広島大学学術情報リポジトリ
Hiroshima University Institutional Repository

Title	Time-resolved measurement of the quantum states of photons using two-photon interference with short-time reference pulses
Auther(s)	Changliang, Ren; Hofmann, Holger Friedrich
Citation	Physical Reviews A , 84 (3) : 32108
Issue Date	2011
DOI	10.1103/PhysRevA.84.032108
Self DOI	
URL	http://ir.lib.hiroshima-u.ac.jp/00034730
Right	(c) 2012 American Physical Society
Relation	

Hiroshima University Institutional Repository

Time-resolved measurement of the quantum states of photons using two-photon interference with short-time reference pulses

Changliang Ren and Holger F. Hofmann*
Graduate School of Advanced Sciences of Matter, Hiroshima University, Kagamiyama 1-3-1, Higashi Hiroshima 739-8530, Japan and JST, CREST, Sanbancho 5, Chiyoda ku, Tokyo 102-0075, Japan
(Received 23 May 2011; published 13 September 2011)

To fully utilize the energy-time degree of freedom of photons for optical quantum-information processes, it is necessary to control and characterize the temporal quantum states of the photons at extremely short time scales. For measurements of the temporal coherence of the quantum states beyond the time resolution of available detectors, two-photon interference with a photon in a short-time reference pulse may be a viable alternative. In this paper, we derive the temporal measurement operators for the bunching statistics of a single-photon input state with a photon from a weak coherent reference pulse. It is shown that the effects of the pulse shape of the reference pulse can be expressed in terms of a spectral filter selecting the bandwidth within which the measurement can be treated as an ideal projection on eigenstates of time. For full quantum tomography, temporal coherence can be determined by using superpositions of reference pulses at two different times. Moreover, energy-time entanglement can be evaluated based on the two-by-two entanglement observed in the coherences between pairs of detection times.

DOI: 10.1103/PhysRevA.84.032108 PACS number(s): 03.65.Wj, 42.50.Dv, 42.65.Re, 03.67.Mn

I. INTRODUCTION

Photons are one of the most versatile systems for the implementation of quantum information processes. The most accessible degrees of freedom are polarization and the spatial degrees of freedom defined by optical paths. In addition, it is possible to encode quantum information in the different frequency modes along the direction of propagation. Encoding information in this energy-time degree of freedom may be more robust against mechanical instabilities than the alternatives and could therefore be especially useful in long-distance quantum communication [1,2].

Energy-time entanglement was first demonstrated by Franson [3] and has been applied in quantum cryptography [4,5] and quantum communication [6–8]. Originally, these approaches are based on comparatively long time scales, so that the necessary time resolution can be achieved by the photon-detection systems directly. However, recent technological developments make it possible to generate entanglement on time scales much shorter than the time resolution of available detectors [9–11]. To fully utilize the potential of such broadband sources of entangled photons, it is necessary to develop measurement systems that can characterize the temporal quantum states of photons on extremely short time scales.

In previous experimental work, short-time correlations between entangled photons were confirmed by observing the rate of two-photon absorption in second harmonic generation, effectively reversing the role of the source to serve as a detector [12,13]. However, the measurement information so obtained is insufficient for a complete characterization of the quantum state. More detailed information can be obtained by the two-photon interference between a pair of down-converted photons, where the shape of the Hong-Ou-Mandel dip can provide spectral information about the photon pairs [14,15]. However, this method only demonstrates that the photons have some time dependent correlation, without deciding whether this correlation is between detection times or represent a time dependence of energy, for example, in chirped pulses.

An independent time standard can be realized by using short-time reference pulses from a coherent laser source. If the coherent light is not phase locked with the signal light, the randomization of the relative phase converts the coherent superposition into a mixture of photon number states. For sufficiently attenuated laser light, the reference pulses can then be treated as a mixture of the vacuum and a single-photon state with the spectral and temporal properties of the reference laser. We can then utilize the achievements of ultrafast optics for measurements on single-photon states. Specifically, it is possible to use the photon bunching effect of the two-photon interference between the signal photon and one photon from the coherent reference pulse to achieve a time-resolved measurement at the time scale determined by the pulse time of the reference pulse. Intuitively, bunching should only occur when the reference photon and the input photon coincide in time. However, bunching is also sensitive to the temporal coherence of the reference pulse. To optimize the information gained about the state of the input photon, it is essential to understand these effects of temporal coherences in the reference pulses on the bunching statistics. In the following, we therefore analyze the quantum statistics obtained from appropriate short-time reference pulses and explore the possibilities of quantum tomography and entanglement verification in the time domain based on direct measurements of temporal coherence by two-photon interference with reference photons in the appropriate temporal states.

In Sec. II, we describe the coincidence rates in the output ports of the two-photon interference in terms of a projective measurement operator defined by the optical quantum state of the photon from the reference pulse. In Sec. III, the effects of time delays used to scan the temporal features of the signal photon state are considered. Significantly, the measurement

*hofmann@hiroshima-u.ac.jp

 ©2011 American Physical Society

is equivalent to the projection on an ideal eigenstate of time if the bandwidth of the input state is narrower than that of the reference pulse. It is therefore possible to optimize the reference pulse shape according to the basic optical properties of the input. In Sec. IV, we show how a superposition of reference pulses at two times can be used to obtain the temporal coherence of the input state. We can then achieve full quantum tomography in the time domain. In Sec. V, we apply the results to derive an experimental criterion for the verification of the temporal entanglement of down-converted photon pairs.

II. TWO-PHOTON INTERFERENCE WITH A REFERENCE PHOTON

Photon bunching of the Hong-Ou-Mandel type is observed when two photons are incident on a 50:50 beam splitter from opposite sides. If the photons share the same temporal and transverse coherence, two-photon interference eliminates the possibility that they will exit the beam splitter on opposite sides. However, any mismatch in the temporal or transverse coherence will result in the appearance of photons exiting at opposite sides. Coincidence detection of photons on opposite sides of the beam splitter is therefore a direct measure of the mismatch between the quantum states of the photons that entered the beam splitter.

To obtain temporal features of a signal photon, we consider the use of a reference photon from a short-time pulse, as illustrated in Fig. 1. The transformation of the light field at the beam splitter can be represented by the transformation of the temporal annihilation operators,

$$\hat{b}_1(t) = \frac{1}{\sqrt{2}}[\hat{a}_1(t) + \hat{a}_2(t)],$$
$$\hat{b}_2(t) = \frac{1}{\sqrt{2}}[\hat{a}_1(t) - \hat{a}_2(t)], \tag{1}$$

where $\hat{a}_1(t)$ and $\hat{a}_2(t)$ are the input field operators and $\hat{b}_1(t)$ and $\hat{b}_2(t)$ are the output field operators for a given input time t. Since the photon in input port 1 originates from the reference pulse, its input state is known and given by the reference state $|\Phi_{\text{ref}}\rangle$. In the single-photon time basis, this state is represented by

$$|\Phi_{\text{ref}}\rangle = \int \phi(t)\hat{a}_1^\dagger(t)dt|0\rangle, \tag{2}$$

where $\phi(t) = \langle t|\Phi_{\text{ref}}\rangle$ is the time dependence of the reference pulse amplitude. Usually, the reference pulse will be generated

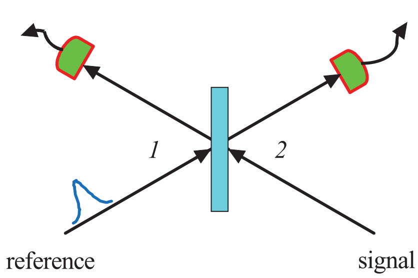

FIG. 1. (Color online) Illustration of a bunching measurement of an unknown photon state using a short-time reference photon and a 50:50 beam splitter.

by weakening a coherent pulse emitted from a laser to a small amplitude of $\alpha \ll 1$. As a result, two-photon bunching only occurs with a probability of $|\alpha|^2$, corresponding to the probability of finding a reference photon in the weakened pulse. Although it would seem desirable to make $|\alpha|^2$ as large as possible, care must be taken to avoid the unintended effects of generating two or more reference photons, since these components can alter the pattern of two-photon correlations obtained in the multiphoton interference.

The two-photon interference effects in the output of the beam splitter can be observed by coincidence detections of one photon in the output mode $\hat{b}_1$ and one photon in the output mode $\hat{b}_2$. The experimental rate of coincidence counts observed with reference intensities of $|\alpha|^2$ and detectors of quantum efficiency η is given by

$$G^{(2)}(t_1,t_2) = |\alpha|^2\eta^2\langle\hat{b}_1^\dagger(t_1)\hat{b}_2^\dagger(t_2)\hat{b}_2(t_2)\hat{b}_1(t_1)\rangle. \tag{3}$$

The rate of coincidence counts therefore depends on an expectation value of the input state defined by the reference and the signal. Using the beam-splitter transformation Eq. (1) and the pulse shape of the reference pulse given by Eq. (2), this expectation value can be expressed in terms of a projection operator on the single-photon state in the signal port 2. The projection operator for coincidence counts at times t_1 and t_2 can then be given by

$$\begin{aligned}\hat{\Pi}_s(t_1,t_2) = \frac{|\alpha|^2\eta^2}{4}[&\phi(t_1)\phi^*(t_1)|t_2\rangle\langle t_2| - \phi(t_1)\phi^*(t_2)\\ &\times|t_2\rangle\langle t_1| - \phi(t_2)\phi^*(t_1)|t_1\rangle\langle t_2| + \phi(t_2)\phi^*(t_2)\\ &\times|t_1\rangle\langle t_1|].\end{aligned} \tag{4}$$

However, this measurement operator assumes perfect time resolution of the detection. In realistic photon detectors, the detection times that can be resolved are usually much longer than the pulse times of the available reference pulses. We can therefore assume that the photon time will be completely unknown, so that the actual measurement operator of the photon coincidence is obtained by integrating over all measurement times,

$$\begin{aligned}\hat{M}_s &= \int\int \hat{\Pi}_s(t_1,t_2)dt_1dt_2\\ &= |\alpha|^2\eta^2\left(\frac{1}{2} - \frac{1}{2}|\Phi_{\text{ref}}\rangle\langle\Phi_{\text{ref}}|\right).\end{aligned} \tag{5}$$

Thus, the observation of coincidence counts from two-photon interferences between a weak coherent reference pulse and an unknown source of individual photons provides measurement information on the single-photon states in the input corresponding to a projection on the photon wave function described by the classical coherence of the reference. We can therefore exploit the control over temporal coherence achieved in laser systems directly in the characterization of temporal features of single-photon sources and entangled-photon states.

In most practical cases, the input state will not be an ideal single-photon state, but a spontaneously generated photon mixed with or superimposed onto a large vacuum component. It is therefore useful to consider the effects of a limited emission efficiency r for the signal photons. If $r \gg |\alpha|^2$, the two-photon interference is simply weakened by the additional factor of r, as two-photon interference is only observed when a source photon becomes available. However, low-efficiency sources would then limit $|\alpha|^2$ to extremely low levels, so that it may be difficult to achieve acceptable count rates. In this case, it may be possible to use higher levels of reference inputs, accepting the possibility of registering two-photon coincidences from photon pairs originating only from the reference. Since these events show no two-photon interferences, they result in a constant background of coincidence rates given by $|\alpha|^4\eta^2/4$. A similar background can originate if there is a possibility of finding two photons in the input. However, this probability depends on the precise photon statistics of the source and needs to be evaluated based on a separate characterization of bunching or antibunching characteristics [16].

Experimentally, the total effect of detector efficiency η, reference intensity $|\alpha|^2$, and emission efficiency of the source r can be determined by measuring the coincidence rates for a reference pulse that is definitely orthogonal to the source state. If the multiphoton background is negligibly low, it is then possible to normalize all other detection rates with this factor to identify the actual quantum state overlap of signal and reference.

III. TIME-RESOLVED MEASUREMENT AND BANDWIDTH LIMITATION

Time-resolved measurements can be realized by scanning the peak time of the reference pulse using an appropriate time delay. To represent this time delay in our formalism, we can simply shift the temporal wave function, so that the amplitude at input time t' for a delay time of t corresponds to

$$\langle t'|\Phi(t)\rangle = \phi(t' - t). \tag{6}$$

Alternatively, this time shift can be expressed by a phase factor in the frequency representation,

$$\langle\omega|\Phi(t)\rangle = e^{i\omega t}\langle\omega|\Phi(0)\rangle. \tag{7}$$

An ideal measurement of time would project on the state $|t\rangle$ defined by a delta function in time or, in the frequency representation, by

$$\langle\omega|t\rangle = \frac{1}{\sqrt{2\pi}}e^{i\omega t}. \tag{8}$$

The comparison of real state (7) and ideal state (8) shows that the difference can be expressed by a time-shift independent attenuation of the frequency components, so that

$$|\Phi(t)\rangle = \left(\int \sqrt{2\pi}\,\langle\omega|\Phi(0)\rangle\;\; |\omega\rangle\langle\omega|\mathrm{d}\omega\right)|t\rangle{=}\hat{F}\;|t\rangle. \tag{9}$$

The problem of time resolution can therefore be expressed completely in terms of the filter operation $\hat{F}$ that converts the ideal time eigenstate into the reference pulse. The measurement operator for a reference time t is given by

$$\hat{M}_s(t) = |\alpha|^2\eta^2\big(\tfrac{1}{2} - \tfrac{1}{2}\hat{F}|t\rangle\langle t|\hat{F}^\dagger\big). \tag{10}$$

To determine the overlap between $|\Phi(t)\rangle$ and an unknown input state $|\psi\rangle$, we can first apply the adjoint filter operator $\hat{F}^\dagger$ to modify the frequency amplitudes of the input state. The overlap of the original state with the reference state is then equal to the overlap of the modified state with an eigenstate of time.

Experimentally, this means that it is not necessary to know the details of the reference pulse shape if the distortions of the input state by the filter operator $\hat{F}$ are negligibly small. This limit is achieved if the spectral width of the pulse is sufficiently broader than the spectral width of the signal photons, and there is no significant chirp within the bandwidth of the signal input. It is then possible to interpret the raw data directly as projections onto eigenstates of time. On the other hand, the temporal resolution problem is now represented in detail by the eigenvalues of the filter operator $\hat{F}^\dagger$. These eigenvalues are equal to $\sqrt{2\pi}$ times the complex conjugate of the frequency amplitude $\langle\omega|\Phi(0)\rangle$ of the reference pulse. The effects of the reference pulse on the time resolution can then be understood in terms of dispersion and transmission of the input state before a fully time-resolved detection. The dispersion component is given by the chirp of the reference pulse and can be avoided by using transform-limited pulses with real and positive amplitudes. The transmission component is given by the absolute values of the frequency amplitudes and describe the effects of a dispersion-free band-pass filter.

In general, any kind of reference pulse can be used to obtain temporal and spectral information. For the purpose of quantum state reconstruction, the method is only limited by the spectral width of the reference pulse, since the signal-to-noise ratio of data concerning frequency components with very low amplitudes in the reference pulse will prevent a reliable reconstruction. However, it may be convenient to design a measurement so that no additional data processing is needed. In this case, the reference pulse should be approximately rectangular, resulting in constant transmission of a specific bandwidth. The temporal measurement will then be completely accurate for all input fields within that bandwidth, and photon bunching can provide direct experimental evidence for the temporal shape of the photon states.

It may also be worth noting that the depth of the dip in the coincidence count rate described by the measurement operator in Eq. (10) depends on the bandwidth $\Delta\omega$ of the reference pulse. For a rectangular spectrum of the reference pulse, the average number of coincidence counts N_c from an input state $|\psi\rangle$ with a frequency spectrum entirely within the bandwidth of $\Delta\omega$ is given by

$$N_c = |\alpha|^2\eta^2\left(\frac{1}{2} - \frac{\pi}{\Delta\omega}|\langle t|\psi\rangle|^2\right). \tag{11}$$

To optimize the signal-to-noise ratio of the measurement, it is therefore desirable to use the minimal bandwidth acceptable for the respective input states.

IV. MEASUREMENT OF TEMPORAL COHERENCE

As shown in the previous section, the probability distribution of photons in time can be obtained from the photon bunching characteristics of two-photon interference with photons in a single short-time reference pulse. In terms of the temporal density matrix, we can then reconstruct the diagonal elements $\langle t|\hat{\rho}|t\rangle$. A complete tomographic reconstruction of an unknown quantum state can be achieved by determining

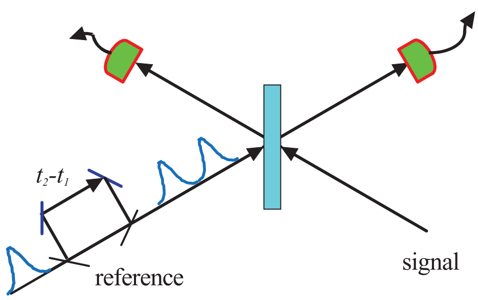

FIG. 2. (Color online) Setup for the measurement of temporal coherence using coherent superpositions of reference pulses at two different times, t_1 and t_2.

all elements $\langle t_1|\hat{\rho}|t_2\rangle$, including the off-diagonal elements that describe coherences between two different times t_1 and t_2.

Since photon bunching with a reference corresponds to a projection on the state of this reference, coherences between two times can be probed by using superpositions of two short-time pulses as a reference. The setup for this kind of measurement is shown in Fig. 2. The coherent reference is generated by interference between the original pulse and a time-delayed pulse, so that the reference is in an equal superposition of input times t_1 and t_2,

$$|\Phi_{\text{ref}}\rangle = \frac{1}{\sqrt{2}}\hat{F}\left(|t_1\rangle + |t_2\rangle\right), \tag{12}$$

where we used Eq. (9) to express the superposition in terms of time eigenstates and the filter operator $\hat{F}$ representing the permitted bandwidth.

Unfortunately, the phase in this superposition is fixed to zero, because the shape of the initial reference pulse defines the precise phase at peak time t_i. To achieve complete quantum tomography, we need to consider the possibility of obtaining phase information from the dependence of coherence on the time difference between t_1 and t_2. Since our measurement has a limited bandwidth centered around a carrier frequency of ω_0, we can assume that small shifts in the time difference do result in a corresponding phase shift $\varphi = \Delta t/\omega_0$. Although the time eigenstates $|t\rangle$ and $|t + \frac{\varphi}{\omega_0}\rangle$ are orthogonal, they are nearly indistinguishable within the bandwidth considered, as long as φ is sufficiently smaller than $\omega_0/\Delta\omega$. Specifically, the application of the filter function $\hat{F}$ ensures that

$$\hat{F}\left|t + \frac{\varphi}{\omega_0}\right\rangle \approx e^{-i\varphi}\hat{F}|t\rangle. \tag{13}$$

Arbitrary phase shifts can then be realized with $-\pi \leqslant \varphi \leqslant \pi$. In this range, it is usually reasonable to consider the above approximation as valid, so that the state $|t + \frac{\varphi}{\omega_0}\rangle$ can be replaced by $e^{-i\varphi}|t\rangle$. A quantitative test of this assumption can be obtained from the normalized overlap σ of the filtered states,

$$\sigma = \frac{\left|\langle t|\hat{F}^\dagger\hat{F}\left|t + \frac{\varphi}{\omega_0}\right\rangle\right|^2}{\langle t|\hat{F}^\dagger\hat{F}|t\rangle\left\langle t + \frac{\varphi}{\omega_0}\right|\hat{F}^\dagger\hat{F}\left|t + \frac{\varphi}{\omega_0}\right\rangle}. \tag{14}$$

In most cases, σ will be close to one for the whole range of phase shifts. It is then safe to assume that we can generate superpositions of different times with arbitrary phases. The measurement operators obtained with these superpositions read

$$\hat{M}_s(t_1,t_2,\varphi) = |\alpha|^2\eta^2\left[\frac{1}{2} - \frac{1}{4}\hat{F}\left(|t_1\rangle\langle t_1| + e^{-i\varphi}|t_1\rangle\langle t_2| + e^{i\varphi}|t_2\rangle\langle t_1| + |t_2\rangle\langle t_2|\right)\hat{F}^\dagger\right]. \tag{15}$$

Following the discussion at the end of Sec. III, it is possible to identify the coincidence rates directly with the coherences between times t_1 and t_2 if the spectrum of the reference pulses that determines $\hat{F}$ is sufficiently broad. The phase dependence of the coincidence counts then represent direct experimental evidence for the temporal coherence between two specific times. Consequently, the temporal representation of the density matrix can be obtained without further data processing, simply by scanning the detection times t_1, t_2. Oppositely, it is always possible to correct the effects of temporal resolution of the reference pulse by first determining the filtered density matrix from the coincidence counts, followed by a mathematical deconvolution of the effects of the filter function $\hat{F}$. Assuming that the eigenvalues of $\hat{F}$ are nonzero, such an inversion of the filter operation is achieved by Fourier transforming the temporal representation and multiplying with the appropriate inverse eigenvalues. In practice, the resolution is then limited by the statistical noise that makes reconstruction of frequencies with very low transmission coefficients very difficult and unreliable. Moreover, a reliable characterization of the reference pulse is necessary to determine the correct reconstruction formalism. Nevertheless, the method is quite general and can be applied to a variety of reference pulse shapes, achieving temporal resolutions that might actually exceed the bandwidth limit if sufficiently large numbers of counts can be obtained.

V. VERIFICATION OF ENERGY-TIME ENTANGLEMENT

In principle, it is a straightforward matter to apply the quantum tomography scheme described in the previous section to photon pairs in energy-time entangled states. As shown in Fig. 3, it is merely necessary to extend the two-photon coincidence measurements for a signal photon and a reference photon to the four-photon coincidences observed for two signal photons with two separate references. The measurement operator of the fourfold coincidence then reads

$$\hat{M}_p(t_{1A},t_{2A},\varphi_A;t_{1B},t_{2B},\varphi_B) = \hat{M}_s(t_{1A},t_{2A},\varphi_A) \otimes\hat{M}_s(t_{1B},t_{2B},\varphi_B), \tag{16}$$

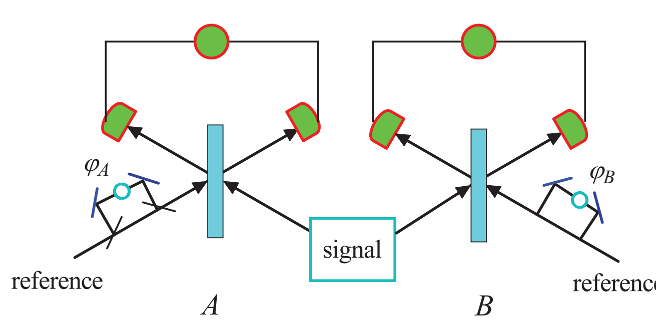

FIG. 3. (Color online) Setup for correlated measurements on energy-time entangled-photon pairs.

where the operators $\hat{M}_s$ describe the single-photon results according to Eq. (15). This operator can be further separated into a positive background, negative contributions from local bunching at either A or B, and a positive term from the correlation between bunching effects. Since only the correlations between bunching effects are required for the characterization of entanglement, we can focus on the last term, which depends only on the projection on the product of the two references in A and in B,

$$|\Phi_A;\Phi_B\rangle = \frac{1}{2}\hat{F}\otimes\hat{F}(|t_{1A},t_{1B}\rangle + e^{i\varphi_A}|t_{2A},t_{1B}\rangle + e^{i\varphi_B}|t_{1A},t_{2B}\rangle + e^{i(\varphi_A+\varphi_B)}|t_{2A},t_{2B}\rangle). \quad (17)$$

If the spectral width of the reference pulse is broad enough, the correlation of coincidences for temporal coherences between times t_{1A},t_{2A} in path A and t_{1B},t_{2B} in path B can be interpreted as temporal quantum statistics for well-defined photon times. The projection then exists in a discrete four-dimensional subspace of Hilbert space that refers exclusively to the entanglement between the selected times in path A and path B. In this two-by-two Hilbert space, the projector of the state can be given by the matrix

$$\hat{P}(\varphi_A,\varphi_B) = \frac{1}{4}\begin{pmatrix} 1 & e^{-i\varphi_B} & e^{-i\varphi_A} & e^{-i(\varphi_A+\varphi_B)} \\ e^{i\varphi_B} & 1 & e^{-i(\varphi_A-\varphi_B)} & e^{-i\varphi_A} \\ e^{i\varphi_A} & e^{i(\varphi_A-\varphi_B)} & 1 & e^{-i\varphi_B} \\ e^{i(\varphi_A+\varphi_B)} & e^{i\varphi_A} & e^{i\varphi_B} & 1 \end{pmatrix}, \quad (18)$$

where the basis states are $\{|t_{1A},t_{1B}\rangle,|t_{1A},t_{2B}\rangle,|t_{2A},t_{1B}\rangle,|t_{2A},t_{2B}\rangle\}$. By varying the phases in A and in B, it is possible to distinguish contributions with different phase factors. In particular, it is possible to identify the coherence between $|t_{1A},t_{1B}\rangle$ and $|t_{2A},t_{2B}\rangle$ from the coincidence data, since it is the only term that depends on the sum of φ_A and φ_B. This term describes a genuine two-photon coherence and is particularly useful for the verification of temporal entanglement.

For single-photon states, the coherence between $|t_1\rangle$ and $|t_2\rangle$ is limited by $|\langle t_1|\hat{\rho}|t_2\rangle| \leqslant 1/2$. Therefore, the two-photon coherence between $|t_{1A},t_{1B}\rangle$ and $|t_{2A},t_{2B}\rangle$ achieved by separable states is limited to

$$|\langle t_{1A},t_{1B}|\hat{\rho}_{\text{sep}}|t_{2A},t_{2B}\rangle| \leqslant \frac{1}{4}. \quad (19)$$

For entangled states, the same coherence can be as high as 1/2. It is therefore possible to verify temporal entanglement directly by detecting values of $|\langle t_{1A},t_{1B}|\hat{\rho}|t_{2A},t_{2B}\rangle| > 1/4$ at a fixed set of detection times. In typical experiments using parametric down-conversion, photon pairs will be emitted simultaneously, so entanglement can be verified for $t_{1A} = t_{1B}$ and $t_{2A} = t_{2B}$. For time differences of $t_2 - t_1$ smaller than the correlation time of the down-conversion process, single-photon coherences will emerge between t_1 and t_2, resulting in a reduction of the observed entanglement. This transition between two-photon coherence and single-photon coherences can be used to identify the time scale of the entanglement. Specifically, it is possible to plot the dependence of the coherence $|\langle t_1,t_1|\hat{\rho}|t_2,t_2\rangle|$ on the time difference $t_2 - t_1$. For typical down-conversion sources, we then expect a value around 1/4 at $t_2 - t_1 = 0$ that gradually rises until it achieves a level of 1/2 for large time differences. The time scale of entanglement is then given by the temporal width of the rise from 1/4 to 1/2. In the spirit of linewidth definitions based on the half width at half maximum, it may therefore be reasonable to define the time scale achieved by broadband entanglement sources as the time difference $t_2 - t_1$ at which the coherence $|\langle t_1,t_1|\hat{\rho}|t_2,t_2\rangle|$ crosses the value of 3/8, halfway between maximal entanglement and separability. Thus, bunching with coherent superpositions of short-time reference pulses provides an experimental method of identifying the ultrashort quantum correlation times of broadband entanglement sources by direct detection of the coherences between well-defined photon arrival times.

VI. CONCLUSIONS

We have shown how two-photon interference with photons from a weak coherent reference pulse can be used to detect the temporal coherence of an unknown input photon. Specifically, the raw data obtained from coincidence counts in the output ports is directly proportional to the projection probabilities of eigenstates of time and their corresponding superpositions. The limitation of time resolution caused by the finite pulse duration of the reference pulse can be accounted for in terms of a filter function that eliminates frequencies outside of the bandwidth of the reference pulse and introduces a dispersion effect for chirped references. To obtain an optimal signal-to-noise ratio, it is desirable to make this bandwidth as narrow as possible without losing too much of the input signal.

Two-time coherences can be evaluated using superpositions of two pulses shifted to appropriate times. The phase relation between these pulses can be controlled in the conventional manner by time shifts shorter than one period of the central frequency of the reference pulse. It is then possible to obtain direct experimental evidence of the coherence between two specific times. By scanning the detection times in the superposition, the temporal density matrix can be mapped completely in terms of the dependence of count rates on the phase shifts induced by short-time shifts. Significantly, the temporal density matrix is recovered directly from the amplitude and the phase shifts of the temporal interference fringes observed in the coincidence counts. Thus, density-matrix elements can be determined from the measurement results without further data processing. However, further data processing can be used to optimize the temporal resolution of the method by inverting the effect of the spectral properties of the reference pulse. It is then possible to achieve temporal quantum tomography at temporal resolutions that may even exceed the limit set by the bandwidth of the reference pulses.

To evaluate the time scales of temporal entanglement, it may be particularly useful that the present method makes possible a direct experimental determination of temporal two-photon coherences by fourfold coincidences between the entangled photons and two independent reference pulses. The amount of entanglement observed in the two-photon coherences at specific coherence times can therefore be evaluated from a comparatively small set of coincidence rates. It is then possible to obtain the entanglement at different coherence times in separate measurements, providing an efficient method of experimentally confirming the transition from single-photon coherence to two-photon coherence that is characteristic for short-time entanglement.

In conclusion, photon bunching with short-time references may prove to be a helpful addition to the available experimental methods for the study of energy-time entanglement and related quantum effects on ultrashort timescales. Our results show that the experimental count rates obtained in such measurements correspond directly to the density-matrix elements in the time basis and therefore provide an intuitively clear image of temporal quantum coherence without any need for further data processing. On the other hand, our analysis also shows how the sensitivity of the method can be improved by further data processing, if details of the reference pulse shapes are taken into account. The present work may thus lay the foundations for detailed experimental investigations of the energy-time degrees of freedom in multiphoton quantum statistics.

ACKNOWLEDGMENTS

Part of this work has been supported by the Grant-in-Aid program of the Japanese Society for the Promotion of Science (JSPS).

[1] H. Zbinden, J. Brendel, N. Gisin, and W. Tittel, Phys. Rev. A **63**, 022111 (2001).
[2] W. Tittel, J. Brendel, H. Zbinden, and N. Gisin, Phys. Rev. Lett. **81**, 3563 (1998).
[3] J. D. Franson, Phys. Rev. Lett. **62**, 2205 (1989).
[4] W. Tittel, J. Brendel, H. Zbinden, and N. Gisin, Phys. Rev. Lett. **84**, 4737 (2000).
[5] G. Ribordy, J. Brendel, J.-D. Gauthier, N. Gisin, and H. Zbinden, Phys. Rev. A **63**, 012309 (2000).
[6] J. Brendel, N. Gisin, W. Tittel, and H. Zbinden, Phys. Rev. Lett. **82**, 2594 (1999).
[7] I. Marcikic, H. de Riedmatten, W. Tittel, V. Scarani, H. Zbinden, and N. Gisin, Phys. Rev. A **66**, 062308 (2002).
[8] I. Marcikic, H. de Riedmatten, W. Tittel, H. Zbinden, and N. Gisin, Nature (London) **421**, 509 (2003).
[9] S. E. Harris, Phys. Rev. Lett. **98**, 063602 (2007).
[10] M. B. Nasr, S. Carrasco, B. E. A. Saleh, A. V. Sergienko, M. C. Teich, J. P. Torres, L. Torner, D. S. Hum, and M. M. Fejer, Phys. Rev. Lett. **100**, 183601 (2008).
[11] M. Hendrych, X. Shi, A. Valencia, and J. P. Torres, Phys. Rev. A **79**, 023817 (2009).
[12] S. Sensarn, I. Ali-Khan, G. Y. Yin, and S. E. Harris, Phys. Rev. Lett. **102**, 053602 (2009).
[13] K. A. O'Donnell and A. B. U'Ren, Phys. Rev. Lett. **103**, 123602 (2009).
[14] C. K. Hong, Z. Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987).
[15] R. Okamoto, S. Takeuchi, and K. Sasaki, Phys. Rev. A **74**, 011801(R) (2006).
[16] A more detailed analysis of multiphoton errors in the two-photon interference between single-photon states and weak coherent pulses can be found in P. Kolenderski and K. Banaszek, Proceedings of IEEE 2008 6th National Conference on Telecommunication Technologies and IEEE 2008 2nd Malaysia Conference on Photonics, NCTT-MCP 2008, art. no. 4814246, pp. 94-97.