Emission Regulations In the Electricity Market: An analysis from consumers, producers and central planner perspectives.

by

Cristian Ricardo Figueroa Rodriguez

Submitted to the Sloan School of Management
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Abstract

In the first part of this thesis, the objective is to identify optimal bidding strategies in the wholesale electricity market. We consider asymmetric producers submitting bids to a system operator. The system operator allocates demand via a single clearing price auction. The highest accepted bid sets the per unit market price payed by consumers. We find a pure Nash equilibrium to the bidding strategies of asymmetric producers unattainable in a symmetric model. Our results show that producers with relatively large capacities are able to exercise market power. However, the market may seem competitive due to the large number of producers serving demand.

The objective of the second part of the thesis, is to compare two regulation policies: a fixed transfer price, such as tax regulation, and a permit system, such as cap-and-trade. For this purpose, we analyze an economy where risk neutral manufacturers satisfy price sensitive demand. The objective of the regulation established by the central planner is to achieve an external objective, e.g. reduce pollution or limit consumption of scarce resource. When demand is uncertain, designing these regulations to achieve the same expected level of the external objective results in the same expected consumer price but very different manufacturers' expected profit and central planner revenue. For instance, our results show that when the firms are price takers, the manufacturers with the worst technology always prefer a tax policy. Interestingly, we identify conditions under which the manufacturers with the cleanest technology benefit from higher expected profit as tax rate increases.

In the third part of the thesis, we investigate the impact labeling decisions have on the supply chain. We consider a two stage supply chain consisting of a supplier and a retailer. Demand is considered stochastic, decreasing in price and increasing in a quality parameter, e.g. carbon emissions. The unit production cost for the supplier is increasing in the quality level chosen. We identify two different contracts that maximize the efficiency of the supply chain while allowing the different parties to achieve their objectives individually.

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Chapter 1

Introduction

1.1 Motivation

The use of natural resources is part of the daily life of individuals of the modern society. Trees, are used as paper, such as in store receipts, cups and airplane tickets, trees are also used as furniture and fuel. Petroleum and oil, are used as fuel for vehicles, heating and fuel. However, figures presented in the BP Statistical Review of World Energy [1] suggest that oil would not last more than 47 years, should global production remain at the same rate.

Furthermore, the depletion of natural resources is not the only concern. Increasing amounts of carbon dioxide and other greenhouse gases are constantly being released. In fact, electricity generation and transportation account for 61% of the U.S. greenhouse gas emissions in 2011[2]. The concern over these gases has been such that in 1997 the united nations established an international treaty where many developed countries agreed to legally binding reductions in their emissions of greenhouse gases.

The depletion of natural resources and the concern over climate change have motivated businesses and governments to use their resources in such a way as to not only meet human needs, but also preserve the environment. The objective of sustainable operations is to be able to continue production with little, or manageable, impact on the environment.

In the past 30 years, different initiatives have been implemented with the purpose of improving sustainability. Examples of these initiatives include car manufacturers improving the technology of their hybrid vehicles, big retailers like Walmart demanding that their suppliers label the carbon emissions associated with their products, televisions being labeled according to energy consumption, washing machines being labeled according to water efficiency, increasing sales in solar panels, etc.

[1]Source: British Petroleum, bp.com
[2]Source: Environmental Protection Agency, epa.gov

Larger scale initiatives are also being implemented, for example, governments are establishing environmental policies in order to regulate emissions to the environment. The two most common policies are the implementation of a tax to be associated with the carbon emissions, and an emission trading scheme where producers acquire emissions rights prior to production and are allowed to trade these permits among each other. With the implementation of a carbon tax, fuels yielding high amount of emissions become more costly, while fuels with low level of emissions are not affected as much. With the implementation of an emissions trading schemes, it is possible to restrict the total amount of emissions to the environment by limiting the amount of permits.

Implementations of these two environmental policies can be found in practice. For example, Australia implemented a carbon tax policy in July 2012, with the intention of transitioning to an emission trading scheme by 2015. An example of a successful implementation of an emission trading scheme can be found in the New England area of the United States, where after the implementation of the Regional Greenhouse Gas Initiative, the usage of fossil fuels has been reduced, see Figure 1-1.

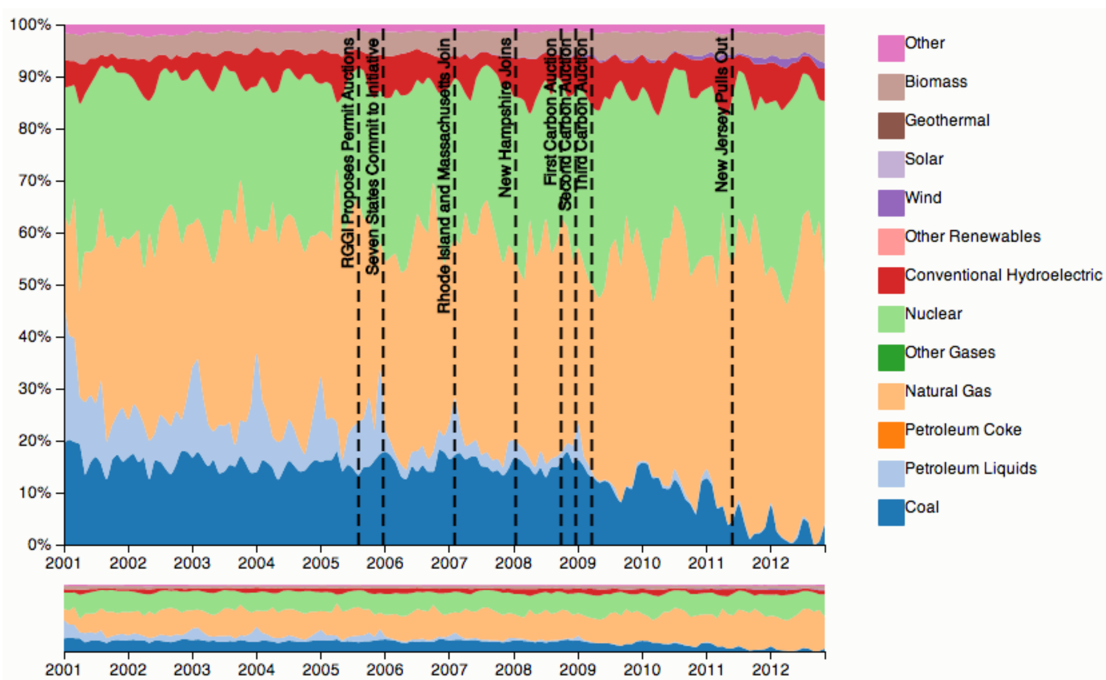


Figure 1-1: Evolution of sources of energy for the New England area after the implementation of the RGGI. (Source: http://www.eia.gov/)

Unfortunately, not all of these initiatives have the desired effect. The initial implementation of an emission trading scheme in the European Union is largely considered a failure by the media. Serious flaws have been detected in the design of the trading scheme. First, permits were given free of charge to businesses, therefore, some companies were able to earn additional profits by trading excess permits. Second, the target emissions was larger than the current production for many businesses and as such the permits had little effect in reducing emissions. An auction mechanism was introduced later to mitigate these effects.

The field of operations research has been actively involved in studying different aspects of sustainability. For example, Benjaafar et al. (2011) study the effect of integrating carbon emission constraints into operational decision making. By modifying classic decision making models, such as facility location problem, the authors find that firms could effectively reduce their carbon emissions without significantly increasing their costs by making only operational adjustments and by collaborating with other members of their supply chain. With the growing interest in local food procurement, Ata et al. (2012), analyze the operational trade-offs of a retailer and farmers in a fresh produce supply chain, studying mechanisms that allow local farmers to increase their competitiveness. Regarding the use of electric vehicles, Avci et al. (2012), provide an analysis suggesting the incorporation of stations where users are able to quickly switch batteries, as an effective way to reduce the low battery life limitations of electric vehicles.

In a global supply chain context, Keskin and Plambeck (2011), analyze the effect imports from a region with no climate policy into a state with climate policy have on greenhouse gas accounting. By considering processes that yield co-products, where only some of the co-products are imported, the authors show that accounting for emissions on imports can increase global emissions when the allocation rule for emissions generated from the co-products is not chosen appropriately.

Motivated by the increase of wind power initiatives, Nair et al. (2012), use a news vendor type model to analyze energy procurement strategies in the presence of renewable, intermittent and unpredictable energy sources. Similarly, the subsidies given in Germany towards solar photovoltaic technology, motivated Lobel and Perakis (2011), to develop a framework for policy makers to find optimal subsidies in order to achieve an adoption target at minimum cost to the system. By assuming consumers purchase solar panels according to a discrete choice model, the authors analyze the structure of an optimal subsidy policy and use it to analyze Germany's current policy.

This thesis analyzes three different problems motivated by sustainability. Two of these topics are related to the energy market, while the last topic explores the impact environmental labels have on supply chain performance. A brief description of the particular problems is as follows.

1.1.1 Bertrand competition over a homogeneous service

The first topic, developed in Chapter 2, is motivated by the electricity market. Traditionally, the electric power industry acted as a vertically integrated, highly regulated monopoly. Over the last three decades, deregulation has been introduced in the electric power market in many countries. Interestingly, different electricity market have implemented decentralization in different ways. For example, prior to 2001, in the England and Wales wholesale electricity market every day generators submit bids to supply demand for electricity, and the highest accepted bid sets the market price received by all winners. In March 2001, a reform was implemented and the payment mechanism was changed to a pay-per-bid scheme. Much research has been produced in comparing these two payment schemes.

In this chapter, we analyze the optimal bidding strategies in a simplified model of a wholesale electricity market. For this purpose, we consider asymmetric producers submitting bids to a system operator. The system operator allocates demand based on the bids submitted – the producer with the lowest bid will satisfy demand up to its capacity and remaining demand is allocated to the second lowest-bid producer until its capacity is exhausted and so-on. The highest accepted bid sets the per unit market price payed by consumers and received by all participating producers. This is known as a uniform price payment scheme.

We find a pure Nash equilibrium to the bidding strategies of asymmetric producers. Our results under full information show that producers with relatively large capacities are able to exercise market power by raising per unit market price far above the perfect competition price. However, the market may seem competitive due to the large amount of producers serving demand.

1.1.2 Emissions Regulation: Carbon Tax and Cap-and-trade

The second topic, developed in Chapter 3, is motivated by the discussion among policy makers regarding the best way to regulate emissions to the environment. The objective of this chapter is to help policy makers compare two regulation policies: a fixed transfer price, such as tax regulation, and a permit system, such as cap-and-trade. For this purpose, we analyze an economy where risk neutral manufacturers satisfy price sensitive demand. The focus of the regulation established by the central planner is on achieving an external objective, e.g. reduce pollution or limit consumption of scarce resource.

We show that when demand is deterministic, and under full information, the two regulation policies are equivalent: the manufacturers, central planner and consumers are indifferent between the two policies. However, when demand is uncertain, designing these regulations to achieve the same expected level of the external objective results in the same expected consumer price but very different manufacturers' expected profit and central planner revenue. For instance, our results show that when the firms are price takers, the manufacturers with the worst technology always prefer a tax policy, while the central planner prefers a permit policy. Interestingly, we identify conditions under which the manufacturers with the cleanest technology benefit from higher expected profit as tax rate increases.

1.1.3 Carbon Footprint Labeling

The third topic, developed in Chapter 4 , investigates the impact the labeling decision has on the supply chain. In particular, our research focuses on whether the information provided by the label, e.g. energy consumption or CO_2 emissions in the case of the environmental labels, should be decided by the manufacturer or imposed by the retailer. Such a decision, like product quality, is typically made by the manufacturer. However, retailers have more direct contact with the consumers, and we consider the possibility of a retailer to be influential enough such that it can suggest a quality level to the manufacturer. The manufacturer bears the cost of quality decisions, that is the cost of reducing emissions, yet the retailer faces the risk of uncertain demand. The question we are looking to analyze is the following:

In an environment where demand is affected by a quality measure, which party should decide on the quality level to be labeled: the manufacturer or the retailer? What kind of contracts are able to achieve the levels and profits of a vertically integrated supply chain?

We identify two different contracts that maximize the efficiency of the supply chain while allowing the different parties to achieve their objectives individually. Additionally, we compare the environmental impacts of two models: one where the retailer determines the level of emissions the manufacturer should follow, and a second model where these decisions are made by the manufacturer.

Chapter 2

Bertrand Competition With Capacities Over a Homogeneus Service

2.1 Introduction

Traditionally, the electric power industry acted as a vertically integrated, highly regulated, monopoly. Over the last three decades, with the objective of introducing competition and improving economic efficiency, deregulation has been introduced in the electric power market in many countries. In these deregulated markets, the schedule of power generation is determined by market mechanisms such as auctions. Producers are asked to submit bids to a system operator which allocates demand based on the bids submitted – the producer with the lowest bid will satisfy demand up to its bid quantity and remaining demand is allocated to the second lowest-bid producer until its bid quantity and so-on. The highest accepted bid sets the per unit market price received by all participating producers. Producers identify their optimal bids in this auction so as to maximize revenue.

Our objective in this work is to describe (pure) optimal bidding strategies for the producers, allowing us to identify possible sources of inefficiencies of this type of auction compared with a more centralized system. We focus on sealed bid single auctions of multiple identical items. In these auctions, all bidders are asked to simultaneously submit multiple bids to the auctioneer, one bid for each item desired, and the highest bids wins the items.

A considerable amount of research has been focused on auction design. Important in our research are sealed bid auctions for the sale of identical objects. Three types of auction design are of special interest: discriminatory auction, uniform-price auction and Vickrey auction. In a discriminatory auction, bidders pay theirs winning bids. For example, U.S. Treasury 2-year and 5-year notes were sold by means of a discriminatory auction prior to 1992. In a uniform-price auction, all items are sold at a single price, and his price is equal to the lowest winning bid or the highest losing bid. For example, U.S. Treasury 2-year and 5-year notes are currently sold in uniform-price auctions. A report from the U.S. Treasury by Archibald and Malvey (1998), summarizes empirical findings comparing the performance of these two auction mechanisms when implemented to sell U.S. Treasury Notes. Interestingly, the report does not show a significant advantage of one auction over the other.

Truthful bidding, the bidding of participants own valuation, is a desired property for an auction as it discourages participants from gaming the auction. Neither of the auctions described earlier, discriminatory or uniform-price, provide incentives for bidders to submit their true values for the items. In Vickrey (1961), the author proposes a multiunit auction, later known as Vickrey auction, that under the assumption of i.i.d. bidders, guarantees that truthful bidding is a weakly dominant strategy. Despite this attractive property of Vickrey auction, the U.S. Treasury and several electricity markets apply uniform-price auctions. They argue that this is an attractive auction from a few points of views: revenue, efficiency, strategic simplicity, and susceptibility to collusion, see Krishna (2009).

The literature supporting this argument is limited, for instance, Milgrom (1989), analyzed a multiunit auction where the valuations of bidders are independently drawn from the same distribution. In his model, each bidder desires at most one unit of the good being auctioned, finding that a sealed bid uniform-price auction yields more revenue than a sealed bid discriminatory price auction. However, Back and Zender (1993), compare a sealed bid uniform-price auction with a discriminatory auction, when the item is perfectly divisible and bidders valuations are drawn from the same distribution. The authors show that results developed under single unit demand assumption, as in Milgrom (1989), do not generalize to auctions in which bidders desire multiple units.

The application of auction theory to electricity markets is not straightforward. There are two characteristics in the electricity market that differ from the typical auctions framework. First, the system operator's objective is not necessarily the maximization of revenue. Second, participating in the electricity market requires an immense capital investment. Therefore, the set of market participants in each auction does not change much over time, moreover there exists public information about the participants' technology.

Regarding the study of auctions in the electricity market, Son et al. (2004) analyze a two player auction game with full information. Under the assumptions of a big player with market power (large capacity) competing against a small player to serve inelastic demand, the authors compare the performance of uniform pricing and pay-as-bid pricing in electricity markets. The authors show that unlike a single item auction, the Nash equilibrium under pay as bid pricing yields less total revenue in expectation than under uniform pricing. Hu et al. (2010), analyze an auction with asymmetrical bidders, each with a specific linear cost function. In their model, suppliers bid a quantity-price pair, the auctioneer collects the bids and satisfies an inelastic demand at minimal cost. The demand allocated to a supplier is limited by the quantity stated in their bid, and similarly to a discriminatory auction, the price-per-unit paid to each supplier is the price stated in their bid. The authors are able to characterize the optimal bidding strategies for suppliers in their model.

In Hu et al. (2011), the authors analyze an auction where suppliers have linear production cost, and a fixed capacity. Suppliers independently submit a bid representing a per-unit price to the auctioneer. The auctioneer then satisfies demand – the supplier with the lowest bid will satisfy demand up to its capacity and remaining demand is allocated to the second lowest-bid supplier up to capacity and so-on. The authors then compare market price volatility when the payment scheme is uniform-price versus a pay-per-bid payment. The authors completely characterize equilibria in a symmetric supplier setting, extending results to account for uncertain demand. The results obtained show that a pay-per-bid payment scheme yields lower price volatility than a uniform-price payment scheme.

In this chapter, we consider asymmetric producers bidding to serve a price sensitive demand. Similar to Hu et al. (2011), each producer submits a single bid representing the per-unit price they charge for energy, up to their capacity. The system operator collects the bids and allocates demand as in a uniform-price auction, the price paid to each producer is the highest accepted bid. We find an intuitive equilibrium for the optimal bidding strategies of producers under the assumption of equal capacities. However, this strategy is not necessarily optimal when producers have different capacities; modifications to the aforementioned strategy are introduced in order to find an equilibrium.

2.2 Model

Consider n producers serving market demand for a homogeneous product, that is, customers do not distinguish between products, say electricity, provided by different producers. Each producer i has a linear production cost $c_i(q) = c_i q$, for delivering q units. Associated with each producer is a capacity constraint, k_i, establishing that producer i is unable to produce more than k_i units.

We consider a market demand $D(p)$, which is a decreasing convex function of the price paid by consumers. We assume demand to be such that $pD(p)$ is concave. Each supplier submits a bid p_i, for the price they are willing to sell their product, but of course they are limited by capacity. We assume that there exists a large upper bound $\mu \gg \max_{i=1,\dots,n} \{c_i\}$ to the possible bids. The central planner then collects the bids from the suppliers. Market equilibrium is achieved when the central planner finds the smallest price p such that demand, which is a function of price, can be satisfied by the capacity available at that price. More precisely, the central planner is looking for the smallest market price p such that:

$$\sum_{i=1}^{n} k_i \delta(p_i < p) < D(p) \leq \sum_{i=1}^{n} k_i \delta(p_i \leq p), \tag{2.1}$$

where δ is the indicator function. Finally, the central planner assigns the portion of demand, z_i, to be satisfied by each supplier based on their bids.

- If supplier i bid is such that $p_i < p$, then supplier i is assigned its capacity k_i.
- If supplier i bid is such that $p_i = p$, then supplier i is assigned,

  $$\frac{k_i}{\sum\limits_{l \in \mathcal{E}(p)} k_l}(D(p) - \sum_{j=1}^{n} k_j \delta(p_j < p)),$$

  where $\mathcal{E}(p)$ is the set of suppliers that bid p.
- If supplier i bid is such that $p_i > p$, then supplier i does not receive any allocation.

To simplify notation we introduce the following definitions. We refer to the vector of bids $\vec{p} = (p_1, ..., p_n)$, where the ith component is the bid submitted by producer i, as a bidding profile. Let $\mathcal{N} = \{1, 2, ..., n\}$ be the set of all producers, $L(p) = \{i \in \mathcal{N} | p_i < p\}$ be the set of producers whose bidding prices is lower than the market price and finally let $G(p) = \{i \in \mathcal{N} | p_i > p\}$ be the set of producers whose bidding price is larger than the market price, p. Of course, the following identity holds $\mathcal{E}(p) = \mathcal{N} \setminus (L(p) \cup G(p))$.

The objective of this study is to find an equilibrium to the suppliers bidding decisions under full information. Interestingly, as we shall see, traditional strategies such as bidding supplier own cost, or undercutting the lowest cost supplier, whose cost is higher than the cost of the bidder, have deviations that prevent them from being an equilibrium.

2.3 Equilibrium Analysis

In this section we develop results allowing us to characterize producers' optimal bidding strategy. For this purpose, we develop four propositions. Proposition 2.3.1, begins by stating a simple, yet useful, observation regarding the equilibrium market price for consumers. Next, Proposition 2.3.2, shows that considering producers with asymmetric costs greatly simplifies the analysis. The third proposition, Proposition 2.3.3, finds optimal bidding strategies under the assumption of equal capacities. Finally, Theorem 2.3.1, finds optimal bidding strategies for the case where capacities are not necessarily equal.

We begin the analysis by stating a proposition that reduces the candidates for equilibrium market price for a given bidding profile $\vec{p} = (p_1, ..., p_n)$.

Proposition 2.3.1. *Let $\vec{p} = (p_1, ..., p_n)$ be a bidding profile submitted by producers. Then the market price to consumers p is such that $p \in \{p_i\}_{i=1}^n$.*

Proof. Given the bidding profile $\vec{p} = (p_1, ..., p_n)$, there is only one value p_l such that p_l satisfies (2.1). Observe that there may be multiple suppliers i bidding p_l, but the value of the bid is unique. Let's consider a price $p^- = p_l - \varepsilon$, where $\varepsilon > 0$. In this situation,

$$\sum_{i=1}^{n} k_i \delta(p_i \leq p^-) \leq \sum_{i=1}^{n} k_i \delta(p_i < p_l) < D(p_l) \leq D(p^-).$$

The inequalities yield, $\sum_{i=1}^{n} k_i \delta(p_i \leq p^-) < D(p^-)$, therefore p^- cannot be the equilibrium market price. Hence p_l is the smallest price satisfying (2.1). □

Proposition 2.3.1 allows us to characterize supplier's profit in a simple expression. Given a bidding profile $\vec{p} = (p_1, ..., p_n)$, supplier i's profit is given by the following expression:

$$u_i(\vec{p}) = \begin{cases} (p_l - c_i)k_i & i \in L(p_l) \\ \frac{k_i}{\sum\limits_{r \in \mathcal{E}(p_l)} k_r}(p_l - c_i)(D(p_l) - \sum\limits_{j \in L(p_l)} k_j) & i \in \mathcal{E}(p_l) \\ 0 & i \in G(p_l) \end{cases}$$

where p_l is the bid satisfying (2.1).

Next, we proceed to show that if suppliers have different costs, then there is exactly one producer that bids p_l in equilibrium, namely $|\mathcal{E}(p_l)| = 1$.

Proposition 2.3.2. *If suppliers costs are such that $c_1 < c_2 < ... < c_n$, then in equilibrium there is exactly one supplier whose bid satisfies (2.1).*

Proof. Let $\vec{p}$ be the equilibrium bid, p_l is the bid satisfying (2.1). Let $i \in \arg\min\limits_{j \in \mathcal{E}(p_l)} \{c_j\}$, the profit for producer i is:

$$u_i(\vec{p}) = \frac{k_i}{\sum\limits_{r \in \mathcal{E}(p_l)} k_r}(p_l - c_i)(D(p_l) - \sum_{j \in L(p_l)} k_j).$$

Assume $|\mathcal{E}(p_l)| > 1$. If $p_l \leq c_i$, then under our assumption of different costs, other suppliers bidding p_l receive negative profits. Therefore there exists a deviation for these producers towards bidding higher, and $\vec{p}$ could not be an equilibrium.

To conclude the proof we consider the case $p_l > c_i$. If supplier i deviates to $\tilde{p}_i = p_l - \varepsilon$, we distinguish between two different cases:

- $\sum_{j \in L(p_l) \cup \{i\}} k_j < D(p_l)$, in this case supplier's i profit is given by $u_i(\vec{p}) = (p_l - c_i)k_i$, meaning supplier i is better off by reducing its bid.

- $\sum_{j \in L(p_l) \cup \{i\}} k_j \geq D(p_l) > \sum_{j \in L(p_l)} k_j$: In this case, for a small value of ε, supplier's i profit is given by $u_i(\vec{p}) = (p_l - \varepsilon)(D(p_l - \varepsilon) - \sum_{j \in L(p_l)} k_j)$. By considering a value of ε, small enough such that,

$$(p_l - \varepsilon - c_i)(D(p_l - \varepsilon) - \sum_{j \in L(p_l)} k_j) > \frac{k_i}{\sum\limits_{r \in \mathcal{E}(p_l)} k_r}(p_l - c_i)(D(p_l) - \sum_{j \in L(p_l)} k_l),$$

  or equivalently,

$$\varepsilon(D(p_l - \varepsilon) - \sum_{j \in L(p_l)} k_j) < (p_l - c_i)(D(p_l - \varepsilon) - D(p_l) + (1 - \frac{k_i}{\sum\limits_{r \in \mathcal{E}(p_l)} k_r})(D(p_l) - \sum_{j \in L(p_l)} k_j)),$$

  we can guarantee that supplier i has a profitable deviation. To prove the existence of such ε, it is enough to notice that for $\varepsilon = 0$ the inequality is satisfied strictly, and the elements of the inequality are continuous. □

Without loss of generality we can index the producers such that $c_1 \leq \ldots \leq c_n$. For the remaining of the analysis we restrict ourselves to the case when the inequalities are strict, namely $c_1 < \ldots < c_n$, because as illustrated in Proposition 2.3.2, under this assumptions there is only one producer setting the price of the auction.

In this framework, the cost revealing strategy, $p_i = c_i$, is never an equilibrium. Indeed, let p_l be the bid satisfying (2.1). For suppliers in $G(p_l)$ there are no profitable deviations, as lowering the bid would yield negative profits. Let q be the supplier such that $p_l = c_q$, then supplier k in $L(p_l) \cup \{q\}$, has incentive for changing its bid to $p_k = c_q + \frac{c_{q+1} - c_q}{2} < c_{q+1}$. Therefore, the bidding profile where producers bid their true cost is not an equilibrium.

One may be inclined to think that the shifted cost revealing strategy, $p_i = c_{i+1}$, where $c_{n+1} = \mu$, and μ is the upper bound on all bids, is an equilibrium. Unfortunately, this strategy won't necessarily be an equilibrium, as the dependency of demand on price implies that suppliers may not necessarily want to increase price.

The next proposition presents an strategy that is an equilibrium when suppliers all have the same capacity.

Proposition 2.3.3. *Assume all suppliers have capacity* $k_i = k$*. Let* $p_i^* = \arg\max_{p \leq c_{i+1}} \{(p - c_i)(D(p) - (i-1)k)\}$*. Let* p_l *be the solution of (2.1) for the bidding profile* $\vec{p} = (p_1^*, \ldots, p_n^*)$*, and* l *the supplier bidding* p_l*. Then, the bidding profile:*

$$p_i = \begin{cases} p_i^* & \text{for } i \in \mathcal{N} \setminus G(p_l) \\ c_i & \text{for } i \in G(p_l) \end{cases}$$

is an equilibrium.

Proof. Given that p_i^* is the result of a maximization problem over a compact set, the existence of p_i^* is guaranteed and the concavity assumption implies a unique, well-defined p_i^*. Notice that under the concavity assumption $p_i^* \leq p_{i+1}^*$.

For suppliers in $G(p_l)$ there are no possible deviations, bidding a lower price would yield negative profits while bidding any higher won't increase profit.

For supplier l, given that he keeps its position as a leader, then it's bidding optimally, by definition. If the leader reduces its price enough to change its position, then the market-per-unit price is set by a producer $k \in L(p_l)$. Therefore, the consumer price, p_k^*, is such that $p_k^* \leq c_l$, and yields non positive profits to supplier l. If the leader increases its price enough to change its position, then a producer in $G(p_l)$ takes its leader position and supplier l is not allocated any demand. Therefore, supplier l has no incentive to deviate.

Let j be a supplier in $L(p_l)$, its profit won't depend on the bidding price unless it replaces supplier l, or a supplier in $G(p_l)$. If it substitutes a supplier in $G(p_l)$, the new profit is zero as it would receive no allocated demand, therefore there is no incentive to deviate. If it replaces supplier l then the problem faced by supplier j is:

$$\max_{p_l^* \leq p \leq c_{l+1}} \{(p - c_j)(D(p) - (l-1)k)\}$$

The solution to this optimization problem will also be p_l^*. For this purpose, we analyze two cases.

- If p_l^* is such that $\frac{d}{dp}\{(p - c_l)(D(p) - (l-1)k)\}|_{p_l^*} = 0$, i.e. interior optimum due to the concavity assumption. Then we have that

$$p_l^* D'(p_l^*) + D(p_l^*) - (l-1)k - c_l D'(p_l^*) = 0$$

  And since $D(p)$ is non increasing, and $c_j \leq c_l$, we have that $-c_l D'(p_l^*) \geq -c_j D'(p_l^*)$.

Hence:

$$p_l^* D'(p_l^*) + D(p_l^*) - (l-1)k - c_j D'(p_l^*) \leq p_l^* D'(p_l^*) + D(p_l^*) - (l-1)k - c_l D'(p_l^*) = 0$$

Therefore by raising the price over p_l^* the payoff of the jth player decreases, and due to the concavity, the optimal strategy for this player is $p_j = p_l^*$.

- If p_l^* is such that $\frac{d}{dp}\{(p - c_l)(D(p) - (l-1)k)\}|_{p_l^*} > 0$, i.e. $p_l^* = c_{l+1}$. Then the space of the feasible solutions for the strategies of j is the singleton $\{p_l^*\}$.

Therefore supplier j receives the same price, but a smaller allocation of demand. And hence supplier j has no incentive to deviate. □

When suppliers do not necessarily have the same capacities, some non intuitive deviations can occur. For example, consider the case of four suppliers with the following characteristics:

- Costs: $c_1 = 1$, $c_2 = 2.45$, $c_3 = 2.5$ and $c_4 = 4$.
- Capacities: $k_1 = 10$, $k_2 = 2$, $k_3 = 1$ and $k_4 = 1$.
- Demand: $D(p) = 16 - p$

The strategy presented in Proposition 2.3.3, $\vec{p} = (2, 2.5, 3.25, 4)$, yields profits of \$22.5 for producer 1 which is allocated 10 units of demand, profits of \$1.6 for producer 2 which is allocated 2 units of demand, profits of \$0.5625 to producer 3 which is allocated 0.75 units of demand, and producer 4 is not allocated any demand. However, producer 1 may increase his profit by bidding a higher price, in exchange for serving less demand. Indeed, a bidding profile $\vec{p} = (6.5, 2.5, 3.25, 4)$ yields profits of \$30.25 for producer 1, and a demand allocation of 6 units of demand. This deviation increases profits for producers 2, 3 and 4 to \$8.1, \$4 and \$2.5, respectively. This sort of deviation involves giving up a portion of demand in exchange for the opportunity to increase consumer price. This deviation is only profitable for producers that have relatively large capacity.

To analyze the general case, where producers have different capacities, let $b^0 = (b_1^0, \ldots, b_n^0)$, defined as:

$$b_i^0 = \begin{cases} p_i^* & \text{for } i \in \mathcal{N} \setminus G(p_l) \\ c_i & \text{for } G(p_l) \end{cases}$$

where p_l is defined by Equation (2.1) and the bidding profile b^0, and l is the producer such that $b_l^0 = p_l$. Producer i's profit under this allocation is:

$$u_i(b^0) = \begin{cases} (p_l - c_i)k_i & \text{for } i \in L(p_l) \\ (p_l - c_l)(D(p_l) - \sum\limits_{j \in L(p_l)} k_j) & \text{for } i = l \\ 0 & \text{for } i \in G(p_l) \end{cases}$$

Let S the set of providers who can make a profitable unilateral deviation from the bidding profile b^0. Note that $S \subseteq \{1, 2, \ldots, l\}$, since for the providers $i > l$ offering a lower price will yield negative profit as the price they would be paid would be less than or equal to c_{l+1}. If $S = \emptyset$ then b^0 would be an equilibrium, since no player has a profitable deviation. Consider the case $S \neq \emptyset$. Unfortunately, in this case b^0 is not an equilibrium, however, we can modify it to become one.

Theorem 2.3.1. *Assume $S \neq \emptyset$. Denote l the producer whose bid satisfies* (2.1) *for the bidding profile b^0. Given $i \in S$, denote:*

$$p_i^j = \arg \max_{b_j^0 \leq p \leq c_{l+j+1}} \{(p - c_i)(D(p) - \sum_{\substack{v=1:l+j \\ v \neq i}} k_v)\}$$

Consider:

$$\begin{aligned} \sigma(i) \in \arg \quad \max \quad & (p_i^j - c_i)(D(p_i^j) - \sum_{\substack{v=1:l+j \\ v \neq i}} k_v) \\ s.t. \quad & j \in S(i) = \{j | D(p_i^j) > \sum_{\substack{v=1:l+j \\ v \neq i}} k_v\} \end{aligned}$$

Let $\tau \in \arg\max_{i \in S}\{p_i^{\sigma(i)}\}$. The bidding profile $b^\tau = (b_1^\tau, \ldots, b_n^\tau)$, defined by

$$b_i^\tau = \begin{cases} p_i^*, & \text{for } i \leq l, i \neq \tau \\ p_\tau^{\sigma(\tau)}, & \text{for } i = \tau \\ c_i, & \text{for } i > l, \end{cases}$$

is an equilibrium.

Observe that p_i^j in the statement of Theorem 2.3.1, is the optimal bid for producer i if it becomes leader by bidding over supplier j. Similarly, $S(i)$ is the set of suppliers such that supplier i can bid over them and still have allocated demand . In addition, supplier i maximum profit from a deviation, is obtained by bidding over supplier $\sigma(i)$. Finally, τ is the producer, among those with profitable deviations, whose most profitable deviation yields the highest price to the consumer, and therefore, the highest per-unit price for all producers.

Theorem 2.3.1 states that such deviation is an equilibrium.

Proof. Given that $S \neq \emptyset$, and $\forall i \in S$, $S(i) \neq \emptyset$, then there exists a solution $\sigma(i)$ for all $i \in S$. Hence, there exists a $\tau \in \arg\max_{i \in S}\{\sigma(i)\}$.

Notice that $\sigma(\tau) > l \geq \tau$, as producer τ is taking the position of leader. Therefore, producer τ's bid defines the per-unit price that is paid to providers $j = 1, \ldots, \sigma(\tau)$. Observe that b^τ is different from the previous profile only on the component τ. To check that the bidding profile b^τ is an equilibrium, we analyze profitable deviations for supplier i:

- If $i = \tau$, given that $\sigma(\tau)$ is chosen as

  $$\arg\max\{(p_\tau^j - c_\tau)(D - \sum_{\substack{v=1:l+j \\ v \neq \tau}} k_v) | j \in S(\tau)\},$$

  by definition, provider τ cannot do better by deviating from the profile b^τ.

- If $i \geq \sigma(\tau) + 1$, then supplier i won't be better of by changing its bid, as then it would get negative profit since $p_\tau^{\sigma(\tau)} \leq c_{\sigma(\tau)+1}$.

- If $l < i \leq \sigma(\tau)$, then given the definition of l, we have that $\sum_{j=1}^{l-1} k_j < D(p_l^*) \leq \sum_{j=1}^{l} k_j$. Given that demand is decreasing, we have that if supplier i bids above producer τ, no demand is allocated to it. If supplier i bids any lower than $p_\tau^{\sigma(\tau)}$, supplier's i profit is not changed. Therefore, there is no incentive to deviate.

- If $i \leq l$ and $i \neq \tau$, supplier i gets allocated its full capacity k_i. To achieve a higher profit provider i has to bid higher than provider τ otherwise the price does not change, and therefore if it were allocated any demand it would be a fraction of its capacity. If that were to be the case, the profit cannot be higher than the most profitable unilateral deviation for i, the one obtained by bidding $p_i^{\sigma(i)}$. To see this, denote $v(i)$ the profit that supplier i obtains when deviating to $p_i^{\sigma(i)}$. Given the choice of τ, then $p_i^{\sigma(i)} \leq p_\tau^{\sigma(\tau)}$, and therefore $(p_i^{\sigma(i)} - c_i)k_i \leq (p_\tau^{\sigma(\tau)} - c_i)k_i$. Hence $v(i) \leq (p_\tau^{\sigma(\tau)} - c_i)k_i$. Therefore i has no incentive to deviate.

□

Revisiting the example presented prior to Theorem 2.3.1, it was shown that the allocation $\vec{p} = (2, 2.5, 3.25, 4)$ had a profitable deviation for producer 1. In fact, the set of producers with a profitable deviation is $S = \{1, 2\}$, as a deviation to $\vec{p} = (2, 3.725, 3.25, 4)$ increases profits for producer 2. In this example, producer 1 is able to bid over producers 2, 3 and 4 and still be allocated demand, therefore $S(1) = \{2, 3, 4\}$. If producer 1 decides to take the position of producer 2, 3 or 4, then the optimal bidding price for producer 1 is $p_1^2 = \$2.5$, $p_1^3 = \$4$ or $p_1^4 = \$6.5$ respectively. By computing the profits, it is easy to see that producer 1 benefits the most from bidding above producer 4, therefore $\sigma(1) = 4$, and $p_1^{\sigma(1)} = \$6.5$.

In the example, producer 2 can only benefit from bidding above producer 3, therefore, $S(2) = \{3\}$. The optimal bid for producer 2, if it becomes leader, is $p_2^3 = \$3.725$. Finally, as there is only one candidate, $\sigma(2) = 3$, and $p_2^{\sigma(2)} = \$3.725$.

The largest $p_i^{\sigma(i)}$ corresponds to producer 1, therefore, Theorem 2.3.1 states that the bidding profile $b^1 = (6.5, 2.5, 3.25, 4)$ is an equilibrium.

Theorem 2.3.1, constructs an equilibrium when producers have different capacities, and the strategy from Proposition 2.3.3 fails to be an equilibrium. The intuition behind the strategy is that a producer that may benefit from letting other producers undercut him, will do so, reaching a point where profit is maximized. Interestingly, this strategy increases the number of suppliers serving market demand, as well as the price to the consumer.

2.4 Conclusions

The objective of this work is to identify producers' optimal bidding strategies in a market where a system operator allocates demand based on price-per-unit submitted – the lowest price provider will satisfy demand up to its capacity and remaining demand is allocated to the second low-price provider until its capacity is exhausted and so-on. The price-per-unit paid to each of the suppliers is the same, and equal to the largest price bid placed by an assigned supplier. Under a full information framework, we are able to characterize the optimal bidding strategies for producers. This framework is appropriate for situations where producers repeat multiple times the auction process. For example in the electricity market, auctions are repeated periodically, and given the high capital requirements, the set of market participants does not change much from auction to auction.

Under the assumption of equal capacities the optimal bidding strategy for a producer is intuitive. Each producer will bid the price that maximizes his/her profit, under the constraint that the producer won't be undercut by a producer with a higher cost. This strategy guarantees that if the producer sets the price, then he/she is receiving optimal profits, alternatively if the producer is not setting the price, there is no incentive to change the bid. A direct implication of this strategy is that producers will receive profits by biding above their marginal cost, this is in contrast with the case of a regulated market. However, the profit per unit is limited, as a high bid will be undercut by producers willing to satisfy demand. Observe that due to the nature of the pricing mechanism, there exists multiple equilibria that achieve the same consumer price and allocation of demand.

When the equal capacity assumption is removed, interesting deviations show up. Producers with large capacities are able to exercise market power. By letting other producers undercut them, producers with large capacities are able to raise their bid to further increase profits. This exercise of market power is particularly interesting as it increases the number of producers participating in the market, yet the market is not behaving as a competitive market. Moreover, given that the deviation increases profits for all producers, it may be confused with collusion.

Chapter 3

Emissions Regulation In The Electricity Market

3.1 Introduction

Businesses often face regulations imposed by governments or business partners when making production decisions. These regulations are set in order to achieve some external objectives, for example reduce pollution or limit consumption of a scarce resource such as water. Controlling the external objective can be achieved in different ways, for instance through price penalties, i.e., taxation (see Pigou (1920)), or by permit regulation, that is, the enforcement of a quota associated with the external objective through permits (see Coase (1960)). Thus, by changing the "rules of the game," governments or business partners let players optimize their profit, and at the same time achieve the external objective (see Mas-Colell and Green (1995)).

Our objective in this chapter is to address the following question. In an environment where the production process delivers an undesirable effect, such as waste or overuse of a scarce resource, which is the better way to regulate production in a decentralized fashion: incorporate a fixed penalty similar to a tax, or require manufacturers to purchase permits that limit the amount of the undesired effect?

Specifically, we are trying to identify conditions under which one approach is preferable to the other, from various points of views: manufacturers, consumers and the central planner. For this purpose, consider a model with a number of producers that compete in the market by delivering the same kind of product or service, albeit may use different technologies. Given a market price, each producer determines the amount she is willing to produce by solving an optimization problem associated with revenue and costs. This optimization problem defines the producer's supply function which is the capacity she is willing to provide the market as a function of market price for the finished product.

We assume that customer demand is linear with price. For each manufacturer the level of externality, for example, emission level, is linear with the amount produced, while manufacturing cost is quadratic with this amount. Such quadratic cost functions are known to well approximate supply chain costs (Samuelson (1970)).

To establish the market price paid by consumers, as well as the quantities that each producer delivers to the market, we search for the market equilibrium. This equilibrium is achieved at the point of agreement between the demand function and aggregated supply function, aggregated across all manufacturers. When the aggregate supply and demand agree, the market price and quantities produced are determined. This is the common theme that we apply, putting emphasis on how regulation affect the market's equilibrium price, manufacturers profits and the external objective.

We consider two types of regulations to be imposed in this market: tax regulation and permit regulation. In tax regulation we impose a monetary penalty proportional to the amount of the undesired product. By contrast, in permit regulation we allow manufacturers to purchase permits that limit the amount of waste produced, or the use of scarce resource. We consider three different types of permit regulations: (i) cap regulation, where manufacturers have an ability to purchase permits; (ii) cap-and-buyback, where manufacturers have an ability to purchase permits and sell surplus permits back to the central planner at the end of the horizon; and (iii) cap-and-trade, where manufacturers have an ability to purchase permits as well as trade permits with other manufacturers. Our objective is to compare the performance of each of these permit mechanisms.

Our starting point is to determine how regulations perform in a setting where producers can anticipate customer demand, i.e., in a deterministic setting. We show, in tax regulation, the existence of a price penalty that allows the central planner to achieve the desired external objective. Since in permit regulations, manufacturers will purchase exactly the permits they need, the three versions of permits regulations are all equivalent. More importantly, *we show that under deterministic setting the two regulation policies, tax and permit regulations, are equivalent from manufacturers, consumers and the central planner points-of-views.* That is, whatever is achieved in the tax regulation can also be achieved under the permit regulation and hence from this model point of view, it is not possible to recommend one regulation policy over the other.

However, under stochastic demand the situation is different and more challenging. First, in tax regulation, the central planner needs to identify the tax level that achieves the external objective on expectation. This is a challenge since demand uncertainty affects market price and thus it affects production decisions. Second, in permit regulation, and unlike the deterministic counterpart, producers purchase permits before observing demand but may not use all their permits. This implies that the three permit regulations, cap, cap-and-buyback and cap-and-trade, yield different performance measures. To evaluate the four strategies, we analyze the impact of a given tax rate or permit price on expected externality level, expected price faced by consumers, expected revenue collected by the central planner and manufacturers profits.

We consider two types of competition between manufacturers. In the first one, we focus on price taking firms, that is, manufacturers that do not consider the effect of their supply functions on market price, while in the second one we focus on those that take into account the effect their supply functions have on market price. These types of competition have been analyzed extensively in the literature, see Section 3.1.1. In the price taking firms case, we explore the difference between tax regulation and permit regulations such as cap and cap-and-buyback. In the price making case, we directly compare tax and cap-and-trade regulations.

Surprisingly, *we show that under the stochastic model, selling permits to control the external objective is identical to finding an appropriate tax level in terms of expected externality level and expected price to the consumer.* This implies that from the consumer price and external objective points of view there is no difference between tax regulation and the three permit regulations discussed earlier.

Manufacturers preference of regulations depends on their technology and cost structure. Interestingly, under tax regulation, *manufacturers with the cleanest technology are able to increase expected profit as taxation rate increases,* while producers with the worst technology (e.g., high emission rate) always prefer tax regulation over cap or cap-and-buyback. By contrast, the central planner expected revenue is always higher under cap and cap-and-buyback than under taxation.

This analysis is motivated by discussions among policy makers, industry executives and economists regarding effective regulations to reduce carbon emission. Some governments are increasingly taking unilateral legislative steps to enforce compliance, Simchi-Levi (2010). For example, on July 1st, 2012, Australia introduced a fixed price carbon tax policy, transitioning to a cap-and-trade emission trading system on July 1st, 2015, see Future (2012). This suggests that at least some governments prefer permit over tax regulation. The academic community has focused on various regulation policies in the last four decades, with a large portion of the research being conducted after the implementation of the Kyoto protocol. We provide a brief review of this and related research in the next section.

3.1.1 Literature Review

Our research is directly related to the seminal work by Weitzman (1974) who answered the following question: Given that an organization wants to control a specific outcome, should they administer the activity by specifying a production level, or just fix transfer prices and rely on self-interested profit or utility maximization to achieve the same objective. Weitzman's starting point is that while under some conditions these mechanisms are theoretically equivalent, organizations and economists seem to favor one over the other. He first developed a model that shows that in the absence of uncertainty there is no difference between the two planning instruments. But, if there is uncertainty in the benefits or the costs, then it is possible to estimate the difference of the expected benefits minus cost between the two approaches and therefore decide which one is more appropriate.

Weitzman's approach is different than our approach in at least two ways. First, we model competition between manufacturers while Weitzman model a single organization. Second, the central planner in our model controls tax rate or initial permit price but not production quantities or market price, while the central planner in Weitzman's model control market price or quantity produced by the organization. Thus, our perspective is different from Weitzman's, as we are interested in operational implications of the regulation policy, for the producers, the central planner and consumers, as opposed to a single performance measure (expected benefit minus cost), as in Weitzman.

The work of Weitzman motivated significant follow up research; a review can be found in Hepburn (2006). Most notably, regarding green house gas emissions, Pizer (1997) builds on Weitzman's research, and uses a global integrated climate economy model to simulate different scenarios at the macroeconomic level. Pizer estimates the net welfare gains of alternative tax and permit policies for different values of tax and cap levels. He then compares the optimal tax instrument with the optimal permit policy, finding that a tax policy generates greater social welfare. Pizer's result is influenced by the fact that permits are allocated at no cost to the manufacturers, generating additional profit for the producers at expense of the consumers.

More recently, Carmona et al. (2010) introduced a model where manufacturers have an initial allocation of permits and produce goods to satisfy inelastic customer demand. Trading of permits enables firms to increase production level or generate revenue by selling permits to others. They show the existence of market equilibrium that specifies permit trading price as well as the presence of windfall profit, a phenomena observed in practice after the implementation of the Kyoto protocol. Our approach is different than their approach because in our model demand is elastic and market price for goods is determined by demand and supply.

Design of an effective permit regulation has also attracted some attention. For instance, Zhao et al. (2010) study the effect that different permit allowance allocation systems have on the long run market equilibrium. By solving nonlinear complementarity problems, the authors study whether permits should be purchased from their regulators, grandfathered in fixed amounts free of charge based on past (emission) performance, or based on current output, investments or other decisions.

Carbon emission constraints have also been studied in the area of operations management with regards to procurement, production and inventory management. Benjaafar et al. (2011), study how emission control policies affect the cost and emissions in traditional inventory models, as well as the impact of collaboration among firms within the same supply chain. The authors investigate how operational decisions can work as an alternative to investments in technology.

Models representing supply function competition have been introduced and extensively analyzed in the literature. The use of supply functions allow producers to adapt better to an uncertain environment. In this model, we distinguish between two settings: price taking firms settings and price making firms settings.

In the price making firms setting, we follow the ideas presented by Klemperer and Meyer (1989), where an oligopoly faces uncertain demand and each firm chooses as its strategy a supply function, a function mapping market price to production, given the supply functions of the other producers. Under uncertain demand, the authors prove the existence of a Nash equilibrium for the supply functions in a symmetric oligopoly, i.e., a model where all producers have the same cost function, and provide sufficient conditions for its uniqueness. Additionally, the authors show that the steeper the equilibrium supply functions, the more closely competition resembles the Cournot competition model; by contrast, the flatter the equilibrium supply functions, the closer the competition is to a Bertrand competition model.

By contrast, in our price taking model, producers submit a supply function by maximizing profits given a fixed market price. This choice of supply functions have been used in the literature. For example, Dixon (1992) proposes a modification and generalization of Edgeworth's price setting duopoly model, which yields the competitive outcome as the only possible equilibrium. In his paper, the model provides an account of price-setting firms behaving as if they were price-taking firms, and setting the competitive price. The supply function resulting from his model is the same as the one used in our price taking firms setting. For additional examples see Maskin (1986) or Klemperer and Meyer (1989).

Supply functions approximate well the electricity market. Green (2007) uses a supply function model to predict electricity producers' profits, given input costs and the level of demand. The author argues, similarly to Green and Newbery (1992), that supply function equilibrium is a close approximation to the mechanism of an electricity auction. Green then compares the effect an emission trading scheme and a carbon tax policy would have in the United Kingdom electricity market. For this purpose, the prices of gas, coal, and oil were modeled as stochastic random variables with means equal to the Department of Trade and Industry's central predictions for 2020. In the model runs for the emission trading scheme, the price of carbon is set to vary randomly around the level that would equalize the cost of electricity generation from a coal-fired and a gas-fired station. Green argues that a price of carbon will vary around that level, as it keeps plants of both types, coal and gas, on the margin. By estimating an appropriate price per tone of carbon dioxide, the author uses a carbon tax of £20.18 per tonne. Green concludes, empirically, that carbon tax reduces the volatility faced by nuclear generators, but raises the volatility faced by fossil fuel stations, and consequently optimal portfolios would contain a higher proportion of nuclear energy if a carbon tax was adopted.

Finally, much like Baldick et al. (2004), we focus on an electricity market with a linear demand function and producers with quadratic costs. Interestingly, Baldick et al. (2004) show that a supply function equilibrium model, that is, a model involving price making firms, approximates the England and Wales electricity market better than previous models.

3.2 Model setting

Consider n producers serving a market demand for an homogeneous good. Each producer, i, has a strictly quadratic cost of production, $c_i(q) = \frac{1}{2}a_i q^2$ for delivering q units. Associated with each producer is a technology parameter t_i, which is the rate at which an undesired effect, for example carbon dioxide emissions, is produced. In our model, technology has a linear effect on the externality produced, that is, producer i will have an impact of $t_i q$ when producing q units.

In our deterministic setting, we consider a linear demand, $D(p) = B - Ap$, which is a decreasing function of the price paid by consumers. Each supplier submits a supply function s_i to the central planner, which determines the amount of demand the producer is willing to satisfy at a given price. That is, given a market price p, supplier i is willing to provide $s_i(p)$. Market equilibrium is achieved when demand, which is a function of price, equals the aggregate supply at that price, defining the price faced by consumers. More precisely, market price is defined by the equation:

$$\sum_{i=1}^{n} s_i(p) = D(p).$$

In the model under uncertainty, we consider demand, $D(p, \theta) = B - Ap + \theta$, to depend on an additive noise, θ, representing an uncertain market size. We assume θ to have a lower bound, meaning $\theta \in [\theta_0, +\infty)$ almost surely. In this setting, each producer submits a supply function before demand is realized. Price is determined after demand is realized by equalizing the aggregate supply function with realized demand. More precisely, for a given realization of the random variable θ, and the aggregate supply function $\sum_{i=1}^{n} s_i(p)$, the realized equilibrium price $p(\theta)$ is such that:

$$\sum_{i=1}^{n} s_i(p(\theta)) = D(p(\theta), \theta). \tag{3.1}$$

The concept of market equilibrium that is applied in both settings, deterministic and stochastic demand, can be stated as follows,

Definition 3.2.1. *Consider n producers, each producer i with cost function $c_i(\cdot)$, technology t_i, and a (stochastic) demand function D. A market supply function equilibrium price p^* is a price such that:*

- *At price p^*, (realized) demand is equal to the aggregate supply of producers for a price p^*.*
- *The supply function $s_i(p)$ maximizes the (expected) profit of producer i.*

We consider two different models representing the way suppliers determine their supply functions: price taking settings and price making settings. In the price taking setting, suppliers maximize their respective profits considering a fixed market price, thus, in this case producers do not take into account the effect their submitted supply function has on changing market price. By contrast, in the price making setting, suppliers strategize over their supply functions, taking into account the effect their supply functions, and other producers supply functions, have on market price.

Following Arrow (1986) we identify two aspects in which a price taking model is beneficial. First, computing a manufacturer's supply function requires only knowledge of its own cost, not information about the supply functions of other producers. Second, under this setting we are able to characterize the supply functions, and hence the optimal decisions for asymmetric firms with minimal assumptions on the random noise. This setting is appropriate when all manufacturers represent small electricity producers.

The advantage of considering price making producers is of course the ability to capture market power. Indeed, some electricity markets feature a small number of producers supplying the majority of electricity. In these markets, where producers decisions have a noticeable effect on market price, a model that captures market power, that is a price making model, is appropriate. The challenge in this setting is that each producer strategy depends on other producers behaviors, and in order to obtain analytical solutions, more restrictive assumptions are needed, in particular regarding the random noise. Moreover, under deterministic demand there is no uniqueness of equilibria, see Klemperer and Meyer (1989).

Two methods for regulation are considered: tax regulation and permit regulation. In tax regulation, the central planner announces a tax rate ξ per unit of externality produced. To illustrate, producers total cost of delivering q units in this setting would be: $c_i(q) + \xi t_i q$. Producers then decide on their supply function and submit it to the central planner. The central planner then collects the supply functions of producers and balances (realized) market demand, finding an equilibrium price.

In permit regulation, the central planner announces a permit price π that allows suppliers to generate units of the externality. Prior to satisfying demand, suppliers are required to purchase permits in order to participate in the market. To illustrate, if supplier i purchases x_i permits, then the cost of delivering q units in that period is: $c_i(q) + \pi x_i$, and q is limited to no more than $\frac{x_i}{t_i}$ units. We consider three different types of permit regulation policies, depending on the role of permits after they are purchased:

- **Cap regulation**: Permits can only be used by the producer that purchased them, and have no value at the end of the period.

- **Cap-and-buyback**: Permits can only be used by the producer that purchased them. Additionally, the central planner announces a buyback price for the permits, producers can sell surplus permits back to the central planner at the end of the period.

- **Cap-and-trade**: Permits can be traded among producers after seeing realized demand, but prior to serving demand, and have no value at the end of the period.

In this study, we aim to compare the effects of tax regulation and cap-and-trade regulation, the most popular emission trading system implemented in practice. In a cap-and-trade regulation, we can distinguish two features that affect producers decisions: an investment in permits prior to production, and a market where permits are traded. In our analysis of permits regulations, we start by isolating the effect of investment in permits, i.e. cap regulation, in a price taking setting, as a stepping stone to understand cap-and-trade. Then we introduce cap-and-buyback as a bridge between cap regulation and tax regulation. Finally, we compare cap-and-trade and carbon tax in a price making setting.

Observe that in the cap-and-buyback regulation policy, the buyback price plays an important role. As this price increases, suppliers risk of buying too many permits, decreases, and hence they are willing to purchase more and more permits. This implies that for high buyback price, producers are not limited by permits and hence, in this case, this model acts like a tax regulation model. Of course, when the buyback price is zero, the model is equivalent to the cap policy. Thus, cap-and-buyback is a model that includes tax policy and cap policy as two special cases.

Throughout the analysis we assume the tax rate is small enough so that producers participate in the market almost surely. Similarly, the permit price is not too high to prevent producers from purchasing at least some permits. This assumption can be satisfied by imposing conditions to the lower bound of the noise, θ_0. The assumption mainly states that the regulation is not strict enough to exclude suppliers from the market and is required in order to obtain results that, as we shall see in Sections 3.3 and 3.4, do not depend on the distribution of the noise.

3.3 Price Taking Firms

We start by considering the case of price taking firms. In this setting, firms do not take into account the effect their submitted supply functions have on changing market price. To illustrate, consider the case where no regulation is imposed. When finding the optimal value for the supply function given a market price of p, producer i solves the following concave optimization problem:

$$s_i(p) \in \arg\max_{q \geq 0} \{pq - c_i(q)\}. \tag{3.2}$$

In the price taking firms setting, the first order conditions to the problem, used to determine the optimal value for the supply function for price p, are defined by:

$$p - c_i'(s_i(p)) = 0.$$

There are two frameworks to be considered: deterministic demand and stochastic demand. This section starts by first stating the results obtained under deterministic demand, afterwards results for stochastic demand are developed illustrating the differences between these two settings.

3.3.1 Deterministic demand framework

In the absence of regulation, the existence and uniqueness of a market equilibrium price are guaranteed for price taking producers facing a deterministic demand. This holds true even for a more general set of assumptions, see Appendix 3.D.

Under tax regulation, the monetary penalty affects production costs and hence limits the level of externality. By contrast, in permit regulations, once permits are acquired, they serve as a hard constraint on the level of externality. Theorem 3.3.1 summarizes our results for deterministic model.

Theorem 3.3.1. *Given a target externality level X identified by the central planner, the following propositions hold:*

1. *Let $s_i(p)$ be the optimal supply function for producer i in the absence of regulation. Under tax regulation with a tax rate ξ, the new optimal supply function for producer i is $s_i(p - t_i\xi)$. The larger the tax rate, the higher the market equilibrium price.*

2. *If $X < \sum_{i=1}^{n} t_i q_i$, where q_i is the production level of producer i in the absence of regulation. Then there exists a tax $\xi > 0$ such that the new market equilibrium under tax regulation achieves X. The larger target externality level, the smaller the tax rate required.*

3. *The central planner can achieve a target externality level, X, under a permit regulation, by setting the permit price to π equal to the tax rate that achieves the same target, ξ. In addition, the following is true:*

    - *The permits purchased by producer i are $t_i q_i^t$, where q_i^t are the production quantities in market equilibrium under tax regulation.*

    - *Producers consume all of the purchased permits.*

    - *The consumer price under permit regulation is the same as the consumer price resulting from a tax regulation with a tax rate ξ.*

Theorem 3.3.1 is proven in Appendix 3.A where we show that the results hold under strictly convex cost functions.

The first property illustrates the effect tax regulation has on the supply function submitted by producers. In order to determine the optimal supply function for a specific producer, it's enough to shift the supply function under no regulation, and the magnitude of the shift depends on producer's technology. This property guarantees that properties of the original supply functions (e.g. convexity, monotonicity or linearity) are not affected by the tax rate. It also presents a first indication of how asymmetry in technology affects producers when regulations are in place. The first property also indicates that market (consumer) price increases with the tax level, but the property is silent about producers profit. Indeed, the effect of tax on producer's profit can not be established in general, as it depends, not only on consumer price but also on the technology and cost of structure of each producer.

The second property indicates that the target externality level can be achieved with a tax policy.

Finally, the third property suggests that in order to achieve the central planner's external objective, tax regulation or permit regulation can be implemented. Moreover, both regulations achieve the same consumer price, quantities produced, penalty paid and producer profit. This equivalence is a consequence of the ability of producers to perfectly anticipate demand, and purchase the exact amount of permits required, a characteristic that is valid in a deterministic setting.

The equivalence between tax regulation and cap-and-trade policy under deterministic setting motivates an analysis of a stochastic demand model, where producers are forced to make an investment in permits prior to the realization of demand, and therefore we can expect to see a difference between permit regulations and tax regulation.

3.3.2 Stochastic demand framework

In this section we compare the impact of tax and permit regulations on market equilibrium under uncertain demand. For this purpose, we develop four propositions. Proposition 3.3.1 analyzes how a particular tax rate ξ affects the market under tax regulation. Next, Proposition 3.3.2 analyzes the effect a permit price π has on the market under a cap regulation. The third proposition, Theorem 3.3.2, examines how the two policies, cap regulation and tax policy, compare to each other. Specifically, given a tax and a permit price that induce the same target expected externality level, we analyze the effect these two policies on expected market price, expected manufacturer profit as a function of manufacturer's cost and technology, and the central planner revenue. The final proposition of this section, Proposition 3.3.3, analyzes the effects of offering producers a buyback price for unused permits, the so called cap-and-buyback regulation.

Given uncertain demand, and using the assumptions stated in Section 3.2, one can establish the following expressions for the relevant performance measures under a tax regulation:

Proposition 3.3.1 (Tax Regulation)**.** *Consider a tax regulation under stochastic demand. Let ξ be the tax price. Then:*

- *The realized price to the consumer, $p(\theta)$, is a linear function of the uncertainty, θ.*
- *The expected price to the consumer is:* $\mathbb{E}[p] = \frac{B+\xi\sum_{i=1}^{n}\frac{t_i}{a_i}}{A+\sum_{i=1}^{n}\frac{1}{a_i}}$.
- *The expected externality level is:* $\mathbb{E}[e] = \frac{\sum_{i=1}^{n}\frac{t_i}{a_i}(B+\xi(\sum_{j=1}^{n}\frac{t_j-t_i}{a_j}-t_iA))}{(A+\sum_{i=1}^{n}\frac{1}{a_i})}$.

- *The expected profit for producer i is:* $\mathbb{E}[\Pi_i] = \frac{1}{2a_i}\mathbb{E}[(p(\theta) - \xi t_i)^2]$.

- *Producer's i expected profit is increasing in ξ if and only if:* $\sum_{j=1}^{n} \frac{t_j - t_i}{a_j} > t_i A$.

- *The expected revenue collected by the central planner is a concave function of ξ.*

The first property of the theorem is key in order to obtain expressions for expected price and expected externality level that are independent of the distribution of the noise, θ. The next three properties establish analytical expressions that we shall use to compare tax regulation and cap regulation. The last two properties are surprising and thus require more discussion.

The first among these two properties, implies that *some producers can face increasing profits as the tax rate increases.* This is explained by the fact that as the tax rate increases, consumer price increases, and therefore manufacturer's revenues may increase. If the increase in revenues dominates the manufacturer's increase in cost, then a producer may experience increasing profits with higher tax rates. The conditions identified in this property, $\sum_{j=1}^{n} \frac{t_j - t_i}{a_j} > t_i A$, implies that *if any manufacturer is going to face an increase in profit, it is the producer with the cleanest technology.* In addition, an immediate corollary of this expression is that not all producers receive increasing profits, as the producer with the highest amount of emissions per unit produced is never going to face an increase in revenue as the tax rate increases.

The last property of Proposition 3.3.1 suggests that increasing tax price does not always lead to higher revenue for the central planner in exchange to lower profits for producers. Indeed, concavity in the central planner revenue can lead to situations where increasing the tax beyond a certain level lowers expected profits for both, manufacturers and central planner. This phenomenon motivates the idea that *subsidies by the central planner can be potentially more beneficial than increasing tax rates* in certain circumstances. We discuss this property further in our concluding remarks.

Having stated the main result for tax regulation, what follows are similar results for permit regulations. We begin by developing a similar proposition for cap regulation.

Proposition 3.3.2 (Cap Regulation)**.** *Consider a cap regulation and uncertain demand. Let π be the permit price, then:*

- *Let p be the realized consumer price and δ be the indicator function. Then the optimal amount of permits purchased by producer i is $x_i = \frac{t_i p(\theta_i)}{a_i}$, where θ_i satisfies:*

$$\mathbb{E}[(p(\theta) - p(\theta_i))\delta(\theta > \theta_i)] = \pi t_i. \tag{3.3}$$

- *The total amount of permits purchased,* $\sum_{i=1}^{n} x_i$*, is decreasing on the permit price,* π*.*

- *The realized price to the consumer,* $p(\theta)$*, is a convex piecewise linear function of the uncertainty,* θ*.*

- *The expected price to the consumer is* $\mathbb{E}[p] = \frac{B+\pi \sum_{i=1}^{n} \frac{t_i}{a_i}}{A+\sum_{i=1}^{n} \frac{1}{a_i}}$.

- *The expected externality level is:* $\mathbb{E}[e] = \frac{\sum_{i=1}^{n} \frac{t_i}{a_i}(B+\pi(\sum_{j=1}^{n} \frac{t_j - t_i}{a_j} - t_i A))}{(A+\sum_{i=1}^{n} \frac{1}{a_i})}$.

- *The expected profit for producer* i *is:* $\mathbb{E}[\Pi_i] = \frac{1}{2a_i}\mathbb{E}[\min\{p(\theta), p(\theta_i)\}^2]$.

- *The revenue for the central planner is:* $R = \sum_{i=1}^{n} \frac{t_i \pi}{a_i} p(\theta_i)$.

The first property of Proposition 3.3.2 suggests a way to transform the cap regulation model from optimizing on x_i to characterizing the smallest value of the uncertainty, θ, such that producer i reaches its cap. This transformation enables the characterization of various performance measures including expected market price, expected externality level, expected producer's profit and the central planner revenue.

In order to compare the policies, tax policy and cap regulation, we apply the following approach. Given a tax, that induces the same level of expected externality level as an appropriate permit price, we characterize the difference between the performance measures induced by each policy including expected market price, expected manufacturer profit and (expected) central planner revenue. This is done in the next theorem.

Theorem 3.3.2. *Consider a tax regulation with tax rate* ξ*, and a cap regulation with permit price* π *that achieve the same expected externality level under their respective regulations. We have,*

- *The tax rate has to be equal to the permit price.*

- *The expected price to the consumer is the same under both regulations.*

- *Revenue for the central planner is greater under cap regulation.*

- *The difference between cap regulation profits and tax regulation profits for producer* i *is given by:*

$$\Delta_i = \frac{1}{2a_i}\{V(\min\{p(\theta), p(\theta_i)\}) - \frac{V(\theta)}{(A + \sum_{i=1}^{n} \frac{1}{a_i})^2}\},$$

where p *denotes the consumer price in permit regulation, and* V *the variance of a random variable.*

- *Let i be a producer such that $t_i = \max_{j=1,..,n} \{t_j\}$. We have that for producer i,*

$$\Delta_i \leq 0.$$

Moreover, in the case of symmetric producers (producers with similar costs and technologies), $\Delta_k \leq 0 \quad \forall k$.

The theorem states that in order to achieve the same impact on expected externality level, it is sufficient to equalize the tax rate and the permit price (per unit of emission). We provide some intuition explaining this surprising result in Appendix 3.E.

Figures 3-1 and 3-3 illustrate the power of the theorem through numerical examples with two producers, see details in Appendix 3.B, where the x-coordinate always represents the tax level / permit price. That is, in this numerical examples we make sure that expected impact on the environment is the same under both regulations.

For example, in Figure 3-1 we observe the difference between the expected revenue collected under tax regulation (presented as blue x) and the revenue collected under cap regulation (presented as green circles) as we increase the tax/ permit price. We see that revenue collected from a cap regulation dominates the expected revenue collected from the tax policy for every tax/permit price. Concavity of the revenue can also be appreciated in the figure.

With respect to the difference in producers' profits, denoted by Δ_i in Theorem 3.3.2, these differences may be positive or negative. Figures 3-2 and 3-3 present profits, under both policies, for the producer with the best technology. In Figure 3-2, the parameters are chosen such that this producer (one with cleanest technology) experiences decreasing profits as the tax/ permit price increases. In this case, we observe a preference for tax regulation. By contrast, in Figure 3-3, parameters are chosen such that the higher the tax/permit price, the higher this producer profit. Here the producer satisfies the condition developed in Proposition 3.3.1, guaranteeing increasing profits in tax regulation. Note that in this case, the producer with best technology prefers a cap policy. Thus, the producer policy preference in general depends on cost and technology information. However, the last statement of the theorem says that in case of producers with similar costs and technology, everyone prefers a tax regulation over a cap policy. Finally, the Theorem also suggests that when producers are asymmetric, the producers having the worst technology always have a preference for a tax policy.

To conclude, the Theorem implies that *it is not possible to obtain a preference between a tax regulation and a cap policy using performance measures such as expected consumer price, expected demand satisfied, or expected externality level.* That is, there is no difference between the two policies along these three performance measures. However, there is a difference

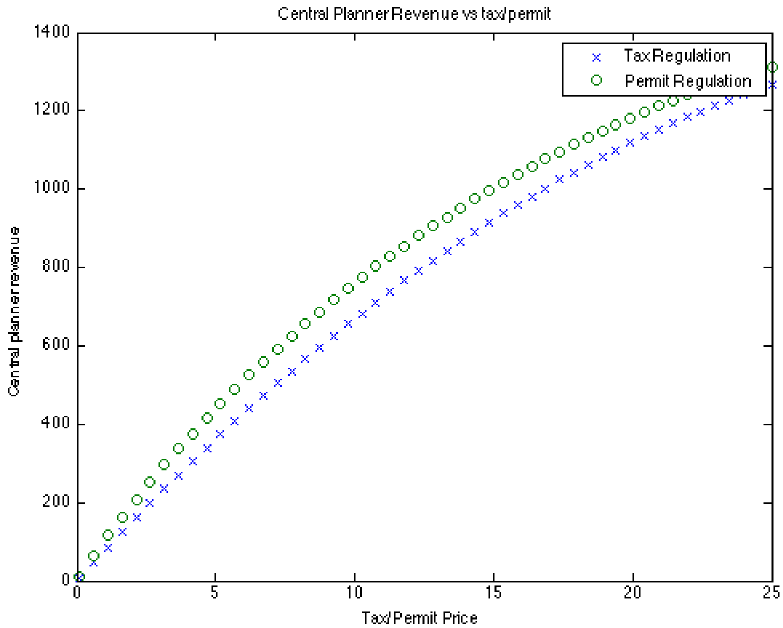


Figure 3-1: Impact on the central planner revenue for different targets of regulation.

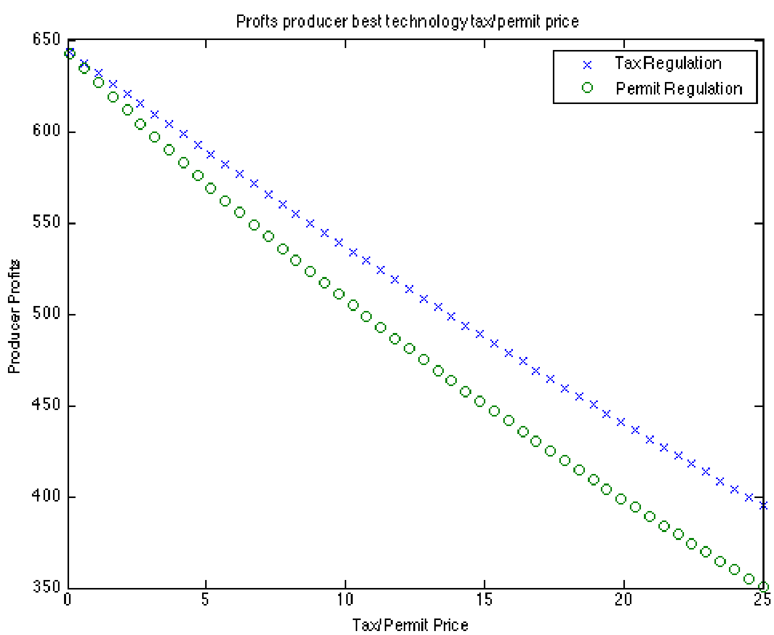


Figure 3-2: Producer showing preference for tax regulation.

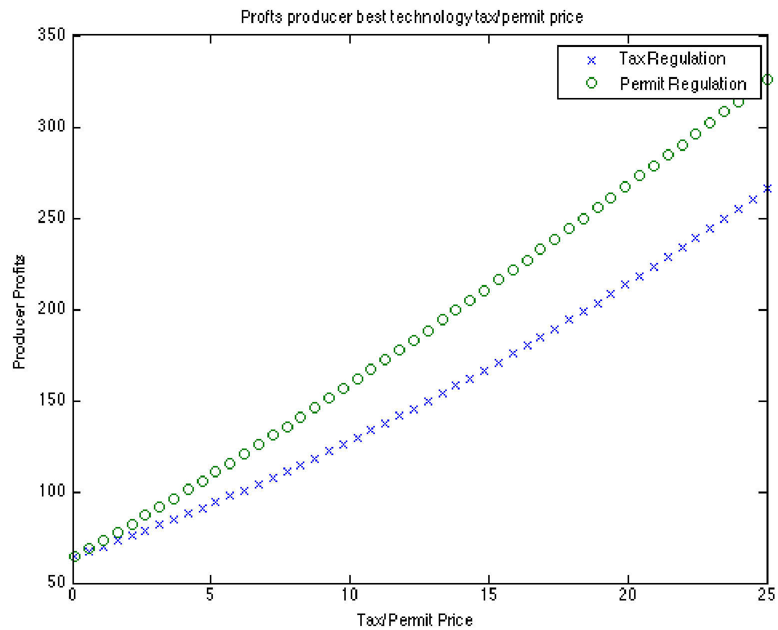


Figure 3-3: Producer showing preference for cap regulation.

between a tax regulation and a cap policy with respect to the central planner's collected revenue and the producers expected profit.

As observed in Proposition 3.3.1 for a tax policy, producer's i expected profit is increasing in ξ if and only if: $\sum_{j=1}^{n} \frac{t_j - t_i}{a_j} > t_i A$. The question is whether a similar statement is true for a cap policy. Appendix 3.B provides a counterexample that shows that this is not true in general. However, it is easy to show, by application of Jensen's (see Appendix 3.C), that there exists a lower bound to producer i's expected profit that increases linearly with $\sum_{j=1}^{n} \frac{t_j - t_i}{a_j} - t_i A$.

It is possible that a different condition exists for a cap policy which will ensure that a producer's expected profit is increasing in π. However, given the equivalence result presented in the deterministic model (see Section 3.3.1), and the counterexample of Appendix 3.B, which involves uncertain demand, one can expect that a condition guaranteeing increasing profits with higher permit price under cap regulation must depend on the distribution of the noise, θ.

Having established the main similarities and differences between tax and cap regulations, one might suggests that reducing the risk faced by producers in a cap policy will reduce the difference between a cap regulation and a tax policy. This motivates an analysis of a third regulation: Cap-and-buyback. Here the central planner announces a buyback price for unused permits, ρ, and commits to buying those permits at the end of the season, after demand is served. The following proposition, which is a counterpart for Proposition 3.3.2, establishes the values of the various performance measures under a cap-and-buyback policy.

Proposition 3.3.3 (Cap-and-buyback). *Let π be there permit price, and $\rho \in [0, \pi]$ be the buyback price for surplus permits. Then:*

- *Let p be the realized consumer price and δ be the indicator function. Then the optimal amount of permits purchased by producer i is $x_i(\rho) = t_i \frac{p(\theta_i) - \rho t_i}{a_i}$, where θ_i satisfies : $\mathbb{E}[(p(\theta) - p(\theta_i))\delta(\theta \geq \theta_i)] = (\pi - \rho)t_i$.*

- *For each producer, realized revenue from selling permits is decreasing in θ while realized profits from production are increasing in θ.*

- *The total amount of permits purchased, $\sum_{i=1}^{n} x_i(\rho)$, is increasing in the buyback price, ρ.*

- *The realized price to the consumer, $p(\theta)$, is a convex piecewise linear function of the uncertainty, θ.*

- *The expected price is: $\mathbb{E}[p] = \frac{B + \pi \sum_{i=1}^{n} \frac{t_i}{a_i}}{A + \sum_{i=1}^{n} \frac{1}{a_i}}$*

- *The expected externality level is: $\mathbb{E}[e] = \sum_{i=1}^{n} \frac{t_i}{a_i}(\mathbb{E}[p] - \pi t_i)$.*

- *The expected profit collected of producer i is: $\mathbb{E}[\Pi_i] = \frac{1}{2a_i}\mathbb{E}[\min\{p(\theta) - \rho t_i, p(\theta_i) - \rho t_i\}^2]$.*

- *The expected revenue collected by the regulator is :$\mathbb{E}[R] = \sum_{i=1}^{n} \frac{t_i}{a_i}\{(p(\theta_i) - \rho t_i)(\pi - \rho) + \rho(\mathbb{E}[p] - \pi t_i)\}$.*

Observe that a buyback price affect the total amount of permits purchased, yet the expected externality level remains the same as in a cap policy. To explain this, note that when realized demand is high, producers will have a higher production level under cap-and-buyback than under a cap policy. However, when realized demand is low, producers under a cap-and-buyback policy are better of selling some of the permits back to the central planner. Thus, in this case, production level under cap-and-buyback will be smaller than production level under a cap policy.

An interesting interpretation of the results is identifying cap-and-buyback regulation as an interpolation between a tax policy and a cap regulation. For example, when the buyback price equals the permit price, then there is no risk in stocking as many permits as desired, and the policy effectively mimics a tax regulation (see Proposition 3.3.1). By contrast, when

Policy	Tax regulation	Cap-and-buyback regulation	Cap regulation
$\mathbb{E}[p]$	$\frac{B+\xi\sum_{i=1}^{n}\frac{t_i}{a_i}}{A+\sum_{i=1}^{n}\frac{1}{a_i}}$	$\frac{B+\pi\sum_{i=1}^{n}\frac{t_i}{a_i}}{A+\sum_{i=1}^{n}\frac{1}{a_i}}$	$\frac{B+\pi\sum_{i=1}^{n}\frac{t_i}{a_i}}{A+\sum_{i=1}^{n}\frac{1}{a_i}}$
$\mathbb{E}[e]$	$\sum_{i=1}^{n}\frac{t_i}{a_i}(\mathbb{E}(p)-\xi t_i)$	$\sum_{i=1}^{n}\frac{t_i}{a_i}(\mathbb{E}(p)-\pi t_i)$	$\sum_{i=1}^{n}\frac{t_i}{a_i}(\mathbb{E}(p)-\pi t_i)$
$\mathbb{E}[\Pi_i]$	$\frac{1}{2a_i}\mathbb{E}((p(\theta)-t_i\xi)^2)$	$\frac{1}{2a_i}\mathbb{E}[\min\{p(\theta)-\rho t_i, p(\theta_i)-\rho t_i\}^2]$	$\mathbb{E}[\frac{\min\{p(\theta),p(\theta_i)\}^2}{2a_i}]$
$\mathbb{E}[R]$	$\xi\sum_{i=1}^{n}\frac{t_i}{a_i}(\mathbb{E}(p)-\xi t_i)$	$(\pi-\rho)\sum_{i=1}^{n}x_i(\rho)+\rho\sum_{i=1}^{n}\frac{t_i}{a_i}(\mathbb{E}[p]-\pi t_i)$	$\sum_{i=1}^{n}x_i$

Table 3.1: Comparison of cap, cap-and-buyback and tax regulation for a given π, ξ.

the buyback price is zero, this policy is effectively a cap regulation as permits have no resale value (see Proposition 3.3.2).

Table 3.1 shows the performance measures for the three policies studied for price taking firms. Importantly, the results suggest that the three policies are identical from expected market price and expected emission as long as the tax rate is equal to the permit price. This implies that the conclusion of Theorem 3.3.2 still holds: *a preference on which policy to implement cannot be established solely by considering consumer price, expected satisfied demand or expected externality level.*

Of course, all our results are established under the assumption of a risk neutral central planner. In this case, the three regulations have the same expected emission (externality) level but realized emission is not the same. Therefore, a risk averse central planner will have a regulation preference depending on its utility function.

Even though our results assume a limited support for the distribution of θ, in order to guarantee that producers participate almost surely, our results do not make any assumptions regarding the distribution of the noise within this range.

3.4 Price Making Firms

In this section we consider price making producers. That is, when choosing a supply function that maximizes their expected profits, producers take into account the effect their supply function has on market price. This immediately implies that producers are required to anticipate other producers' supply functions. To illustrate, consider the case where no regulation is imposed. Given other producer's supply function, producer i can determine the amount that can be sold to the market at a given price p by means of the market equilibrium equation (3.1) :

$$s_i(p) = D(p,\theta) - \sum_{j\neq i} s_j(p) \tag{3.4}$$

The right hand side in Equation (3.4) is key to the producer's supply function decision problem, as it quantifies the residual demand, the portion of demand not satisfied by other producers at a given price. As in Klemperer and Meyer (1989), maximization of expected profit can be replaced by the maximization of profit with respect to p for each realization of θ. Therefore, given a realization of demand θ, producer i finds the market price that would result in maximum profit by solving:

$$\max_p (p - c_i)[D(p,\theta) - \sum_{j \neq i} s_j(p)] \tag{3.5}$$

The producer then submits $s_i(p^*) = D(p^*, \theta) - \sum_{j \neq i} s_j(p^*)$, where p^* denotes the solution to the optimization problem (3.5). Considering the first order optimality condition of problem (3.5), for each producer i, yields the system of equations:

$$s_i(p) - [p - c_i'(s_i(p))][A + \sum_{j \neq i} s_j'(p)] = 0 \quad \forall i. \tag{3.6}$$

Inspired by Baldick et al. (2004), we find a solution that satisfies the system of equations by proposing a solution of a reasonable form, for example in this unregulated case $s_i(p) = b_i p$, and then determine appropriate values for the constants $\{b_i\}_{i=1}^n$ by solving the system of equations defined by (3.6). Having found a solution, we proceed to obtain performance measures as in Section 3.3.

We compare the impact of tax and cap-and-trade regulations. For this purpose, we develop two propositions. Proposition 3.4.1 analyzes how a particular tax rate ξ affects the market under tax regulation. Proposition 3.4.2 analyzes the effect a permit price π has on the market under cap-and-trade regulation.

To introduce Proposition 3.4.1 and 3.4.2, we start with the following definition.

Definition 3.4.1. *Consider the following function $b_i : \mathbb{N} \to \mathbb{R}$, defined as:*

$$b_i(m) = \frac{1}{2(m-1)} \left\{ \frac{(m-2)}{a_i} - A + \sqrt{(\frac{m-2}{a_i} - A)^2 + \frac{4(m-1)A}{a_i}} \right\}.$$

One interesting property of $b_i(m)$ is that as m grows to infinity, $b_i(m)$ approaches $\frac{1}{a_i}$, a factor commonly used in the expressions derived in Section 3.3. Moreover, as we shall see, many of the results derived in the price taking setting can be recovered just by replacing $b_i(m)$ by $\frac{1}{a_i}$.

Given uncertain demand, and using the assumptions stated in Section 3.2, one can establish the following expressions for the relevant performance measures under a tax regulation:

Proposition 3.4.1 (Oligopoly tax regulation)**.** *Consider a tax regulation under stochastic demand. Let ξ be the tax price. Then, there exists a supply function equilibrium such that:*

- *The expected price to the consumer is:* $\mathbb{E}[p] = \frac{B+\xi \sum_{i=1}^{n} t_i b_i(n)}{A+\sum_{i=1}^{n} b_i(n)}$
- *The expected externality level is:* $\mathbb{E}[e] = \sum_{i=1}^{n} t_i b_i(n)(\mathbb{E}[p] - t_i\xi)$
- *The expected profit for producer i is:* $\mathbb{E}[\Pi_i] = b_i(n)(1 - \frac{1}{2}a_i b_i(n))\mathbb{E}[(p(\theta) - t_i\xi)^2]$.
- *Producer's j expected profit is increasing in ξ if and only if:* $\sum_{i=1}^{n} b_i(n)(t_i - t_j) \geq t_j A$
- *The expected revenue collected by the regulator is a concave function of ξ.*

Proposition 3.4.1, is the counterpart of Proposition 3.3.1 from Section 3.3. As such, much of the analysis from Proposition 3.3.1 can be extended to this setting.

Having stated the main result for tax regulation, what follows is a similar result for cap-and-trade regulation. Our model of cap-and-trade, involves adding a trading stage to the cap regulation studied in Section 3.3. To be precise, we start by clarifying the sequence of events and the decisions involved:

1. The regulator announces a permit price π.

2. Producers purchase permits that maximize their expected profits. By introducing some additional notation we can write the optimization problem faced by producers as:

$$\max_{x_i \geq 0} \mathbb{E}[p(\theta)s_i(p(\theta)) - c_i(s_i(p(\theta))) - \rho(\theta)\tau_i(\theta)] - \pi x_i,$$

   where $\tau(\theta)$ denotes the realized transferences that occur in the trading. For example, if $\tau_i(\theta)$ is positive it means producer i purchased $\tau_i(\theta)$ permits, while if the value is negative it means producer i sold $\tau_i(\theta)$ permits. Here $\rho(\theta)$ is the realized trading price of the permits.

3. The additive noise, θ is realized. In this stage producers are allowed to trade permits among themselves. To determine the outcome of trading, as well as the production quantities for producers after trading is finished, each producer solves the problem:

$$\max_{\substack{u_i \geq 0 \\ t_i u_i \leq x_i + \tau_i}} \; p u_i - c_i(u_i) - \rho\tau_i.$$

Here u_i is the realized production quantity, τ_i, the realized traded amount and ρ the realized trading price. This problem yields a production quantity, u_i, and a trading amount, τ_i, both of these quantities depend on the market price, p, and realized trading price, ρ. Market clearing conditions are used to determine the realized permit trading price, ρ, and the realized market price, p.

- To determine the realized permit trading price ρ, the permit market clearing condition is $\sum_{i=1}^{n} \tau_i = 0$.
- To determine the realized market price, p, The market clearing condition is $\sum_{i=1}^{n} u_i = D(p, \theta)$.

4. Realized demand is served. That is, producer i delivers u_i units to satisfy consumer demand, and sells (buys) τ_i permits at a price ρ.

In this setting the value of θ as well as x_i are known before trading occurs. The outcome of the trading stage can be characterized by three parameters: $\{u_i\}_{i=1}^n$ the amount of realized demand satisfied by each producer, $\{\tau_i\}_{i=1}^n$ the realized permits purchased (sold if the value is negative) and ρ the realized permit trading price.

The outcome of the trading stage for a given θ can be found by solving an integer program. Moreover, closed form solutions can be derived depending on the sign, positive or zero, of the permit trading price and whether the constraint $t_i u_i \leq \tau_i$ is binding or not. In order to obtain a simple closed form solution, enabling us to compare results with tax policy, we restrict the support of the distribution of θ to guarantee that no producer sells all of its permits and the permit trading price is positive, almost surely. The results are summarized in Proposition 3.4.2.

Proposition 3.4.2 (Cap-and-trade)**.** *Consider a cap-and-trade regulation and stochastic demand. Let π be the permit price. Then there exists an interval, such that if $\theta \in [\theta_0, \theta_\infty]$ almost surely, then:*

- *Each producer satisfies a portion of demand, namely, $u_i > 0 \quad \forall i$ almost surely.*
- *Permit trading price, ρ, is positive almost surely.*
- *The expected price satisfies:*

$$\mathbb{E}[p(\theta)] = \frac{B + \bar{\rho} \sum_{i=1}^{n} t_i b_i(n)}{A + \sum_{i=1}^{n} b_i(n)},$$

where $\bar{\rho}$ is the expected permit trading price.

- *The expected externality level is:* $\mathbb{E}[e] = \sum_{i=1}^{n} t_i b_i(n)(\mathbb{E}[p] - \bar{\rho} t_i)$.
- *Producer's i expected profit is:* $\mathbb{E}[\Pi_i] = \mathbb{E}[b_i(n)(p(\theta) - \rho(\theta) t_i)^2 (1 - \frac{1}{2} a_i b_i(n))] - (\pi - \bar{\rho}) x_i$, *where x_i denotes the amount of permits purchased by producer i.*

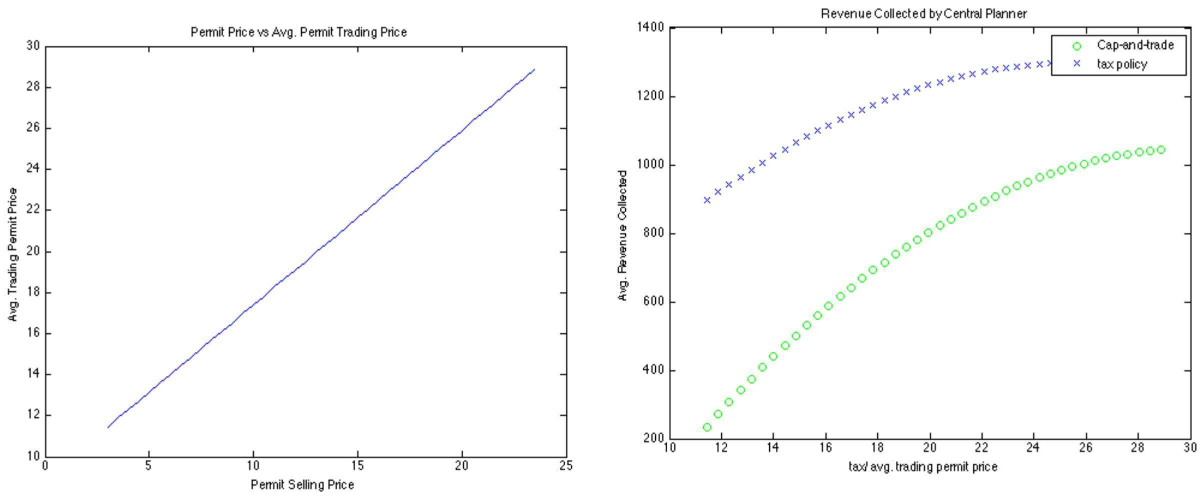
Table 3.2 summarizes expected consumer price and expected externality level under both tax policy and cap-and-trade, as provided by Propositions 3.4.1 and 3.4.2.

Policy	Tax	Cap-and-trade
$\mathbb{E}[p]$	$\frac{B+\xi \sum_{i=1}^{n} t_i b_i(n)}{A+\sum_{i=1}^{n} b_i(n)}$	$\frac{B+\bar{\rho} \sum_{i=1}^{n} t_i b_i(n)}{A+\sum_{i=1}^{n} b_i(n)}$
$\mathbb{E}[e]$	$\sum_{i=1}^{n} t_i b_i(n)(\mathbb{E}(p) - \xi t_i)$	$\sum_{i=1}^{n} t_i b_i(n)(\mathbb{E}(p) - \bar{\rho} t_i)$

Table 3.2: Comparison of cap-and-trade and tax regulation.

As one can see, if the tax rate, ξ, equals the expected permit trading price, $\bar{\rho}$, then the two policies have exactly the same expected externality levels and the same expected market price. This result is similar to the one developed in Section 3.3 for the price taking model. Indeed, Table 3.1 shows that if tax rate is equal to permit price then expected externality level and expected consumer price are the same under tax policy, cap policy and cap-and-buyback policy.

Unfortunately, unlike the price taking model, we cannot establish that expected revenue is higher for the central planner under a cap-and-trade regulation. In fact, Figure 3-4 illustrates two observations obtained by numerical examples with four producers, see details in Appendix 3.F. First, the permit selling price can be much smaller than the expected trading price. Second, the expected revenue collected by the central planner under the tax policy is not necessarily smaller than the revenue collected under the cap-and-trade policy.



(a) Relation between expected trading price for permits and permit price.

(b) Comparison of (expected) revenue collected by the central planner under cap-and-trade and tax policy.

Figure 3-4: Numerical results for cap-and-trade and tax regulation.

3.5 Conclusion

The objective of this work is to identify differences between permit regulations, or quota systems, and a tax regulation. We show that in the absence of uncertainty both methods are equivalent. That is, if the tax rate and permit price are such that both regulations deliver the same externality level, then the consumer price, demand satisfaction, producer profits and revenue to the central planner is the same under both regulations. Of course, from an implementation point of view, these two policies may be quite different, see Avi-Yonah and Uhlmann (2009).

By contrast, when demand is stochastic, the various policies agree on some dimensions and differ on others. For example, when considering price taking producers, a permit regulation such as cap or cap-and-buyback, and a tax regulation that achieve the same expected externality level lead to the same expected consumer price. However, these policies lead to different central planner's revenue and producers expected profits. Importantly, producers with high emission rates always prefer tax to permit regulations while those with clean technology sometimes prefer permit policies. In addition, under some conditions that are easy to verify, producers with clean technology will face higher expected profits as the tax rate increases, which is quite a surprising insight of this analysis.

Finally, in the price taking model, the central planner's expected revenue is always higher under a permit policy, such as cap or cap-and-buyback, than under a tax policy. In addition, in the case of tax regulation the expected revenue collected by the central planner is concave in the tax rate, and therefore experiences decreasing returns on scale. This implies that as the tax rate increases, the additional revenue collected in this policy decreases. This is noteworthy, as the decrease may get to a point where the central planner loses revenue by increasing the tax rate. Therefore, a marginal increase in tax beyond this point would decrease the central planner's revenue, and possibly producers' profit. In a situation such as this, *combining tax regulation with other externality reducing policies, such as subsidies to invest in cleaner technology, should be considered.*

In an economy where few producers dominate the market, price making producers provide a better model. Under such framework, we are able to compare the performance of cap-and-trade and tax regulation. We found, under some conditions, that if we select the permit price/ tax rate such that both policies achieve the same externality level, the expected price to the consumer is the same under both policies. This is consistent with our results from the price taking framework, suggesting that differences in the environmental policies are not in the expected consumer price, but rather in the producers' profits as well as revenue collected by the regulator.

An important limitation of our model is that the permit policy requires the central planner to anticipate the price of the permit that would lead to the desirable expected level of externality. In practice this is done through an auction mechanism that provides an effective way to distribute the permits. One possible way to extend our model to this setting is to apply a uniform price auction where the central planner hold back some of the permits when the permit price is too low. In this case, Back and Zender (2001) shows that the central planner can extract all the benefit of the permit, the central planner can sell all the permits and bidders cannot benefit from gaming the system, when there is a large number of bidders. This result suggests that when in the presence of a large number of firms, our results would not be affected by the introduction of an auction mechanism.

3.A Proofs

3.A.1 Theorem 3.3.1

1. Under tax regulation producer i faces the production problem:

$$\max_{q_i \geq 0} pq_i - c_i(q_i) - \xi t_i q_i.$$

Denote $\tilde{s}_i(p, \xi)$ the supply function in this setting. Grouping the terms with q_i and recognizing terms, the following identity can be established:

$$\tilde{s}_i(p, \xi) = s_i(p - t_i \xi),$$

where s_i is the optimal supply function in the unregulated case. The definition of market equilibrium implies that:

$$F(p, \xi) = \sum_{i=1}^{n} s_i(p - t_i \xi) - D(p) = 0. \tag{3.7}$$

Notice that $F(p, \xi)$ is increasing in p, decreasing in ξ. Therefore, $p(\xi)$ the equilibrium price is increasing in ξ.

2. In order to prove the existence of the appropriate tax we make use of the following lemma, proven below.

Lemma 3.A.1. *Consider a function $F(p, \xi)$ continuous, nondecreasing in p and non increasing in ξ, and a function $G(p, \xi)$, continuous, nondecreasing in p and non increasing in ξ, $X \in (0, +\infty)$. If F and G satisfies the following properties:*

(a) $F(0, 0) < 0$.

(b) $\lim_{p \to +\infty} F(p, \xi) > 0$, $\forall \xi$.

(c) $\lim_{\xi \to +\infty} G(p, \xi) = 0$, $\forall p$.

(d) $\lim_{p \to +\infty} G(p, \xi) > X$, $\forall \xi$.

Then either there exists p^* and ξ^* such that $F(p^*, \xi^*) = 0$ and $G(p^*, \xi^*) = X$ or there exists nondecreasing sequences $(p^j)_{j \geq 0}$ and $(\xi^j)_{j \geq 0}$ such that $F(p^j, \xi^j) = 0$, $G(p^j, \xi^{j+1}) = X$, $p^j \to +\infty, \xi^j \to +\infty$.

Using the lemma, consider $F(p, \xi) = \sum_{i=1}^{n} \tilde{s}_i(p, \xi) - D(p)$, $G(p, \xi) = \sum_{i=1}^{n} t_i \tilde{s}_i(p, \xi)$. Both F and G are continuous, and satisfy the monotonicity properties required by

Lemma 3.A.1. Also notice that $F(0,0) = -D(0) < 0$ and checking the rest of the properties are straightforward as they are defined by the supply functions. Finally, we have to check that the prices and taxes cannot approach infinity.

Assume we have nondecreasing sequences $(p^j)_{j\geq 0}$ and $(\xi^j)_{j\geq 0}$ such that $F(p^j, \xi^j) = 0$, $G(p^j, \xi^{j+1}) = X$, $p^j \to +\infty, \xi^j \to +\infty$. This implies:

$$F(p^j, \xi) = \sum_{i=1}^{n} \tilde{s}_i(p^j, \xi) - D(p^j) \quad \forall \xi$$

$$X = G(p^j, \xi^{j+1}) = \sum t_i \tilde{s}_i(p^j, \xi^{j+1}) \leq \max_k \{t_k\} \sum \tilde{s}_i(p^j, \xi^{j+1})$$

Since $t_i > 0$ $\forall i$, we obtain that $\sum \tilde{s}_i(p^j, \xi^{j+1}) \geq \frac{X}{\max_k\{t_k\}}$ and hence:

$$0 = F(p^j, \xi^j) \geq F(p^j, \xi^{j+1}) = \sum \tilde{s}_i(p^j, \xi^{j+1}) - D(p^j) \geq \frac{X}{\max_k\{t_k\}} - D(p^j)$$

Since $X > 0$, and $D(p^j)$ is smaller than $\epsilon = \frac{X}{2\max_k\{t_k\}}$ for large enough j, we obtain a contradiction. Hence the result of the theorem follows.

To prove the relation between X and ξ, we prove it in a more general context of stochastic demand and strictly convex cost. In this setting we denote:

(a) $\tilde{s}_i(p, \xi) = \arg\max_{q\geq 0} (p - t_i\xi)q - c_i(q)$

(b) Let $p(\xi, \theta)$ be a solution of the equation $\sum_{i=1}^{n} \tilde{s}_i(p, \xi) = D(p, \theta)$.

Observe that many of the properties are inherited from the deterministic setting, such as the super modularity of the objective function. the main difference is the added parameter in demand, as well as now the externality level is a random variable.

Hence from (a), we get that:

$$\tilde{s}_i(p, \xi) = s_i(p - t_i\xi).$$

This together with (b) implies that

$$f(p, \xi, \theta) = \sum_{i=1}^{n} s_i(p - t_i\xi) - D(p, \theta) = 0. \tag{3.8}$$

Notice that $f(p, \xi, \theta)$ is increasing in p, decreasing in ξ and θ. Therefore, $p(\xi, \theta)$ is increasing in ξ and θ.

Let $g(\xi) = \mathbb{E}[\sum_{i=1}^{n} t_i s_i(p(\xi,\theta) - t_i\xi)]$, be the expected externality level, we want to prove that this is decreasing in ξ. To show that $g(\xi)$ is decreasing in ξ, assume without loss of generality that $t_1 \leq t_2 \leq \cdots \leq t_n$. We rewrite $g(\xi)$

$$g(\xi) = \mathbb{E}[\sum_{i=1}^{n}(t_i - t_{i-1})S_i(\xi,\theta)],$$

where $t_0 = 0$ and

$$S_i(\xi,\theta) = \sum_{l=i}^{n} s_l(p(\xi,\theta) - t_l\xi).$$

The equation (3.8) implies that $S_1(\xi,\theta)$ is decreasing in ξ. We will prove that $S_i(\xi,\theta)$ is decreasing in ξ. To see this, consider any two points ξ and ξ' with $\xi < \xi'$. Since $S_1(\xi,\theta) \geq S_1(\xi',\theta)$ and $p(\xi,\theta) - t_i\xi$ and $p(\xi',\theta) - t_i\xi'$ are decreasing in i, there exists an index i_0 such that

$$p(\xi,\theta) - t_i\xi \geq p(\xi',\theta) - t_i\xi'$$

for $i \geq i_0$ and

$$p(\xi,\theta) - t_i\xi \leq p(\xi',\theta) - t_i\xi'$$

for $i \leq i_0$. Since $s_i(p)$ is increasing in p, we have that

$$S_i(\xi,\theta) - S_i(\xi',\theta) = \begin{cases} \sum_{l=i}^{n}(s_l(p(\xi,\theta) - t_l\xi) - s_l(p(\xi',\theta) - t_l\xi')), \text{if } i \geq i_0 \\ S_1(\xi,\theta) - S_1(\xi',\theta) - \sum_{l=1}^{i}(s_l(p(\xi,\theta) - t_l\xi) - s_l(p(\xi',\theta) - t_l\xi')), \text{if } i \leq i_0, \end{cases}$$

The above formula immediately implies that $S_i(\xi,\theta) \geq S_i(\xi',\theta)$ for $i = 1,\ldots,n$. Therefore, $g(\xi)$ is decreasing in ξ, which implies that the solution $g(\xi) = X$ is decreasing in X. That is, the larger the externality level required, the smaller the tax rate.

Proof of Lemma 3.A.1

We create sequences that will converge p^*, and ξ^*. First, consider $\xi^0 = 0$. Given that $F(0,\xi^0) < 0$ by property 2a, $\lim_{p\to+\infty} F(p,\xi^0) > 0$ by property2b, and continuity, then the intermediate value theorem implies that there exists $p^0 \in (0,+\infty)$ such that $F(p^0,\xi^0) = 0$. In case there exists more than one possibility we choose the smallest value, given this value there are 3 possibilities:

(a) $G(p^0,\xi^0) = X$.

(b) $G(p^0,\xi^0) < X$.

(c) $G(p^0,\xi^0) > X$.

In the first case the lemma is proven. In the second case we will find ξ^{n+1} such that $G(p^n, \xi^{n+1}) = X$, due to the monotonicity of G we know that $\xi^{n+1} \in (-\infty, \xi^n)$. Due to the monotonicity of F we have $F(p^n, \xi^{n+1}) \geq 0$, and therefore, using the intermediate values theorem for F we have that there exists $p^{n+1} \in [0, p^n]$ such that $F(p^{n+1}, \xi^{n+1}) = 0$, and due to the monotonicity of G, we have $G(p^{n+1}, \xi^{n+1}) \leq X$. If $G(p^{n+1}, \xi^{n+1}) = X$ we are done, otherwise we continue to generate elements. Since p^j is bounded within $[0, p^0]$ and non increasing we have that $p^j \to p^*$, using this fact and property 2c we find that ξ is also bounded, and hence $\xi^j \to \xi^*$. Now to prove the required property, we use the fact that F and G are continuous and therefore:

$$0 = F(p^j, \xi^j) \to F(p^*, \xi^*), \qquad X = G(p^j, \xi^{j+1}) \to G(p^*, \xi^*)$$

The third case is more complex, and will generate sequences in a very similar form. Suppose that we have $p^n \in [p^{n-1}, +\infty), \xi^n \in (\xi^{n-1}, +\infty)$ such that $F(p^n, \xi^n) = 0$ and $G(p^n, \xi^n) > X$, then using the intermediate value theorem for G, there exists ξ^{n+1} such that $G(p^n, \xi^{n+1}) = X$, due to the monotonicity of G we know that $\xi^{n+1} \in (\xi^n, +\infty)$. Due to the monotonicity of F we have $F(p^n, \xi^{n+1}) \leq 0$, and therefore, using the intermediate values theorem for F we have that there exists $p^{n+1} \in [p^n, +\infty)$ such that $F(p^{n+1}, \xi^{n+1}) = 0$, and due to the monotonicity of G, we have $G(p^{n+1}, \xi^{n+1}) \geq X$. If $G(p^{n+1}, \xi^{n+1}) = X$ we are done, otherwise we continue to generate elements.

Hence either we found (p^*, ξ^*) or we have $(p^j)_{j \geq 0}$ and $(\xi^j)_{j \geq 0}$, both non decreasing sequences, and therefore with well defined limits (that may be infinite). If the limit of p^j is finite, then it has a limit $p^j \to p^*$. Using this fact and property 2c we find that ξ is also bounded, and hence $\xi^j \to \xi^*$. Now to prove the required property, we use the fact that F and G are continuous and therefore:

$$0 = F(p^j, \xi^j) \to F(p^*, \xi^*), \qquad X = G(p^j, \xi^{j+1}) \to G(p^*, \xi^*)$$

It remains to prove that if the limit is not finite, then there is no finite solution such that $F(p^*, \xi^*) = 0$ and $G(p^*, \xi^*) = X$. Suppose there exists p^*, ξ^* such that $F(p^*, \xi^*) = 0$ and $G(p^*, \xi^*) = X$, we will prove that $p^j \leq p^*$ and $\xi^j \leq \xi^*$, hence giving a contradiction, as then the sequences would be bounded and monotone, meaning they converge. Notice that we are still in case 3, therefore given our choice of p^0 as the smallest such that $F(p^0, 0) = 0$ and case 3, we have that $\xi^* > 0 = \xi^0$. Notice that $F(p^*, \xi^0) \geq F(p^*, \xi^*) = 0 = F(p^0, \xi^0)$, given our choice of p^0 we find that $p^* \geq p^0$. Suppose $p^* \geq p^j$, $\xi^* \geq \xi^j, \forall j \leq n$. Then, $G(p^n, \xi^{n+1}) = G(p^*, \xi^*) = X \leq G(p^*, \xi^{n+1})$

due to monotonicity of G. Due to our choice of ξ^{n+1} as the smallest that satisfies said property, we have that $\xi^{n+1} \leq \xi^*$. Now, $F(p^*, \xi^{n+1}) \geq F(p^*, \xi^*) = 0 = F(p^{n+1}, \xi^{n+1})$ then given our choice of the smallest, such that satisfies $F(p^{n+1}, \xi^{n+1}) = 0$, we conclude $p^{n+1} \leq p^*$. And therefore the sequences would converge. □

3. Under cap regulation, given π producers face the problem:

$$v_i(p, x_i) = \max_{0 \leq q \leq \frac{x_i}{t_i}} \{pq - c_i(q)\}$$

Denote the solution $s_i(p, x_i)$. Additionally, x_i satisfies:

$$x_i \in \arg\max\{v_i(p, x_i) - \pi x_i\}$$

And the coupling constraint balancing supply and demand:

$$\sum_{i=1}^{n} \hat{s}_i(p, x_i) = D(p).$$

Now we restrict ourselves to producers that participate in the market. Given that $\pi > 0$, $s_i(p, x_i) = \frac{x_i}{t_i}$. Under a cap-and-buyback regulation, where the buyback price is smaller than the permit price, this remains true.

Through a change of variables, and rewriting the maximization problem for x_i, the optimal $u_i = \frac{x_i}{t_i}$ then satisfies:

$$u_i \in \arg\max\{pu_i - c_i(u_i) - t_i \pi u_i\}$$

A posteriori we can see this is satisfied also when the producer does not have incentive to participate in the market. From here we can see that when $\pi = \xi$ the two regulations coincide. □

3.A.2 Proposition 3.3.1

Production almost surely can be guaranteed by the condition $\theta_0 \geq ((A + \sum_{i=1}^{n} \frac{1}{a_i}) \max_{j=1,\ldots,n}\{t_j\} - \sum_{i=1}^{n} t_i \frac{1}{a_i})\xi - B$

Under a tax regulation:

$$\sum_{i=1}^{n} s_i(p(\theta) - t_i \xi) = D(p(\theta), \theta)$$

Hence under quadratic cost linear demand assumptions we have:

$$\sum_{i=1}^{n}(\frac{p(\theta)-t_i\xi}{a_i})^+ = B - Ap(\theta) + \theta. \tag{3.9}$$

Under assumption that producers participate for all realizations of uncertainty, we have that the price as a random variable satisfies:

$$\sum_{i=1}^{n}\frac{p-t_i\xi}{a_i} = B - Ap + \theta$$

Using equation (3.9), the expected price:

$$p(\theta) = \frac{(B+\theta+\xi\sum_{i=1}^{n}\frac{t_i}{a_i})}{(A+\sum_{i=1}^{n}\frac{1}{a_i})} \qquad \forall\theta$$

Thus $\mathbb{E}[p] = \frac{B+\xi\sum_{i=1}^{n}\frac{t_i}{a_i}}{A+\sum_{i=1}^{n}\frac{1}{a_i}}$.

The expected externality level under these assumptions satisfy:

$$\mathbb{E}[e] = \mathbb{E}[\sum_{i=1}^{n} t_i s_i(p(\theta)-t_i\xi)] = \mathbb{E}[\sum_{i=1}^{n} t_i\frac{p(\theta)-t_i\xi}{a_i}] = \sum_{i=1}^{n}\frac{t_i}{a_i}(\mathbb{E}[p]-\xi t_i)$$

Replacing the expected price yields the result.

The profit for a producer is:

$$\mathbb{E}[\Pi_i] = \mathbb{E}[p(\theta)\frac{(p\theta)-t_i\xi)}{a_i} - \frac{1}{2}a_i\frac{(p(\theta)-t_i\xi)^2}{a_i^2} - t_i\xi\frac{(p\theta)-t_i\xi)}{a_i}] = \frac{1}{2a_i}\mathbb{E}[(p(\theta)-t_i\xi)^2].$$

Let $\nu_i = (A+\sum_{j=i}^{n}\frac{1}{a_j})^{-1}$. We can expand $p(\theta)$ in the profits expression and we obtain:

$$\begin{aligned}
\mathbb{E}[\Pi_i] &= \frac{1}{2a_i}\mathbb{E}[\rho_1^2(B+\theta+\xi\sum_{j=1}^{n}\frac{t_j}{a_j}))^2 - 2\rho_1(B+\theta+\xi\sum_{j=1}^{n}\frac{t_j}{a_j})t_i\xi + t_i^2\xi^2] \\
&= \frac{\rho_1^2}{2a_i}(B^2+V(\theta)+2B\sum_{j=1}^{n}\frac{t_j\xi}{a_j} + (\sum_{j=1}^{n}\frac{t_j\xi}{a_j})^2 - 2\rho_1^{-1}(B+\sum_{j=1}^{n}\frac{t_j\xi}{a_j})t_i\xi + \rho_1^{-2}(t_i^2\xi^2) \\
&= \frac{\rho_1^2}{2a_i}(V(\theta)+(B+\xi(\sum_{j=1}^{n}\frac{t_j-t_i}{a_j} - t_iA))^2)
\end{aligned}$$

Where V denotes the variance of the random variable. From here by taking derivatives, we can see that the condition for increasing profits on the tax is:

$$\sum_{j=1}^{n} \frac{t_j - t_i}{a_j} > t_i A.$$

To see the concavity in the central planner expected revenue observe that after replacing the expected price, the expected revenue is:

$$\mathbb{E}[R_t] = \frac{\xi}{A + \sum_{i=1}^{n} \frac{1}{a_i}} (B \sum_{i=1}^{n} \frac{t_i}{a_i} + \xi((\sum_{i=1}^{n} \frac{t_i}{a_i})^2 - \sum_{i=1}^{n} \frac{t_i^2}{a_i} \sum_{j=1}^{n} \frac{1}{a_j} - A \sum_{i=1}^{n} \frac{t_i^2}{a_i})).$$

We consider a random variable t such that $\mathbb{P}(t = t_i) = \frac{\frac{1}{a_i}}{\sum_{j=1}^{n} \frac{1}{a_j}}$. Then:

$$V(t) = \frac{\sum_{i=1}^{n} \frac{t_i^2}{a_i}}{\sum_{i=1}^{n} \frac{1}{a_i}} - \frac{(\sum_{j=1}^{n} \frac{t_j}{a_j})^2}{(\sum_{i=1}^{n} \frac{1}{a_i})^2} \geq 0$$

And hence the expected revenue is concave. □

3.A.3 Proposition 3.3.2

- We start by finding the optimality conditions in the case of permit regulation. We do this in a more general case of convex cost and linear demand with an additive noise.

  Given π there are three conditions that are satisfied in market equilibrium:

  - Let $v_i(p, x_i) = \max_{0 \leq q \leq \frac{x_i}{t_i}} \{pq - c_i(q)\}$. Let $\hat{s}_i(p, x_i)$ be the unique optimal solution.
  - Let $p(\theta)$ be a solution of $\sum_{i=1}^{n} \hat{s}_i(p(\theta), x_i) = D(p(\theta), \theta)$
  - $x_i \in \arg\max_{x_i \geq 0} E[v_i(p(\theta), x_i)] - \pi x_i$

  Consider $s_i(p) = \arg\max\{pq - c_i(q)\}$, and denote θ_i, the value of a realization of uncertainty such that $s_i(p(\theta_i)) = \frac{x_i}{t_i}$. Such value is uniquely defined as we are considering a continuous random variable. Observe that the optimization problem $v_i(p, x_i)$, can be rewritten depending on the value of the multiplier associated with the inequality $q \leq \frac{x_i}{t_i}$. For the values of p such that the multiplier is zero, we have that $\hat{s}_i(p, x_i) = s_i(p)$, for the values of p such that the multiplier is positive we have $\hat{s}_i(p(\theta), x_i) = \frac{x_i}{t_i}$. Then we can use a change of variables and rewrite $\hat{s}_i(p(\theta), x_i) = \min\{s_i(p(\theta)), s_i(p(\theta_i))\}$. Rewriting the condition for optimal permits yield:

$$\theta_i \in \arg\max_{s_i(p(\theta_i)) \geq 0} E[p(\theta) \min\{s_i(p(\theta)), s_i(p(\theta_i))\} - c_i(\min\{s_i(p(\theta)), s_i(p(\theta_i))\})] - \pi t_i s_i(p(\theta_i)).$$

We can rewrite the objective function into:

$$E[(p(\theta)s_i(p(\theta))-c_i(s_i(p(\theta)))\delta(\theta \leq \theta_i)]+E[p(\theta)\delta(\theta > \theta_i)]s_i(p(\theta_i))-c_i(s_i(p(\theta_i)))\mathbb{P}(\theta \geq \theta_i)-\pi t_i s_i(p(\theta_i)).$$

Where δ denotes the indicator function. We can then solve the optimization problem similar to a news-vendor problem to obtain the condition:

$$\mathbb{E}[(p(\theta) - p(\theta_i))\delta(\theta > \theta_i)] = \pi t_i.$$

- Under quadratic costs, we can write the supply function as:

$$s_i(p) = \frac{p^+}{a_i}.$$

We consider a extension of s_i :

$$r_i(p) = \frac{p}{a_i}.$$

To prove the existence of the optimal values θ_i, we make use of the following lemma first.

Lemma 3.A.2. *Assume the equilibrium p is increasing. Order the producer's such that $t_1 \geq t_2 \geq \ldots \geq t_n$, and consider the values $\theta_{b(1)} \leq \theta_{b(2)} \leq \ldots \leq \theta_{b(n)}$ as described before. Then $b(i) = i$.*

Proof.[Lemma] From equation (3.3) and the assumption that all producers participate in the market:

$$\pi t_i = \mathbb{E}[(p(\theta) - p(\theta_i))\delta(\theta > \theta_i)]. \tag{3.10}$$

We define a function using the right hand side $f(\bar{\theta}) = \mathbb{E}[(p(\theta) - p(\bar{\theta}))\delta(\theta > \bar{\theta}))]$, and decrease its values until we find a producer such that (3.3) is satisfied. Since f does not depend on i given the equilibrium price function, and the producer's are ordered increasing in technology, using (3.10), we obtain that $b(i) = i$. □

Consider $\theta' \geq \mathbb{E}[\theta] - A\pi t_n$, Jensen's inequality tells us $A\pi t_n \geq \mathbb{E}[(\theta - \theta')\delta(\theta > \theta')] = \mathbb{E}[(\theta - \theta')^+] \geq \mathbb{E}[\theta - \theta'] = \mathbb{E}[\theta] - \theta'$, We define $p^0(\theta)$ in the range $\theta \geq \theta'$, as the solution to $D(p^0(\theta), \theta) = 0$. Using the Lemma and (3.10), we obtain that θ_n solves:

$$g(p^0, \theta_n) = \pi t_n.$$

Where $g(p, \theta^*) = \mathbb{E}[(p(\theta) - p(\theta^*))\delta(\theta > \theta^*)]$. Hence we extend p^0 until we reach the equality. Now assume that we know the values $\theta_j, j = i, \ldots, n$ and $p^0(\theta)$ for $\theta \geq \theta_i$. We determine θ_{i-1} and $p^0(\theta)$ for $\theta \in [\theta_{i-1}, \theta_i)$ by setting

$$\sum_{j=i}^{n} r_j(p^0(\theta)) = D(p^0(\theta), \theta) + \sum_{j=i}^{n} r_j(p^0(\theta_j)),$$

or equivalently

$$\sum_{j=i}^{n} (r_j(p^0(\theta)) - r_j(p^0(\theta_j))) = D(p^0(\theta), \theta). \tag{3.11}$$

We stop at the first θ, denoted as θ_{i-1}, such that

$$\pi t_{i-1} = g(p^0, \theta).$$

Given the quadratic costs and linear demand, $r_j(\cdot)$ becomes linear in the domain of our concern. Thus, solving the above equation, we have that $p^0(\theta)$ is linear for $\theta \in [\theta_{i-1}, \theta_i)$ with a slope

$$\nu_i = (A + \sum_{j=i}^{n} \frac{1}{a_i})^{-1}.$$

Repeat this processes until we assign θ_1.

Observe that θ_i is invariant to linear shifts of p^0. We prove this using induction on i. For θ_n, as we define $p^0(\theta)$ in this range as the solution to $D(p^0(\theta), \theta) = 0$, we have that is linear, and hence the solution θ_n to $g(p^0, \theta_n) = \pi t_n$, remains the same for a shift in the function p^0. Given that θ_j remain unchanged for $j \geq i + 1$, by observing that:

$$\begin{aligned} g(p^0, \theta_i) &= \mathbb{E}[(p^0(\theta) - p^0(\theta_i))\delta(\theta \geq \theta_i)] \\ &= \mathbb{E}[(p^0(\theta) - p^0(\theta_i))\delta(\theta \in [\theta_i, \theta_{i+1}))] + \mathbb{E}[(p^0(\theta) - p^0(\theta_{i+1}))\delta(\theta \geq \theta_{i+1})] \\ &\quad + \mathbb{E}[(p^0(\theta_{i+1}) - p^0(\theta_i))\delta(\theta \geq \theta_{i+1})] \end{aligned}$$

We can see that terms are invariant to shifts due to the piecewise linear structure of $p^0(\theta)$ and the induction hypothesis. And hence, θ_i is invariant to shifts in $p^0(\theta)$.

As a conclusion, we notice that the values $(\theta_i)_{i=1}^n$ depend only on the uncertainty, the technologies and π. Another thing to notice is that θ_i is independent of $(\theta_j)_{j<i}$.

In order to find the appropriate value for $p(\theta_i)$ we solve the following system of linear equations:

$$\sum_{i=1}^{n} s_i(\min\{p(\theta_i), p(\theta_k)\}) = D(p(\theta_k), \theta_k) \qquad \forall k = 1, \ldots, n.$$

Subject to the constraints that $p(\theta_i) \geq 0$. This can be written as:

$$M\vec{p} = B\vec{\mathbb{1}} + \vec{\theta}$$

$$p \geq 0$$

Where:

$$M_{ij} = \begin{cases} 0 & \text{if } i < j \\ \frac{1}{\nu_i} & \text{if } i = j \\ \frac{1}{a_j} & \text{if } i > j \end{cases}$$

Observe that we have n l.i. constraints for n variables, hence we have unique solution in case of existence. On the other hand as M is a triangular matrix with nonzero diagonal, we have existence.

Having defined $p(\theta_i)$ to find $p(\theta)$ for a general θ we can again use the equation:

$$\sum_{i=1}^{n} s_i(\min\{p(\theta_i), p(\theta)\}) = D(p(\theta), \theta).$$

Moreover we can write :

$$\begin{aligned} p(\theta) = \quad & \min \quad p \\ & s.t. \quad \textstyle\sum_{i=1}^{n} y_i = B - Ap + \theta \\ & a_i y_i \leq p \\ & t_i y_i \leq x_i \\ & p \geq 0 \\ & y \geq 0 \end{aligned}$$

Where $t_i x_i = p(\theta_i)$, linear programming results show that p is piecewise linear and convex in θ. This proves the first item of the results.

We now prove that θ_i^π is decreasing in π. Notice that

$$\begin{aligned} \pi(t_{i-1} - t_i) &= g(p, \theta_{i-1}) - g(p, \theta_i) \\ &= E[(p(\theta) - p(\theta_i))\delta(\theta \in (\theta_{i-1}, \theta_i])] + (p(\theta_i) - p(\theta_{i-1}))E[\delta(\theta > \theta_{i-1})] \end{aligned}$$

Since $p(\theta)$ is linear for $\theta \in [\theta_{i-1}, \theta_i]$ with slope ν_i, we have that

$$\frac{\pi(t_{i-1} - t_i)}{\nu_i} = E[(\theta - \theta_{i-1})\delta(\theta \geq \theta_{i-1})] - E[(\theta - \theta_i)\delta(\theta \geq \theta_i)]$$

Hence by denoting $G(q) = E[(\theta - q)\delta(\theta \geq q)]$, we can generate a recursion and obtain:

$$G(\theta_i) = \sum_{j=i+1}^{n+1} \frac{\pi(t_{j-1} - t_j)}{\nu_j}$$

From here we can see that θ_i is decreasing on π since G is decreasing.

Using the previous polyhedral characterization we can prove monotonicity on the permits bought by each producer.

We have:

$$M\vec{p} = B\vec{\mathbb{1}} + \vec{\theta}$$

And hence:

$$D^{-1}\vec{p} = (MD)^{-1}(B\vec{\mathbb{1}} + \vec{\theta}),$$

where D is a diagonal matrix with a_i in ith element of the diagonal. Using this equation and the monotonicity of θ proven before, we can prove monotonicity for the total amount of permits purchased $\sum_{i=1}^{n} x_i$.

Consider the matrix:

$$Q = \begin{cases} 0 & \text{if } i < j \\ \tilde{\nu_i} & \text{if } i = j \\ -\tilde{\nu_i}\tilde{\nu_j}\prod_{l=j+1}^{i-1}(1 - \tilde{\nu_l}) & \text{if } i > j \end{cases}$$

Where $\tilde{\nu_i} = \frac{\nu_i}{a_i}$. It is easy to check that $(MDQ)_{ii} = 1 \quad \forall i$, for a general term, we have:

$$(MDQ)_{ij} = \tilde{\nu_i}(1 - \prod_{l=j+1}^{i-1}(1 - \tilde{\nu_l}) - \sum_{k=j+1}^{i-1} \tilde{\nu_k} \prod_{l=k+1}^{i-1}(1 - \tilde{\nu_l}))$$

The term in parenthesis is zero, we can see this by interpreting each of $\tilde{\nu_l}$ as the probability of heads of an independent coin. The term in the sum is the probability of a head occurring (either the first coin was heads, or it was tails and the second was heads, and so forth and so on). The product term is the probability of no heads occurring. Hence any outcome of the coins is included here, and the sum of these probabilities has to add up to one, meaning the term in the parenthesis is zero. Hence

$Q = (MD)^{-1}$. Replacing in the system of equations we have that the ith equation is:

$$\frac{p_i}{a_i} = (\tilde{\nu_i} + \sum_{j=1}^{i-1} (MD)^{-1}_{ij})B + \tilde{\nu_i}\theta_i + \sum_{j=1}^{i-1} (MD)^{-1}_{ij}\theta_j.$$

Hence,

$$\frac{x_i}{t_i} = B\tilde{\nu_i}(1 - \sum_{j=1}^{i-1} \tilde{\nu_j} \prod_{l=i+1}^{j-1} (1 - \tilde{\nu_l})) + \tilde{\nu_i}\theta_i(1 - \sum_{j=1}^{i-1} \tilde{\nu_j} \prod_{l=i+1}^{j-1} (1 - \tilde{\nu_l})\frac{\theta_j}{\theta_i}).$$

Therefore,

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} t_i B\tilde{\nu_i}(1 - \sum_{j=1}^{i-1} \tilde{\nu_j} \prod_{l=i+1}^{j-1} (1 - \tilde{\nu_l})) + \tilde{\nu_i}(\theta_i - \sum_{j=1}^{i-1} \tilde{\nu_j} \prod_{l=i+1}^{j-1} (1 - \tilde{\nu_l})\theta_j).$$

We invert the order of the sum for the last term, to be precise we use:

$$\sum_{i=1}^{n} t_i\tilde{\nu_i}(\theta_i - \sum_{j=1}^{i-1} \tilde{\nu_j} \prod_{l=i+1}^{j-1} (1 - \tilde{\nu_l})\theta_j) = \sum_{i=1}^{n} \tilde{\nu_i}\theta_i t_i - \sum_{j=1}^{n-1} \sum_{i=j+1}^{n} t_i\tilde{\nu_i}\tilde{\nu_j}\theta_j \prod_{l=i+1}^{j-1} (1 - \tilde{\nu_l}))$$

Finally,

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} t_i B\tilde{\nu_i}(1 - \sum_{j=1}^{i-1} \tilde{\nu_j} \prod_{l=i+1}^{j-1} (1 - \tilde{\nu_l})) + \sum_{i=1}^{n} \tilde{\nu_i}\theta_i t_i(1 - \sum_{j=i+1}^{n-1} \frac{t_j}{t_i}\tilde{\nu_j} \prod_{l=i+1}^{j-1} (1 - \tilde{\nu_l}))$$

Finally Lemma 3.A.2, states that $t_j < t_i$ for $j > i$. Combining this with:

$$\sum_{j=1}^{i-1} \tilde{\nu_j} \prod_{l=i+1}^{j-1} (1 - \tilde{\nu_l}) < 1,$$

$$\sum_{j=i+1}^{n-1} \tilde{\nu_j} \prod_{l=i+1}^{j-1} (1 - \tilde{\nu_l})) < 1,$$

we conclude the result. This last observations come from the fact that if we see each $\tilde{\nu_l}$ as the probability of heads for an independent coin, then this expression is the probability that a head occurs. Therefore as θ_i increases, $\sum_{i=1}^{n} x_i$ increases.

- So far, we have developed the following:

– Optimal permits, can be translated to a condition on θ_i:

$$\mathbb{E}[(p(\theta) - p(\theta_i)\delta(\theta \geq \theta_i)] = \pi t_i.$$

– Price as random variable satisfies the equation:

$$\sum_{i=1}^{n} \frac{\min\{p(\theta), p(\theta_i)\}}{a_i} = B - Ap(\theta) + \theta. \quad \text{a.s.}$$

Observe that his expressions are valid when all producers participate in the market. Otherwise the optimal permits for the producer are zero.

Given that for all θ, we have:

$$\sum_{i=1}^{n} \frac{\min\{p(\theta), p(\theta_i)\}}{a_i} = B - Ap(\theta) + \theta.$$

Then the expected value of the left hand side, should be equal to the expected value of the right hand side.

The expected value of the left hand side:

$$\begin{aligned}
\mathbb{E}[\sum_{i=1}^{n} \frac{\min\{p(\theta), p(\theta_i)\}}{a_i}] &= \sum_{i=1}^{n} \mathbb{E}[\frac{\min\{p(\theta), p(\theta_i)\}}{a_i}] \\
&= \sum_{i=1}^{n} \frac{1}{a_i} \mathbb{E}[p(\theta)\delta(\theta \leq \theta_i) + p(\theta_i)\delta(\theta \geq \theta_i)] \\
&= \sum_{i=1}^{n} \frac{1}{a_i} \mathbb{E}[p(\theta)\delta(\theta \leq \theta_i) + p(\theta)\delta(\theta \geq \theta_i) - \pi t_i] \\
&= \sum_{i=1}^{n} \frac{\mathbb{E}[p] - \pi t_i}{a_i}
\end{aligned}$$

The expected value of the right hand side:

$$\mathbb{E}[B - Ap(\theta) + \theta] = B - A\mathbb{E}[p].$$

Using these two expressions we find that:

$$\mathbb{E}[p] = \frac{B + \pi \sum_{i=1}^{n} \frac{t_i}{a_i}}{A + \sum_{i=1}^{n} \frac{1}{a_i}}$$

- The expected externality level in this case are:

$$\mathbb{E}[\sum_{i=1}^{n} t_i \frac{\min\{p(\theta), p(\theta_i)\}}{a_i}] = \sum_{i=1}^{n} \frac{t_i}{a_i}(\mathbb{E}[p] - \pi t_i)$$

Where we used again that $\mathbb{E}[\min\{p(\theta), p(\theta_i)\}] = \mathbb{E}[p] - \pi t_i$. Replacing the value of the expected price yields the result.

- Under quantity regulation the expected profit is:

$$\begin{aligned}
\mathbb{E}[\Pi_c] &= \mathbb{E}[p(\theta)\frac{\min\{p(\theta), p(\theta_i)\}}{a_i} - \frac{1}{2}a_i(\frac{\min\{p(\theta), p(\theta_i)\}}{a_i})^2] - \pi t_i \frac{p(\theta_i)}{a_i} \\
&= \mathbb{E}[\frac{1}{2a_i}p(\theta)^2\delta(\theta \leq \theta_i) + \frac{p(\theta_i)}{a_i}(p(\theta)\delta(\theta \geq \theta_i) - \pi t_i - \frac{1}{2}p(\theta_i)\delta(\theta \geq \theta_i))] \\
&= \mathbb{E}[\frac{1}{2a_i}p(\theta)^2\delta(\theta \leq \theta_i) + \frac{p(\theta_i)^2}{2a_i}\delta(\theta \geq \theta_i))] \\
&= \mathbb{E}[\min\{p(\theta), p(\theta_i)\}^2]
\end{aligned}$$

- The central planner revenue is the total amount of permits sold times the permit price:

$$R_c = \pi(\sum_{i=1}^{n} \frac{t_i}{a_i} p(\theta_i))$$

Thus proving all the results. □

3.A.4 Theorem 3.3.2

- If we want the same expected externality level under both regulations, we then require:

$$\sum_{j=1}^{n} \frac{t_j}{a_j}(\frac{B + \pi \sum_{i=1}^{n} \frac{t_i}{a_i}}{A + \sum_{i=1}^{n} \frac{1}{a_i}} - \pi t_j) = \sum_{j=1}^{n} \frac{t_j}{a_j}(\frac{(B + \xi \sum_{i=1}^{n} \frac{t_i}{a_i})}{A + \sum_{i=1}^{n} \frac{1}{a_i}} - \xi t_j)$$

Or equivalently:

$$(\pi - \xi)(\sum_{j=1}^{n} \frac{t_j}{a_j}(\sum_{i=1}^{n} \frac{t_i - t_j}{a_i})) = 0$$

Which we can rewrite into:

$$(\pi - \xi)((\sum_{j=1}^{n} \frac{t_j}{a_j})^2 - (\sum_{i=1}^{n} \frac{t_i^2}{a_i})(\sum_{i=1}^{n} \frac{1}{a_i}) - A \sum_{i=1}^{n} t_i) = 0$$

Using the same reasoning as in Proposition 3.3.1, we prove that the second factor is negative. And hence $\pi = \xi$.

- Having that $\pi = \xi$, replacing in the expressions from Proposition 3.3.1 and 3.3.2, we observe that the expected price is the same under both regulations.

- Given that demand is linear and our assumption of all producers participating, we see that the expected demand served is the same under both regulations.

- Using this and the fact that, $\min\{p(\theta), p(\theta_i)\} \leq p(\theta_i)$ we can prove that the central planner revenue is higher in cap-and-trade.

- To see the profit expression.

  Observe that:

$$\begin{aligned} 2a_i\mathbb{E}[\Pi_c - \Pi_t] &= \mathbb{E}[\min\{p_c(\theta), p_c(\theta_i)\}^2] - (V(p_t) + \mathbb{E}[p_t]^2 - 2\mathbb{E}[p_t]t_i\pi + t_i^2\pi^2) \\ &= \mathbb{E}[\min\{p_c(\theta), p_c(\theta_i)\}^2] - (V(p_t) + (\mathbb{E}[p] - t_i\pi)^2) \\ &= \mathbb{E}[\min\{p_c(\theta), p_c(\theta_i)\}^2] - (V(p_t) + \mathbb{E}[\min\{p_c(\theta), p_c(\theta_i)\}]^2) \\ &= V(\min\{p_c(\theta), p_c(\theta_i)\}) - V(p_t) \end{aligned}$$

  Where we've used that both policies induce the same expected price, as well as the characterizing equation for permits. Under the assumptions for the tax, we can calculate the variance for the price under tax.

$$V(p_t) = \frac{V(\theta)}{(A + \sum_{i=1}^{n} \frac{1}{a_i})^2}.$$

  Then the expression becomes:

$$2a_i\mathbb{E}[\Pi_c - \Pi_t] = V(\min\{p_c(\theta), p_c(\theta_i)\}) - \frac{V(\theta)}{(A + \sum_{i=1}^{n} \frac{1}{a_i})^2}.$$

  Thus,

$$\Delta_i = \frac{1}{2a_i}(V(\min\{p_c(\theta), p_c(\theta_i)\}) - \frac{V(\theta)}{(A + \sum_{i=1}^{n} \frac{1}{a_i})^2}).$$

- Let i be the producer with the worst technology, i.e. highest t_i. As a result of Lemma 3.A.2 from Proposition 3.3.2, the producer with the worst technology is the first producer to reach its cap. Therefore using the market equilibrium equation:

$$\sum_{j=1}^{n} s_j(p) = D(p, \theta)$$

We find:

$$\min\{p_c(\theta), p_c(\theta_i)\} = \frac{1}{(A + \sum_{i=1}^{n} \frac{1}{a_i})}(B + \min\{\theta, \theta_i\})$$

Thus,

$$\Delta_i = \frac{1}{2a_i(A + \sum_{i=1}^{n} \frac{1}{a_i})^2}(V(\min\{\theta, \theta_i\}) - V(\theta))$$

Similarly, In the case of symmetric suppliers, we find:

$$\Delta_i = \frac{1}{2a(A + \frac{n}{a})^2}(V(\min\{\theta, \theta^*\}) - V(\theta)) \quad \forall i$$

All that remains to prove is that:

$$V(\min X, c) \leq V(X)$$

Where X is a random variable of zero mean and c a constant.

We start by stating,

$$\min\{X, c\} + \max\{X, c\} = X + c. \tag{3.12}$$

This means that the variance satisfies:

$$V(\min\{X, c\}) + V(\max\{X, c\}) + 2(\mathbb{E}[\min\{X, c\} \max\{X, c\}] - \mathbb{E}[\min\{X, c\}]\mathbb{E}[\max\{X, c\}]) = V(X)$$

Rearranging the terms, and using that $\mathbb{E}[\min\{X, c\} \max\{X, c\}] = c\mathbb{E}[X] = 0$, we find:

$$V(\min\{X, c\}) - V(X) = -V(\max\{X, c\}) + 2\mathbb{E}[\min\{X, c\}]\mathbb{E}[\max\{X, c\}]$$

We want to prove that the right hand side is negative. To do this, we separate in two cases, positive c and negative c. If $c \geq 0$, we take expected value in Equation (3.12), and find:

$$\mathbb{E}[\max\{X, c\}] = c - \mathbb{E}[\min\{X, c\}].$$

Therefore,

$$\mathbb{E}[\min\{X, c\}]\mathbb{E}[\max\{X, c\}] = c\mathbb{E}[\min\{X, c\}] - \mathbb{E}[\min\{X, c\}]^2$$

Given the fact that $\min\{X,c\} \leq X$, we know that $\mathbb{E}[\min\{X,c\}] \leq 0$, proving that the right hand side is negative.

If $c < 0$, following the same reasoning in Equation (3.12), we obtain,

$$\mathbb{E}[\min\{X,c\}] = c - \mathbb{E}[\max\{X,c\}].$$

Therefore,

$$\mathbb{E}[\min\{X,c\}]\mathbb{E}[\max\{X,c\}] = c\mathbb{E}[\max\{X,c\}] - \mathbb{E}[\max\{X,c\}]^2$$

Given that $\max\{X,c\} \geq X$, we know that $\mathbb{E}[\max\{X,c\}] \geq 0$, and therefore:

$$c\mathbb{E}[\max\{X,c\}] \leq 0.$$

Proving the result for the case $c < 0$.

3.A.5 Proposition 3.3.3

The assumption that producers produce almost surely can be guaranteeing by the condition $\theta_0 \geq ((A + \sum_{i=1}^n \frac{1}{a_i}) \max\limits_{j=1,\dots,n}\{t_j\} - \sum_{i=1}^n t_i \frac{1}{a_i})\rho - B$

- Following a similar methodology as in the cap regulation case, the optimization problem faced by the producers is:

  $$s_i^b(p) \in \arg \max_{0 \leq q \leq \frac{x_i}{t_i}} \{pq - c_i(q) + \rho(x_i - t_i q)\}.$$

  Under our assumptions in cost structure this yields:

  $$s_i^b(p) = \min\{\frac{p - \rho t_i}{a_i}, \frac{x_i}{t_i}\}.$$

- To choose the optimal x_i we do a change of variable just as in the cap regulation case, and instead search for θ_i such that:

  $$s_i^b(p) = \min\{\frac{p(\theta) - \rho t_i}{a_i}, \frac{p(\theta_i) - \rho t_i}{a_i}\}.$$

  Where p represents the price as a function of uncertainty using the optimal values of $(\theta_i)_{i=1}^n$.

The optimal value of θ_i satisfies:

$$\theta_i \in \arg\max_{\theta_i} \mathbb{E}[p(\theta)\frac{\min\{p(\theta),p(\theta_i)\}-\rho t_i}{a_i} - \frac{1}{2}a_i(\frac{\min\{p(\theta),p(\theta_i)\}-\rho t_i}{a_i})^2 - \rho t_i \frac{\min\{p(\theta),p(\theta_i)\}-\rho t_i}{a_i})] +$$

$$(\rho-\pi)t_i\frac{p(\theta_i)-\rho t_i}{a_i}.$$

The first order condition for θ_i yields:

$$\mathbb{E}[(p(\theta)-p(\theta_i))\delta(\theta > \theta_i)] = (\pi-\rho)t_i.$$

- Given a realization θ, the profit obtained by selling permits can be written as:

$$\rho\frac{t_i}{a_i}(p(\theta_i) - \min\{p(\theta),p(\theta_i)\})$$

Similarly the profit obtained by production is:

$$(\rho t_i + p(\theta) - \frac{1}{2}\min\{p(\theta),p(\theta_i)\})\frac{\min\{p(\theta),p(\theta_i)\}-\rho t_i}{a_i}$$

From the expressions we can see that the revenue from selling permits decreases as θ increases, and the reverse is true for the profits from production.

- The convexity of p is obtained just as in Proposition 3.3.2. To see this, note that,

$$\sum_{i=1}^{n}\min\{p(\theta),p(\theta_i)\} = D(p(\theta),\theta) + \rho\sum_{i=1}^{n}\frac{t_i}{a_i}.$$

Hence the market equilibrium equation is just a shift from the structure from cap regulation, and hence convexity is preserved.

- Similar to Proposition 3.3.2, we now prove that θ_i is increasing in ρ. Notice that

$$(\pi-\rho)(t_{i-1}-t_i) = E[(p(\theta)-p(\theta_i))\delta(\theta\in(\theta_{i-1},\theta_i])] + (p(\theta_i)-p(\theta_{i-1}))E[\delta(\theta > \theta_{i-1})]$$

The market equilibrium equation establishes that $p(\theta)$ is linear for $\theta\in[\theta_{i-1},\theta_i]$ with slope ν_i, therefore,

$$\frac{\pi(t_{i-1}-t_i)}{\nu_i} = E[(\theta-\theta_{i-1})\delta(\theta\geq\theta_{i-1})] - E[(\theta-\theta_i)\delta(\theta\geq\theta_i)]$$

Hence by denoting $G(q) = E[(\theta - q)\delta(\theta \geq q)]$, we can generate a recursion and obtain:

$$G(\theta_i) = \sum_{j=i+1}^{n+1} \frac{(\pi - \rho)(t_{j-1} - t_j)}{\nu_j}$$

Given that G is a decreasing function, the larger the value of ρ, the larger the value of θ_i.

To prove the monotonicity of the total amount of permits we proceed as in Proposition 3.3.2. First, by considering the market equilibrium equation for $\theta_1, ..., \theta_n$ we obtain n equations,

$$\sum_{i=1}^{n} \frac{\min\{p(\theta_j), p(\theta_i)\} - \rho t_i}{a_i} = B - Ap(\theta_j) + \theta_j \quad j = 1, ..., n.$$

Which, following the notation and definitions used in the proof of Proposition 3.3.2, can be rewritten into,

$$M\vec{p} = (B + \rho \sum_{i=1}^{n} \frac{t_i}{a_i})\vec{\mathbb{1}} + \vec{\theta}.$$

Therefore,

$$\vec{t}' D^{-1}\vec{p} = (B + \rho \sum_{i=1}^{n} \frac{t_i}{a_i})\vec{t}'(MD)^{-1}\vec{\mathbb{1}} + \vec{t}'(MD)^{-1}\vec{\theta}$$

$$\sum_{i=1}^{n} x_i = \vec{t}' D^{-1}\vec{p} - \rho t' \vec{D^{-1}} t = B\vec{t}'(MD)^{-1}\vec{\mathbb{1}} + \vec{t}'(MD)^{-1}\vec{\theta} + \rho(\mathbb{1}' D^{-1}\vec{t}\vec{t}'(MD)^{-1}\vec{\mathbb{1}} - t'\vec{D^{-1}}t)$$

$$\sum_{i=1}^{n} x_i = B\vec{t}'(MD)^{-1}\mathbb{1} + \vec{t}'(MD)^{-1}(\vec{\theta} + \rho(\mathbb{1}\mathbb{1}' - (MD))D^{-1}\vec{t})$$

In Proposition 3.3.2, we proved that $t'(MD)^{-1} \geq 0$. By proving that $\vec{\theta} + \rho(\mathbb{1}\mathbb{1}' - (MD))D^{-1}\vec{t}$ is increasing in ρ, we finish proving the monotonicity of the permits purchased. We do this in two steps: first we find $\frac{d}{d\rho}\theta_i$ and then we conclude the result.

We know,

$$\theta_i = G^{-1}((\pi - \rho) \sum_{j=i+1}^{n+1} \frac{(t_{j-1} - t_j)}{\nu_j})$$

Using the inverse function theorem, we have:

$$\frac{d}{d\rho}\theta_i = -\frac{\sum_{j=i+1}^{n+1} \frac{(t_{j-1}-t_j)}{\nu_j}}{G'((\pi - \rho) \sum_{j=i+1}^{n+1} \frac{(t_{j-1}-t_j)}{\nu_j})}.$$

Given that $G(q) = E[(\theta - q)\delta(\theta \geq q)]$, it is easy to show:

$$G'(q) = -F_\theta(q)$$

Let $\alpha_i(\rho) = \frac{1}{F_\theta((\pi-\rho)\sum_{j=i+1}^{n+1}\frac{(t_{j-1}-t_j)}{\nu_j})} > 1$.

Then we have:

$$\frac{d}{d\rho}\sum_{i=1}^{n} x_i = t'(MD)^{-1}z$$

Where,

$$\begin{aligned}
z_i &= \alpha_i(\rho)\sum_{j=i+1}^{n+1}\frac{(t_{j-1}-t_j)}{\nu_j} + \sum_{l=1}^{n}(\mathbb{1}\mathbb{1}' - MD)_{il}\frac{t_l}{a_l} \\
&= \alpha_i(\rho)\sum_{j=i+1}^{n+1}\frac{(t_{j-1}-t_j)}{\nu_j} + (1 - \frac{a_i}{\nu_i})\frac{t_i}{a_i} + \sum_{l>i}^{n}\frac{t_l}{a_l} \\
&= \alpha_i(\rho)\sum_{j=i+1}^{n+1}\frac{(t_{j-1}-t_j)}{\nu_j} - t_i(A + \sum_{j=i}^{n}\frac{1}{a_j}) + \sum_{l\geq i}^{n}\frac{t_l}{a_l} \\
&= \alpha_i(\rho)\sum_{j=i+1}^{n+1}\frac{(t_{j-1}-t_j)}{\nu_j} - t_iA + \sum_{l\geq i}^{n}\frac{t_l - t_i}{a_l} \\
&= \alpha_i(\rho)\sum_{l>i}^{n+1}(t_{l-1}-t_l)(A + \sum_{j=l}^{n}\frac{1}{a_j}) - t_iA + \sum_{l>i}^{n}\frac{t_l - t_i}{a_l} \\
&= \alpha_i(\rho)At_i + \alpha_i(\rho)\sum_{l>i}^{n+1}\frac{(t_{l-1}-t_l)}{a_l} + \alpha_i(\rho)\sum_{l>i}^{n+1}\sum_{j>l}^{n}(t_{l-1}-t_l)\frac{1}{a_j} - t_iA + \sum_{l>i}^{n}\frac{t_l - t_i}{a_l} \\
&= At_i(\alpha_i(\rho)-1) + \alpha_i(\rho)\sum_{l>i}^{n+1}\frac{(t_{l-1}-t_i+t_i-t_l)}{a_l} + \alpha_i(\rho)\sum_{l>i}^{n+1}\sum_{j>l}^{n}(t_{l-1}-t_l)\frac{1}{a_j} + \sum_{l>i}^{n}\frac{t_l - t_i}{a_l} \\
&= At_i(\alpha_i(\rho)-1) + \alpha_i(\rho)\sum_{l>i}^{n+1}\frac{t_{l-1}-t_i}{a_l} + \alpha_i(\rho)\sum_{l>i}^{n+1}\sum_{j>l}^{n}(t_{l-1}-t_l)\frac{1}{a_j} + (\alpha_i(\rho)-1)\sum_{l>i}^{n}\frac{t_i - t_l}{a_l}
\end{aligned}$$

Given that all the terms are positive we conclude the result.

- The rest of the expressions are derived analogous to Proposition 3.3.2. □

3.A.6 Proposition 3.4.1

Following the analysis from Klemperer and Meyer (1989), in order for producer to decide on their supply functions, it solves the supply function equilibrium problem. That is the solution satisfies:

$$\max_{p\geq 0}(p - t_i\xi)(D(p,\theta) - \sum_{j\neq i} s_j(p)) - \frac{1}{2}a_i(D(p,\theta) - \sum_{j\neq i} s_j(p))^2$$

The first order conditions to this problem are:

$$s_i(p) - (p - t_i\xi - a_i s_i(p))(A + \sum_{j\neq i} s'_j(p)) = 0$$

We propose a solution of the form: $s_i(p) = c_i(p - t_i\xi)$. Then we have c_i should satisfy the system:

$$\frac{c_i}{(1 - a_i c_i)} = (A + \sum_{j\neq i} c_j) \quad \forall i$$

In order to find a solution to the system, we can sum the equations to obtain the relation:

$$\sum_{i=1}^{n} \frac{c_i}{(1 - a_i c_i)} - (n-1)c_i = nA$$

We propose a solution such that:

$$\frac{c_i}{(1 - a_i c_i)} - (n-1)c_i = A \quad \forall i$$

This yields a quadratic equation for c_i. The positive solution to this system is $b_i(n)$, as defined in Definition 3.4.1.

In Klemperer and Meyer (1989), establishes that if the solution to the quadratic problem we showed, exists then it's the solution to the supply function equilibrium. Hence we derived:

$$s_i(p) = b_i(n)(p - t_i\xi)$$

Under this conditions following the algebra as in Proposition 3.3.1, we can find the expected price:

$$\mathbb{E}[p] = \frac{B + \xi \sum_{i=1}^{n} t_i b_i(n)}{A + \sum_{i=1}^{n} b_i(n)}$$

The expected emissions:

$$\mathbb{E}[e] = \sum_{i=1}^{n} t_i b_i(n)(\mathbb{E}[p] - t_i\xi)$$

expected revenue collected by the regulator:

$$\mathbb{E}[R] = \xi \mathbb{E}(e).$$

The expected profit for the regulator:

$$\mathbb{E}[\Pi_i] = b_i(n)(1 - \frac{1}{2}a_i b_i(n))\mathbb{E}[(p(\theta) - t_i\xi)^2].$$

Throughout the paper we assume producers are active for all uncertainties. This is equivalent to adding the additional constraint in θ:

$$p(\theta) = \frac{(B + \theta + \xi \sum_{i=1}^n t_i b_i(n))}{A + \sum_{i=1}^n b_i(n)} \geq \min_j \{t_j\}\xi$$

3.A.7 Proposition 3.4.2

In this setting the value of θ as well as x_i are known when trading occurs. In order to determine the result from trading, we need to find the value to two parameters: $s_i(p)$ the amount produced and τ_i the permits purchased (sold if the value is negative).

When trading occurs each producer faces the following optimization problem:

$$\max_{\substack{s_i(p) \geq 0 \\ t_i s_i(p) \leq x_i + \tau_i}} \quad p s_i(p) - \frac{1}{2} a_i s_i(p)^2 - \rho \tau_i.$$

Subject to the market clearing condition:

$$\sum_{i=1}^n \tau_i = 0. \tag{3.13}$$

Given that :

$$\sum_{i=1}^n s_i(p) = B - Ap + \theta \tag{3.14}$$

We can replace the price in the optimization problem, the first order conditions yield:

$$\begin{aligned}
s_i(p) - \{p - a_i s_i(p) - \lambda_i t_i\}(A + \sum_{j \neq i} s_j'(p)) &= 0 \\
\rho - \lambda_i &= 0 \\
s_i(p)\mu_i &= 0 \\
\lambda_i(\tau_i + x_i - t_i s_i(p)) &= 0 \\
s_i(p) &\geq 0 \\
\mu_i &\geq 0 \\
\lambda_i &\geq 0 \\
t_i s_i(p) - \tau_i &\leq x_i
\end{aligned}$$

We restrict ourselves to the cases when ρ the trading price is positive almost surely, as well as the production from producers. We later characterize the range of θ that is consistent with our assumptions. This simplification implies $\lambda_i = \rho$ and $\mu_i = 0$.

Hence the system then becomes:

$$\begin{aligned}
s_i(p) - \{p - a_i s_i(p) - \rho t_i\}(A + \sum_{j \neq i} s_j'(p)) &= 0 \\
t_i s_i(p) - \tau_i &= x_i
\end{aligned}$$

We proceed as in Proposition 3.4.1 and propose a solution $s_i(p(\theta)) = c_i(p(\theta) - \rho(\theta)t_i)$. In this case the equation we obtain for c_i is the same as in Proposition 3.4.1, and hence the appropriate value for c_i is $b_i(n)$. Note that ρ is now a random variable, function of the consumer price $p(\theta)$, defined by:

$$\rho(\theta) = \frac{t_i b_i(n) p(\theta) - \sum_{i=1}^n x_i}{\sum_{i=1}^n t_i^2 b_i(n)}$$

- One can show using (3.14), that:

$$\mathbb{E}[p(\theta)] = \frac{B + \mathbb{E}[\rho(\theta)] \sum_{i=1}^n t_i b_i(n)}{A + \sum_{i=1}^n b_i(n)}.$$

$$\rho(\theta) \geq 0$$

Resembling previous results.

- Using (3.13), we find:

$$\sum_{i=1}^{n} t_i b_i(n)(\mathbb{E}[p] - \mathbb{E}[\rho]t_i) = \sum_{i=1}^{n} x_i.$$

  Thus we have can determine the expected price, and the expected market trading price in terms of $\sum_{i=1}^{n} x_i$, by solving these two equations.

- In order to characterize the permits bought, once can find the producer's profits are:

$$\mathbb{E}[\Pi_i] = \mathbb{E}[b_i(n)(p(\theta) - \rho(\theta)t_i)^2(1 - \frac{1}{2}a_i b_i(n)) + \rho(\theta)x_i] - \pi x_i$$

  Under this assumption an expression characterizing the optimal value of x_i may be obtained with some algebra.

- Lastly, to characterize the bounds for θ, we want to find x_i, the amount of permits purchased. As established before x_i comes from a maximization problem. In this scenario, the maximization problem is:

$$\max_{x_i \geq 0} \mathbb{E}[(1 - \frac{a_i b_i(n)}{2})b_i(n)(p(\theta) - t_i\rho(\theta))^2 + \rho(\theta)x_i] - \pi x_i.$$

  To find the first order condition, we can use the equations previously developed to observe that the dependency of ρ and p on x_i is linear. In fact, it has the same slope for all i. Hence:

$$\frac{\partial}{\partial x_i}\mathbb{E}[\rho(\theta)] = \rho' \quad \forall i$$

$$\frac{\partial}{\partial x_i}\mathbb{E}[p(\theta)] = p' \quad \forall i$$

  Moreover to find the values of ρ' and p' we solve the system of equations:

$$\sum_{i=1}^{n} t_i b_i(n)(p' - \rho' t_i) = 1.$$

$$\sum_{i=1}^{n} b_i(n)(p' - \rho' t_i) = -Ap'.$$

  And thus, we see they do not depend on the value of π. After observing this we can determine the first order conditions to the maximization problem:

$$2(1 - \frac{a_i b_i(n)}{2})(p' - t_i\rho')(\mathbb{E}[p(\theta)] - t_i\mathbb{E}[\rho(\theta)])b_i(n) + \mathbb{E}[\rho(\theta)] + x_i^*\rho' = \pi \qquad (3.15)$$

This condition depends on the expected price and the expected trading price, ρ, hence it depends on the other x_j, $j \neq i$. Observe that the interchange between expected value and derivatives is justified when θ has a bounded range.

In order to obtain the optimal amount of permits that each producer purchases, we first determine ρ' and p', as they do not depend on π or x_i, and then solve the following system of equations in the variables $\mathbb{E}(p)$,$\mathbb{E}(\rho)$ and $\{x_i\}_{i=1}^n$:

$$\begin{aligned}
2(1-\frac{a_i b_i(n)}{2})(p'-t_i\rho')b_i(n)(\mathbb{E}[p]-t_i\mathbb{E}[\rho])+\mathbb{E}[\rho]+x_i^*\rho' &= \pi \quad \forall i \\
\sum_{i=1}^n t_i b_i(n)(\mathbb{E}[p]-\mathbb{E}[\rho]t_i) &= \sum_{i=1}^n x_i^* \\
\sum_{i=1}^n b_i(n)(\mathbb{E}[p]-\mathbb{E}[\rho]t_i) &= B - A\mathbb{E}[p].
\end{aligned}$$

Once the solution to the system of linear equations is established, one can characterize the ranges for θ by imposing, $s_i(p), \rho(\theta) \geq 0$.

Observe that if we are only concerned with the expected price and the total amount of permits purchased, we can reduce the problem to two variables and two equations as follow:

$$\begin{aligned}
\sum_{i=1}^n \{2(1-\frac{a_i b_i}{2})(p'-\rho' t_i)+\rho' t_i\} b_i(\mathbb{E}[p]-t_i\mathbb{E}[\rho]) &= n(\pi-\mathbb{E}[\rho]) \\
\sum_{i=1}^n b_i(\mathbb{E}[p]-\mathbb{E}[\rho]t_i) &= B - A\mathbb{E}[p].
\end{aligned}$$

Here we summed the first n equations of the previous system, and replace the value for the sum of the permits using the second equation of the previous system. □

3.B Counterexamples

Consider a random variable θ uniformly distributed in the interval $[-f, f]$, B, A, t_2, t_1, a_2, a_1 and π are parameters. Consider $t_2 < t_1$.

Then if we choose the parameters such that both producers purchase permits, we would have:

$$\theta_1 = f - 2\sqrt{f\pi((t_2-t_1)(A+\frac{1}{a_2})+t_2},$$

$$\theta_2 = f - 2\sqrt{f\pi t_2}.$$

Price as a random variable for quantities regulation is:

$$p(\theta) = \begin{cases} \frac{B+\theta}{(A+\frac{1}{a_1}+\frac{1}{a_2})} & \theta \in [-f, \theta_1) \\ \frac{(\theta-\theta_1)}{A+\frac{1}{a_2}} + \frac{B+\theta_1}{(A+\frac{1}{a_1}+\frac{1}{a_2})} & \theta \in [\theta_1, \theta_2) \\ \frac{\theta-\theta_2}{A} + \frac{(\theta_2-\theta_1)}{A+\frac{1}{a_2}} \frac{B+\theta_1}{(A+\frac{1}{a_1}+\frac{1}{a_2})} & \theta \in [\theta_2, f] \end{cases}$$

The expected profits under permit regulation are :

$$\frac{1}{2a_1}\mathbb{E}[\min\{p(\theta), p(\theta_1)\}^2] = \frac{1}{4a_1 f(A+\sum_{i=1}^{2}\frac{1}{a_i})^2}(\frac{(B+\theta_1)^3}{3} - \frac{(B-f)^3}{3} + (B+\theta_1)^2(f-\theta_1))$$

$$\frac{1}{2a_2}\mathbb{E}[\min\{p(\theta), p(\theta_2)\}^2] = \frac{1}{4a_2 f}\{\frac{1}{(A+\sum_{i=1}^{2}\frac{1}{a_i})^2}(\frac{(B+\theta_1)^3}{3} - \frac{(B-f)^3}{3}) + (\frac{(\theta_2-\theta_1)}{A+\frac{1}{a_2}} +$$

$$\frac{B+\theta_1}{(A+\frac{1}{a_1}+\frac{1}{a_2})})^2(f-\theta_2)) + \frac{A+\frac{1}{a_2}}{3}((\frac{(\theta_2-\theta_1)}{A+\frac{1}{a_2}} + \frac{B+\theta_1}{(A+\frac{1}{a_1}+\frac{1}{a_2})})^3 - (\frac{B+\theta_1}{(A+\frac{1}{a_1}+\frac{1}{a_2})})^3)\}.$$

Price as a random variable for tax regulation is:

$$p(\theta) = \frac{B+\theta+\pi\sum_{i=1}^{2}\frac{t_i}{a_i}}{A+\sum_{i=1}^{2}\frac{1}{a_i}}$$

The expected profits under tax regulation are:

$$\frac{1}{2a_2}\mathbb{E}[(p(\theta)-t_2\xi)^2] = \frac{1}{12fa_2(A+\sum_{i=1}^{2}\frac{1}{a_i})^2}\{(B+f+\pi(\sum_{i=1}^{2}\frac{t_i-t_2}{a_i}-t_2A))^3-(B-f+\pi(\sum_{i=1}^{2}\frac{t_i-t_2}{a_i}-t_2A))^3\}.$$

$$\frac{1}{2a_1}\mathbb{E}[(p(\theta)-t_1\xi)^2] = \frac{1}{12fa_1(A+\sum_{i=1}^{2}\frac{1}{a_i})^2}\{(B+f+\pi(\sum_{i=1}^{2}\frac{t_i-t_1}{a_i}-t_1A))^3-(B-f+\pi(\sum_{i=1}^{2}\frac{t_i-t_1}{a_i}-t_1A))^3\}.$$

All our figures have in common $B = 100$, $A = 1$ By choosing $B = 100$, $A = 1$, $f = 50$ and maximum $\pi = 4$.

- In Figure 3-1 and Figure 3-2, the parameters chosen were $t_1 = 1$, $t_2 = 0.5$, $a_1 = 1.6$ and $a_2 = 2$. These parameters guarantee that both producers have decreasing profits as the tax rate/permit price increases.

- In Figure 3-3, the parameters chosen were $t_1 = 1$, $t_2 = 0.1$, $a_1 = 0.2$ and $a_2 = 2$. These parameters guarantee that the producer 2 receives increasing profits under both regulations. The producer experiences higher profits under cap regulation.

- Below, in Figure 3-5, the parameters chosen were $t_1 = 1$, $t_2 = 0.1$, $a_1 = 0.2$ and $a_2 = 2$. These parameters guarantee that the producer 2 receives increasing profits under both regulations. For low permit prices the producer experiences higher profits under cap regulation, but after a certain value a tax policy delivers higher profits.

- Below, in Figure 3-6, the parameters chosen were $t_1 = 16$, $t_2 = 3$, $a_1 = 4$ and $a_2 = 6$. These parameters guarantee that the producer 2 receives increasing profits under tax regulation, and decreasing profits under cap regulation.

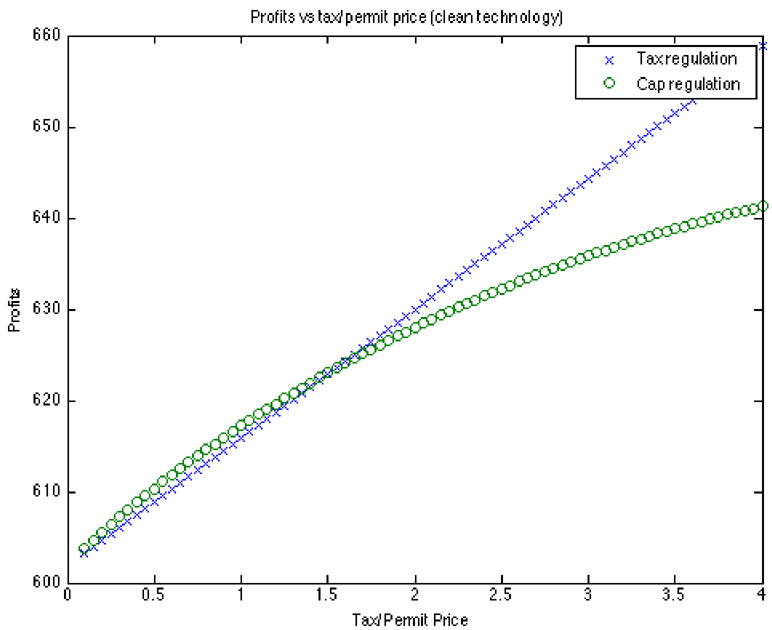


Figure 3-5: Producer 2 profits crossing each other.

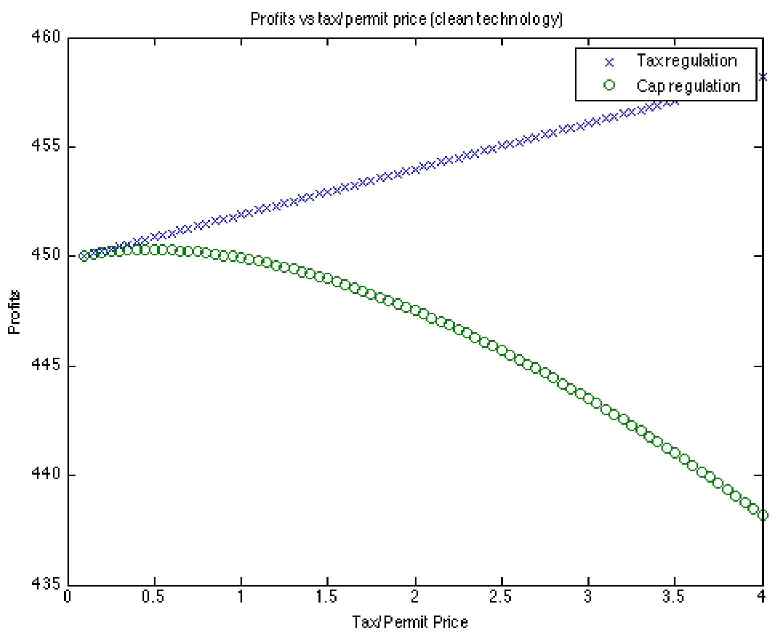


Figure 3-6: Producer 2 profits increasing in tax and decreasing in permit price.

3.B.1 Matlab Code

Below is the code used to generate the graphics in Matlab.

```
function [c,t]=checkProfits()
m=2;
%both decreasing
%t=[1;0.5];
%c=[0.8;1];

%Disjoint
%t=[16; 3];
%c=[2; 3];

%Both increasing
%t=[1;0.1];
%c=[0.1; 1];

%Cross
%t=[15;2];
%c=[2; 2];

a=2*c;

cond=sum(t./a)*ones(m,1)-t*sum(1./a)-t;

range=0.1:0.05:4;
PT=zeros(m,length(range));
PCT=zeros(m,length(range));
i=1;
for tt=range
    P=theoProfits(tt);
    PT(:,i)=P(:,2);
    PCT(:,i)=P(:,1);
    i=i+1;
end

plot(range,PT(2,:),'x',range,PCT(2,:),'o')
title('Profits vs tax/permit price (clean technology)')
legend('Tax regulation','Cap regulation')
xlabel('Tax/Permit Price')
ylabel('Profits')

function P=theoProfits(tx)
B=100;
f=50;
A=1;
tp=zeros(m,1);
cp=zeros(m,1);
for j=1:m
        tp(j)=(1/(2*a(j)))*(1/(6*f*(A+sum(1./a))^2))*((B+f+tx*(cond(j)))^3-(B-f+tx*(cond(j)))^3);
end

tth=[f-2*sqrt(f*tx*((t(1)-t(2))*(1+1/a(2))+t(2))) ;f-2*sqrt(f*tx*t(2))];
cp(2)=(1/(2*a(2)))*(1/(2*f))*((1/(A+sum(1./a))^2)*( ((B+tth(1))^3)/3 - ((B-f)^3)/3 )
+((tth(2)-tth(1))/(A+1/a(2)) + (B+tth(1))/(A+sum(1./a)))^2*(f-tth(2))
+((A+1/a(2))/3)*( ( (tth(2)-tth(1))/(A+1/a(2))
+ (B+tth(1))/(A+sum(1./a)) )^3 - ( (B+tth(1))/(A+sum(1./a)) )^3 )  );
cp(1)=(1/(2*a(1)))*(1/(2*f*(A+sum(1./a))^2))*( ((B+tth(1))^3)/3 -((B-f)^3)/3
```

```
+(f-tth(1))*(B+tth(1))^2);
P=[cp tp];
end
end
```

3.C Lower bound for profits in the case of Cap regulation

From Proposition 3.3.2, the profits for cap regulation for producer i are given by,

$$\mathbb{E}[\Pi_i] = \frac{1}{2a_i}\mathbb{E}[\min\{p(\theta), p(\theta_i)\}^2].$$

Using Jensen's inequality we can obtain a lower bound:

$$\begin{aligned}
\mathbb{E}[\Pi_i] &\geq \frac{1}{2a_i}\mathbb{E}[\min\{p(\theta), p(\theta_i)\}]^2 \\
&= \frac{1}{2a_i}(\mathbb{E}[p] - \pi t_i)^2 \\
&= \frac{1}{2a_i(A + \sum_{j=1}^n \frac{1}{a_j})^2}(B + \pi(\sum_{j=1}^n \frac{t_j - t_i}{a_j} - At_i))^2
\end{aligned}$$

Taking derivatives it can be verified that the lower bound is increasing in π if and only if $\sum_{j=1}^n \frac{t_j - t_i}{a_j} > At_i$.

3.D Existence and uniqueness in the unregulated framework

We prove this in a more general framework where the cost is strictly convex and $\lim_{q\to+\infty} c_i(q) = +\infty$, additionally demand is decreasing in p.

Under this assumptions, there is a unique optimal solution to the optimization problem defining $s_i(p)$, and hence it is well defined. Moreover, since the objective function defining $s_i(p)$ is super modular in p and q and the set $\{(p,q) : q \geq 0, p \geq 0\}$ is a lattice, $s_i(p)$ is increasing in p.

Let $f(p) = D(p) - \sum_{i=1}^n s_i(p)$, which given the assumptions is a decreasing function. $f(0) = D(0) > 0$, plus the monotonicity of s_j, $\lim_{p\to+\infty} f(p) < 0$. Hence there exists a p^M such that

$f(p^M) < 0$. Then using the intermediate value theorem we have the existence of p^* the market equilibrium. Using the strict monotonicity of f, uniqueness follows. □

3.E Intuition on Theorem 3.3.2

Next, we provide some intuition on why the permit price has to be equal to the tax rate in order to compare regulations. Consider the problem:

$$\begin{aligned} \min_{x \in X} \quad & f(x) \\ \text{s.t.} \quad & Ax \leq b \end{aligned}$$

Where we are minimizing an objective function, f, subject to some resource constraints. For example, think about x as production, f to be minus the social welfare, X to include all technical constraints and $Ax \leq b$ represent emission limits. This is the problem the central planner is interested in solving. Lagrangean duality, under proper assumptions, establishes the absence of a duality gap as well as existence of a dual multiplier, λ, such that the optimal value to the problem coincides with the optimal value to the relaxed problem, therefore,

$$\min_{x \in X, Ax \leq b} f(x) = \min_{x \in X} \{f(x) + \lambda'(b - Ax)\}.$$

To see the connection with our results, we rewrite the equality as:

$$\min_{x \in X, Ax \leq b} \{f(x)\} - \lambda' b = \min_{x \in X} \{f(x) - \lambda'(Ax)\}.$$

The left hand side mimics a situation where resources are purchased prior to production at a price λ, while the right hand side mimics the situation where resources are purchased in conjunction with production at a price λ. Lagrangean duality states that when using the appropriate value of λ the production objective can be achieved by either system.

3.F Numerical results for cap-and-trade

From Appendix 3.A.7, we can find ρ', p' by solving:

$$\sum_{i=1}^{n} t_i b_i(n)(p' - \rho' t_i) = 1.$$

$$\sum_{i=1}^{n} b_i(n)(p' - \rho' t_i) = -Ap'.$$

Once we have these values, we solve the following system of equations:

$$\begin{aligned}
2(1 - \frac{a_i b_i(n)}{2})(p' - t_i\rho')b_i(n)(\mathbb{E}[p] - t_i\mathbb{E}[\rho]) + \mathbb{E}[\rho] + x_i^*\rho' &= \pi \quad \forall i \\
\sum_{i=1}^{n} t_i b_i(n)(\mathbb{E}[p] - \mathbb{E}[\rho]t_i) &= \sum_{i=1}^{n} x_i^* \\
\sum_{i=1}^{n} b_i(n)(\mathbb{E}[p] - \mathbb{E}[\rho]t_i) &= B - A\mathbb{E}[p].
\end{aligned}$$

Thus, obtaining the values for $\mathbb{E}[p], \mathbb{E}[\rho]$ and $\{x_i\}_{i=1}^n$. Given these values average emissions is, $\sum_{i=1}^n x_i$, and the revenue collected by the central planner is $\pi \sum_{i=1}^n x_i$.

The case of tax is completely analogous to Appendix 3.B.

The parameters used in Figure 3-4 are:

- $a = 1, 1.2, 1.8, 2.2.$
- $t = 2, 1.5, 1, 0.8.$
- $D(p, \theta) = 100 - p + \theta.$
- $\theta \sim Uniform[-20, 20].$

Chapter 4

Supply Chain Impact Of Labeling Decisions

4.1 Introduction

Consumers are increasingly using dimensions beyond price to make purchasing decisions of goods and services. For example, diverse quality measures such as CO_2 emissions and nutritional values of goods can be significant sources of competitive advantage for products and services, and poor performance in these dimensions can lead to potential loss of customers and profitability. Although certain types products' quality (e.g., products that are *credence goods*) cannot be easily observed by consumers, firms are increasingly making quality more discernible through *quality labels* in order to differentiate their products and inform consumers' purchasing decisions. Examples of carbon labels come in different contexts, such as environmental (e.g., the Carbon Trust's carbon footprint labels, the U.S. Environmental Protection Agency's Energy Star ratings for electrical appliances, and the Forest Stewardship Council (FSC) certified labels for responsibly sourced timber), health (e.g., nutritional labels), and safety (e.g., Nike's Code of Conduct).

In this chapter, we investigate the impact the labeling decision has on the supply chain. Because labeling makes previously unobservable characteristics of products visible to consumers, there are potential benefits in terms of increased sales to be obtained from labeling, independent of which party in the supply chain leads the initiative to label the products. An increase in potential sales generates additional revenue for the retailer, while at the same time it produces an increase in the orders the supplier receives from the retailer. Given that both parties may benefit from labeling, the design of an appropriate supply-chain contract that maximizes potential benefit is of the essence. Our research focuses on whether the quality described in the label (e.g., energy consumption, CO_2 emissions, or nutritional facts) should be decided by the supplier or imposed by the retailer. In particular, the question we are looking to analyze is the following: *In an environment where demand is affected by a quality measure, which party should decide on the quality level to be labeled: the supplier or the retailer? What kind of contracts are able to achieve the levels and profits of a vertically integrated supply chain?*

For this purpose, we use a stylized model to represent the strategic interaction between a retailer and a supplier. The supplier produces goods and decides the wholesale price at which it supplies its goods to the retailer. The supplier's production cost is increasing in the quality of the product. The retailer is a price-setter facing one-period uncertain demand which is decreasing in retail price and increasing in the supplier's quality level. The retailer chooses retail price as well as the stocking quantity (amount purchased from the supplier) prior to realization of demand. We assume that unsatisfied demand over the period is filled with an emergency order from an alternative source. The retailer faces an expedite cost for demand shortage, and a salvage/disposal cost for additional leftover units. Note that the supplier and the retailer bear different sets of costs: the supplier bears the cost of quality decisions, whereas the retailer faces the risk of uncertain demand.

We analyze decisions about the optimal quality of a product made under three models that reflect different decision-making parties. We begin by considering a vertically integrated supply chain making decisions on both the quality level and the retail price. This serves as a *benchmark (B)* model by giving the maximum amount of profits that can be obtained in the supply chain. Then, we consider two different models to compare with the benchmark: the *Supplier-Choice (SC)* model, and the *Retailer-Choice (RC)* model. It is natural for the supplier to make a decision about the quality level of its product; the SC model, in which the supplier decides the product's quality level as well as its wholesale price to the retailer, reflects this reality. However, retailers have more direct contact with the consumers, and we also consider the possibility of a retailer to be influential enough such that it can suggest a quality level to the supplier. This interaction is captured in our RC model, in which the retailer enforces a quality level on the supplier's product, and the supplier then decides the wholesale price to the retailer.

We show that when market demand is quasilinear and deterministic, the SC, RC, and B models yield identical optimal quality levels. We also show that the optimal quality levels under the SC and the RC models that are identical under deterministic demand can differ in the presence of demand uncertainty. We characterize two different contracts coordinating the supply chain. These contracts coordinate the supply chain independent of assumptions on demand or costs. In the case of linear demand we characterize the optimal solution for each of the models, as well as finding a system of ordinary differential equations characterizing the effect of demand variance. In the majority of realistic scenarios, the RC model leads to higher chosen quality levels than the SC model; thus, the retailer faces reduction in payoff when leaving quality level decisions to the supplier.

In Chapter 4 of Jira (2013), this model is analyzed under the additional assumption of uniformly distributed demand. This Chapter extends the model in order to extend results for general distributions of demand.

4.2 Literature Review

Our work builds upon and contributes to several streams of literature that explore additional product attributes beyond price that are desirable to end customers but are costly to provide. In particular, our work contributes to three streams of literature: (1) the literature on vertical differentiation of products and services, especially those that contain interactions between supply chain members, (2) the inventory management literature that feature decision choices among members of the supply chain, such as the literature on Vendor-Managed Inventory, and (3) the literature on the interaction between product quality and supply chain performance.

There is a large body of work in economics and operations management on vertical differentiation models, in which a firm's products and services compete on both price and another aspect beyond price ("quality") that is desirable to consumers. A higher level of quality increases consumer demand, but costs more for a firm to provide. Examples of these studies include various models of duopolistic price and quality competition as multistage games in which quality and price are chosen in different stages (Choi and Shin, 1992; Wauthy, 1996; Moorthy, 1988; Motta, 1993; Lehmann-Grube, 1997; Wang, 2003). Within this stream of literature, there are studies pertaining to specific contexts of *quality*, such as environmental management (e.g., Amacher et al., 2004; Conrad, 2005; Rodriguez-Ibeas, 2007; Craig, 2012), advertising budget (e.g., Dorfman and Steiner, 1954), and production (e.g., Banker et al., 1998; Chambers et al., 2006; Bernstein and Federgruen, 2004). These streams of research, like ours, model consumer demand as linearly decreasing in price and increasing in quality, and model production cost as increasing in quality. However, these streams of research differ from ours in several ways. Firstly, in these studies, both price and quality decisions are made by one firm. As such, these studies do not touch upon supply chain interactions between the supplier and the retailer. Secondly, with the exception of Raz et al. (2012) and Bernstein and Federgruen (2004), these studies do not consider stochasticity in consumer demand, like ours does. Thirdly, these studies focus almost exclusively on horizontal duopolistic or oligopolistic competition between firms, which is not the focus of our work.

Unlike the above studies, which do not contain a supply chain interaction, another body of work focuses on the interaction between a supplier and the retailer about price and "quality" decisions. In operations management, there are various streams of such literature. One example is the body of work about price and service competition (e.g., Desiraju and Moorthy, 1997; Tsay and Agrawal, 2000; Cachon and Harker, 2002; Boyaci and Gallego, 2004; Bernstein and Federgruen, 2007; Allon and Federgruen, 2009; Krishnan and Winter, 2010). Like our study, many of these studies consider the optimal decisions as well as contractual alignments associated with the interaction between a retailer and a supplier facing stochastic consumer demand. However, the quality (i.e., service) levels are driven by the retailer's actions, whereas the quality levels in our model are the property of the supplier's products and thus driven by the supplier's production technologies. These studies also focus on the horizontal competition between retailers, which is not the focus of our analysis. Another stream of work is on price and lead time decisions (e.g., Liu et al. 2007, Pekgun et al. 2008, Ha et al. 2010, Hua et al. 2010). While these studies, like ours, explore the difference between the centralized versus decentralized decision-making structures for price and quality, the first three studies do not consider the possibility that the retailer can make decisions on quality as our work does, while the last study only focuses on deterministic demand. Outside of operations management, similar models exist in various contexts such as private labels, Gomez-Arias and Bello-Acebron (2008) and environmental labels Craig (2012). These studies have two main differences from our work: (1) quality is determined exogenously and thus is not part of the suppliers' or the retailers' decisions, and (2) unlike our models, which focus on *who* chooses the level of quality, these models focus on the competition between the suppliers or the supply chains.

Our work is also related to the literature on Vendor Managed Inventory (e.g., Choi et al. 2004, Aviv 2002, Kim 2008, Mishra and Raghunathan 2004, Nagarajan and Rajagopalan 2008, and Kraiselburd et al. 2004) because of the similar focus on choices made by retailers versus suppliers. However, in this stream of literature, the choice is on stocking quantity decisions rather than quality, and the only "quality" dimension mentioned is effort, which is not contractible and not observable. More generally, because we explicitly model inventory decisions in the supply chain, our work also extends the operations and inventory management literature that focuses on the integration of pricing and inventory (e.g., Simchi-Levi et al., 2005).

Lastly, there exists an extensive literature on quality management in the supply chain. However, the focuses of these papers are different from ours in a significant way. For example, Baiman, Fisher, Rajan (2000) focused on prevention and appraisal costs for defective products and contractual terms in the face of moral hazard. Whang et al (2006) examine the buyers problem of inducing the suppliers quality effort through appraisal and certification. Gans (2002) focused on customer choice in response to random variation in quality. Hosanagar et al (2005) focuses on the decision of quality of service and price in relations to the cache sizes. Zhu et al (2007) models quality at the unit product level and considers choices of investing in quality-improvement efforts from both the retailer and the supplier. Rayniers and Tapiero (1995) model the effect of contract parameters such as price rebates and after-sales warranty costs on the choice of quality by a supplier, the inspection policy of a producer, and the resulting end product quality.

4.3 Model

We consider a sequential two-stage supply chain consisting of one supplier and one retailer, facing customer demand. The supplier supplies good to the retailer under a wholesale price contract in which the wholesale price is specified by the supplier. The supplier's goods have associated with it a quality level (e.g., carbon emissions) which is determined by the supplier's production process. The risk-neutral retailer has to decide on its stock level and the selling price of a single product. Demand is stochastic, and depends on the selling price and a quality parameter. As expected, the supplier anticipates the reaction of the retailer and takes it into account when deciding on the quality level and wholesale price.

The setting in this model is as follows. Demand is stochastic and endogenously determined. In particular, for a given selling price p, and a given quality level x, demand has the following form.

$$D(p, x, \varepsilon) = D(p, x) + \varepsilon,$$

where ε is a continuous random variable with $\mathbb{E}[\varepsilon] = 0$, $\mathbb{E}[\varepsilon^2] = \sigma^2$, probability density function f and cumulative distribution function F. In the analysis under deterministic demand, we consider a quasilinear demand, $D(p, x) = -bp + X(x)$, where $b > 0$ and X is a continuously differentiable function. In the analysis under stochastic demand, we consider a linear demand $D(p, x) = A - bp + rx$, where A, b and r are positive coefficients. An implicit assumption is that realized demand $D(p, x, \varepsilon)$ is always non negative, which imposes conditions on the selling price, quality parameter and random variable.

The retailer places an order from the supplier before realization of demand, and sells the product to its customers at a unit price p. The unit ordering cost is the wholesale price from the supplier w, and unsatisfied demand is filled with an emergency order. Let $h(y)$ be the inventory holding/disposal cost or the emergency ordering cost when the inventory level after satisfying demand is y. We assume $h(y)$ of the following form.

$$h(y) = h^+ \max\{y, 0\} + h^- \max\{-y, 0\},$$

where h^+ is the unit inventory holding/disposal cost if h^+ is nonnegative or the unit salvage value if it is negative, and h^- is the unit cost for the emergency order. Additionally, we require $h(y)$ to be convex, and zero to be a minimizer of $wy + h(y)$, or equivalently,

$$h^- \geq w \geq \max\{0, -h^+\},$$

namely, the salvage value is no more than the wholesale cost from the supplier, which in turn is no more than the unit cost of an emergency order. These assumptions guarantee that production of the units is more profitable than expediting all units, or over purchasing to cover demand almost surely. For a given stock level y and a selling price p, the expected profit of the retailer has the following form.

$$\Pi_r = \mathbb{E}[pD(p, x, \varepsilon) - wy - h(y - D(p, x, \varepsilon))].$$

The supplier receives orders from the retailer before realization of demand. The wholesale price the supplier specifies to the retailer is w, and the unit production cost is $c(x)$. We assume unit production cost to be increasing in the quality level x. For a given order quantity y, wholesale price w and quality level x, the profit for the supplier has the following form.

$$\Pi_s = (w - c(x))y.$$

Before proceeding to establish the maximization problem that each of the parties faces, we differentiate between three different models, depending on the interaction between supplier and retailer.

- *Supplier-choice model*: The supplier and the retailer act as a two-stage supply chain. The supplier decides on the quality level, x.

- *Retailer-choice model*: The supplier and the retailer act as a two-stage supply chain. The retailer decides on the quality level, x.

- *Integrated model*: The retailer and the supplier act as a vertically integrated supply chain.

In the **supplier-choice** model, the supplier chooses the quality level to be used. This model is the most natural, as the supplier experiences the cost of increasing quality. In this model, a two-stage Stackleberg game determines the optimal level of quality x. In the second stage, the maximization problem faced by the retailer is the following.

$$\max_{p,y\geq 0} \mathbb{E}[pD(p,x,\varepsilon) - wy - h(y - D(p,x,\varepsilon))]. \tag{4.1}$$

In the first stage, the supplier anticipates the behavior of the retailer when setting the wholesale price and quality level. The objective of the supplier is to maximize its own profit, which can be written as a function of the ordering quantity of the retailer, $y(w,p)$.

$$\max_{\substack{w\geq 0\\ x\in\mathbb{R}}} (w - c(x))y(w,x). \tag{4.2}$$

By contrast, in the **retailer-choice** model, the retailer enforces a quality level on the retailer. This model is more natural in settings where the retailer has bargaining power over the supplier, or is more acquainted with customer demand. In this model, a three stage Stackelberg game determines the optimal quality level x. In the last stage, the retailer decides on the consumer price as well as the order quantity, the maximization problem faced by the retailer is as follows.

$$\max_{p,y\geq 0} \mathbb{E}[pD(p,x,\varepsilon) - wy - h(y - D(p,x,\varepsilon))],$$

In the middle stage, the supplier anticipates the behavior of the retailer and decides on the wholesale price given the quality level suggested by the retailer. Given the order quantity from the retailer $y(w)$, the maximization problem faced by the supplier in this stage is as follows.

$$\max_{w\geq 0} (w - c(x))y(w), \tag{4.3}$$

Finally, in the first stage, the retailer anticipates the behavior of the supplier in the middle stage. Given p and $y(w)$ the solutions to the last stage, and w the solution to the middle stage, the retailer solves the following optimization problem in order to find a quality level x that maximizes its profits.

$$\max_{x\in\mathbb{R}} \mathbb{E}[p(x)D(p(x),x,\varepsilon) - w(x)y(w(x)) - h(y(w(x)) - D(p(x),x,\varepsilon))], \tag{4.4}$$

Lastly, in the **integrated** model, the supplier and the retailer act as a vertically integrated supply chain. In this model, the objective is to maximize the system expected profit. The profit for the system has the following form.

$$\max_{\substack{p,y\geq 0\\ x\in\mathbb{R}}} \mathbb{E}[pD(p,x,\varepsilon) - c(x)y - h(y - D(p,x,\varepsilon))]. \tag{4.5}$$

The next section characterizes the solution in the absence of demand uncertainty. Then, Section 4.5 illustrates two different contracts that allow a supplier-choice model to achieve the profits of the integrated model.

4.4 Deterministic Demand Analysis

In this section we restrict the model to the case when demand is deterministic. Under deterministic demand, and given the assumptions on $h(y)$, the retailer's optimal ordering quantity is the demand. The next proposition, specifies the optimal quality level in the supplier-choice model, the retailer-choice model and the integrated supply chain system.

Theorem 4.4.1. *Consider a quasilinear demand $D(p,x) = -bp+X(x)$. Assume the optimal quality level is finite and the supplier and retailer experience positive profits. Then, the optimal quality level under the integrated supply chain, supplier-choice and retailer-choice the models satisfy the equation:*

$$c'(x) = \frac{X'(x)}{b}. \tag{4.6}$$

Theorem 4.4.1 characterizes the optimal quality level for the three different models. As it turns out, in the absence of uncertainty the three models use the same quality level. Two observations can be made from the expression in the theorem. First, the optimal quality level is increasing when the marginal increase in demand, $X'(x)$, is higher. That is, if the potential increase in demand is higher, then the quality level used is higher. Second, if the effect of retail price on consumer demand, b, is much larger than the effect of quality on consumer demand, $X'(x)$, the optimal quality level is smaller.

As we shall see, when demand uncertainty is introduced this equivalency between the models is broken, due to a *double maginization* behavior. However, in the next section we present contracts that are able to coordinate the supply chain under general assumptions on demand.

4.5 Coordination Contracts

In the supplier-choice model as well as the retailer-choice model, the retailer bears all the risks of overstocking and backorders. Therefore, as we shall see in the next section, the retailer tends to be conservative in its order amounts, and the supplier tends to use a quality level for the product that is not the optimal for the entire system. Thus, if we design contracts that address these two factors, the increasing cost of quality and the retailer's risk, one can expect to improve the performance of the supply chain.

In this section we show two different contracts that are able to achieve the performance of a vertically integrated supply chain. In the first contract, the retailer and the supplier agree on a wholesale price that depends on the quality level. This wholesale price is smaller than the production costs. In return supplier receives a given fraction of the revenue from each unit sold by the retailer. In order to be able to coordinate the supply chain the retailer also has to share the costs of misaligned demand with the supplier.

In our second contract, the supplier specifies a wholesale price, a buyback price and an expedite cost. The retailer is able to reduce its risk of misaligned demand by means of the buyback price and expedite costs offered by the supplier. The following proposition specifies the contracts that coordinate the supply chain.

Theorem 4.5.1 (Coordination). *Let $\phi \in (0,1)$, the following two contracts are able to coordinate the supply chain in order to obtain the profits of a vertically integrated supply chain.*

- **Sharing**: *The retailer shares a portion ϕ of the revenues, as well as costs of misaligned demand, with the supplier. The supplier guarantees a wholesale price of $w(x) = (1-\phi)c(x)$ to the retailer.*
- **Buyback**: *Supplier offers a buyback price $b = -h^+ - \phi(p - h^+)$ for unsold units, expedite cost $\tilde{h}^- = h^- - \phi(h^- + p)$ for units required, and a wholesale price $w(p,x) = c(x) + \phi(p - c(x))$.*

Under either of these contracts, the supplier receives a fraction ϕ of the profits of the integrated supply chain, and the retailer receives a fraction $1-\phi$ of the profits of the integrated supply chain.

For any given value of ϕ, the contracts from Theorem 4.5.1 are able to coordinate the supply chain. However, for a contract to be implementable it must guarantee profits higher than in the absence of such contract, and hence not every value of ϕ is implementable. Notice the differences between the usual revenue sharing and buyback contracts. In the case of the sharing contract, the wholesale price has to depend on the quality level. This is understandable as there has to exist additional incentives for the supplier in order to provide high quality. Observe that a high value of ϕ reduces the wholesale price, but at the same time increases the percentage of revenue shared.

In the case of buyback contracts, the wholesale price is also increasing with the quality level. However, in this case a higher value of ϕ increases the wholesale price, but at the same time increases the costs from misaligned demand that are transferred to the supplier.

Theorem 4.5.1 is valid for general demand models. In the next three sections, we consider linear demand with the objective of characterizing solutions to all of the three models. In the following section, we begin by analyzing the vertically integrated supply chain.

4.6 Vertically Integrated Supply Chain

In a vertically integrated supply chain, retailer and supplier maximize profits as one system. Therefore, the profit obtained as a vertically integrated supply chain acts as an upper bound to the sum of the retailer profits and the supplier profits under the other two models. Understanding the solution of the integrated supply chain allows us to better understand the profits of a coordinated supply chain.

In this section, we characterize the optimal quality level in a vertically integrated supply chain. In addition, we develop a proposition that characterizes how the optimal quality level changes as the variance in the consumer demand increases.

The following Lemma proves useful in deriving the first order conditions to the optimization problem faced by the vertically integrated supply chain, as well as the supplier-choice model and the retailer-choice model.

Lemma 4.6.1 (Interchange). *Consider a function h as described by our model, and ε a continous (non atomic) random variable. Then:*

$$\frac{d}{dx}\{\mathbb{E}_{\varepsilon}[h(x-\varepsilon)]\} = \mathbb{E}[\partial h(x-\varepsilon)],$$

where ∂ denotes the sub differential operator.

Lemma 4.6.1 guarantees that we can interchange expectation and derivatives. This property greatly simplifies the analysis in the general stochastic demand model. Next, we present the main theorem of this section, characterizing the optimal quality level for a vertically integrated supply chain.

Theorem 4.6.1 (Integrated). *Assume demand is linear, namely, $D(p, x, \varepsilon) = A - bp + rx + \varepsilon$, denote $D(p, x)$ the expected demand. Assume average profits are concave and finite. Then* *the optimal quality level in a vertically integrated supply chain is given by:*

$$c'(x)\left(1+2\frac{F^{-1}\left(\frac{h^- - c(x)}{h^- + h^+}\right)}{D(c(x),x)}\right)=\frac{r}{b}. \tag{4.7}$$

And thus, the retailer stocking quantity is given by:

$$y = D(p,x) + F^{-1}\left(\frac{h^- - c(x)}{h^- + h^+}\right).$$

Theorem 4.6.1 characterizes the optimal quality level x, as well as the optimal stocking quantity y. Observe that as consumer demand variance approaches zero, we recover the deterministic demand solution.

An interesting question to explore is how the optimal quality level is affected by increasing variance in consumer demand. Interestingly, by means of the implicit function theorem, we can derive an ordinary differential equation that characterizes the evolution of the quality level as the variance increases, as presented in the following proposition.

Proposition 4.6.1. *Consider a vertically integrated supply chain. Assume demand is linear, namely, $D(p,x,\varepsilon) = A - bp + rx + \varepsilon$, denote $D(p,x)$ the expected demand. Assume average profits are concave. Then, the evolution of the optimal quality level as the variance increases is given by the following ordinary differential equation:*

$$x'(\sigma) = \frac{\frac{\partial}{\partial\sigma}F^{-1}(\frac{h^- - c(x)}{h^+ + h^-})}{\frac{c'(x)}{f(F^{-1}(\frac{h^- - c(x)}{h^+ + h^-}))(h^+ + h^-)} + F^{-1}(\frac{h^- - c(x)}{h^+ + h^-})\frac{-bc'(x)+r}{D(c(x),x)} - \frac{c''(x)D(c(x),x)r}{2c'(x)^2 b}} \tag{4.8}$$

$$x(0) = (c')^{-1}(\frac{r}{b}) \tag{4.9}$$

Proof. We know the solution for deterministic demand, meaning $\sigma = 0$, this proves the expression for $x(0)$. We are interested in studying the effect of increasing variance on the quality level.

Equation (4.7) guarantees that the set of optimal quality level for different values of σ can be described as :

$$X = \{(x,\sigma) \in [0,+\infty)^2 | g(x,\sigma) = \frac{r}{b}\},$$

where $g(x,\sigma) = c'(x)\left(1+2\frac{F^{-1}(\frac{h^- - c(x)}{h^- + h^+})}{D(c(x),x)}\right)$. Observe that g is as differentiable as F^{-1} and c'. By means of the analytic implicit function theorem we prove the existence of a solution

$x(\sigma)$, and moreover:

$$x'(\sigma) = -\frac{g_\sigma(x(\sigma), \sigma)}{g_x(x(\sigma), \sigma)} \tag{4.10}$$

Substituting the expressions in (4.10), we obtain the result. □

Equation 4.8 characterizes the evolution of the optimal quality level as the variance increases. Of course, the initial condition to the differential equation is the solution from the deterministic demand model. Unfortunately, an analytical solution to this equation is not easy to derive. However, differential equations allows for easy numerical implementations. In this particular case, given the simplicity of the numerator, we can establish some analytical properties as well.

Corollary 4.6.1. *Let $x_0 = (c')^{-1}(\frac{r}{b})$, the deterministic solution, and ε a symmetric distribution. If $c(x_0) = \frac{1}{2}(h^+ - h^-)$, then $x(\sigma)$ is constant for all σ.*

Proof. Given the hypothesis we have,

$$\frac{h^- - c(x_0)}{h^- + h^+} = \frac{1}{2}.$$

Given that the distribution is symmetric $\frac{\partial}{\partial\sigma}F^{-1}(\frac{1}{2}) = 0$ for all σ. Therefore, $x'(\sigma) = 0$. concluding the result. □

Corollary 4.6.1 characterizes a special type of solution. If the deterministic solution is such that the cost of its quality level is an average between h^+ and $-h^-$, then the solution is not affected by the variance. In other words, it is possible for the risk of misaligned demand to be balanced by a particular value of h^+ and h^-. The only requirement for the existence of this solution is for the distribution to be symmetric. Given that for this particular case we can obtain analytical expressions for all the relevant quantities, this solution proves useful as a first step to identify relations between parameters.

In the following sections we explore the solutions for the supplier-choice and the retailer-choice model.

4.7 Supplier-Choice Model

In the supplier-choice model, we have two different parties each maximizing its own profit. The supplier decides the quality level for the product and the wholesale price to the retailer. The retailer decides on the stocking quantity and the price to the consumer. A higher quality level comes at a greater cost to the supplier. However, quality increases expected demand

and thus retailer's stocking quantity, affecting supplier's payoff. We begin by characterizing the optimal wholesale price, consumer price and quality level used by the respective parties.

The following proposition characterizes the quality level, price to the consumer and wholesale price, by means of a system of equations.

Theorem 4.7.1 (Supplier-Choice). *Assume linear consumer demand with an additive noise, $D(p,x,\varepsilon) = A - bp + rx + \varepsilon$. Consider a supplier-choice setting. If the supplier is receiving non-zero profits, then the optimal selling price p, and the optimal quality level x satisfy the following system:*

$$c'(x)\left[1 + \frac{2}{bf(F^{-1}(\xi(p,x)))(h^+ + h^-)}\right] = \frac{r}{b} \tag{4.11}$$

$$2(D(p,x) + F^{-1}(\xi(p,x))) = (p - \frac{1}{b}D(p,x) - c(x))(b + \frac{2}{f(F^{-1}(\xi(p,x)))(h^+ + h^-)}),$$

where $\xi(p,x) = \frac{h^- - p + \frac{D(p,x)}{b}}{h^+ + h^-}$ and $D(p,x)$ is the expected demand given p and x. Additionally, given the solution to the system, the wholesale selling price from the supplier to the retailer is given by

$$w = p - D(p,x)/b.$$

Equation (4.11) resembles the condition from Theorem 4.6.1. However in this case, the condition depends explicitly on the price to the customers. Thus, it's not possible to directly compare in general the quality level in a supplier-choice model, with the quality level in the integrated supply chain. However, if ε is distributed uniformly over an interval, the dependency on p in Equation (4.11) disappears and it is possible to compare the policies as done in Jira (2013).

In order to obtain numerical solutions, it is possible to proceed as in the previous section. Given that the quality level is now characterized in a system of equations, applying the procedure from Proposition 4.6.1 yields a system of differential equations. The system of differential equations characterizing the solution as demand variance increases is presented in the next proposition.

Corollary 4.7.1. *The solution to the supplier-choice model, (x,p), satisfies the following differential equation on σ.*

$$\begin{cases} \alpha_1(x,p,\sigma)x'(\sigma) + \beta_1(x,p,\sigma)p'(\sigma) &= \gamma_1(x,p,\sigma) \\ \alpha_2(x,p,\sigma)x'(\sigma) + \beta_2(x,p,\sigma)p'(\sigma) &= \gamma_2(x,p,\sigma) \end{cases}$$

where,

- $\alpha_1(x,p,\sigma) = 2c''(x)(1+\frac{2}{bf(F^{-1}(\xi(x,p)))(h^+ + h^-)}) - 2c'(x)r\frac{f'(F^{-1}(\xi(x,p)))}{b^2(h^- + h^+)^2 f(F^{-1}(x,p))^3}$.
- $\beta_1(x,p,\sigma) = 2c'(x)f'(F^{-1}(\xi(x,p)))\frac{2}{b(h^+ + h^-)^2 f(F^{-1}(\xi(x,p)))^3}$
- $\gamma_1(x,p,\sigma) = 2c'(x)\frac{\partial_\sigma f(F^{-1}(\xi(x,p))) + f'(F^{-1}(\xi(x,p)))\partial_\sigma F^{-1}(\xi(x,p))}{b(h^+ + h^-)f(F^{-1}(\xi(x,p)))^2}$.
- $\alpha_2(x,p,\sigma) = 3r + 4\frac{r}{b(h^+ + h^-)f(F^{-1}(\xi(x,p)))} + c'(x)(b + \frac{2}{(h^+ + h^-)f(F^{-1}(\xi(x,p)))})$
  $+ 2(p - D(p,x)/b - c(x))\frac{rf'(F^{-1}(\xi(x,p)))}{b(h^+ + h^-)^2 f(F^{-1}(\xi(x,p)))^3}$.
- $\beta_2(x,p,\sigma) = -4b - \frac{8}{(h^+ + h^-)f(F^{-1}(\xi(x,p)))} - 4(p - D(p,x)/b - c(x))\frac{f'(F^{-1}(\xi(x,p)))}{(h^+ + h^-)^2 f(F^{-1}(\xi(x,p)))^3}$.
- $\gamma_2(x,p,\sigma) = -2\partial_\sigma F^{-1}(\xi(x,p)) - 2(p - D(p,x)/b - c(x))\frac{\partial_\sigma f(F^{-1}(\xi(x,p))) + f'(F^{-1}(\xi(x,p)))\partial_\sigma F^{-1}(\xi(x,p))}{(h^+ + h^-)f(F^{-1}(\xi(x,p)))^2}$.

The system of differential equations in Corollary 4.7.1, is significantly more complex then the system presented in Corollary 4.6.1. The added complexity is a result of not being able to separate consumer price and quality level in the optimality conditions. However, numerical solutions can be derived from the system of equations by means of approximations of the derivatives.

In order to obtain numerical intuition for the relations between the three models presented, we require to determine a similar system of equations for the retailer-choice model.

4.8 Retailer Choice Setting

In the retailer-choice model we consider a retailer to be influential enough to be able to suggest a quality level to the supplier. In this setting, the retailer chooses the price to the consumer, stocking quantity and quality level to be used by the supplier. The supplier chooses the wholesale price. The retailer has incentives to suggest higher quality level in order to increase demand. However, suggesting higher quality levels leads to higher wholesale price from the supplier.

Similar to the previous sections, the following proposition characterizes the wholesale price, quality level and stocking quantity by means of a system of equations.

Theorem 4.8.1 (Retailer-Choice)**.** *Assume a linear demand with an additive noise, $D(p,x) = A - bp + rx + \varepsilon$. Consider a retailer-choice setting. If the supplier is receiving non-zero profits then the optimal selling price p, and the optimal quality level x, satisfy the following system:*

$$\left\{ c'(x)[1+\frac{2}{bf(F^{-1}(\xi(p,x)))(h^+ + h^-)}] \quad = \frac{r}{b}(1 - 2\frac{D(p.x)}{D(p,x)+F^{-1}(\xi(p,x))}(\frac{F^{-1}(\xi(p.x))}{D(p,x)} - \frac{2}{bf(F^{-1}(\xi(p,x)))(h^+ + h^-)} \right.$$

$$-(p - D(p,x)/b - c(x))\frac{f'(F-1(\xi(p,x)))}{bf(F^{-1}(\xi(p,x)))^3(h^+ + h^-)^2})) \tag{4.12}$$

$$2(D(p,x) + F^{-1}(\xi(p,x))) = (p - \frac{1}{b}D(p,x) - c(x))(b + \frac{2}{f(F^{-1}(\xi(p,x)))(h^+ + h^-)}),$$

where $\xi(p,x) = \frac{h^- - p + \frac{D(p,x)}{b}}{h^+ + h^-}$. *Additionally, given the solution to the system, the wholesale price from the supplier to the retailer is given by*

$$w = p - D(p,x)/b.$$

Observe that the system is similar to Theorem 4.7.1. In fact, the only difference is in the right hand side of Equation (4.12). However, as in Theorem 4.7.1, the explicit dependency on price in Equation (4.12) hinder us from comparing the conditions from Theorem 4.7.1 and 4.8.1 analytically for a general distribution of customer demand. In the particular case when ε is distributed uniformly over an interval, it is possible to extract analytical solutions and compare the three models analytically, see Jira (2013). Additionally, as in the previous section, it is possible to derive a system of differential equations that characterizes the evolution of the optimal quality level and price to the consumer as the variance increases. The system of differential equations is remitted to the appendix.

4.9 Numerical Example

Figures 4-1–4-2 illustrate the power of the system of differential equations through numerical examples, see details in Appendix 4.C. The x-coordinate always represents the standard deviation σ. That is, in this numerical example for $x = 0$, we have the deterministic solution.

In Figure 4-1 in the top right corner, we observe the difference between the expected revenue of the retailer under a retailer-choice model (presented in green), and a supplier-choice model (presented in blue). Of course, the retailer's profit is always higher in a retailer-choice model. Interestingly, the profit of the retailer initially increases with small values of the variance, and proceeds to decrease after a certain point. There are many factors interacting in the model, however as it can be seen from the plot in the top left corner the increase in profits has to be connected to the increase in expected demand. The plot in the bottom left corner shows the profits for the supplier. Obviously, the supplier profit is higher under a supplier-choice model. Finally, it can be seen from the plot in the bottom right corner that the benefits of coordinating the supply chain are of the order of 28% in this example.

In Figure 4-2, it can be seen that the greatest difference between the models is in the quality level. In the retailer-choice model, the retailer tends to demand a higher level of quality than that of the benchmark. On the other hand, in the supplier-choice model, the supplier tends to use a quality level that is lower than the benchmark. As seen in Section 4.5, the lack of incentive to invest in quality for the supplier can be balanced by having a wholesale price depending on the quality of the product.

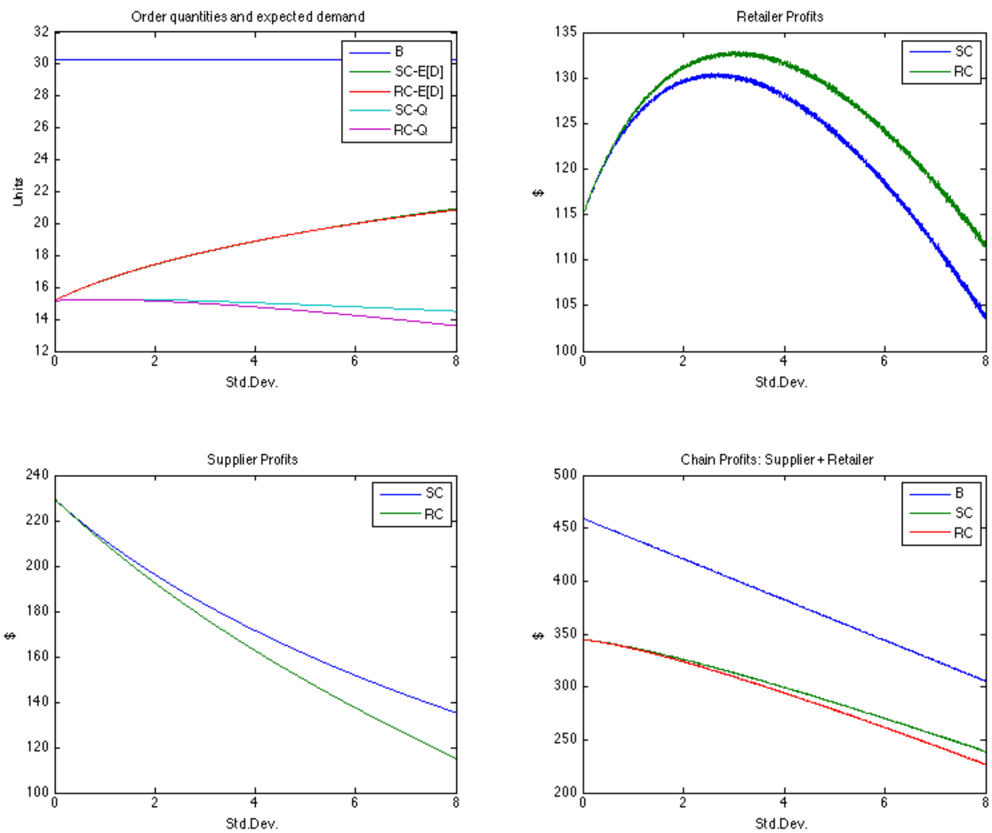


Figure 4-1: Numerical performance of the policies, SC=supplier-choice, RC= retailer-choice, B=Integrated supply chain. E[D] = Expected Demand, Q=order quantity.

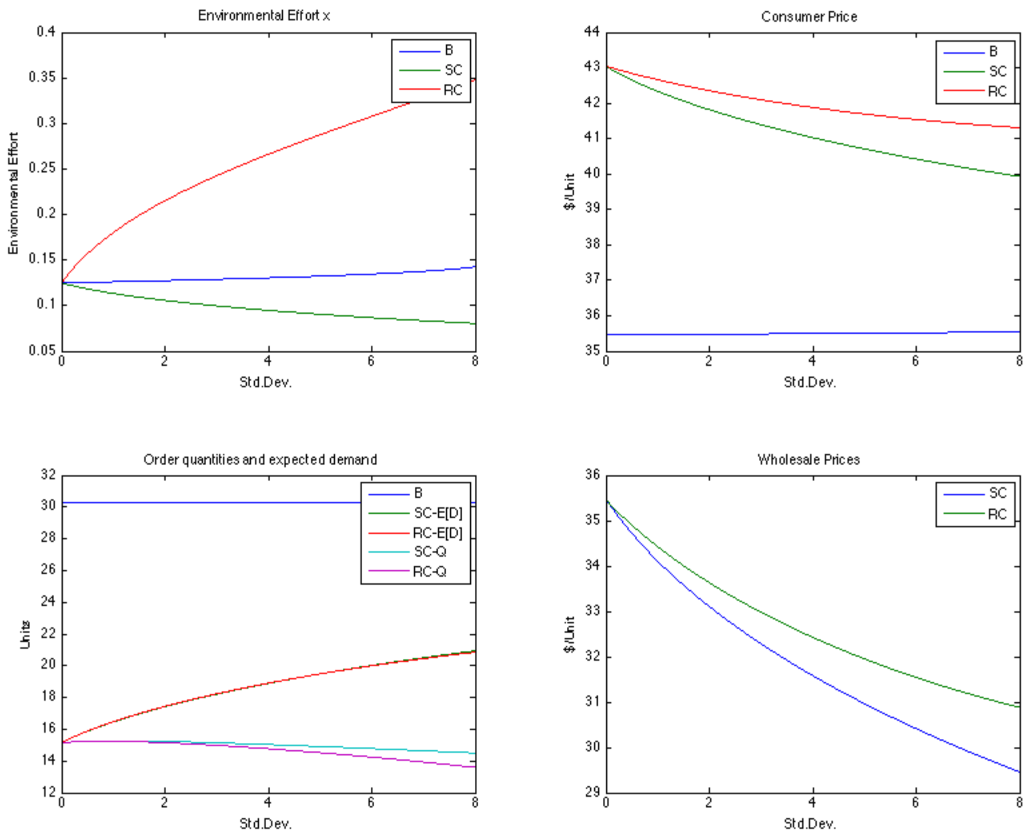


Figure 4-2: Numerical performance of the policies, SC=supplier-choice, RC= retailer-choice, B=Integrated supply chain. E[D] = Expected Demand, Q=order quantity.

Interestingly, the retailer demanding a higher quality level than the supplier is not always the case. When the difference between the expediting costs and the unit holding cost is too high, it is possible to reverse the quality preference in the model, that is, a retailer would desire less quality than what the supplier would normally choose. This unusual phenomenon is a result of the high risk of misaligned demand faced by the retailer. When the risk is too high, the benefits of additional consumer demand for the retailer can be surpassed by the increased costs of misaligned demand.

4.10 Conclusions

Whether it is lower carbon emissions, greater nutritional value, or luxury add ons, there is an opportunity to increase consumer demand by increasing the quality of a product. Of course, an increase in quality comes at a higher cost to the supplier. However, there is potential for increasing profits for both parties, the supplier and the retailer.

In this chapter, we investigated the impact quality investment decisions have on the supply chain. We analyzed three different models, (i) a supplier-choice model, where the supplier is in charge of deciding the optimal product quality level, (ii) a retailer-choice model, where a retailer demands a quality level from the supplier, and (iii) an integrated supply chain, where supplier and retailer act as a vertically integrated firm.

Under a deterministic demand model, we found that the optimal quality level in all three models is the same. However, when demand uncertainty is introduced a difference in the optimal quality level in each model is observed.

For the supplier- and retailer-choice model we characterized the conditions for optimal quality level and price to the consumer. Through a system of differential equations we derived the changes in quality level and price to the consumer as the variance in consumer demand increases. The large number of parameters defining the problem, as well as the interaction among these, are an obstacle to obtaining general analytical results that are valid for any given range of parameters. Nonetheless, these systems of differential equations provide a basis for algorithms to obtain numerical solutions efficiently. In most of the examples studied, the quality level demanded by the retailer in the retailer-choice model was higher than the quality level decided by the supplier in a supplier-choice model. However, there were exceptions when the costs associated to misaligned demand (expediting/ holding costs) where much higher than the price to the consumer.

The profits in an integrated supply chain were significantly higher than the profits in either the supplier- or retailer-choice model. There are two factors contributing to this difference. First, there is the added benefit from eliminating the *double marginization* induced by the wholesale contract. Second, there is the added benefit of choosing a quality level that is optimal for the entire supply chain. Depending on the sensitivity of demand to the quality level, and the relation between supplier cost and quality, this second effect can be very significant.

A final result to emphasize is the extension of buyback contracts and revenue sharing contracts to coordinate the supply chain in our model. There are two elements that are key in these contracts. The first element is reducing risks of misaligned demand to the retailer. This is a feature present in many contracts that coordinate the supply chain. The second element is particular to our model, and is to allow the wholesale price to be proportional to the supplier cost. This added feature effectively provides incentives to the supplier for higher investment in quality.

4.A Proofs

4.A.1 Theorem 4.4.1

Proof. The positive profits assumption guarantees that the optimization problems, (4.1)-(4.5), have interior optimal solutions.

Given that demand is deterministic, and the constraints in h^+ and h^-, the optimal purchase amount for the retailer is $y = D(p, x)$.

- The optimal technology level in the vertically integrated supply chain is given by,

$$\max_{\substack{p\geq 0\\ x\in\mathbb{R}}}(p - c(x))D(p, x).$$

  Given the positivity of costs, the boundary constrain $p = 0$, yields negative profit. Therefore, given that the optimal technology level is finite, the solution lies in the interior and satisfies the following conditions:

$$\begin{aligned} D(p, x) - (p - c(x))b &= 0 \\ -c'(x)D(p, x) + (p - c(x))X'(x) &= 0 \end{aligned}$$

  By multiplying the first equation by $\frac{X'(x)}{b}$, and adding the conditions, we obtain (4.6).

- In the supplier choice model, by finding the optimal conditions for p in (4.1), the following relation for the wholesale price w can be established,

$$w = p - \frac{D(p, x)}{b}. \tag{4.13}$$

  Therefore, we can pose the problem faced by the supplier, (4.2), in terms of p and x,

$$\max_{\substack{p\geq \frac{D(p,x)}{b}\\ x\in\mathbb{R}}} (p - \frac{D(p, x)}{b} - c(x))D(p, x).$$

  The optimal interior conditions for this problem are:

$$2D(p, x) - (p - \frac{D(p, x)}{b} - c(x))b = 0 \tag{4.14}$$

$$-c'(x)D(p, x) + (p - \frac{D(p, x)}{b} - c(x))X'(x) = 0 \tag{4.15}$$

By multiplying the first equation by $\frac{X'(x)}{b}$, this yields the condition for the optimal level of x.

- In the retailer choice model, solving the third stage problem yields the condition (4.13). The second stage problem yields the condition (4.14). Finally, by means of (4.13) and using $p(x)$ implicitly, we write the last stage problem as,

$$\max_{x} -\frac{D(p(x),x)}{b}.$$

Hence, we obtain the condition: $X'(x) = bp'(x)$. By means of the implicit function theorem, we can differentiate (4.14) implicitly. By doing this, we obtain,

$$(p'(x) - \frac{1}{b}(bp'(x) - X'(x)) - c'(x))D(p(x),x) + (2 + p(x) - \frac{1}{b}D(p(x),x) - c(x))(-bp'(x) + X'(x)) = 0$$

Finally, using that $X'(x) = bp'(x)$, we obtain the condition for the optimal level of x. □

4.A.2 Theorem 4.5.1

Proof. We show that the maximization problem faced by the retailer and supplier is the same maximization problem as the vertically integrated supply chain.

- **Sharing:** Under this contract the retailer's decision problem, (4.1), becomes:

$$\max_{p,y\geq 0} \mathbb{E}[pD(p,x,\varepsilon) - (1-\phi)c(x)y - h(y - D(p,x,\varepsilon)) - \phi pD(p,x,\varepsilon) + \phi h(y - D(p,x,\varepsilon))],$$

or equivalently,

$$\max_{p,y\geq 0}(1-\phi)\mathbb{E}[pD(p,x,\varepsilon) - c(x)y - h(y - D(p,x,\varepsilon))].$$

Therefore, given x, the retailer chooses the price and order quantity of the vertically integrated supply chain. The problem faced by the supplier in this contract is:

$$\max_{x\in\mathbb{R}}(-\phi c(x))y(x) + \phi\mathbb{E}[p(x)D(p(x),x,\varepsilon) - h(y(x) - D(p(x),x,\varepsilon))],$$

or equivalently,

$$\max_{x\in\mathbb{R}} \phi\mathbb{E}[p(x)D(p(x),x,\varepsilon) - c(x)y(x) - h(y(x) - D(p(x),x,\varepsilon))].$$

And hence the supplier choose the technological level x of the vertically integrated supply chain.

- **Buyback** Given the new buyback and expedite costs, the retailer problem is:

$$\max_{p,y\geq 0} \mathbb{E}[pD(p,x,\varepsilon)-(c(x)+\phi(p-c(x))y+b\max\{0,y-D(p,x,\varepsilon)\}-\tilde{h}^{-}\max\{0,D(p,x,\varepsilon)-y\}],$$

Given the definitions in the contract, the problem can be rewritten into,

$$\max_{p,y\geq 0}(1-\phi)\mathbb{E}[pD(p,x,\varepsilon)-c(x)y-h(y-D(p,x,\varepsilon))],$$

Similarly, the profit for the supplier under this contract is:

$$\max_{x\in\mathbb{R}} \phi(p(x)-c(x))y(x)-b\max\{0,y(x)-D(p(x),x,\varepsilon)\}-\tilde{h}^{-}\max\{0,D(p(x),x,\varepsilon)-y(x)\},$$

which, given the definitions in the statement, can be rewritten into,

$$\max_{x\in\mathbb{R}} \phi\mathbb{E}[p(x)D(p(x),x,\varepsilon)-c(x)y(x)-h(y(x)-D(p(x),x,\varepsilon))].$$

Therefore, implementing this contract yields the same result as a vertically integrated supply chain. □

4.A.3 Lemma 4.6.1

Proof. A quick calculation yields:

$$\partial h(x) = \begin{cases} h^{+} & \text{if } x > 0 \\ h^{-} & \text{if } x \leq 0 \end{cases}$$

Note that the value at zero for ∂h is irrelevant as there is an expected value of a continuous random variable, and hence the value at a specific point can be ignored.

We begin by using the definition of derivative:

$$\frac{d}{dx}\{\mathbb{E}_{\varepsilon}[h(x-\varepsilon)]\} = \lim_{\delta\to 0}\frac{1}{\delta}(\mathbb{E}_{\varepsilon}[h(x+\delta-\varepsilon)]-\mathbb{E}_{\varepsilon}[h(x-\varepsilon)]) = \lim_{\delta\to 0}\frac{1}{\delta}(\mathbb{E}_{\varepsilon}[h(x+\delta-\varepsilon)-h(x-\varepsilon)])$$

Let's consider the case $\delta \searrow 0$, meaning δ positive. The case for negative is symmetric. Once we show that both limits (left and right) exists and are the same, we have that the derivative exists. For $\delta > 0$, we have:

$$h(x+\delta-\varepsilon)-h(x-\varepsilon)=\begin{cases} h^+\delta & \text{if } \varepsilon\le x \\ (h^++h^-)(x-\varepsilon)+h^+\delta & \text{if } x<\varepsilon\le x+\delta \\ h^-\delta & \text{if } \varepsilon > x+\delta \end{cases}$$

Hence:

$$\begin{aligned} \mathbb{E}_\varepsilon[h(x+\delta-\varepsilon)-h(x-\varepsilon)]) &= h^+\delta\mathbb{P}(\varepsilon\le x)+h^-\delta\mathbb{P}(\varepsilon>x+\delta) \\ &+ h^+\delta\mathbb{P}(x<\varepsilon\le x+\delta)+(h^++h^-)\mathbb{E}[(x-\varepsilon)\mathbb{1}_{(x,x+\delta]}(\varepsilon)] \end{aligned}$$

Observe that as $\delta\searrow 0$, monotone convergence theorem guarantees $\mathbb{P}(\varepsilon > x+\delta)\nearrow\mathbb{P}(\varepsilon > x)$, and hence the first two terms converge to $\mathbb{E}[\partial h(x-\varepsilon)]$. It remains to see that the other terms converge to zero.

- Given that $h^+\mathbb{P}(x<\varepsilon\le x+\delta)\le h^+$, by means of the dominated convergence theorem we prove that $h^+\mathbb{P}(x<\varepsilon\le x+\delta)\to\mathbb{P}(x=\varepsilon)=0$.
- Finally, observe that:

  $$-\delta\mathbb{P}(x<\varepsilon\le x+\delta)\le\mathbb{E}[(x-\varepsilon)\mathbb{1}_{(x,x+\delta]}(\varepsilon)]\le 0$$

  Therefore, we know that $\frac{1}{\delta}\mathbb{E}[(x-\varepsilon)\mathbb{1}_{(x,x+\delta]}(\varepsilon)]\to 0$, since it's bounded by two limits that go to zero.

□

4.A.4 Proposition 4.6.1

Proof. The optimization problem in this setting is:

$$\max_{\substack{p,y\ge 0\\ x\in\mathbb{R}}} pD(p,x)-c(x)y-\mathbb{E}[h(y-D(p,x)-\varepsilon)]$$

Given that a price, $p=0$ no revenue is collected, it is not going to be optimal. Increasing price provides revenue, and reduces demand, therefore less cost from unsatisfied demand. An order quantity, $y=0$ can be verified to not be optimal given that $c(x)<h^-$, i.e. as long as expedite cost is greater than production cost we won't expedite everything. Finally, given the restriction of finite profits, x has to be finite.

Given that average profits are concave by hypothesis, and $g(z)=\mathbb{E}[h(z-\varepsilon)]$ is convex in ε (see Simchi-Levi et al. (2005)), we have that the objective function is concave. Therefore, the optimal is in the interior, first order conditions are sufficient.

The first order conditions are:

$$\begin{aligned}
y &: -c(x) - \frac{\partial}{\partial y}\mathbb{E}[h(y - D(p,x) - \varepsilon)] = 0. \\
p &: -pb + D(p,x) - \frac{\partial}{\partial p}\mathbb{E}[h(y - D(p,x) - \varepsilon)] = 0. \\
x &: pr - c'(x)y - \frac{\partial}{\partial x}\mathbb{E}[h(y - D(p,x) - \varepsilon)] = 0.
\end{aligned}$$

Lemma 4.6.1, states that we can interchange derivative and expectation, therefore:

$$y : -c(x) = \mathbb{E}[\partial h(y - D(p,x) - \varepsilon)] \tag{4.16}$$

$$p : -pb + D(p,x) + c(x)b = 0 \tag{4.17}$$

$$x : pr - c'(x)y - c(x)r = 0 \tag{4.18}$$

By using the definition of ∂h in (4.16), the following expression for the order quantity can be derived:

$$y = D(p,x) + F^{-1}\left(\frac{h^- - c(x)}{h^- + h^+}\right)$$

Given equation (4.17) and that demand is linear, by adding and subtracting $D(c(x),x)$, the following identity can be derived:

$$2D(p,x) = D(c(x),x).$$

Now multiplying (4.17) by $-\frac{r}{b}$ and subtracting (4.18) we find:

$$-D(p,x)\frac{r}{b} + c'(x)\left[D(p,x) + F^{-1}\left(\frac{h^- - c(x)}{h^- + h^+}\right)\right] = 0$$

Thus concluding the theorem. □

4.A.5 Theorem 4.7.1

Proof. Consider the supplier choice model. To find w, p, x and y, we start by solving the problem the retailer faces.

$$\max_{p,y\geq 0} pD(p,x) - wy - \mathbb{E}[h(y - D(p,x) - \varepsilon)]$$

Similar to Theorem 4.6.1, boundary conditions cannot be optimal. Given that $pD(p,x)$

is concave in p, then an optimal solution satisfies the following first order conditions.

$$\begin{aligned} y &: -w - \frac{\partial}{\partial y}[\mathbb{E}[h(y - D(p,x) - \varepsilon)]] = 0. \\ p &: pD_p + D(p,x) - \frac{\partial}{\partial p}[\mathbb{E}[h(y - D(p,x) - \varepsilon)]] = 0. \end{aligned}$$

In this situation we can apply Lemma 4.6.1 to simplify the first order conditions:

$$\begin{aligned} y &: -w = \mathbb{E}[\partial h(y - D(p,x) - \varepsilon)] \\ p &: -pb + D(p,x) + wb = 0 \end{aligned}$$

Therefore we can characterize y in terms of w, and with the second equation, characterize w in terms of p:

$$y(p) = F^{-1}\left(\frac{h^- - w(p)}{h^+ + h^-}\right) + D(p,x) \tag{4.19}$$

$$w(p) = p - \frac{1}{b}D(p,x) \tag{4.20}$$

Given these expressions we can formulate the problem faced by the retailer as:

$$\max_{\substack{p \geq D(p,x)/b \\ x \in \mathbb{R}}} (w(p) - c(x))y(p) = \max_{\substack{p \geq D(p,x)/b \\ x \in \mathbb{R}}} (p - \frac{1}{b}D(p,x) - c(x))(D(p,x) + F^{-1}(\xi(p,x))).$$

Given the hypothesis of non-zero profits, the solution has to be in the interior. An interior solution satisfies:

$$\begin{aligned} p: \quad & 2(D(p,x) + F^{-1}(\xi(p,x))) &&= (p - \frac{1}{b}D(p,x) - c(x))(b + \frac{2}{f(F^{-1}(\xi(p,x)))(h^+ + h^-)}) \\ x: \quad & (-\tfrac{r}{b} - c'(x))(D(p,x) + F^{-1}(\xi(p,x))) &&= (p - \frac{1}{b}D(p,x) - c(x))(-r + \frac{r}{D_p f(F^{-1}(\xi(p,x)))(h^+ + h^-)}) \end{aligned}$$

Given the first equation, we can replace $(D(p,x) + F^{-1}(\xi(p,x)))$ in the second equation to find,

$$(w(p) - c(x))\left(\frac{1}{2}(-\frac{r}{b} - c'(x))\left(b + \frac{2}{f(F^{-1}(\xi(p,x)))(h^+ + h^-)}\right) + r + \frac{r}{bf(F^{-1}(\xi(p,x)))(h^+ + h^-)}\right) = 0$$

Given the supplier is receiving positive profits, we have,

$$\frac{1}{2}(-\frac{r}{b} - c'(x)) \left(b + \frac{2}{f(F^{-1}(\xi(p,x)))(h^+ + h^-)} \right) + r + \frac{r}{bf(F^{-1}(\xi(p,x)))(h^+ + h^-)} = 0$$

Rearranging the terms we find:

$$c'(x) \left[1 + \frac{2}{bf(F^{-1}(\xi(p,x)))(h^+ + h^-)} \right] = \frac{r}{b}$$

Therefore proving the statement. □

4.A.6 Proposition 4.8.1

Proof. To solve this setting we start with the latest stage, the retailer solves,

$$\max_{p,y \geq 0} pD(p,x) - wy - \mathbb{E}[h(y - D(p,x) - \varepsilon)]$$

As we can see the problem is exactly the same as in the supplier choice setting, and therefore the first order conditions are the same:

$$\begin{aligned} y(p) &= F^{-1}\left(\frac{h^- - w(p)}{h^+ + h^-}\right) + D(p,x) \\ w(p) &= p - \frac{1}{b}D(p,x) \end{aligned}$$

Next the supplier determines the sell price w. As done in the supplier choice problem, we formulate the optimization problem in terms of p,

$$\max_{w(p) \geq 0} (w(p) - c(x))y(p) = \max_{p \geq D(p,x)/b} (p - \frac{1}{b}D(p,x) - c(x))(D(p,x) + F^{-1}(\xi(p,x))).$$

Given the non-zero profits assumption, the solution has to be an interior solution. Therefore, the first order condition is:

$$2(D(p,x) + F^{-1}(\xi(p,x))) = (p - \frac{1}{b}D(p,x) - c(x))(b + \frac{2}{f(F^{-1}(\xi(p,x)))(h^+ + h^-)}). \quad (4.21)$$

This condition is the same as in the supplier choice setting, and characterizes $p(x)$.

For the rest of the analysis we work with $p(x)$ as an implicit variable.

The last stage to consider is the decision of x by the retailer. This decision comes from the optimization problem:

$$\max_x p(x)D(p(x),x) - w(p(x))y(p(x)) - \mathbb{E}[h(F^{-1}(\xi(p(x),x)) - \varepsilon)]. \tag{4.22}$$

By means of implicit derivation, we can establish the following identities:

$$\frac{d}{dx}D(p(x),x) = -bp'(x) + r$$
$$\frac{d}{dx}\xi(p(x),x) = \frac{-2p'(x) + \frac{r}{b}}{h^+ + h^-}$$

From the optimal conditions of the retailer's last stage we know $w(p(x)) = \mathbb{E}[\delta h(y(p(x)) - D(p(x),x) - \varepsilon)]$. Using this identities we can rewrite the first order condition of (4.22) into :

$$p'(x) = \frac{r}{b}[1 - \frac{F^{-1}(\xi(p(x),x))}{2(F^{-1}(\xi(p(x),x)) + D(p(x),x))}]. \tag{4.23}$$

In order to find an expression similar to the previous section, we consider equation (4.21), which is valid for the different values that x may take, and differentiate the equality with respect to x. This yields the following expression,

$$\begin{aligned} -bc'(x)\left(1 + \frac{2}{bf(F^{-1}(\xi(p(x),x)))(h^+ + h^-)}\right) &= -4bp'(x) + 3r - 4\frac{2p'(x)b - r}{bf(F^{-1}(\xi(p(x),x))(h^+ + h^-)} \\ &+ (w(p(x)) - c(x))\frac{2(-2p'(x)b + r)f'(F^{-1}(\xi(p,x))}{bf(F^{-1}(\xi(p(x),x)))^3(h^+ + h^-)^2} \end{aligned}$$

By means of equation (4.23), we replace $p'(x)$ and divide by $-b$, yielding the following expression expressed in functional form:

$$\begin{aligned} c'(x)(1 + \frac{2}{b(h^+ + h^-)f \circ F^{-1} \circ \xi}) &= \frac{r}{b} - \frac{2r}{b}\frac{F^{-1} \circ \xi}{(D + F^{-1} \circ \xi)} + \frac{2r}{b^2}\frac{D}{(D + F^{-1} \circ \xi)}\frac{2}{(h^+ + h^-)f \circ F^{-1} \circ \xi} \\ &+ (p - D/b - c)\frac{2r}{b^2}\frac{D}{D + F^{-1} \circ \xi}\frac{f' \circ F^{-1} \circ \xi}{(f \circ F^{-1} \circ \xi)^3(h^+ + h^-)^2} \end{aligned}$$

Therefore, concluding the result. □

4.B System of differential equations Retailer Choice Model

By using the implicit function theorem in the system of equations in Theorem 4.8.1. We find a system of equations,

$$\left\{\begin{array}{rcl} \alpha_2(x,p,\sigma)x'(\sigma)+\beta_2(x,p,\sigma)p'(\sigma) & = & \gamma_2(x,p,\sigma) \\ \alpha_3(x,p,\sigma)x'(\sigma)+\beta_3(x,p,\sigma)p'(\sigma) & = & \gamma_3(x,p,\sigma) \end{array}\right.$$

where α_2, β_2 and γ_2, are the same as in Corollary 4.7.1, as the equation that yield them is the same. To identify the remaining terms we separate them in four.

It can be shown that $\alpha_3 = \alpha_{31} - \alpha_{32} - \alpha_{33} - \alpha_{34}$, $\beta_3 = \beta_{31} - \beta_{32} - \beta_{33} - \beta_{34}$ and $\gamma_3 = -\gamma_{31} + \gamma_{32} + \gamma_{33} + \gamma_{34}$, where,

$$\begin{array}{rcl}
\alpha_{31} & = & c''\left(1+\frac{2}{b(h^{+}+h^{-})f\circ F^{-1}\circ\xi}\right) - 2rc'\frac{f'\circ F^{-1}\circ\xi}{b^2(h^{+}+h^{-})^2 f\circ F^{-1}\circ\xi^3} \\
\alpha_{32} & = & -2\frac{r^2}{b}\frac{D}{(D+F^{-1}\circ\xi)^2}\left(\frac{1}{b(h^{+}+h^{-})f\circ F^{-1}\circ\xi} - \frac{F^{-1}\circ\xi}{D}\right) \\
\alpha_{33} & = & 4\frac{r^2}{b^2}\frac{D}{(D+F^{-1}\circ\xi)}\frac{\frac{1}{(D+F^{-1}\circ\xi)}(\frac{F^{-1}\circ\xi}{D}-\frac{1}{b(h^{+}+h^{-})f\circ F^{-1}\circ\xi})-\frac{f'\circ F^{-1}\circ\xi}{(h^{+}+h^{-})(f\circ F^{-1}\circ\xi)^2}}{(h^{+}+h^{-})f\circ F^{-1}\circ\xi} \\
aux & = & \left(\left(\frac{f''\circ F^{-1}\circ\xi}{f'\circ F^{-1}\circ\xi} - 3\frac{(f'\circ F^{-1}\circ\xi)}{(f\circ F^{-1}\circ\xi)}\right) + \frac{(f\circ F^{-1}\circ\xi)}{(D+F^{-1}\circ\xi)}\left(b(h^{+}+h^{-})\frac{F^{-1}\circ\xi}{D} - \frac{1}{f\circ F^{-1}\circ\xi}\right)\right) \\
aux_2 & = & -(\frac{r}{b}+c')(f\circ F^{-1}\circ\xi) + \frac{r(p-D/b-c)}{b(h^{+}+h^{-})}aux \\
\alpha_{34} & = & \frac{D}{(D+F^{-1}\circ\xi)}\frac{2r}{b^2(h^{+}+h^{-})^2}\frac{f'\circ F^{-1}\circ\xi}{(f\circ F^{-1}\circ\xi)^4}aux_2 \\
\beta_{31} & = & 4c'\frac{f'\circ F^{-1}\circ\xi}{b(h^{+}+h^{-})^2(f\circ F^{-1}\circ\xi)^3} \\
\beta_{32} & = & 2r\frac{D}{(D+F^{-1}\circ\xi)^2}\left(\frac{2}{b(h^{+}+h^{-})f\circ F^{-1}\circ\xi} - \frac{F^{-1}\circ\xi}{D}\right) \\
\beta_{33} & = & -4\frac{r}{b}\frac{D}{D+F^{-1}\circ\xi}\frac{\frac{1}{D+F^{-1}\circ\xi}\left(\frac{F^{-1}\circ\xi}{D}-\frac{2}{b(h^{+}+h^{-})f\circ F^{-1}\circ\xi}\right)-2\frac{f'\circ F^{-1}\circ\xi}{b(h^{+}+h^{-})(f\circ F^{-1}\circ\xi)^2}}{(h^{+}+h^{-})f\circ F^{-1}\circ\xi} \\
aux & = & \frac{2f'\circ F^{-1}\circ\xi}{(f\circ F^{-1}\circ\xi)^3} - \frac{(p-D/b-c)}{(h^{+}+h^{-})}aux_2 \\
aux_2 & = & (2(\frac{f''\circ F^{-1}\circ\xi}{(f\circ F^{-1}\circ\xi)^4} - 3\frac{(f'\circ F^{-1}\circ\xi)^2}{(f\circ F^{-1}\circ\xi)^5}) + \frac{f'\circ F^{-1}\circ\xi}{(f\circ F^{-1}\circ\xi)^3}\frac{(\frac{F^{-1}\circ\xi}{D}b(h^{+}+h^{-})-\frac{2}{f\circ F^{-1}\circ\xi})}{(D+F^{-1}\circ\xi)}) \\
\beta_{34} & = & 2\frac{r}{b^2(h^{+}+h^{-})^2}\frac{D}{(D+F^{-1}\circ\xi)}aux \\
\gamma_{31} & = & \frac{2}{h^{+}+h^{-}}c'\frac{(\partial_\sigma f)\circ F^{-1}\circ\xi+(f'\circ F^{-1}\circ\xi)((\partial_\sigma F^{-1})\circ\xi)}{(-b)(f\circ F^{-1}\circ\xi)^2} \\
\gamma_{32} & = & -2\frac{r}{b}(\partial_\sigma F^{-1})\circ\xi\frac{D}{(D+F^{-1}\circ\xi)^2} \\
\gamma_{33} & = & -4\frac{r}{b^2}\frac{D}{(D+F^{-1}\circ\xi)}\frac{\frac{(\partial_\sigma F^{-1})\circ\xi}{D+F^{-1}\circ\xi}+\frac{(\partial_\sigma f)\circ F^{-1}\circ\xi}{f\circ F^{-1}\circ\xi}+f'\circ F^{-1}\circ\xi\frac{(\partial_\sigma F^{-1})\circ\xi}{f\circ F^{-1}\circ\xi}}{(h^{+}+h^{-})f\circ F^{-1}\circ\xi} \\
aux_1 & = & \frac{(\partial_\sigma f')\circ F^{-1}\circ\xi}{(f\circ F^{-1}\circ\xi)^3} - 3\frac{(f'\circ F^{-1}\circ\xi)}{(f\circ F^{-1}\circ\xi)^4}((\partial_\sigma f)\circ F^{-1}\circ\xi) \\
aux_2 & = & ((\partial_\sigma F^{-1})\circ\xi)(\frac{(f''\circ F^{-1}\circ\xi)}{(f\circ F^{-1}\circ\xi)^3} - 3\frac{(f'\circ F^{-1}\circ\xi)^2}{(f\circ F^{-1}\circ\xi)^4}) \\
aux_3 & = & (f'\circ F^{-1}\circ\xi)\frac{((\partial_\sigma F^{-1})\circ\xi)}{((D+F^{-1}\circ\xi)*(f\circ F^{-1}\circ\xi^3)} \\
aux_4 & = & aux_1 + aux_2 - aux_3 \\
\gamma_{34} & = & \frac{2r}{b^2(h^{+}+h^{-})^2}(p-D/b-c)\frac{D}{D+F^{-1}\circ\xi}aux_4
\end{array}$$

4.C Parameters Figures 4-1–4-2

The following parameters were used in the creation of Figures 4-1–4-2.

- Demand : $D(p, x, \varepsilon) = 100 - 2p + 10x + \varepsilon$, $\varepsilon\ N(0, \sigma^2)$.
- Cost : $c(x) = 20 + 20x^2$.
- Misaligned demand costs: $h^- = 40$, $h^+ = 10$.

Chapter 5

Concluding Remarks

In this thesis we analyzed three different problems associated with sustainability. In Chapter 2, we studied optimal bidding strategies in a wholesale electricity market. This market works as an auction where producers bid a price they are willing to sell their capacity, and demand is allocated to producers based on their bid. Even though the market operates as an auction, in the analysis we did not use the classic auction framework. In most of the auction literature, the main assumption involves symmetric players with valuations for items drawn from a common distribution. In the case of the wholesale electricity market, this would translate to assuming producers' cost to be drawn from a common distribution. This assumption, while suitable for auctions where players have little information about each other, e.g. antiques auctions, is not appropriate for the wholesale electricity market where, (i) different players have different technologies, (ii) information about players technology is accessible to all players, and (iii) players are not likely to switch technology. For these reasons we model the wholesale electricity market as an asymmetric game with public information. Surprisingly, we find that with asymmetry of producer's costs, there exists a pure strategy equilibrium which could not be obtained in a symmetric model.

In Chapter 3, the problem of comparing carbon tax with quota based policies was studied, taking into account the perspective of consumers, producers and central planner. We show that in a deterministic setting there is no differences between the two policies, however, these two policies differ considerably under stochastic demand. There are two results that deserve special attention. One is that it is possible for producers with clean technology to offset the additional cost of a tax, and in fact, obtain higher profits with an increasing tax value. The intuition behind this result is that the introduction of a carbon tax policy increases consumer prices, and if the producer's technology is much cleaner than the technology of the competition, the increase in cost would be offset by the increase in price. It is important to note that this result is unattainable in a symmetric producers model, as in such model it can be proved that producers receive decreasing profits with higher tax values. Second, our analysis shows that the producer with the highest amount of emissions per unit produced is always better of in a tax policy as opposed to a cap policy. However, this is not always true for all producers, as the producer with the cleanest technology may receive higher profits under a cap policy. As in the previous result, this observation is only possible to obtain in an asymmetric producers model. Indeed, in a symmetric producer model, all producers are better off under a tax policy.

Chapter 4 follows models more consistent with the supply chain management literature. In this chapter, we analyzed a sequential two-stage supply chain. In the model, demand is increasing in a quality measures such as emissions level, nutritional values or luxury features, which can be increased at higher cost to the supplier. Typically, the supplier chooses the quality level to apply, however, in some cases retailers impose a quality requirement on the supplier. When comparing these two models, we found there was no difference for the supply chain performance under deterministic demand. However, these models differ under stochastic demand.

Obviously, in both of these environments supply chain profit is lower than under an integrated model. We derived contracts capable of coordinating the supply chain, implying the contracts are able to achieve the profits and quality level of a vertically integrated supply chain. The contracts found resembled buyback and revenue sharing contracts, but include a dependency on the quality level. In the performance analysis, the implicit function theorem was used in a non traditional way as means for characterizing the evolution of the quality parameter when the variance on the demand increases. This characterization allowed for simple algorithms that compute profits, quality level, consumer price, etc. This approach can be useful in many models where an analytical solution is hard to compute, e.g. news vendor model with pricing.

This thesis would be incomplete if we don't point out other ways to model and compare cap-and-trade and carbon tax. For example, consider a deterministic demand model where suppliers have linear production cost and linear technology. Suppliers bid a supply function, and there is a central planner that allocates demand at minimum cost to consumers. Assume that there is a fixed number of permits that are distributed among the players, and the players are engaged in a market mechanism that allows them to trade with each other. For a given electricity market price and demand allocation, cooperative game theory can be applied to determine the resulting price of permits and the number of permits purchased/sold by each player. However, finding the optimal bidding strategies for the suppliers requires solving a non convex optimization problem.

Bibliography

Allon, G. and Federgruen, A. (2009). Competition in service industries with segmented markets. *Management Science*, 55(4):619–634.

Amacher, G., Koskela, E., and Ollikainen, M. (2004). Environmental quality competition and eco-labeling. *Journal of Environmental Economics and Management*, 47:284–306.

Archibald, C. and Malvey, P. (1998). *Uniform-Price Auctions: Update of the Treasure Experience*. Report, Office of Market Finance, U.S. Treasury.

Arrow, K. J. (1986). Rationality of self and others in an economic system. *The Journal of Business*, 59(4):pp. S385–S399.

Ata, B., Lee, D., and Tongarlak, M. H. (2012). *Got Local Food?* Harvard Business School.

Avci, B., Girotra, K., and Netessine, S. (2012). Electric vehicles with a battery switching station: Adoption and environmental impact. *Available at SSRN: 2005092*.

Avi-Yonah, R. and Uhlmann, D. (2009). Combating global climate change: why a carbon tax is a better response to global warming than cap and trade. *Stanford Envir. Law*.

Back, K. and Zender, J. (1993). Auctions of divisible goods: on the rationale for the treasury experiment. *Review of Financial Studies*, 6(4):733–764.

Back, K. and Zender, J. F. (2001). Auctions of divisible goods with endogenous supply. *Economics Letters*, 73(1):29 – 34.

Baldick, R., Grant, R., and Kahn, E. (2004). Theory and application of linear supply function equilibrium in electricity markets. *Journal of Regulatory Economics*, 25(2):143–167.

Banker, R., Khosla, I., and Sinha, K. (1998). Quality and competition. *Management Science*, 44(9):1179–1192.

Benjaafar, S., Y. L., and Daskin, M. (2011). Carbon footprint and the management of supply chains: Insights from simple models. *In Review*.

Bernstein, F. and Federgruen, A. (2004). A general equilibrium model for industries with price and service competition. *Operations Research*, 52(6):868–886.

Bernstein, F. and Federgruen, A. (2007). Coordination mechanisms for supply chains under price and service competition. *Manufacturing and Service Operations Management*, 9(3):242–262.

Boyaci, T. and Gallego, G. (2004). Supply chain coordination in a market with customer service competition. *Production and Operations Management*, 13(1):3–22.

Cachon, G. P. and Harker, P. T. (2002). Competition and outsourcing with scale economies. *Management Science*, 48(10):1314–1333.

Carmona, E., Fehr, M., Hinz, J., and Porchet, A. (2010). Market design for emission trading schemes. *Siam Review*, 52.

Chambers, C., Kouvelis, P., and Semple, J. (2006). Quality-based competition, profitability, and variable costs. *Management Science*, 52:1884–1895.

Choi, C. T. and Shin, H. S. (1992). A comment on a model of vertical product differentiation. *The Journal of Industrial Economics*, 40(2):229–231.

Coase (1960). The problem of social cost. *Journal of Law and Economics*, 3:1–44.

Conrad, K. (2005). Price competition and product differentiation when consumers care for the environment. *Environmental and Resource Economics*, 31:1–19.

Craig, A. J. (2012). Measuring Supply Chain Carbon Efficiency: A Carbon Label Framework. Ph.D. Dissertation, Massachusetts Institute of Technology.

Desiraju, R. and Moorthy, S. (1997). Managing a distribution channel under asymmetric information with performance requirements. *Management Science*, 43(12):1628–1644.

Dixon, H. D. (1992). The competitive outcome as the equilibrium in an edgeworthian price-quantity model. *Economic Journal*, 102(411):301–09.

Dorfman, R. and Steiner, P. O. (1954). Optimal advertising and optimal quality. *The American Economic Review*, 44:826–836.

Future, C. E. (2012). An overview of the clean energy legislative package. Internet.

Gomez-Arias, J. T. and Bello-Acebron, L. (2008). Why do leading brand manufacturers supply private labels? *Journal of Business and Industrial Marketing*, 23(4):273–278.

Green, R. (2007). Carbon tax or carbon permits: The impact on generators' risks. Discussion Papers 07-02, Department of Economics, University of Birmingham.

Green, R. J. and Newbery, D. M. (1992). Competition in the british electricity spot market. *Journal of political economy*, pages 929–953.

Hepburn, C. (2006). Regulation by prices, quantities, or both: A review of instrument choice. *Oxford Review of Economic Policy*, 22(2):226–247.

Hu, S., Kapuscinski, R., and Lovejoy, W. (2010). Bertrand-edgeworth auction with multiple asymmetric bidders. *Available at SSRN 1659996*.

Hu, S., Kapuscinski, R., and Lovejoy, W. (2011). Price dispersion in electricity auctions: The strategic foundation and implication to market design. *Available at SSRN 1660609*.

Jira, C. (2013). Engaging supply chains in environmental initiatives: Adoption and information sharing. Harvard Business School Ph.D. Thesis Dissertation.

Keskin, N. and Plambeck, E. (2011). Greenhouse gas emissions accounting: allocating emissions from processes to co-products. *Available at SSRN 1956211*.

Klemperer, P. D. and Meyer, M. A. (1989). Supply function equilibria in oligopoly under uncertainty. *Econometrica*, 57(6):pp. 1243–1277.

Krishna, V. (2009). *Auction theory*. Academic press.

Krishnan, H. and Winter, R. A. (2010). Inventory dynamics and supply chain coordination. *Management Science*, 56:141–147.

Lehmann-Grube, U. (1997). Strategic choice of quality when quality is costly: The persistence of the high-quality advantage. *The RAND Journal of Economics*, 28(2):372–384.

Lobel, R. and Perakis, G. (2011). Consumer choice model for forecasting demand and designing incentives for solar technology. *Available at SSRN 1748424*.

Mas-Colell, W. and Green (1995). *Microeconomic Theory*. Oxford Univ. Press.

Maskin, E. (1986). The existence of equilibrium with price-setting firms. *The American Economic Review*, 76(2):pp. 382–386.

Milgrom, P. (1989). Auctions and bidding: A primer. *The Journal of Economic Perspectives*, 3(3):3–22.

Moorthy, K. S. (1988). Product and price competition in a duopoly. *Marketing Science*, 7(2):141–168.

Motta, M. (1993). Endogenous quality choice: Price vs. quantity competition. *The Journal of Industrial Economics*, 41(2):113–131.

Nair, J., Adlakha, S., and Wierman, A. (2012). Energy procurement strategies in the presence of intermittent sources. Technical report, Caltech Technical Report.

Pigou, A. C. (1920). *The economics of welfare / by A. C. Pigou*. Macmillan, London :.

Pizer, W. (1997). Prices vs. quantities revisited: The case of climate change. Discussion Papers dp-98-02, Resources For the Future.

Raz, G., Druehl, C. T., and Blass, V. D. (2012). Design for the Environment: Life Cycle Approach Using a Newsvendor Model. Working Paper.

Rodriguez-Ibeas, R. (2007). Environmental product differentiation and environmental awareness. *Environmental and Resource Economics*, 36:237–254.

Samuelson, P. A. (1970). The fundamental approximation theorem of portfolio analysis in terms of means, variances and higher moments. *Review Of Economic Studies*.

Simchi-Levi, D. (2010). *Operations Rules: Delivering Customer Value through Flexible Operations*. The MIT Press.

Simchi-Levi, D., Chen, X., and Bramel, J. (2005). *The Logic of Logistics: Theory, Algorithms, and Applications for Logistics and Supply Chain Management*. Springer, New York.

Son, Y. S., Baldick, R., Lee, K.-H., and Siddiqi, S. (2004). Short-term electricity market auction game analysis: uniform and pay-as-bid pricing. *Power Systems, IEEE Transactions on*, 19(4):1990–1998.

Tsay, A. A. and Agrawal, N. (2000). Channel dynamics under price and service competition. *Manufacturing and Service Operations Management*, 2(4):372–391.

Vickrey, W. (1961). Counterspeculation, auctions, and competitive sealed tenders. *The Journal of Finance*, 16(1):8–37.

Wang, H. (2003). A note on the high-quality advantage in vertical differentiation models. *Bulletin of Economic Research*, 55(1):91–99.

Wauthy, X. (1996). Quality choice in models of vertical differentiation. *The Journal of Industrial Economics*, 44(3):345–353.

Weitzman, M. (1974). Prices vs. quantities. *Review Of Economic Studies*.

Zhao, J., Hobbs, B. F., and Pang, J.-S. (May/June 2010). Long-run equilibrium modeling of emissions allowance allocation systems in electric power markets. *Operations Research*, 58(3):529–548.