Modified divergence theorem for analysis and optimization of wall reflecting cylindrical UV reactor

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Abstract

In this paper, the modified divergence theorem (MDT), known in earlier literature as the Gauss–Ostrogradsky theorem, was formulated and proposed as a general approach to electromagnetic (EM) radiation, especially ultraviolet (UV) radiation reactor modeling. A formulated mathematical model, based on MDT, for a multilamp UV reactor was applied to all sources in a reactor in order to obtain intensity profiles at chosen surfaces inside the reactor. Applied modification of MDT means that intensity at a real opaque or transparent surface or through a virtual surface, opened or closed, from different sides of the surface are added and not subtracted as in some other areas of physics. The derived model is applied to an example of the multiple UV sources reactor, where sources are arranged inside a cylindrical reactor at the coaxial virtual cylinder, having the radius smaller than the radius of the reactor. In this work, optimization of a reactor means maximum transfer of EM energy sources into the fluid for given fluid absorbance and fluid flow-dose product. The obtained results, for water quality known in advance, give a unique solution for an optimized model of a multilamp reactor geometry. As everyone can easily verify, MDT is a very good starting point for every reactor modeling and analysis.

Keywords: UV reactor; UV reactor model; UV intensity; EM flux; divergence theorem.

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UV reactors applications

Electromagnetic (EM) radiation reactors, and especially ultraviolet (UV) reactors, are widely applied in many areas of technological processes. Here we will mention some that are most prominent [1–5]: 1) biochemical processes, *i.e.*, de-germination: disinfection, sterilization and conditioning of UV transparent fluids (air, potable or wastewater, water solutions, serums, etc.); 2) ozone generation (in wavelength range from X--ray band of EM radiation to 200 nm, but optimal wavelength is around 185 nm, or ≈ 6.7 eV in energy spectrum) and disintegration of ozone (optimal wavelength is around characteristic mercury line, *i.e.*, 254 nm, or ≈ 4.9 eV in energy spectrum); 3) decomposition of peroxide and hypochlorite compounds or free chlorine in water; 4) advanced oxidation processes (AOP), applied to oxygen, ozone, hydrogen peroxide, and other substances with or without presence of TiO_2 as catalyst; 5) UV spectrophotometers for determination the content and concentration of impurities of fluids (air, water, water solutions, etc), and determination of transparency or absorbance of fluids prior UV treatment, or for UV reactor modeling; 6) polymerization of colors (automobile industry, graphic and publishing studios), plastics, dentistry, etc; 7) accelerated artificial ageing of wood, paper, paintings, alcoholic drinks (forbidden by law), and whitening of textiles, wool, etc.; 8) induced photo electronic emission from surfaces of metals.

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UV Reactors and types

The main reactor properties are defined by: 1) fluid characteristics, firstly fluid depth influenced by its absorbance, 2) geometry shape of the reactor (cylindrical, plan-parallel, etc.), 3) lamps effective radiation power, 4) their operating working spectrum coupled with their 5) geometrical disposition in the reactor and lamp dimensions, 6) required fluid flow (capacity of the reactor) and 7) transferred dose into the treated fluid. Product of fluid flow and transferred dose to the fluid is simply denoted as dose-flow reactor characteristics.

According to its shape, UV reactors could be cylindrical, plan-parallel (PP), or elliptical [1]. They can have negative, positive and neutral irradiation geometry, as it is seen in Figures 1–6.

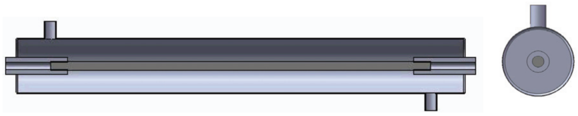

Figure 1. One lamp cylindrical reactor with negative irradiation geometry, longitudinal and transverse cross section.

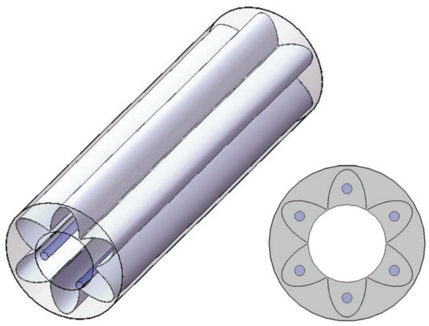

Figure 2. Cylindrical multilamp reactor with positive irradiation geometry.

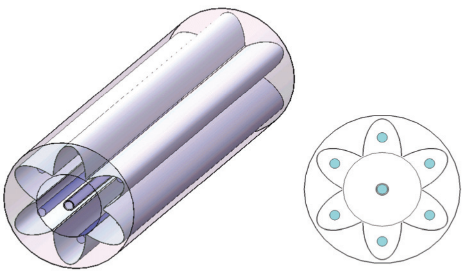

Figure 3. Cylindrical multilamp reactor with neutral irradiation geometry.

Figure 4. High capacity cylindrical multilamp reactor with negative irradiation geometry (above 1 m^3/s).

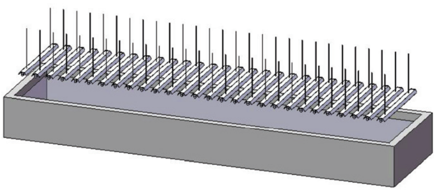

Figure 5. Plan-parallel reactor with lamps over fluid and negative (pseudo neutral) irradiation geometry.

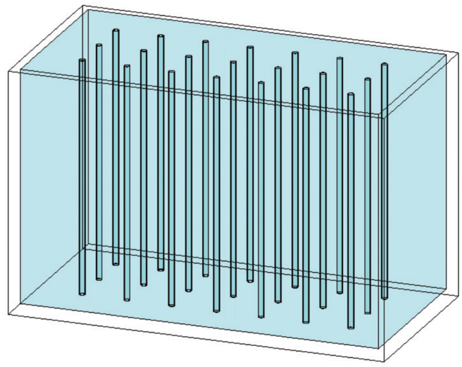

Figure 6. Plan-parallel reactor with lamps immersed in fluid and negative irradiation geometry.

UV reactor mathematical models – short historical review

Although the first UV reactor for water treatment was built in 1910 in France, there was no previous modeling for that facility, for it was made by experience. Much later, Luckiesh [6] made a proposed solution for an inexpensive flow-through UV reactor for water disinfection with one low-pressure lamp over the water surface and reflector above the lamp. Luckiesh calculated intensity profiles versus water depth for various absorption coefficients of water, and concluded that a battery of the described reactors could increase the capacity of treated water. Eight years later, Kamimura and Suzuki [7] made amplan-parallel (PP) flow-through reactor for seawater disinfection with six lamps above water surface. Kelly [8] made a similar UV reactor, based on the previous experience, with 13 UV lamps above the water surface. According to the author's calculations, the total power of UV radiation was 57.1 W, UV power, calculated per unit surface of water, was 3.84 mW/cm^2, water depth was 18 cm and output dose (for water on the surface) with flow rate of 9 m^3/h and after 15 s retention was 3.84 $mW/cm^2 \times 15$ s = 57.6 mJ/cm^2. These were among the first announced rough calculations of a reactor. Eight years later, Hill *et al*. [9], at the same reactor, made experiments on UV inactivation of Poliovirus type 1. Cassano *et al*. [10] gave general approach to photochemical reactors intensity profiles, based on Lambert's law. Jacob and Dranoff [11] analyzed a cylindrical UV reactor with negative irradiation geometry and intensity profile, for the case of perfect mixing of fluid in it. They divided lamp into *N* parts treating them as spherical sources, and apply Lambert's law. That approach was later on established as multiple point source summation (MPSS) method. Mathematical model of reactor with UV source that have the same characteristics through the whole lamp length, and without effects of reflection, refraction and absorption of fluid in the reactor was analyzed by Irazoqui *et al*. [12]. Intensity and dose expressions were derived. They concluded that for high ratios of length to radius of the finite length lamp, the intensity profile is the same as for infinite sources of radiation. Schenck [1], in the chapter of UV sterilization of water, presented all models for UV reactors known up to that moment for reactors that have negative, positive or neutral irradiation geometry. Models are mostly based on Lambert's law. Suidan and Severin [13] compared two models for intensity and dose distribution of radiation. One in approximation of infinite source length, and another with finite source length in cylindrical reactor, based on Lambert's law, under the condition that both models can give the same results. The obtained results, for average values of intensity, for both models are very similar. The conclusion of the article is very important and is worth to be cited: "*For situations where the average intensity within the reactor is adequate to define kinetic rates, there is no practical reason to use the more complex finite-length lamp model instead of the infinite-length lamp model. In many cases, especially for reactors with large length to lamp radius ratios and overall reactor radius to lamp radius ratios, the finite-length model can predict higher average intensities than can the infinite length model.*" Blatchley [14] gave numerical model of intensity applied to collimated-beam reactors and continuous-flow systems. Chiu *et al*. [15], dealing with integral model of UV disinfection, gave the estimate values dose distribution in an open PP reactor, with vertically immersed lamps, protected with quartz jackets. Absorption of quartz tube is taken into account, but not Fresnell's coefficients on relation air––quartz–water. Bolton [16] made a model for calculation intensity distribution in a reactor with one source, taking into account both absorption of quartz and Fresnell's coefficients on relation air–quartz–water. Milanović [2] formulated a model for accurate calculation of intensity distribution in the plan-parallel reactor of Luckiesh and Kamimura–Suzuki (or later on Kelly's) type. Jin *et al*. [17] analyzed the influence of shadowing of lamps in a system with many lamps. Mathematical model and calculations are made for air (no absorption) as a working fluid, only in order to get a rough picture about the examined effect. Pareek [18] made the computer analysis of a cylindrical reactor with two coaxial lamps in homogenous and heterogeneous media, and concluded that the optimal distance between lamps depends on the optical characteristics of the medium. Sozzi and Taghipour [19] analyzed the influence of hydrodynamics on reactor characteristics. Intensity model distribution, with some modifications is accepted from Jacob and Dranoff. Van Mourik *et al*. [20] analyzed dynamics of a reactor and automatics for controlling its work, applying a model based on Lambert's law, which is slightly changed by multiplying it with a factor, in order to suite it to one type of microorganisms. Milanović [5] gave review of UV reactors and their basic characteristics primarily for high capacities aimed at drinking water disinfection, for closed as well as for opened UV reactors. Milanović *et al*. [21] gave a mathematical model and an approach for optimization of a mono-lamp cylindrical (annular) reactor, obtaining that the water depth is equal to the inverse value of the water absorbance.

In this paper, we analyze a cylindrical multilamp reactor and derive an intensity profile, as well as optimization of reactor diameter and the virtual cylinder radius where the sources are positioned. Through the whole analysis we will assume low pressure mercury (LPM) sources in the approximation of the infinite length, in a way explained by Suidan and Severin [13]. The mathematical model for calculating intensity distribution is based on the modified divergence theorem (MDT). Above all, we also assume that there is only absorption and no scattering in the water in laminar flow.

ANALYSIS

Modified divergence theorem formulation

In order to obtain formulation of MDT we can analyze the cross section of a tubular reactor that has a radius R_R, with N tubular sources of radiation (lamps) in quartz sleeves, which have a radius r_Q, randomly distributed, as it is shown in Figure 7.

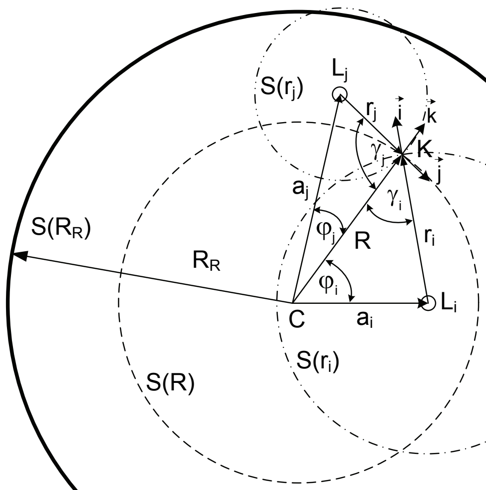

Figure 7. Cross section of a tubular reactor of radius R_R with sources randomly distributed in the reactor volume. S denotes surface of the sub-cylinder with radius $R < R_R$, and $L_{i,j}$ are i-th and j-th lamp in the reactor.

Let us form a virtual sub-cylinder S with radius $R < R_R$, so that inside it there are I lamps, and outside there are

J lamps (satisfying the relation $I + J = N$), so that we can denote with *n* any of those lamps $n = 1,2,...,N$.

In this paper, we will assume that our mathematical reactor model has no wall thickness. Further on, in the text and figures we will define R ($0 \le R < R_R$), as "inside reactor radius", or "inside reactor radius R", or "R" alone, but "outside reactor radius", or "outside reactor radius R" or "R_R" alone, when $R = R_R$.

Let us choose a point (*i.e.*, line) *K* on *S*, as in Figure 7, and analyze the *i*-th ($i \in [1,I]$) and *j*-th lamp ($j \in [1,J]$), or *n*-th lamp ($n \in [1,N]$) in the reactor.

Since there are many different definitions of the power radiation flux in the literature, in this paper we will assume power flux to be power across an unit surface, multiplied with cosine of the angle between incident radiation path and an orthogonal ort of analyzed surface, having the unit W/m^2.

Now we can define power radiation flux (further on only flux) of two eccentrically placed *n*-th lamp, through a virtual surface *S*, which is orientated by ort $\vec{k}$, so that $\vec{S} = S \cdot \vec{k}$.

If we denote distance from the centers of *i*-th or *j*-th lamp to the line *K* as $r_{n,k}$, than we can define the flux of the *n*-th lamp Φ_n, according to the MDT, through whole surface *S* as:

$$\oint_S \Phi_n \cdot d\vec{S} = \oint_V \nabla\Phi_n \cdot dV = \oint_V \vec{\rho}_{Pn} \cdot dV = \vec{P}_n \quad (1)$$

$$\vec{P}_n = \eta \cdot \vec{P}_0 \cdot T_n \quad (2)$$

where P_0 is the initial power of every lamp, T_n transmission coefficient from lamp to point *K*, and $\vec{P}_n$ is power vector from *n*-th lamp to point *K*, and:

$$\eta = \eta_A \eta_T \eta_V T_Q T_F \quad (3)$$

$$T_Q = \exp(-\alpha_Q q)\ ,\ T_F = [1 - R_{F1}][1 - R_{F2}] \quad (4)$$

$$T_W = \exp(-\alpha_W (r_n - r_Q)) \triangleq T_n \quad (5)$$

where η is coefficient of losses, including η_A - lamp ageing factor, η_T - power transmission factor from the lamp to the fluid as described by Suidan and Severin [13], and η_V - voltage line factor, which could have great influence to lamp operation. T_F is Fresnell's transmission coefficient on surfaces air–quartz and quartz–water, and T_Q transmission coefficient of quartz. $T_W = T_n$ is the transmission coefficient of water from quartz *n*-th sleeve to the point *K*. R_{F1} and R_{F2} are reflection coefficients according to Born and Wolf [22], *q* is thickness of the quartz, α_Q quartz absorption factor, α_W water absorption factor, $r_n - r_Q$ water layer depth between quartz sleeve and a point in the reactor.

Denoting the cylindrical surface of every lamp as S_n ($n = i,j$) and the *n*-th lamp's flux through S_n with Φ_n, then by multiplying expression (1) with ort $\vec{k}$, one can obtain:

$$\oint_S \Phi_n \cdot d\vec{S}_n \cdot \vec{k} = \oint_V \Phi_n dS_{n,k} = \vec{P}_n \cdot \vec{k} = P_n \left|\cos(\gamma_{n,k})\right| \quad (6)$$

$$\gamma_{n,k} = \sphericalangle(\vec{k}, \vec{P}_n)$$

where the cosine of the angle between ort $\vec{k}$ and incident radius from lamp *n* is always positive and it is accented with absolute value of cosine, for fluxes through virtual are added, as it was explained earlier, wherever the source is placed inside or outside *S*. Then, radiation power flux of *n*-th lamp through whole surface $S = S_n$, in every point $k \in [1,K]$ at *S*, is given by an expression which is obtained from Eq. (6):

$$\Phi_{n,k} = \frac{P_n \left|\cos(\gamma_{n,k})\right|}{S_{n,k}} \quad (7)$$

where $\Phi_{n,k}$ means flux from lamp *n* through *S* at point *k*, and:

$$S_{n,k} = 2\pi L_e r_{n,k} \quad (8)$$

where $S_{n,k}$ denotes surface *S* where at point *k* exists a flux from lamp *n*, and:

$$r_{n,k} = \sqrt{a_n^{\ 2} + R^2 - 2a_n R\cos(\varphi_{n,k})} \quad (9)$$

where $r_{n,k}$ is distance from center of lamp *n* to point *k* at *S* (S_n), $\Phi_{n,k}$ relative angle between *n*-th lamp and *k*-th point, for whole angle range of *n*-th lamp $\varphi_{n,k} \in (0,2\pi]$, and total flux Φ_k through point *k*, from all lamps is given by:

$$\Phi_k = \sum_{n=1}^{N} \Phi_{n,k} = \sum_{n=1}^{N} \frac{P_{n,k} \left|\cos(\gamma_{n,k})\right|}{S_{n,k}} \quad (10)$$

Radiation intensity profile, Ψ_k, or fluence, according to IUPAC [23], originated directly from the sources, one can obtain from (10), where $\cos(\gamma_{n,k}) = 1$:

$$\Psi_k = \sum_{n=1}^{N} \Psi_{n,k} = \sum_{n=1}^{N} \frac{P_{n,k}}{S_{n,k}} = \frac{\eta P_0}{2\pi L_e} \sum_{n=1}^{N} \frac{T_{n,k}}{r_{n,k}} \quad (11)$$

where $T_{n,k}$ represents transparency from *n*-th source quartz tube surface to *k*-th point, at surface *S*(*R*) according to Eq. (5). Coefficient η is defined by Eqs. (3) and (4).

Including average effective reflection of the reactor wall, ρ, at $R = R_R - \varepsilon$, ($\varepsilon \to 0$), it is easily seen that intensity Ψ_k and dose D_k profile near the wall can be expressed as follows:

$$\Psi_k = (1+\rho)\sum_{n=1}^{N} \Psi_{n,k} = (1+\rho)\sum_{n=1}^{N} \frac{P_{n,k}}{S_{n,k}} = \frac{(1+\rho)\eta P_0}{2\pi L_e} \sum_{n=1}^{N} \frac{T_{n,k}}{r_{n,k}} \quad (12)$$

$$D_k = t_r \Psi_k = \frac{V_R}{Q_F}\Psi_k = \frac{(1+\rho)\eta P_0}{2}\frac{(R_R^{\,2} - Nr_Q)}{Q_F}\sum_{n=1}^{N}\frac{T_{n,k}}{r_{n,k}} \qquad (13)$$

$$D_k(R<a) = (1+\rho)t_r \Psi_k = \frac{(1+\rho)\eta P_0}{2}\frac{(R_R^{\,2} - Nr_Q)}{Q_F}\sum_{n=1}^{N}\frac{T_{n,k}(R<a)}{r_{n,k}(R<a)} \qquad (14)$$

$$t_r = \frac{V_R}{Q_F} = \frac{S_e L_e}{Q_F} = \frac{\pi(R_R^{\,2} - Nr_Q^{\,2})L_e}{Q_F} \qquad (15)$$

where t_r is retention fluid time in the reactor, V_R, S_e and L_e are effective reactor volume, cross section surface and lamp length, respectively, and Q_F is stationary flow rate of the fluid (water).

Optimization of a cylindrical multilamp UV reactor

In order to optimize the multilamp cylindrical reactor having the lamps disposition at virtual cylinder of radius $a_n = a$, as shown in Figure 8, we have to define a criterion for optimization. Having in mind that we are using commercial low power lamps, *i.e.*, LPM sources, we will have slow water velocity through reactor, and we can assume it to be very close to laminar flow. It means that retention time will be nearly the same in every axis of the reactor, parallel to the central axis. In that case we can assume that intensity of radiation should be the same in the central axis of the reactor (line C in Figure 8) and on the wall of the reactor, between two lamps (line A in Figure 8). In lines like A, there are minimum of intensities on the reactor wall, as it will be seen, although it is obvious at the very first sight.

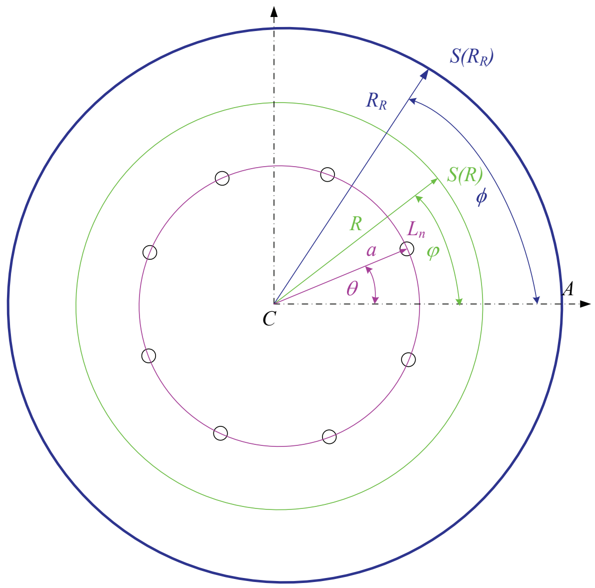

Figure 8. Cross section of a reactor with sources distributed at the virtual coaxial cylinder surface inside the reactor where S is virtual coaxial surface, L_n n-th lamp, R radius of virtual surface, and R_R outside reactor radius.

Radiation intensity, ψ_C, at the central axis (line C in Figure 8) of the reactor is given by the expression:

$$\Psi_C = \frac{\eta P_0}{2\pi L_e}\sum_{n=1}^{N}\frac{T_{n,c}}{r_{n,c}} = N\frac{\exp(-\alpha_W(a-r_Q))}{a} \qquad (16)$$

According to the defined criterion, for k = A from Eqs. (12), (13) and (16), one can obtain the reactor function expression or dose-flow product:

$$f(a,R_R) = (D_A - D_C)Q_F = (\Psi_A - \Psi_C)Q_F \qquad (17)$$

$$f(a,R_R) = (1+\rho)\sum_{n=1}^{N}\frac{T_{n,A}}{r_{n,A}} - N\frac{\exp(-\alpha_W(a-r_Q))}{a} = 0 \qquad (18)$$

$$r_{n,A} = \sqrt{a^2 + R_R^{\,2} - 2aR_R\cos(\varphi_{n,A})}\ ,\ \varphi_{n,A} = \frac{\pi}{N} = \frac{\pi}{8} \qquad (19)$$

$$T_{n,A} = [T_Q]T_F[T_{WA}] = [\exp(-\alpha_Q q)]T_F[\exp(-\alpha_W(r_{n,A} - r_Q))] \qquad (20)$$

$$T_F = \left\{1 - \left[\frac{n_a - n_q}{n_a + n_q}\right]^2\right\}\left\{1 - \left[\frac{n_q - n_w}{n_q + n_w}\right]^2\right\} \qquad (21)$$

where T_F is Fresnell's coefficient for normal incidence (in the case of infinite source approximation, when radiation from the source is propagating orthogonally from the cylindrical lamp and to the quartz sleeve too). Indexes of refraction for air, quartz and water are denoted as n_a, n_q and n_w, respectively.

Roots of the reactor function, $f(a,R_R)$, given in Eq. (18), for defined quality of the water α_W, reflection coefficient ρ and quartz tubes radius r_Q give us optimal values for eccentricity, $a = a_O$, and reactor radius $R_R = R_O$. As seen, roots of reactor function could not be obtained analytically, but only numerically, applying iterative procedure by varying values of a and R_R in relation (18).

RESULTS AND DISCUSSION

Now we are going to apply previous general results to analyze and optimize reactors with 6, 8, 10 and 12 lamps, without and with wall reflection, and obtain maximum dose for given water characteristics (transmission) and flow. Generally, it is possible to define various criterion types for reactor optimization, according to accepted fluid flow and its characteristics. All of them undergo mini-max or maximum efficiency principle. In our case it means maximum dose for minimum input energy, but in the every point of fluid exiting reactor it must be higher than minimal required. Mini-max principle could be applied according to: the average dose transferred to the fluid flowing through the reactor, dose equality in the points of minimum intensity of the reactor, and so on, depending on the application. In this paper, we accept optimization criterion that minimum dose-flow product in axis A (D_AQ_F) at the reactor wall should be equal to dose-flow product in the central axis C (D_CQ_F). We still have in mind laminar flow and small water velocities which will be seen through calculations later.

Results of iterative calculation from relations (13) and (18) are given in Table 1 for constant input values: water flow Q = 25 l/s, quartz tube radius r_Q = 16 mm, water absorbance (α_W) range 0.02–0.05 cm^{-1}, number of lamps in reactor (N) 6–12, having effective length (length of the lamps arc) L_e = 111 cm, power of a lamp at mercury dominant wavelength 254 nm, P_0 = 26 W. Indexes of refraction for air, water and quartz at 254 nm (wavelength of LP mercury source) are: n_a = 1, n_W = = 1.376 [24], n_q = 1.52 [16], respectively. Coefficient of losses is obtained to be η = 0.756 according to Eq. (3) from assumed or calculated values: η_A = 0.8, η_T = 0.994 [19], η_V = 1. The value for η includes Fresnell's transmission air-quartz-water coefficient T_F = 0.955, and transmission losses through quartz tube T_Q = 0.999, calculated for quartz tube wall thickness of q = 1.5 mm, and absorbance of quartz α_Q = 0.005 cm^{-1} [16]. The minimum required and accepted dose for water that leaves reactor is 40 mJ/cm^2. Analysis was done for values of reactor wall reflection ρ = 0 or ρ = 0.4 in order to observe influence of wall reflectance to optimal values of reactor radius, R_O, and radius of cylinder for lamps displacement (eccentricity of lamps), a_O.

From Table 1 it is obvious that a reactor with 8 lamps will satisfy required conditions, without regard to wall reflectance, and that reactors with more than 8 lamps are not rational solutions, or they can be applied to higher flow requirements. Also it is seen that higher the wall reflectance the lower the reactor radius is required for the same water quality and higher dose for the same water flow.

Functions of the reactor, Eq. (18), in point A (Figure 8), for various values of eccentricity, a, as a parameter (17, 27, 37 and 47 cm) are plotted in Figures 9 and 10 for ρ = 0 and ρ = 0.4, respectively. It is seen that for every chosen eccentricity, there exists a zero of reactor function, *i.e.*, corresponding reactor radius value R and that pair of values gives the highest value of dose-flow product, but there is only one pair of values (a_O,R_O) that gives the maximum dose, *i.e.*, optimal reactor characteristics.

Dose values for various eccentricity parameter a and related reactor radius R are plotted in Figures 11 and 12 forρ = 0 and ρ = 0.4, respectively. It is now explicitly seen that every eccentricity has its related optimal radius, but there is only one optimal radius value, at dose maximum (see values in Table 1 for N = 8 and α_W = 0.005 cm^{-1}) satisfying condition of doses equilibrium at wall point A and axis line C (at the exit of the reactor).

Figures 13a and 13b clearly show that the relation between optimal reactor radius and optimal eccentricity is almost linear.

Dose profiles at the reactor wall and in the reactor axis are given in Figures 14 and 15 forρ = 0 and ρ = 0.4, respectively. As expected, average values of dose at reactor wall are 47.7 and 56 mJ/cm^2 for ρ = 0 and 0.4, respectively.

In Figure 16, intensity profiles at various virtual cylinders radius are given together with dose at central axis C, for comparison doses among each other. As expected, minimal dose is at axis C. Other dose values increases with R, *i.e.*, when R becomes closer to optimal eccentricity, a_O. Dose variation versus radius R is given in Figure 17, and it can be seen that increases for $R < a_O$, reaches its maximum for $R = a_O$, and then decreases for $R > a_O$. In Figure 17, dose dependence *versus* radius R is given at zero azimuth angle, *i.e.*, at the line between A and C (Figure 8) under the condition of laminar fluid (water) flow, from Eq. (14). In the case of

Table 1. Minimum dose, D, at reactor wall and central axis as a function water quality, α_W, of optimized values for reactor radius, R, and eccentricity, a_O, for various number of lamps, N, and for non-reflective and reflective walls

Parameter	*N*															
	6				8				10				12			
	α_W / cm^{-1}															
	0.02	0.03	0.04	0.05	0.02	0.03	0.04	0.05	0.02	0.03	0.04	0.05	0.02	0.03	0.04	0.05
	ρ = 0															
R_O / mm	795	534	404	328	819	549	419	340	823	554	422	343	831	559	426	346
a_O / mm	642	431	326	264	624	418	318	258	614	413	314	255	614	413	314	255
D / mJ cm^{-2}	66	45	34	28	100	68	51	41	131	88	67	54	160	108	81	65
	ρ = 0.4															
R_O / mm	746	785	800	806	502	529	541	544	381	402	411	416	309	327	334	339
a_O / mm	528	536	540	541	355	361	365	365	269	274	277	279	218	223	225	227
D / mJ cm^{-2}	89	128	163	197	60	86	110	133	46	65	83	100	37	53	67	81

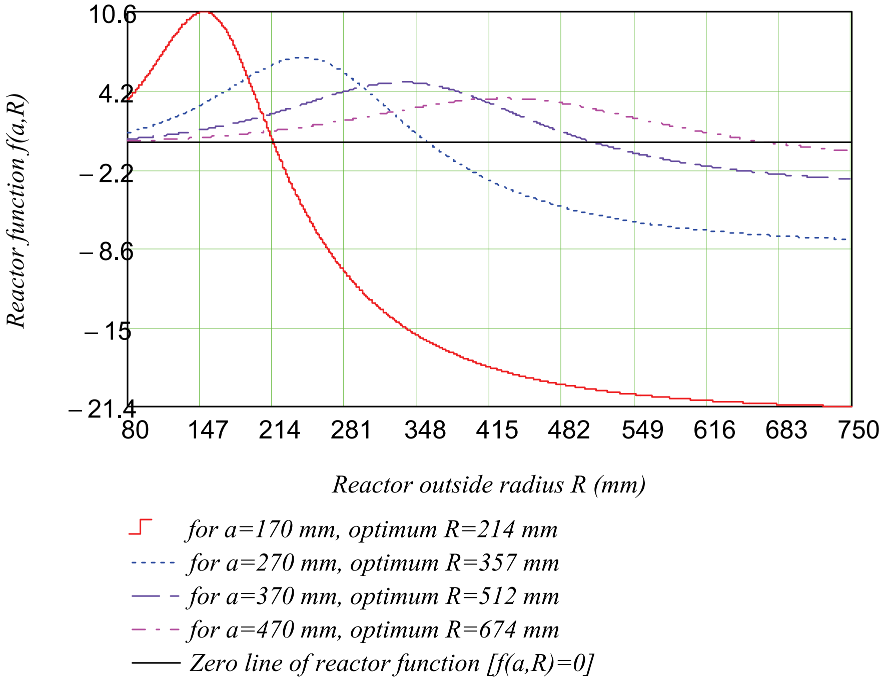

Figure 9. Values of dose distribution function f(a,R) versus eccentricity, a, and outside reactor radius, R, for various values of eccentricity (17, 27, 37 and 47 cm, respectively) and without wall reflectance $\rho = 0$ at the line A.

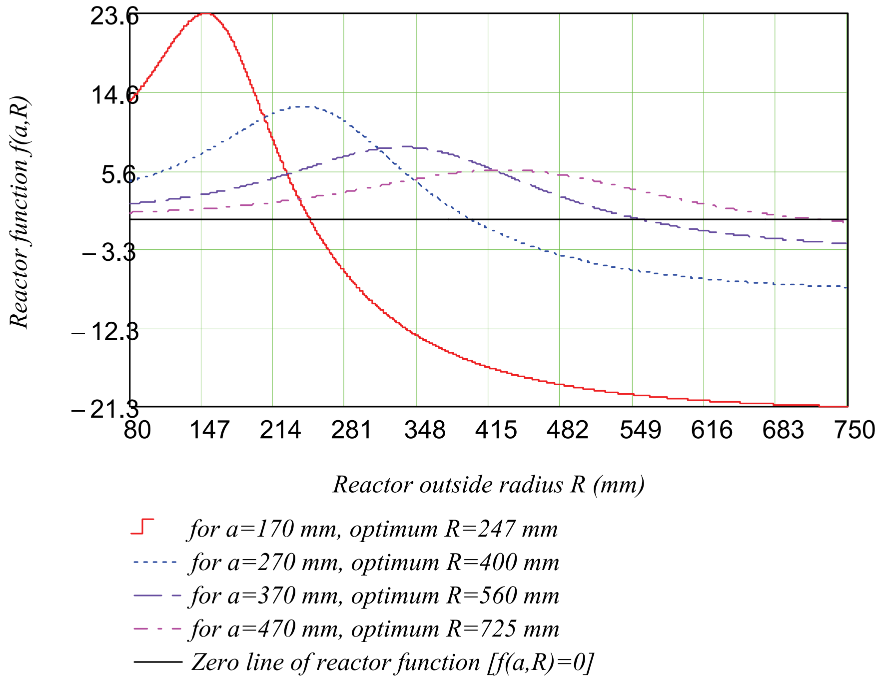

Figure 10. Values of dose distribution function f(a,R) versus eccentricity, a, and outside reactor radius, R, for various values of eccentricity (17, 27, 37 and 47 cm, respectively) and with wall reflectance ($\rho = 0.4$) at the line A.

turbulence, one will have averaged values of dose. Turbulence is applied whenever it is possible in order to obtain doses that are almost the same in every point of fluid volume exiting reactor.

Comparing minimal doses from Table 1 (for $N = 8$ reactor and $\alpha_W = 0.05$ cm^{-1}), for $\rho = 0$ and $\rho = 0.4$, respectively, it is seen that the reactor with reflection, transfers higher dose 52.6 mJ/cm^2, with smaller reactor radius (327 mm), then the reactor without wall reflection with dose 41.3 mJ/cm^2, and greater reactor radius (340 mm), giving the rate 52.6/41.3 = 1.273, *i.e.*, 27% higher dose for reactor with smaller reactor radius, or higher flow rate for the same dose! For nonreflecting walls, or low walls reflection, we have negative irradia-

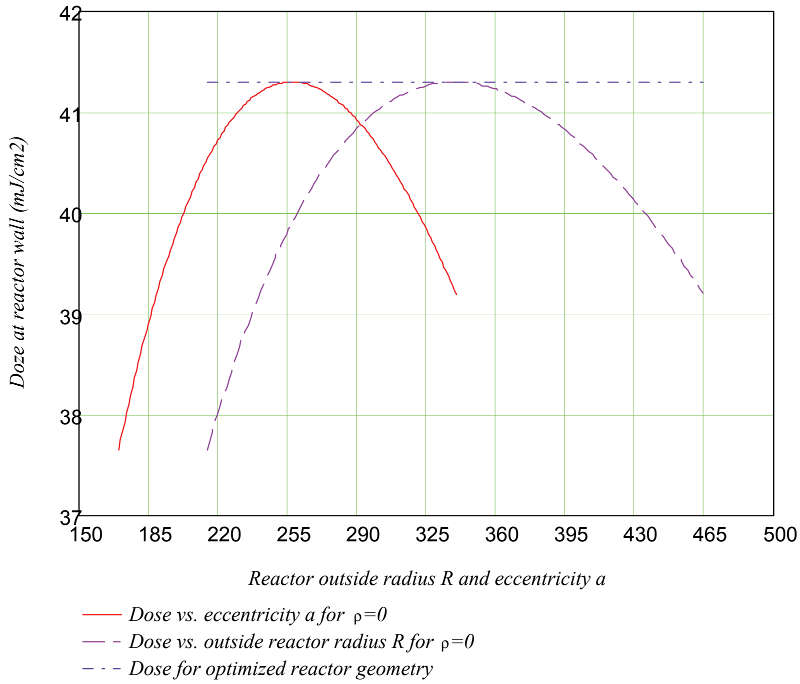

Figure 11. Dose, D, versus outside reactor radius, R, for N = 8 lamps at reactor exit for water absorbance α_W *= 0.05 cm*$^{-1}$ *and* ρ *= 0 at the line A.*

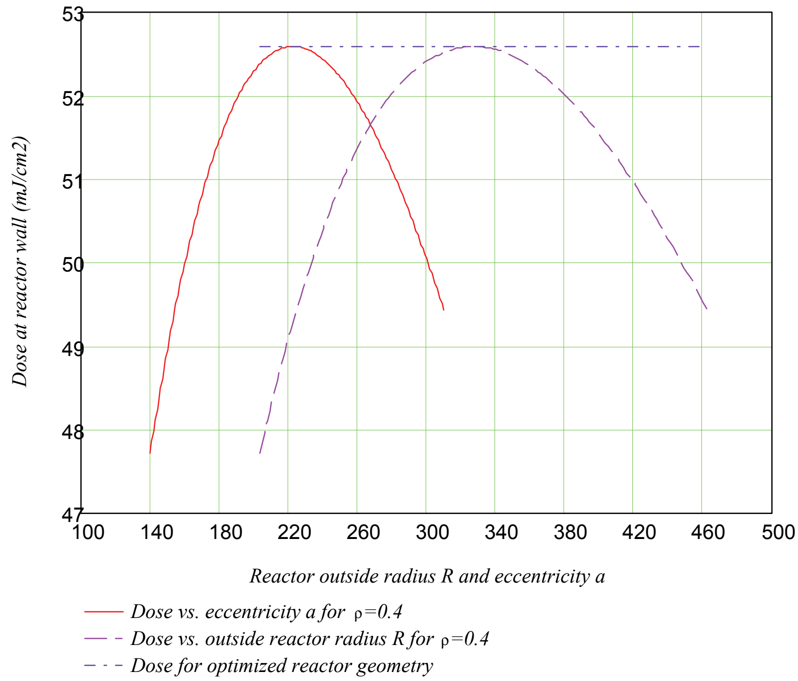

Figure 12. Dose, D, versus outside reactor radius, R, for N = 8 lamps at reactor exit for water absorbance α_W *= 0.05 cm*$^{-1}$ *and with wall reflectance* ρ *= 0.4 at the line A.*

tion geometry, but in the case of reactor walls high reflection, we will have pseudo-neutral irradiation geometry of the reactor, *i.e.*, we will have the situation which becomes closer to existence of outside positioned lamps radiating through transparent wall into the reactor volume. To increase greatly the reactor wall reflection (theoretically up to 80%), in some cases in practice it is possible to obtain very good results, or nearly neutral radiation geometry.

If we calculate velocity of water through reactor under the worst conditions, for given requirement, which is achieved for N = 8 reactor, with reflective walls when the optimal radius, R_O, is minimal, water velocity,

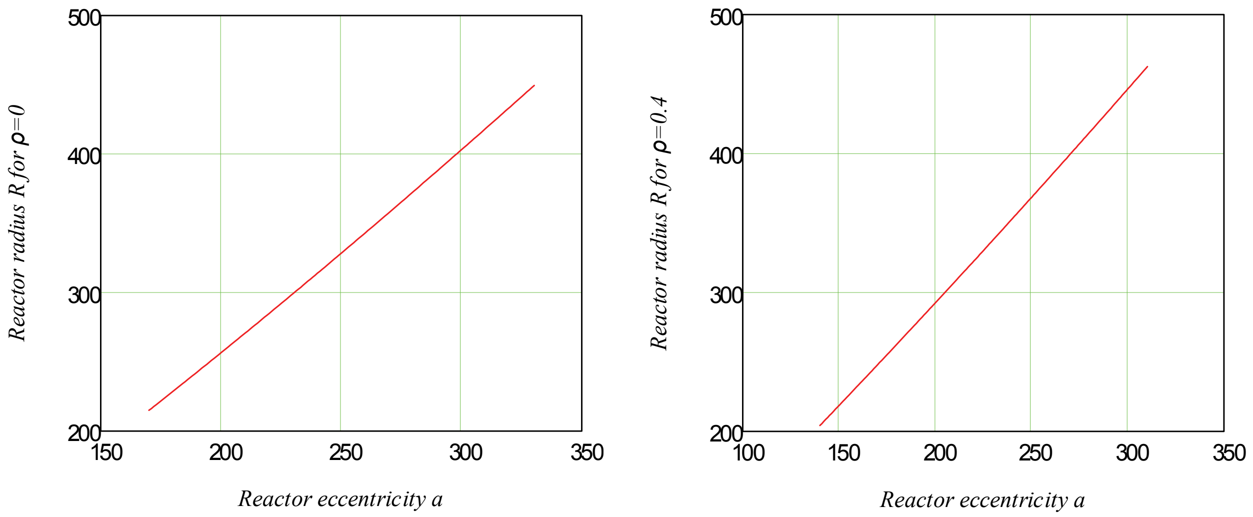

Figure 13. Optimal outside reactor radius, R, versus lamp eccentricity parameter, a and with wall reflectance: a) $\rho = 0$ and b) $\rho = 0.4$.

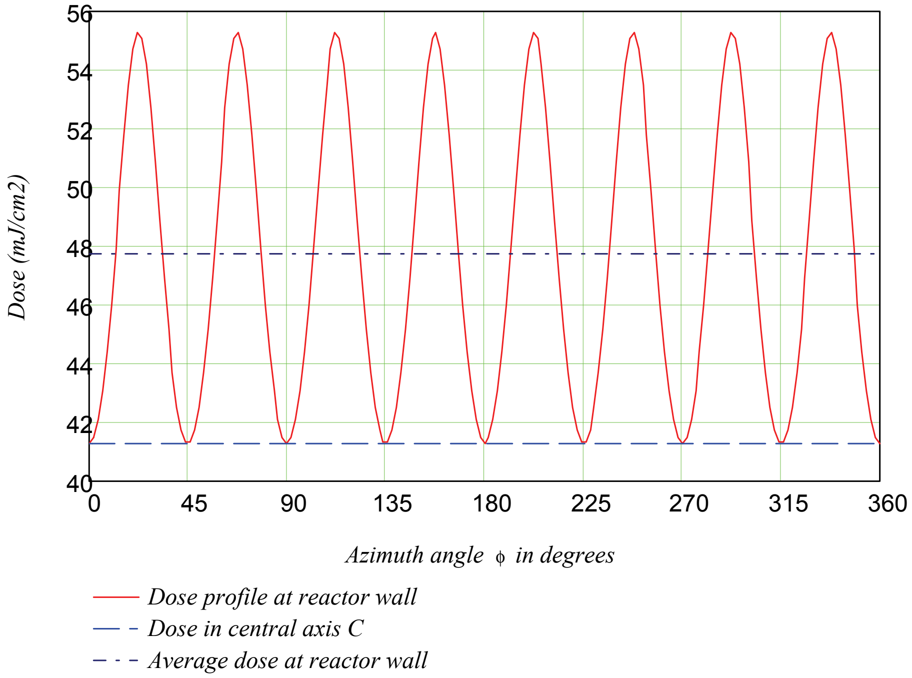

Figure 14. Dose profile, D, at reactor wall (wall reflectance $\rho = 0$) before water leaves reactor and dose value in the central axis of the reactor D_C.

V_W, under the steady state conditions and laminar flow is given as:

$$V_W = \frac{Q_F}{S_e} = \frac{Q_F}{\pi(R_O^2 - Nr_Q^2)} \approx 7.6 \text{ cm/s} \quad (22)$$

where S_e is effective surface of the reactor cross section.

This result justifies the advanced assumed condition about small fluid velocities for LPL (LPM lamps), in accordance with the assumption of laminar flow (see very beginning of the paragraph "Optimization of a cylindrical multilamp UV reactor").

CONCLUSIONS

The main contributions of the presented article are: 1) formulation and 2) application of the modified divergence theorem for analysis of UV reactors and 3) optimization of a cylindrical UV reactor.

This model allows the design of an appropriate reactor for water disinfection and other purposes, taking into account the reactor wall reflection close to the wall. When taking wall reflection into account, it is readily seen that the reactor radius decreases when reflection of the wall increases, for the same reactor characteristics (represented by flow-dose product), in

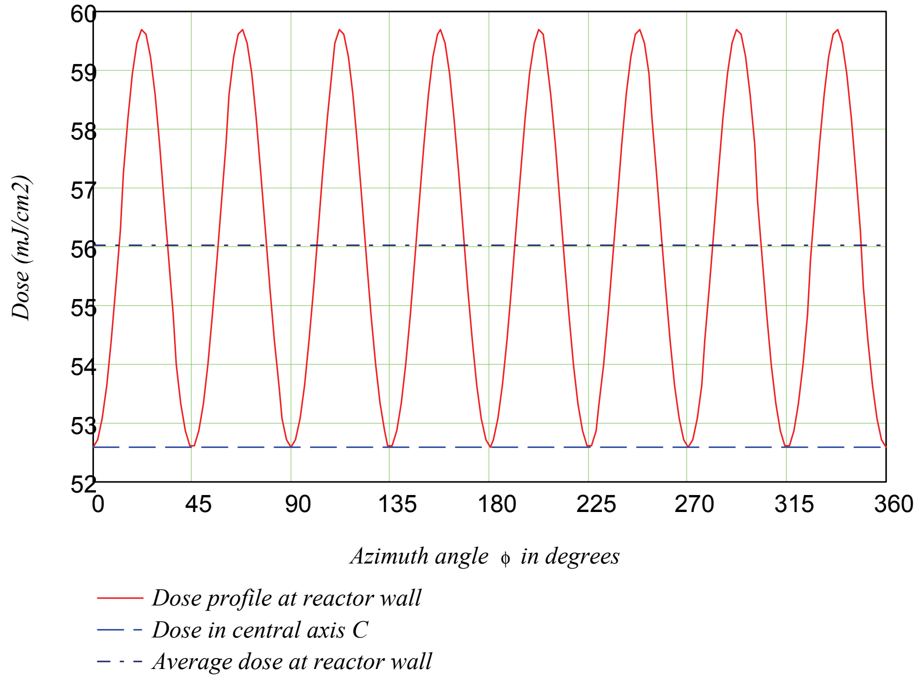

Figure 15. Dose profile, D, at reactor wall (wall reflectance ρ = 0.4) before water leaves reactor and dose value in the central axis of the reactor.

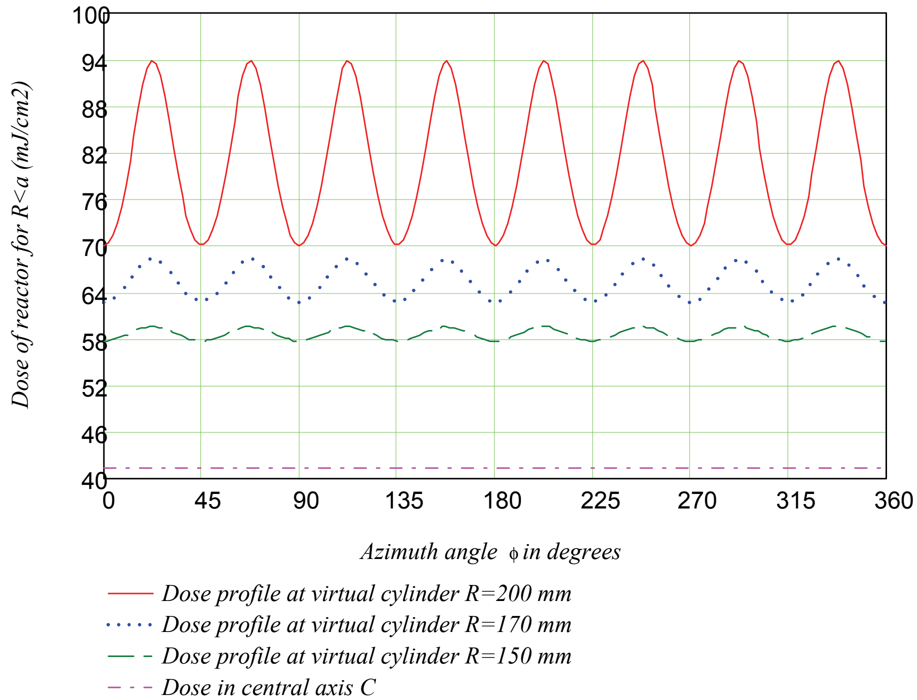

Figure 16. Dose profile, D, in the optimized reactor at virtual cylinders that have inside reactor radius smaller then optimal eccentricity a_O = 258 mm (wall reflectance ρ = 0).

our case the same output dose for the water flowing through it. Besides, for the same reactor diameter and required dose characteristic, higher water flow (of the same quality) can be achieved by application of reactor with higher reflection of the wall. This model and MDT can be easily applied for reactor optimization purposes of various geometries and various requirements according to flow-dose product.

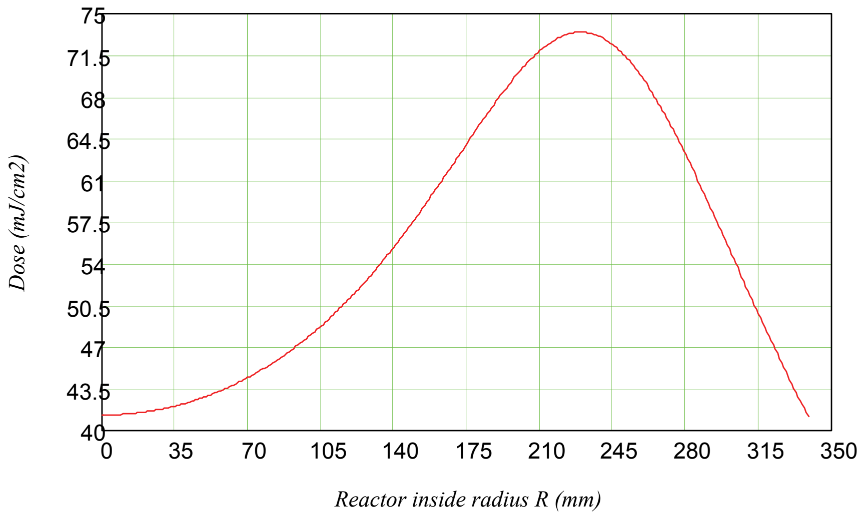

Figure 17. Dose profile, D, versus inside reactor radius, R, at azimuth angle $\phi = 0$ at the line connecting axis C and A for the optimized reactor at the exit of the reactor.

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IZVOD

MODIFIKOVANA TEOREMA O DIVERGENCIJI ZA ANALIZU I OPTIMIZACIJU CILINDRIČNOG UV REAKTORA

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(Naučni rad)

U ovom radu formulisana je i proširena Modifikovana teorema o divergenciji (MTD), u starijoj literaturi poznatija kao Teorema Gauss–Ostrogradski, kao generalni pristup za modelovanje reaktora elektromagnetnog zračenja, konkretno ultravioletnog (UV) zračenja. Na osnovu dobijenog modela iz MTD, formulisan je matematički model UV reaktora sa više izvora UV zračenja za određivanje raspodele intenziteta UV zračenja i optimizaciju. MTD se primenjuje na sve izvore zračenja u reaktoru, radi određivanja raspodele intenziteta zračenja na izabranoj površini u unutrašnjosti reaktora. Primenjena modifikacija MTD znači da se intenzitet na realnoj neprozračnoj ili transparentnoj površini, ili kroz virtuelnu površinu, otvorenu ili zatvorenu, sa raznih strana površine, sabiraju a ne oduzimaju, kao u nekim drugim oblastima fizike. Izvedeni model je primenjen na slučaj UV reaktora sa više izvora zračenja, gde su izvori raspoređeni u cilindričnom reaktoru na koaksijalnom virtuelnom cilindru, koji ima radijus manji od radijusa reaktora. U ovom radu, pod optimizacijom reaktora podrazumeva se transfer maksimalne energije elektromagnetnog (EM) izvora u fluid za zadatu apsorpciju fluida i proizvod doza-protok fluida.

Ključne reči: UV reaktor • Model UV reaktora • UV intenzitet • EM fluks • Teorema o divergenciji