COMPARISON OF THE EMPIRICAL VARIANCES AND MEAN VALUES OF NORMALLY DISTRIBUTED POPULATIONS OF NUCLEAR COUNTS

by

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This paper discusses the possibility of applying the F-test and double t-test in problems related to the identification of number of radioactive isotopes in a contaminated area by using only counters for radiation detection. The descriptions of the F-test and the double t-test are given along with the corresponding tabular values that enable their implementation. Finally, the experiment is presented via two radioactive samples. The results of the experiment were treated in the manner proposed in the paper and satisfactory results were obtained.

Key words: nuclear count, F-test, double t-test, variance

INTRODUCTION

The development of mathematical algorithms, no matter how complex they might look, enables us to quickly adopt adequate conclusions on a statistical behavior of random variable. Since the environment is increasingly contaminated by electromagnetic and particle radiation, one of the most important tasks, in the field of radiation protection, is to identify the sources of radiation.

This implies radiation detection and the determination of the source of radiation. It is particularly important to determine the number of radiation sources and separate the effects of irradiation related to the particular source in a contaminated area. This is of particular interest for the proper functioning of electronic systems in areas exposed to radiation [1-3].

The field of radiation protection often involves the detection of nuclear radiation in contaminated environments. Such measurements are used to determine whether there is a radioactive source (one or more) in order to eliminate its effect in the process of decontamination. However, in practice, the investigated area is frequently contaminated with several radioactive sources [4, 5].

For this reason, it is useful to develop a mathematical method that can determine whether the detected radiation comes from one or several sources. The aim of this paper is to develop an algorithm for establishing two or more normally distributed populations of nuclear counting results that originate from one or more of the radioactive sources by using only radiation counter as a detector [6, 7].

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COMPARISON OF EMPIRICAL VARIANCE AND MEAN VALUES

To determine if two normally distributed populations originate from a radioactive source involves proving the statistical equality of their variances and mean values. An F-test is used for the comparison of variances and a double t-test for the comparison of mean values.

The basis of the F-test is F-distribution, defined in the following way: if y and z are independent variables that have χ_2 distribution with degrees of freedom m_1 and m_2, the ratio $X = (y/m_1)/(z/m_2)$ is the F-distribution with degrees of freedom m_1 and m_2. Its probability density function is described as following

$$f_F(x) = \begin{cases} 0, & x \le 0 \\ \dfrac{\left(\dfrac{m_1}{m_2}\right)^{\frac{m_1}{2}} x^{\frac{m_1}{2}-1}}{B\left(\dfrac{m_1}{2};\dfrac{m_2}{2}\right)} \left(1+\dfrac{m_1}{m_2}x\right)^{-\frac{m_1+m_2}{2}}, & x > 0 \end{cases} \tag{1}$$

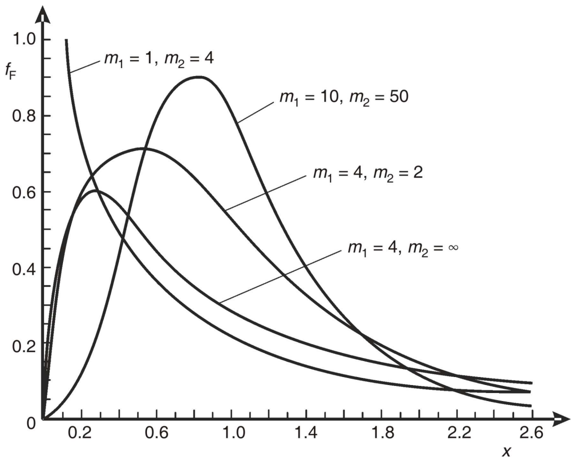

Figure 1. Probability density function characteristic $f_F(x)$ of F distribution with m_1 and m_2 degrees of freedom as parameters

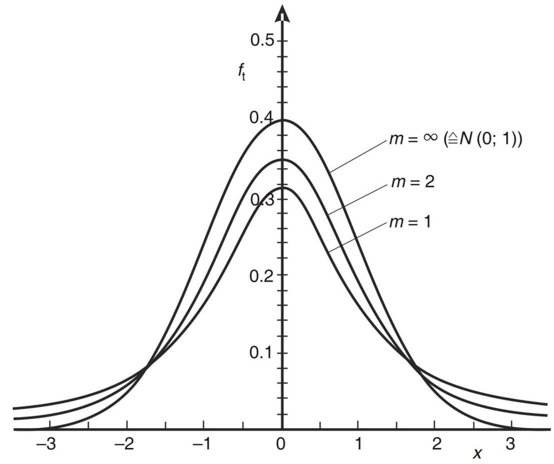

Figure 2. Probability density function $f_t(x)$ of t – distribution with degrees of freedom m as parameter

Table 1. Quantiles q of F-distribution $F_{m_1;m_2;q}$

m_2	m_1					
	6	8	10	13	16	20
(a) q = 0.95						
6	4.28	4.15	4.06	3.98	3.92	3.87
8	3.58	3.44	3.35	3.26	3.20	3.15
10	3.22	3.07	2.98	2.89	2.83	2.77
13	2.92	2.77	2.67	2.58	2.51	2.46
16	2.74	2.59	2.49	2.40	2.33	2.28
20	2.60	2.45	2.35	2.25	2.18	2.12
(b) q = 0.975						
6	5.82	5.60	5.46	5.33	5.24	5.17
8	4.65	4.43	4.30	4.16	4.08	4.00
10	4.07	3.85	3.72	3.58	3.50	3.42
13	3.60	3.39	3.25	3.11	3.03	2.95
16	3.34	3.12	2.99	2.85	2.76	2.68
20	3.13	2.91	2.77	2.64	2.55	2.46

Table 2. t-distribution quantiles $t_{m;q}$

m	Quantiles $t_{m;q}$		
	q = 0.95	q = 0.975	q = 0.99
1	6.31	12.7	31.8
2	2.92	4.30	6.96
3	2.35	3.18	4.54
4	2.13	2.78	3.75
5	2.02	2.57	3.37
7	1.89	2.36	3.00
9	1.83	2.26	2.82
14	1.76	2.14	2.62
19	1.73	2.09	2.54
24	1.71	2.06	2.49
40	1.68	2.02	2.42
120	1.66	1.98	2.36
∞	1.64	1.96	2.33

where $B(r,s) = \int_0^1 x^{r-1}(1-x)^{s-1}\mathrm{d}x$ represents a Beta function.

The probability density function characteristic is given in fig. 1 with m_1 and m_2 degrees of freedom as parameters. Table 1 shows quantiles q of F-distribution $F_{m_1;m_2;q}$ [8, 9].

The basis of the double t-test is t-distribution, which is defined in the following way: if y and z are independent random variables, and if y has a normal distribution N (0; 1), and if z has a χ^2 distribution with m degrees of freedom, the quotient $x = y/(2/m)^{1/2}$ has a t-distribution with m degrees of freedom. In addition, the quotient of the two independent variables having a normal N (0; 1) distribution has a t-distribution with m = 1 degrees of freedom. The probability density function of the t-distribution is

$$f_1(x) = \frac{\Gamma\left(\frac{m+1}{2}\right)\left(1+\frac{x^2}{m}\right)^{-\frac{(m+1)}{2}}}{\Gamma\left(\frac{m}{2}\right)\sqrt{\pi m}} \qquad (2)$$

The probability density function of the t-distribution is shown in fig. 2. Quantiles $t_{m;q}$. of t-distribution are given in tab. 2.

The comparison of the two empirical variances in the F-test is performed on the basis of the following hypotheses: the variance of two normally distributed populations σ_x^2 and σ_y^2 represented by the size of two samples u_x and u_y; the empirical s_x^2 and s_y^2 variances are equal if $\sigma_x^2 = \sigma_y^2$. To test the hypothesis the size taken is

$$t = \frac{s_x^2}{s_y^2} \qquad (3)$$

where x and y have to be chosen so that $s_x^2 \geq s_y^2$.

The critical value of $F_{m_1;m_2;q}$ is based on tab. 1, from the F distribution, as a quantile with a degree of freedom $m_1 = n_x - 1$ and $m_2 = n_y - 1$, and an order of $q = 1 - \alpha/2$ (double t-test at α level of significance). The hypothesis is discarded if $t > F_{m_1;m_2;q}$.

The comparison of the mean values, by using the two double t-tests, is based on the following hypotheses.

The two mean values μ_x and μ_y normally distributed with a population of the equal variance (assuming a double t-test does not discard the F-test), which can assume a size of the two samples n_x and n_y, the mean values $\bar{x}$ and $\bar{y}$, and empirical variances s_x^2 and s_y^2,are equal to $\mu_x = \mu_y$.

To test the hypothesis, the size taken is

$$t \quad \frac{(\bar{x} \quad \bar{y})\sqrt{\dfrac{n_x n_y}{n_x \quad n_y}}}{\sqrt{\dfrac{(n_x \quad 1)s_x^2 \quad (n_y \quad 1)s_y^2}{n_x \quad n_y \quad 2}}} \tag{4}$$

The critical value of $t_{m;q}$ is obtained on the basis of tab. 2, degree of freedom $m = n_x + n_y - 2$ and order $q = 1 - \alpha/2$ (two-sided test at α significance level). The hypothesis is rejected if $|t| \quad t_{m;q}$ [10, 11].

EXPERIMENT

A radioactive source was placed in the laboratory (^{137}Cs). After that, 21 pulse measurements with the GM counter were performed at intervals of 5 minutes. The procedure was then repeated, with the exception that, in the laboratory, another source of radiation was added (^{241}Am) [12, 13].

RESULTS AND DISCUSSION

At first, two measurements of empirical variances s_1= 64,4 and s_2 = 57,8 of two normally distributed populations were obtained. Since $s_1 > s_2$ then $s_1^2 \quad s_x^2, s_2^2 \quad s_y^2, n_1 \quad n_x$, $n_1 = n_x$ and $n_2 = n_y$. On the basis of expression (3), the test statistic t = 1.24 is obtained. The hypothesis has been tested at a significance level α = 0.05 (q = 0.975) with $m_1 = m_2 = 20$. From tab. 1, a critical value $F_{20;20;0975}$ was obtained. Having that 1.24 < 2.46, the hypothesis that these two samples were obtained from populations with the same variance cannot be rejected.

As already mentioned, there was no rejection of the hypothesis on the basis of the F-test that was performed. The hypothesis of equality of the mean values was then validated by the double t-test. On the basis of expression (4) and the fact that the mean values are 2980 and 3032, and that the variance can be considered the same, a test statistic that equals 1.12 was obtained, while critical value $t_{40;0.975}$ = 2.02 is taken from tab. 2.

Considering that 1.12 < 2.02, the hypothesis cannot be rejected. The samples are therefore obtained from the population distributed in the same manner.

Further, the measurements of two samples with two radiation source included were taken. Through measurements of two samples, variances s_1 = 130.37 and s_2 = 68.9 were obtained with the mean values being 3120 and 2310. Since $s_1 > s_2$ then $s_1^2 \quad s_x^2, s_2^2 \quad s_y^2, n_1 \quad n_x$, and $n_2 = n_y$. On the basis of expression (3), a test statistic t = 3.58 is obtained. If the hypothesis is tested at a significance level of with $m_1 = m_2 = 20$, from tab. 1 the critical value $F_{20;20;0975}$ = 2.46 is obtained. Since, 3.58 > 2.46, the hypothesis that these two samples were obtained from the same population can be rejected.

CONCLUSION

The proposed method of applying the F-test and double t-test provides a simple experimental procedure and an even simpler method of mathematical statistics as a basis for determining the number and type (under certain conditions) of radioactive isotopes that have contaminated a facility or an area.

The proposed method could, in addition to these applications, find use in other, similar areas of dosimetry and radiation protection.

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AUTHOR CONTRIBUTIONS

Theoretical analysis carried out by Dj. R. Lazarević and M. D. Obrenović. Simulations and calculations were carried out by Dj. R. Lazarević and I. S. Fetahović. All authors analyzed and discussed the results. The manuscript was written by Dj. R. Lazarević and M. D. Obrenović, and the figures and tables were prepared by Dj. R. Lazarević.

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ПОРЕЂЕЊЕ ДОБИЈЕНЕ ЕМПИРИЈСКЕ ВАРИЈАНСЕ И СРЕДЊИХ ВРЕДНОСТИ НОРМАЛНО РАСПОДЕЉЕНИХ ПОПУЛАЦИЈА НУКЛЕАРНОГ ОДБРОЈА

У раду се разматра могућност примене F-теста и t-дуплог теста у проблемима везаним за идентификацију броја радиоактивних изотопа, који се налазе у контаминираним подручјима, када се за детекцију зрачења користе бројачи. Описи F-теста и t-дуплог теста су дати заједно са одговарајућим табеларним вредностима које омогућавају њихово спровођење. Експеримент је спроведен са два радиоактивна извора. Резултати експеримента су третирани на начин предложен у самом раду при чему су добијени задовољавајући резултати.

Кључне речи: нуклеарни одброј, F-тест, t-дупли тест, варијанса