Supplementary Material for

Reproducing The Supershear Portion Of The 2002 Denali Earthquake Rupture In Laboratory

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1. Development of the Scaling Relationship

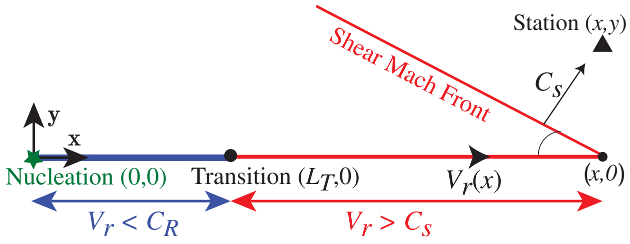

Figure 1: Problem Geometry

Consider the problem geometry shown in Fig. 1. We have a fault (horizontal straight line) where the rupture nucleates at the origin and transitions to supershear speed at $x = L_T$. The station is located at (x, y). Let $V_r(x)$ be the rupture velocity history on the fault where $V_r(x) < C_R$, the Rayleigh wave speed, for $x < L_T$ and $V_r(x) > C_s$, the shear wave speed, for $x \geq L_T$. Then the time taken for the rupture to arrive at $(x, 0)$ is simply:

$$\int_0^{L_T} \frac{dx}{V_{r,SR}(x)} + \int_{L_T}^{x} \frac{dx}{V_{r,SS}(x)} \tag{1}$$

We have intentionally written the integral in the above form so that the sub-Rayleigh (SR) and the supershear (SS) parts of the rupture are separated.

By the time the rupture arrives at $(x, 0)$ it is already supershear and has a Mach front associated with it. Assuming that the rupture speed is constant after transition, the Mach front is straight and the speed of the normal to the Mach front is the shear wave speed, C_s (see figure above). As an aside, note that if the rupture front accelerates or decelerates the Mach front is convex or concave respectively and the speed of the normal to the local tangent of the Mach front is still the shear wave speed. Thus the time taken for the Mach front to reach the station is simply $y\cos\theta/C_s$ where $\sin\theta = C_s/V_{r,SS}$. Thus the time taken for the main rupture information to reach the station, t_{SS}, is given by

$$t_{SS} = \int_0^{L_T} \frac{dx}{V_{r,SR}(x)} + \frac{x - L_T}{V_{r,SS}} + \frac{y\cos\theta}{C_s} \tag{2}$$

Lets now consider the time taken for the trailing Rayleigh pulse of a supershear rupture to arrive at the station. We can imagine that this pulse existed from the nucleation site and traveled at the speed of the main rupture until transition point. From here on this pulse travelled exactly at the Rayleigh wave speed, C_R. Also, lets assume that if the station is close to the fault then the pulse arrives at $(x, 0)$ and the station (x, y) simultaneously i.e. the time taken for this pulse to arrive at the station (x, y) is equal to the time taken for it to propagate along the fault to a location that corresponds to the projection of the station on the fault plane, $(x, 0)$. Thus the time taken for this pulse to reach the station (x, y), t_R, is,

$$t_R = \int_0^{L_T} \frac{dx}{V_{r,SR}(x)} + \int_{L_T}^{x} \frac{dx}{C_R} = \int_0^{L_T} \frac{dx}{V_{r,SR}(x)} + \frac{x - L_T}{C_R} \tag{3}$$

Hence, the difference in the rupture arrival time and the Rayleigh pulse arrival time, $\Delta t_{SS,R} = t_R - t_{SS}$, is given by

$$\Delta t_{SS,R} = \frac{x - L_T}{C_R} - \frac{x - L_T}{V_{r,SS}} - \frac{y}{C_s}\sqrt{1 - \frac{C_s^2}{V_{r,SS}^2}} \tag{4}$$

where $\cos\theta = \sqrt{1 - C_s^2/V_{r,SS}^2}$. From here on we will drop the subscript SS and simply denote the supershear rupture velocity by V_r. Solving for x we obtain,

$$x = L_T + C_s \Delta t_{SS,R} + y\sqrt{1 - \frac{C_s^2}{V_r^2}}\left(\frac{C_s}{C_R} - \frac{C_s}{V_r}\right)^{-1} \tag{5}$$

Define the following quantities,

$$\begin{aligned} \tan\beta &= \left(\frac{C_s}{C_R} - \frac{C_s}{V_r}\right)\left(1 - \frac{C_s^2}{V_r^2}\right)^{-1/2} \\ L_* &= (L_T + C_s \Delta t_{SS,R})\tan\beta \end{aligned} \tag{6}$$

Rearranging eqn. (5) and using eqn. (6) we get a linear relationship between x and y which is the locus of stations that give the same $\Delta t_{SS,R}$ given by

$$y = x\tan\beta - L_* \tag{7}$$

This locus consists of a pair of straight lines inclined at an angle $\pm\beta$ and intersecting the fault at $L_T + C_s\Delta t_{SS,R}$ (See

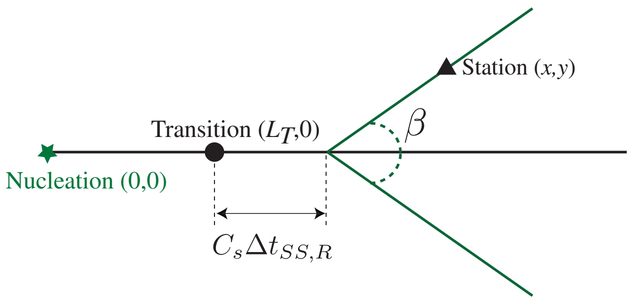

Figure 2: Locus of stations with the same $\Delta t_{SS,R}$ assuming a fixed transition length and constant supershear rupture velocity. β is defined in eqn. (6).

Fig. 2). To further constrain the above solutions we impose that geometric scaling is retained. It is assumed that in both cases, Laboratory and Denali, the ratio of the fault normal co-ordinate of the station, y, to the fault parallel distance relative to the supershear transition location, $(x - L_T)$, remain the same. That is

$$\frac{y^D}{(x^D - L_T^D)} = \frac{3\ km}{18\ km} = \frac{y^L}{(x^L - L_T^L)} = S_L \tag{8}$$

Here the superscripts D and L correspond to the Denali Pump Station 10 (PS10) and the Laboratory Station respectively. Solving the above equation with the equation for the locus, eqn. (7), we get the co-ordinates for the experimental station.

$$x^L = \frac{L_*^L - L_T^L S_L}{\tan\beta^L - S_L} \quad ; \quad y^L = S_L(x^L - L_T^L) \tag{9}$$

where

$$L_*^L = (L_T^L + C_s^L \Delta t_{SS,R}^L)\tan\beta^L$$

$$\tan\beta^L = \left(\frac{C_s^L}{C_R^L} - \frac{C_s^L}{V_r^L}\right)\left[1 - \left(\frac{C_s^L}{V_r^L}\right)^2\right]^{-1} \tag{10}$$

To be very precise the above expressions determine a geometrically equivalent station, in terms of the arrival times of the rupture and the trailing Rayleigh, in a medium with different elastic properties than typical rock. One needs to know, *a priori*, the transition length, L_T^L, for the experiment (often set by a far field load level or by seeding transition by a stress perturbation) and the difference in the arrival time, $\Delta t_{SS,R}^L$.

The transition length in the laboratory is known as it is constructed by design. To determine $\Delta t_{SS,R}^L$ we make the following temporal scaling argument. We start from the Denali PS10 record which provides the equivalent time difference, $\Delta t_{SS,R}^D$. This is now scaled temporally by ensuring that the trailing Rayleigh signature in the laboratory experiment would match the same in the Denali PS10 record when multiplied by an appropriate temporal scaling factor. This requires some confidence in the constancy of the temporal width of the trailing Rayleigh signature, Δt_R^L, in the experiments. We determine Δt_R^L to be $17.8\mu s$ based on a series of past experiments which indeed show trailing Rayleigh pulses of a remarkably consistent width at near fault distances. Figure 3F of the main text indeed confirms a trailing Rayleigh pulse (shaded in blue) width of 17.8 μs as stated above. The corresponding value for Denali PS10 record, Δt_R^D is $6.9s$. Thus the temporal scaling factor, S_T, is

$$S_T = \frac{\Delta t_R^D}{\Delta t_R^L} = \frac{6.9s}{17.8\mu s} = 3.87 \times 10^5 \tag{11}$$

We can now use this temporal scaling factor to determine $\Delta t_{SS,R}^L$ based on the corresponding value for Denali PS10 i.e. $\Delta t_{SS,R}^L = \Delta t_{SS,R}^D / S_T$. Substituting this into first equation of eqn. (10) one now obtains,

$$L_*^L = \left(L_T^L + \frac{C_s^L \Delta t_{SS,R}^D}{S_T}\right)\tan\beta^L \tag{12}$$

Substituting this into eqn. (9) along with $L_T^L = 39$mm, $C_s^L = 1.28$km/s, $C_R^L = 0.92 C_s^L$ and $V_r^L = 1.7 C_s$ one obtains $(x^L, y^L) = (57, 3)$mm. Once the experimental records are obtained they are then subjected to temporal scaling using the relation, $t^{scaled} = t^L \times S_T$.