Model-based Aeroservoelastic Design and Load Alleviation of Large Wind Turbines

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This paper presents an aeroservoelastic modeling approach for dynamic load alleviation in large wind turbines with trailing-edge aerodynamic surfaces. The tower, potentially on a moving base, and the rotating blades are modeled using geometrically non-linear composite beams, which are linearized around reference conditions with arbitrarily-large structural displacements. Time-domain aerodynamics are given by a linearized 3-D unsteady vortex-lattice method and the resulting dynamic aeroelastic model is written in a state-space formulation suitable for model reductions and control synthesis. A linear model of a single blade is used to design a Linear-Quadratic-Gaussian regulator on its root-bending moments, which is finally shown to provide load reductions of about 20% in closed-loop on the full wind turbine non-linear aeroelastic model.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$M^{SS}$</td>
<td>Discrete mass matrix</td>
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<tr>
<td>$C^{SS}$</td>
<td>Gyroscopic damping matrix</td>
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<tr>
<td>$K^{SS}$</td>
<td>Gyroscopic stiffness matrix</td>
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<tr>
<td>$K^{stif}$</td>
<td>Stiffness matrix</td>
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<tr>
<td>$Q_{ext}$</td>
<td>External forces</td>
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<tr>
<td>$M^{SR}$</td>
<td>Coupling mass matrix</td>
</tr>
<tr>
<td>$C^{syr}$</td>
<td>Coupling damping matrix</td>
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<tr>
<td>$\eta$</td>
<td>Nodal dispacements and rotations</td>
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<td>$\nu$</td>
<td>Rigid body velocities</td>
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<td>$\lambda$</td>
<td>Lagrange multipliers</td>
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<tr>
<td>$A_c$</td>
<td>Aerodynamic influence coefficient matrix</td>
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<td>$\Gamma$</td>
<td>Vortex circulation strength</td>
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<tr>
<td>$\Phi$</td>
<td>Constraint function</td>
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<tr>
<td>$\phi$</td>
<td>Nonholonomic constraint</td>
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<tr>
<td>$x$</td>
<td>State variable</td>
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<tr>
<td>$w$</td>
<td>Aerodynamic downwash</td>
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<tr>
<td>$\beta$</td>
<td>Flap deflection angle</td>
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<td>$\delta$</td>
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<td>$\Omega$</td>
<td>Rotational speed</td>
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<td>$\theta$</td>
<td>Azimuth angle</td>
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<tr>
<td>$n$</td>
<td>Time step</td>
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<tr>
<td>$t$</td>
<td>Time</td>
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Sub-, Superscripts

(•)_0  Equilibrium conditions

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I. Introduction

The size of wind turbines has doubled each decade for the past 30 years, both in terms of tower height and rotor diameter. The largest wind turbines in operation have rotors measuring above 120m in diameter, and rotors of up to 160m are already being developed.\(^1\) Aeroelastic effects previously not seen in smaller rotors are beginning to surface as a result of increased flexibility from the longer blades.\(^2\) This brought about new needs in terms of modeling requirements, as blade flexibility may have a significant effect in the performance characteristics of turbines. It also poses new technological challenges, in particular, with respect to an increased need for methods of load control to avoid unnecessary loadings and oscillations of the flexible blades that would impact their fatigue life.\(^3\)

A number of design codes have been developed in recent years for aeroelastic analysis of wind turbines, including NREL’s open source FAST, DTU’s FLEX5, DTU Wind’s HAWC2, TU Delft’s DU_SWAMP, and GL Garrad Hassan’s Bladed, among others.\(^4\)\textsuperscript{–}\textsuperscript{11}\) Their structural models are based on either direct finite-element solutions or assumed modes approaches, while their aerodynamics relies on blade-element momentum theory with various, mostly empirical, corrections to account for dynamic inflow, stall and tip loss effects.\(^2\)\textsuperscript{,}\textsuperscript{12}\) For larger and typically offshore turbines, there will be larger aeroelastic couplings, tower motions, and structural deformations on the more flexible blades, and consequently a potential for increased 3-D aerodynamic effects. Hence, an alternative description based on a time-domain Unsteady Vortex-Lattice Method (UVLM)\(^13\) formulation could provide a better characterization of the aeroelastic responses under attached flow conditions. This will be explored in this paper. As it will also be shown, such an approach can be directly formulated in a suitable manner for model reductions, control synthesis and design.

Existing turbines are fitted with pitch control systems for speed regulation. Their actuation is however relatively slow\(^14\) and their use as load alleviation system is therefore limited to low-frequency excitations. Distributed load alleviation systems, placed along the blade span, can overcome such limitations and these concepts, which derive from well-known solutions in air vehicles, are gaining momentum in wind turbine research. They can be designed to complement existing pitch control mechanisms by tackling higher frequency loadings, or even be used to exploit aeroelastic twisting of the rotor blades. Using trailing-edge flaps for load reduction, Frederick \textit{et al.},\(^15\) Riziotis \textit{et al.},\(^16\) and Basualdo\(^17\) were able to achieve significant reduction in loadings on simple 2-D aeroelastic models, while Barlas \textit{et al.}\(^7\)\textsuperscript{,}\textsuperscript{18} and Wilson \textit{et al.}\(^19\) have demonstrated the performance of multiple flaps on a full rotor. Concepts for conformable trailing-edge flaps\(^20\)\textsuperscript{–}\textsuperscript{23} and microtabs\(^24\) have also shown great potential. Comparisons between various active control concepts have been recently investigated by various authors.\(^1\)\textsuperscript{,}\textsuperscript{25}\textsuperscript{–}\textsuperscript{27}\) To date, there is no commercial application of the active flap concept on existing wind turbines, but a recent full-scale experiment was conducted by Castaignet \textit{et al.}\(^28\) on a Vestas V27 wind turbine using frequency-weighted model predictive control (MPC), and demonstrated the
potential of trailing-edge flaps by reducing average flapwise blade root load by 14%. Also, Sandia National Laboratories have designed and built turbine blades equipped with aerodynamic flaps for testing.

Barlas et al. have reviewed the most recent developments in the use of active flaps on wind turbines for load reduction, and have reported the performance predicted through the use of different control methods. A wide spectrum of numerical results was found: various authors have reported reductions in root-bending moments (RBM) ranging from 10% up to 30%, depending on the type of controller used and also the size and distribution of flaps. To the best of our knowledge, the only work that involved the aeroelastic modeling of wind turbine blades using vortex methods was by Riziotis et al., who used a vortex panel code coupled with a structural module. Using PID control and trailing-edge flaps with lengths ranging between 15% and 50% span, the authors demonstrated a load reduction of up to 30% under an exponential wind shear profile. In fact, most of the works in active aeroelastic control of wind turbines has relied on classical control methods, such as PD and PID, with only a few of the more recent works considering modern control methods, such as LQR or predictive control. This is probably because the intention has normally been to show the potential of feedback control in enhancing the aeroelastic performance of the blades, but in a recent study, we have shown that PD may need up to 70% more actuation power than a robust controller under similar load reduction targets.

Within this context, this paper will extend the previous study of modeling and load alleviation on a single rotating blade to the full wind turbine. It will introduce a model-based control design by first implementing the vortex-lattice method in an efficient state-space representation. This is subsequently coupled to a linearized structural dynamics description of the turbine rotor to produce a compact form suitable for aeroservoelastic analysis. The approach will then be used to model the NREL 5-MW reference offshore wind turbine rotor, which is attached to the tower through Lagrange multipliers for prescribed rotations. We will then use the model to demonstrate the use of flaps in reducing the root-mean-square (rms) values of RBM, tip deflections and fatigue using LQG controllers.

II. Methodology

The aeroservoelastic modeling of wind turbines will be based upon the integrated framework for the Simulation of High Aspect Ratio Planes (SHARP), which has been developed in previous works for flexible aircraft applications, including static aeroelastic analyses, linear stability analyses, control synthesis and non-linear open-loop time-marching simulations. The following description provides an overview of the underlying structural and aerodynamic models, which have been tailored in this work for application to large wind turbines.

A. Composite beam model

Taking advantage of the slenderness of the tower and blades, their structural deformations will be modeled using a composite beam formulation written in a rotating frame of reference. In its original implementation, the structural model can account for large static and transient deformations of the blades, which has been reduced to a 1-D representation in the 3-D space, using an appropriate cross-sectional analysis methodology. For the purpose of efficient control synthesis, this work will focus on the linearized approach that provides a compact form of the equations of motion (EoM) around a possibly geometrically non-linear steady-state equilibrium.

As shown in Figure 1, the deformation of the structure is described in terms of a hub-fixed (structural) reference coordinate system, S, which moves according to the hub translational and angular velocities, $v_S(t)$ and $\omega_S(t)$, respectively. The global motions of the rotor blades are given by the hub rigid-body velocities $\nu^T = [v^T_S \omega^T_S]$, with respect to an inertia frame G. To account for large blade deformations, the local orientation of each cross section along the beam reference line is defined by a local coordinate system B and is parametrized at time $t$ by the Cartesian Rotation Vector $\Psi(s, t)$, where $s$ is the arc-length along the beam reference line. Hence, the nodal positions $R_S(s, t)$, expressed in the hub-fixed frame S, and the cross-sectional orientations, $\Psi(s, t)$, form the independent set of variables for the structural problem.

After a finite-element discretization, the equations of motion for the structural subsystem are then obtained as:

$$\mathcal{M}^{SS} (\eta) \ddot{\eta} + \mathcal{M}^{SR} (\eta) \dot{\nu} + Q_{gyr} (\eta, \dot{\eta}, \nu) + Q_{stiff} (\eta) = Q_{ext},$$

where the discrete mass matrix $\mathcal{M}^{SS}$ and the discrete gyroscopic, elastic and external generalized forces,
\( Q_{gyr}, Q_{stif} \) and \( Q_{ext} \), respectively, are obtained through a finite element discretization of the primary variables, with \( \eta \) the column matrix with all the nodal displacements and rotations. Equation (1) captures the possibly large deformations of the blades by balancing the discrete inertial and elastic forces with the external aerodynamic forces. The inertial couplings due to \( \nu \) and \( \dot{\nu} \), which are functions of time, are incorporated through the coupling mass matrix \( M^{SR} \) and the gyroscopic forces \( Q_{gyr} \). The superscripts \( S \) and \( R \), denoting structural and rigid-body contributions, respectively, highlight the coupling between the blade structural dynamics and the overall (rigid-body) motion of the rotor.

To arrive at a compact state-space form of the aeroelastic EoM suitable for control synthesis, the structural dynamics equations are linearized around a geometrically non-linear steady-state equilibrium condition \( (\eta_0, \dot{\eta}_0 = 0, \nu_0) \). This steady-state solution is obtained by neglecting all time derivatives in Equation (1).

The incremental form of the beam EoM is:

\[
\mathcal{M}^{SS} (\eta_0) \Delta \dot{\eta} + \mathcal{M}^{SR} (\eta_0) \Delta \dot{\nu} + C^{SS}_{gyr}(\eta_0, \nu_0) \Delta \dot{\eta} + C^{SR}_{gyr}(\eta_0, \nu_0) \Delta \nu + \left[ K^{SS}_{gyr}(\eta_0, \nu_0) + K^{SS}_{stif}(\eta_0) \right] \Delta \eta = \Delta Q_{ext},
\]

where the contribution to the (constant) mass, damping and stiffness matrices has been obtained through direct linearization of the different generalized forces with respect to \( \eta \) and its time derivatives. The effects of tower oscillation is accounted for in the coupling mass and damping matrices, \( \mathcal{M}^{SR} \) and \( C^{SR}_{gyr} \), respectively. A detailed derivation of Equation (2) can be found in Hesse and Palacios. A linearized beam equations also preserve the rotational effect due to the prescribed angular velocity \( \Omega \) through the contribution of the linearized gyroscopic forces to the damping and stiffness matrices \( C^{SS}_{gyr} \) and \( K^{SS}_{gyr} \), respectively.

The resulting linear system can be cast into state-space form from which we can define an eigenvalue problem that includes rotational effects and geometrically non-linear equilibrium deformations. At last, the state-space system is discretized in time using a standard Newmark-\( \beta \) discretization method for the integration of the structural dynamics equations with the discrete-time unsteady aerodynamic formulation.

**B. Flexible multi-body dynamics**

The inclusion of tower dynamics in the modeling of large wind turbines constitutes a multi-body dynamics problem. Similar to the blades, the tower is modeled using beam elements and its structural dynamics are described with respect to a global frame \( A \) at the tower base. The motion of this frame is further described
with respect to the inertia frame G and parametrized through quaternions, to account for any motion of the tower found in floating structures. The tower and rotor are connected using Lagrange multipliers, which constrain the nodal velocities of the local frame B\textsubscript{t} at the tower top to the translational and angular velocities of frame S fixed to the rotor hub, as shown in Figure 2. Next, the extension of the beam equations to multiple bodies is given in the context of large wind turbines assuming the problem is posed as an open kinematic chain.

The constraints between N connected flexible bodies are conveniently enforced in the weak form of the equations of motions using Lagrange multipliers. Starting from Hamilton’s principle, we get:

\[
\int_{t_1}^{t_2} \left\{ \sum_{i=1}^{N} \left[ \int L_i \delta T_i - \delta U_i + \delta W_i \right] ds \right\} + \sum_{i=1}^{N} \sum_{j=1}^{N} \left[ \delta \Phi_{ij} \lambda_{ij} + \delta \lambda_{ij} \Phi_{ij} \right] dt = 0, \tag{3}
\]

where \( T_i \) and \( U_i \) are the kinetic and internal energy densities, respectively, and \( \delta W_i \) is the virtual work per unit length of body \( i \). These virtual quantities have been defined in Refs.\textsuperscript{33,34} in terms of the infinitesimal beam kinematics, i.e. the local beam strains and velocities. Each constraint between two connected bodies (tower and rotor) is enforced using six Lagrange multipliers \( \lambda \), for three translational and three rotational constraints, while \( \Phi \) is the corresponding constraint function. Due to the parameterization of large rotations in the present geometrically non-linear description of flexible bodies, it is convenient to enforce the constraints in terms of velocities, which leads to nonholonomic (bilateral) constraints, written in general as:\textsuperscript{38}

\[
\Phi_{ij} \begin{pmatrix} \dot{\eta}_i, \dot{\eta}_i, \nu_i, \nu_j \end{pmatrix} = \Phi_{ij}^{nh} \begin{pmatrix} \dot{\eta}_i \\ \nu_i \\ \nu_j \end{pmatrix} + \phi_{ij} = 0, \tag{4}
\]

where \( \dot{\eta}_i \) are the nodal displacements and rotations at the end of body \( i \). In the context of the wind turbine model, \( i \) and \( j \) denote the tower and rotor degrees of freedom, respectively, and \( \dot{\eta}_i \) corresponds to the displacements and rotations of tower top frame \( B_t \), as defined in Figure 2. From the kinematic description of
the beam displacements and velocities in Hesse and Palacios,33 we can write the nonholonomic constraints as:

\[
\Phi_{ij}^{nh} = \begin{bmatrix} \Lambda(\dot{\eta}_i) & A_{RC}(\dot{\eta}_i) & -A_{CC}(\dot{\eta}_i, \nu_{0,j}) \end{bmatrix}
\]

and

\[
\Phi_{ij} = A_{CC}(\dot{\eta}_i, \nu_{0,j}) \nu_{0,j}(t),
\]

where \(\Lambda\), \(A_{RC}\), and \(A_{CC}\) are transformation matrices to enforce the velocity constraints in the local frame at the end of body \(i\). At last, \(\nu_{0,j}(t)\) accounts for possible (time-varying) prescribed velocities of body \(j\), which will be used in this paper to introduce the (constant) angular velocity of the rotor. The resulting change in orientation between the rotor frame \(S\) and the tower top frame \(B_t\) is accounted for in matrix \(A_{CC}\) which transforms the velocities of the rotor hub to the tower top frame \(B_t\).

As for a single body, a finite element discretization of the primary variables, the displacements and rotations of all blades, finally leads to the discrete form of the augmented system equations, given as:

\[
\mathcal{M}(\eta) \begin{bmatrix} \dot{\eta} \\ \nu \end{bmatrix} + Q_{gyr}(\eta, \dot{\eta}, \nu) + Q_{stif}(\eta) = Q_{ext} - \Phi_{nh}^\top \lambda - J \lambda
\]

\[
\Phi_{nh} \begin{bmatrix} \dot{\eta} \\ \nu \end{bmatrix} + \phi = 0
\]

where \(\eta\) is the resulting column matrix with all the nodal displacements and rotations of the \(N\) bodies and \(\nu = [\nu_1^\top, \cdots, \nu_N^\top]^\top\). Hence, Equation (6) extends the original flexible-body dynamics equations with the additional constraints, where \(\Phi_{nh}\) represents the nonholonomic quantity of all constraints and \(\phi\) is the vector of all prescribed velocities, both defined in Equation (5) for a constraint between two bodies \(i\) and \(j\). Finally, \(J\) is the tangent matrix of all algebraic constraints defined between two bodies as \(J_{ij} = \partial \Phi_{ij}/\partial \dot{\eta}_i\). Note that the discrete mass matrix, \(\mathcal{M}\), and the discrete generalized forces, \(Q_{gyr}, Q_{stif}\) and \(Q_{ext}\), respectively, now include the structural and rigid-body contributions of all bodies.

The augmented equations for the coupled wind turbine problem, Equation (6), can also be linearized around the steady-state equilibrium with constant rotor velocity leading to a compact form of the equations coupling the tower and rotor dynamics, written as:

\[
\begin{bmatrix}
\mathcal{M}_t^{SS} & 0 & 0 & -\Lambda^T \\
0 & \mathcal{M}_r^{SS} & \mathcal{M}_r^{SR} & 0 \\
0 & \mathcal{M}_r^{RS} & \mathcal{M}_r^{RR} & A_{CC}^T(\theta) \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} \Delta \dot{\eta}_t \\ \Delta \dot{\eta}_r \\ \Delta \nu_r \\ \Delta \lambda \end{bmatrix}
+ \begin{bmatrix}
C_t^{SS} & 0 & 0 & 0 \\
0 & C_r^{SS} & C_r^{SR} & 0 \\
0 & C_r^{RS} & C_r^{RR} & 0 \\
\Lambda & 0 & -A_{CC}(\theta) & 0
\end{bmatrix}
\begin{bmatrix} \Delta \eta_t \\ \Delta \eta_r \\ \Delta \nu_r \\ \Delta \lambda \end{bmatrix}
+ \begin{bmatrix}
K_t^{SS} & 0 & 0 & 0 \\
0 & K_r^{SS} & 0 & 0 \\
0 & K_r^{RS} & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} \Delta \eta_t \\ \Delta \eta_r \\ \Delta \nu_r \\ \Delta \lambda \end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \\ \int Q_{ext} \end{bmatrix}
\]

with the subscripts \(t\) and \(r\) corresponding to the tower and rotor, respectively. The prescribed rotor velocity and azimuth angles of the rotor blades at steady-state, \(\Omega\) and \(\theta = \Omega t\), respectively, leads to gyroscopic contributions to the damping and stiffness matrices with non-negligible effects on the vibration frequencies of the coupled system.32 Linearization around steady-state greatly simplifies the incremental form of the constraint function included in the last row of Equation (7). Note however that the transformation matrix \(A_{CC}(\theta)\), which projects the incremental hub velocities to the local frame \(B_t\) at the tower top, becomes a periodic function due to the prescribed rotor velocity, \(\Omega\).

C. Unsteady aerodynamics model

The aerodynamics are modeled using the discrete-time UVLM36,41 with a prescribed helicoidal wake, which allows non-stationary aerodynamics to be captured in low-speed, high-Reynolds-number attached-flow conditions for arbitrary blade kinematics. The UVLM uses vortex rings as fundamental solutions, which are located in lattices that represent the blades (modeled as a lifting surface) and their wakes (modeled as thin shear layers). The leading segment of the vortex ring is placed along the quarter chord of each panel. The
collocation points are then placed at the three-quarter chord of each panel where boundary conditions are imposed. In the current implementation, the aerodynamics of the tower is neglected.

The wake vortex rings are convected downstream by both the freestream velocity and also the velocity induced by all other vortex rings. The influence of the latter is commonly known as wake roll-up and introduces non-linearities in the model. To obtain linear models for control methods, we could freeze the model with wake roll-up and linearize around that point. Alternatively, the effect of wake roll-up can be ignored. In this paper, the latter method was adopted such that the wake was assumed to be helicoidal.

In the UVLM, Neumann boundary conditions are imposed on the lifting surface. Hence, the normal velocity at each collocation point due to vortex rings (blade and wake) and motion of the blade has to be zero. This relationship is given by:

$$A_{c,b} \Gamma_b^{n+1} + A_{c,w} \Gamma_w^{n+1} + A_{c,x} \Gamma_x^{n+1} + w^{n+1} = 0,$$

where $\Gamma_b$ and $\Gamma_w$ denotes the circulation strengths of the bound and wake vortex rings respectively (shown in Figure 3), and $\Gamma_x$ accounts for circulations (both bound and wake) due to multiple blades. $A_{c,b}$, $A_{c,w}$ and $A_{c,x}$ are the influence coefficients that gives the induced normal velocity to blade surface at collocation points (resolved using the Biot-Savart law) due to bound and wake vortex rings, and $n$ is the time step. The last term $w$ is the downwash at collocation points and is generated by the motion of the lifting surface $w_b$ (the blade), rigid body motions $w_\nu$ (such as the deflection of the tower), the actuators $w_\beta$ (such as trailing-edge flaps) and external disturbances $w_\delta$ (such as gusts). The downwash are all in the same time step and the first two contributions $w_b$ and $w_\nu$ will disappear in the process of coupling with the structural dynamics. The last two contributions $w_\beta$ and $w_\delta$ will appear as inputs to the system and in order for the final equations of motion to be in explicit form, they are written in time step $n$ instead of $n + 1$, assuming effects on the solution will be minimal if time steps are kept small:

$$w^{n+1} = w_b^{n+1} + w_\nu^{n+1} + w_\beta^{n} + w_\delta^{n}.$$  

In particular, the downwash due to motion of the lifting surface $w_b$ is mapped from the structural beam model of the rotor blades and can be represented using:

$$w_b^{n+1} = [T_{\Delta\eta} \ T_{\Delta\dot{\eta}}] \begin{bmatrix} \Delta\eta^{n+1} \\ \Delta\dot{\eta} \end{bmatrix}.$$  

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where $T_{\Delta \eta}$ and $T_{\Delta \dot{\eta}}$ project the contribution of the nodal displacements $\Delta \eta$ and velocities $\Delta \dot{\eta}$ onto the aerodynamic lifting surface as downwash terms.

The downwash due to rigid body motion of the rotor is expressed as:

$$ w_r^{n+1} = T_{\Delta \nu}(\theta) \Delta \nu_r^{n+1}, $$

where in the context of the wind turbine, $T_{\Delta \nu}$ is varying with the azimuth location $\theta$ of the rotor blades with respect to the tower.

Pressure distribution across each panel on the lifting surface can be computed using the unsteady Bernoulli equation,\textsuperscript{41} from which the aerodynamic forces in each panel are obtained. This can be expressed in compact form as:

$$ \Delta P_k^{n+1} = \Phi_k^{n+1} \Gamma_b^{n+1} + \Phi_k^n \Gamma_b^n, $$

where $\Gamma_b^{n+1}$ contains circulation strengths of all vortex rings on the lifting surface.

Since the UVLM is based on thin wing theory, special care is needed to obtain the leading-edge suction.\textsuperscript{43} In steady flow, pressure forces in the direction of flow are canceled by leading-edge suction, arising in zero drag, which is commonly known as the d’Alembert’s paradox. In the case of unsteady flow, drag may be present and an approximation given by Katz et al.\textsuperscript{41} is used, expressed compactly as:

$$ D_k^{n+1} = \Upsilon_k^{n+1} \Gamma_b^{n+1} + \Upsilon_k^n \Gamma_b^n, $$

D. Aeroelastic equations

The continuous-time structural equations of motion of the wind turbine are discretized using the Newmark-$\beta$ method and the blades are subsequently coupled with the discrete-time UVLM. As each lifting surface is comprised of panels while the structure is modeled using beams that run along the span of the blades, aerodynamic loads are mapped onto the beam nodes as shown in Figure 4.

In the coupled model description, the structural beams representing the rotor blades are linearized around a prescribed rotational velocity, and non-linearity enters the system through the constraint equation (4), which is azimuth dependent in the transformation matrix. The UVLM is also linear in the rotor description but the coupling term in Equation 11 injects an azimuth dependent downwash due to tower deflection onto the panel collocation points. The aeroelastic formulation accommodates for rotor anisotropy such as imbalance and also external anisotropy pertaining to operating conditions such as wind shear, rotor imbalance, yaw error and gravity.\textsuperscript{44}

With each bound and wake circulation in the UVLM model representing a state, the coupled equations of motion are huge, causing simulation to be computationally expensive. Furthermore, the rotation of the rotor imposes non-linearity in both the constrained structural equations of motion and the coupling terms with aerodynamics, rendering model reduction on the final aeroelastic system infeasible. Hence, the structural and UVLM models are reduced separately prior to coupling. Firstly, the structural degrees of freedom are truncated using classical modal decomposition\textsuperscript{40} on individual beams. Next, the UVLM model is reduced
through balanced truncation which Hesse and Palacios\textsuperscript{45} have demonstrated its effectiveness in reducing large flexible aircraft models.

The resulting equations of motion between the constrained structural model and UVLM provide the full aeroelastic system in state-space representation.\textsuperscript{35} The equations can be written in the standard form:

\[
\begin{align*}
\mathbf{x}^{n+1} &= \mathbf{A}(\theta^n)\mathbf{x}^n + \mathbf{B}\mathbf{w}_\beta^n + \mathbf{G}\mathbf{w}_\delta^n, \\
\mathbf{y}^n &= \mathbf{C}\mathbf{x}^n + \mathbf{D}\mathbf{w}_\beta^n + \mathbf{H}\mathbf{w}_\delta^n,
\end{align*}
\]

where the state matrix \( \mathbf{A} \) is a function of the azimuth angle \( \theta \), which defines the orientations of the rotor blades with respect to the tower at each time step \( n \). The state vector that completely defines the aeroelastic system is:

\[
\mathbf{x}^T = \begin{bmatrix}
\Delta \Gamma_b^T & \Delta \Gamma_w^T & \Delta \eta_i^T & \Delta \dot{\eta}_i^T & \Delta \eta_r^T & \Delta \dot{\eta}_r^T & \Delta \nu^T & \Delta \lambda^T
\end{bmatrix},
\]

containing the aerodynamics, structures and rigid body states. The inputs to Equation (14) are from the downwash terms in Equation (9). Note that the downwash due to motion of lifting surface \( \mathbf{w}_b \) and rigid body motion \( \mathbf{w}_v \) does not appear, since it is coupled to the structural dynamics. The control input \( \mathbf{w}_\beta \) represents the downwash due to flap motion and the external disturbance \( \mathbf{w}_\delta \) is the downwash due to gust. The output vector, \( \mathbf{y} \), includes the desired output (e.g., blade root-bending moments, tip deflection, tower top deflection).

### III. Closed-loop model

The control input for Equation (14) contains the flap deflection angle \( \beta(t) \) and its time derivative. Hence, a double integrator\textsuperscript{46} is introduced such that the control input is now the flap acceleration, \( \ddot{\beta}(t) \). The inflow speed to the rotor is constant and disturbance enters the system in the form of turbulence in the longitudinal direction that is assumed to be homogeneous in the rotor disk. It is simulated by passing white Gaussian noise through a filter such that the signal output will have a von Kármán turbulence spectrum that is not affected by the mean flow field of the rotor. A simplified closed-loop block diagram of the aeroelastic model is illustrated in Figure 5. In the same figure, \( \mathbf{w}_\delta \) represents the external disturbance (Gaussian) that is passed through the von Kármán filter to produce the turbulence signal \( \delta \), and \( \nu \) accounts for any external measurement noise.

Due to the characteristics of the disturbance (Gaussian), LQG controls will be considered in this paper. For a Gaussian disturbance, a LQG controller minimizes the expected value of the quadratic cost function:\textsuperscript{47,48}

\[
J = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} \left( (\mathbf{x}^n)^T Q \mathbf{x}^n + (\ddot{\beta}^n)^T R \ddot{\beta}^n \right),
\]

where \( Q \) and \( R \) are appropriately chosen weighting matrices.

Performance will be measured in terms of the percentage reduction in rms values of blade flapwise RBM as well as tip deflections, while keeping maximum flap deflection angles and rates within the prescribed limits. Fatigue will also be measured using the percentage reduction in Damage Equivalent Load (DEL).\textsuperscript{49,50}

### IV. Numerical results

The aeroelastic code SHARP has been extensively verified in the context of flexible aircraft aeroelasticity and flight dynamics.\textsuperscript{36,32} Ng et al.\textsuperscript{32} extended it to problems with a single rotating blade, in which gyroscopic effects have been compared to analytical solutions\textsuperscript{51} and flutter speeds have been investigated. Additional studies are presented here for the full wind turbine including coupling of rotor and tower.

#### A. NREL 5-MW reference wind turbine

Using the structural formulation described above, the NREL 5-MW reference wind turbine is modeled. The tower and blades with cross sectional properties described in Jonkman et al.\textsuperscript{52} are modeled as beams connected through Lagrange multipliers. The nacelle and hub are modeled as point masses on the tower top at prescribed offset locations. As for the drivetrain, it is modeled as a beam with a single element connecting
Figure 5: Closed-loop block diagram.

Figure 6: The NREL 5-MW offshore wind turbine.\textsuperscript{52} The gearbox and generator are merged with the nacelle as a single point mass in the modeling, but are shown here for completeness.

the tower to the rotor hub. The rotor and shaft are pitched at 5° as documented. A graphical representation of the NREL 5-MW wind turbine is shown in Figure 6, describing the locations and dimensions of various components based on our interpretation of the written turbine description in Jonkman \textit{et al.}\textsuperscript{52}

For the aerodynamics model, vortex panels are placed on the outer 80\% span of the blade and a helicoidal
Figure 7: Aerodynamic model of NREL 5-MW reference wind turbine rotor with prescribed helicoidal wake. (For the purpose of clarity in visualization of wake profile, 4 chordwise panels (instead of 10) are used to generate the figure.)

Wake profile is prescribed to enable a linear UVLM representation, as shown in Figure 7. The inboard segment of the blade with cylindrical cross-sections is not modeled here but can be included as additional drag forces. The aerodynamic forces as a result of the pre-twist in the NREL 5-MW reference wind turbine can be modeled as additional downwash on the lifting surface.

In all the cases that follows, unless otherwise stated, the model of the NREL 5-MW wind turbine rotor and tower is used. The operating conditions are under rated inflow of 11 m/s, time step of 0.007 seconds, Tip Speed Ratio (TSR) of around 7 and turbulence intensity of 10%.

B. Numerical implementation and validation

The implementation of the aeroelastic code to model a single NREL 5-MW wind turbine blade has been performed in a previous study. Progressing from the single rotating blade to the full rotor, we will first investigate the effects of wake interactions due to the presence of multiple surfaces on the aerodynamic loads. Next, the Lagrange multipliers are verified to ensure that the constraints between the tower and rotor are implemented correctly. This will be done by comparing the response of the turbine to static loads as well as matching the natural frequencies of the full wind turbine in parked conditions with Jonkman et al. Finally, we will study the effects of tower flexibility on the frequency response of the system and the effects of rotation on individual blades.

Firstly, in modeling the rotor, three similar NREL 5-MW blades are placed at 120° azimuth from each other with cross influence of circulation included in the UVLM model. The effect of the multiple lifting surfaces is to generate additional backflow from the rotating wake, potentially reducing loads on the blades. Figure 4 shows the flapwise aerodynamic load on one of the blades in the full rotor subject to an impulsively started gust and the vertical dotted lines indicate each time the blade have rotated 120°. It is evident that in the transient stage, each time the blade passes through the wake shed by another blade or by itself, the aerodynamic loads are reduced. Also plotted on the same figure is a single rotating blade for comparison, showing a slight reduce in load when it passes by its own shed wake after a full rotation. In steady state, the loads in the full rotor configuration are about 15% lower than in a single rotating blade.

Next, the Lagrange multipliers constraining the tower and rotor in the augmented structural equations of motion were validated, where the kinematic results for the constrained problem compared well to a multi-
Figure 8: Normalized flapwise aerodynamic loads for a single rotating blade and for the full rotor, subjected to an impulsively started gust. (Vertical dotted lines indicate each time a blade have rotated 120°)

Table 1: Full system natural frequencies (Hz) of the NREL 5-MW wind turbine (stationary) for structural discretization of (a) 10 tower elements and 48 elements on each blade (same as documentation\textsuperscript{52}) and (b) 10 tower elements and 12 elements on each blade (interpolated blade properties).

<table>
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<tr>
<th>Mode</th>
<th>Description</th>
<th>FAST\textsuperscript{52}</th>
<th>ADAMS\textsuperscript{52}</th>
<th>SHARP (10t, 48b)</th>
<th>SHARP (10t, 12b)</th>
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<td>0.315</td>
<td>0.316</td>
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<tr>
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<td>0.662</td>
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<td>0.670</td>
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<td>3.040</td>
</tr>
</tbody>
</table>

beam configuration of the turbine with similar properties. The natural frequencies of the full wind turbine with rotor and tower connected through the Lagrange multipliers in parked configuration are then computed using the same discretization as in Jonkman \textit{et al.}\textsuperscript{52} of 10 tower elements and 48 elements for each of the blades. They are listed in the second last column of Table 1 and match very closely to those using FAST and ADAMS\textsuperscript{52}. The corresponding mode shapes are shown in Figure 9, showing low frequency modes being dominated by tower motion as well as blade out-of-plane flapping.

Next, we study the effects of tower flexibility on the overall structural response of the wind turbine. Figure 10 shows the Bode diagram from gust input to blade flapwise tip deflection for a stationary turbine with different degrees of tower flexibility. Comparing the solid (NREL tower) and dashed lines (rigid tower),
Figure 9: Stationary mode shapes of the NREL 5-MW reference wind turbine in parked configuration.
Figure 10: Bode plot of full aeroelastic system from gust input to blade flapwise tip deflection on one of the rotor blades (stationary). Flexible tower has stiffness in the NREL 5-MW tower reduced by factor of three.

Figure 11: Time series showing the flapwise tip deflection of the three rotor blade in response to homogeneous gust, under rated conditions and 10% turbulence intensity.

it is evident that the tower introduces dynamics around 0.3 Hz and 2.9 Hz (the first and second tower fore-aft frequencies) and not affecting much of the blade flapwise frequency of around 0.6 Hz. Increasing the flexibility of the tower further by reducing the stiffness of the NREL tower by factor of three, we observe that tower frequencies are reduced further as expected and the blade flapwise frequency is also shifted to the left. Figure 11 shows flapwise tip deflection of the three blades in rotation when subject to homogeneous gust. Depending on the azimuth location of the blades with respect to the tower during rotation, their tip deflections are altered slightly from one another due to the structural response of the tower.

The implementation of the UVLM relies closely on the discretization of bound and wake panels for convergence, and the number of wake panels to capture unsteadiness in the aerodynamics and wake interaction among the blades. A convergence study revealed that an equal spanwise discretization of 10 panels, equal chordwise discretization of 10 panels and keeping a wake profile of a quarter rotor diameter downstream is sufficient to capture the dynamics. This chordwise discretization is closely linked to the time step as a row of wake panels is shed in the UVLM model at every time instant. This results in a time step of 0.007 seconds using the chord $c_m$ and relative velocity $v_m$ at blade mid-span as reference, with the relationship given by $dt = c_m / (n_c v_m)$, where $n_c$ is the number of chordwise panels.

The finite element discretization of the beam model is the same as the spanwise panel discretization of the UVLM. Including the inner span of the blade with cylindrical cross section in the structural beam model, and maintaining regular discretization, a total of 12 elements for each blade is used with properties listed
in Table 2 of the Appendix - these properties are obtained using a linear interpolation on the documented structural properties in Jonkman et al.\textsuperscript{52} which had 48 elements for the blades. The natural frequencies for the turbine with 12 elements on each blade and 10 tower elements are listed in the last column of Table 1, showing marginal difference from the base model. The resulting coupled aeroelastic system using this discretization contains a total of over 9500 states. However, using the model reduction techniques described in Section II, D, the number of states is brought down to 300, including additional states from the coupling, Newmark-{$\beta$} discretization, turbulence filter and also double integrator.

The reduction of the structural model through modal decomposition is achieved through a parametric study on the full aeroelastic system suggesting that it is sufficient to keep only the lowest 15 tower modes and 40 blades modes. For the aerodynamics model, balanced truncation is implemented. The Hankel Singular Values (HSV)\textsuperscript{48} of the balanced UVLM model with respect to the largest HSV is shown in Figure 12. HSV provides a measure of the energy contribution of each state to the input/output behavior of the system and in the current balanced UVLM model, relative states that are smaller than three orders of magnitude are truncated. The reduced UVLM model is then coupled with the truncated structural equations and simulated using turbulent input signals. The time marching solutions compared well with the full system, providing confidence that the reduced UVLM model retains key dynamical characteristics of the full aeroelastic model.

We have analyzed the solution of the truncated UVLM in the fully coupled system to avoid truncating states that appear insignificant in the aerodynamics model but may still be crucial in the coupling process. A Bode plot from gust input to tip deflection on one of the stationary rotor blades is shown in Figure 13, which is evident that the reduced model of keeping 150 aerodynamic states, 15 tower modes and 40 blade structural modes is able to capture the dominant lower frequency responses. Also plotted on the same figure is the Bode diagrams for keeping only 5 tower and 20 blade modes (while keeping 150 aerodynamic states) which shows a slight shift in the first flapwise mode which otherwise is sufficient to also capture the dominant modes. In another Bode plot shown in Figure 14, we demonstrate the effects of truncating aerodynamic states below 150. With only 20 aerodynamic states, we observe a smoothening effect across most frequencies with missing dynamics. This is due to having gust as input to the Bode plot, which in the model itself is imposed through downwash on all the bound aerodynamic panels. Hence, truncating away more of the aerodynamics states would imply less excitation of modes in the input/output frequency response of the system. Also plotted is the Bode diagram of keeping 50 aerodynamic states which from Figure 12, is to keep states of relative energy just before the plateau between 50 to 150 states. Comparing the 50 and 150 aerodynamic states models, we could see significant differences across most frequencies, although some of the lower frequency modes have been weakly captured.

To speed up computation even further (due to the long turbulence signal), the state matrix in Equation 14 is updated only after every five time steps. This is shown to have no effect on the computed response as we
Figure 13: Bode plot from gust input to blade flapwise tip deflection on one of the rotor blades (stationary). Comparing full model with reduced models of (a) 5 tower and 20 blade structural modes, 150 aero states, (b) 15 tower and 40 blade structural modes, 150 aero states.

Figure 14: Bode plot from gust input to blade flapwise tip deflection on one of the rotor blades (stationary). Comparing effect of reducing aerodynamic states (20, 50 and 150), all with 15 tower and 40 blade structural modes.

are sampling at over 142 Hz which is well above the modes of interest in the model as tabulated in Table 1. Also, with the rotor blades returning to the same azimuth location after each round of rotation, all the state matrices in the first rotation can be stored and extracted for use in subsequent loops. These methods, including model reductions in the structures and aerodynamics as described, helped to save computation time by more than two orders of magnitude with minimal impact in the accuracy of the simulations.

C. Gust load alleviation

The model of the 5-MW NREL reference wind turbine is fitted with one flap on each blade and simulated. Based on a previous study, a flap occupying 20% of the span of the lifting surface and 10% of the local chord is chosen and located at a mean position of 80% span. This location is found to provide the largest
reductions in RBM and tip deflection and agrees with the flap sizing used in Refs.\textsuperscript{7,16} Better performances could be expected with multiple and distributed flaps\textsuperscript{7,19} but since we are assuming a homogeneous gust model, a single flap will be sufficient.

Five statistically significant turbulence signals of 20 length scales each are generated for the simulation. The variance for the LQG controller is taken from the turbulence intensity (standard deviation) and the weights on states $Q$ in Equation (16) are increased relative to $R$ until either the flap deflection angle or rate limits of $|\beta| \leq 10^\circ$ and $|\dot{\beta}| \leq 100^\circ$/s reported by Berg et al.\textsuperscript{53} are encountered.

The controller is synthesized from a clamped single rotor blade described in a rotating frame and written in linear state-space representation.\textsuperscript{32} Also, the von Kármán turbulence filter is included in the system shown in Figure 5 (dotted block) for controller synthesis, thus enabling the controller to have knowledge of the disturbance spectrum. When placed in closed-loop with the full turbine model, it forms a distributed control system in which three independent controllers act on the flaps on the three blades.

Using blade RBM feedback, on average, we are observing around 22% rms reduction in RBM and 23% rms reduction in tip deflection. The limit on the flap deflection angle of $\pm 10^\circ$ is met while the limit on flap deflection rate was less of a concern as it was well below $\pm 100^\circ$/s. The Damage Equivalent Load (DEL) was reduced by 17%. For the fatigue analysis, a S-N slope of 10 is selected, typical for composite materials.\textsuperscript{49,50} A section of the time series for the RBM and tip deflections on the three blades is shown in Figure 15a and Figure 15b respectively, where it is evident that peaks are reduced in the closed-loop system. The flap deflection angle $\beta$ is shown in Figure 15c which are within prescribed limits. On the same figure, the fluctuation of the inflow is also plotted in which we observe a phase lag in the response of the flaps to the inflow turbulence. The tower top out-of-plane deflection is plotted in Figure 15d, showing the additional benefits of flaps in reducing tower loads as well. As mentioned, the control model had with it the structure of the von Kármán turbulence filter. Interestingly, the closed-loop performance deteriorated slightly by 10% when the control model had the filter removed.

The results were similar when a centralized control method was adopted in which the control model was a rotor with three blades rigidly clamped at the hub. This would now include aerodynamics cross-influence due to multiple surfaces but does not have any structural interaction between the blades due to the rigid joint. Hence, the closed-loop performance is similar to the original case, except for a smaller amplitude of loads due to aerodynamic backflow.

In an effort to enhance the performance of the control action, the single rotating blade model used for controller synthesis was also augmented with rigid body motion at its root, which can be representative of tower motions. The controller synthesized from this model was then placed in closed-loop with the full wind turbine model. However, despite efforts in adjusting the relative importance of the gust against rigid body motions in the model, the results were no better than the base case controller synthesized without rigid body motions, indicating gust disturbance dominating the response of the system.

A final study compared the results with a controller synthesized from the full wind turbine model at an instantaneous arbitrary azimuth location. As observed previously in Figure 11, the response of the three blades in rotation is relatively similar, despite the non-linearity imposed through different azimuth location of the rotor blades with respect to the tower. The resulting closed-loop performance was less than 5% different from previous cases, further demonstrating the dominance of gust disturbance and that it is sufficient to use a single rotating blade for controller synthesis.

V. Conclusions

The aerelastic response and gust load alleviation of a large wind turbine using active control surfaces is presented using a model-based aeroservoelastic tool, coupling composite beam models and unsteady aerodynamics in a state-space description. The finite-element solution of the rotor blades are linearized around large geometrically non-linear rotating steady-state equilibriums. The connection between the rotor hub and tower, including prescribed angular velocities are enforced through velocities with the Lagrange multipliers. The aerodynamics modeled through the Unsteady Vortex-Lattice Method allows the response of large flexible blades to be captured with better fidelity than blade-element momentum theory and control surfaces to be modeled directly. While the standard UVLM implementation have a relatively large computation cost, the state-space representation presented here can be coupled to any structural model easily, allows for standard model reduction techniques and provides a computationally efficient route for control synthesis as illustrated. Through modal decomposition of the structural equations of motion and balanced model truncation of the
Figure 15: Section of time series for the full wind turbine configuration under rated inflow conditions and 10% turbulence intensity.
aerodynamics, the size of the resulting coupled aeroelastic model can be reduced to speed up computation time by more than two orders of magnitude. This includes making use of similarity in the cyclic behavior of the wind turbine in which system matrices in one rotation can be stored and reloaded.

Using the aeroelastic formulation presented, the NREL 5-MW offshore reference wind turbine is modeled and validated. Subsequently, the turbine blades are attached with trailing-edge flaps and using LQG controllers with RBM feedback, an average rms reduction of 22% for RBM, 23% for tip deflection and DEL reduction of 17% is observed, with flap deflections angles kept within the limits of ±10°. The current description of the model can be extended to include cyclic loads such as gravity, wind shear, base excitation and also the use of multiple flaps modeled directly in the UVLM model, which will be considered in a further work.

Appendix

Table 2: Interpolated blade structural properties for NREL 5-MW wind turbine.

<table>
<thead>
<tr>
<th>Node</th>
<th>Radius (m)</th>
<th>StrTwst (°)</th>
<th>MassDen (kg/m)</th>
<th>FlpStff (N-m²)</th>
<th>EdgStff (N-m²)</th>
<th>GJStff (N)</th>
<th>EAStff (Kg-m)</th>
<th>FlpIner (m)</th>
<th>EdgIner (m)</th>
<th>EdgcgOf (m)</th>
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<td>1</td>
<td>1.50</td>
<td>13.31</td>
<td>678.94</td>
<td>1.81×10¹⁰</td>
<td>1.81×10¹⁰</td>
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<td>9.73×10⁵</td>
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Acknowledgments

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References


