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(2,0) Chern-Simons Supergravity Plus Matter Near the Boundary of AdS_3

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Abstract

We examine the boundary behaviour of the gauged $N = (2, 0)$ supergravity in $D = 3$ coupled to an arbitrary number of scalar supermultiplets which parametrize a Kähler manifold. In addition to the gravitational coupling constant, the model depends on two parameters, namely the cosmological constant and the size of the Kähler manifold. It is shown that regular and irregular boundary conditions can be imposed on the matter fields depending on the size of the sigma model manifold. It is also shown that the super AdS transformations in the bulk produce the transformations of the $N = (2, 0)$ conformal supergravity and scalar multiplets on the boundary, containing fields with nonvanishing Weyl weights determined by the ratio of the sigma model and the gravitational coupling constants. Various types of (2,0) superconformal multiplets are found on the boundary and in one case the superconformal symmetry is shown to be realized in an unconventional way.

1 Introduction

In probing various aspects of the remarkable connections between anti de Sitter and conformal supergravity theories, the AdS_3/CFT_2 correspondence in particular provides a relatively more manageable case to study. At the same time, some novel features arise due to the fact that AdS supergravity in $D = 3$ is essentially non-dynamical. Nonetheless, AdS_3 supergravity plays a significant role in the description of the matter fields to which it couples. As a step towards a detailed study of the amplitudes, anomalies and other significant properties of this type of theories, it is useful to determine precisely the behaviour of the AdS supersymmetry transformations at the boundary. This problem has been examined for pure $N \leq 2$ AdS_3 supergravity in [1], pure $N = 4$ AdS_3 supergravity in [2] [1], the maximal AdS_7 supergravity in [5], the maximal $F(4)$ AdS_6 supergravity in [6] and the minimal AdS_5 supergravity in [7], where it was shown that the correct transformations rules of the boundary conformal supergravities indeed follow from a careful study of the bulk super AdS transformations. However, a similar analysis does not seem to have been carried out so far for matter coupled AdS supergravities which should shed further light on the AdS/CFT duality questions in the context of M-theory in backgrounds with less than maximal supersymmetry. Our aim in this paper is to fill this gap.

AdS supergravities are based on AdS superalgebras. Given the fact that the AdS group in $2+1$ dimensions is a product of two factors as $SO(2,2) = SO(2,1)_L \times SO(2,1)_R$, the super AdS group itself has the factored form $G_L \times G_R$. It turns out that there are many choices for $G_{L,R}$, the most typical case being $OSp(2,p) \times OSp(2,q)$, where p and q are not necessarily equal. The supergravity theories based on these algebras will be referred to as the $N = (p,q)$ AdS_3 supergravities. They have been constructed as Chern-Simons gauged theories long ago by Achucarro and Townsend [8]. However, very little is known so far about their matter couplings. In fact, the only cases studied until now seem to be the $N = (2,0)$ AdS_3 supergravity coupled to an arbitrary number of scalar supermultiplets [9, 10] and $N = (16,0)$ AdS_3 supergravity with an exceptional sigma model sector [11]. The models constructed in [9] and [10] are significantly different from each other, stemming from the fact that the scalar fields are neutral under the U(1) R-symmetry group in the model of [9], but charged in the model of [10]. The Izquierdo-Townsend model has only one free parameter, namely the cosmological constant, in addition to the gravitational constant, unlike the model of [10], where there is the additional parameter that measures the size of the sigma model manifold. In fact, the U(1) charge carried by the scalar fields is related to this size, and as we will show in this paper, the limit in which the U(1) charge vanishes implies a flat sigma model manifold, and the models of [9] and [10] do indeed agree in that case.

The model studied here is expected to arise from a compactification of M-theory. The much studied compactification of Type IIB theory on $AdS_3 \times S^3 \times K$, where K is essentially T^4 or $K3$, gives rise to $N = (4,4)$ or $N = (4,0)$ AdS_3 supergravities coupled to matter. The spectra of these theories are known [12, 13] but not their actions so far. Whether our $N = (2,0)$ model arises as a consistent truncation of such theories remains to be seen.

[1] Various aspects of the AdS/CFT correspondence for pure supergravities in $D = 3$, in particular the asymptotic symmetries, have been studied in [3, 4].

The super AdS transformations in the bulk theory studied here are shown to produce the transformations of $N = (2,0)$ conformal supergravity coupled to scalar multiplets with nonvanishing Weyl weight determined by the ratio of the Kähler sigma model manifold and the gravitational coupling constant. In doing so, the so called regular and irregular boundary conditions are utilized [14, 15]. These choices of boundary conditions result in the phenomenon in which scalar fields in AdS space of sufficiently negative mass-squared can be associated with CFT operators of two possible dimensions. An example of this has been discussed in [15] in the context of $AdS_5 \times T^{1,1}$ compactification of Type IIB string theory. Here, we provide another example of this phenomenon and show explicitly the resulting CFT supergravity plus matter symmetry transformations. In doing so, we find an interesting conformal supermultiplet structure that involves a submultiplet of fields that transform into each other. In this novel multiplet the superconformal symmetry is realized in an unconventional fashion.

The $(2,0)$ model is described in the next section. The relation between the models of [9] and [10] is described in Section 3. The boundary conditions and the linearized field equations are given in Section 4, and the bosonic and fermionic symmetries of the boundary CFT are obtained in Section 5. Concluding remarks are contained in Section 6.

2 The Matter Coupled N=(2,0) AdS$_3$ Supergravity

The $N = (2,0)$ AdS_3 supergravity multiplet consists of a graviton $e_\mu{}^a$, two Majorana gravitini ψ_μ (with the $SO(2)$ spinor index suppressed) and an $SO(2)$ gauge field A_μ. The n copies of the $N = (2,0)$ scalar multiplet, on the other hand, consists of $2n$ real scalar fields $\phi^\alpha (\alpha = 1, ..., 2n)$ and $2n$ Majorana fermions λ^r ($r = 1, ..., n$ and the $SO(2)$ spinor indices are suppressed).

In [10], the sigma model manifold M was taken to be a coset space of the form $G/H \times SO(2)$ where G can be compact or non-compact and $H \times SO(2)$ is the maximal compact subgroup of G, where $SO(2)$ is the R-symmetry group. In particular, the following cases are considered [10]

$$M_+ = \frac{SO(n+2)}{SO(n) \times SO(2)} \ , \qquad M_- = \frac{SO(n,2)}{SO(n) \times SO(2)} \ . \tag{2.1}$$

The results can be readily translated to the case of $G/H \times U(1)$ with $G = SU(n+1)$ or $SU(n,1)$ and $H = SU(n)$.

Key ingredients in the description of the model are the matrices $(L_I{}^i, L_I{}^r)$ where $I = 1, ..., n+2$, $i = 1, 2$, $r = 1, .., n$, which form a representative of the coset $M_\pm$. It follows that

$$\begin{aligned} &L_I{}^i L^{Ij} = \pm\delta^{ij} \ , \qquad L_I{}^r L^{Is} = \delta^{rs} \ , \qquad L_I{}^i L^{Ir} = 0 \ , \\ &\pm L_I{}^i L^{Ji} + L_I{}^r L^{Jr} = \delta_I^J \ , \end{aligned} \tag{2.2}$$

where $\pm$ correspond to the scalar manifolds $M_\pm$. The $SO(n)$, $SO(2)$ and $SO(n+2)$ vector indices are raised and lowered with the Kronecker deltas and the $SO(n,2)$ vector indices with the metric $\eta_{IJ} = \mathrm{diag}(++\ldots+--)$.

Other important ingredient of the model is the $SO(2)$ gauged pull-back of the Maurer-Cartan form on $M_\pm$ which is decomposed into the $SO(n)\times SO(2)$ connections Q_μ^{rs} and Q_μ^{ij}, and the nonlinear covariant derivative P_μ^{ir} as follows:

$$P_\mu^{ir} = \left(L^{-1}D_\mu L\right)^{ir}\ , \quad Q_\mu^{ij} = \left(L^{-1}D_\mu L\right)^{ij}\ , \quad Q_\mu^{rs} = \left(L^{-1}D_\mu L\right)^{rs}\ , \tag{2.3}$$

where the $SO(2)$ covariant derivative is defined as

$$D_\mu L = \left(\partial_\mu + \tfrac{1}{2}A_\mu^{ij}T_{ij}\right)L\ , \tag{2.4}$$

The anti-hermitian $SO(2)$ generator T_{ij} occurring in this definition is realized in terms of an $(n+2)\times(n+2)$ matrix, which can be chosen as $(T_{ij})_I{}^J = (\pm\,\delta_{Ii}\,\delta_j^J - i\leftrightarrow j\,)$. Introducing the coordinates $\phi^\alpha\ (\alpha = 1,\ldots,2n)$ which parametrize the scalar manifold G/H, we can also define the coset vielbein V_α^{ir} and the $SO(2)\times SO(n)$ connections $A_\alpha^{ij}, A_\alpha^{rs}$ on G/H as

$$V_\alpha^{ir} = \left(L^{-1}\partial_\alpha L\right)^{ir}\ , \quad A_\alpha^{ij} = \left(L^{-1}\partial_\alpha L\right)^{ij}\ , \quad A_\alpha^{rs} = \left(L^{-1}\partial_\alpha L\right)^{rs}\ , \tag{2.5}$$

where $\partial_\alpha \equiv \frac{\partial}{\partial\phi^\alpha}$. From the above relations it follows that

$$P_\mu^{ir} = \partial_\mu\phi^\alpha V_\alpha^{ir} + A_\mu S^{ir}\ , \tag{2.6}$$

$$Q_\mu = \partial_\mu\phi^\alpha A_\alpha + A_\mu C\ , \tag{2.7}$$

$$Q_\mu^{rs} = \partial_\mu\phi^\alpha A_\alpha^{rs} + A_\mu C^{rs}\ , \tag{2.8}$$

where $A_\alpha^{ij} = A_\alpha\epsilon^{ij}$, $A_\mu^{ij} = A_\mu\epsilon^{ij}$, $Q_\mu^{ij} = Q_\mu\epsilon^{ij}$ and the (C, S^{ir}) functions are defined as

$$\begin{aligned}
\epsilon_{ij}\epsilon^{kl}C &= (L^{-1}T_{ij}L)^{kl}\ ,\\
\epsilon_{ij}C^{rs} &= (L^{-1}T_{ij}L)^{rs}\ ,\\
\epsilon_{ij}S^{kr} &= (L^{-1}T_{ij}L)^{kr}\ .
\end{aligned} \tag{2.9}$$

The matter coupled $N=(2,0)$ Chern-Simons supergravity Lagrangian which makes use of these ingredients has been obtained in [10]. Up to quartic fermions the Lagrangian is as follows [10]: [2]

[2]Conventions: $\eta_{ab} = (-++)$, $\bar\varepsilon = \varepsilon^\dagger i\gamma_0$, $\gamma^\mu C$ and $\gamma^{\mu\nu}C$ are symmetric and $\gamma^{\mu\nu\rho} = \frac{1}{\sqrt{-g}}\epsilon^{\mu\nu\rho}$. The $SO(2)$ charge conjugation matrix is unity, Γ^i is symmetric and $\{\Gamma^i,\Gamma^j\} = 2\delta^{ij}$. A convenient representation is $\Gamma_1 = \sigma_1$, $\Gamma_2 = \sigma_3$. We define $\Gamma_3 = \Gamma_1\Gamma_2$. Note that $(\Gamma^3)^2 = -1$.

$$
\begin{aligned}
e^{-1}\mathcal{L} &= \frac{1}{4}R+\frac{e^{-1}}{2}\epsilon^{\mu\nu\rho}\bar\psi_\mu D_\nu\psi_\rho-\frac{e^{-1}}{16ma^4}\,\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho-\frac{1}{4a^2}P_\mu^{ir}P^\mu_{ir}\\
&+\frac{1}{2}\,\bar\lambda_r\gamma^\mu D_\mu\lambda^r+\frac{1}{2a}\,\bar\lambda_r\gamma^\mu\gamma^\nu\Gamma_i\psi_\mu P_\nu^{ir}-\frac{m}{2}\,\bar\psi_\mu\gamma^{\mu\nu}\psi_\nu C^2\\
&-2ma\,\bar\psi_\mu\gamma^\mu\Gamma_i\Gamma^3\lambda_r CS^{ir}-\frac{1}{2}m(1+4\epsilon a^2)\,\bar\lambda^r\lambda_r C^2\\
&+2ma^2\,\bar\lambda_r\Gamma^3\lambda_s C^{rs}C+2ma^2\,\bar\lambda_r\Gamma_i\Gamma_j\lambda_s S^{ir}S^{js}\\
&+2m^2C^2(C^2-2a^2S^{ir}S_{ir})\ ,
\end{aligned}
\tag{2.10}
$$

which has the local $N=2$ supersymmetry

$$
\begin{aligned}
\delta e_\mu{}^a &= -\bar\varepsilon\gamma^a\psi_\mu\ ,\\
\delta\psi_\mu &= D_\mu\varepsilon+m\gamma_\mu C^2\varepsilon\ ,\\
\delta A_\mu &= 4ma^2\,(\bar\varepsilon\Gamma^3\psi_\mu)\,C-4ma^3\,(\bar\lambda_r\gamma_\mu\Gamma_i\varepsilon)\,S^{ir}\ ,\\
L_i{}^I\delta L_I{}^r &= a\bar\varepsilon\,\Gamma_i\lambda^r\ ,\\
\delta\lambda^r &= \left(-\frac{1}{2a}\gamma^\mu P_\mu^{ir}+2ma\Gamma^3CS^{ir}\right)\Gamma_i\varepsilon\ .
\end{aligned}
\tag{2.11}
$$

The functions C and S^{ir} are defined in (2.9) and [3]

$$
\begin{aligned}
D_\mu\varepsilon &= \left(\partial_\mu+\tfrac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}-\frac{1}{2a^2}\,Q_\mu\Gamma^3\right)\varepsilon\ ,\\
D_\mu\lambda^r &= \left(\partial_\mu+\tfrac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}+\left(\epsilon+\frac{1}{2a^2}\right)Q_\mu\Gamma^3\right)\lambda^r+Q_\mu{}^{rs}\lambda^s\ .
\end{aligned}
\tag{2.12}
$$

The parameter $\epsilon=\pm1$ corresponds to the manifolds $M_\pm$ defined in (2.1), and the constant a is the characteristic curvature of $M_\pm$ (e.g. $2a$ is the inverse radius in the case of $M_+=S^2$). The gravitational coupling constant κ has been set equal to one, but it can easily be introduced by dimensional analysis. The constant m is the AdS_3 cosmological constant. Unlike in a typical anti de Sitter supergravity coupled to matter, here the constants κ, a, m are not related to each other for non-compact scalar manifolds, while a is quantized in terms of κ in the compact case as [10].

To conclude this section, and for later purposes, we list the equations of motion which follow from the Lagrangian (2.10):

[3] The ϵ term in $D_\mu\lambda^r$ was inadvertently omitted in [10].

$$R_{\mu\nu} - a^{-2} P^{ir}_\mu P^{ir}_\nu + 8m^2 C^2 \left(C^2 - 2a^2 S^{ir} S_{ir}\right) g_{\mu\nu} \;\; = \;\; 0 \;, \tag{2.13}$$

$$\psi_{\mu\nu} + 2m\gamma_{[\mu}\,\psi_{\nu]}\, C^2 + 2ma\gamma_{\mu\nu}\Gamma_i\Gamma^3\lambda_r C S^{ir} - \frac{1}{2a}\Gamma_i\gamma^\rho\gamma_{\mu\nu}\lambda_r P^{ir}_\rho \;\; = \;\; 0 \;, \tag{2.14}$$

$$F_{\mu\nu} - 4ma^2\sqrt{-g}\epsilon_{\mu\nu\rho}P^\rho_{ir}S^{ir} \;\; = \;\; 0 \;, \tag{2.15}$$

$$\begin{aligned}\gamma^\mu D_\mu\lambda^r - m(1+4\epsilon a^2)\,\lambda^r C^2 + \frac{1}{2a}\,\gamma^\mu\gamma^\nu\Gamma_i\psi_\mu P^{ir}_\nu + 4ma^2\Gamma^3\lambda_s C^{rs} C \\ +4ma^2\Gamma_i\Gamma_j\lambda_s S^{ir}S^{js} - 2ma\gamma^\mu\Gamma^3\Gamma_i\psi_\mu C S^{ir} \;\; = \;\; 0 \;,\end{aligned} \tag{2.16}$$

$$\begin{aligned}D^\mu P^{ir}_\mu + 16m^2a^2\epsilon_{ij}S^{jr}C\left((1+\epsilon a^2)C^2 - a^2\, S^{ks}S_{ks}\right) \\ +16m^2a^4\, C^{rs}S_{is}C^2 \;\; = \;\; 0 \;,\end{aligned} \tag{2.17}$$

where the fermion bilinears in the bosonic field equations have been suppressed and

$$\psi_{\mu\nu} \;\; = \;\; D_\mu\psi_\nu - D_\nu\psi_\mu \;, \tag{2.18}$$

$$D_\mu P^{\mu ir} \;\; = \;\; \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}P^{ir}_\nu\right) + \epsilon\, Q^{ik}_\mu P^{\mu\, kr} + Q^{rs}_\mu P^{\mu\, ir} \;. \tag{2.19}$$

3 Connection with the Izquierdo-Townsend Model

The model reviewed above [10] differs from the one constructed by Izquierdo and Townsend [9], in all the terms containing the C and S-functions. These differences stem from the fact that the scalar fields in the model above are charged under the R-symmetry group $SO(2)$ while in the model of [9] they are neutral. Given that this charge is related to the sigma model radius, taking the zero charge limit in order to compare the two models is expected to constrain the scalar manifold. Here, we will show the relation between the two models and show that they indeed agree only in the limit in which the scalar manifold is flat.

We begin by parametrizing the coset representative L as follows

$$L = exp\begin{pmatrix} 0 & \phi^{ir} \\ -\epsilon(\phi^{ir})^T & 0 \end{pmatrix} \;, \tag{3.1}$$

where ϕ^{ir} are $2n$ real coordinates on $M_\pm$. Next, we perform the rescalings

$$A_\mu \to a^2\, A_\mu \;, \qquad \phi^{ir} \to a\,\phi^{ir} \tag{3.2}$$

and consider the limit $a^2 \to 0$. From the definitions in (2.3) we find

$$
\begin{aligned}
P_\mu^{ir} &= a\partial_\mu\phi^{ir} + \cdots \ , \\
Q_\mu &= a^2\left(A_\mu - \frac{1}{2}\phi^{ir}\partial_\mu\phi_r{}^j\epsilon_{ij}\right) + \cdots \ , \\
Q_\mu^{rs} &= a^2(\partial_\mu\phi^{ri}\phi_i{}^s - \phi^{ri}\partial_\mu\phi_i{}^s) + \cdots \ ,
\end{aligned}
\tag{3.3}
$$

where $\cdots$ denote higher order terms in positive powers of a^2. Let us define

$$
d\phi^\alpha A_\alpha = \frac{1}{2}\phi^{ir}d\phi^j{}_r\,\epsilon_{ij} \ , \tag{3.4}
$$

so that $Q_\mu = a^2(A_\mu + \partial_\mu\phi^\alpha A_\alpha)$, where the index α represents a pair of indices (ir). We have

$$
\frac{1}{2}d\phi^\alpha \wedge d\phi^\beta F_{\alpha\beta} = \frac{1}{2}d\phi^{ir}\wedge d\phi^j{}_r\,\epsilon_{ij} \ , \tag{3.5}
$$

where $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$. In the limit $a^2 \to 0$ the Lagrangian becomes

$$
\begin{aligned}
e^{-1}\mathcal{L} \;=\; & \frac{1}{4}R + \frac{1}{2}e^{-1}\epsilon^{\mu\nu\rho}\bar\psi_\mu D_\nu\psi_\rho - \frac{1}{16}\frac{e^{-1}}{m}\,\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - \frac{1}{4}\partial_\mu\phi^{ir}\partial^\mu\phi_{ir} \\
& +\frac{1}{2}\,\bar\lambda_r\gamma^\mu D_\mu\lambda^r + \frac{1}{2}\,\bar\lambda_r\gamma^\mu\gamma^\nu\Gamma_i\psi_\mu\partial_\nu\phi^{ir} \\
& -\frac{1}{2}m\,\bar\psi_\mu\gamma^{\mu\nu}\psi_\nu - \frac{1}{2}m\,\bar\lambda^r\lambda_r + 2m^2 \ ,
\end{aligned}
\tag{3.6}
$$

and the transformation rules become

$$
\begin{aligned}
\delta e_\mu{}^a &= -\bar\varepsilon\gamma^a\psi_\mu \ , \\
\delta\psi_\mu &= D_\mu\varepsilon + m\gamma_\mu\varepsilon \ , \\
\delta A_\mu &= 4m\,\bar\varepsilon\,\Gamma^3\psi_\mu \ , \\
\delta\phi^{ir} &= \bar\varepsilon\,\Gamma^i\lambda^r \ , \\
\delta\lambda^r &= -\frac{1}{2}\gamma^\mu\partial_\mu\phi^{ir}\Gamma_i\varepsilon \ ,
\end{aligned}
\tag{3.7}
$$

where the covariant derivatives are defined by

$$
\begin{aligned}
D_\mu \psi_\nu &= \left(\nabla_\mu - \frac{1}{2}\,\partial_\mu \phi^\alpha A_\alpha \Gamma^3 - \frac{1}{2}\,A_\mu \Gamma^3\right)\psi_\nu \ , \\
D_\mu \lambda^r &= \left(\nabla_\mu + \frac{1}{2}\partial_\mu \phi^\alpha A_\alpha \Gamma^3 + \frac{1}{2}A_\mu \Gamma^3\right)\lambda^r \ , \\
D_\mu \varepsilon &= \left(\nabla_\mu - \frac{1}{2}\,\partial_\mu \phi^\alpha A_\alpha \Gamma^3 - \frac{1}{2}\,A_\mu \Gamma^3\right)\varepsilon \ ,
\end{aligned}
\tag{3.8}
$$

and $\nabla_\mu = \partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}$. Introducing

$$
\begin{aligned}
A'_\mu &= A_\mu + \partial_\mu \phi^\alpha A_\alpha \\
&= A_\mu + \frac{1}{2}\phi^{ir}\partial_\mu \phi^j{}_r \epsilon_{ij} \ ,
\end{aligned}
\tag{3.9}
$$

the Lagrangian (3.6) becomes

$$
\begin{aligned}
e^{-1}\mathcal{L} &= \frac{1}{4}R + \frac{1}{2}e^{-1}\epsilon^{\mu\nu\rho}\bar\psi_\mu D_\nu \psi_\rho - \frac{1}{16}\frac{e^{-1}}{m}\,\epsilon^{\mu\nu\rho}A'_\mu \partial_\nu A'_\rho - \frac{1}{4}\partial_\mu \phi^{ir}\partial^\mu \phi_{ir} \\
&\quad -A'_\mu J^\mu + \frac{1}{2}\partial_\mu \phi^\alpha A_\alpha J^\mu + \frac{1}{2}\,\bar\lambda_r \gamma^\mu D_\mu \lambda^r + \frac{1}{2}\,\bar\lambda_r \gamma^\mu \gamma^\nu \Gamma_i \psi_\mu \partial_\nu \phi^{ir} \\
&\quad -\frac{1}{2}m\,\bar\psi_\mu \gamma^{\mu\nu}\psi_\nu - \frac{1}{2}m\,\bar\lambda^r \lambda_r + 2m^2 \ ,
\end{aligned}
\tag{3.10}
$$

where the current J^μ is defined as

$$
J^\mu = -\frac{1}{16me}\epsilon^{\mu\nu\rho}\partial_\nu \phi^\alpha \partial_\rho \phi^\beta F_{\alpha\beta} \ ,
\tag{3.11}
$$

with $F_{\alpha\beta}$ defined in (3.5). The transformations rules (3.7), on the other hand, become

$$
\begin{aligned}
\delta e_\mu{}^a &= -\bar\varepsilon \gamma^a \psi_\mu \ , \\
\delta \psi_\mu &= D_\mu \varepsilon + m\gamma_\mu \varepsilon \ , \\
\delta A'_\mu &= 4m\,\bar\varepsilon\,\Gamma^3 \psi_\mu - \delta\phi^\alpha \partial_\mu \phi^\beta F_{\beta\alpha} \\
\delta \phi^{ir} &= \bar\varepsilon\,\Gamma^i \lambda^r \ , \\
\delta \lambda^r &= -\frac{1}{2}\gamma^\mu \partial_\mu \phi^{ir}\Gamma_i \varepsilon \ ,
\end{aligned}
\tag{3.12}
$$

where we have discarded a term in δA_μ which can be expressed as a gauge transformation $\delta A_\mu = \partial_\mu \Lambda$. Of course, the above transformations are up to cubic fermion terms in the transformation rules of ψ_μ and λ^r, since the Lagrangian (3.10) is up to quartic fermion terms. Note also that the covariant derivatives have now simplified to

$$\begin{aligned} D_\mu \psi_\nu &= \left(\nabla_\mu - \frac{1}{2}\, A'_\mu \Gamma^3\right) \psi_\nu \ , \\ D_\mu \lambda^r &= \left(\nabla_\mu + \frac{1}{2} A'_\mu \Gamma^3\right) \lambda^r \ , \\ D_\mu \varepsilon &= \left(\nabla_\mu - \frac{1}{2}\, A'_\mu \Gamma^3\right) \varepsilon \ . \end{aligned} \tag{3.13}$$

The formulae (3.10), (3.12) and (3.13) agree with those of the Izquierdo-Townsend model [9] for the flat sigma model. Note that in trying to set the U(1) charge of the scalar fields equal to zero, we have been forced to flatten the sigma model manifold. This is due to the fact that the U(1) charge is related to the radius of the scalar manifold. The flat model discussed here will be used in Section 5.3.

4 Boundary Conditions and Linearized Field Equations

In order to examine the properties of the model described above near the boundary, we shall begin by fixing certain gauges and studying the behaviour of the linearized field equations near the boundary.

The AdS_3 spacetime can be covered by two regions each of which is parametrized by a set of Poincaré coordinates (x^0, x^1, x^2) in $\mathbf{R}^3$ with $x^2 > 0$. We shall use the notation $x^\mu = (x^0, x^1)$ and $x^2 \equiv r$. This patch contains half the boundary of AdS_3 in the form of the Minkowskian plane at $r = 0$. The other region is behind the horizon at $r = \infty$. In what follows we shall work only within one of the regions.

Following [1], we choose the following gauge conditions

$$\begin{aligned} &{e_r}^2 = \frac{1}{2mr} \ , \quad {e_r}^a = 0 \ , \quad {e_\mu}^2 = 0 \ , \\ &\psi_r = 0, \qquad A_r = 0 \ , \end{aligned} \tag{4.1}$$

where $a = 0, 1$ is the tangent space index in $D = 2$. Note that the second coordinate is labeled as r in curved space and as 2 in tangent space. The metric in this gauge takes the form

$$ds^2 = \frac{1}{(2mr)^2}(dr^2 + dx^\mu dx^\nu \hat{g}_{\mu\nu}) \ , \tag{4.2}$$

where $\hat{g}_{\mu\nu} = \hat{e}_\mu{}^a \hat{e}_\nu{}^b \eta_{ab}$. The SO(2,2) invariant AdS metric corresponds to the case $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$. The components of the spin connection following from the metric (4.2) are

$$\begin{aligned} \omega_\mu{}^{ab} &= \hat{\omega}_\mu{}^{ab} , \qquad \omega_r{}^{ab} = -\hat{e}^{[b\nu}\partial_r \hat{e}_\nu{}^{a]} , \\ \omega_\mu{}^{a2} &= -\frac{1}{r}\hat{e}_\mu{}^a + \hat{e}^{(b\nu}\partial_r \hat{e}_\nu{}^{a)}\hat{e}_{\mu b} , \qquad \omega_r{}^{a2} = 0 . \end{aligned} \tag{4.3}$$

When $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$, the only nonvanishing component is $\omega_\mu{}^{a2} = -\delta^a_\mu/r$.

We next study the asymptotic behaviour of the solutions of the linearized field equations near the boundary $r = 0$. We are going to do this in Euclidean signature. In this signature the AdS space consists of a single region covered by Poincaré coordinates plus a point at $r = \infty$. This point is actually a boundary point and the boundary has the topology of the two sphere, represented in the Poincaré coordinates by the Euclidean plane at $r = 0$ plus the point at infinity.

We will assume that the dreibein $e_\mu{}^a$ behaves as r^{-1} as in the $SO(2,2)$ invariant case. To determine the asymptotic behaviours of the remaining fields, we need to examine their linearised field equations as expanded around the supersymmetric AdS background in which the only nonvanishing fields are

$$\hat{g}_{\mu\nu} = \eta_{\mu\nu} , \qquad L = 1 . \tag{4.4}$$

Next, we use the coset representative L given in (3.1) which leads to the following expressions at the linearized level

$$P_\mu^{ir} \to \partial_\mu \phi^{ir}, \quad C \to 1, \quad C^{rs} \to 0, \quad S^{ir} \to \epsilon\ \epsilon^{ij}\phi^{jr} . \tag{4.5}$$

The field equations for A_μ and ψ_μ linearized around the background (4.4) are [4]

$$\partial_r A_\mu = 0 , \qquad \partial_{[\mu}A_{\nu]} = 0 , \tag{4.6}$$

$$\partial_r \psi_{\mu\pm} = \mp\tfrac{1}{2} r^{-1}\, \psi_{\mu\pm} . \tag{4.7}$$

where the suffixes $\pm$ indicate the eigenvalues of γ^2, which in turn indicate the chiralities of the spinors on the boundary. The equations involving radial derivatives are readily solved to all orders in r, and in a convenient normalization we have

$$A_\mu = A_{(0)\mu} , \qquad \psi_\mu = (2mr)^{-\frac{1}{2}}\, \psi_{\mu(0)+} + (2mr)^{\frac{1}{2}}\, \psi_{\mu(0)-} . \tag{4.8}$$

To give the proper boundary condition for the vector field, we define [5]

[4] We use the convention $\gamma^{\mu\nu} = \frac{1}{\sqrt{-g}}\epsilon^{\mu\nu}\gamma^2$.

[5] We use a notation in which the chiralities and Hodge dualities are labeled by lower $\pm$ indices and the regular and irregular nature of boundary conditions are labeled by upper $\pm$ indices.

$$A_{(0)\mu\pm} \;=\; P_{\mu\pm}{}^{\nu}A_{(0)\nu} \;=\; \frac{1}{2}(g_{\mu\nu} \pm \sqrt{-g}\epsilon_{\mu\nu})A^{\nu}_{(0)} \; . \tag{4.9}$$

The remaining equation for A_μ in (4.6) then amounts to

$$\partial^{\mu}_{-}A_{\mu+} \;=\; \partial^{\mu}_{+}A_{\mu-} \; . \tag{4.10}$$

As we shall show in the next section, the anti self dual component $A_{(0)\mu-}$ forms an off-shell $d = 2$ supermultiplet together with $\psi_{(0)\mu+}$ and $e_{(0)\mu}{}^{a}$. Thus it is natural to treat $A_{(0)\mu-}$ as the independent boundary field, and let $A_{(0)\mu+}$ be determined from (4.10). The fact that only one of the Hodge dualities of the vector field is independent can also be understood by considering the Hamiltonian formulation of the bulk Chern-Simons theory, where $A_{(0)\mu\pm}$ form a pair of canonically conjugate variables. Thus, the proper boundary conditions for the supergravity multiplet are:

$$e_{\mu}{}^{a} \sim (2mr)^{-1}e_{(0)\mu}{}^{a} \; , \qquad \psi_{\mu+} \sim (2mr)^{-\frac{1}{2}}\,\psi_{(0)\mu+} \qquad A_{\mu-} \sim A_{(0)\mu-} \; . \tag{4.11}$$

We now turn to the discussion of the boundary conditions on the matter fields, starting with the scalar fields.

4.1 Matter Scalars

The linearized scalar field equation near the boundary is given by (the r-dependence is shown explicitly and the $SO(2) \times SO(n)$ indices of ϕ^{ir} and λ^{r} are suppressed):

$$r^2\partial^{\mu}\partial_{\mu}\phi + r^3\partial_r\left(r^{-1}\partial_r\phi\right) - m^2_{\phi}\phi = 0 \; , \tag{4.12}$$

where

$$m^2_{\phi} = 4a^2(a^2+\epsilon) = \Delta(\Delta-2) \; , \tag{4.13}$$

and Δ equals Δ_+ or Δ_- defined by

$$\Delta_{\pm}(\phi) = 1 \pm \sqrt{1+m^2_{\phi}} \; . \tag{4.14}$$

Thus, in terms of ϵ and a^2:

$$\epsilon \;=\; 1 \;\; : \;\; \begin{cases} \Delta_+(\phi) \;=\; 2+2a^2 \; , \\ \\ \Delta_-(\phi) \;=\; -2a^2 \; , \end{cases} \tag{4.15}$$

$$\epsilon = -1 \quad : \quad \begin{cases} \Delta_+(\phi) = 1 + |1-2a^2| \ , \\ \\ \Delta_-(\phi) = 1 - |1-2a^2| \ . \end{cases} \tag{4.16}$$

A free scalar field ϕ behaves near the boundary as

$$\phi(r,\vec{x}) \sim (2mr)^{\Delta_-(\phi)} \left[\phi^+_{(0)} + (2mr)^2 \phi^+_{(2)} + \cdots\right] + (2mr)^{\Delta_+(\phi)} \left[\phi^-_{(0)} + (2mr)^2 \phi^-_{(2)} + \cdots\right] \tag{4.17}$$

for $2a^2 \notin \mathbf{Z}$, and

$$\phi(r,\vec{x}) \sim (2mr)^{\Delta_-(\phi)} \left[\phi^+_{(0)} + (2mr)^2 \phi^+_{(2)} + \cdots\right] + (2mr)^{\Delta_+(\phi)} \ln(2mr) \ \left[\phi^-_{(0)} + (2mr)^2 \phi^-_{(2)} + \cdots\right] \tag{4.18}$$

for $2a^2 \in \mathbf{Z}$. The expansion coefficients $\phi^\pm_{(2n)}$ depend only on $\vec{x}$. For $2a^2 \notin \mathbf{Z}$ and for $\epsilon = -1, a^2 = \frac{1}{2}$, that is $\Delta_+ = 1$, the coefficients $\phi^\pm_{(2n)}$, $n \geq 1$, are determined through the linearized field equations as local expressions in terms of $\phi^\pm_{(0)}$. For other values of ϵ and a^2, that is for $\Delta_+ = 2, 3, ...$, the coefficients $\phi^+_{(2n)}, n \leq \Delta_+ - 2$ are given in terms of $\phi^+_{(0)}$ while $\phi^+_{(2\Delta_+-2)}$ is undetermined and thus independent. At higher order in r one then finds that $\phi^-_{(0)}, \phi^-_{(2)}, \ldots$ are given in terms of $\phi^+_{(0)}$ and that $\phi^+_{(2\Delta_+)}, \phi^+_{(2\Delta_++2)}, \ldots$ are given in terms of $\phi^+_{(2\Delta_+-2)}$. The above results follow from the small z expansion of the modified Bessel functions [16]. Similar results hold for small perturbations around the anti-de Sitter background [17].

There are two types of boundary conditions that may be imposed on the scalars: regular conditions which amount to specifying the leading component at the boundary and irregular conditions which amount to specifying the independent subleading component described above [14, 15], and which are possible when $\Delta_- \geq 0$. Thus, for $2a^2 \notin \mathbf{Z}$, a regular boundary condition amounts to specifying $\phi^+_{(0)}$ while an irregular boundary condition amounts to specifying $\phi^-_{(0)}$. For $\Delta_+ = 1$, that is, for $\epsilon = -1, a^2 = \frac{1}{2}$, a regular boundary condition amounts to specifying $\phi^-_{(0)}$ and an irregular condition amounts to specifying $\phi^+_{(0)}$.

Given that $\phi^\pm_{(0)}$ are associated with conformal operators of weight $\Delta_\pm(\phi)$, the requirement of unitarity imposes the following restriction for irregular boundary conditions:

$$\Delta_-(\phi) \geq 0 \ . \tag{4.19}$$

For regular boundary conditions, the unitarity condition is automatically satisfied, while for irregular conditions (4.19) restricts the possible values of a^2. Thus it follows that the following boundary conditions are possible:

$$\epsilon = 1 \quad : \quad \text{regular:} \qquad \phi \sim (2mr)^{\Delta_-(\phi)} \phi^+_{(0)} \quad \text{for } a^2 \geq 0 \tag{4.20}$$

$$\text{irregular:} \quad \phi \sim (2mr)^2 \phi^+_{(2)} \quad \text{for } a^2 = 0 \ . \tag{4.21}$$

$$\epsilon = -1 \quad : \quad \text{regular:} \quad \phi \sim \begin{cases} (2mr)^{\Delta_-(\phi)} \phi^+_{(0)} & \text{for } a^2 \neq \frac{1}{2} \ , \\ (2mr) \ln(2mr) \phi^-_{(0)} & \text{for } a^2 = \frac{1}{2} \ . \end{cases} \tag{4.22}$$

$$\text{irregular:} \quad \phi \sim \begin{cases} (2mr)^{\Delta_+(\phi)} \phi^-_{(0)} & \text{for } 0 < a^2 < 1 \ , \ a^2 \neq \frac{1}{2} \ , \\ (2mr) \phi^+_{(0)} & \text{for } a^2 = \frac{1}{2} \ , \\ (2mr)^2 \phi^+_{(2)} & \text{for } a^2 = 0, 1 \ . \end{cases} \tag{4.23}$$

4.2 Matter Fermions

We now turn to the boundary conditions on the matter fermions. The linearized equations obeyed by the matter fermions near the boundary is

$$r\gamma^\mu \partial_\mu \lambda + r\gamma_2 \partial_r \lambda - \gamma_2 \lambda - m_\lambda \lambda = 0 \ , \tag{4.24}$$

where the fermion mass is given by

$$m_\lambda = \frac{1}{2}(1 + 4\epsilon a^2) \ . \tag{4.25}$$

We find that for $m_\lambda \notin \mathbf{Z} + \frac{1}{2}$ a solution to (4.24) is given by

$$\lambda \ = \ (2mr)^{1-m_\lambda} \left[\lambda_{(0)-} + 2mr\lambda_{(1)+} + \cdots \right] + (2mr)^{1+m_\lambda} \left[\lambda_{(0)+} + 2mr\lambda_{(1)-} + \cdots \right] \ , \tag{4.26}$$

where $\pm$ refers to the γ^2 eigenvalue. For $m_\lambda = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, ...$, the solution takes the form

$$\lambda = (2mr)^{1-m_\lambda} \left[\lambda_{(0)-} + 2mr\lambda_{(1)+} + \cdots \right] + (2mr)^{1+m_\lambda} \ln 2mr \ \left[\lambda_{(0)+} + 2mr\lambda_{(1)-} + \cdots \right] \ . \tag{4.27}$$

For $m_\lambda = -\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2}, ...$, the solution is given by (4.27) with $m_\lambda \to -m_\lambda$ and all the chiralities flipped. For later use, we also record the following relation

$$\lambda_{(1)\pm} \ = \ \frac{1}{2m(2m_\lambda \mp 1)} \gamma^\mu \partial_\mu \lambda_{(0)\mp} \ , \quad 2m_\lambda \mp 1 \neq 0 \ . \tag{4.28}$$

Note that unlike for the scalars, the coefficients in the logarithmic branch in (4.27) is never undetermined. Next, following [18], we define the conformal weights of the fermions as

$$\Delta_\pm(\lambda) \;=\; 1 \pm |m_\lambda| \;=\; 1 \pm \tfrac{1}{2}|1+4\epsilon a^2| \;. \tag{4.29}$$

Thus, in terms of ϵ and a^2:

$$\epsilon \;=\; 1 \;\;:\;\; \begin{cases} \Delta_+(\lambda) \;=\; \frac{3}{2}+2a^2 \\ \\ \Delta_-(\lambda) \;=\; \frac{1}{2}-2a^2 \;. \end{cases} \tag{4.30}$$

$$\epsilon \;=\; -1 \;\;:\;\; \begin{cases} \Delta_+(\lambda) \;=\; 1+\frac{1}{2}|1-4a^2| \;, \\ \\ \Delta_-(\lambda) \;=\; 1-\frac{1}{2}|1-4a^2| \;. \end{cases} \tag{4.31}$$

Thus, the regular boundary conditions are associated with $r^{\Delta_-(\lambda)}$ behaviour and the irregular boundary conditions with $r^{\Delta_+(\lambda)}$ behaviour (note that this holds for all values of a^2). It follows from (4.26) and (4.27) that the chirality of the regular and irregular boundary spinors $\lambda^\pm_{(0)}$ is equal to minus the sign of the fermion mass:

$$\left(\gamma^2 \pm \frac{m_\lambda}{|m_\lambda|}\right)\lambda^\pm_{(0)} \;=\; 0 \;, \tag{4.32}$$

where the superscript "+" refers to regular and "−" to irregular boundary conditions. Thus, in the case of regular boundary conditions the chirality is negative for positive fermion mass, and positive for negative fermion mass. In the case of irregular boundary conditions the chirality is positive for positive mass and negative for negative mass.

Imposing the following unitarity condition

$$\Delta_-(\lambda) \geq \tfrac{1}{2} \;, \tag{4.33}$$

it follows that the allowed boundary conditions for the matter fermions are:

$$\epsilon = 1 \;\;:\;\; \text{regular:} \quad \lambda \sim (2mr)^{\Delta_-(\lambda)}\lambda_{(0)-} \quad \text{for } a^2 \geq 0 \;, \tag{4.34}$$

$$\text{irregular:} \quad \lambda \sim (2mr)^{\frac{3}{2}}\lambda_{(1)+} \quad \text{for } a^2 = 0 \;. \tag{4.35}$$

$$\epsilon = -1 \;\;:\;\; \text{regular:} \quad \lambda \sim \begin{cases} (2mr)^{\Delta_-(\lambda)}\lambda_{(0)-} & \text{for } 0 \leq a^2 \leq \frac{1}{4} \;, \\ \\ (2mr)^{\Delta_-(\lambda)}\lambda_{(0)+} & \text{for } a^2 \geq \frac{1}{4} \;. \end{cases} \tag{4.36}$$

$$\text{irregular:} \quad \lambda \sim \begin{cases} (2mr)^{\frac{3}{2}}\lambda_{(1)+} & \text{for } a^2 = 0 \ , \\ (2mr)^{\Delta_+(\lambda)}\lambda_{(0)+} & \text{for } 0 < a^2 < \frac{1}{4} \ , \\ (2mr)^{\Delta_+(\lambda)}\lambda_{(0)-} & \text{for } \frac{1}{4} < a^2 < \frac{1}{2} \ , \\ (2mr)^{\frac{3}{2}}\lambda_{(1)-} & \text{for } a^2 = \frac{1}{2} \ . \end{cases} \tag{4.37}$$

Note that for $\epsilon = -1$ and $a^2 = \frac{1}{4}$ the regular boundary condition can be imposed on either of the chiralities.

5 The Local Conformal Supersymmetry on the Boundary

In this section we shall derive the realization of the $d = 2$, $N = (2, 0)$ conformal supersymmetry on the boundary supergravity multiplet $(e_{(0)\mu}{}^a, \psi_{(0)\mu+}, A_{(0)\mu-})$ and boundary chiral multiplets involving fields that are to be specified case by case in accordance with the boundary conditions.

The $d = 2$ symmetries are found by examining the nature of the bulk transformation rules close to the boundary. To analyze this we first find the $D = 3$ transformation parameters which preserve the $D = 3$ gauge conditions (4.1) near the boundary. We then evaluate the resulting $D = 3$ transformations of a solution to the $D = 3$ field equations with given set of boundary data $\Phi_{(0)}$. By matching powers of r in the limit when $r \to 0$ we thus obtain the resulting $d = 2$ transformations $\delta\Phi_{(0)}$. In specifying the boundary data we have to choose between regular and irregular boundary conditions such that $\delta\Phi_{(0)}$ is a local expression in terms of $\Phi_{(0)}$ and its derivatives.

For $\epsilon = 1$ and $a^2 > 0$, the scalar fields diverge at the boundary and the perturbative expansion breaks down, therefore we shall exclude the case $\epsilon = 1$ from now on. For $\epsilon = -1$ and $a^2 > 0$, the matter scalars diverge for $a^2 > 1$ and the matter fermions diverge for $a^2 > \frac{3}{4}$. Moreover, for $\frac{1}{2} < a^2 < \frac{3}{4}$, the supersymmetry transformation of the vector field involve matter contributions which diverge as $r \to 0$ (see footnote below (5.9)), which is not consistent with its r expansion. For $0 < a^2 \leq \frac{1}{2}$ the r expansion is well-defined, and in fact, since all matter fields have positive Weyl weight for this case, the nonlinearities vanish at the boundary. Finally, for $a^2 = 0$, the appropriate model to consider is the $\mathbf{R}^{2n}$ sigma model, in which case the scalars have Weyl weight zero. In summary, the perturbative expansion makes sense only for

$$\epsilon = -1 \ , \quad 0 \leq a^2 \leq \tfrac{1}{2} \ , \qquad \text{or} \quad \epsilon = +1 \ , \quad a^2 = 0 \ . \tag{5.1}$$

We remark that in the case of single scalar multiplet coupling, namely when the sigma model manifold is S^2 for $\epsilon = 1$ and H^2 for $\epsilon = -1$, the excluded range of the parameter a^2 coincides with the fact that the scalar potential has the form of a confining well. In the allowed range, however, the potential is unbounded from below. It would be interesting to study if the potential exhibits qualitatively similar behaviour for arbitrary number of scalar multiplets.

Importantly, regularity of the $D = 3$ solutions determines the irregular boundary conditions in terms of the regular, or vice versa, which leads to the usual interpretations of the anti-de Sitter/conformal field theory duality. This leads to subtleties, however, in the case of irregular boundary conditions in the matter sector, where the nonlinearities appear to lead to mixings between the regular and irregular fields in the transformations of the irregular fields (at least for certain rational values of a^2). As a first step towards understanding this, it is reasonable to begin by examining the nature of the transformations of the irregular fields among themselves by formally putting the regular fields to zero in the case of irregular boundary conditions. This is our approach when $0 < a^2 < \frac{1}{2}$, which we refer to as case 1 below. For the special values $a^2 = \frac{1}{2}$ and $a^2 = 0$, which we treat separately as case 2 and 3 below, the nonlinearities are however more manageable, and for these two cases we therefore keep both regular and irregular fields. Thus, in summary, the boundary conditions in the matter sector are taken as follows:

$$\begin{array}{lcl} \text{Case 1:} & \epsilon = -1 \ , \quad 0 < a^2 < \frac{1}{2} & \text{Either regular or irregular matter fields.} \\ \text{Case 2:} & \epsilon = -1 \ , \quad a^2 = \frac{1}{2} & \text{Both regular and irregular matter fields.} \\ \text{Case 3:} & a^2 = 0 & \text{Both regular and irregular matter fields.} \end{array} \tag{5.2}$$

5.1 Case 1 : $\epsilon = -1$ and $0 < a^2 < \frac{1}{2}$

We begin by determining the asymptotic behaviors of the local symmetries of the bulk which preserve the gauge conditions (4.1) near the boundary. Using the asymptotic behavior of the supergravity multiplet fields given in (4.11), we find (without linearizing in the fields) that the residual gauge symmetries are

$$\begin{aligned} \xi^r &= -r\Lambda_{D(0)} \ , \quad \xi^\mu = \xi^\mu_{(0)} + \mathcal{O}(r^2) \ , \\ \Lambda_L^{a2} &= \mathcal{O}(r^2) \ , \quad \Lambda_L^{ab} = \Lambda_{L(0)}\epsilon^{ab} + \mathcal{O}(r^2) \ , \\ \Lambda &= \Lambda_{(0)} + \mathcal{O}(r^{4a^2}) \ , \\ \varepsilon_\pm &= (2mr)^{\mp\frac{1}{2}} \left[\varepsilon_{(0)\pm} + \mathcal{O}(r^2)\right] \ , \end{aligned} \tag{5.3}$$

where $\Lambda_{D(0)}$, $\xi_{(0)}$, $\Lambda_{L(0)}$, $\Lambda_{(0)}$ and $\varepsilon_{(0)\pm}$) denote the parameters of dilatation, reparametrization, Lorentz rotation, $SO(2)$ rotation and supersymmetry, respectively, and the fields with suffix (0) are arbitrary functions of x^μ. Note that the parameter ξ^r is determined fully and it has only linear r-dependence, while the other parameters have series expansions in r. The form of ξ^r, ξ^μ and Λ_L^{a2} come from the variations of the gauge conditions involving the dreibein and the form of Λ_L^{ab} can be deduced from the requirement of residual Lorentz transformations on the boundary. The last two results come from the variation of the gauge conditions $A_r = 0$ and $\psi_r = 0$, respectively.

To derive the bosonic transformations in $d = 2$, we insert (5.3) together with the expansions (4.11), (4.17-4.18) and (4.26-4.27) into the bosonic transformations in $D = 3$. These do not mix different chiralities and powers of r. It is therefore straightforward to read off the transformation for the leading components. We find the usual general coordinate transformations of all the fields with parameter $\xi^\mu_{(0)}$, and (in the rest of subsection we have dropped the (0) labels for notational simplicity)

$$
\begin{aligned}
\delta e_\mu{}^a &= \left(\Lambda_D \eta^{ab} - \Lambda_L \epsilon^{ab}\right) e_{\mu b} \ , \\
\delta \psi_{\mu+} &= \left(\tfrac{1}{2}\Lambda_D + \tfrac{1}{2}\Lambda_L + \frac{1}{2a^2}\Lambda\Gamma_3\right) \psi_{\mu+} \ , \\
\delta A_{\mu-} &= \tfrac{1}{2}\left(g_{\mu\nu} - \sqrt{-g}\epsilon_{\mu\nu}\right)\partial^\nu \Lambda \ , \\
\delta \phi^{\pm ir} &= -\Delta_{\mp}(\phi)\Lambda_D \phi^{\pm ir} + \Lambda \epsilon^{ij} \phi^{\pm jr} \ , \\
\delta \lambda^{\pm r} &= \left[-\Delta_{\mp}(\lambda)\Lambda_D + \tfrac{1}{2}\Lambda_L \gamma_2 + (1 - \frac{1}{2a^2})\Lambda\Gamma_3\right] \lambda^{\pm r} \ .
\end{aligned}
\tag{5.4}
$$

Note, the superscripts on the matter fields refer to regular/irregular boundary conditions, and the chiralities of the matter fermions are given by (4.32).

To find the $d = 2$ supersymmetry transformation rules we substitute the expression for the supersymmetry parameter given in (5.3) into the $D = 3$ supersymmetry transformation rules (2.11) and take the limit $r \to 0$. In the supergravity sector we find:

$$
\begin{aligned}
\delta e_\mu{}^a &= -\bar{\varepsilon}\gamma^a \psi_\mu \ , \\
\delta \psi_\mu &= D_\mu \varepsilon + 2\gamma_\mu \eta \ , \\
\delta A_{\mu-} &= \tfrac{1}{2}a^2 \bar{\varepsilon}\Gamma_3 \gamma_\mu \gamma^{\rho\sigma}\psi_{\rho\sigma} + 2a^2 \bar{\eta}\Gamma^3 \gamma^\nu \gamma_\mu \psi_\nu \ ,
\end{aligned}
\tag{5.5}
$$

where we have introduced the notation

$$
\begin{aligned}
\gamma^\mu &= \gamma^a e_a{}^\mu \ , \\
\varepsilon &= \varepsilon_+ \ , \\
\eta &= m\varepsilon_- - \frac{1}{8a^2}\Gamma_3 \gamma^\mu A_{\mu+}\varepsilon_+ \ , \\
D_\mu \varepsilon &= \left(\partial_\mu - \tfrac{1}{2}\omega_\mu - \frac{1}{2a^2} A_{\mu-}\Gamma_3\right)\varepsilon
\end{aligned}
\tag{5.6}
$$

and the $d = 2$ gravitino field strength $\psi_{\mu\nu}$ is defined as in (2.18) but with the covariant derivative defined above. We note the correction to the special supersymmetry parameter η. In the gravitino transformation rule, this correction arises from the $A_{\mu+}$ contribution to the three dimensional $D_\mu\varepsilon$, while in the vector transformation rule it arises from the $D=3$ covariant derivative in the gravitino field strength and from the varying the self-duality projector according to the following:

$$\delta A_{\mu\pm} \equiv \delta(P_{\mu\pm}{}^\nu A_\nu) \;=\; P_{\mu\pm}{}^\nu \delta A_\nu \mp \bar\varepsilon\gamma_\mu\psi^\nu A_{\nu+} \ , \tag{5.7}$$

where the projection is defined in (4.9). We also note that in obtaining the vector transformation rule we have eliminated the anti self dual component of $\psi_{\mu-}$ using the boundary limit of the $\mu\nu$ component of the $D=3$ gravitino equation (2.14) as follows:

$$\frac{m}{2}\gamma_\mu\gamma^\nu\psi_{\nu-} \;=\; \tfrac{1}{4}\gamma^\nu\psi_{\mu\nu} + \frac{1}{8a^2}\Gamma^3\gamma_\mu\psi^\nu A_{\nu+} + ma\gamma_\mu\Gamma^i\lambda^r_+\phi^{+ir} \ , \tag{5.8}$$

where $\psi_{\mu\nu} = D_\mu\psi_\nu - D_\nu\psi_\mu$ with the covariant derivative defined as in (5.6). In deriving (5.8) one notices that the last two terms in the $\mu\nu$ component of the $D=3$ gravitino equation (2.14) add up in the leading order, which follows from the fact that for $0 < a^2 < \frac{1}{2}$ the $D=3$ scalar fields obey

$$r\partial_r\phi^{ir} \;=\; 2a^2(\phi^{ir} + \mathcal{O}(r)) \ . \tag{5.9}$$

In the $r\mu$ component of the $D=3$ gravitino equation, however, these terms cancel, which means that there is no matter contribution to the leading order. Thus we have obtained a local realization of the boundary supersymmetry on the $(2,0)$ conformal supergravity multiplet in $d=2$ which is off-shell and decoupled from matter [6].

We next study the transformations of the matter fields. We recall from (4.22-4.23) and (4.36-4.37) that both regular and irregular boundary conditions are admissible for $0 < a^2 < \frac{1}{2}$. There are two sets of combined regular and irregular boundary conditions which lead to two types of conformal $(2,0)$ supermultiplets which will be referred to as Type 1 and Type 2. The Type 1 multiplet consists of regular scalars, and fermions which are regular for $0 < a^2 < \frac{1}{4}$ and irregular for $\frac{1}{4} < a^2 < \frac{1}{2}$. The Type 2 multiplet, on the other hand, is a novel multiplet which consists of irregular scalars, and fermions which are irregular for $0 < a^2 < \frac{1}{4}$ and regular for $\frac{1}{4} < a^2 < \frac{1}{2}$.

To find the transformations at the linearized level, we insert (4.11), (4.17) and (4.26) into the $D=3$ transformation rules in (2.11) and match powers of r. In the case of Type 2, we also use the Dirac equation (4.28) in obtaining the scalar transformation. The results are as follows:

$$\text{Linearized Type 1:} \quad \begin{cases} \delta\phi^{+\,ir} \;=\; a\bar\varepsilon_+\Gamma^i\lambda^r_- \ , \\ \delta\lambda^r_- \;=\; -\frac{1}{2a}\gamma^\mu\partial_\mu\phi^{+\,ir}\Gamma_i\varepsilon_+ + 4ma\phi^{+\,ir}\Gamma_i\varepsilon_- \ , \end{cases} \tag{5.10}$$

[6] At this point we can see why the range $\frac{1}{2} < a^2 < \frac{3}{4}$ does not yield a local realization of the boundary supersymmetry: although the matter fields vanish at the boundary, so that we can trust the r-expansion, (5.9) is now replaced by $r\partial_r\phi^{ir} = (2-2a^2)(\phi^{ir} + \mathcal{O}(r))$ which results in divergent, matter dependent contributions to $\delta A_{\mu-}$.

$$\text{Linearized Type 2:} \quad \begin{cases} \delta\phi^{-\,ir} &= \frac{a}{4m(1-2a^2)}\bar{\varepsilon}_+\Gamma^i\gamma^\mu\partial_\mu\lambda^r_+ + a\bar{\varepsilon}_-\Gamma^i\lambda^r_+ \ , \\ \delta\lambda^r_+ &= -2ma^{-1}(1-2a^2)\phi^{-\,ir}\Gamma_i\varepsilon_+ \ . \end{cases} \tag{5.11}$$

We now turn to the full transformation rules. In the case of the Type 1 multiplet all the nonlinearities vanish except the $A_{\mu+}$ contribution to the special supersymmetry transformation, and we find the local transformation rules

$$\text{Full Type 1:} \quad \begin{cases} \delta\phi^{+\,ir} &= a\bar{\varepsilon}\Gamma^i\lambda^r_- \ , \\ \delta\lambda^r_- &= -\frac{1}{2a}\gamma^\mu\partial_\mu\phi^{+\,ir}\Gamma_i\varepsilon + 4a\phi^{+\,ir}\Gamma_i\eta \ , \end{cases} \tag{5.12}$$

where η is given by (5.6). Note that the gauge field A_μ does not appear in the derivative of the scalars ϕ^{ir} in the above formula because the chirality of ε projects it to its positive Hodge dual, which is then absorbed into the special supersymmetry parameter η. In other words, the above formulae are $U(1)$ covariant modulo η transformations.

In the Type 2 case the first nontrivial order the $D = 3$ Dirac equation now yields the following expression for the negative chirality spinor:

$$\lambda^r_{(1)-} = \frac{1}{4m(1-2a^2)}\gamma^\mu\widehat{D}_\mu\lambda^r_+ + \frac{1}{8ma^2}\Gamma^3\gamma^\mu A_{\mu+}\lambda^r_+ \ , \tag{5.13}$$

where $\widehat{D}_\mu\lambda$ is the supercovariant derivative defined by

$$\widehat{D}_\mu\lambda^r = \nabla_\mu\lambda^r + 2ma^{-1}(1-2a^2)\phi^{ir}\Gamma_i\psi_\mu \ . \tag{5.14}$$

The fact that matter fermions have the $U(1)$ charge $-1+\frac{1}{2a^2}$ has been used and ∇_μ is the ordinary Lorentz covariant derivative. Thus the full transformation rules for the Type 2 multiplet reads:

$$\text{Full Type 2:} \quad \begin{cases} \delta\phi^{-\,ir} &= \frac{a}{4m(1-2a^2)}\bar{\varepsilon}\Gamma^i\gamma^\mu\widehat{D}_\mu\lambda^r_+ + \frac{a}{m}\bar{\eta}\Gamma^i\lambda^r_+ \ , \\ \delta\lambda^r_+ &= -2ma^{-1}(1-2a^2)\phi^{-\,ir}\Gamma_i\varepsilon \ . \end{cases} \tag{5.15}$$

In summary, the transformation rules (5.4) and (5.5) are those of $N = (2,0)$ conformal supergravity [19] consisting of fields $(e_\mu{}^a, \psi_{\mu+}, A_{\mu-})$ coupled to either one of the following matter multiplets:

- Type 1: scalar multiplets consisting of scalar fields ϕ^{ir} with Weyl weight $2a^2$ and negative chirality spinors λ^r_- with Weyl weight $\frac{1}{2} + 2a^2$.
- Type 2: scalar multiplets consisting of scalar fields ϕ^{ir} with Weyl weight $2 - 2a^2$ and positive chirality spinors λ^r_+ with Weyl weight $\frac{3}{2} - 2a^2$.

The supersymmetry transformation rules (5.5), (5.12) and (5.15) close off-shell as follows

$$
\begin{aligned}
[\delta_{\varepsilon_1}, \delta_{\varepsilon_2}] &= \delta_\xi(\xi^\mu) + \delta_{\Lambda_L}(-\xi^\mu\omega_\mu) + \delta_\varepsilon(-\xi^\mu\psi_\mu) \ , && (5.16) \\
[\delta_\eta, \delta_\varepsilon] &= \delta_{\Lambda_D}(-2\bar\varepsilon\eta) + \delta_{\Lambda_L}(-2\bar\varepsilon\eta) + \delta_\Lambda(4a^2\bar\varepsilon\Gamma_3\eta) + \delta_\eta(\tfrac{1}{2}\bar\eta\psi^\mu\gamma_\mu\varepsilon) \ , && (5.17)
\end{aligned}
$$

where $\xi^\mu = \bar\varepsilon_1\gamma^\mu\epsilon_2$. The arguments on the right hand side are the composite parameters of the relevant transformations. Note the absence of the usual field dependent $U(1)$ gauge transformation in the commutator of two supersymmetry transformations. In obtaining (5.16-5.17) we have used

$$
\delta\omega_\mu = \tfrac{1}{2}\bar\varepsilon\gamma^\lambda\psi_{\lambda\mu} + 2\bar\psi_\nu\gamma_\mu\gamma^\nu\eta \ , \tag{5.18}
$$

which directly follows from ${\omega_\mu}^{ab} = \omega_\mu\epsilon^{ab}$ which is determined from its algebraic equation of motion as

$$
\omega_\mu = e^{-1}\epsilon^{\rho\sigma}{e_\mu}^a\partial_\rho e_{\sigma a} + \bar\psi_\mu\gamma^\nu\psi_\nu \ . \tag{5.19}
$$

The result (5.16-5.17) is up to cubic fermion terms that may arise through some of the composite ε and η-transformations, since the transformations (2.11) were themselves up to that order. In the case of Type 1 we have supercovariantized the derivative of ϕ^{+ir}. In the case of Type 2, the supercovariant derivative of λ^r_+ is already present, due to the fact that the three-dimensional fermionic matter field equation (2.16) is already supercovariant. However, in this case the closure of two supersymmetries on the fermion requires $\delta\phi\lambda$ type terms in $\delta\lambda$ that are expected to arise in the complete transformation rules.

5.2 Case 2 : $\epsilon = -1$ and $a^2 = \frac{1}{2}$

In this case, we recall the boundary behaviors of the matter fields from (4.18) and (4.27) as follows:

$$
\begin{aligned}
\phi^{ir} &= 2mr\left[\phi^{+ir}_{(0)} + (2mr)^2\phi^{+ir}_{(2)} + \cdots\right] + 2mr\ln(2mr)\left[\phi^{-ir}_{(0)} + (2mr)^2\phi^{-ir}_{(2)} + \cdots\right] , \\
\lambda^r &= (2mr)^{\frac{1}{2}}\left[\lambda^r_{(0)+} + 2mr\lambda^r_{(1)-} + \cdots\right] + (2mr)^{\frac{3}{2}}\ln(2mr)\left[\lambda^r_{(0)-} + 2mr\lambda^r_{(1)+} + \cdots\right] \quad (5.20)
\end{aligned}
$$

As we shall show, the regular boundary fields $(\phi^{-ir}_{(0)}, \lambda^r_{(0)+})$ form a supermultiplet of Type 1, using the terminology introduced above, according to which multiplets containing the regular scalars are called Type 1 and those containing the irregular scalars are called Type 2. As for the irregular fields $(\phi^{+ir}_{(0)}, \lambda^r_{(1)-})$, they will be shown to form an extended multiplet, together with the fields of the Type 1 multiplet.

Substituting (5.20) into the negative chirality component of the λ field equation we find that there are indeed no conditions on $\lambda^r_{(0)+}$ and $\lambda^r_{(1)-}$ and that $\lambda^r_{(0)-}$ is determined as follows:

$$\lambda^r_{(0)-} = \frac{1}{2m}\gamma^\mu \hat{D}_\mu \lambda^r_{(0)+}, \tag{5.21}$$

where

$$\hat{D}_\mu \lambda^r_{(0)+} = \nabla_\mu \lambda^r_{(0)+} + \sqrt{2} m \phi^{-ir}_{(0)} \Gamma_i \psi_{(0)\mu+} \ . \tag{5.22}$$

As we shall see later, $\hat{D}_\mu \lambda^r_{(0)+}$ is supercovariant. Note that the U(1) charge $(-1+\frac{1}{2a^2})$ vanishes here since $a^2 = \frac{1}{2}$.

Let us now examine the boundary behaviour of the local symmetry transformations, starting with the bosonic ones. The bosonic transformations of the matter fields are the same as in the $0 < a^2 < \frac{1}{2}$ case, except for the dilatation which acts as follows:

$$\text{Extended Type 1:} \qquad \begin{cases} \delta\phi^{-ir}_{(0)} = -\Lambda_D \phi^{-ir}_{(0)} \ , \\ \delta\lambda^r_{(0)+} = -\frac{1}{2}\Lambda_D \lambda^r_{(0)+} \ , \\ \delta\phi^{+ir}_{(0)} = -\Lambda_D \phi^{+ir}_{(0)} - \Lambda_D \phi^{-ir}_{(0)} \ , \\ \delta\lambda^r_{(1)-} = -\frac{3}{2}\Lambda_D \lambda^r_{(1)-} - \frac{1}{2m}\Lambda_D \gamma^\mu \hat{D}_\mu \lambda^r_{(0)+} \ . \end{cases} \tag{5.23}$$

We see that due to the logarithmic terms in the expansion and the fact that $\Delta_+ = \Delta_-$ there is an admixture of the Type 1 fields in the transformations of the irregular fields $(\phi^{+ir}_{(0)}, \lambda^r_{(1)-})$. Thus, the full set of fields are considered to form an extended Type 1 multiplet. This multiplet structure will also emerge in the conformal supersymmetry transformation rules.

We next turn to the boundary limits of the supersymmetry transformations. Those of the supergravity multiplet take the same form as given in (5.5), with the replacement $A_{\mu-} \to A'_{\mu-}$ defined as (we suppress the (0) labels in the supergravity sector):

$$A'_{\mu-} = A_{\mu-} - \frac{1}{4}\bar{\lambda}^r_{(0)+}\gamma_\mu \Gamma^3 \lambda^r_{(0)+} \ . \tag{5.24}$$

To see this, we begin by noting that there is an additional log term in the expansion of the Rarita-Schwinger field. By substituting (5.20) into the $r\mu$ component of the Rarita-Schwinger field equation we find

$$\begin{aligned} \psi_{\mu+}(r,\vec{x}) &= (2mr)^{-\frac{1}{2}}\psi_{\mu+} + \cdots, \\ \psi_{\mu-}(r,\vec{x}) &= (2mr)^{\frac{1}{2}}\psi_{\mu-} + \frac{1}{\sqrt{2}}(2mr)^{\frac{1}{2}} \ln(2mr)\gamma_\mu \phi^{-ir}_{(0)}\Gamma_i \lambda^r_{(0)+} + \cdots. \end{aligned} \tag{5.25}$$

The $\mu\nu$ component of the Rarita-Schwinger field equation gives

$$\frac{m}{2}\gamma_\mu\gamma^\nu\psi_{\nu-} = \tfrac{1}{4}\gamma^\nu\psi_{\mu\nu} + \frac{1}{4}\Gamma^3\gamma_\mu\psi^\nu A_{\nu+} + \frac{1}{\sqrt{2}}m\gamma_\mu\left(\phi^{+ir}_{(0)} + \frac{1}{2}\phi^{-ir}_{(0)}\right)\Gamma_i\lambda^r_{(0)+} \ , \tag{5.26}$$

where $\psi_{\mu\nu} = D_\mu\psi_\nu - D_\nu\psi_\mu$ and $D_\mu\varepsilon = (\nabla_\mu - A_\mu\Gamma_3)\varepsilon$. Using this the fermionic transformation of $A_{\mu-}$ becomes

$$\delta A_{\mu-} = \frac{1}{2}\bar\varepsilon\Gamma^3\gamma^\nu\psi_{\mu\nu} + \bar\eta\Gamma^3\gamma^\nu\gamma_\mu\psi_\nu - \frac{1}{\sqrt{2}}m\bar\varepsilon\gamma_\mu\Gamma_i\Gamma^3\lambda^r_{(0)+}\phi^{-ir}_{(0)} \ . \tag{5.27}$$

The matter dependence can be removed by the redefinition (5.24). The field $A'_{\mu-}$ then transforms as in (5.5) upon the use of $\lambda^r_{(0)+}$ given below. Altogether, we find the conformal supergravity multiplet transformations (5.5), with $a^2 = \frac{1}{2}$.

We next study the supersymmetry transformations of the matter fields near the boundary. We find after rescaling $\phi^\pm \to \frac{1}{\sqrt{2}}\phi^\pm$ (and dropping the (0)'s for notational simplicity)

$$\text{Extended Type 1:}\quad \left\{\begin{array}{lcl} \delta\phi^{-ir} & = & \frac{1}{2m}\bar\varepsilon\Gamma^i\gamma^\mu\hat{D}_\mu\lambda^r_+ \ , \\ \delta\lambda^r_+ & = & -m\phi^{-ir}\Gamma_i\varepsilon \ , \\ \delta\phi^{+ir} & = & \bar\varepsilon\Gamma^i\lambda'^r_- + \frac{1}{m}\bar\eta\Gamma^i\lambda^r_+ \ , \\ \delta\lambda'^r_- & = & -\frac{1}{2}\gamma^\mu\partial_\mu\phi^{+ir}\Gamma_i\varepsilon + 2\Gamma_i\eta\left(\phi^{+ir} + \frac{1}{2}\phi^{-ir}\right) \ , \end{array}\right. \tag{5.28}$$

where we have used the field equation (5.21) and made the field redefinition

$$\lambda'^r_- = \lambda^r_{(1)-} - \frac{1}{4m}\gamma^\mu A_{\mu+}\Gamma^3\lambda^r_{(0)+}. \tag{5.29}$$

We observe that $(\phi^{-ir}, \lambda^r_+)$ form a Type 1 submultiplet, whose fields are inert under the conformal supersymmetry transformations, and whose supersymmetry transformations can be obtained by rescaling the fields of the Type 2 multiplet (5.15) as $\phi^{-ir} \to (1-2a^2)^{-1}\phi^{-ir}$ and $\lambda_+ \to 2\sqrt{2}\lambda_+$, and taking the limit $a^2 \to \frac{1}{2}$. Correspondingly, the superalgebra closes as in (5.16-5.17) by setting $a^2 = \frac{1}{2}$. The fields $(\phi^{+ir}, \lambda'^r_-)$, on the other hand, transform into each other under the ordinary supersymmetry transformations, but transform into $(\phi^{-ir}, \lambda^r_+)$ under the conformal supersymmetry transformations. (This is similar to the situation of the dilatation transformation laws (5.23). Therefore, we view the enlarged set of fields $(\phi^{+ir}, \phi^{-ir}, \lambda'^r_-, \lambda^r_+)$ as forming a conformal supermultiplet, with closure given by (5.16-5.17) for $a^2 = \frac{1}{2}$ (with dilatation transformation (5.23)). This extended Type 1 multiplet can be truncated consistently to an ordinary Type 1 multiplet by setting the Type 2 submultiplet equal to zero, but the reverse is not consistent.

5.3 Case 3 : $a^2 = 0$

In this case the boundary behavior of the matter fields is given by

$$\begin{aligned}
\phi &= \left[\phi^+_{(0)} + (2mr)^2\phi^+_{(2)} + \cdots\right] + (2mr)^2\ln(2mr)\left[\phi^-_{(0)} + (2mr)^2\phi^-_{(2)} + \cdots\right], \\
\lambda &= (2mr)^{\frac{1}{2}}\left[\lambda_{(0)-} + 2mr\lambda_{(1)+} + (2mr)^2\lambda_{(2)-} + \cdots\right] \\
&\quad +(2mr)^{\frac{3}{2}}\ln(2mr)\left[\lambda_{(0)+} + 2mr\lambda_{(1)-} + \cdots\right] .
\end{aligned} \tag{5.30}$$

The supersymmetric variations of the zweibein and gravitino are as in (5.5) while the transformation of $A_{\mu-}$ has a subtlety due to the fact that there is an additional log term in the expansion of the Rarita-Schwinger field. Taking this into account, from the Rarita-Schwinger field equation we find

$$\begin{aligned}
\psi_{\mu+}(r,\vec{x}) &= (2mr)^{-\frac{1}{2}}\psi_{\mu+} + \cdots, \\
\psi_{\mu-}(r,\vec{x}) &= (2mr)^{\frac{1}{2}}\psi_{(0)\mu-} + \frac{1}{4m}(2mr)^{\frac{1}{2}}\ln(2mr)\Gamma_i\gamma^\nu\gamma_\mu\lambda^r_{(0)-}\partial_\nu\phi^{+ir}_{(0)} + \cdots \;, && (5.31) \\
\frac{m}{2}\gamma_\mu\gamma^\nu\psi_{(0)\nu-} &= \tfrac{1}{4}\gamma^\nu\psi_{\mu\nu+} + \frac{1}{8}\Gamma^3\gamma_\mu\psi^\nu_+A_{\nu+} - \frac{1}{8}\Gamma_i\gamma_\mu\gamma^\nu\lambda^r_{(0)-}\partial_\nu\phi^{+ir}_{(0)} \;, && (5.32)
\end{aligned}$$

where $\psi_{\mu\nu} = D_\mu\psi_\nu - D_\nu\psi_\mu$ with the U(1) connection term in $D_\mu\psi$ shifted as in (3.8). Using this the fermionic transformation of $A_{\mu-}$ becomes

$$\delta A_{\mu-} = \bar{\varepsilon}\Gamma^3\gamma^\nu\psi_{(0)\mu\nu+} + 2\bar{\eta}\Gamma^3\gamma^\nu\gamma_\mu\psi_{(0)\nu+} - \frac{1}{2}\bar{\varepsilon}\Gamma^3\Gamma_i\gamma_\mu\gamma^\nu\lambda^r_{(0)-}\partial_\nu\phi^{+ir}_{(0)} \;, \tag{5.33}$$

where we have used (5.7). To cancel the matter contributions, we first need to examine the transformations of the matter fields. ¿From the matter field equations and the boundary conditions stated earlier for the case at hand, we find that the independent fields at the boundary are $(\phi^{+ir}_{(0)}, \lambda^r_{(0)-}, \phi^{+ir}_{(2)}, \lambda^r_{(1)+})$. Under dilatations, these fields are found to transform as

$$\begin{aligned}
\delta\phi^{+ir}_{(0)} &= 0, \\
\delta\lambda^r_{(0)-} &= -\frac{1}{2}\Lambda_D\lambda^r_{(0)-}, \\
\delta\phi^{+ir}_{(2)} &= -2\Lambda_D\phi^{+ir}_{(2)} - \Lambda_D\phi^{-ir}_{(0)}, \\
\delta\lambda^r_{(1)+} &= -\frac{3}{2}\Lambda_D\lambda^r_{(1)+} - \Lambda_D\lambda^r_{(0)+} \;,
\end{aligned} \tag{5.34}$$

where, again, a mixing of the type observed earlier in the case of $a^2 = \frac{1}{2}$ arises here. The fermionic transformation of the matter fields also exhibit this kind of mixing:

$$
\begin{aligned}
\delta\phi^{+ir}_{(0)} &= \bar{\varepsilon}_{(0)+}\Gamma^i\lambda^r_{(0)-} \ , \\
\delta\lambda^r_{(0)-} &= -\frac{1}{2}\Gamma_i\gamma^\mu\varepsilon_{(0)+}\partial_\mu\phi^{+ir}_{(0)} \ , \\
\delta\phi^{+ir}_{(2)} &= \bar{\varepsilon}_{(0)+}\Gamma^i\lambda^r_{(2)-} + \bar{\varepsilon}_{(0)-}\Gamma^i\lambda^r_{(1)+} \ , \\
\delta\lambda^r_{(1)+} &= -\frac{1}{2}\Gamma_i\gamma^\mu\varepsilon_{(0)-}\partial_\mu\phi^{+ir}_{(0)} - 2m\Gamma_i\varepsilon_{(0)+}\left(\phi^{+ir}_{(2)} + \frac{1}{2}\phi^{-ir}_{(0)}\right) \ ,
\end{aligned}
\tag{5.35}
$$

where $\phi^{-ir}_{(0)} = -\frac{1}{8m^2}\nabla^\mu\partial_\mu\phi^{+ir}_{(0)}$ and $\lambda^r_{(2)-}$ is a more complicated function of the independent fields.

Armed with this result, we first redefine A_μ as in (3.9) so that the newly defined field $A'_{\mu-}$ transforms precisely as in (5.5), that is without any matter contributions, as expected from an off-shell conformal supergravity multiplet. As for the interpretation of the matter multiplet transformations, surprisingly enough, the story is somewhat more complicated. While the $\varepsilon_{(0)-}$ parameter can be redefined into the special supersymmetry parameter η as in (5.6), a close examination of the transformation rules $\delta\phi^{+ir}_{(2)}$ and $\delta\lambda^r_{(1)+}$ shows that the dependence of the result on $A_{\mu+}$ and the special supersymmetry gauge field, which is an appropriately redefined $\psi_{\mu-}$, cannot be removed unless certain equations of motion are imposed. However, these gauge fields are determined in terms of the independent fields as nonlocal expressions. Therefore, in order to realize the conformal supersymmetry on the boundary in a local fashion, we need to remove the dependence on these dependent gauge fields. This can be achieved by setting

$$
\left(g^{\mu\nu} - \frac{\epsilon^{\mu\nu}}{\sqrt{-g}}\right)\hat{D}_\nu\phi^{+ir}_{(0)} = 0 \ , \qquad \gamma^\mu\hat{D}_\mu\lambda^r_{(0)-} = 0 \ ,
\tag{5.36}
$$

where the supercovariant derivatives are defined in a standard way in accordance with the supersymmetry variations (5.35), and $\hat{D}_\nu\lambda^r_{(0)-}$ contains the shifted field $A'_{\mu-}$. These field equations transform into each other under the supersymmetry variations (5.35), as they should.

Imposing the on-shell conditions (5.36), and recalling that the derivatives on (5.35) need to be supercovariantized when considering the higher order fermion terms, we find that the $\varepsilon_{(0)-}$ term in the last equation in (5.35) drops out, and that the boundary evaluation of the $D = 3$ matter field equations imply that

$$
\phi^{-ir}_{(0)} = 0 \ , \qquad \lambda^r_{(0)+} = 0 \ ,
\tag{5.37}
$$

$$
\lambda^r_{(2)-} = \frac{1}{4m}\gamma^\mu\left(\hat{D}_\mu\lambda^r_{(1)+} + \frac{1}{2}A_{\mu+}\Gamma^3\lambda^r_{(1)+}\right) \ ,
\tag{5.38}
$$

with the supercovariant derivatives defined in standard way. Substituting these results in (5.35), the $A_{\mu+}$ dependent term can be absorbed into redefinition of $\varepsilon_{(0)-}$ to yield the special supersymmetry parameter η, and all in all, the supersymmetry transformation rules (5.35) for the independent field disentangle into those of two separate multiplet of fields as follows (dropping the (0) and chirality labels on the conformal supergravity fields and parameters):

$$\text{Type 1:} \qquad \begin{cases} \delta\phi^{+ir}_{(0)} = \bar{\varepsilon}\Gamma^i\lambda^r_{(0)-} \ , \\ \delta\lambda^r_{(0)-} = -\frac{1}{2}\Gamma_i\gamma^\mu\varepsilon\partial_\mu\phi^{+ir}_{(0)} \ , \end{cases} \tag{5.39}$$

$$\text{Type 2:} \qquad \begin{cases} \delta\phi^{+ir}_{(2)} = \frac{1}{4m}\bar{\varepsilon}\Gamma^i\gamma^\mu\hat{D}_\mu\lambda^r_{(1)+} + \frac{1}{m}\bar{\eta}\Gamma^i\lambda^r_{(1)+} \ , \\ \delta\lambda^r_{(1)+} = -2m\Gamma_i\varepsilon\phi^{+ir}_{(2)} \ . \end{cases} \tag{5.40}$$

Again, we have used the terminology of Type 1 and Type 2, according to whether the multiplet contains regular or irregular scalar fields. The result for Type 1 agrees with that of [19], where (2,0) conformal supergravity and its coupling to a sigma model in $D = 2$ is constructed. All the fields occurring in both multiplets above now have definite Weyl weights since the mixings in the dilatation transformations (5.34) disappear upon the use of (5.37-5.38).

The algebra (5.39-5.40) closes as in (5.17). In interpreting the composite U(1) transformation, the composite parameter must be rescaled by a^2, since A_μ has been rescaled by a^2, prior to taking the limit $a^2 \to 0$. The closure of the algebra can be seen from the fact that the Type 1 and Type 2 multiplets here can be obtained from the Type 1 and Type 2 multiplets found Section 5.1 for $0 < a^2 < \frac{1}{2}$, by first rescaling the scalar fields as $\phi \to a\phi$ and then taking $a^2 \to 0$.

6 Conclusions

In this paper we have analyzed the behaviour of (2,0) gauged supergravity coupled to matter in $D = 3$ near the boundary of AdS. We have exhibited the role of the bulk supergravity and matter field equations in determining the realization of conformal supersymmetry on the boundary of AdS. We have found that various types of matter multiplets emerge at the boundary in addition to a universal (2,0) conformal supergravity multiplet These multiplets involve fields whose conformal dimensions depend on the radius of the Kählerian sigma model coset space and on the gravitational coupling constant (set equal to 1 in most of the paper). The nature of the boundary conformal multiplets found depends crucially on the ratio of these constants. Interestingly, the local supersymmetry of the $D = 3$ theory does not fix this ratio nor the sign of the sigma model curvature constant, though the most interesting boundary conditions turn out to be possible for noncompact sigma model coset space whose curvature scalar is restricted to lie in a finite range in units of the $D = 3$ Planck length, as discussed in Section 5. In the case of flat sigma model manifold, we find a connection between the model of [10] and that of Izquierdo and Townsend [9].

We have seen that there are several subtleties in choosing the boundary conditions for the matter fields. In particular, we find that both regular and irregular boundary conditions can be imposed on the matter fields as a consequence of the fact that scalar fields with sufficiently negative mass-squared can be associated with CFT operators of two possible dimensions on the boundary. In fact, this phenomenon has already been observed in [15] in the context of $AdS_5 \times T^{1,1}$ compactification of Type IIB string theory. Here, we provide another example of this phenomenon, and we find the resulting CFT supergravity plus matter symmetry transformations. Somewhat surprisingly, we also find an interesting conformal supermultiplet structure on the boundary that involve fields which do not have definite Weyl weights but rather mix with other fields of the multiplet under dilatations. In this novel multiplet the superconformal symmetry is also realized in an unconventional fashion.

In the case of irregular boundary conditions the analysis had to be restricted for certain values of the sigma model radius, referred to as Case 1 in Section 5, such that the effects of the nonlinear contributions from the regular fields to the transformations of the irregular fields were omitted. The inclusion of both regular and irregular fields is necessary for the interpretations of the AdS/CFT correspondence. The study of these effects is intimately connected with the identification of the boundary conformal field theory, which lies beyond the scope of this paper.

We conclude by commenting on some of the interesting open problems. Firstly, it is clearly desirable to find an M-theoretic origin of the model studied here. The structure of the conformal supermultiplets that we have found on the boundary provide information on a class of operators which the boundary CFT must contain but do not provide the full data required to specify uniquely the the CFT in question. It is conceivable that an M-theoretic origin of the model exists only for a certain critical value of the sigma model curvature constant. At any rate, many of the features encountered in the analysis of the $(2,0), AdS_3$ supergravity plus matter system studied here are likely to arise in the $(4,4), AdS_3$ supergravity plus matter system which arises in the $AdS_3 \times S^3$ compactification of $(2,0), D = 6$ supergravity coupled to tensor multiplets, whose embedding in M-theory is known. We hope that the results presented here may give a flavor of what to expect in that case. Indeed, these results may also prove useful in analysing higher dimensional AdS supergravity plus matter systems as well.

It would also be interesting to extend the above analysis to a generalized setup in which the boundary conditions are imposed on a surface which is a finite distance away from the AdS boundary. This is expected to provide an understanding of how a supergravity plus matter system can be localized on a brane worldvolume in a Randall-Sundrum like scenario. This leads to normalizable bulk modes which correspond to fluctuating boundary modes. The boundary CFT, which is dual to the matter coupled supergravity in the bulk, should therefore be supplemented by an off-shell Lagrangian for the (fluctuating) boundary supergravity/matter modes. Thus, the total dynamics is that of the boundary CFT plus the localized, matter coupled bulk supergravity. In this context the unexpected result of Section 5.3, in the form of the on-shell constraint given in (5.36), indicates that ordinary off-shell as well as chiral two-dimensional matter systems may be localized on the brane. We finally remark that we expect the proper vacuum for this setup to be the black string (domain wall) solution of [10], rather than the anti-de Sitter vacuum. It would therefore be interesting to extend the bulk theory by the inclusion of two-form potentials and to give the supersymmetric coupling of these black strings to the bulk supergravity [20]

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