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FAST INTERPLANETARY MISSIONS WITH LOW-THRUST PROPULSION SYSTEMS

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ABSTRACT

A simple family of indirect transfer trajectories between circular orbits is used to evaluate the mass ratio required to complete round-trip interplanetary missions using low-thrust propulsion systems. The results indicate that indirect interplanetary trajectories yield substantial reductions in total round-trip time for low-thrust as well as high-thrust vehicles, and that space vehicles propelled with electric rockets may produce greater reductions in trip time, for a given initial weight, than those propelled by high-thrust nuclear rockets.

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SUMMARY

A simple family of indirect transfer trajectories between circular orbits is used to evaluate the mass ratio required to complete round-trip interplanetary missions using low-thrust propulsion systems. These trajectories, although not optimum, yielded very substantial reductions in total round-trip time for Mars missions with moderate increases in initial weight. For a powerplant specific weight α of 10 pounds per kilowatt of jet power, trip times were reduced from 1200 to 600 days, for a typical manned mission, with an initial weight increase of a factor of two. Comparison with a nuclear rocket with 1000-second specific impulse indicated that the electric-propulsion system required less initial weight for trip times as low as 550 days with $\alpha = 10$ and as low as 400 days with $\alpha = 5$ pounds per kilowatt. Further weight reductions would be expected with more nearly optimum trajectories.

INTRODUCTION

The study of round-trip interplanetary missions using low-thrust propulsion systems (such as electric propulsion) is more difficult than similar studies for high-thrust systems, because the trajectory cannot, in general, be represented in closed form. To determine the propellant mass required, a complete integration of the trajectory equations is generally necessary for each trajectory chosen. It is naturally desirable to choose those trajectories that minimize the total initial weight required to complete a mission in a given length of time. For one-way trips to Mars, such an optimization program was reported in reference 1. A discussion was also given in reference 1 of round-trip missions consisting of optimum one-way trajectories; but, since no attempt was made to include the effect of waiting time at the destination planet (which is required to effect rendezvous with the Earth), no valid conclusions could be drawn on the propellant requirements as functions of overall mission time. It is, in fact, clear from studies of high-thrust trajectories (refs. 2 to 4) that direct Earth-planet trajectories such as those of reference 1 are unlikely to produce the largest mission-time reductions for a given initial weight.

Another study of low-thrust round-trip missions was included in reference 5, wherein constant-tangential-thrust trajectories were used. However, as was pointed out in that reference, an extensive program of trial-and-error computations would be required to make these trajectories useful for round-trip missions much faster than the minimum-energy mission. Consequently, it has hitherto not been determined whether faster round-trip interplanetary missions are possible using electric-propulsion systems, or what weight penalties are likely to be involved to achieve such time reductions. The purpose of the present report is to answer these questions in a preliminary way.

The approach used herein is to select a particular family of continuous-thrust trajectories (not necessarily optimum) for which it is easy to vary trip time, transit angle, and perihelion or aphelion distance. Combinations of members of this family of trajectories are then found that minimize the initial weight for a given total trip time. The results are then applied to typical manned and unmanned Mars missions to show how initial weight varies with mission time and to compare this weight with that possible using high-thrust nuclear rockets.

MISSION ANALYSIS

General Discussion

Before considering the particular trajectories selected, it is well to derive general expressions for determining initial weight required for any round-trip interplanetary mission. These missions can generally be divided into seven phases: (1) Earth escape, (2) Earth-planet transfer, (3) descent to planetary orbit (or surface), (4) waiting time, (5) escape from planet, (6) planet-Earth transfer, and (7) descent to Earth orbit (or surface). For one-way interplanetary probes, only the first two or three of these phases need be considered; while for round-trip missions, all must be included. With high-thrust propulsion systems, only phases (1), (3), (5), and (7) involve propulsion periods; while with low-thrust systems all phases except phase (4) may require continuous propulsion.

For preliminary studies of mission capabilities and feasible trajectories, such as those contained herein, it is assumed that:

(1) Only a single gravitational mass need be considered during each phase (the Earth in phases (1) and (7), the destination planet in phases (3), (4), and (5), and the Sun in phases (2) and (6)).

(2) The orbits of the Earth and the destination planet are circular and coplanar.

These assumptions are justifiable on the grounds that the major energy changes involved in the actual mission are included in this simplified model.

The escape and descent phases of the mission (phases (1), (3), (5), and (7)) can be adequately handled using the charts of reference 5. These charts present trajectory parameters for constant specific impulse and constant thrust directed parallel to the instantaneous velocity vector. It has frequently been indicated (e.g., refs. 1 and 5) that constant-tangential-thrust trajectories are close to optimum with respect to propellant consumption for accelerations from satellite orbits to escape velocity.

The Earth-planet transfer phases (2) and (6) will be evaluated using a particular trajectory family to be derived. These phases consist of transfers between two circular orbits (those of the Earth and the destination planet) with the boundary condition that the vehicle velocity is equal in magnitude and direction to that of the Earth at the Earth's orbit and that of the destination planet at its orbit. For such transfers, reference 1 indicated that propellant consumption is minimized if maximum jet power is applied at all times (as in the constant-thrust case) but with the thrust varied to follow certain optimum trajectories. Phases (2) and (6) will therefore be called variable-thrust phases; and the trajectory family to be derived, although not optimum, is of the constant-power, variable-thrust type.

Mass Ratio for Constant-Thrust Phases

As pointed out in reference 5, mass losses other than propellant consumption can sometimes become important in calculating the trajectories and performance of low-thrust vehicles. Particularly for manned missions, the consumption and ejection of subsistence supplies may significantly affect the trajectory during the lengthy propulsion periods. The mass-ratio expressions, both for constant-thrust and variable-thrust trajectories, will therefore be derived for the case where supplies, other than propellant, are being ejected without generating thrust. It is assumed that this ejection takes place at a constant rate, since discontinuous ejection of sizable mass requires stepwise integration of the equation of motion.

To determine the trajectory as a function of time from the charts of reference 5, the two parameters that must be specified are a_0 and V_j, where $a_0 = F/m_0 g_0$ is the initial thrust acceleration, and $V_j = v_j/v_{c,0}$

is the effective jet velocity parameter. (Subscript 0 is used to designate values in the reference orbit, which is either the departure orbit for escape phases or the destination orbit for descent phases.) The expression for effective jet velocity parameter is derived as follows:

$$v_j = \frac{F}{\dot{m}} = \frac{F}{\dot{m}_{pr} + \dot{m}_s} \tag{1}$$

where v_j is effective jet velocity, F is thrust, $\dot{m}_{pr}$ is propellant consumption rate, and $\dot{m}_s$ is supply ejection rate. (All symbols are defined in appendix A.) Now

$$\dot{m}_{pr} = \frac{F}{v_{j,pr}}$$

where $v_{j,pr}$ is propellant jet velocity, so that

$$v_j = \frac{v_{j,pr}}{1 + \frac{\dot{m}_s}{\dot{m}_{pr}}} \tag{2}$$

Making all velocities dimensionless with the reference circular velocity $v_{c,0}$ yields

$$V_j = \frac{V_{j,pr}}{1 + \frac{\dot{m}_s}{\dot{m}_{pr}}} \tag{3}$$

With V_j and a_0 specified, constant-thrust phases of the trip can be determined from the charts of reference 5. The mass ratio for these propulsion periods is given by

$$\frac{m_b}{m_a} = 1 - \frac{a_0\tau}{V_j}\frac{m_0}{m_a} \tag{4}$$

where $\tau = v_{c,0}t/r_0$, the dimensionless propulsion-time parameter, and where $m_0 = m_a$ for outward paths and $m_0 = m_b$ for inward paths.

For electric-propulsion systems, thrust and propellant jet velocity (or a_0 and $v_{j,pr}$) are not independent. For constant jet power, they are, in fact, related by

$$\dot{m}_{pr}v_{j,pr}^2 \equiv Fv_{j,pr} = 1476P_j \tag{5}$$

where F is expressed in pounds, $v_{j,pr}$ in feet per second, and P_j (the jet power) in kilowatts. In dimensionless form this becomes

$$a_0 V_{j,pr} = \frac{1476}{v_{c,0}} \frac{W_i}{m_0 g_0} \frac{P_j}{W_i} \tag{6}$$

where $W_i = m_i g_{00}$ is the initial space vehicle weight. This initial weight is introduced to permit use of two design parameters: the specific powerplant weight α (= W_{pp}/P_j, lb/kw) and the ratio of powerplant weight to initial gross weight β (= W_{pp}/W_i). With these parameters, equation (6) becomes

$$a_0 V_{j,pr} = \frac{1476}{v_{c,0}} \frac{W_i}{m_0 g_0} \frac{\beta}{\alpha} \tag{7}$$

The mass ratio for the propulsion period can be expressed in a more useful form, in terms of the propulsion time and the parameters α and β. Thus,

$$\frac{m_b}{m_a} = 1 - \frac{m_0}{m_a} \frac{a_0 \tau}{V_{j,pr}} - \frac{\dot{m}_s t_f}{m_a} \tag{8}$$

where t_f is the propulsion time. But

$$V_{j,pr} = \frac{1476}{a_0 v_{c,0}} \frac{W_i}{m_0 g_0} \frac{\beta}{\alpha} \tag{9}$$

and

$$a_0 = \frac{(a_0 \tau)}{\tau_f} = \frac{(a_0 \tau)}{\frac{v_{c,0} t_f}{r_0}} \tag{10}$$

Consequently,

$$\frac{m_b}{m_a} = 1 - 4.14\times10^{-5}\ r_0 (a_0 \tau)^2 \left(\frac{m_0}{m_a}\right)^2 \frac{m_a g_0}{W_i} \frac{\alpha}{\beta t_f} - \frac{\dot{m}_s t_f}{m_a} \tag{11}$$

where r_0 is expressed in miles and t_f in days.

For the outward trajectory ($m_0 = m_a$), equation (11) is the solution for m_b/m_a. For the inward trajectory ($m_0 = m_b$), the solution of the quadratic for m_b/m_a is

$$\frac{m_b}{m_a} = \frac{\sqrt{1 + 4C\left(1 - \frac{\dot{m}_s t_f}{m_a}\right)} - 1}{2C}$$

where

$$C = 4.14\times10^{-5} \; r_0(a_0\tau)^2 \frac{m_a g_0}{W_i} \frac{\alpha}{\beta t_f}$$

For values of C and $\dot{m}_s t_f/m_a$ much less than unity (which is generally the case), expansion of the radical yields, to first order in C and $\dot{m}_a t_f/m_a$,

$$\frac{m_b}{m_a} \approx \left(1 - \frac{\dot{m}_s t_f}{m_a}\right)(1 - C)$$

This expression can also be written in the following more convenient form (for later derivations):

$$\frac{m_b}{m_a} \approx \left(1 - \frac{\dot{m}_s t_f}{m_a}\right)\left[1 + 4.14\times10^{-5} \; r_0(a_0\tau)^2 \frac{m_a g_0}{W_i} \frac{\alpha}{\beta t_f}\right]^{-1} \tag{11a}$$

Equation (11) (with $m_0 = m_a$) will be used for computations of mass ratio for outward paths, and equation (11a) for inward paths. Use of equation (11a) is conservative, in that values of m_b/m_a obtained are lower than those resulting from the exact equation.

The dimensionless total-impulse parameter $(a_0\tau)$ is determined from reference 5 for the energy change required during the propulsion period. For values of a_0 and V_j corresponding to escape from, and descent to, circular orbits near the planets with electric-propulsion systems, the value of $a_0\tau$ is approximately 0.9 and is rather insensitive to moderate variations in a_0 and V_j in the range of interest for electric propulsion. This value will therefore be used for the escape and descent phases of interplanetary missions.

It is clear from equation (11) that, for each constant-thrust propulsion period, an optimum propulsion time exists that minimizes the sum of the propellant weight and supply weight required. Representing equation (11) by

$$\frac{m_b}{m_a} = 1 - a_1 t_f^{-1} - a_2 t_f \tag{12}$$

and differentiating with respect to t_f yield the following values for optimum propulsion time and maximum mass ratio:

$$t_{opt} = \sqrt{a_1/a_2}, \quad \text{days} \tag{13}$$

$$\left(\frac{m_b}{m_a}\right)_{max} = 1 - 2\sqrt{a_1 a_2} \tag{14}$$

For most missions of interest, t_{opt} is unreasonably large. In particular, if a_2 is negligible (very small supply ejection rate), equation (12) leads to the obvious conclusion that the propulsion time should be as large as possible to reduce propellant consumption. This optimization is therefore not a useful one when reductions in total trip time are being sought.

Mass Ratio for Variable-Thrust Phases

For variable-thrust propulsion phases, the derivation of convenient expressions for the mass ratio proceeds in a manner similar to that of reference 1, except that supply consumption and ejection are considered. The differential mass loss can be expressed as follows:

$$\begin{aligned} dm &= -dm_{pr} - dm_s \\ &= -\frac{F}{v_{j,pr}}\,dt - \dot{m}_s\,dt \\ &= -\left(\frac{m^2 a^2}{Fv_{j,pr}} + \dot{m}_s\right) dt \end{aligned} \tag{15}$$

where the constant-power condition

$$Fv_{j,pr} = 1476P_j \tag{16}$$

has been used. Equation (15) can be written in integral form as follows:

$$\frac{1}{m_b} - \frac{1}{m_a} - \dot{m}_s \int_0^{t_f} \frac{dt}{m^2} = \int_0^{t_f} \frac{a^2\,dt}{1476P_j} \tag{17}$$

or

$$\frac{0.28P_j}{m_a g_a v_{c,a}}\left(\frac{m_a}{m_b} - 1 - \dot{m}_s m_a \int_0^{t_f} \frac{dt}{m^2}\right) = \int_0^{\tau_f} \left(\frac{a}{g_a}\right)^2 d\tau = \gamma \tag{18}$$

where $v_{c,a}$ is expressed in miles per second. The integral of the square of the nondimensional acceleration γ is the quantity to be determined from the particular trajectories selected.

To obtain the mass ratio in a reasonably simple manner from equation (18), the mass in the denominator of the supply-consumption integral is represented by a mean value, given by

$$\bar{m} = k(m_a + m_b) = km_a\left(1 + \frac{m_b}{m_a}\right) \tag{19}$$

With this substitution, equation (18) becomes

$$\frac{0.28P_j}{m_a g_a v_{c,a}}\left[\frac{m_a}{m_b} - 1 - \frac{\frac{\dot{m}_s t_f}{m_a}}{k^2\left(1 + \frac{m_b}{m_a}\right)^2}\right] = \gamma \tag{20}$$

In terms of the parameters α and β,

$$\frac{0.28P_j}{m_a g_a v_{c,a}} = \frac{0.28P_j}{W_i v_{c,a}} \frac{m_i}{m_a} \frac{g_{00}}{g_a} = \frac{0.28}{v_{c,a}} \frac{g_{00}}{g_a} \frac{m_i}{m_a} \frac{\beta}{\alpha} \tag{21}$$

so that equation (20) can be written

$$\frac{m_a}{m_b} - 1 - \frac{\frac{\dot{m}_s t_f}{m_a}}{k^2\left(1 + \frac{m_b}{m_a}\right)^2} = \frac{m_a}{m_i} \frac{\alpha}{\beta} \Gamma \tag{22}$$

where

$$\Gamma = \frac{v_{c,a}}{0.28} \frac{g_a}{g_{00}} \gamma, \qquad \frac{kw}{lb} \tag{23}$$

Equation (22) is the required relation between mass ratio and the trajectory parameters. It is plotted in figure 1 for several values of the supply-consumption ratio $\dot{m}_s t_f/m_a$ for the particular value $k = 1/2$. That this value of k is a reasonable one for moderate supply-consumption rates was verified by comparing the curves of figure 1 with values obtained for two special cases for which the m^{-2} integral in equation (18) is easily evaluated. These cases are (1) $m/m_a = 1 - ct$, and (2) $(m/m_a)^2 = 1 - ct$. For the first case (constant mass-consumption rate), equation (22) becomes

$$\frac{m_b}{m_a} = \frac{1 - \dfrac{\dot{m}_s t_f}{m_a}}{1 + \dfrac{m_a}{m_i}\dfrac{\alpha}{\beta}\Gamma} \tag{22a}$$

For the second case, equation (22) becomes

$$\frac{m_a}{m_b} - 1 - \frac{\dfrac{\dot{m}_s t_f}{m_a} \ln\left(\dfrac{m_a}{m_b}\right)^2}{\left(1 - \dfrac{m_b}{m_a}\right)^2} = \frac{m_a}{m_i}\frac{\alpha}{\beta}\Gamma \tag{22b}$$

Values obtained from equations (22a) and (22b) are indicated in figure 1 for the case $\dot{m}_s t_f/m_a = 0.3$. The comparison indicates that, at least for values of $\dot{m}_s t_f/m_a \leq 0.3$, the curves obtained for $k = 1/2$ are adequate for preliminary mission studies. These curves do not, however, yield the correct values of m_b/m_a at $\Gamma = 0$ $\left[(m_b/m_a)_{\Gamma=0} = 1 - (\dot{m}_s t_f/m_a)\right]$. The dashed curves were therefore faired into the correct value to improve the accuracy near the abscissa.

With the curves of figure 1, the mass ratio required to follow constant-power trajectories can be determined when the value of γ for those trajectories is found. For the particular case of heliocentric transfer from the orbit of the Earth, the relation between γ and Γ is

$$\Gamma = 0.04\,\gamma \tag{24}$$

[It should be noted that the γ of this report is not the same as that of reference 1. The relation between them is

$$\alpha\Gamma = (\gamma^2)_{\text{Ref. 1}}$$

The expression Υ^2/α plotted in reference 1 is related to present parameter by

$$\Gamma = 0.454\left(\frac{\Upsilon^2}{\alpha}\right)_{\text{Ref. 1}}, \quad \frac{\text{kw}}{\text{lb}}$$

where the factor 0.454 converts the α of reference 1 (which is in kg/kw) into pounds per kilowatt.]

Mass Ratio for Interplanetary Round Trips

The mass ratios required for the seven phases of a round-trip interplanetary mission can now be summarized as follows:

Phase (1). - Escape from circular orbit of radius r_i around Earth (from eq. (11)):

$$\frac{m_1}{m_i} = 1 - 3.35\times10^{-5}\, r_i \frac{g_i}{g_{00}} \frac{\alpha}{\beta t_1} - \frac{\dot{m}_s t_1}{m_i} \tag{25}$$

Phase (2). - Earth-planet transfer (eq. (22)):

$$\frac{m_1}{m_2} - 1 - \frac{\dfrac{4\dot{m}_s t_2}{m_1}}{\left(1 + \dfrac{m_2}{m_1}\right)^2} = \frac{m_1}{m_i} \frac{\alpha}{\beta} \Gamma_2 \tag{26}$$

(The solution of this equation is given in fig. 1.)

Phase (3). - Descent to circular orbit at radius r_3 around planet (eq. (11)):

$$\frac{m_3}{m_2} = 1 - 3.35\times10^{-5}\, r_3 \frac{m_3 g_3}{m_i g_{00}} \frac{m_3}{m_2} \frac{\alpha}{\beta t_3} - \frac{\dot{m}_s t_3}{m_2} \tag{27}$$

or, from equation (11a):

$$\frac{m_3}{m_2} \approx \left(1 - \frac{\dot{m}_s t_3}{m_2}\right)\left(1 + 3.35\times10^{-5}\, r_3 \frac{m_2}{m_i} \frac{g_3}{g_{00}} \frac{\alpha}{\beta t_3}\right)^{-1} \tag{27a}$$

Phase (4). - Waiting or exploration period:

$$\frac{m_4}{m_3} = 1 - \frac{\dot{m}_s t_4}{m_3} - \frac{m_{ex}}{m_3} \tag{28}$$

where m_{ex} is the mass, other than provisions, that is used during this period and left behind (such as landing and exploration equipment).

Phase (5). - Escape from circular orbit at radius r_3 around planet (eq. (11)):

$$\frac{m_5}{m_4} = 1 - 3.35\times10^{-5}\ r_3 \frac{m_4}{m_i} \frac{g_3}{g_{00}} \frac{\alpha}{\beta t_5} - \frac{\dot{m}_s t_5}{m_4} \tag{29}$$

Phase (6). - Planet-Earth transfer (eq. (22)):

$$\frac{m_5}{m_6} - 1 - \frac{\dfrac{4\dot{m}_s t_6}{m_5}}{\left(1 + \dfrac{m_6}{m_5}\right)^2} = \frac{m_5}{m_i} \frac{\alpha}{\beta} \Gamma_6 \tag{30}$$

(The solution of this equation is plotted in fig. 1.)

Phase (7). - Descent to circular orbit at radius r_7 around Earth (eq. (11)):

$$\frac{m_7}{m_6} = 1 - 3.35\times10^{-5}\ r_7 \frac{m_7}{m_6} \frac{m_7}{m_i} \frac{g_7}{g_{00}} \frac{\alpha}{\beta t_7} - \frac{\dot{m}_s t_7}{m_6} \tag{31}$$

or, from equation (11a):

$$\frac{m_7}{m_6} \approx \left(1 - \frac{\dot{m}_s t_7}{m_6}\right)\left(1 + 3.35\times10^{-5}\ r_7 \frac{m_6}{m_i} \frac{g_7}{g_{00}} \frac{\alpha}{\beta t_7}\right)^{-1} \tag{31a}$$

The mass m_7 is the mass returned to an orbit around the Earth, and includes (1) the return payload mass (crew quarters and supplies, except consumed provisions, crew shielding, communication and navigation equipment, instrumentation, etc.), (2) the powerplant mass m_{pp}, and (3) the reserve propellant and provisions m_{res}. If the basic payload and reserve are grouped together and denoted by m_p, the return payload mass, then

$$\frac{m_p}{m_i} = \frac{m_7}{m_i} - \beta \tag{32}$$

All structure weight is assumed to be included in the powerplant weight.

It is worth noting that the parameters α and β appear in equations (25) to (31a) only as the ratio α/β, which is equal to W_i/P_j. Consequently, the final- to initial-weight ratio m_7/m_i can be determined without separately specifying α and β. To find the portion of the final mass m_7 that is payload, of course, requires specification of powerplant weight ratio β.

For the general manned mission, the payload must be calculated by the step-by-step process, starting with an assumed initial mass m_i. If the payload turns out to be less than or greater than that needed for the mission, other values of m_i must be assumed, and the correct value can be obtained by plotting a curve of m_p against m_i.

For preliminary calculations, aimed at determining optimum values of the many parameters involved in equations (25) to (32), it is better to simplify these equations by neglecting m_{ex} and the supply consumption $\dot{m}_s$. It is also convenient to consider a specific mission. The mission chosen for consideration herein is a round-trip Mars journey starting and ending in an orbit around the Earth at radius $r_i = r_7 = 4360$ miles ($g_i = g_7 = 26.6$ ft/sec^2). At Mars, a waiting orbit at $r_3 = 2500$ miles is assumed ($g_3 = 8.7$ ft/sec^2). With these simplifications, equations (25) to (32) can be combined to yield the following expression for the ratio of payload weight to initial weight:

$$\frac{m_p}{m_i} = \frac{\frac{m_1}{m_i}\left\{1 + \frac{m_1}{m_i}\frac{\alpha}{\beta}\left[\Gamma_2 + 0.0226\left(\frac{1}{t_3} - \frac{1}{t_5}\right)\right]\right\}}{\left[1 + \frac{m_1}{m_i}\frac{\alpha}{\beta}\left(\Gamma_2 + \frac{0.0226}{t_3}\right)\right]^2 + \frac{m_1}{m_i}\frac{\alpha}{\beta}\left(\Gamma_6 + \frac{0.1207}{t_7}\right)\left\{1 + \frac{m_1}{m_i}\frac{\alpha}{\beta}\left[\Gamma_2 + 0.0226\left(\frac{1}{t_3} - \frac{1}{t_5}\right)\right]\right\}} - \beta \tag{33}$$

where

$$\frac{m_1}{m_i} = 1 - 0.1207\,\frac{\alpha}{\beta t_1} \tag{34}$$

$$\Gamma_2 = 0.04\,\gamma_2, \quad \text{kw/lb} \tag{35}$$

$$\Gamma_6 = 0.04\left(\frac{r_E}{r_M}\right)^{5/2}\gamma_6 = 0.014\,\gamma_6, \quad \text{kw/lb} \tag{36}$$

Further simplification is permissible for those trips for which the terms involving t_3 and t_5 are negligible in comparison with Γ_2, as they will be for the faster trips to be considered. (This is equivalent to ignoring the mass used for phases (3) and (5).) Equations (33) and (34) then yield

$$\frac{m_p}{m_i} = \frac{\dfrac{m_1}{m_i}}{1 + \dfrac{\alpha}{\beta}\dfrac{m_1}{m_i}\left(\Gamma_2 + \Gamma_6 + \dfrac{0.1207}{t_7}\right)} - \beta$$

$$\equiv \frac{\dfrac{m_1}{m_i}}{1 + \dfrac{m_1}{m_i}\dfrac{\alpha}{\beta}\Gamma'} - \beta \qquad (37)$$

where

$$\Gamma' = \Gamma_2 + \Gamma_6 + \frac{0.1207}{t_7} \qquad (38)$$

Equation (37) can be used for initial estimates of mass ratio both for one-way trips and round trips (for the former, Γ' is simply Γ_2). This equation indicates also that an optimum value of β exists for fixed values of m_1/m_i and Γ'. Differentiation with respect to β yields

$$\beta_{opt} = \frac{m_1}{m_i}\sqrt{\alpha\Gamma'}\left(1 - \sqrt{\alpha\Gamma'}\right) \qquad (39)$$

and substitution of this value of β into equation (37) yields

$$\left(\frac{m_p}{m_i}\right)_{max} = \frac{m_1}{m_i}\left(1 - \sqrt{\alpha\Gamma'}\right)^2 \qquad (40)$$

For the missions considered herein, typical values of m_1/m_i range between 0.80 and 0.95.

Equations (39) and (40) are plotted in figure 2 for $m_1/m_i = 1.0$. The results for this case are the same as in reference 1. With equations (34) to (39), initial estimates for weight ratios as functions of trip times can be made when Γ_2 and Γ_6 have been determined from the equations for transfer between circular orbits.

TRAJECTORY ANALYSIS

The differential equations for the trajectory followed by a vehicle propelled continuously in a central gravitational field are (ref. 5)

$$\rho'' - \rho(\theta')^2 + \frac{1}{\rho^2} = \frac{a_r}{g_a} \tag{41}$$

and

$$\rho\theta'' + 2\rho'\theta' = \frac{a_\theta}{g_a} \tag{42}$$

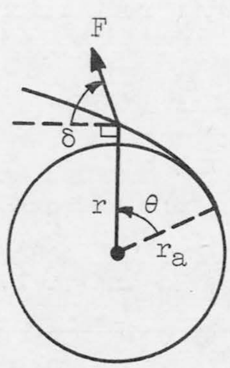
where

$$a_r = \frac{F \sin \delta}{m} \tag{43}$$

and

$$a_\theta = \frac{F \cos \delta}{m} \tag{44}$$

and where δ is the direction angle of thrust vector, measured outward from the circumferential direction (see sketch (a)). The radius



Sketch (a)

ratio ρ is r/r_a, and the derivatives are with respect to the time parameter $\tau = v_{c,a}t/r_a$.

Equations (41) and (42) have been integrated in the past for several types of trajectories. In reference 5, the thrust is constant and directed at all times parallel to the velocity vector (tangential thrust). In reference 6, the acceleration F/m is constant, again with tangential thrust. In reference 1, auxiliary equations are derived, using variational methods, whose solutions yield trajectories that minimize γ for a given one-way trip time between two circular orbits. Other integrations have been carried out for circumferential thrust ($\delta = 0^\circ$) and radial thrust

(δ = 90°). None of these approaches are satisfactory for fast round-trip analyses, because, as pointed out previously, the trajectories most likely to reduce total round-trip time with minimum propellant consumption are not direct transfers between circular orbits. Although indirect transfer trajectories can be obtained by trial-and-error patching of various constant-thrust and zero-thrust trajectories, the procedure is much too laborious for preliminary analyses.

The approach used herein is to prescribe a family of fairly simple trajectories, calculate the thrust and acceleration programing required to follow these trajectories, and determine the combinations of members of this family that yield minimum values of the two-way mass-ratio parameter $(\Gamma_2 + \Gamma_6)$.

Two families of trajectories were considered. For the first, the equations of motion were left intact, and an attempt was made to find simple equations for the path that were capable of satisfying the boundary conditions for transfer between circular orbits. A reasonable form for the trajectory is obtained if ρ is assumed to be a sine function of θ. The angle θ, in turn, was represented by a power series in τ. The resulting equations for the trajectory were

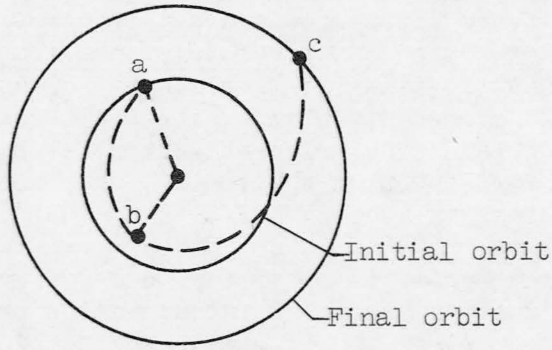
$$\theta = \tau\left[\theta_0' + \frac{\tau}{\tau_f}\left(3\,\frac{\theta_f}{\tau_f} - 2\theta_0' - \theta_f'\right) + \left(\frac{\tau}{\tau_f}\right)^2\left(\theta_0' - 2\,\frac{\theta_f}{\tau_f} + \theta_f'\right)\right] \tag{45}$$

$$\rho = 1 + \frac{\rho_f - 1}{2}\left[1 + \sin\frac{\pi}{2}\left(\frac{2\theta}{\theta_f} - 1\right)\right] \tag{46}$$

where subscript f refers to conditions at the final orbit. These equations satisfy the boundary conditions

$$\left.\begin{aligned} &\text{At } \tau = 0: && \theta = 0,\ \rho = 1,\ \rho' = 0 \\ &\text{At } \tau = \tau_f: && \theta = \theta_f,\ \rho = \rho_f,\ \rho' = 0 \end{aligned}\right\} \tag{47}$$

Substitution of these expressions for θ and ρ and their derivatives into equations (41) and (42) and subsequent integration of the square of the acceleration yield γ as a function of θ_f, τ_f, and the initial and final circumferential-velocity parameters θ_0' and θ_f'. If circular velocity is desired at both extremities, then $\theta_0' = 1$ and $\theta_f' = \rho_f^{-3/2}$. These values produce a direct transfer between two circular orbits. For such transfers, an optimum θ_f is found that minimizes γ for each transit time τ_f. For indirect transfers, combinations of paths can be used as illustrated in sketch (b). Both portions $\overline{ab}$ and $\overline{bc}$ could be



Sketch (b)

transfers between circular orbits, but lower values of γ for the combined path can be obtained by varying the circumferential-velocity parameters θ_b' at the intermediate point b. A number of one-way and round-trip trajectories were calculated for this family, but it was found that lower values of γ were obtained with the second family tried. Since the second family is also simpler to deal with in that closed expressions are obtained for the γ's, no further discussion of the first family will be presented.

For the second family of trajectories, the assumption was made that at all points the relation

$$\theta' = \rho^{-3/2} \tag{48}$$

was maintained. This assumption implies that the circumferential component of the velocity is equal to the local circular velocity at all times. For this family of trajectories, the equations of motion (eqs. (41) and (42)) assume the particularly simple form

$$\frac{a_r}{g_a} = \rho'' \tag{49}$$

$$\frac{a_\theta}{g_a} = \frac{1}{2} \rho^{-3/2}\rho' \tag{50}$$

The expression for γ then becomes

$$\gamma = \int_0^{\tau_f} \left(\frac{a}{g_a}\right)^2 d\tau = \int_0^{\tau_f} \left[(\rho'')^2 + \frac{1}{4} \rho^{-3}(\rho')^2\right] d\tau \tag{51}$$

The assumption $\theta' = \rho^{-3/2}$ defines the relation between the two independent variables θ and ρ, but still permits arbitrary variations of ρ as a function of time. It is now only necessary to find an expression for ρ as a function of τ that satisfies the following boundary conditions for transfer between two circular orbits:

$$\left.\begin{aligned} &\text{At } \tau = 0: && \rho = 1,\ \rho' = 0 \\ &\text{At } \tau = \tau_f: && \rho = \rho_f,\ \rho' = 0 \end{aligned}\right\} \tag{52}$$

The simplest expression that satisfies these conditions is

$$\rho = 1 + (\rho_f - 1)\left(\frac{\tau}{\tau_f}\right)^2\left(3 - 2\,\frac{\tau}{\tau_f}\right) \tag{53}$$

Denoting τ/τ_f by ξ, the expressions for ρ and its derivatives are

$$\rho = 1 + (\rho_f - 1)\xi^2(3 - 2\xi) \tag{54}$$

$$\rho' = \frac{6(\rho_f - 1)}{\tau_f}\,\xi(1 - \xi) \tag{55}$$

$$\rho'' = \frac{6(\rho_f - 1)}{\tau_f^2}\,(1 - 2\xi) \tag{56}$$

For indirect trips, such as those of sketch (b), combinations of two transfers between circular orbits can again be employed, but there is no choice of the magnitude of the circumferential velocity of the intermediate radius, because this velocity is circular, for these trajectories, when $\rho' = 0$. Substitution of equations (54) to (56) into (51) yields

$$\gamma = \frac{3(\rho_f - 1)^2}{\tau_f}\left[\frac{4}{\tau_f^2} + 3F(\rho_f)\right] \tag{57}$$

where

$$F(\rho_f) = \int_0^1 \frac{\xi^2(1 - \xi)^2\,d\xi}{\left[1 + (\rho_f - 1)\xi^2(3 - 2\xi)\right]^3} \equiv \int_0^1 f(\xi,\rho_f)d\xi \tag{58}$$

and where $f(\xi,\rho_f)$ is equal to the integrand of the second term.

The value of θ_f for these trajectories is, from equations (48) and (54),

$$\theta_f = \int_0^{\tau_f} \rho^{-3/2}\, d\tau = \tau_f \int_0^1 g(\xi,\rho_f)d\xi$$

$$= \tau_f G(\rho_f) \qquad (59)$$

where

$$G(\rho_f) \equiv \int_0^1 g(\xi,\rho_f)d\xi = \int_0^1 \frac{d\xi}{\left[1 + (\rho_f - 1)\xi^2(3 - 2\xi)\right]^{3/2}} \qquad (60)$$

The functions $F(\rho_f)$ and $G(\rho_f)$ are plotted in figure 3. For the entire range of ρ_f shown, an excellent approximate expression for $F(\rho_f)$ is

$$F(\rho_f) = 0.033\ \rho_f^{-3/2} \qquad (61)$$

For a limited range of ρ_f $(0.3 < \rho_f < 3.0)$, which is of interest for Mars and Venus trips, the function $G(\rho_f)$ is given to good approximation by

$$G(\rho_f) = \rho_f^{-3/4} \qquad (62)$$

Substituting these expressions into equations (57) and (59),

$$\gamma = \frac{3(\rho_f - 1)^2}{\tau_f}\left(\frac{4}{\tau_f^2} + 0.1\ \rho_f^{-3/2}\right) \qquad (63)$$

$$\left.\begin{aligned} \theta_f &= \tau_f\rho_f^{-3/4}, \text{ radians} \\ &= 57.3\ \tau_f\rho_f^{-3/4}, \text{ deg} \end{aligned}\right\} \qquad (64)$$

Equations (63) and (64) are the equations for the trajectory parameters upon which the remainder of the discussion will be based.

Direct Transfer Between Circular Orbits

Although much of the present report is devoted to round-trip interplanetary missions, it is of interest to discuss briefly the use of $\theta' = \rho^{-3/2}$ trajectories for one-way direct transfers between circular orbits. Such transfers are needed for missions such as raising or lowering satellites and for one-way planetary probes. Furthermore, combinations of such direct transfers will subsequently be used for analysis of round-trip missions. For heliocentric paths starting from the Earth's orbit, Γ is given by

$$\Gamma = 0.04\ \gamma \tag{65}$$

and the trip time parameter is

$$\tau_f = \frac{v_{c,a} t_f}{r_a} = \frac{18.5\times 0.864\times 10^5\ t_f}{92.9\times 10^6} = 0.0172\ t_f \tag{66}$$

where t_f is expressed in days.

Values of Γ calculated from equations (63) and (65) are shown in figure 4 as a function of τ_f for various radius ratios ρ_f. Shown for comparison is the curve for the optimum Earth-Mars transfer taken from reference 1. It is apparent that the $\theta' = \rho^{-3/2}$ trajectories are far from optimum for one-way interplanetary transfers. For an Earth-Mars transfer time of 175 days, for example, and for a value of $\alpha/\beta = 50$, the ratio of initial to final mass m_1/m_2 is about 1.007 for the trajectories of reference 1 and about 1.25 for the $\theta' = \rho^{-3/2}$ trajectories. Although it appears doubtful that the low mass ratios obtained for the optimum trajectory can be realized in practice, even for one-way trips, the difference is so large that there is adequate basis for assuming that round-trip mass ratios calculated with $\theta' = \rho^{-3/2}$ trajectories are quite pessimistic, and that subsequent trajectory studies will produce considerably lower values for given trip times.

For geocentric missions involving transfer to higher orbits from a near-Earth orbit (r_0 = 4360 miles), Γ is given by (see eq. (23))

$$\Gamma = 13.82\ \gamma \tag{67}$$

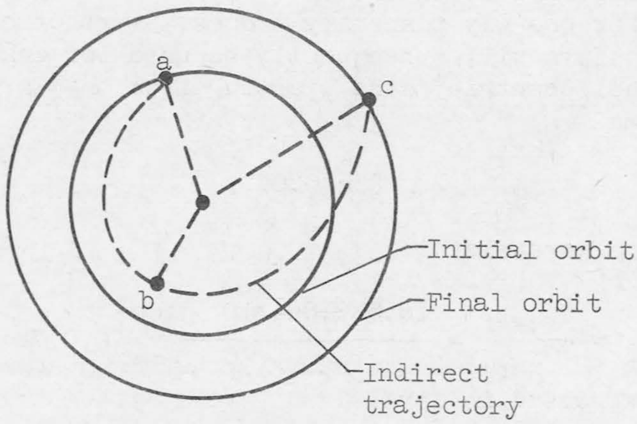
The relation between τ_f and t_f is

$$\tau_f = \frac{4.69\times 0.864\times 10^5\ t_f}{4360} = 93\ t_f \tag{68}$$

where t_f is again in days. Obviously, the range of τ_f of interest for geocentric missions differs greatly from that of figure 4.

Indirect Transfers Between Circular Orbits

To evaluate mass ratios for indirect transfers with the $\theta' = \rho^{-3/2}$ family of trajectories, two members of this family are combined, as shown in sketch (c):



Sketch (c)

Let

$$\left.\begin{aligned} \rho_a &= \frac{r_b}{r_a} \\ \rho_b &= \frac{r_c}{r_b} \\ \rho_c &= \frac{r_c}{r_a} \\ \rho_1 &= \frac{r}{r_a} \\ \rho_2 &= \frac{r}{r_b} \end{aligned}\right\} \qquad (69)$$

Then the expression for γ appropriate for the entire transfer $\overline{abc}$ is derived as follows:

$$\gamma = \int_0^{\tau_c} \left(\frac{a}{g_a}\right)^2 d\tau_1$$

$$= \int_0^{\tau_c} \left[(\rho_1'')^2 + \frac{1}{4}\rho_1^{-3}(\rho_1')^2\right] d\tau_1$$

$$= \int_0^{\tau_a} \left[(\rho_1'')^2 + \frac{1}{4}\rho_1^{-3}(\rho_1')^2\right] d\tau_1 + \rho_a^{-5/2} \int_0^{\tau_b} \left[(\rho_2'')^2 + \frac{1}{4}\rho_2^{-3}(\rho_2')^2\right] d\tau_2 \qquad (70)$$

where

$$\left.\begin{aligned}
\tau_a &= \frac{v_{c,a}}{r_a} t_{a,b} \\
\tau_b &= \frac{v_{c,b}}{r_b} t_{b,c} \\
\tau_c &= \frac{v_{c,a}}{r_a} t_{a,c} \\
\tau_1 &= \frac{v_{c,a}}{r_a} t \\
\tau_2 &= \frac{v_{c,b}}{r_b} t
\end{aligned}\right\} \qquad (71)$$

The integrals in this expression for γ are the same as those which led to equation (63). Consequently, the value of γ for an indirect trip becomes

$$\gamma = \frac{3(1 - \rho_a)^2}{\tau_a}\left[\left(\frac{2}{\tau_a}\right)^2 + 0.10\,\rho_a^{-3/2}\right] + \frac{3(1 - \rho_b)^2\rho_a^{-5/2}}{\tau_b}\left[\left(\frac{2}{\tau_b}\right)^2 + 0.10\,\rho_b^{-3/2}\right] \qquad (72)$$

Let $\kappa = t_{a,b}/t_{a,c}$ be the fraction of the total transit time alloted to the portion $\overline{ab}$ of the trajectory. Then, since $\rho_b = \rho_c/\rho_a$, equation (72) becomes

$$\gamma = \frac{3(1 - \rho_a)^2}{\kappa\tau_c}\left[\left(\frac{2}{\kappa\tau_c}\right)^2 + 0.1\,\rho_a^{-3/2}\right] + \frac{3(\rho_c - \rho_a)^2}{(1 - \kappa)\tau_c}\left\{\left[\frac{2}{(1 - \kappa)\tau_c}\right]^2 + 0.1(\rho_a\rho_c)^{-3/2}\right\}$$

$$= \frac{12(1 - \rho_a)^2}{(\kappa\tau_c)^3}\left[1 + \left(\frac{\kappa}{1 - \kappa}\right)^3 A^2\right] + \frac{0.30(1 - \rho_a)^2}{\kappa\tau_c\rho_a^{3/2}}\left(1 + \frac{\kappa}{1 - \kappa}\,\frac{A^2}{\rho_c^{3/2}}\right) \quad (73)$$

where

$$A = \left|\frac{\rho_c - \rho_a}{1 - \rho_a}\right|$$

For a given total one-way trip time $t_{a,c}$ and a given destination radius ratio ρ_c, equation (73) expressed γ in terms of the intermediate radius ratio ρ_a and the proportion of the transit time allowed for the first portion of the trip κ. For each ρ_a, ρ_c, and t_c, an optimum value of κ exists that minimizes γ for the trip. Differentiation of equation (73) with respect to κ yields the following equation for this optimum κ (denoted by κ_0):

$$\frac{36}{\tau_c^2}(1 - \kappa_0)^{-2}\left[A^2 - \left(\frac{1 - \kappa_0}{\kappa_0}\right)^4\right] + \frac{0.30}{\rho_a^{3/2}}\left[\frac{A^2}{\rho_c^{3/2}} - \left(\frac{1 - \kappa_0}{\kappa_0}\right)^2\right] = 0 \quad (74)$$

The solutions of this equation for $\tau_c = 0$ and $\tau_c = \infty$ are

$$\text{For } \tau_c = 0: \qquad \frac{1 - \kappa_0}{\kappa_0} = A^{1/2} \quad (75a)$$

$$\text{For } \tau_c = \infty: \qquad \frac{1 - \kappa_0}{\kappa_0} = \frac{A}{\rho_c^{3/4}} \quad (75b)$$

These solutions for optimum κ_0 are plotted in figure 5. Also shown are values obtained for $\tau_c = \sqrt{120}$. These calculations show that, for values of τ_c of interest for interplanetary missions, there is no appreciable change in κ_0 from the $\tau_c = 0$ value. It is, therefore, a good approximation to use the value of κ_0 for $\tau_c = 0$ in the expression for γ (eq. (73)). This expression then becomes

$$\gamma = \tau_c^{-3} f_1(\rho_c,\rho_a) + \tau_c^{-1} f_2(\rho_c,\rho_a) \tag{76}$$

where

$$f_1(\rho_c,\rho_a) = 12(1 - \rho_a)^2(1 + \sqrt{A})^4 \tag{77a}$$

$$f_2(\rho_c,\rho_a) = 0.30(1 - \rho_a)^2\rho_a^{-3/2}(1 + \sqrt{A})\left[1 + (A/\rho_c)^{3/2}\right] \tag{77b}$$

The limiting forms of the functions for $\rho_a = 1$ and $\rho_a = \rho_c$ (direct transfers) are

$$f_1(\rho_c,1) = f_1(\rho_c,\rho_c) = 12(1 - \rho_c)^2 \tag{78a}$$

$$f_2(\rho_c,1) = f_2(\rho_c,\rho_c) = 0.30\ \rho_c^{-3/2}(1 - \rho_c)^2 \tag{78b}$$

The resulting expressions for γ are, of course, identical with the direct-transfer values of equation (63).

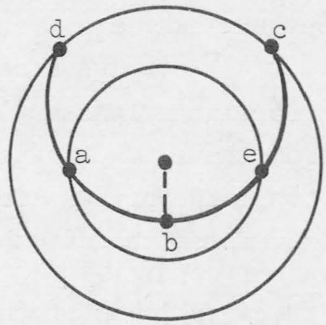
Considerable simplification in making round-trip calculations results from noting that

$$A(\rho_c^{-1},\rho_c^{-1}\rho_a) = \left[A(\rho_c,\rho_a)\right]^{-1} \tag{79}$$

$$f_1(\rho_c^{-1},\rho_c^{-1}\rho_a) = \rho_c^{-2} f_1(\rho_c,\rho_a) \tag{80}$$

$$f_2(\rho_c^{-1},\rho_c^{-1}\rho_a) = \rho_c f_2(\rho_c,\rho_a) \tag{81}$$

The transformations $\rho_c \to \rho_c^{-1}$, $\rho_a \to \rho_c^{-1}\rho_a$ correspond to inward and outward trips between the same circular orbits along trajectories with the same intermediate radius r_b (sketch (d)). If the value of γ for the



Sketch (d)

outward trip (abc) is denoted by γ_2, and for the inward trip (dbe) by γ_6, equations (79) to (81), together with equation (76), yield the result that

$$\gamma_6\left(\frac{r_a}{r_d}, \frac{r_b}{r_d}\right) = \left(\frac{r_c}{r_a}\right)^{5/2} \gamma_2\left(\frac{r_c}{r_a}, \frac{r_b}{r_a}\right) \tag{82}$$

where $r_d = r_c$. This equation, in turn, when used in equation (36), yields the required result that

$$\Gamma_6\left(\frac{r_a}{r_c}, \frac{r_b}{r_c}\right) = \Gamma_2\left(\frac{r_c}{r_a}, \frac{r_b}{r_a}\right) \tag{83}$$

Values of f_1 and f_2 are shown in figure 6 for $\rho_c = 1.524$ (Earth-Mars trip) and $\rho_c = 0.656$ (Mars-Earth trip). With these values, mass ratios required for various combinations of round-trip Mars trajectories can be calculated.

Another quantity needed for these round-trip calculations is the angular distance traveled. From equations (59) and (62),

$$\begin{aligned}\theta &= \tau_a\rho_a^{-3/4} + \tau_b\rho_b^{-3/4} \\ &= \frac{v_{c,a}\kappa t_{a,c}}{r_a}\rho_a^{-3/4} + \frac{v_{c,b}}{r_b}(1 - \kappa)t_{a,c}\left(\frac{\rho_c}{\rho_a}\right)^{-3/4} \\ &= \frac{\tau_c\rho_a^{-3/4}}{1 + \sqrt{A}}\left(1 + \sqrt{A}\,\rho_c^{-3/4}\right), \text{ radians}\end{aligned} \tag{84}$$

or

$$\frac{\theta}{\tau_c}(\rho_c,\rho_a) = \frac{1 + \rho_c^{-3/4}\sqrt{A}}{\rho_a^{3/4}(1 + \sqrt{A})} \tag{85}$$

where, as before, the optimum value of κ for $\tau_c = 0$ has been used.

The relation between values of θ/τ for inward and outward transfer between the same circular orbits with the same intermediate radius is

$$\frac{\theta}{\tau}(\rho_c^{-1},\rho_c^{-1}\rho_a) = \rho_c^{3/2}\frac{\theta}{\tau}(\rho_c,\rho_a)$$

so that

$$\theta(\rho_c^{-1},\rho_c^{-1}\rho_a) = \theta(\rho_c,\rho_a)$$

Even more useful for interplanetary mission studies is the lead angle φ acquired during the transit by the vehicle relative to the initial planet (Earth). Any lead angle acquired during the outward trip (and during the time spent in the vicinity of the destination planet) must be reduced to zero or augmented to 2π if the rendezvous with the initial planet is to be achieved. The lead angle acquired during the departure phase (phase (2)) is

$$\begin{aligned}\varphi_2 &= \theta_2 - \frac{v_{c,E}}{r_E}t_2 \\ &= \frac{v_{c,E}}{r_E}t_2\left[\frac{\theta}{\tau}\left(\frac{r_P}{r_E}, \frac{r_{b,2}}{r_E}\right) - 1\right]\end{aligned} \tag{86}$$

where $v_{c,E}$ and r_E are the orbital speed and radius of the Earth, r_P is the orbital radius of the destination planet, and $r_{b,2}$ is the intermediate radius on the outward trip. The lead angle for the return trip (phase (6)) is

$$\begin{aligned}\varphi_6 &= \theta_6 - \frac{v_{c,E}}{r_E}t_6 \\ &= \frac{v_{c,E}}{r_E}t_6\left[\left(\frac{r_P}{r_E}\right)^{-3/2}\frac{\theta}{\tau}\left(\frac{r_E}{r_P}, \frac{r_{b,6}}{r_P}\right) - 1\right] \\ &= \frac{v_{c,E}}{r_E}t_6\left[\frac{\theta}{\tau}\left(\frac{r_P}{r_E}, \frac{r_{b,6}}{r_E}\right) - 1\right]\end{aligned} \tag{87}$$

where $r_{b,6}$ is the intermediate radius for the return trip. The lead angle acquired during the time spent in the vicinity of the destination planet (phases (3), (4), and (5)) is

$$\varphi_w = (\dot{\theta}_P - \dot{\theta}_E)t_w$$

$$= \frac{v_{c,E}}{r_E} t_w \left[\left(\frac{r_E}{r_P}\right)^{3/2} - 1\right] \tag{88}$$

where $t_w = t_3 + t_4 + t_5$.

For the Earth-Mars trip ($v_{c,E}/r_E$ = 0.0172 radian/day = 0.986°/day), these lead angles become

$$\frac{\varphi_2}{t_2} = 0.986\left[\frac{\theta}{\tau}\left(1.524, \frac{r_{b,2}}{r_E}\right) - 1\right], \text{ deg/day} \tag{89}$$

$$\frac{\varphi_6}{t_6} = 0.986\left[\frac{\theta}{\tau}\left(1.524, \frac{r_{b,6}}{r_E}\right) - 1\right], \text{ deg/day} \tag{90}$$

$$\frac{\varphi_w}{t_w} = -0.462, \text{ deg/day} \tag{91}$$

The function $\varphi/t = 0.986\left[\frac{\theta}{\tau}\left(1.524, \frac{r_b}{r_E}\right) - 1\right]$ is plotted in figure 7. With this curve, the radius ratio required for the return trip, for a given outward trip and waiting time, can be determined with the relation

$$\varphi_6 + \varphi_2 + \varphi_w = 2n\pi \qquad (n = 0,1,2,3) \tag{92}$$

For fast trips, the case $n = 0$ is of most interest.

Weight Ratios for Mars Round Trips

To evaluate the weights needed as functions of trip time for interplanetary round-trip missions, either with the step-by-step method (eqs. (25) to (32)) or with approximate methods, the values of Γ_2 and Γ_6 must be determined for combinations of trajectories that satisfy equation (92). For Earth-Mars missions, the expressions for Γ_2 and Γ_6 are

$$\Gamma_n = 0.04 \left(\frac{58.2}{t_n}\right)\left[\left(\frac{58.2}{t_n}\right)^2 f_1\left(1.524, \frac{r_{b,n}}{r_E}\right) + f_2\left(1.524, \frac{r_{b,n}}{r_E}\right)\right] \tag{93}$$

where $n = 2$ for the outward trajectory and $n = 6$ for the inward trajectory. This function of transit time and r_b/r_E is plotted in figure 8. Since Γ has a sharp minimum at $r_b/r_E = 1.0$, it was thought likely that the lowest values of $\Gamma_2 + \Gamma_6$ for a given total trip time would be obtained when either the outward or return trip is a direct one ($r_b/r_E = 1.0$). Subsequent calculations confirmed this conjecture, and further calculations were therefore made for $r_{b,2}/r_E = 1.0$. The first step was to calculate $\Gamma_2 + \Gamma_6$ as a function of t_2 for several fixed values of t_w and t', where t' is the total round-trip time with the exceptions of the Earth escape and descent phases; that is,

$$t' = t_2 + t_w + t_6 \tag{94}$$

Equations (94) and (92) and figure 7 determined φ_6, t_6, and $r_{b,6}/r_E$ as a function of t_2; Γ_2 and Γ_6 were then obtained from figure 8. This calculation yielded the optimum distribution between t_2 and t_6 for given t' and t_w, together with the values of Γ_2, Γ_6, and $r_{b,6}/r_E$ corresponding to these optimum trips. The results are shown in figure 9 for $t_w = 50$ days. The curves show that the outward (direct) trip time t_2 is about one-quarter of t', and that the optimum intermediate radius for the return trip $r_{b,6}$ is about 57×10^6 miles and is almost independent of t'.

Using the values from figure 9, a calculation was next undertaken using equations (34) and (37) to determine the best distribution between t_1 and t' for a given total trip time. Values for β of 0.2 and α of 10 were used for this part of the calculation. The ratio m_p/m_i was found to be rather insensitive to small redistributions of total trip time between t_1, t', and t_7. The resulting breakdown of trip times is shown in table I.

Using these values, the ratio of initial mass to payload mass was calculated from equation (33), which is the more precise equation for the case $m_{ex} = 0$, $\dot{m}_s = 0$. Results are shown in figure 10 for $\alpha = 5$ and 10 pounds per kilowatt. Also shown for comparison are mass ratios for the same mission using nuclear rockets with specific impulses of 800 and 1000 seconds. The computation procedure and trajectories used for the nuclear rocket are described in appendix B. The two branches of the curves for the nuclear rockets result from use of a different type of trajectory for low and high trip times.

Figure 10 shows that, contrary to general opinion, electric rockets are potentially as capable as nuclear rockets of making fast interplanetary round trips. In fact, for mass ratios less than 10, it appears that electric rockets can accomplish the mission considerably faster than nuclear rockets. These comparisons, of course, depend on practical achievement of the assumed performance parameters for both systems, but the ones chosen are generally regarded as being attainable. Another interesting result shown in figure 10 is that the mass ratio increases very slowly as trip time is reduced quite drastically below the minimum-energy values.

Thrust and Specific-Impulse Program

It is of interest to calculate, for some typical missions, the thrust and specific impulse required to follow the assumed trajectories. For constant-power trajectories, such as those considered herein, thrust and specific impulse (or jet velocity) are related by equation (5), which can be expressed as

$$\frac{F}{m_i g_{00}} I = 45.9 \frac{P_j}{m_i g_{00}} = 45.9 \frac{\beta}{\alpha} \tag{95}$$

If $F/m_i g_{00}$ is denoted by a_{00}, equation (95) becomes

$$I = \frac{45.9}{a_{00}} \frac{\beta}{\alpha} \tag{96}$$

For constant-thrust phases (phases (1), (3), (5), and (7)),

$$a_{00} = a_0 \frac{m_0 g_0}{m_i g_{00}} = \frac{a_0 \tau}{\tau} \frac{m_0 g_0}{m_i g_{00}} = \frac{0.9}{\frac{v_{c,a} t}{r_a}} \frac{m_0 g_0}{m_i g_{00}} \tag{97}$$

where $m_0 g_0 = m_a g_a$ for phases (1) and (5), and $m_0 g_0 = m_b g_b$ for phases (3) and (7). Thus, for phase (1) (Earth escape), with $g_a/g_{00} = 26.6/32.2 = 0.826$, and $m_a = m_i$,

$$a_{00,1} = \frac{0.008}{t_1} \tag{98}$$

and

$$I_1 = 5740 \frac{\beta t_1}{\alpha} \tag{99}$$

For phase (3) (Mars descent), $g_b/g_{00} = 8.7/32.2 = 0.270$,

$$a_{00,3} = \frac{0.00335}{t_3}\frac{m_3}{m_i} \tag{100}$$

and

$$I_3 = 13,700\,\frac{\beta t_3}{\alpha}\frac{m_i}{m_3} \tag{101}$$

For phase (5) (Mars escape),

$$a_{00,5} = \frac{0.00335}{t_5}\frac{m_4}{m_i} \tag{102}$$

and

$$I_5 = 13,700\,\frac{\beta t_5}{\alpha}\frac{m_i}{m_4} \tag{103}$$

For phase (7) (Earth descent),

$$a_{00,7} = \frac{0.008}{t_7}\frac{m_7}{m_i} \tag{104}$$

and

$$I_7 = 5740\,\frac{\beta t_7}{\alpha}\frac{m_i}{m_7} \tag{105}$$

For variable-thrust phases (phases (2) and (6)), the accelerations are obtained from equations (49), (50), (54), (55), and (56), and the thrust is calculated from

$$a_{00} \equiv \frac{F}{m_i g_{00}} = \frac{m g_a}{m_i g_{00}}\sqrt{\left(\frac{a_r}{g_a}\right)^2 + \left(\frac{a_\theta}{g_a}\right)^2} \tag{106}$$

Thus, for phase (2) (direct Earth-Mars trajectory),

$$\frac{a_{r,2}}{g_a} = \frac{10,650}{t_2^2}(1 - 2\xi) \tag{107}$$

and

$$\frac{a_{\theta,2}}{g_a} = \frac{91.4}{t_2} \sqrt{f(1.524,\xi)} \tag{108}$$

where $f(1.524,\xi)$ is the function under the integral sign in equation (58).

For phase (6) (Mars-Earth indirect return), the accelerations for the first portion are

$$\frac{a_{r,6}}{g_a} = \frac{-42,300}{(0.42t_6)^2} (1 - 2\xi) \tag{109}$$

and

$$\frac{a_{\theta,6}}{g_a} = -\left(\frac{193}{0.42t_6}\right)\sqrt{f(\rho_{a,6},\xi)} \qquad \left(\rho_{a,6} = \frac{r_b}{r_M}\right) \tag{110}$$

and for the second portion,

$$\frac{a_{r,6}}{g_a} = \frac{2980}{(0.58t_6)^2} (1 - 2\xi) \tag{111}$$

and

$$\frac{a_{\theta,6}}{g_a} = \frac{51.2}{0.58t_6}\sqrt{f\left(\frac{r_E}{r_{b,6}},\xi\right)} \tag{112}$$

The thrust and specific-impulse programs are shown in figure 11(a) for a 500-day trip and in figure 11(b) for an 800-day trip. Values of α and β used were: $\alpha = 10$ pounds per kilowatt and $\beta = 0.20$. These figures show that the specific impulse ranges from 2,500 to 37,000 for the 500-day trip, and from 4,000 to 38,000 for the 800-day trip. The mass-flow rates required for a given power vary as I^{-2}, so that mass-flow variation by a factor of 100 is needed if I varies by a factor of 10. Although these large variations are not impossible with ion accelerators, a reduction in the range of variation will certainly be desirable. It is possible that trajectories can be found that do not require such large thrust variations and that will not significantly increase the mass ratio. Considerable investigation to find such trajectories is certainly warranted.

The time variations of several trajectory variables for the Earth-Mars transfer portion (phase (2)) of the trips shown in figure 11 are plotted in figure 12. Of particular interest are the curves for the direction of the thrust vector δ, which show that the thrust is radial at both ends of the trajectory and is inclined at large angles to the velocity vector throughout most of the transfer. The fact that the thrust is radial at both ends of the trajectory for all $\theta' = \rho^{-3/2}$ transfers can also be seen from equations (49), (50), and (56), which show that the radial component of the thrust vector decreases linearly with ξ from its maximum positive value at $\xi = 0$ to its maximum negative value at $\xi = 1$, while the circumferential component is zero at both ends ($\rho' = 0$) and reaches its maximum value near the middle of the transfer path.

MANNED MARS MISSIONS

Using the near-optimum parameters of table I, calculations were made of the weights required for a manned Mars mission similar to that discussed in references 5 and 7; that is, an 8-man expedition capable of landing on Mars. The constants of the journey are the same as used for the unmanned trips of the previous section, and the additional constants are $\dot{m}_s g_{00}$ = 80 pounds per day (based on a consumption rate of 10 lb per man per day) and exploration equipment $m_{ex} g_{00}$ equal to 40,000 pounds.

Equations (25) to (32) for this mission become, with $\beta = 0.2$:

Phase (1):

$$\frac{W_1}{W_i} = 1 - \frac{0.604\alpha}{t_1} - \frac{80t_1}{W_i} \tag{113}$$

Phase (2):

$$\frac{W_1}{W_2} - 1 - \frac{4\left(\frac{80t_2}{W_1}\right)}{\left(1 + \frac{W_2}{W_1}\right)^2} = 5\alpha\Gamma_2 \frac{W_1}{W_i} \qquad \text{(solution plotted in fig. 1)} \tag{114}$$

Phase (3):

$$\frac{W_3}{W_2} = \frac{1 - \frac{80t_3}{W_2}}{1 + \frac{0.113\alpha}{t_3}\frac{W_2}{W_i}} \tag{115}$$

Phase (4):

$$\frac{W_4}{W_3} = 1 - \frac{80t_4}{W_3} - \frac{40,000}{W_3} \tag{116}$$

Phase (5):

$$\frac{W_5}{W_4} = 1 - \frac{0.113\alpha}{t_5}\frac{W_4}{W_i} - \frac{80t_5}{W_4} \tag{117}$$

Phase (6):

$$\frac{W_5}{W_6} - 1 - \frac{4\left(\frac{80t_6}{W_5}\right)}{\left(1 + \frac{W_6}{W_5}\right)^2} = 5\alpha\Gamma_6 \frac{W_5}{W_i} \qquad \text{(solution plotted in fig. 1)} \tag{118}$$

Phase (7):

$$\frac{W_7}{W_6} = \frac{1 - \frac{80t_7}{W_6}}{1 + \frac{0.604\alpha}{t_7}\frac{W_6}{W_i}} \tag{119}$$

The return payload ratio is, as before,

$$\frac{W_p}{W_i} = \frac{W_7}{W_i} - 0.20 \tag{120}$$

Using equations (113) to (120) and the parameters of table I, the return payload weight W_p was calculated as a function of initial weight for several trip times and for values of specific powerplant weight of 10 and 5 pounds per kilowatt. Results are shown in figure 13. For a return payload of 50,000 pounds, the initial weights are shown as functions of trip time in figure 14. Also shown in figure 14, for comparison, are initial weights required for the same mission with a nuclear rocket having a specific impulse of 1000 seconds and a powerplant weight of 20,000 pounds. Appendix B describes the calculation procedure for the nuclear rocket.

Among the features of interest in figure 14 is the result that the initial weight increases very slowly, with electric propulsion, as the trip time is reduced from the minimum-energy value of about 1200 days (refs. 5 and 7) to about 700 days for $\alpha = 10$ or about 550 days for $\alpha = 5$. The rate of increase is even slower than that for the unmanned mission (fig. 10), because reduction in supply weight needed tends to compensate for the increased propellant weight as trip time is reduced. The relative position of the curves for electric and nuclear rockets remains about as for the unmanned-mission calculation.

CONCLUDING REMARKS

The results of this study show that, even with a simple, nonoptimum family of trajectories, it is possible to produce drastic reductions in the time required for round-trip interplanetary missions using continuous, low-thrust propulsion. It is to be hoped that further trajectory optimization studies will produce even greater trip-time reductions for given initial mass. It is also desirable, although perhaps not essential, to find trajectories that require less variation in thrust and specific impulse than those used herein.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, April 22, 1960

APPENDIX A

SYMBOLS

A	$\left\lvert\dfrac{\rho_c - \rho_a}{1 - \rho_a}\right\rvert$
a	thrust acceleration
a_r	radial thrust acceleration
a_θ	circumferential thrust acceleration
a_0	thrust acceleration in reference circular orbit for constant-thrust phases
C	$4.4\times10^{-5}\ r_0(a_0\tau)^2(m_a g_0/W_i)(\alpha/\beta t_f)$
F	thrust, lb
g	gravitational acceleration
g_0	gravitational acceleration in reference circular orbit
g_{00}	gravitational acceleration at Earth surface (32.2 ft/sec^2)
I	specific impulse, sec
k	mean-value parameter
m	mass, slugs
m_{ex}	mass, other than provisions, used and left behind (phase (4))
P_j	jet power, kw
r	distance from center of gravitational body
r_b	intermediate radius for indirect trajectory
r_E	Earth orbital radius, 92.9×10^6 miles
r_M	Mars orbital radius, 141.5×10^6 miles
t	time, days

t_w — time for phases (3), (4), and (5), $t_3 + t_4 + t_5$

t' — total round-trip time except phases (1) and (7), $t_2 + t_w + t_6$

V_j — jet velocity parameter, $v_j/v_{c,0}$

v_c — circular velocity

$v_{c,0}$ — circular velocity in reference orbit (departure orbit for escape phases and destination orbit for descent phases)

v_j — jet velocity

W — weight, mg_{00}

W_i — $m_i g_{00}$

α — specific powerplant weight, W_{pp}/P_j, lb/kw

β — ratio of powerplant weight to initial vehicle weight, W_{pp}/W_i

Γ — $(v_{c,a}g_a/0.28g_{00})\ \gamma$, kw/lb

γ — integrated acceleration parameter for variable-thrust phases

δ — angle between thrust vector and circumferential direction

θ — trajectory angle (anomaly)

ψ — angle between velocity vector and circumferential direction

κ — fraction of time alloted to first portion of indirect transfer

ξ — τ/τ_1

ρ — radius ratio

τ — time parameter, $v_{c,0}t/r_0$

τ_c — see eq. (71)

φ — lead angle

Subscripts:

a — values at start of phase

b — values at end of phase

E	Earth
f	final values at end of propulsion period
i	initial
p	payload
pp	powerplant
pr	propellant
s	subsistence supplies
w	wait time (phases (3), (4), and (5))
0	reference orbit
1-7	phases of interplanetary mission

Superscript:

'	indicates differentiation with respect to τ

APPENDIX B

MISSION WEIGHT CALCULATIONS FOR NUCLEAR ROCKET

For a high-thrust interplanetary mission, starting and ending in an orbit around the Earth, there are four propulsion phases ((1), (3), (5), and (7)), each characterized by velocity increment Δv required to follow the chosen trajectories. If only the propellant mass reduction resulting from each Δv is considered, the mass ratio for each firing is

$$\frac{m'_{b,n}}{m_{a,n}} = e^{-\Delta v_n/v_j} \qquad (n = 1,3,5,7) \tag{B1}$$

For the nuclear rocket, it is assumed that a single nuclear reactor and thrust chamber will be used throughout the mission, so that no motor staging will take place. However, after each firing, some propellant tankage can be disposed of. If it is assumed that the tankage weight is proportional to the propellant used, the weight after each propulsion period can be represented as

$$m_{b,n} = m'_{b,n} - \epsilon_n(m_{a,n} - m'_{b,n}) \tag{B2}$$

or

$$\frac{m_{b,n}}{m_{a,n}} = (1 + \epsilon_n)e^{-\Delta v_n/v_j} - \epsilon_n \qquad (n = 1,3,5,7) \tag{B3}$$

where ϵ_n is the ratio of disposed propellant tankage to propellant used. A value of ϵ_n of 0.05 was used in the present computations. If the mass ratio of equation (B3) is denoted by m'_n, the initial weight for a complet manned interplanetary mission can be written as follows:

$$W_i = \frac{W_p + W_{pp} + m'_7\dot{W}_s t_6 + m'_5 m'_7(\dot{W}_s t_4 + W_{ex}) + m'_3 m'_5 m'_7 \dot{W}_s t_2}{m'_1 m'_3 m'_5 m'_7} \tag{B4}$$

where $\dot{W}_s$ = 80 pounds per day for the 8-man mission considered, and W_{ex} = 40,000 pounds. For the unmanned mission ($\dot{W}_s = W_{ex} = 0$), equation (B4) reduces to

$$\frac{W_i}{W_p} = \frac{1 + \frac{W_{pp}}{W_p}}{m'_1 m'_3 m'_5 m'_7} \tag{B5}$$

For the nuclear-rocket curves of figure 10, equation (B5) was used, with W_{pp}/W_p assumed negligible; while for the curves of figure 14, equation (B4) was used with W_{pp} assumed to be 20,000 pounds.

The values of Δv and trip times were obtained from reference 4, which contains results of systematic calculation of Earth-Mars round-trip trajectories. For each range of total trip time, the trajectories yielding minimum total Δv were used. For trip times between the minimum-energy value of 970 days and 650 days, the best routes found in reference 4 were of the direct type for the outward trip and an aphelion route (passing outside the Mars orbit) for the return trip. For the trip time range below 600 days, lowest Δv's were found when outward and return trip were both along perihelion routes (passing inside the Earth's orbit). Some interpolation was necessary in the times and Δv's of reference 4, since, for some of the trajectories of interest, values were given only for 100 days' and zero days' waiting time (t_4) in the vicinity of Mars. To obtain the weight ratio comparable with those for the low-thrust missions, a value of $t_w(= t_4)$ of 25 days was assumed unless larger waiting times produced smaller Δv's. The resulting values for the Δv's and times are given in table II. The times required for the propulsion phases were assumed to be negligible. A slight correction was made in the values of Δv_3 and Δv_5 obtained from reference 4, to allow for the difference between the 2500-mile orbit assumed herein and the orbit at 1.1 r_M used in reference 4. This correction amounted to changes of the order of 0.06 mile per second or less from the values of Δv_3 and Δv_5 obtained from reference 4.

Table II shows that, for trip times greater than 810 days, the waiting time at Mars is greater than 25 days for the optimum trajectories. No calculations for $t_4 > 25$ days were made for the low-thrust propulsion system; however, the results of references 5 and 7 for minimum-energy paths (t_4 = 450 days, $t_{tot} \approx$ 1200 days) yielded initial weights comparable to those obtained herein for t_4 = 25 days, t_{tot} = 800 days. Consequently, it is expected that trajectories can be found, in the range of t_{tot} from 800 to 1200, that yield longer waiting times than 25 days with little, if any, increase in initial weight.

The round-trip missions analysed herein are not necessarily the most economical for either nuclear rockets or electric rockets. Other missions possible with nuclear rockets might include nuclear boost from the Earth as part of the escape phase (phase (1)), whereby some improvement in overall mass ratio might be realized relative to electric rockets. Such a mission should be compared with one in which electric rockets are launched into orbit with nuclear rockets to determine whether there is sufficient advantage in switching to electric rockets for the interplanetary part of the mission. Use of atmospheric braking, both at the destination planet and upon return to Earth, would reduce the weight ratio needed for both nuclear and electric rockets for a given total trip time. The reduction might be greater, percentagewise, for nuclear rockets than for electric rockets, because reductions in the effective Δv tend to be more significant with systems having lower specific impulse. Such analyses, as well as discussion of the relative optimism or pessimism in the assumed performance parameters of the two systems, are beyond the scope of this report.

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TABLE I. - NEAR-OPTIMUM PARAMETERS FOR EARTH-MARS ROUND TRIPS

FOR t_w = 50 DAYS USING $\theta' = \rho^{-3/2}$ TRAJECTORIES

Total trip time, days	t_1	t'	t_2	t_3	t_4	t_5	t_6	t_7	Γ_2, kw/lb	Γ_6, kw/lb	$\frac{r_{b,6}}{r_E}$
400	15	370	98	15	25	10	222	15	0.0315	0.063	0.61
450	20	410	110				250	20	.023	.044	.62
500	25	455	122				283	20	.0165	.031	.62
550	30	495	130				315	25	.0132	.0235	.625
600	35	540	148				342	25	.0095	.0185	.63
700	50	620	160				410	30	.0060	.0124	.63
800	70	700	190				460	30	.0043	.0090	.63

TABLE II. - TRAJECTORY PARAMETERS USED FOR NUCLEAR-ROCKET CALCULATIONS

Total trip time, days	t_2	t_w	t_6	Δv_1	Δv_3	Δv_5	Δv_7	Δv_{tot}, miles/sec	Route
970	260	450	260	2.20	1.36	1.36	2.20	7.12	Minimum-energy
900	260	225	415	2.20	1.36	2.62	2.32	8.50	Direct-aphelion
850	260	110	480	2.20	1.36	3.22	2.44	9.22	Direct-aphelion
810	260	25	525	2.20	1.36	3.64	2.53	9.73	Direct-aphelion
650	160	25	465	4.48	2.05	4.84	4.58	15.95	Direct-aphelion
550	263	25	262	4.60	2.19	2.68	5.00	14.47	Perihelion-perihelion
500	226	25	249	4.70	2.78	2.68	4.30	14.46	Perihelion-perihelion
400	155	25	220	4.70	3.56	3.17	4.00	15.43	Perihelion-perihelion
350	123	25	202	4.60	4.36	3.76	4.90	17.62	Perihelion-perihelion

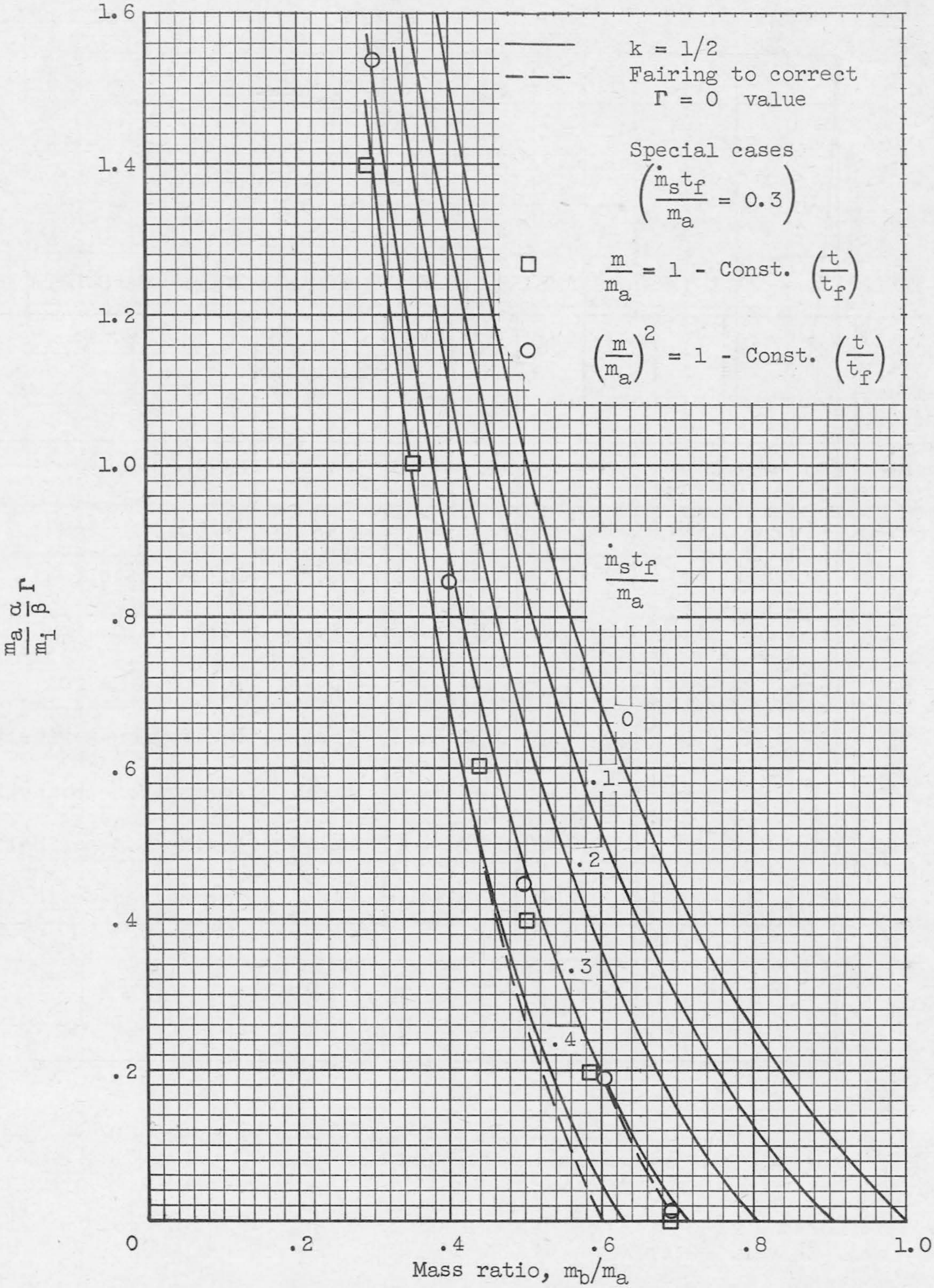


Figure 1. - Relation between mass ratio and Γ for variable-thrust propulsion periods involving mass ejection.

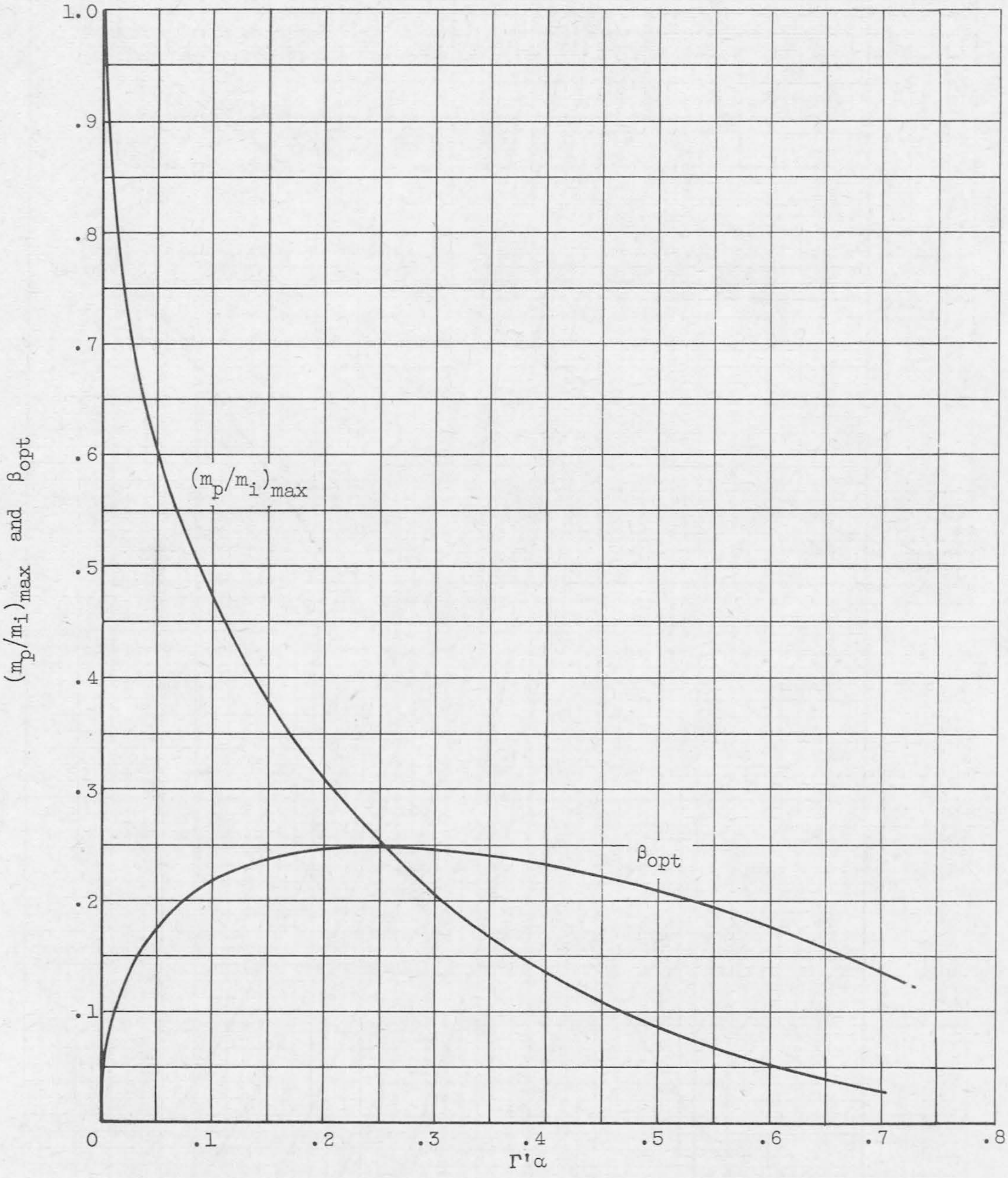


Figure 2. - Optimum powerplant mass ratio and maximum payload ratio as functions of $\Gamma'\alpha$ for $m_l/m_i = 1.0$.

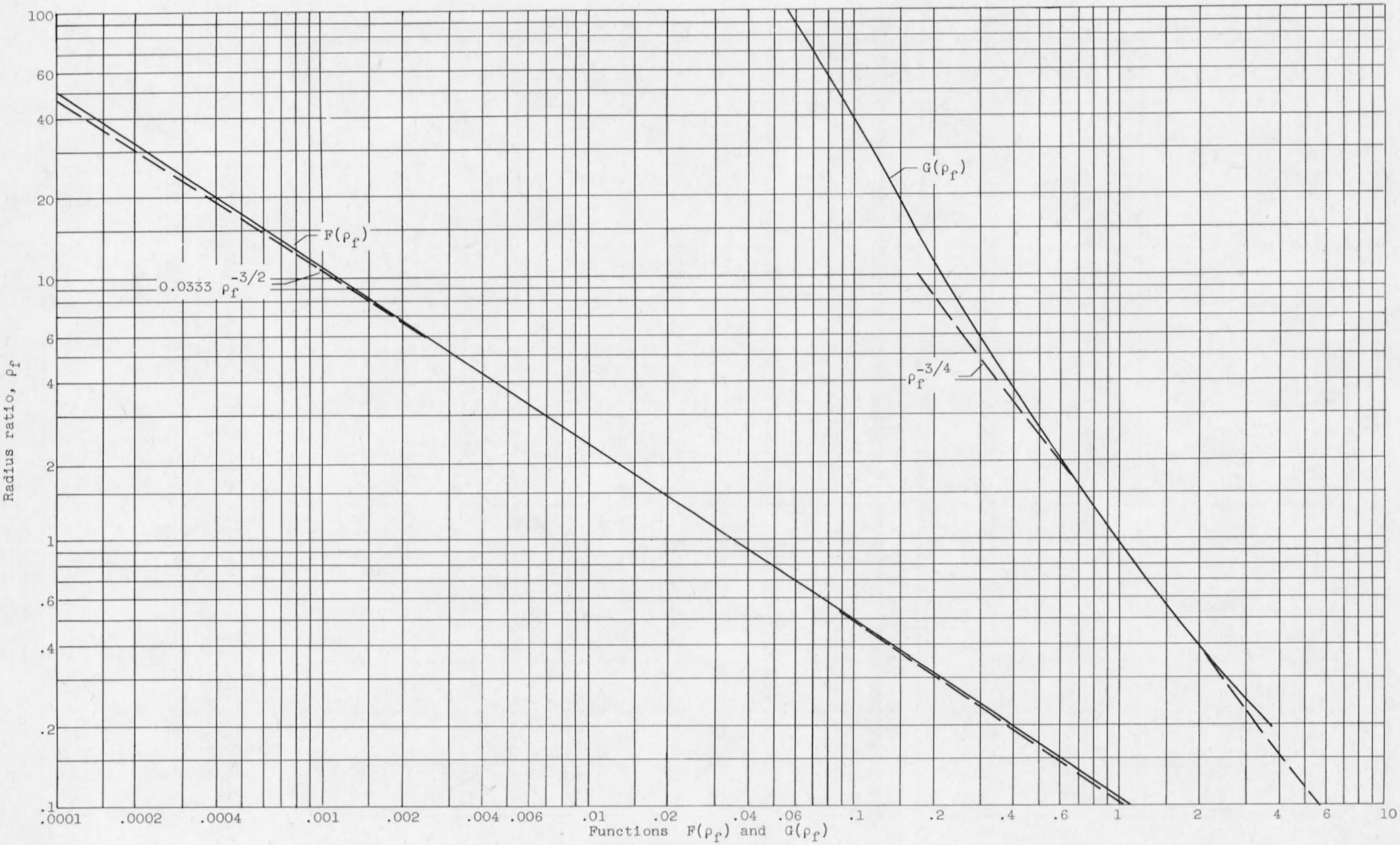


Figure 3. - Exact and approximate values of functions $F(\rho_f)$ and $G(\rho_f)$.

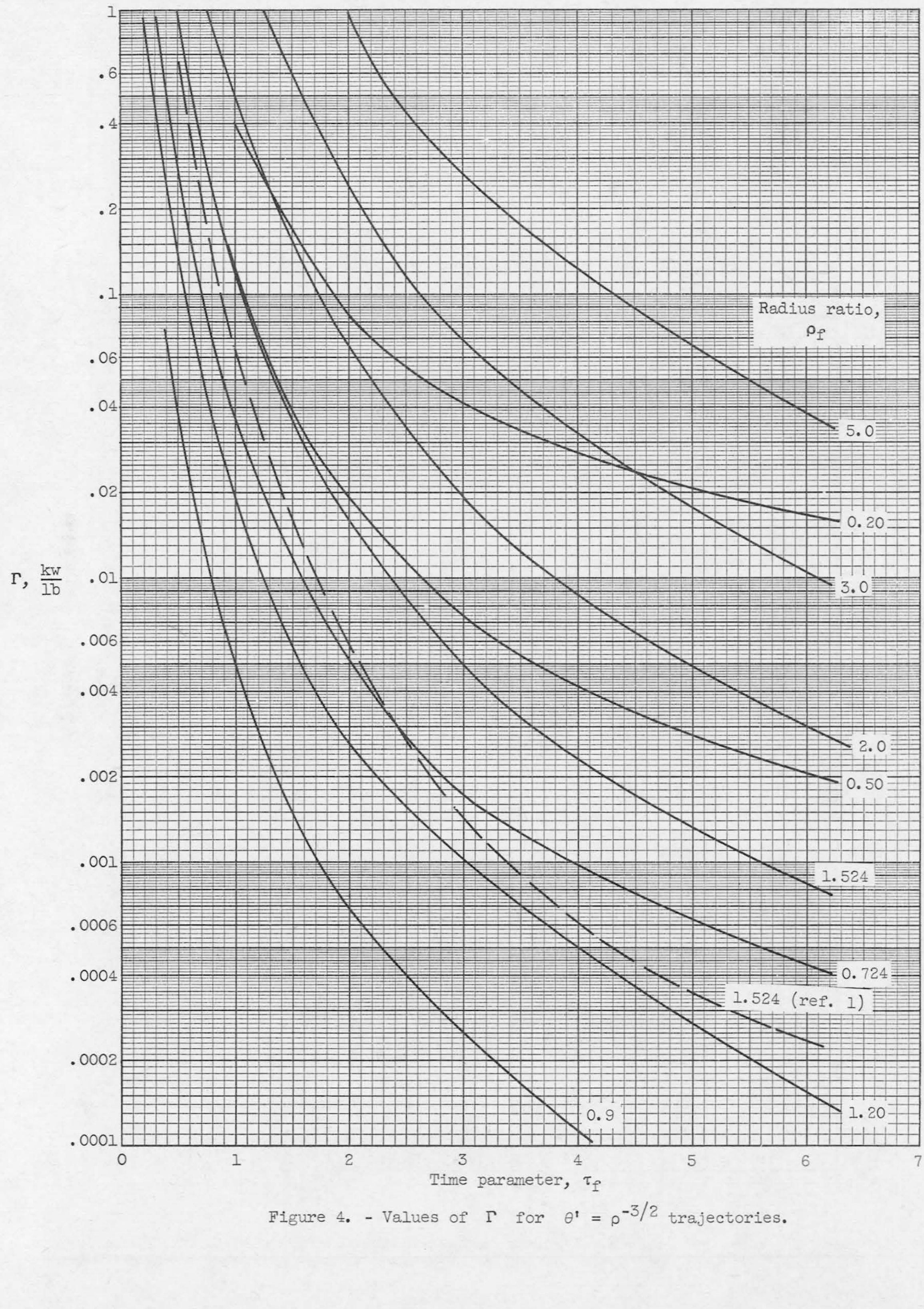


Figure 4. - Values of Γ for $\theta' = \rho^{-3/2}$ trajectories.

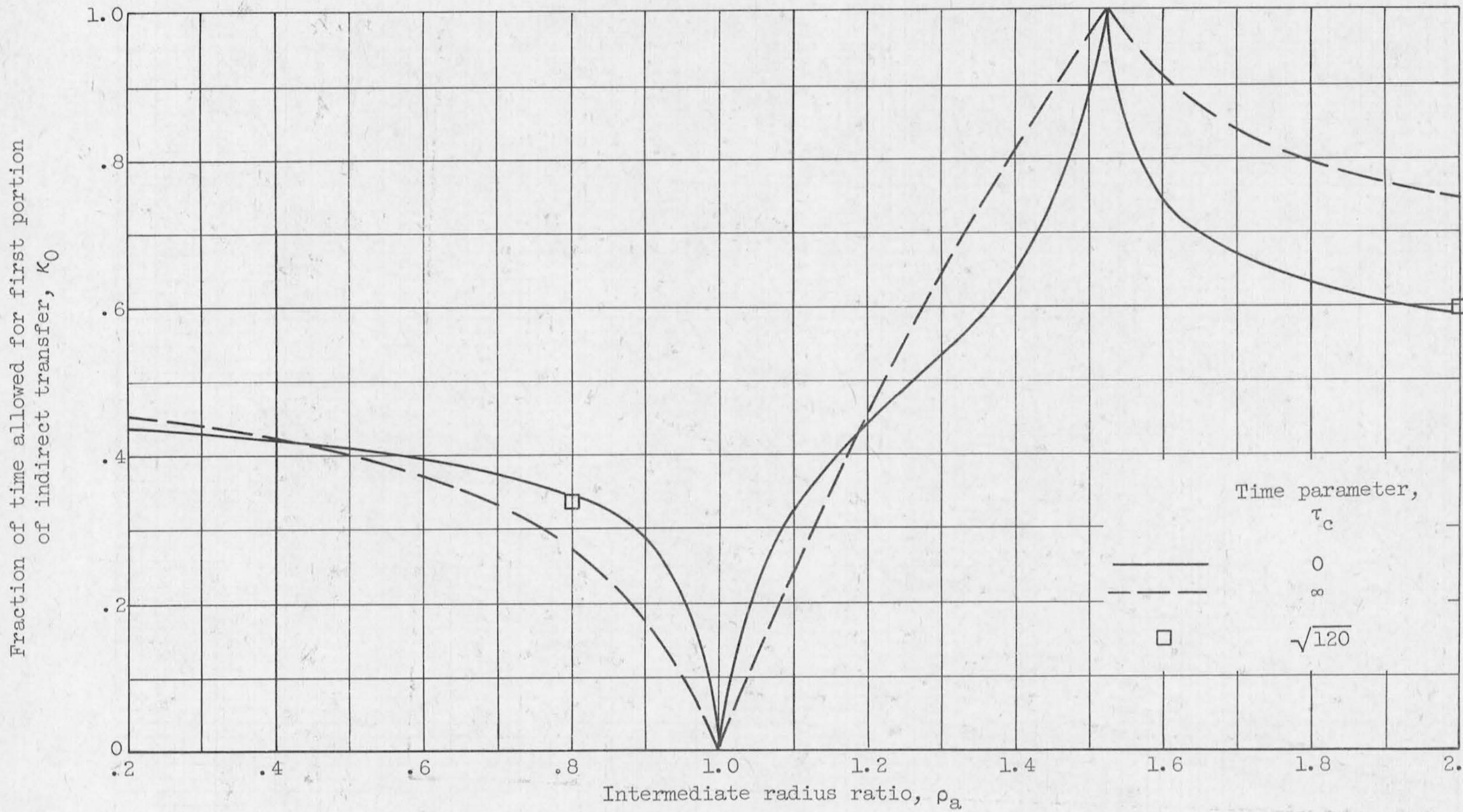


Figure 5. - Limiting values of optimum κ_0 for $\rho_c = 1.524$.

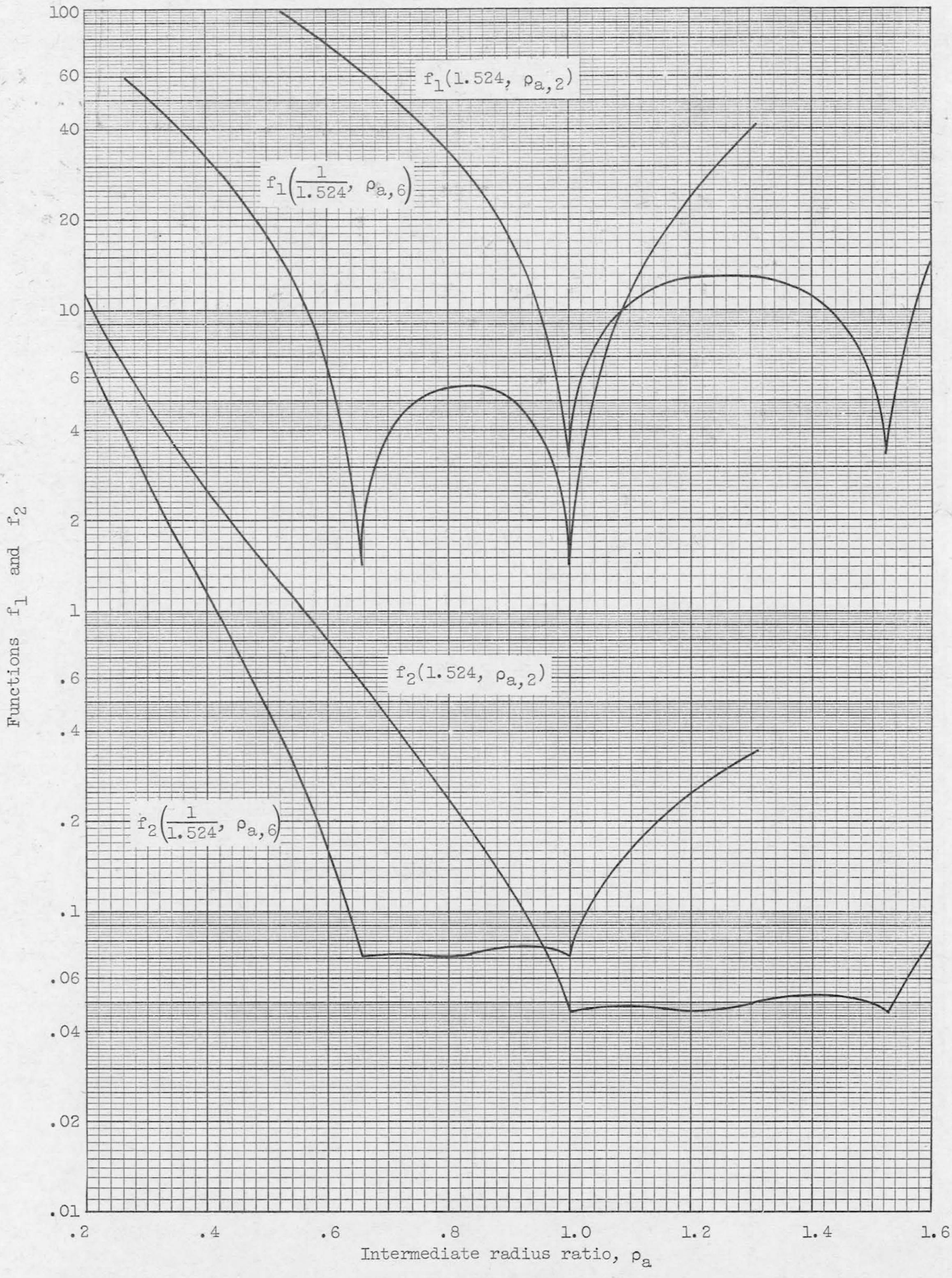


Figure 6. - Values of f_1 and f_2 for Earth-Mars transfers.

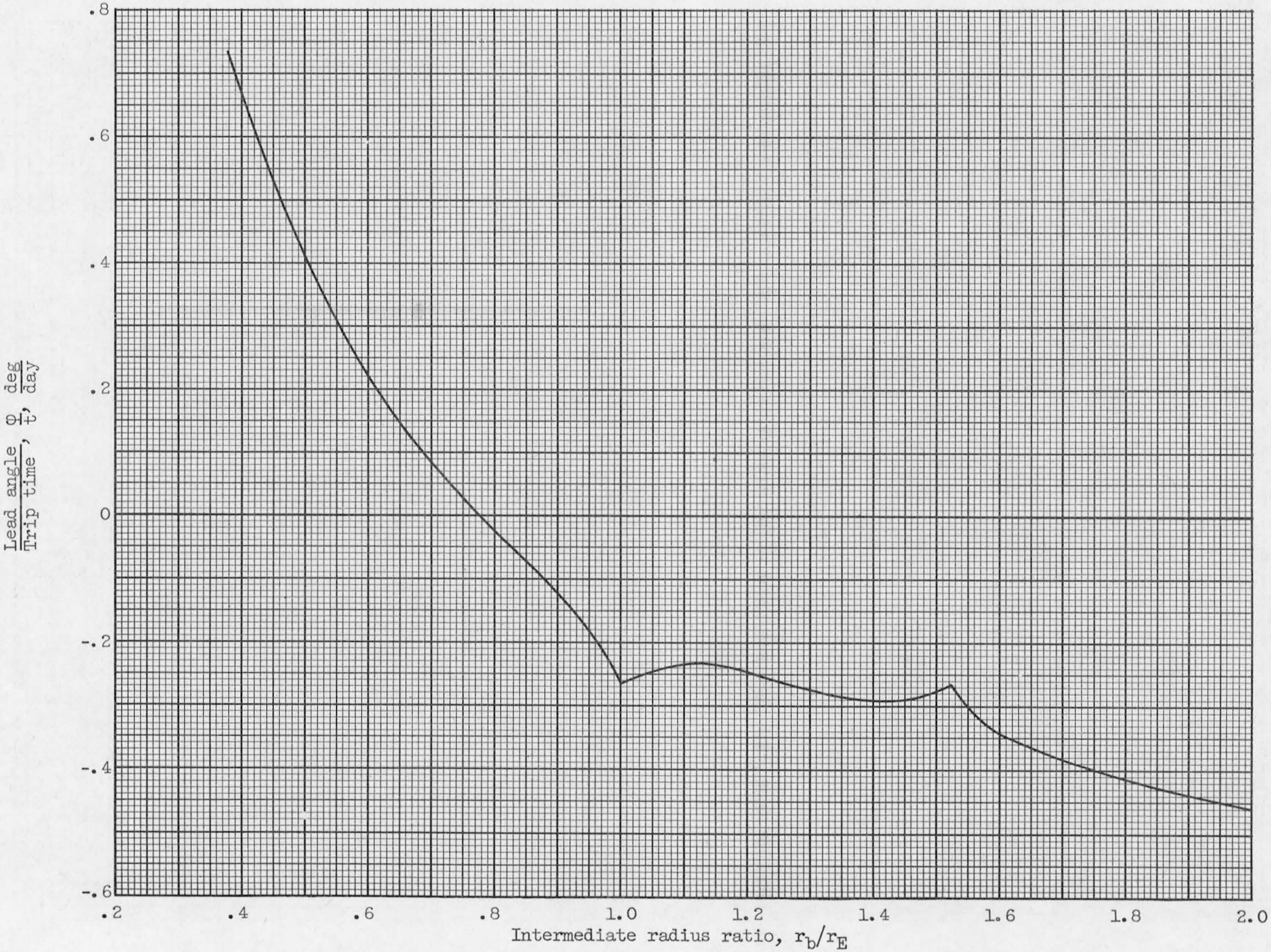


Figure 7. - Lead angles for Earth-Mars transfers with $\theta' = \rho^{-3/2}$ trajectories.

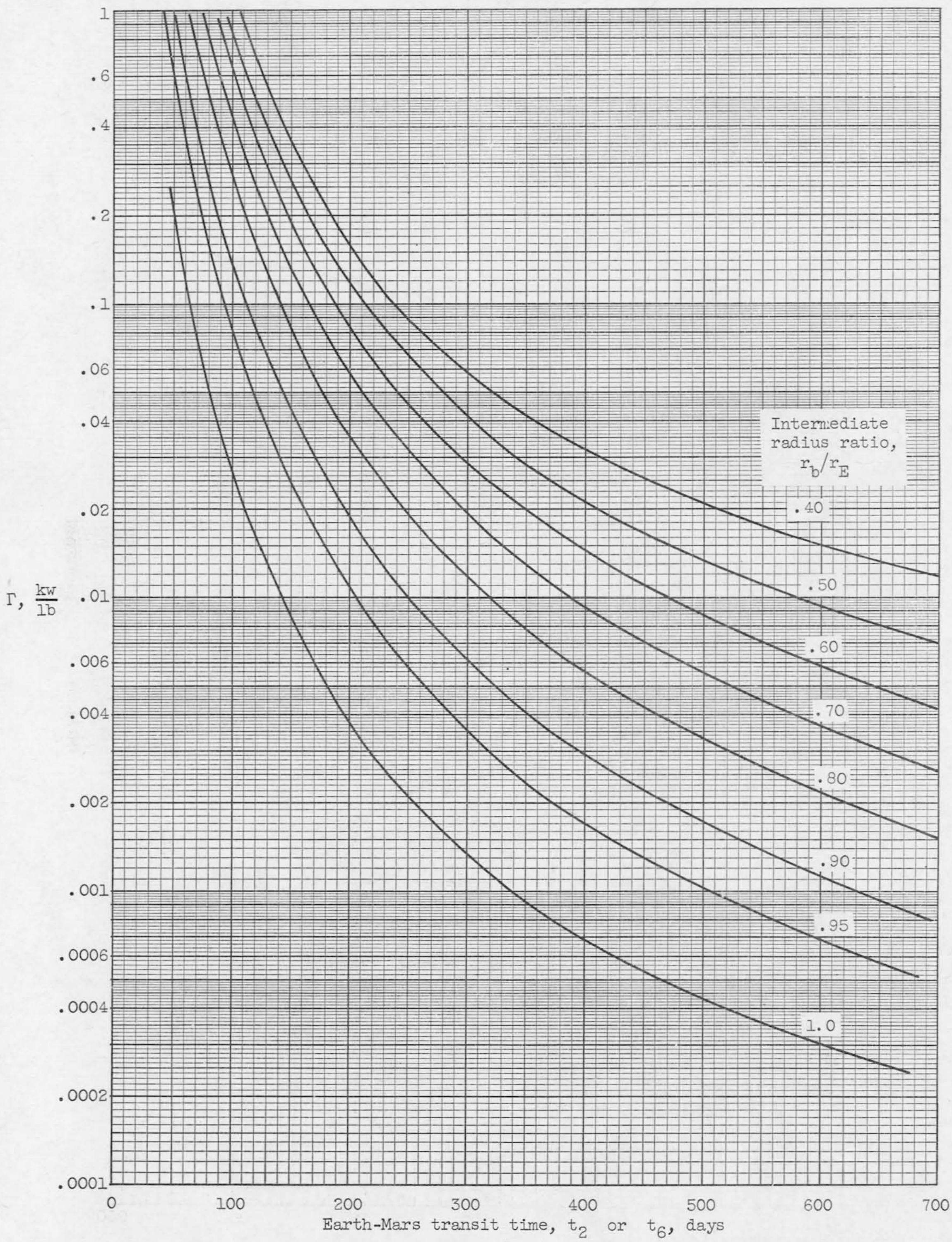


Figure 8. - Values of Γ for Earth-Mars transfers.

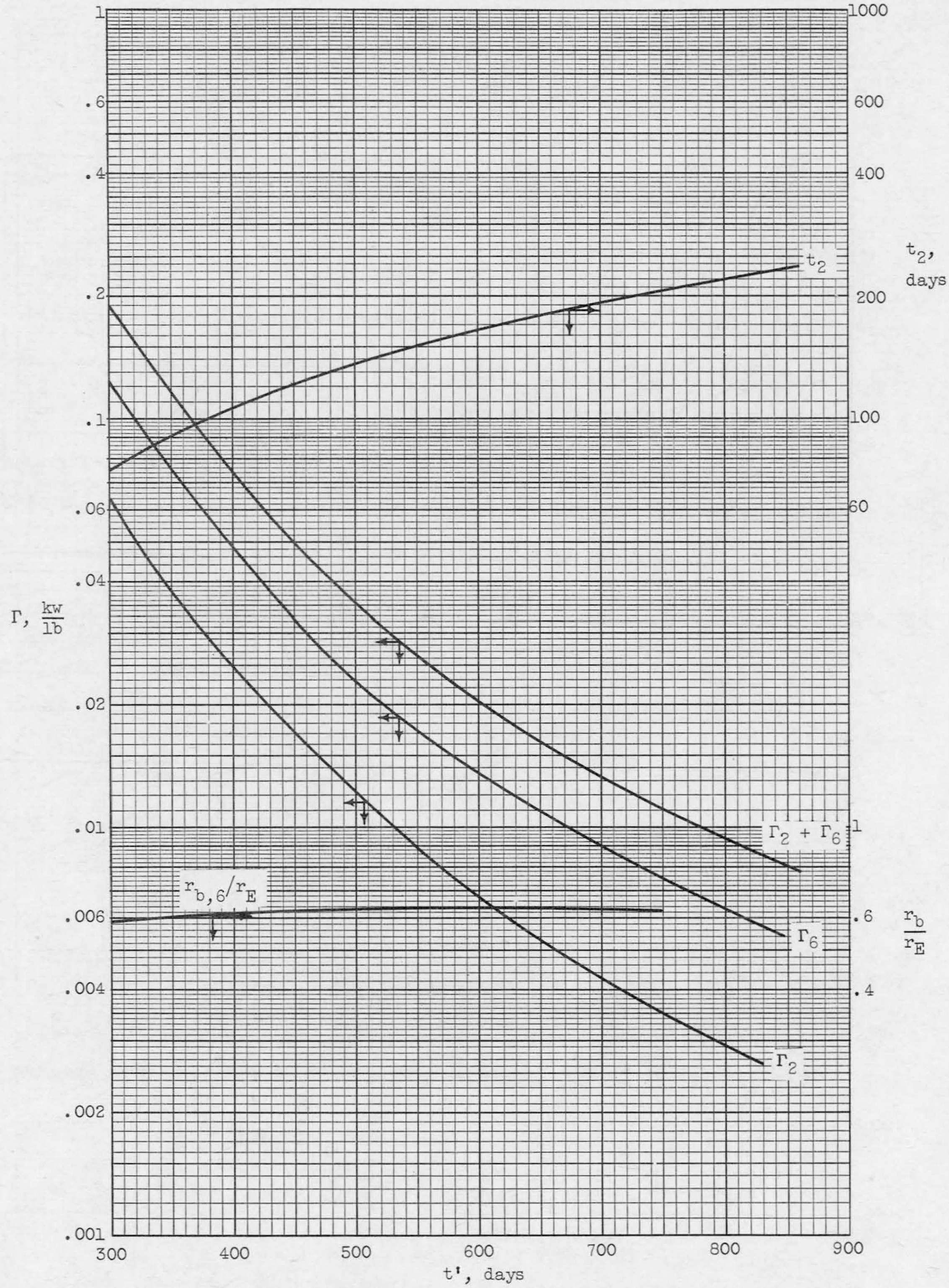


Figure 9. - Optimum values of parameters for Earth-Mars round trips (t_w = 50 days; $t' = t_2 + t_w + t_6$).

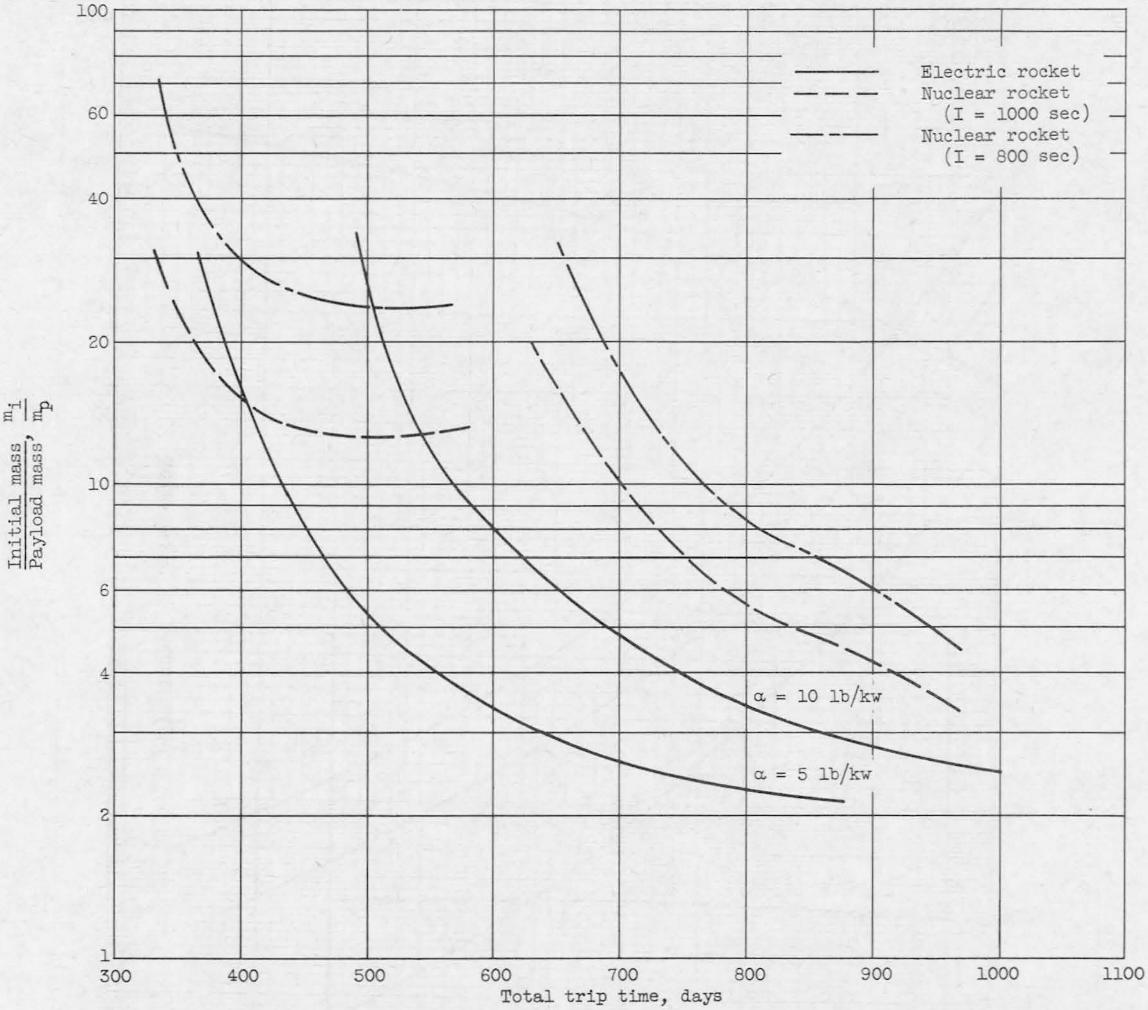
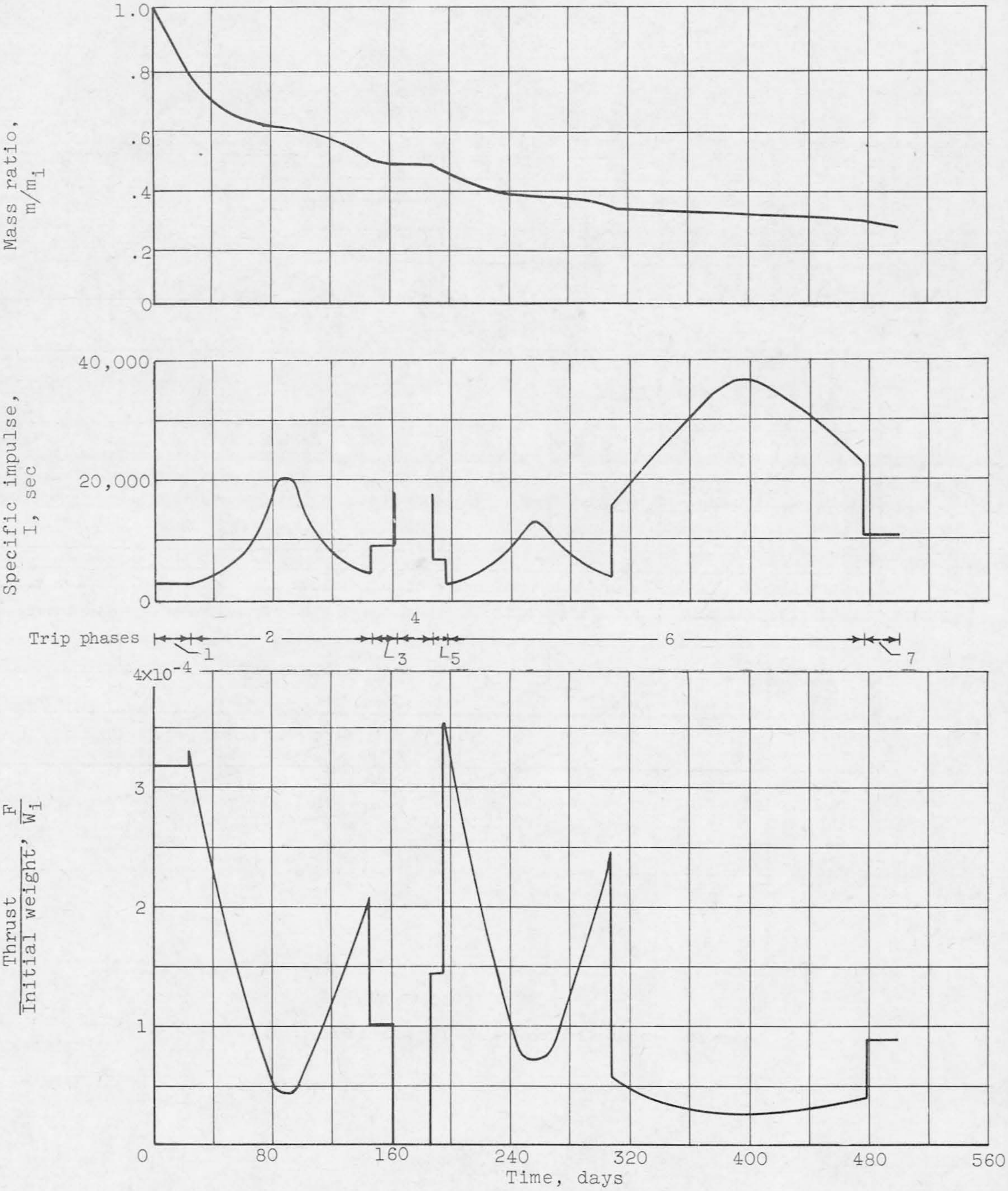
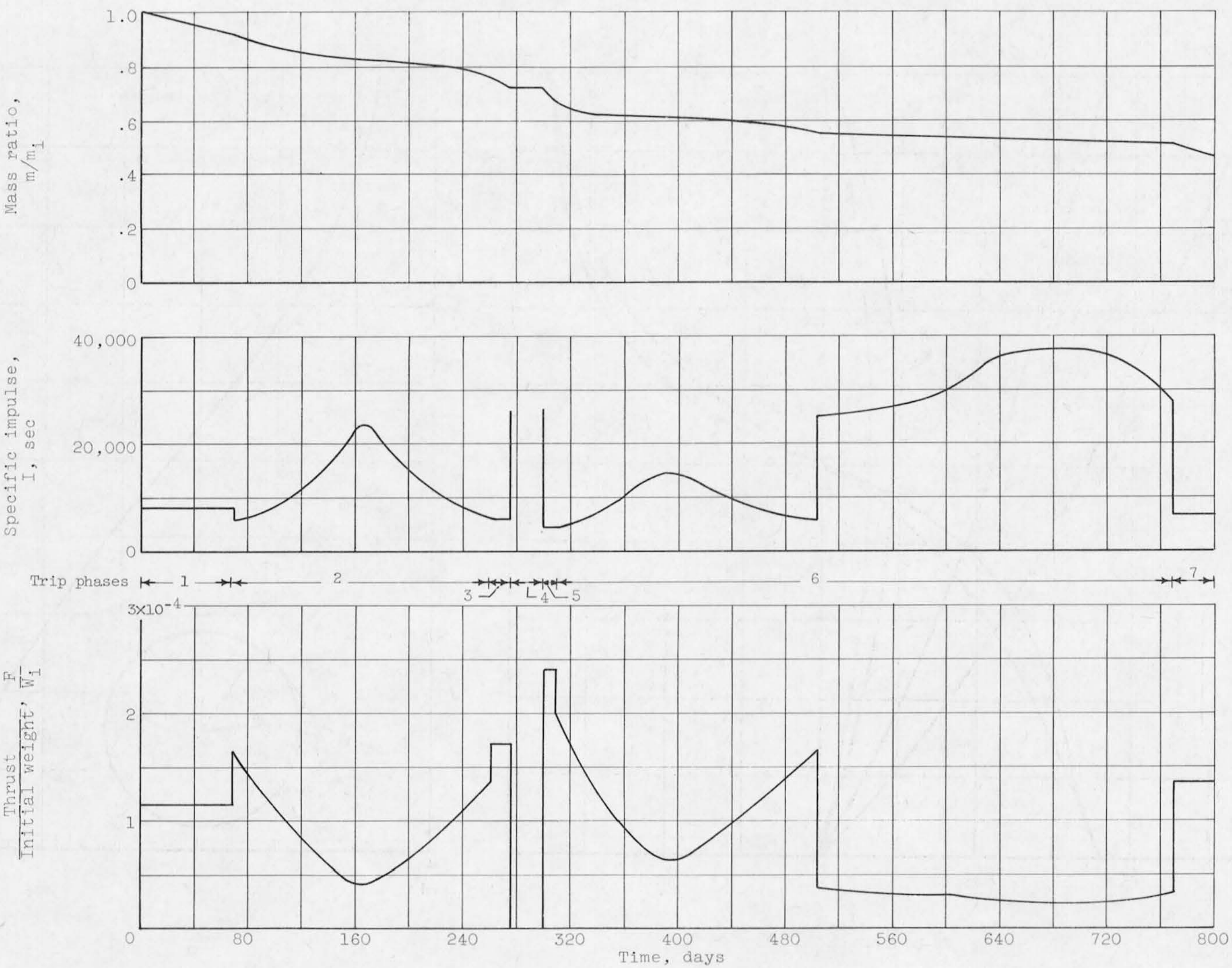


Figure 10. - Mass-ratio comparison for Mars round trips ($m_{ex} = 0$; $\dot{m}_2 = 0$).



(a) 500-Day trip.

Figure 11. - Time history of mass, thrust, and specific impulse for 500-day and 800-day Mars round trips (α = 10 lb/kw, β = 0.20).



(b) 800-Day trip.

Figure 11. - Concluded. Time history of mass, thrust, and specific impulse for 500-day and 800-day Mars round trips (α = 10 lb/kw, β = 0.20).

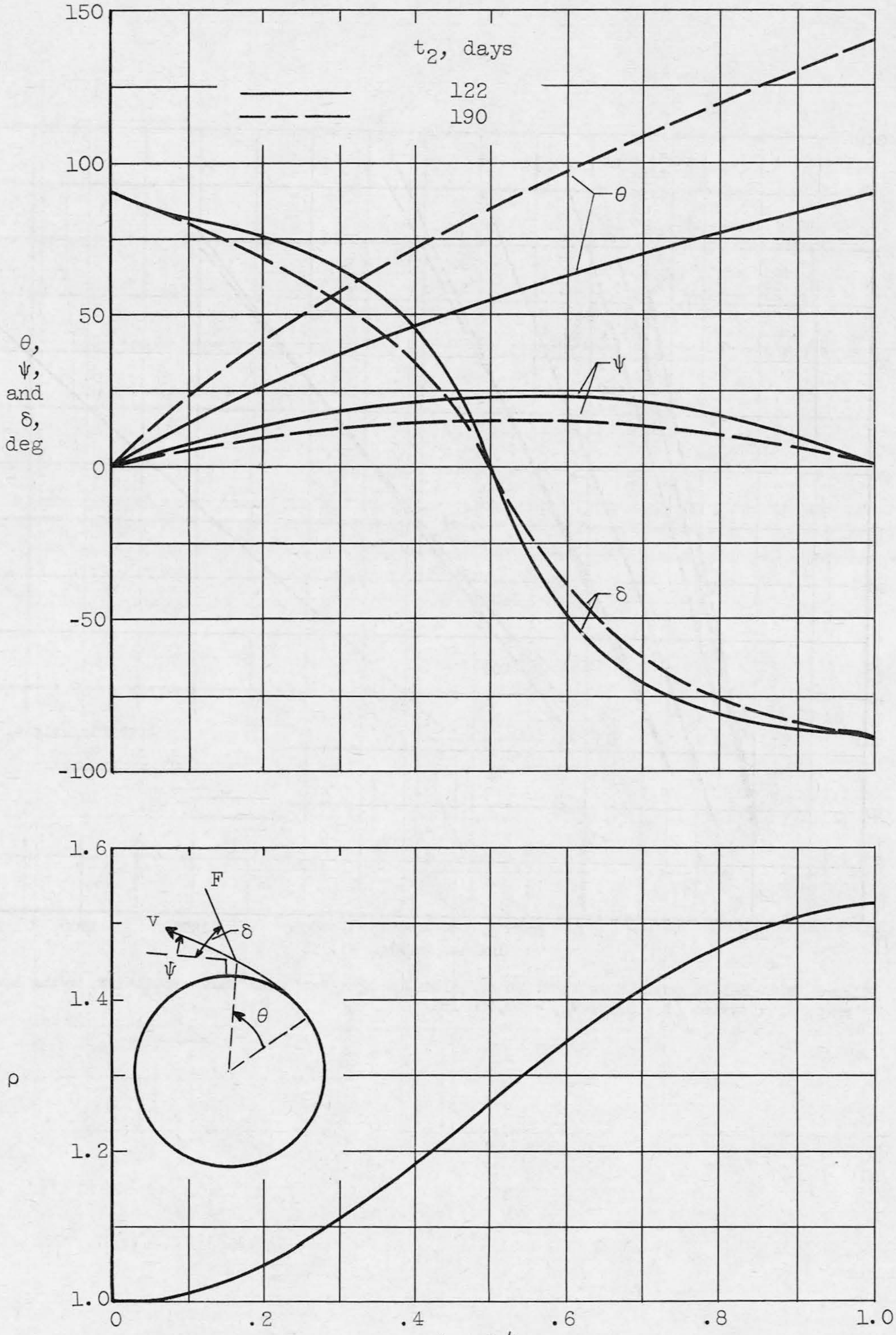


Figure 12. - Time history of several trajectory variables for Earth-Mars transfer along $\theta' = \rho^{-3/2}$ trajectories.

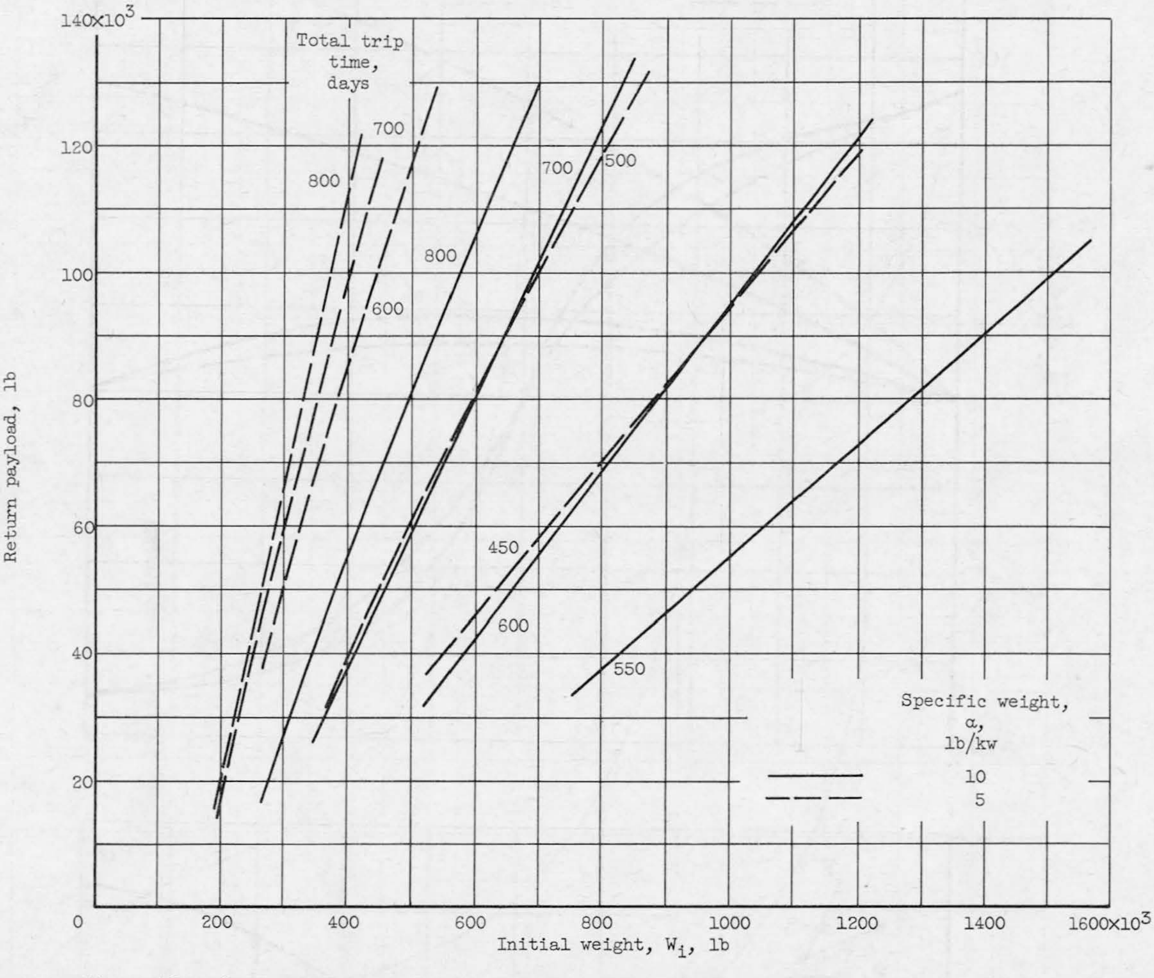


Figure 13. - Return payload as function of initial weight for 8-man Mars expedition, using low-thrust propulsion system (β = 0.20; W_{ex} = 40,000 lb).

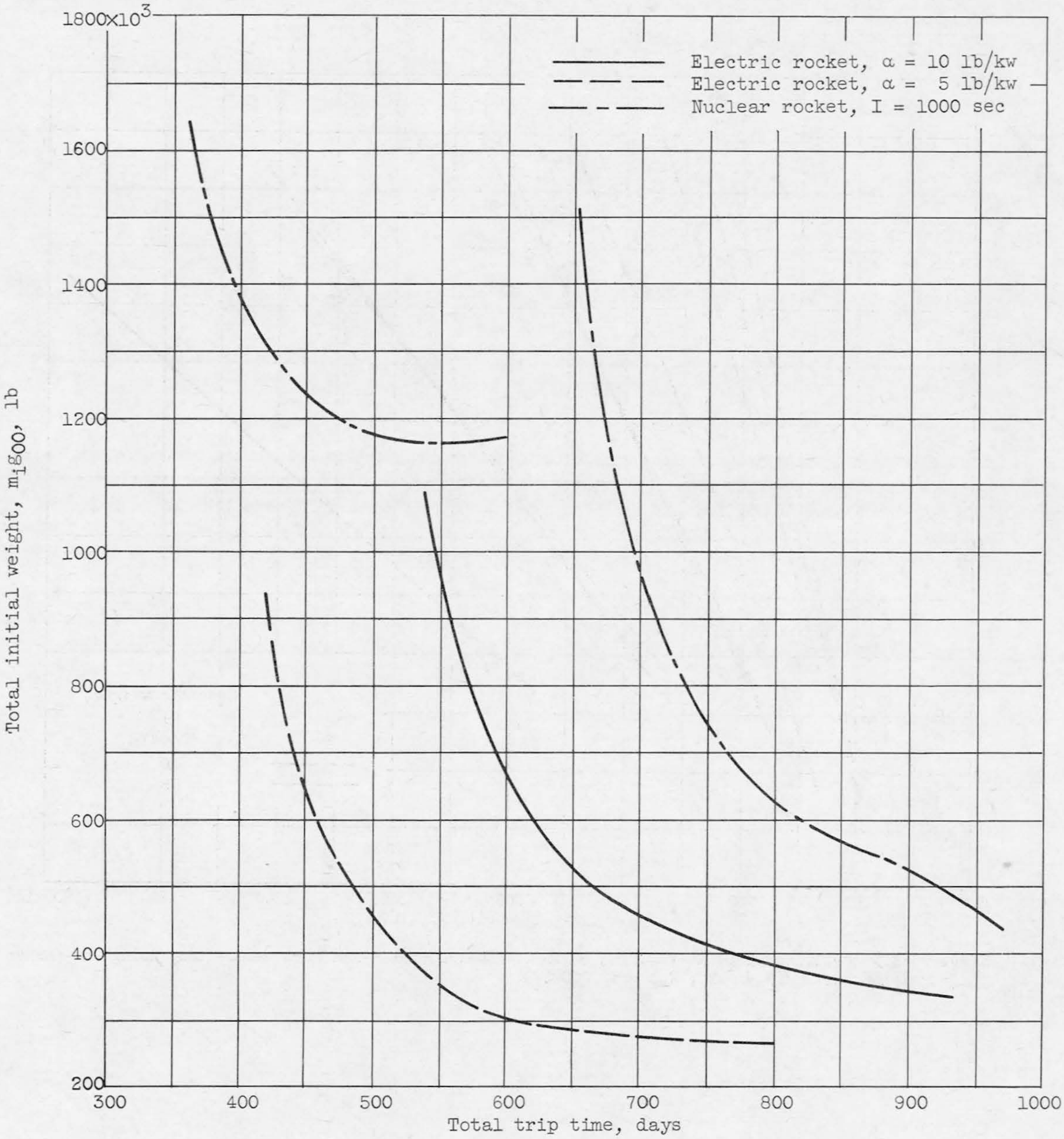


Figure 14. - Comparison of initial weights required for 8-man Mars mission as function of total trip time (return payload = 50,000 lb; W_{ex} = 40,000 lb).