

## Optimal Sizing and Location of Shunt Capacitors in Medium Voltage Underground Power Cables: A Case of Minimum Cost

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**Abstract - This paper introduces a method to determine optimal sizing and location of shunt capacitors in medium voltage Underground Power Cables or in distribution system. To execute this method, two models of standard branch and the Newton-Raphson method are also mentioned to analyze and determine values of bus voltage and currents going through lines in whole system. The EN 50160 standard is introduced to evaluate all operational parameters and propose solutions to reduce risks. In additionally, the cost function method is introduced basing on cost of purchasing compensative device, power loss in own compensators and whole system after compensating. By considering partial derivative of cost function with respect to variables, optimal sizing and location of shunt capacitors can be determined for all buses in the system. Analyzing the system by using the Newton-Raphson method in stable mode, all operational parameters are evaluated to see the benefit of placing capacitors in some criterions: reduce the current going through all lines and increase value of bus voltage in whole system. Theory research is verified by simulation results carried out in Matlab 2016 software.**

**Keywords:** *Compensator, distribution system, optimal sizing, optimal location, shunt capacitor, underground power cable*

### I. INTRODUCTION

Power system has too many voltage levels due to the transmissive distance. Because each voltage affects to mathematical model of line parameters, it must have an effective method to determine all operational parameters for whole system. In medium voltage underground cables, the mathematical model of lines only has two parameters that are reactance and resistance and the low ratio of them is low. So,

it can help to reduce the mathematical volume of whole system.

When the number of bus increases, the number of equations describing the balance of active and reactive power also increases. Some iterative methods are often proposed to solve above system of nonlinear equations. In this context, the Newton-Raphson method is often used as an effectively mathematical tool more popular than the Gauss-Seidel method because it can help the problem converge faster and it provide a scientific method to analyze a large system [1], [2]. In power system problems, the participant of many factors at each bus such as generators, compensative device, and load or transmission lines makes difficult to build admittance matrix. Because of this reason, the standard branch in two models where a transformer or impedance is connected to the considered bus must be used to determine operational parameters by combining Newton-Raphson method and the rated values to estimate exact solutions.

After solving the system of nonlinear equations that describes whole system and comparing the stable rules, that were defined by some international standards such as EN 50160 [3], [4]. When the standard are not ensured in the operational process, it must be used some solutions such as upgrading the sectional area of lines or placing shunt or series capacitors or dismissing load. Series capacitor affects directly to increase the capacity of current going through lines but they can make resonance in low frequency if they aren't controlled. Dismissing load affects badly to electric quality. Above analysis showed that shunt capacitors is the best solution to improve the capacity for whole system.

Each sizing of shunt capacitors can make change of the power flow and all operational parameters in whole power system. The problem of optimal sizing and location of shunt capacitors has been solved by technical and cost factors [5] [6] [7]. The cost factor can help to evaluate the cost for purchasing capacitors, power loss in own capacitors and whole system after compensating when the load varies or not [5], [8]. Moreover, technical and cost problems can be solved

absolutely by considering cost function because almost operational parameters are improved after placing capacitors. So, the third section will introduces a method to determine sizing and location of shunt capacitors in distribution system after introducing the method to analyze whole system and evaluate operational parameters in the second section. In each section, a distribution system, 19-bus system, is used as an example to illustrate the theory. The last section will bring out some conclusions about achievements and contributions.

**II. ANALYZE POWER SYSTEM AND EVALUATE OPERATIONAL PARAMETERS**

**A. Method to build the system of nonlinear Equations for power system**

In distribution system, almost buses are PQ buses except the slack bus. These buses are often connected by lines or branches and interchanged by load, generators, capacitors... To have mathematical description for whole system, two models of the any branch must be used. With the convention of positive sign for input current and negative sign for output current, the first model for branch ij which connects a transformer into the bus i is represented in Fig 1 [9].

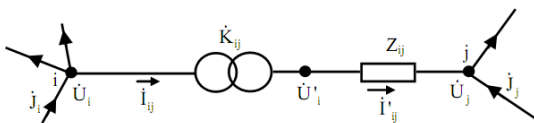


Fig. 1. The first model of branch ij

The complex number of transformer ratio for the 1st model can be calculated by (1):

$$\hat{K}_{ij} = \frac{\hat{U}'_i}{\hat{U}_i} = \frac{\hat{I}'_{ij}}{\hat{I}_{ij}} \tag{1}$$

The equation describing the balance of currents at the bus i and written by Kirchhoff 1 is depicted in (2):

$$\sum_{j=1, j \neq i}^n \hat{I}_{ij} = \hat{J}_i \Leftrightarrow \sum_{j=0, j \neq i}^n \hat{K}_{ij} \hat{I}'_{ij} = \hat{J}_i \tag{2}$$

where: n is the number of buses in whole system, symbol having "hat" represents the conjunction of complex number.

J<sub>i</sub> is the current injecting into the bus i.

Applying Ohm law for the branch, we have the equation (3):

$$\sum_{j=1, j \neq i}^n \hat{K}_{ij} \frac{\hat{U}'_i - \hat{U}_j}{Z_{ij}} = \hat{J}_i \Leftrightarrow Y_{ii} \hat{U}_i + \sum_{j=1, j \neq i}^n Y_{ij} \hat{U}_j = \hat{J}_i \tag{3}$$

where:

$$Y_{ii} = \sum_{j=0, j \neq i}^n \frac{K_{ij}^2}{Z_{ij}} \text{ is individual admittance of the bus } i,$$

$$Y_{ij} = -\frac{\hat{K}_{ij}}{Z_{ij}} \text{ is mutual admittance of branch } ij.$$

The second model for branch ij which connects an impedance into the bus i is represented in Fig 2 [9].

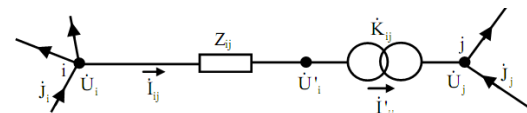


Fig. 2. The second model of branch ij

The complex number of transformer ratio for the 2<sup>nd</sup> model can be calculated by (4):

$$\hat{K}'_{ij} = \frac{\hat{U}'_i}{\hat{U}_j} \tag{4}$$

The equation describing the balance of currents at the bus i and written by Kirchhoff 1 is depicted in (5):

$$\sum_{j=1, j \neq i}^n \hat{I}'_{ij} = \hat{J}_i \Leftrightarrow Y_{ii} \hat{U}_i + \sum_{j=1, j \neq i}^n Y_{ij} \hat{U}_j = \hat{J}_i \tag{5}$$

where:

$$Y_{ii} = \sum_{j=1, j \neq i}^n \frac{1}{Z_{ij}} \text{ is individual admittance of the bus } i,$$

$$Y_{ij} = -\frac{\hat{K}'_{ij}}{Z_{ij}} \text{ is mutual admittance of branch } ij.$$

Using (3) and (5) for any branch model and applying Kirchhoff 1 for whole system, we have system of equation describing the current balance as represented in (6):

$$\begin{cases} Y_{11} \hat{U}_1 + Y_{12} \hat{U}_2 + \dots + Y_{1n} \hat{U}_n = \hat{J}_1 \\ Y_{21} \hat{U}_1 + Y_{22} \hat{U}_2 + \dots + Y_{2n} \hat{U}_n = \hat{J}_2 \\ \dots \\ Y_{n1} \hat{U}_1 + Y_{n2} \hat{U}_2 + \dots + Y_{nn} \hat{U}_n = \hat{J}_n \end{cases} \tag{6}$$

Multiplying each side of each equation in (6) with the conjunction of voltage at corresponding bus, we have the system of power balance equations as represented in (7):

$$\begin{cases} Y_{11} \hat{U}_1^2 + Y_{12} \hat{U}_2 \hat{U}_1 + \dots + Y_{1n} \hat{U}_n \hat{U}_1 = P_1 - j Q_1 \\ Y_{21} \hat{U}_1 \hat{U}_2 + Y_{22} \hat{U}_2^2 + \dots + Y_{2n} \hat{U}_n \hat{U}_2 = P_2 - j Q_2 \\ \dots \\ Y_{n1} \hat{U}_1 \hat{U}_n + Y_{n2} \hat{U}_2 \hat{U}_n + \dots + Y_{nn} \hat{U}_n^2 = P_n - j Q_n \end{cases} \tag{7}$$

Generally, the equation describing the power balance at the bus i is shown in (8):

$$Y_{ii} \hat{U}_i^2 + \sum_{j=1, j \neq i}^n Y_{ij} \hat{U}_j \hat{U}_i = P_i - j Q_i \tag{8}$$

where:  $i=1, 2, \dots n$ .

Representing bus voltage at the bus  $i$  and admittance of branch  $ij$  in trigonometrical function, we have (9):

$$\begin{aligned} \dot{U}_i &= U_i \angle \delta_i = U_i (\cos \delta_i + j \sin \delta_i) \\ \dot{Y}_{ij} &= Y_{ij} \angle \psi_{ij} = Y_{ij} (\cos \psi_{ij} + j \sin \psi_{ij}) \end{aligned} \quad (9)$$

Substituting (9) into (8), we have (10) and (11):

$$U_i^2 Y_{ii} \cos \psi_{ii} + \sum_{j=1, j \neq i}^n U_i U_j Y_{ij} \cos(\delta_i - \delta_j - \psi_{ij}) = P_i \quad (10)$$

$$-U_i^2 Y_{ii} \sin \psi_{ii} + \sum_{j=1, j \neq i}^n U_i U_j Y_{ij} \sin(\delta_i - \delta_j - \psi_{ij}) = Q_i \quad (11)$$

Applying (10) and (11) for any system, the number of nonlinear equations is  $2n$  while the number of variables that are module and angle of bus voltage is also  $2n$ . From module and angle of bus voltage, values of power flow and current going through branch  $ij$  can be determined by (12):

$$\begin{aligned} P_{ij} &= U_i^2 Y_{ii} \cos \psi_{ii} - \sum_{j=1, j \neq i}^n U_i U_j Y_{ij} \cos(\delta_i - \delta_j - \psi_{ij}) \\ Q_{ij} &= -U_i^2 Y_{ii} \sin \psi_{ii} - \sum_{j=1, j \neq i}^n U_i U_j Y_{ij} \sin(\delta_i - \delta_j - \psi_{ij}) \\ \dot{I}_{ij} &= \frac{\dot{Y}_{ij}(\dot{U}_i - \dot{U}_j)}{\sqrt{3}} \end{aligned} \quad (12)$$

Above analysis shows that if variables can be determined, operational parameters are also determined. Because of strong nonlinear equations as represented in (10) and (11), Newton-Raphson is the most suitable method to determine variables. Although it uses an iterative technique but it converges very fast because all parameters only vary near rated value. That is the reason why it is often used to solve system of equations describing power systems.

Table 1. EN 50160 standard about permissible deviation ranges

| No | Parameter                    | Supply voltage characteristics                         |
|----|------------------------------|--|
| 1  | Voltage magnitude variations | $\pm 10\%$ for 95% of week, mean 10 minutes rms values |
| 2  | Rapid voltage changes        | 4% normal  |

**B. Newton-Raphson Method to Determine Operational Parameters**

Consider a curve described by a function  $F(X)$ . From an initial solution, tangent of the curve will cut in the horizontal axis a point that is nearer than the initial point. This process is executed many times to determine the desired solution as represented in Fig 3 [1], [2], [9].

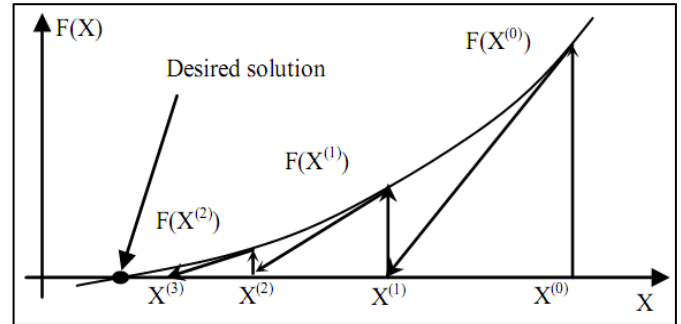


Fig. 3. Newton-Raphson method

In this method, if the derivative of  $f$  functions by variables  $F'(X) \neq 0$ , the next detective solution can be determined by (13):

$$X_{i+1} = X_i - \frac{F(X_i)}{F'(X_i)} \quad (13)$$

The convergence of the Newton-Raphson method is strongly related to the initial point  $F(X(0))$  and  $F'(X) \neq 0$  ( $F'(X)$  is the derivative of  $F(X)$ ).  $F(X(0))$  is closer to unknown root, the convergence will be faster.

To apply Newton-Raphson method to determine variables in power system, it needs to build admittance matrix and calculate the inversion of admittance matrix. Because the number of variables is too large, the calculated volume is also too large. Thus, the detection process can be executed by a software as Matlab software.

**C. EN 50160 Standard to Evaluate Operational Parameters**

EN 50160 is the European standard on power quality in public distribution systems. Specially, the permissible deviation range of voltage in medium voltage grid that is defined by EN 50160 is represented in Table 1 [3], [4].

Another technical factor that is defined by almost standards about power quality is the permissible value of currents going through lines. A rated current will be defined corresponding to each sectional area. It characterizes the limitation for thermal and the endurance of long life for lines. Because of the factor, value of current going through any line must be smaller than rated value.

To see the way to evaluate operational parameters, 19-bus system is used in the next part and the tool to analyze whole system is Matlab software.

**D. An Example to Analyze a Distribution System**

The 19-bus system is one of distribution systems is a 11 kV-cable distribution system shown in Fig 4 [10]. It notes that bus 1 is the slack bus. Parameters of lines and load in this system are represented in Table 2 and Table 3.

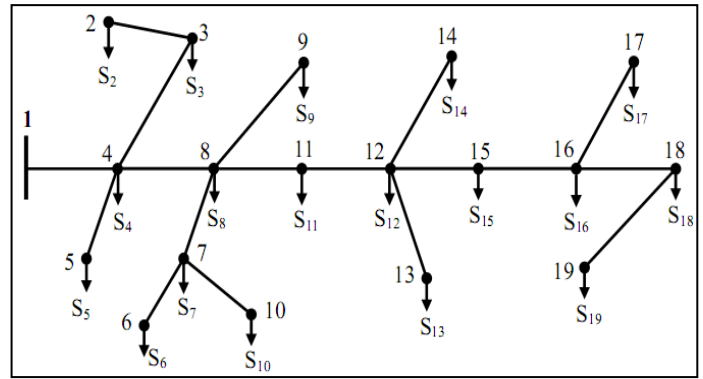


Fig. 4. 19-bus system

Table 2. Parameters of lines in 19-bus system

| Line  | Length (m) | Sectional area (mm <sup>2</sup> ) | Resistor (Ω/km) | Reactance (Ω/km) | Rated current (A) |
|-------|------------|-----------------------------------|-----------------|------------------|-------------------|
| 1-4   | 50         | 1000                              | 0.0376          | 0.17             | 1468              |
| 3-2   | 500        | 35                                | 1.113           | 0.2303           | 164               |
| 4-3   | 200        | 150                               | 0.2645          | 0.1964           | 391               |
| 4-5   | 1000       | 35                                | 1.113           | 0.2303           | 164               |
| 4-8   | 1000       | 800                               | 0.0495          | 0.1708           | 1188              |
| 7-6   | 200        | 35                                | 1.113           | 0.2303           | 164               |
| 8-7   | 500        | 150                               | 0.2645          | 0.1964           | 391               |
| 7-10  | 500        | 35                                | 1.113           | 0.2303           | 164               |
| 8-9   | 800        | 150                               | 0.2645          | 0.1964           | 391               |
| 8-11  | 600        | 500                               | 0.0791          | 0.1761           | 802               |
| 11-12 | 500        | 500                               | 0.0791          | 0.1761           | 802               |
| 12-13 | 1000       | 35                                | 1.113           | 0.2303           | 164               |
| 12-14 | 200        | 35                                | 1.113           | 0.2303           | 164               |
| 12-15 | 2000       | 185                               | 0.2107          | 0.1911           | 449               |
| 15-16 | 1000       | 150                               | 0.2645          | 0.1964           | 391               |
| 16-17 | 200        | 35                                | 1.113           | 0.2303           | 164               |
| 16-18 | 800        | 35                                | 1.113           | 0.2303           | 164               |
| 18-19 | 1000       | 35                                | 1.113           | 0.2303           | 164               |

Table 3. Parameters of load in 19-bus system

| Bus | Load active power (kW) | Load reactive power (kVAr) | Load power factor |
|-----|------------------------|----------------------------|-------------------|
| 2   | 1103.3                 | 683.8                      | 0.85              |
| 3   | 1642.6                 | 930.9                      | 0.87              |
| 4   | 1504.5                 | 932.4                      | 0.85              |
| 5   | 941.6                  | 608.3                      | 0.84              |
| 6   | 1283.2                 | 727.3                      | 0.87              |
| 7   | 1826.6                 | 1083.9                     | 0.86              |
| 8   | 1349.9                 | 728.6                      | 0.88              |
| 9   | 1103.3                 | 683.8                      | 0.85              |
| 10  | 1014.8                 | 602.2                      | 0.86              |
| 11  | 1644.9                 | 1019.4                     | 0.85              |
| 12  | 1765.3                 | 952.8                      | 0.88              |
| 13  | 1504.5                 | 932.4                      | 0.85              |
| 14  | 1014.8                 | 602.2                      | 0.86              |
| 15  | 1434.3                 | 888.9                      | 0.85              |
| 16  | 1061.5                 | 907.6                      | 0.87              |
| 17  | 1349.9                 | 728.6                      | 0.88              |
| 18  | 1474.4                 | 913.8                      | 0.85              |
| 19  | 1404.2                 | 870.2                      | 0.85              |

Table 4: Calculation results of lines in 19-bus system

| Line  | Active power (kW) | Reactive power (kVAr) | Apparent power (kVA) | Root mean square current (A) | %load |
|-------|-------------------|-----------------------|----------------------|------------------------------|-------|
| 1-4   | 21280             | 13430                 | 25162.5              | 1450.41                      | 98.8  |
| 3-2   | 988.5             | 612.6                 | 1162.9               | 67.53                        | 41.18 |
| 4-3   | 2484              | 1458                  | 2880.3               | 166.28                       | 42.53 |
| 4-5   | 839.7             | 542.4                 | 999.6                | 58.18                        | 35.48 |
| 4-8   | 16400             | 9977                  | 19196.4              | 1133.27                      | 95.39 |
| 7-6   | 1093              | 619.1                 | 1256.1               | 74.8                         | 45.61 |
| 8-7   | 3524              | 2059                  | 4080.4               | 242.48                       | 62.02 |
| 7-10  | 859.5             | 510                   | 999.4                | 59.69                        | 36.4  |
| 8-9   | 951.1             | 589                   | 1118.7               | 66.23                        | 16.94 |
| 8-11  | 10660             | 6519                  | 12495.3              | 746.24                       | 93.05 |
| 11-12 | 9222              | 5455                  | 10714.6              | 645.26                       | 80.46 |
| 12-13 | 1215              | 752.9                 | 1429.4               | 87.42                        | 53.3  |
| 12-14 | 841.7             | 499.4                 | 978.7                | 59.06                        | 36.01 |
| 12-15 | 5486              | 3238                  | 6370.3               | 398.24                       | 88.69 |
| 15-16 | 4305              | 2497                  | 4976.7               | 316.72                       | 81    |
| 16-17 | 1001              | 540.5                 | 1137.6               | 72.62                        | 44.28 |
| 16-18 | 2046              | 1266                  | 2406                 | 156.79                       | 95.6  |
| 18-19 | 993.3             | 615.6                 | 1168.6               | 76.37                        | 46.57 |

Using Matlab/Simulink software, calculation results about active power, reactive power, apparent power, root mean square current going through lines are represented in Table 4.

From Table 4, we can see highlight lines that have the level of reserve smaller than 20%: line 1-4 (1.2%), line 4-8 (4.61%), line 8-11 (6.95%), line 11-12 (19.54%), line 12-15 (11.31%), line 15-16 (19%), line 16-18 (4.4%). These lines are on the main transmissive axis of the system. Because of having low reserve, they are highly sensitive to life of cable and easy to meet overheat if the load demand continue to increase. In addition, voltage and current waves going through lines connected to bus 4 are represented in Fig 5 and Fig 6.

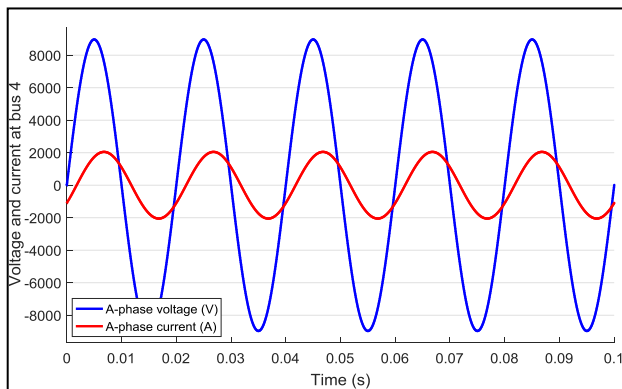


Fig. 5. Voltage and current waves at bus 4

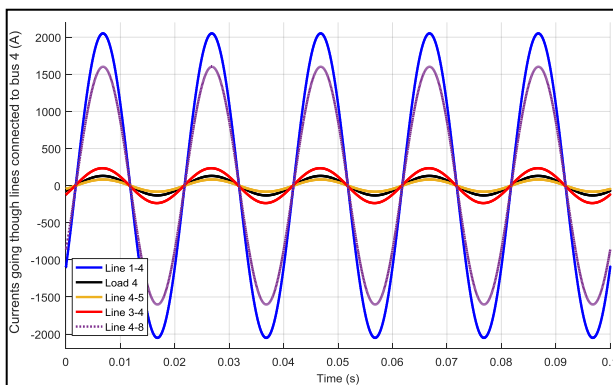


Fig. 6. Current waves s going through line connected to bus 4

Values of bus voltage in this operational scenario are represented in Fig 7.

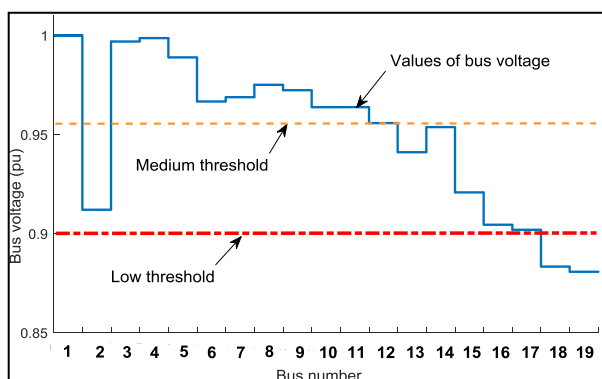


Fig. 7. Values of bus voltage

From Fig 5, voltage and current waves at bus 4 have the same sinusoidal form, where the current wave lagged behind the voltage wave because of being affected by impedance lines. Results in Figure 6 showed the same exactly wave form of currents going through lines connected to bus 4. The peak value of current going through line 1-4 is total of peak values of currents going through line 4-5, line 3-4, line 4-8 and load 4. It means that accuracy of the calculation process is very high and highly ensures the Kirchoff 1 law. From Fig 7, we can see voltage values from bus 12 to bus 19 are smaller than medium threshold (isn't enough 4% reserve). Specially, voltage values of bus 18 and bus 19 are smaller than low threshold (isn't enough 10% reserve). These buses affects highly to power quality.

Above analysis shows that it needs to have a solution to improve voltage quality and increase the level of reserve for transmission lines for 19-bus system. The proposed solution in this paper applied for this diagram is that uses shunt capacitors. The problem that must be solved is their optimal sizing and location in whole system.

### III. OPTIMAL SIZING AND LOCATION OF SHUNT CAPACITORS BY CONSIDERING COST FUNCTION

#### A. Building Cost Function

Cost function for sizing and location of capacitors is based on cost for investment problem of own capacitors ( $Z_1$ ), cost for power loss in own capacitors ( $Z_2$ ) and cost for power loss in whole system after compensating ( $Z_3$ ) [7], [10], [11], [12], [13], [14].

Function  $Z_1$  can be calculated by (14):

$$Z_1 = (a_{co} + a_{cp})k_0 \sum_{i=1}^n Q_{Ci} \quad (14)$$

where:  $a_{co}$  is coefficient of operational cost,

$a_{cp}$  is coefficient of primary recovery cost

$k_0$  is price per kVAr of compensative device (USD/kVAr)

$Q_{Ci}$  is sizing of compensative device at bus  $i$  (kVAr).

Function  $Z_2$  can be calculated by (15):

$$Z_2 = \Delta P_0 TC \sum_{i=1}^n Q_{Ci} \quad (15)$$

where:

$\Delta P_0$  is the rate of active power loss caused by own compensative device (kW/kVAr),

$T$  is the working time of compensative device (h),

$C$  is the electricity price (USD/kWh).

Function  $Z_3$  can be calculated by (16):

$$Z_3 = \frac{\tau C}{U^2} \sum_{i=1, j \neq i}^n R_{ij} (Q_{ij} - Q_{Cij})^2 \quad (16)$$

where:

$R_{ij}$  is resistance of branch  $ij$  ( $\Omega$ ),

$U$  is rated voltage of the grid (kV),

$\tau$  is equivalent time corresponding to the operation at maximum load in considered period.

$Q_{Cij}$  is total compensative capacity in branch  $ij$ .

Using (14), (15) and (16), we have cost function (17):

$$Z = (a_{co} + a_{cp})k_0 \sum_{i=1}^n Q_{Ci} + \Delta P_0 TC \sum_{i=1}^n Q_{Ci} + \frac{\tau C}{U^2} \sum_{i=1, j \neq i}^n R_{ij} (Q_{ij} - Q_{Cij})^2 \quad (17)$$

**B. Algorithm to determine optimal sizing and location of shunt capacitors**

The expected value in economic problem is that receives the minimum value of the cost function. It only achieves if the partial derivatives of the cost function with respect to variables reach to zero [7], [10], [12], [13]. It means that values of variable ( $Q_{Ci}$ ) can be determined by solving system of equations (18):

$$\frac{\partial Z}{\partial Q_{Ci}} = 0 \quad (18)$$

where:  $i=1 \div (n-1)$

(not compensating at slack bus)

From (18), the number of linear equations is  $(n-1)$  corresponding to  $(n-1)$  variables. It is easy to solve above system of equations but there is a small consideration when the results have some negative values. In the economic problem, negative values must be neglected. Because of above reason, an algorithm is proposed in this paper to determine sizing and location of shunt capacitors as depicted in Fig 8.

In this algorithm, the normal stage will collect input data, build and solve system of equations for the economic problem. The iterative stage will assign zero for negative values and resolve system of equations after neglecting equations that were built at buses having negative values.

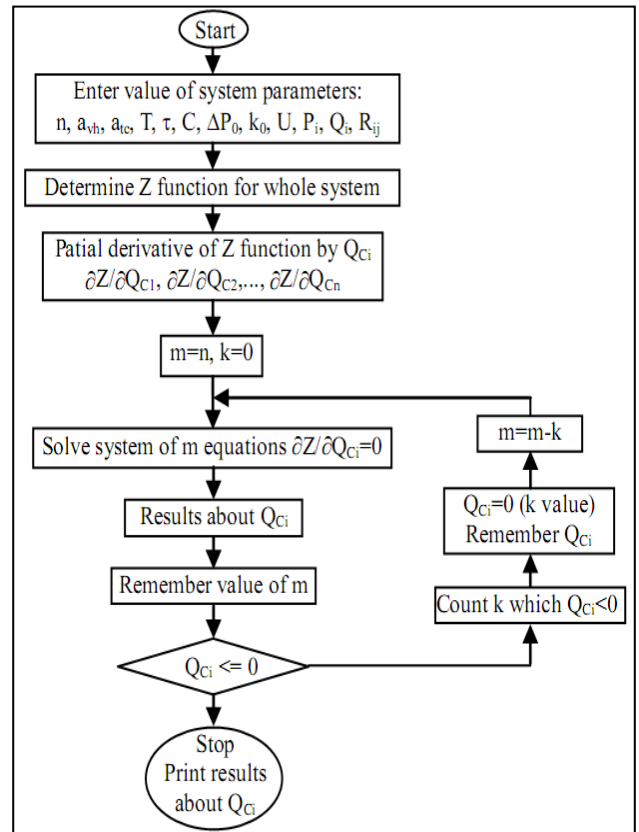


Fig. 8. Proposed algorithm to determine optimal sizing and location of shunt capacitors by considering cost function

**C. Apply to the 19-bus system**

In this example, values of economic parameters are:

- $a_{co} = 0.1$ ;  $a_{cp} = 0.125$ ;  $\Delta P_0 = 0.005$  (kW/kVAr)
- $T = 8760$  hours;  $\tau = 2500$  hours;
- $k_0 = 3$  (USD/kVAr);  $C = 0.09$  (USD/kWh);

Using the algorithm in Fig 8, the results about sizing and location of shunt capacitors are represented in Table 5.

The results about lines in the 19-bus system are represented in Table 6. Values of bus voltage in this operational scenario are represented in Fig 9. A-phase voltage and current waves at bus 4 after compensating are depicted in Fig 10 and Fig 11.

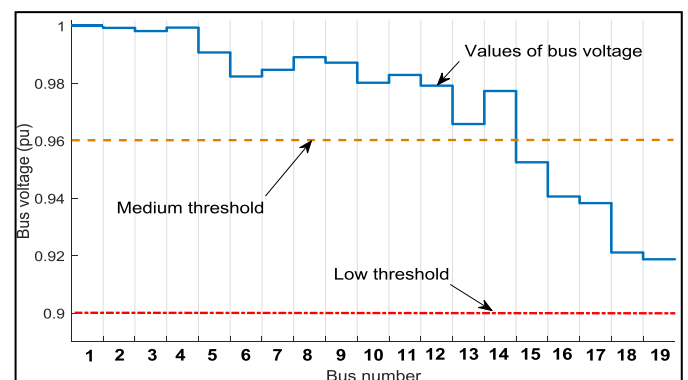


Fig. 9. Bus voltage values

Table 5. Parameters of buses in the 19-bus system after compensating

| Bus | Sizing of shunt capacitor (kVAr) | Load active power (kW) | Load reactive power (kVAr) | $Q_{load} - Q_C$ (kVAr) |
|-----|----------------------------------|------------------------|----------------------------|-------------------------|
| 2   | 579.5                            | 1103.3                 | 683.8                      | 104.3                   |
| 3   | 727                              | 1642.6                 | 930.9                      | 203.9                   |
| 4   | 0                                | 1504.5                 | 932.4                      | 932.4                   |
| 5   | 481.6                            | 941.6                  | 608.3                      | 126.7                   |
| 6   | 616.3                            | 1283.2                 | 727.3                      | 111                     |
| 7   | 809.3                            | 1826.6                 | 1083.9                     | 274.6                   |
| 8   | 537.1                            | 1349.9                 | 728.6                      | 191.5                   |
| 9   | 579.5                            | 1103.3                 | 683.8                      | 104.3                   |
| 10  | 578.1                            | 1014.8                 | 602.2                      | 24.1                    |
| 11  | 863.9                            | 1644.9                 | 1019.4                     | 155.5                   |
| 12  | 1560.8                           | 1765.3                 | 952.8                      | -608                    |
| 13  | 790.2                            | 1504.5                 | 932.4                      | 142.2                   |
| 14  | 510.3                            | 1014.8                 | 602.2                      | 91.9                    |
| 15  | 753.3                            | 1434.3                 | 888.9                      | 135.6                   |
| 16  | 769.2                            | 1061.5                 | 907.6                      | 138.4                   |
| 17  | 617.5                            | 1349.9                 | 728.6                      | 111.1                   |
| 18  | 774.4                            | 1474.4                 | 913.8                      | 139.4                   |
| 19  | 737.5                            | 1404.2                 | 870.24                     | 132.74                  |

Table 6. Parameters of lines in the 19-bus system

| Line  | Active power (kW) | Reactive power (kVAr) | Apparent power (kVA) | Root mean square current (A) | %load |
|-------|-------------------|-----------------------|----------------------|------------------------------|-------|
| 1-4   | 22010             | 2739                  | 22179.8              | 1277.1                       | 87    |
| 3-2   | 992.4             | -0.03                 | 992.4                | 57.52                        | 35.07 |
| 4-3   | 2490              | -0.17                 | 2490                 | 143.57                       | 36.72 |
| 4-5   | 842.9             | 29.48                 | 843.4                | 49                           | 29.88 |
| 4-8   | 17160             | 1392                  | 17216.4              | 1001.6                       | 84.31 |
| 7-6   | 1129              | -88.07                | 1132.4               | 66.37                        | 40.47 |
| 8-7   | 3641              | -50.57                | 3641.3               | 212.8                        | 54.42 |
| 7-10  | 889.1             | -57.61                | 890.96               | 52.3                         | 31.89 |
| 8-9   | 980.5             | 0.07                  | 980.5                | 57.17                        | 14.62 |
| 8-11  | 11260             | 1232                  | 11327.2              | 663                          | 82.67 |
| 11-12 | 9771              | 151.1                 | 9772.2               | 574.4                        | 71.62 |
| 12-13 | 1280              | 0.01                  | 1280                 | 76.3                         | 46.52 |
| 12-14 | 883.9             | 0.04                  | 883.9                | 52.1                         | 31.77 |
| 12-15 | 5895              | 789                   | 5947.6               | 359.4                        | 80.04 |
| 15-16 | 4650              | 10.89                 | 4650.01              | 284.5                        | 72.76 |
| 16-17 | 1084              | 0.01                  | 1084                 | 66.5                         | 40.55 |
| 16-18 | 2224              | 0.58                  | 2224                 | 139                          | 84.76 |
| 18-19 | 1081              | 0.04                  | 1081                 | 67.7                         | 41.28 |



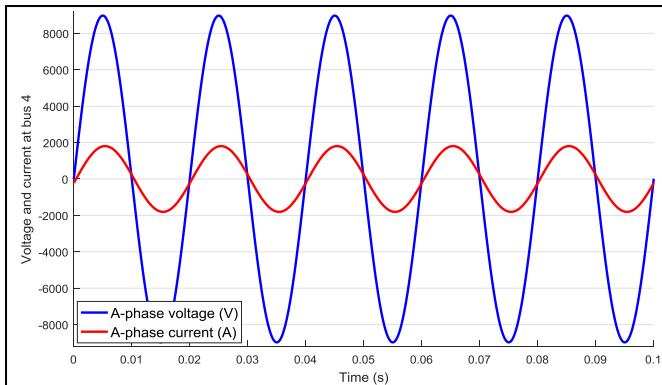


Fig. 10. Voltage and current waves at bus 4 after compensating

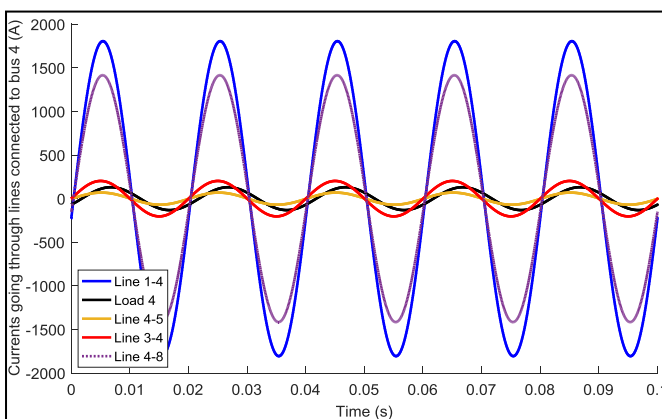


Fig. 11. Current waves going through line connected to bus 4 after compensating.

From Table 4, we can realize that almost buses in this system must be compensated except bus 4. All values of %load are reduced smaller than 90% as represented in Table 6 and all values of voltage drop are smaller than 10% as shown in Fig 9. From Fig 10, voltage and current waves at bus 4 have the same sinusoidal form and are nearly in phase. The current going through line 1-4 is also total of currents going through line 4-5, line 3-4, line 4-8 and load 4 as represented in Fig 11. It means that almost reactive powers are met by capacitors. They also help to reduce power flows in whole system, increase the system reserve and improve voltage quality.

#### IV. CONCLUSION

This paper proposes an algorithm to determine optimal sizing and location of shunt capacitors by considering cost function. This function evaluates three types of cost about the investment for own capacitors, power loss in own capacitors and cost for power loss in whole system after compensating. This method helps to provide information about the optimal sizing of shunt capacitor at each bus and then the optimal locations that must be compensated in whole system. This information ensures the minimum of cost function.

Applying to calculate a 19-bus system, capacitors play an important role in reducing currents flowing lines in whole system and improving voltage quality. Instead of upgrading sectional area of lines, cutting down electric load or using series capacitors, simulation results showed that the reversed power on each transmission line increases because the currents decreased very much and values of bus voltage reached over the limitation. It means the reactive power is generated near fully by the capacitor at each bus.

Considering cost function is only one of methods to determine optimal sizing and location of shunt capacitors. To have truly information about optimal sizing and location, the cost function must be evaluated in many load scenarios. Corresponding to varying of electric load, capacity of each capacitor can be change by using controlled device, called distributed flexible alternative current transmission system (D-FACTS). In this case, the cost function will provide the rated reactive power for capacitors. For the next research, the optimal sizing and location can be combined with the power factor to have exact information about the whole system.

#### V. DECLARATION

All authors have disclosed no conflicts of interest.

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