An Energy Market Modeling Approach for Valuing Real Options

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In this article, we provide a tractable stochastic modeling approach for the valuation of natural gas storage contracts. The model tackles the well-known problem that natural gas futures contracts, similar to swap contracts, provide aggregated price expectations over their delivery periods, which are difficult to incorporate in standard pricing frameworks. We solve this problem by combining a market model with a smooth interpolation function. Our two-step modeling framework provides great flexibility in modeling futures price dynamics and allows model parameters to be calibrated directly to observable market data instead of latent factors. To highlight the convenience of our modeling approach, we discuss an implementation designed for the valuation of a natural gas storage contract in the U.S. market.

The derivatives pricing and hedging literature has made considerable progress during the past few decades. Yet, when applied to energy commodities such as natural gas markets, large gaps remain between theory and practice. First, traditional stochastic modeling approaches rely on the dynamics of (hypothetical) spot contracts referring to a fixed delivery date. However, market participants trade gas not for delivery dates but for delivery periods; that is, real-world contracts such as exchange-traded futures are defined in terms of flows over a given time window instead of a stock on a particular date. Second, traditional models require the specification of a drift and volatility function characterizing the spot price dynamics in the case of a one-factor model or, even more demanding, the dynamics of the latent factors driving the spot price dynamics in a multifactor setting. By contrast, market participants neither observe hypothetical spot prices nor the drift and volatility functions characterizing their dynamics. Instead, they observe futures contracts for different future physical delivery periods and implied volatilities of options written on these futures. Third, recently developed market models seem to close the gap between theory and practice because they specify the dynamics of a set of exchange-traded futures contracts whose volatilities are naturally linked to traded options. From a modeling perspective, market models need to concentrate on a subset of delivery contracts (say, futures contracts with monthly delivery periods) but leave contracts that refer to other delivery periods unspecified (say, day-ahead contracts). Yet, many real-world pricing and hedging applications in natural gas markets, such as those involving delivery agreements or storage contracts, require the dynamics of the complete futures price curve.

In this article, we develop a new stochastic modeling framework that is tailored to market participants' needs in natural gas markets. Instead of modeling hypothetical spot prices, the new approach models the price dynamics of actively traded futures contracts and is completed by applying a well-known smooth interpolation function and deriving the arbitrage-free price dynamics of the complete future price curve. We thus close the gap between theory and practice: First, in contrast to traditional spot price models, our approach accommodates physical delivery periods that are characteristic for physical natural gas trading and can range from a calendar day for short-term contracts up to more than a calendar year for long-term contracts. Second, again in contrast to traditional spot price models, our framework can be efficiently calibrated to market data. Third, in contrast to market models, our approach models the dynamics of the complete future price curve. This feature renders the new stochastic modeling approach flexible enough for many important real options applications.

As an example to clarify these points, suppose the holder of a (embedded) storage option seeks to profit from (1) strongly fluctuating price spreads between day-ahead and futures contracts and (2) a seasonal natural gas futures price curve with higher prices for winter months than for summer months. To fully exploit the contracts' inherent flexibility, storage operators trade in both day-ahead contracts to optimize their physical operation policies and in exchange-traded futures contracts to hedge their price risk efficiently. Thus, stochastic modeling approaches are particularly fruitful when they rely on the complete futures price curve. At the same time, the price dynamics need to reflect the market prices of the actively traded futures contracts. Note that pricing and hedging such embedded storage options play crucial roles in gas markets because storage capacities are limited and costly. Therefore, storage facilities belong to the first-order important assets, and almost all bilateral delivery agreements among market participants are structured with some embedded storage options that allow the holder to react flexibly to fluctuating physical customer demand. Although there is profound knowledge of how to conceptualize these embedded contingencies thanks to Meyers's [1977] real option concept and increasingly advanced optimization techniques allow market participants to efficiently derive exercising or operational decisions, we add to the contracts' valuation and hedging by providing a convenient stochastic modeling approach.

Our approach relates to both strands of the literature on commodity price modeling: spot and futures price models as well as market models. Standard spot price models start with the price dynamics of the unobservable spot contract and use latent risk factors to capture the time-varying spot-futures price relation. For instance, Schwartz [1997], Miltersen and Schwartz [1998], and Casassus and Collin-Dufresne [2005] included a stochastic convenience yield when modeling the spot commodity. In commodity markets, the convenience yield measures the additional benefit of holding a spot commodity as opposed to holding a futures contract. This benefit—or "embedded timing option attached to the commodity" Brennan [1958]—exists because a significant part of the demand in many commodity markets is driven by real needs such that physically holding the commodity allows its holder to circumvent potential shortages in the spot commodity when needed for a production process. Similarly, futures price models start with the price dynamics of the (unobservable) entire futures curve and can reflect the properties of the convenience yield indirectly via the shape and volatility structure of the future curve (see, e.g., Heath, Jarrow, and Morton [1992]). Although we ultimately end up with multifactor spot and futures price processes, our approach does not rely on exogenously specifying a stochastic process for unobservable state variables or contracts. Instead, we start with a market model for exchange-traded futures contracts; this provides the link to the market modeling literature. We do not provide new insights about the number of risk factors required to model multiple futures contracts, nor do we empirically test new volatility functions (see, e.g., Manoliu and Tompaidis [2002]; Casassus and Collin-Dufresne [2005]; Benth and Koekebakker [2008]; Liu and Tang [2010]; Karstanje, van der Wel, and van Dijk [2015]); rather, we show how to extend standard market models for the valuation of more complex real options in physical commodity markets.

The article is organized as follows: In the next section, we analyze the trade-off between tractability and completeness for two canonical modeling approaches in energy commodity markets and make the case for our alternative energy market modeling approach. In the third section, the theoretical modeling framework is introduced and discussed. In fourth section, we carry out an implementation exercise for the U.S. natural gas market. The final section concludes the article.

MODELING PROBLEM: TRADEOFF BETWEEN TRACTABILITY AND COMPLETENESS

There are two main requirements for pricing models of real options in energy markets: (1) tractable price dynamics for exchange-traded futures contracts that allow model parameters to be calibrated to market information to ensure consistent model prices (tractability); (2) appropriate arbitrage-free price dynamics for spot contracts that are essential to value-embedded storage options in physical markets (completeness). Application of existing stochastic modeling approaches suffers from significant trade-offs between tractability and completeness. Similar to classical fixed income models, there are two basic choices: the highly tractable (but incomplete) market models or the complete (but often intractable in practical applications) spot and futures price models. We clarify some important shortfalls of both frameworks for the purpose of real option valuation in the natural gas market and make the case for our alternative energy market modeling approach.

Market models start with the price dynamics of a finite number of traded futures contracts with fixed delivery periods. This model design simplifies parameter estimation because model parameters can be directly fitted to observable market data, but it leads to undefined price dynamics for delivery contracts with delivery periods that are not exogenously modeled. Incompleteness is not a problem for many standard valuation purposes. However, it prevents the use of market models for the valuation of important real options such as natural gas storage contracts.

In contrast to market models, spot and future price models start with the price dynamics of theoretical delivery contracts referring to delivery dates. The dynamics of the spot price s_t and futures prices $f_t(u)$ for any delivery date $u \geq t$ are given by

$$ds_t = df_t(t) = \alpha_t^{(s)} dt + \sigma_t^{(s)} dw_t^{\mathbb{Q}}$$
$$df_t(u) = \sigma_t(u) dw_t^{\mathbb{Q}}$$

where the drift and volatility functions $\alpha_t^{(s)}$, $\sigma_t^{(s)}$, and $\sigma_t(u)$ must satisfy standard suitability conditions[1] and $w_t^{\mathbb{Q}}$ is a standard Wiener process under the risk-neutral measure $\mathbb{Q}$. It is then possible to derive arbitrage-free futures price dynamics for arbitrary delivery periods based on the risk-neutral valuation approach. For instance, if real futures contracts refer to uniform deliveries in their delivery periods, the no-arbitrage relation between theoretical and real futures prices is given by

$$\begin{aligned} f_t(\tau_b, \tau_e) &= \int_{\tau_b}^{\tau_e} g(u;\, \tau_b,\, \tau_e) \mathbb{E}_t^{\mathbb{Q}}[s_u]\, du \\ &= \int_{\tau_b}^{\tau_e} g(u;\, \tau_b,\, \tau_e) f_t(u)\, du \end{aligned} \tag{1}$$

where $f_t(\tau_b,\, \tau_e)$ corresponds to the real futures price for the delivery period $(\tau_b,\, \tau_e]$.[2] The function $g(u;\tau_b,\, \tau_e)$ depends on the settlement procedure of the futures contract. It is equal to

$$g(u;\, \tau_b,\, \tau_e) = \frac{\exp(-ru)}{\int_{\tau_b}^{\tau_e} \exp(-rv)\, dv}$$

if the settlement takes place uniformly during the delivery period, and r is the constant instantaneous risk-free interest rate.[3] To illustrate the problem of fitting spot and futures price dynamics to market data, let us consider a widely used log-normal futures price model:

$$df_t(u) = \sigma_t(u) f_t(u) dw_t^{\mathbb{Q}} \tag{2}$$

Based on the no-arbitrage relation in Equation (1), we can derive the implied price dynamics for $f_t(\tau_b,\, \tau_e)$ given by

$$\begin{aligned} &df_t(\tau_b, \tau_e) \\ &= \left(\int_{\tau_b}^{\tau_e} \sigma_t(u) g(u;\, \tau_b,\, \tau_e) f_t(u)\, du \right) dw_t^{\mathbb{Q}} \\ &= \left(\sigma_t(\tau_e) f_t(\tau_b, \tau_e) - \int_{\tau_b}^{\tau_e} \frac{\partial \sigma_t}{\partial u}(u) \frac{g(u;\, \tau_b, \tau_e)}{g(u;\, \tau_b, u)} f_t(\tau_b, u)\, du \right) dw_t^{\mathbb{Q}} \end{aligned} \tag{3}$$

Equation (3) implies that tradable futures contracts with delivery periods are only log-normal in the unrealistic case that the volatility function does not depend on the delivery date $\left(\frac{\partial \sigma_t}{\partial u}(u) = 0 \right)$. In more realistic cases, futures contracts are not log-normal, and computationally intensive numerical algorithms are required to estimate model parameters from historical return data or current option prices. This estimation problem is not specific to log-normal models and generally emerges for all nonadditive stochastic processes (see Benth, Kallsen, and Meyer-Brandis [2007]).

As a workaround, it is common practice to fit spot or futures price processes to inconsistent proxies for unobservable (theoretical) spot or futures prices either directly or by using a two-stage approach. For instance, Gibson and Schwartz [1990], Schwartz [1997], Schwartz and Smith [2000], Casassus and Collin-Dufresne [2005], Cartea and Williams [2008], and Chen and Forsyth [2010] ignored delivery periods of crude oil or natural gas futures contracts in their empirical studies. Two-stage estimation approaches derive theoretical spot and futures prices from real futures prices based on an interpolation function and then fit model parameters of the underlying spot and futures price processes to these interpolated prices. Doing so does not ensure consistent parameter estimates because two independent interpolation functions are used: the interpolation function applied to extract theoretical futures prices and the endogenous interpolation function (futures price curve) implied by the spot price process (see, e.g., Koekebakker and Ollmar [2005]). In a recent article, Kiely, Murphy, and Cummins [2015b] discussed the problem of fitting Lévy spot price processes to delivery swaption markets and showed that relatively tractable formulas can be obtained for at least the first four moments. However, these formulas can only be applied for discrete payout contracts and are still difficult to fit to time series properties of traded natural gas futures contracts.

For the special case of additive stochastic processes (e.g., affine-linear models), we do obtain tractable price dynamics for real futures contracts (see Bouwman, Raviv, and van Dijk [2012]). This choice simplifies estimation methods but strongly restricts potential model specifications. For instance, the Black or Heston model or widely used models in other studies (see, e.g., Koekebakker and Ollmar [2005], Benth and Koekebakker [2008], or Trolle and Schwartz [2009]) are not contained in the class of additive stochastic processes. In addition, it is not easily possible to capture important price movements such as stochastic summer–winter spreads in the natural gas market within (standard) affine-linear stochastic models.

As an alternative to the widely used market and spot and futures price models, we develop a modeling framework for the common stochastic price behavior of futures contracts with arbitrary delivery periods. Importantly, the model is able to take specific market frictions into account and can be easily calibrated to market data. The core idea behind obtaining a consistent modeling framework is to capture the stochastic behavior of traded futures contracts with fixed nonoverlapping delivery periods through a standard market model and to price all other instruments relative to them based on a smooth interpolation approach.

THE ENERGY MARKET MODEL

In this section, we develop our stochastic term structure model for the entire futures price curve in $[\tau_s, \tau_e]$. We assume a market environment that consists of multiple futures contracts with fixed consecutive delivery periods $\{(\tau_i, \tau_{i+1}]\}_{i=1}^{mf}$ for $i = 1, \ldots, mf$. Each futures contract i is traded at the futures price $F_t^{(i)} = f_t(\tau_i, \tau_{i+1})$ until its first delivery date.[4] For simplicity, we assume that all futures contracts are traded without transaction costs, refer to uniform delivery of the same quantity of natural gas in their delivery periods, and are settled continuously.

We develop our modeling approach in two major steps. We first derive an arbitrage-free and smooth instantaneous futures price curve. We then endogenously derive price dynamics for arbitrary delivery contracts to complete our pricing framework.

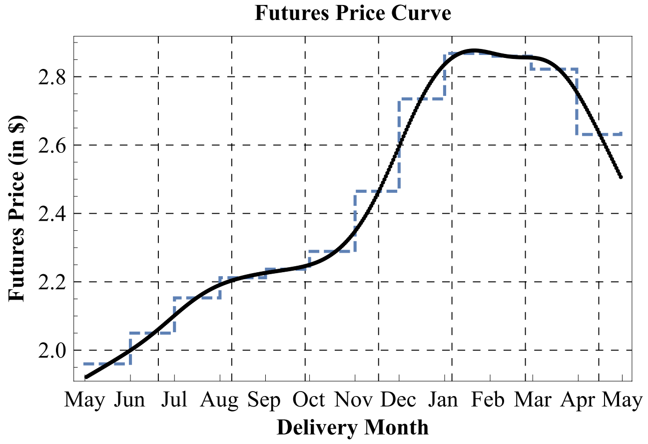
Smooth Interpolation Function

Using futures market information, the first step is to deduce arbitrage-free model prices for futures contracts with arbitrary delivery periods that are not currently traded. We apply an interpolation approach that infers theoretical market prices for instantaneous deliveries from real futures prices referring to delivery periods. Exhibit 1 illustrates this approach. The interpolation function is based on two no-arbitrage conditions and a maximum smoothness criterion that avoids strongly oscillating futures price curves. The first condition on the interpolation function ensures that two portfolios with the same physical delivery flows have the same market value at any point in time.

Condition 1 (static no-arbitrage condition). The futures price curve $f_t(u)$ satisfies the static no-arbitrage relation at any time t:

$$F_t^{(i)} = \int_{\tau_i}^{\tau_{i+1}} g(u; \tau_i, \tau_{i+1}) f_t(u)\, du, \quad i = 1, \ldots, mf \tag{4}$$

Exhibit 1
Interpolation Approach



Notes: The graph shows the (traded) futures price curve (dashed line) and the theoretical futures price curve (smooth interpolation function, solid line). The theoretical futures price curve is defined by the no-arbitrage conditions, the usual spline conditions, and the maximum smoothness criterion.

The static no-arbitrage relation is imposed even when theoretical futures contracts are nontraded instruments to guarantee an arbitrage-free modeling approach for arbitrary physical delivery contracts. Moreover, the no-arbitrage principle requires that endogenous futures price dynamics are martingales under the risk-neutral measure regardless of the price dynamics of the traded futures contracts.

Condition 2 (dynamic no-arbitrage condition). The futures price dynamics satisfy the martingale property

$$f_t(u) = \mathbb{E}_t^{\mathbb{Q}}[f_l(u)], \quad t \le l \le u \tag{5}$$

These two conditions are both necessary for an arbitrage-free complete pricing framework, but they do not ensure that endogenous futures price curves are reasonable. Notably, interpolation functions of higher order, which are required to satisfy the static no-arbitrage relation, tend to be strongly oscillating. To circumvent this problem, we use smooth polynomial splines of order four during the delivery periods of the futures contracts:

$$f_t(u) = a_i + b_i u + c_i u^2 + d_i u^3 + e_i u^4, \quad u \in [\tau_i, \tau_{i+1}], \quad i = 1, \ldots, mf \tag{6}$$

The spline parameters are uniquely determined by the usual spline conditions, the static no-arbitrage conditions, and a maximum smoothness condition. The maximum smoothness condition was first introduced by McCulloch [1971] for yield curves and applied for energy markets by Benth, Koekebakker, and Ollmar [2007]. It minimizes the average second derivative of the interpolation function over the underlying time period.

Condition 3 (maximum smoothness condition). The futures price curve is a piecewise polynomial of order four and satisfies the maximum smoothness criterion at τ_s subject to the two no-arbitrage conditions 1 and 2:

$$\min_{\{(a_i, b_i, c_i, d_i, e_i)\}_{i=1}^{mf}} \int_{\tau_1}^{\tau_{mf+1}} \left(\frac{\partial^2 f_{\tau_s}}{\partial^2 u}(u) \right)^2 du \tag{7}$$

It turns out that the assumption of splines of order four is not restrictive and that piecewise polynomials of order four are the maximal smooth functions subject to weak conditions (see Benth, Koekebakker, and Ollmar [2007]). These three conditions uniquely define the interpolation function.

Lemma 1 (futures price curve). The two no-arbitrage conditions and the maximum smoothness condition yield to the following relation between the theoretical futures price curve and real spot and futures prices at any point in time *t:*

$$f_t(u) = \sum_{j=1}^{mf} \beta^{(j)}(u) F_t^{(j)} \tag{8}$$

$$f_t(\tau_b, \tau_e) = \sum_{j=1}^{mf} \left(\int_{\tau_b}^{\tau_e} g(u; \tau_b, \tau_e) \beta^{(j)}(u)\, du \right) F_t^{(j)} \tag{9}$$

where the weighting functions $\{\beta^{(j)}(u)\}_{j=1}^{mf}$ fulfill the following conditions:

$$\beta^{(j)}(u) = \tilde{a}_{ij} + \tilde{b}_{ij} u + \tilde{c}_{ij} u^2 + \tilde{d}_{ij} u^3 + \tilde{e}_{ij} u^4, \quad u \in [\tau_i, \tau_{i+1}]$$

$$\sum_{j=1}^{mf} \beta^{(j)}(u) \equiv 1$$

$$\int_{\tau_i}^{\tau_{i+1}} g(u; \tau_i, \tau_{i+1}) \beta^{(j)}(u)\, du = \begin{cases} 1 & j = i \\ 0 & j \ne i \end{cases}, \quad \text{for } i, j = 1, \ldots, mf$$

The spline parameters of the weighting functions $\{(\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}, \tilde{d}_{ij}, \tilde{e}_{ij})\}_{i,j=1}^{mf}$ are uniquely determined by the usual spline conditions, the static no-arbitrage condition, and the maximum smoothness criterion.

Proof. See the Appendix.

The linear relation, Equation (8), between the theoretical futures price curve and futures prices exists due to a linear relation between the spline parameters $\{(\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}, \tilde{d}_{ij}, \tilde{e}_{ij})\}_{i,j=1}^{mf}$ and real futures prices (see the Appendix). Note that the dynamic no-arbitrage condition for arbitrary martingale processes of real futures contracts is only satisfied for linear relations between theoretical and real futures prices. Furthermore, the maximum smoothness criterion implies that parallel shifts in futures prices yield to parallel shifts in the theoretical futures price curve. Thus, weighting functions add up to one. The fact that the integral of the weighting function over a delivery period has to be either zero or one follows directly from the static no-arbitrage condition.

Price Dynamics

Now we specify a general modeling framework for the price dynamics of the entire futures price curve based on the linear relation between real futures prices and the instantaneous futures price curve. Because significant physical storage costs distort the classical cost-of-carry relation between futures prices referring to nonoverlapping delivery periods, we can use any appropriate market model approach. In the following, we consider quite general price dynamics for the futures price process, given by

$$dF_t^{(i)} = \sum_{j=1}^{m_d} \sigma_t^{(i,j)} dw_t^{(j,\mathbb{Q})} + \sum_{k=1}^{m_j} \int_{\mathbb{R}} \phi_t^{(i,k)} \, dn^{(k,\mathbb{Q})}(dt, dz) \quad (10)$$

where $\{w_t^{(j,\mathbb{Q})}\}_{j=1}^{m_d}$ are standard Wiener processes and $\{n^{(k,\mathbb{Q})}\}_{k=1}^{m_j}$ are square-integrable Lévy processes independent of $\{w_t^{(j,\mathbb{Q})}\}_{j=1}^{m_d}$ for $i = 1, \ldots, mf$.

Using these specifications and no-arbitrage restrictions and applying Itô's lemma to the futures price curve given in Lemma 1, we obtain the following price dynamics.

Lemma 2 (spot and futures price dynamics). In the energy market model, we obtain the following risk-neutral price dynamics for delivery contracts with theoretical delivery dates and delivery periods:

1. rolling instantaneous delivery date t:

$$ds_t = df_t(t) = \sum_{i=1}^{mf} \left(\frac{\partial \beta^{(i)}}{\partial t}(t) \right) F_t^{(i)} dt + \sum_{i=1}^{mf} \beta^{(i)}(t) dF_t^{(i)} \quad (11)$$

2. fixed future delivery date u:

$$df_t(u) = \sum_{i=1}^{mf} \beta^{(i)}(u) dF_t^{(i)} \quad (12)$$

3. fixed future delivery period $(\tau_b, \tau_e]$:

$$df_t(\tau_b, \tau_e) = \sum_{i=1}^{mf} \left(\int_{\tau_b}^{\tau_e} g(u; \tau_b, \tau_e) \beta^{(i)}(u) \, du \right) dF_t^{(i)}, \quad \tau_b < \tau_e \quad (13)$$

Given some price dynamics for the common behavior of the traded futures contracts (i.e., the market model, Equation (10)), Lemma 2 shows that the spot price dynamics (Equation (11)), the theoretical future price dynamics (Equation (12)), and the dynamics of futures contracts with arbitrary physical delivery periods (Equation (13)) result directly from the weighting functions of Lemma 1. In Equation (11), the price dynamics referring to a rolling delivery date do not have to satisfy the martingale property because trading in such a contract requires storing natural gas physically.[5] Instead, its drift component is equal to the current slope of the futures price curve. In contrast, theoretical and real futures prices refer to fixed delivery dates or periods. Thus, both stochastic processes satisfy the martingale property under the risk-neutral measure.

The key difference in alternative models for instantaneous delivery contracts is that observable instead of theoretical futures price dynamics are modeled exogenously. This means that the model prices for traded futures contracts do not have to be endogenously derived before model parameters can be estimated on market data. Note that the drift component of Equation (11) linearly depends on observable futures prices instead of latent factors.

As a result, no-arbitrage conditions can be met without restricting the price dynamics of traded futures contracts. This allows us to obtain a price process for an instantaneous delivery that results in tractable futures price dynamics for real delivery contracts in contrast to standard modeling approaches. Overall, our modeling approach is a natural and consistent extension of Benth, Koekebakker, and Ollmar's [2007] static approach to value customized linear commodity contracts for all kinds of nonlinear derivative contracts.

APPLICATION TO STORAGE VALUATION

This section illustrates the benefits of our proposed energy market model for the valuation of natural gas storage contracts compared to standard approaches. To this end, we briefly characterize typical storage contracts and their embedded trading options to outline the requirements for pricing models of real options in natural gas markets. We then implement a concrete energy market model, apply it for the valuation of a natural gas storage contract in the U.S. market, and highlight its benefits vis-a-vis standard approaches.

Storage Contracts

In short, storage contracts allow their holders to store natural gas up to certain volumes subject to maximal injection and withdrawal rates depending on the underlying technical constraints. For instance, Centrica offers one-year storage contracts with given maximal withdrawal and injection volumes per day and some total storage capacity. The initial volume in storage is zero, and the storage must be returned with the same volume at the end of the contract period.[6]

To facilitate the understanding of this real option, it is useful to separate the storage value into an intrinsic and an extrinsic real option value. The intrinsic storage value originates from the seasonal pattern of the natural gas futures price curve. For instance, a storage operator can earn the summer–winter spread by injecting natural gas in the warmer summer months, when there is less natural gas demand, storing it, and withdrawing it during the colder winter months when there is higher natural gas demand. This riskless strategy can be implemented without need for a stochastic model, and its value mainly depends on the summer–winter spread implied by the futures market. The extrinsic storage value arises from temporary price shocks, which can have a large impact on day-ahead prices without changing the remaining futures price curve. For instance, storage operators own the option to sell stored natural gas at high day-ahead prices if natural gas prices spike because of, for example, extreme weather conditions as in the United States in February 2014, or to buy natural gas at low day-ahead prices if there is a temporary oversupply in the market. To separate temporary from permanent price movements, it is important to consider spot as well as futures prices. For example, in February 2014, a sharp price increase in the spot market combined with nearly unchanged futures prices clearly signaled a temporary price shock and provided a selling signal for flexible storage operators. In contrast, during the financial crisis in 2008, spot prices decreased about the same amount as the futures price curve such that lower spot prices did not obviously signal a profitable buying opportunity. Therefore, to obtain storage strategies in line with market prices, it is crucially important to capture the common stochastic behavior of spot and futures market information. These observations motivate the specification of our subsequent energy market model.

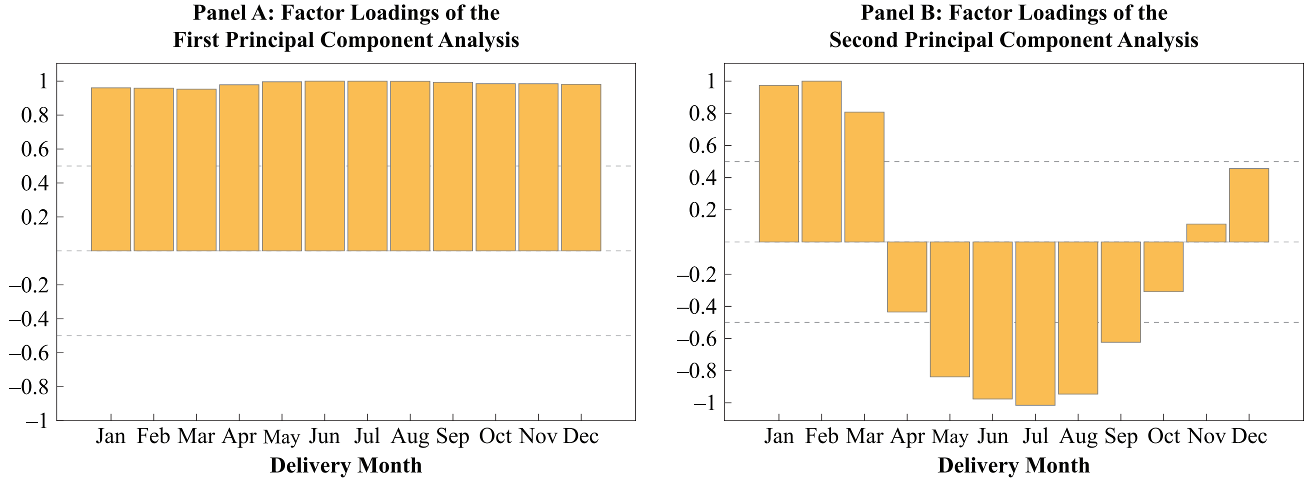
Specification of an Energy Market Model

We specify the uncertain natural gas price dynamics in two steps. First, we consider historical day-ahead and futures return data to select relevant risk factors with simultaneous consideration of their relevance for the storage valuation problem and their statistical importance in the available market data. Second, we calibrate our model to futures and option market data to obtain a consistent pricing framework for storage contracts.

Spot and futures contracts. Our underlying dataset consists of Henry Hub natural gas day-ahead contracts traded over-the-counter and Henry Hub natural gas futures contracts for the next 12 calendar months, traded at the Chicago Mercantile Exchange (CME). Daily settlement prices are obtained from the Bloomberg database from January 1, 1997 through December 31, 2015. These futures contracts refer to nonoverlapping short delivery periods that span the whole contract period of the underlying storage contract. Together with the rolling day-ahead contract, the chosen market contracts provide a good picture of temporary imbalances between supply and demand in the market, reflect market expectations about future prices, and avoid extrapolation errors. Using them to estimate our energy market model thus allows us to capture the joint behavior of truly tradable instruments and ensures consistency with market contracts that are relevant for possible hedging strategies.

To specify an appropriate modeling approach for natural gas price dynamics, it is important to consider

EXHIBIT 2
Factor Loadings



Notes: These graphs show the factor loadings of first principal component of both principal component analyses. The underlying dataset consists of absolute futures price returns from January 1997 to December 2015.

the underlying risk factors of natural gas price dynamics as well as the sensitivity of the natural gas storage value against certain price movements. We start with a statistical analysis of the common stochastic behavior of natural gas futures price dynamics by principal component analysis (PCA). We first sort the absolute return data by delivery month and then apply a zero-mean and unit-variance normalization. The PCA shows that the first two PCA factors explain 94% of the total variation in the futures price curve. The first risk factor is by far the most important, explaining 90% of the total variation of the futures price dynamics. Exhibit 2 shows that the first risk factor has nearly the same price impact on all futures contracts. From a storage valuation perspective, this is important because perfect parallel shifts have no impact on storage values such that the impact of the first PCA factor on the storage value should be minor despite its high statistical explanation power. To exploit this fact, we use perfect parallel shifts instead of the first PCA factor. As a consequence, we do not simply use the second PCA factor as a risk factor of natural gas price dynamics. Rather, we calculate average futures price returns for every trading day as a proxy for perfect parallel shifts, subtract them from individual futures price returns, and rerun the PCA. In this second PCA, the first risk factor explains 40% of the remaining variation of the futures price dynamics and captures the stochastic price spread between summer and winter months in the natural gas futures market (see Exhibit 2). In the following, we denote this factor as stochastic summer–winter spread.

In addition, we consider a short-term risk factor that reflects temporary demand and supply shocks in the market. The short-term risk factor mainly influences the short end of the futures price curve and is modeled via an exponentially decaying volatility function in the market model. Empirically, we require an additional risk factor because less than 10% of the total variation of the day-ahead price dynamics is captured by the price level and the summer–winter spread factors, and temporary price shocks are particularly important for the extrinsic storage value.

Next, we incorporate the summer–winter spread and the short-term risk factor in our modeling approach, taking specific characteristics of the storage option into account.

Risk factors. We jointly model the natural gas futures price curve with a simple two-factor normal market model

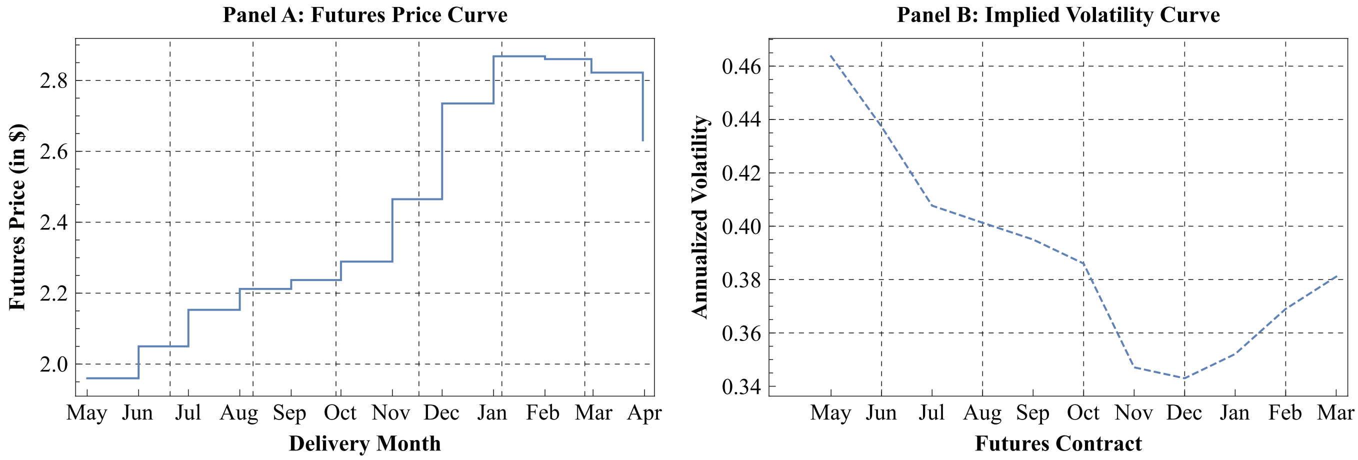
$$dF_t^{(i)} = \sigma^{(i,sw)} dw_t^{(1,\mathbb{Q})} + \sigma_t^{(sh)}(\tau_i, \tau_{i+1}) dw_t^{(2,\mathbb{Q})} \tag{14}$$

where

$$\sigma_t^{(sh)}(\tau_i, \tau_{i+1}) = \sigma^{(sh)} \frac{\exp(-\kappa(\tau_i - t)) - \exp(-\kappa(\tau_{i+1} - t))}{\kappa(\tau_{i+1} - \tau_i)}, \ \sigma^{(i,sw)} \in \mathbb{R}^+$$

EXHIBIT 3

Futures Prices and Volatilities



Notes: Panel A shows the natural gas futures price curve on March 31, 2016. Panel B shows implied volatilities for option contracts expiring one trading day before the underlying futures contracts. The natural gas futures prices refer to physical deliveries at Henry Hub in Louisiana.

with uncorrelated Wiener processes $\{w_t^{(j,\mathbb{Q})}\}_{j=1}^2$ for $i = 1 \ldots, 12$. We omit the price-level factor from the pricing model because it has no impact on the storage value. The summer–winter spread captures movements in the difference between the generally higher prices of winter delivery months compared with summer, and the short-term factor reflects temporary demand shocks in the market. The specification is primarily motivated by the PCA results and the fact that the stochastic summer–winter spread is the decisive factor in the intrinsic storage value, and the short-term factor allows exploitation of the extrinsic storage value using a dynamic trading strategy built upon truly tradable day-ahead contracts.

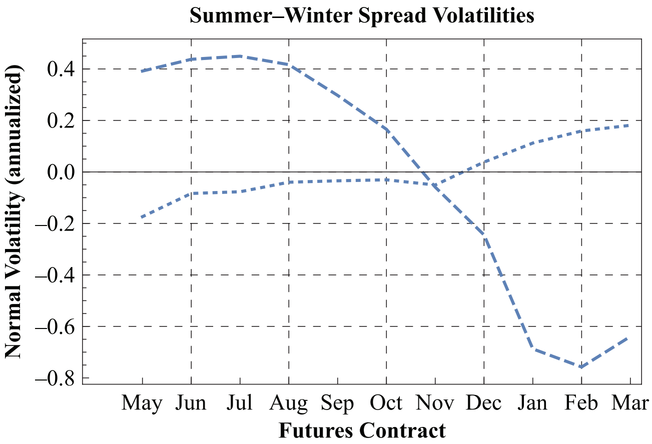
We use a simple estimation approach to fit model parameters to (1) historical return data and (2) market data at the valuation date of the storage contract. The summer–winter spread volatility parameters $\{\sigma^{(i,sw)}\}_{i=1}^{12}$ are estimated based on the PCA outlined in "Spot and futures contracts" and shown in Exhibit 2. For the short-term risk factor, our estimation method relies on two pillars: (1) matching real day-ahead prices and (2) estimating κ such that the impact of the short-term risk factor on front-month futures prices is uncorrelated with the residuals between model-implied and market front-month futures prices. Let's assume that model parameters (κ in particular) are known. We can then use Equations (9) and (14) to filter out the short-term risk factor from real day-ahead prices. Together with the condition that residuals between model-implied and market front-month futures prices should be uncorrelated with the short-term risk factor, this allows us to estimate κ numerically. To estimate the short-term volatility parameter, we scale the short-term risk factor values by the average futures price level at the beginning of every calendar year to account for long-term price level effects and set $\sigma^{(sh)}$ to the standard deviation of short-term risk factor.

To get a consistent pricing model at the valuation date of a storage contract, generally only two types of market contracts provide important information about uncertain natural gas price dynamics: futures and options contracts. Exhibit 3 shows futures prices and at-the-money implied volatilities for the next 12 calendar months at March 31, 2016 obtained from the Bloomberg database. The exhibit shows (1) the seasonal pattern of natural gas prices and volatilities with roughly 40% higher natural gas prices for winter months compared to summer months and (2) that the short end of the futures price curve is more volatile than the long end.

Because we use futures prices as input values, our modeling approach is per se consistent with the future curve shown in Exhibit 3. Implied volatilities give us information about average volatility functions during the lifetime of the underlying option contracts. In more complex stochastic volatility models, such information can be used to extract unobservable states of the volatility process. Here, in our constant volatility

EXHIBIT 4

Historical and Implied Summer–Winter Spread Volatilities



Note: The dashed line corresponds to historical summer–winter spread volatilities and the dotted line to the implied summer–winter spread volatilities on March 31, 2016.

approach, we first adjust the short-term volatility parameter $\sigma^{(sh)}$ to the difference between the front-month and 13-month-ahead implied volatilities and then simply readjust $\{\sigma^{(i,sw)}\}_{i=1}^{12}$ such that model-implied volatilities are consistent with market-implied volatilities (see Exhibit 4). This allows us to obtain a consistent pricing framework that captures the main risk factors in the underlying dataset.

Of course, our market model is very simplistic for most practical applications because futures price returns are far from being normally distributed. Out-of-the-money option prices provide valuable information about univariate non-normal futures price dynamics while separating risk factors, and estimation of correlations parameters must be done based on historical return data because of missing liquid market products that are sensitive to such price dynamic characteristics. We leave this for future research, but because our framework models underlying market contracts directly, it provides the easiest way to incorporate additional risk factors and more complex price dynamics (e.g., stochastic volatility or price jumps, as in work by Christoffersen, Jacobs, and Li [2016]).

Empirical Implementation and Results

Finally, we value an example of a storage contract outlined in "Storage Contracts" based on different specifications of our energy market model to highlight the importance of a consistent pricing framework for real options in natural gas markets. We start with a short description of the concrete storage contract and then outline the numerical estimation approach for the valuation problem.

Optimizing the inventory-trading decisions. Consider a storage operator with access to a storage contract similar to the Standard Bundled Units (SBUs) offered by Centrica and assume that the operator faces price risk captured by our energy market model when making inventory-trading decisions. Formally, this leads to a warehouse problem with both space and injection/withdrawal capacity limits.[7] In our case, the space limit is given by a total storage capacity of 180 mmBtu. Maximal withdrawal and injection volumes are −3 mmBtu/day and 1 mmBtu/day, respectively. The contract design further imposes variable injection and withdrawal costs of \$0.07/mmBtu and \$0.03/mmBtu. No natural gas is lost during the injection or withdrawal process. The initial volume in storage is zero, and the storage must be returned with the same volume at the end of the contract period.

In the first step, we exploit the fact that rational storage operators just need to trade in the spot contract[8] and that feasible trading times and volumes can both be restricted to a finite number. Optimal trading times can be restricted to the rolling dates of the spot contract without reducing the expected storage value under the pricing measure. This is because (1) interest rate effects have no impact on early exercise strategies because payoffs are linked to physical delivery flows instead of trading times, and (2) active trading within the trading period of the spot contract is purely speculative and does not exploit any physical storage options. Moreover, we can restrict attention to a finite number of feasible volumes.[9] These insights allow us to determine the storage value via a manageable discrete stochastic optimization problem.

In the following, we numerically solve the underlying optimization problem using the least squares Monte Carlo simulation approach of Longstaff and Schwartz [2001].[10] We generate 10,000 price paths based on the initial futures price curve from March 31, 2016, shown in Exhibit 3, using different specifications of our energy market model. Starting from the last storage date, we determine optimal trading decisions backward. At each date and for each volume in storage, we approximate the continuation value with

EXHIBIT 5

Daily Parameter Estimates, Storage Values, and Expected Maximal Volumes in Storage for the Different Model Specifications

Empirical Specification—Parameter Estimates and Storage Values						
Specification	**Trad. Cont.**	**κ**	**σ**	**Stor. Val.**	**Max. Vol.**
I: two-factor model historical volatilities	Day-Ahead	0.029	0.143	201	129
II: one-factor model historical volatilities	Day-Ahead	0.029	0.143	156	127
III: two-factor model implied volatilities	Day-Ahead	0.029	0.085	147	164
IV: two-factor model implied volatilities	Futures	0.029	0.085	123	180
Intrinsic Value	Futures			123	180

polynomials of order three, where parameters are estimated via ordinary least squares regressions. For an overview of alternative numerical valuation approaches, see Kiely, Murphy, and Cummins [2015a]; Parsons [2013]; and Loehndorf and Wozabal [2015].

In a first run (specification I), we consider our two-factor normal market model (Equation (14)) and use historical price information only to estimate model parameters (historical volatilities). That is, we use the estimated price dynamic without taking implied volatilities into account. In a second run (specification II), we ignore the stochastic summer–winter spread to compare our approach to standard spot price models. Thus, we simply consider a one-factor specification. As in the previous specification, we only use historical price information. For the next two specifications, we again consider our two-factor normal market model (Equation (14)). To analyze the importance of option market data, we readjust volatility parameters to implied market volatilities in the third run (specification III). Lastly, in the fourth run (specification IV), we optimize the storage value by trading in futures contracts only to highlight the impact of day-ahead trading. Essentially, we assume that the storage operator only trades in the front-month futures contract, not in the (shorter-term) day-ahead contract.

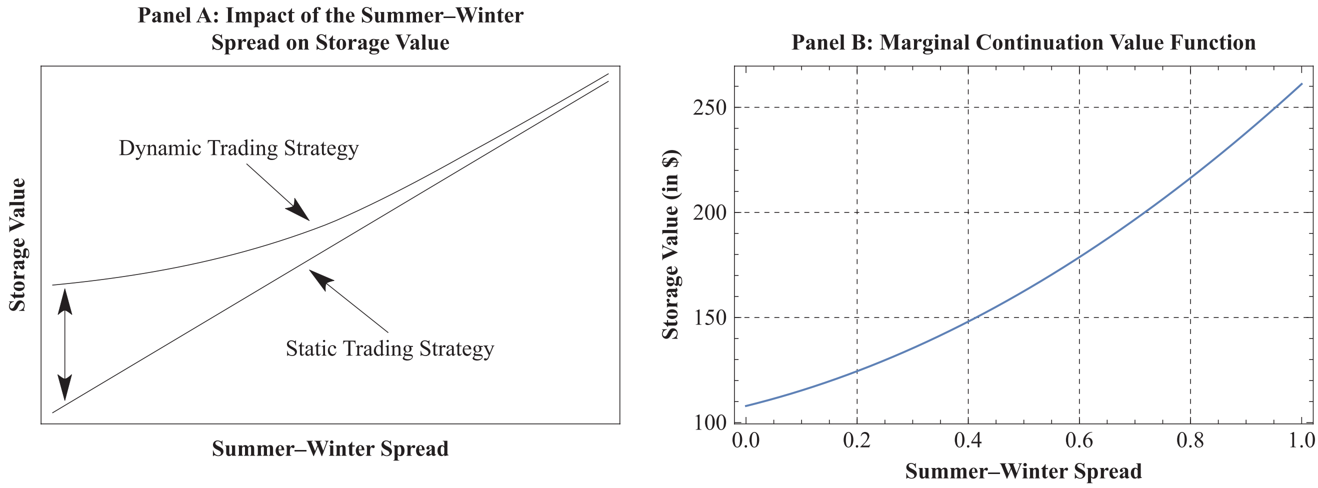
Insights. The empirical analysis provides at least four important insights. First, based on our optimization results, we can conclude that storage operators can profit substantially from short-term optimization. For the given storage, we end up with an overall value of $201 in our first specification while the intrinsic value amounts to only $123 (see Exhibit 5). Thus, dynamic trading increases the expected storage value by more than 60%. Interestingly, the short-term optimization has such a strong impact on the optimal trading strategy that the expected maximal volume in storage is only about 72% of the total storage capacity over the underlying contract period (see Exhibit 5).

Second, the summer–winter spread not only linearly affects the storage value via the intrinsic value, but movements in the summer–winter spread also matter. The intuition behind this result is as follows: If the summer–winter spread narrows, switches within the future curve are more likely. Thus, it is more attractive to trade in the day-ahead market to profit from temporary price shocks. This partially compensates for the loss resulting from a lower intrinsic storage value. On the contrary, if the summer–winter spread widens, the storage operator profits from a larger intrinsic storage value, but short-term trading becomes less profitable. Panel A in Exhibit 6 depicts a sketch of this relation, and Panel B shows that the storage value is indeed a convex function in the summer–winter spread in our empirical implementation. This convex relation reinforces the link between storage valuation and model specification. In particular, this relation reveals that a stochastic summer–winter spread makes a difference that cannot be captured simply by sequentially readjusting a model with a deterministic summer–winter spread on each trading day. Exhibit 5 shows, in our concrete case, that the storage value declines by more than 20% if we ignore the stochastic summer–winter spread.

Third, not surprisingly, the storage value strongly depends on the underlying market risk. If we use implied volatilities instead of historical ones, the storage value is about 27% lower due to the flatter implied volatility curve (see Exhibit 4). This shows the importance of taking option market data into account. Lastly, we optimize the storage strategy by trading in front-month

Exhibit 6

Impact of the Summer–Winter Spread on Storage Value



Notes: This exhibit shows the impact of the summer–winter spread on the storage value. Panel A gives a sketch of the general relation. Panel B shows the impact of the summer–winter spread on the continuation value function derived within our simulation runs. The value function refers to a storage contract with a volume in storage equal to zero at the fifth trading day.

futures contracts only. The underlying storage value is close to the intrinsic value and shows that day-ahead trading is essential to maximize the value of flexible storage contracts.

Overall, our findings disclose that model specification is crucial for the evaluation of a storage's inherent flexibility. Our empirical findings not only show that ignoring relevant price risk factors such as our short-term factor, which tracks the price spread between the day-ahead and the front-month futures contracts or a stochastic summer–winter spread, underestimates a storage's extrinsic value but also that this effect is quantitatively important. It is not least the ease of incorporating these risk factors that highlights the convenience of our proposed modeling approach: In a standard spot rate model, our short-term factor would translate to a highly volatile mean-reversion rate. Clearly, ignoring this factor (i.e., ignoring stochastic mean-reversion) would lead to suboptimal trading decisions and lower the storage value significantly. While accounting for this optimization potential, our proposed approach reaches a market-conforming valuation simply by construction. Such an endeavor proves almost impossible in standard spot rate models unless inconsistent proxies for the unobservable theoretical spot price are used. The shortcomings of standard market models are no less important. They rely on market contracts spanning a given tenor structure to capture the full dynamics of the future term structure. For instance, for a monthly tenor structure, the underlying instruments are exchange-traded futures contracts, referring to delivery in future calendar months. Consequently, day-ahead contracts remain unspecified, and the short-term optimization cannot be exploited.[11] This concern is reinforced by high storage cost and the unforeseen temporary imbalances between supply and demand prevalent in natural gas markets.

CONCLUSION

Our energy pricing approach combines the advantages of market models and the completeness of spot or future price models. Market models, originally developed to price interest-rate derivatives, have several well-known advantages: They are directly based on observable market prices—in our context, exchange-traded liquid natural gas futures contracts having physical delivery periods. They result in convenient pricing formulas for plain vanilla options written on these futures—in the simplest framework, the standard Black pricing formula is applicable. It is very easy to calibrate the models to market prices. We carry over these advantages to the (theoretical) spot and future price dynamics by determining an arbitrage-free and smooth interpolation function that depends on the same observable futures prices in a linear, albeit time-dependent, structure. The proposed approach results in multifactor spot and futures price processes that are fully specified by traded instruments instead of latent factors. The interpolation not only completes the pricing framework and makes it applicable for the valuation of a broad range of real options in energy markets, it also naturally provides central results on sensitivity and risk management.

The complexity of the empirical specification can be chosen in accordance with the pricing or hedging application. As our empirical specification for the valuation of a natural gas storage shows, specific characteristics such as a stochastic summer–winter spread or the common stochastic behavior of day-ahead and futures prices are not a challenge to the approach. Furthermore, incorporating market information from derivatives markets is easily possible using this approach. Finally, our energy market model is neither restricted to a specific commodity nor to an application for storage contracts only because nearly all bilateral delivery agreements refer to delivery periods due to technical constraints in commodity markets. We leave an analysis of such applications for future research.

APPENDIX

SMOOTH FUTURES PRICE CURVE

Benth, Koekebakker, and Ollmar [2007] showed that polynomial splines of order four are required to satisfy both no-arbitrage conditions and the maximum smoothness criterion

$$f_t(u) = \begin{cases} a_1 + b_1 u + c_1 u^2 + d_1 u^3 + e_1 u^4, & u \in [\tau_1, \tau_2] \\ a_2 + b_2 u + c_2 u^2 + d_2 u^3 + e_2 u^4, & u \in [\tau_2, \tau_3] \\ \vdots & \\ a_{mf} + b_{mf} u + c_{mf} u^2 + d_{mf} u^3 + e_{mf} u^4, & u \in [\tau_{mf}, \tau_{mf+1}] \end{cases}$$

The spline parameters are uniquely determined by the following linear equation system:

$$\begin{pmatrix} 2H & A^T \\ A & 0 \end{pmatrix} \begin{pmatrix} \gamma \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ b \end{pmatrix} \qquad \text{(A-1)}$$

In Equation A-1, the matrix A and the vector b are defined by

1. the static no-arbitrage conditions

$$F_t^{(i)} = \int_{\tau_i}^{\tau_{i+1}} g(u; \tau_i, \tau_{i+1}) f_t(u)\, du, \quad i = 1, \ldots, mf$$

2. the usual spline conditions

$$\begin{aligned} 0 &= \Delta_i e(\tau_{i+1})^4 + \Delta_i d(\tau_{i+1})^3 + \Delta_i c(\tau_{i+1})^2 + \Delta_i b \tau_{i+1} + \Delta_i a, \\ 0 &= 4\Delta_i e(\tau_{i+1})^3 + 3\Delta_i d(\tau_{i+1})^2 + 2\Delta_i c \tau_{i+1} + \Delta_i b, \\ 0 &= 12\Delta_i e(\tau_{i+1})^2 + 6\Delta_i d \tau_{i+1} + 2\Delta_i c \end{aligned}$$

where $\Delta_i a = a_{i+1} - a_i$, $\Delta_i b = b_{i+1} - b_i$, $\Delta_i c = c_{i+1} - c_i$, $\Delta_i d = d_{i+1} - d_i$, and $\Delta_i e = e_{i+1} - e_i$ for $i = 1, \ldots, mf - 1$.

The matrix H is derived from the maximum smoothness condition

$$\min_{\gamma} \gamma^T H \gamma$$

where

$$H = \begin{pmatrix} h_1 & & 0 \\ & \ddots & \\ 0 & & h_{m+1} \end{pmatrix}, \quad h_j = \begin{pmatrix} \frac{144}{5}\Delta_j^5 & 18\Delta_j^4 & 8\Delta_j^3 & 0 & 0 \\ 18\Delta_j^4 & 12\Delta_j^3 & 6\Delta_j^2 & 0 & 0 \\ 8\Delta_j^3 & 6\Delta_j^2 & 4\Delta_j & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma = (e_1, d_1, c_1, b_1, a_1, \ldots, e_{mf}, d_{mf}, c_{mf}, b_{mf}, a_{mf}), \quad \text{and}$$

$$\Delta_j^k = (\tau_{j+1})^k - (\tau_j)^k$$

for $k = 1, \ldots, 5$ and $j = 1, \ldots, mf$.

The linear equation system (Equation (A-1)) implies that the spline parameters linearly depend on real futures prices:

$$\begin{aligned} a_i &= \sum_{j=1}^{mf} \tilde{a}_{ij} F^{(j)},\ b_i = \sum_{j=1}^{mf} \tilde{b}_{ij} F^{(j)},\ c_i = \sum_{j=1}^{mf} \tilde{c}_{ij} F^{(j)}, \\ d_i &= \sum_{j=1}^{mf} \tilde{d}_{ij} F^{(j)},\ e_i = \sum_{j=1}^{mf} \tilde{e}_{ij} F^{(j)} \end{aligned}$$

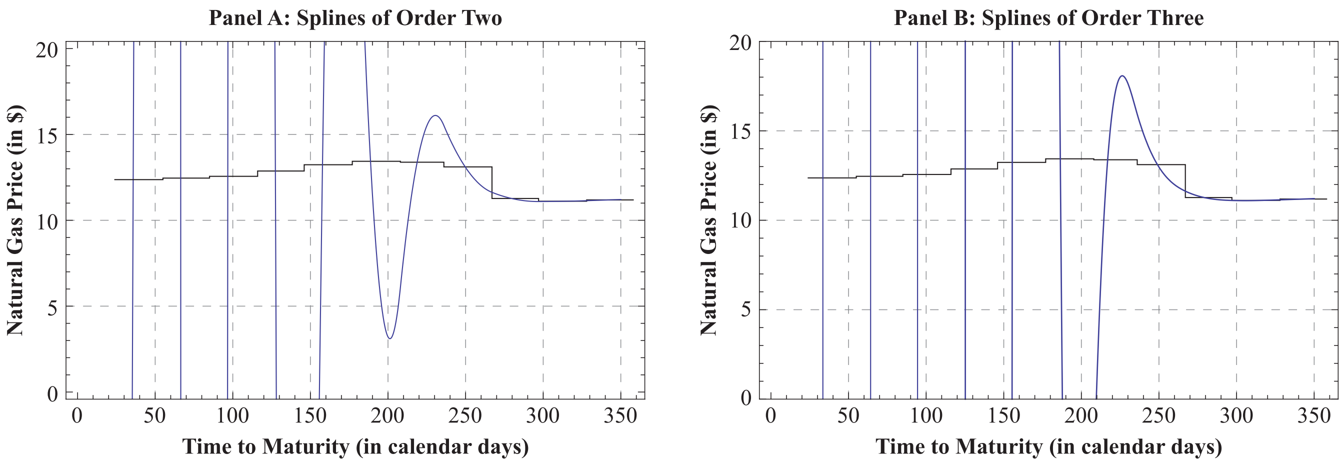
for $i = 1, \ldots, mf$.

We can now obtain the structure of the weighting functions by inserting this linear relation in the spline function

$$\begin{aligned} & a_i + b_i u + c_i u^2 + d_i u^3 + e_i u^4 \\ &= \sum_{j=1}^{mf} \tilde{a}_{ij} F^{(j)} + \sum_{j=1}^{mf} \tilde{b}_{ij} F^{(j)} u + \sum_{j=1}^{mf} \tilde{c}_{ij} F^{(j)} u^2 + \sum_{j=1}^{mf} \tilde{d}_{ij} F^{(j)} u^3 + \sum_{j=1}^{mf} \tilde{e}_{ij} F^{(j)} u^4 \\ &= \sum_{j=1}^{mf} (\tilde{a}_{ij} + \tilde{b}_{ij} u + \tilde{c}_{ij} u^2 + \tilde{d}_{ij} u^3 + \tilde{e}_{ij} u^4) F^{(j)}, \quad i = 1, \ldots, mf \end{aligned}$$

Exhibit A1

Splines without the Maximum Smoothness Criterion



Note: The graphs show two interpolation functions without the maximum smoothness criterion (quadratic in Panel A and cubic splines in Panel B) for real futures price on December 28, 2005.

The role of the maximum smoothness criterion to avoid extreme inter- or extrapolation values can be exemplarily illustrated for quadratic and cubic splines without a maximum smoothness condition (see Exhibit A1).

ENDNOTES

[1]See, for example, Benth and Koekebakker [2008].

[2]Note that Equation (1) could also be stated in the form $f_t(\tau_b, \tau_e) = \int_{\tau_b}^{\tau_e} g(u; \tau_b, \tau_e) e^{\int_t^u (r+q_t(v))dv} s_t du$ with r being the constant instantaneous risk-free interest rate and $q_t(u)$ the market-implied physical storage costs (cost of carry model). In principle, modeling $q_t(u)$ or $f_t(u)$ is equivalent and thus poses the same problem in accounting for delivery periods within a consistent modeling approach.

[3]In real energy markets, futures contracts are settled at discrete dates. This can be easily incorporated by modifying $g(u; \tau_b, \tau_e)$ (see Benth, Koekebakker, and Ollmar [2007] or Benth and Koekebakker [2008]).

[4]In most natural gas markets, futures contracts have fixed delivery periods equal to successive calendar months. In Europe, long-term futures contracts refer to quarters or years.

[5]Note that a simple buy-and-hold strategy cannot be implemented without owning costly physical storage capacities for natural gas.

[6]See http://www.centrica-sl.co.uk.

[7]See, for example, Secomandi [2010].

[8]More generally, trading can be restricted to nonredundant physical delivery contracts with delivery periods that cannot be duplicated by other traded delivery contracts. This is due to the simple no-arbitrage argument that two trading strategies with the same physical delivery flows must have the same expected market value at any point in time. In our market environment, the spot contract is the only nonredundant physical delivery contract, whereas all futures contracts have redundant physical delivery periods.

[9]For a formal proof, see Secomandi [2010] and Unger [2013] in the context of a discrete-time spot price process.

[10]See Boogert and de Jong [2008, 2011] and Neumann and Zachmann [2009].

[11]See, for example, Lai, Margot, and Secomandi [2010]. This is true whether the market modeling approach uses high-dimensional forward models or is based on a low-dimensional representation.

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