ and $\bar{q}=(d_R{}^{c1}, -u_R{}^{c1}, d_R{}^{c2}, -u_R{}^{c2}, d_R{}^{c3}, -u_R{}^{c3})$. Because leptons do not have color indices, they exist as they were. It is interesting that the matter content of both theories is almost the same. The difference is the existence of nine pairs of Higgs superfields M_j^i and N_j^i and their Yukawa terms coupling to ordinary matter superfields,

$$W=-q_L{}^iN_i^ju_{Rj}^c+q_L{}^iM_i^jd_{Rj}^c\,. \tag{6}$$

If one combination of these Higgs scalar fields*)

$$\tilde{H}=\sum_{i,j}^{3}a_j{}^i\tilde{N}_i^j+b_j{}^i(\tilde{M}^c)_i^j\,, \tag{7}$$

where M^c denotes the charge conjugated field of M, has a VEV $\langle H\rangle=(v,0)$, the $SU(2)_L\times U(1)_Y$ symmetry is broken to the electromagnetic gauge group $U(1)_Q$. In this case, ordinary quark mass matrices are determined by the mixing of the Higgs scalar field. It should be noted that at least one quark has a heavy mass, which is almost the order of the weak scale v, if the Yukawa coupling can be taken to be of order one because of the strong dynamics. Namely, the heaviness of the top quark can be naturally understood.

Unfortunately, this model has many phenomenological problems; there is no Yukawa coupling of leptons, there appear Nambu-Goldstone bosons when the Higgs particles have vacuum expectation values, and the SUSY mass terms of Higgs particles vanish.

We break R parity in the above model in order to induce the Yukawa couplings of leptons and to avoid Nambu-Golistone bosons. We introduce the superpotential

$$W=\lambda_{ijk}L^iL^jE_R{}^{ck}\,, \tag{8}$$

which breaks the global symmetry $SU(3)_L\times SU(3)_{ER}\times U(1)_L\times U(1)_{ER}$ for leptons and R parity. Because leptons are singlet under the $SU(3)_{\bar{c}}$, these terms survive in the dual theory. Though to introduce all such terms causes dangerous phenomena which are experimentally excluded, we can introduce only one term among them.[9),10)]

In the theory with $SU(3)_{\bar{c}}$, we introduce the R-violating superpotential

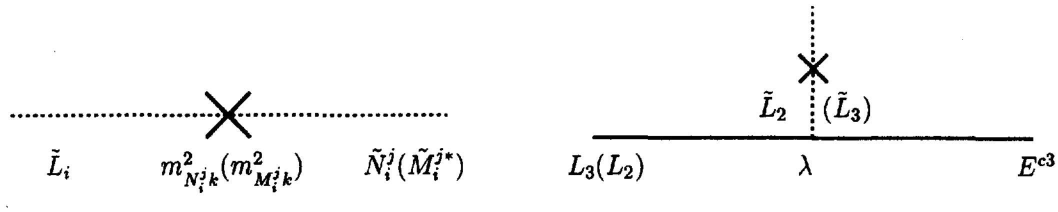
$$W=\lambda L^2L^3E_R{}^{c3}\,, \tag{9}$$

which breaks the global symmetry of the lepton sector $SU(3)_L\times SU(3)_{ER}\times U(1)_L\times U(1)_E$ to $SU(2)_{L23}\times U(1)_{L1}\times SU(2)_{ER}\times U(1)_{(E1+E2)}\times U(1)_{(L2+L3-2E3)}$, where $SU(2)_{L23}$ acts on the multiplet (L_2, L_3), $U(1)_{L1}$ indicates that the charge of L_1 is 1 and those of others' are 0, and so on. The magnitude of the coupling is expected to be $O(0.1)$.[9)]

If the scalar component of the L_i $(\equiv\tilde{L}_i)$ acquire a VEV, then at least one lepton acquires mass. When we consider SUSY breaking terms in the theory, it is naturally understood that the L_i acquire VEVs. We will see this in the following.

First of all, the global symmetry group $SU(3)_{QL}\times SU(3)_{UR}\times SU(3)_{DR}\times U(1)_R$ must be broken explicitly, because otherwise when Higgs fields N_i^j and M_i^j have VEVs, that

*) In this paper we denote the scalar component with a tilde, e.g., $\tilde{A}$ represents the scalar component of the superfield A.



(a) (b)

Fig. 1. (a) A typical graph which gives a VEV to sneutrinos (neutral component of $\tilde{L}_i$). (b) A graph which induces a mass term for lepton at tree level below the electroweak scale.

is the global symmetry is spontaneously broken, massless Nambu-Goldstone bosons appear. One possibility of breaking the global symmetry explicitly is to introduce soft SUSY breaking terms[11),12)] which also break all the global symmetry except $U(1)_B$.*)

$$\sum_{i,j,k}(A_{ijk}\tilde{Q}_i\tilde{U}_j^c\tilde{L}_k+B_{ijk}\tilde{Q}_i\tilde{D}_j^c\tilde{L}_k^*+C_{ijk}\tilde{L}_i\tilde{L}_j\tilde{E}_k)+\text{mass term}\,. \tag{10}$$

Thus there appear mixing terms between the Higgs doublets, $\tilde{N}_i^j$ and $\tilde{M}_i^j$, and the scalar components of lepton doublets $\tilde{L}_i$:

$$m^2_{N_i{}^j k}\tilde{N}_i^j\tilde{L}_k+m^2_{M_i{}^j k}\tilde{M}_i^{j*}\tilde{L}_k\,, \tag{11}$$

as well as self-mass of $\tilde{L}_i$

$$m^2_{\tilde{l}_i}|\tilde{L}_i|^2\,. \tag{12}$$

Then after Higgs fields acquire VEVs, VEVs of the scaler leptons are induced. We assume transition masses are smaller than self-masses because the former breaks not only the global flavor symmetry but also the lepton global symmetry, while the latter breaks only some part of the lepton global symmetry. In this case the induced VEVs are roughly given by $\langle\tilde{L}_i\rangle\sim(m^2_{N_j{}^k i}/m^2_{\tilde{l}_i})\langle N_j{}^k\rangle\sim(m_N{}^2/m^2_{\tilde{l}_i})v$, where $m_N{}^2$ is a typical transition mass and v is the VEV of $\tilde{H}$. Graphically these are expressed in Fig. 1(a). Naively the magnitude of the scalar lepton VEVs is $O(10)$ GeV.**)

By these VEVs, leptons masses arise at the tree level according to (9) (see Fig. 1(b)). The form of the mass matrix for leptons is

*) We introduce SUSY breaking terms which do not respect the holomorphy. Such terms do not cause quadratic divergence, that is, they do not spoil the hierarchy stability unless there is a singlet field. We assume simply that there is no singlet field.

**) Such a scalar VEV induces a mass of μ neutrino $m_{\nu_\mu}\sim(g_1{}^2\mu_2+g_2{}^2\mu_1)\langle\tilde{L}\rangle^2/\mu_1\mu_2$ where μ_i and g_i ($i=1$, 2) are gaugino masses and gauge couplings of $U(1)_Y$ and $SU(2)_L$. We can make the neutrino mass zero by $g_1{}^2\mu_2+g_2{}^2\mu_1=0$.

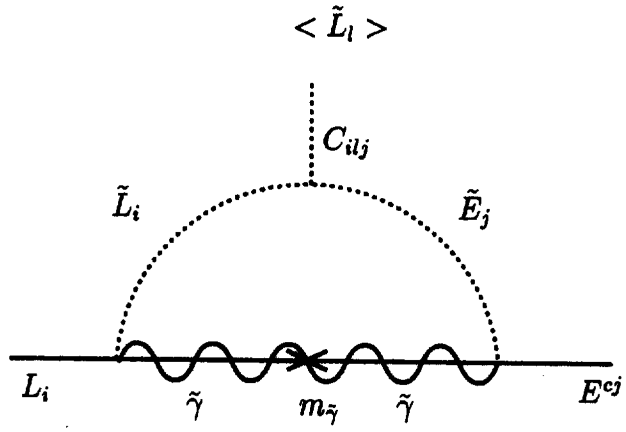


Fig. 2. A typical graph which induces a mass term for lepton at one loop below the electroweak scale.

$$M_l=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & m_2\\ 0 & 0 & m_3\end{pmatrix}, \tag{13}$$

where $m_2=-\lambda\langle\tilde{L}_3\rangle$, $m_3=\lambda\langle\tilde{L}_2\rangle\sim O(1)$ GeV.*) Thus we can understand the τ mass naturally.

Through radiative corrections like Fig. 2[13] the other will be induced. These are strongly dependent on C_{ijk}.**)

In summary, we examined duality of a SUSY model with R-Parity breaking terms. In this model, the Yukawa couplings of the lepton sector as well as the quark sector are induced. Moreover, since all the global symmetries except $U(1)_B$ are broken by SUSY breaking terms, we can avoid the appearance of Nambu-Goldstone bosons when Higgs fields acquire VEVs. The fact that nature does not respect the R-Parity, which might be observed in near future, may suggest that nature uses the duality.

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*) Because there is $SU(2)_{L23}$ global symmetry, we can assume, without loss of generality, that among $\tilde{L}_2$ and $\tilde{L}_3$ only $\tilde{L}_2$ acquires a VEV and hence only m_3 is not zero.

**) In the case that R-Parity is broken, in general, neutrino masses are also induced.[10] These are also dependent on SUSY breaking parameters and here we do not touch the details.

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