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Relation	

Quantum correction to the tiny vacuum expectation value in the two-Higgs-doublet-model for the Dirac neutrino mass

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We study a Dirac neutrino mass model of Davidson and Logan. In the model, the smallness of the neutrino mass is originated from the small vacuum expectation value of the second Higgs of two Higgs doublets. We study the one-loop effective potential of the Higgs sector and examine how the small vacuum expectation is stable under the radiative correction. By deriving formulas of the radiative correction, we numerically study how large the one-loop correction is and show how it depends on the quadratic mass terms and quartic couplings of the Higgs potential. The correction changes depending on the various scenarios for extra Higgs mass spectrum.

I. INTRODUCTION

The smallness of the neutrino mass compared with the other quarks and leptons is one of the mysteries of nature. Recently, a new mechanism generating small Dirac mass terms for neutrino has been proposed [1–3]. The similar mechanism generating the small neutrino Dirac mass term for the TeV seesaw mechanism is also proposed in [4] and phenomenology is studied in [5,6]. There are also models with radiatively generated Dirac mass term in [7,8]. The interesting feature of the model proposed in [1,2] is the tiny vacuum expectation value for an extra Higgs SU(2) doublet [9]. The small neutrino mass is realized without introducing tiny Yukawa coupling for neutrinos. A softly broken global U(1) symmetry guarantees the tiny vacuum expectation value for the extra doublet. In addition to the small softly breaking mass parameter, the mass squared parameter for the extra Higgs is chosen to be positive so that the light pseudo Nambu-Goldstone bosons due to the softly broken global symmetry do not appear. This is a contrast to the mass squared parameter for the standard model like Higgs boson.

In the present paper, we study the global minimum of the tree level Higgs potential by explicitly solving the stationary conditions. There are many studies of the tree level Higgs potential of general two Higgs doublet model [10–15]. (See also [16] for recent review of two Higgs doublet model). It has been shown that the charge neutral vacuum is lower than the charge breaking vacuum [10]. Also, the vacuum energy difference of two neutral minima was derived [12,14]. We make use of the results and identify the vacuum of the present model. When the U(1) symmetry breaking term is turned off, the tree level Higgs potential and the phase structure of the present model is rather similar to the model with Z_2 discrete symmetry [17,18]. In contrast to Z_2 symmetric case, it is essential to keep the soft breaking term when finding the true vacuum. If we set the symmetry-breaking term at zero, then the order parameter corresponding to the softly broken U(1) symmetry becomes redundant parameter and can not be determined. We treat the soft breaking term as small expansion parameter and obtain the vacuum expectation values and the vacuum energies in terms of the parameters of the Higgs potential.

The constraints on the parameters of the model for which the desired vacuum can be realized are derived and they are rewritten in terms of Higgs masses and a few coupling constants, which can not be directly related to the Higgs masses. These constraints are fully used when we study the radiative corrections to the vacuum expectation values numerically.

Beyond the tree level, we study the radiative correction to the Higgs potential and the vacuum expectation values of Higgs. Since the neutrino masses are proportional to the vacuum expectation value of one of Higgs, one can also compute the radiative corrections to neutrino masses. As already noted in [1], the radiative correction to the softly breaking mass parameter is logarithmically divergent and it is renormalized multiplicatively. We derive the formulas for the one-loop corrected vacuum expectation values for two Higgs doublets by studying one-loop corrected effective potential. The corrections are evaluated numerically by exploring the parameter regions allowed from the global minimum condition for the vacuum. We show how the radiative corrections change depending on the extra Higgs spectrum. The radiative corrections are also evaluated for the case that a relation among the coupling constants is satisfied.

The paper is organized as follows. In Sec. II, we derive the condition for the desired vacuum being global minimum. In Sec. III, one-loop effective potential is derived, and one-loop corrections to the vacuum expectation values are obtained in Sec. IV. In Sec. V, the corrections are evaluated numerically for various choices of parameters of the Higgs potential. Section VI is devoted to summary and discussion.

II. MODEL FOR DIRAC NEUTRINO WITH A TINY VACUUM EXPECTATION VALUE

The model of the Dirac neutrino is proposed in [1]. In [1], two Higgs SU(2) doublets are introduced,

$$\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1^1 + i\phi_1^2 \\ \phi_1^3 + i\phi_1^4 \end{pmatrix}, \qquad \Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_2^1 + i\phi_2^2 \\ \phi_2^3 + i\phi_2^4 \end{pmatrix}, \quad (1)$$

where Φ_1's vacuum expectation value is nearly equal to the electroweak breaking scale and the second Higgs Φ_2 has a small vacuum expectation value, which gives rise to neutrino mass. The Higgs potential in [1] is:

$$V_{\text{tree}} = \sum_{i=1,2}\left(m_{ii}^2\Phi_i^\dagger\Phi_i + \frac{\lambda_i}{2}(\Phi_i^\dagger\Phi_i)^2\right) - (m_{12}^2\Phi_1^\dagger\Phi_2 + \text{H.c.})$$

$$+ \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4|\Phi_1^\dagger\Phi_2|^2. \quad (2)$$

U(1)′ charge is assigned to the second Higgs. The U(1)′ global symmetry is broken softly with the term m_{12}^2. In this paper, we introduce the following real O(4) representation for each doublet, because this parametrization is convenient when computing the one-loop corrected effective potential.

$$\phi_1^a = \begin{pmatrix} \phi_1^1 \\ \phi_1^2 \\ \phi_1^3 \\ \phi_1^4 \end{pmatrix}, \qquad \phi_2^a = \begin{pmatrix} \phi_2^1 \\ \phi_2^2 \\ \phi_2^3 \\ \phi_2^4 \end{pmatrix}, \qquad \tilde{\phi}_1^a = \begin{pmatrix} -\phi_1^2 \\ \phi_1^1 \\ -\phi_1^4 \\ \phi_1^3 \end{pmatrix}. \quad (3)$$

Using the notation above, the tree level effective potential introduced in Eq. (2) can be written as:

$$V_{\text{tree}} = m_{11}^2\frac{1}{2}\sum_{a=1}^4(\phi_1^a)^2 + m_{22}^2\frac{1}{2}\sum_{a=1}^4(\phi_2^a)^2$$

$$- m_{12}^2\sum_{a=1}^4\phi_1^a\phi_2^a + \frac{\lambda_1}{8}\left(\sum_{a=1}^4\phi_1^{a2}\right)^2 + \frac{\lambda_2}{8}\left(\sum_{a=1}^4\phi_2^{a2}\right)^2$$

$$+ \frac{\lambda_3}{4}\left(\sum_{a=1}^4\phi_1^{a2}\right)\left(\sum_{a=1}^4\phi_2^{a2}\right) + \frac{\lambda_4}{4}\left(\left(\sum_{a=1}^4\phi_1^a\phi_2^a\right)^2\right.$$

$$+ \left.\left(\sum_{a=1}^4\tilde{\phi}_1^a\phi_2^a\right)^2\right), \quad (4)$$

where one can choose m_{12}^2 real and positive. With the notation of Eq. (3), the softly broken global symmetry U(1)′ corresponds to the following transformation on ϕ_2^a:

$$\phi_2' = O_{\text{U(1)}'}\phi_2$$

$$= \begin{pmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & \cos\phi & -\sin\phi \\ 0 & 0 & \sin\phi & \cos\phi \end{pmatrix}\phi_2. \quad (5)$$

ϕ_1 does not transform under U(1)′. Therefore, U(1)′ is broken softly when m_{12}^2 does not vanish. Without loss of

generality, one can choose the vacuum expectation values of Higgs with the form given as

$$\langle\phi_1\rangle = \begin{pmatrix} 0 \\ 0 \\ v\cos\beta \\ 0 \end{pmatrix}, \qquad \langle\phi_2\rangle = \begin{pmatrix} v\sin\beta\sin\alpha\cos\theta' \\ -v\sin\beta\sin\alpha\sin\theta' \\ v\sin\beta\cos\alpha\cos\theta' \\ -v\sin\beta\cos\alpha\sin\theta' \end{pmatrix}, \quad (6)$$

where the range for θ' is $[0, 2\pi)$ and the range for β and α is $[0, \frac{\pi}{2}]$. We call the four order parameters as $\varphi_I = (v, \beta, \alpha, \theta')$, $(I = 1, 2, 3, 4)$. When m_{12} vanishes, by taking $\phi = \theta'$ in Eq. (5), one can rotate θ' away in Eq. (6). For the most general case, in total, there are four independent order parameters when U(1)′ symmetry is broken.

For completeness of our discussion, we give the constraints on the quartic couplings from condition that the tree level potential is the bounded below[1,10,19]:

$$\lambda_1 > 0, \qquad \lambda_2 > 0, \quad (7)$$

$$-\sqrt{\lambda_1\lambda_2} \le \lambda_3, \quad (8)$$

$$-\sqrt{\lambda_1\lambda_2} \le \lambda_3 + \lambda_4. \quad (9)$$

In addition to the conditions on the quartic terms, one can constrain the parameters, including the quadratic terms so that the desired vacuum satisfies the global minimum conditions of the potential. About the global minimum of the tree potential, it was shown that the energy of charge neutral vacuum is lower than that of the charge-breaking vacuum [10]. We therefore set α zero. We also require the vacuum expectation value of the second Higgs is much smaller than that of the first Higgs, which implies that $\tan\beta$ is small. In terms of the parametrization in Eq. (6) with $\alpha = 0$, the potential can be written as

$$V_{\text{tree}}(v, \beta, \theta') = A(\beta)v^4 + B(\beta, \theta')v^2, \quad (10)$$

where

$$A(\beta) = \frac{\lambda_1}{8}\cos^4\beta + \frac{\lambda_2}{8}\sin^4\beta + \left(\frac{\lambda_3}{4} + \frac{\lambda_4}{4}\right)\cos^2\beta\sin^2\beta,$$

$$B(\beta, \theta') = \frac{m_{11}^2}{2}\cos^2\beta + \frac{m_{22}^2}{2}\sin^2\beta - m_{12}^2\cos\theta'\cos\beta\sin\beta.$$

$$(11)$$

We first find the global minimum of V_{tree}. The stationary conditions $\frac{\partial V_{\text{tree}}}{\partial\varphi_I} = 0$ $(I = 1, 2, 4)$, are written as

$$v(2Av^2 + B) = 0, \quad (12)$$

$$2r_4 = \sin2\beta\frac{(1 - r_1r_2)\cos2\beta + r_2 - r_1r_3}{r_2\cos^22\beta + (r_3 + 1)\cos2\beta + r_2}, \quad (13)$$

$$m_{12}^2\sin\theta'\sin2\beta = 0, \quad (14)$$

where $r_i(i = 1 \sim 4)$ are defined as,

$$r_1 = \frac{m_{11}^2 - m_{22}^2}{m_{11}^2 + m_{22}^2}, \qquad r_2 = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2 - 2\lambda_3 - 2\lambda_4},$$

$$r_3 = \frac{\lambda_1 + \lambda_2 + 2\lambda_3 + 2\lambda_4}{\lambda_1 + \lambda_2 - 2\lambda_3 - 2\lambda_4}, \qquad r_4 = \frac{m_{12}^2 \cos\theta'}{m_{11}^2 + m_{22}^2}. \tag{15}$$

The stationary conditions in Eq. (12) and (13) correspond to Eq. (36) of [14]. Here we solve them explicitly by treating the soft breaking term m_{12} as perturbation. The nonzero solution for v^2 in Eq. (12) is written as

$$v^2 = -\frac{B}{2A}$$

$$= -4 \frac{m_{11}^2 + m_{22}^2}{\lambda_1 + \lambda_2 - 2\lambda_{34}} \frac{1 + r_1 \cos2\beta - 2r_4 \sin2\beta}{\cos^2 2\beta + r_3 + 2r_2 \cos2\beta}, \tag{16}$$

where $\lambda_{34} = \lambda_3 + \lambda_4$. Substituting it into V_{tree}, one obtains,

$$V_{\text{tree}} \ge V_{\text{min}} = -\frac{(m_{11}^2 + m_{22}^2)^2}{2(\lambda_1 + \lambda_2 - 2\lambda_{34})}$$

$$\times \frac{(1 + r_1 \cos2\beta - 2r_4 \sin2\beta)^2}{\cos^2 2\beta + 2r_2 \cos2\beta + r_3}. \tag{17}$$

For nonzero m_{12}^2 and $\sin2\beta$, the solution of Eq. (14) is $\sin\theta' = 0$. One still needs to find β among the solutions of Eq. (13), which leads to the minimum of V_{min}. We solve Eq. (13) and determine β by treating $r_4(m_{12}^2)$ as a small expansion parameter. One can easily find the approximate solutions as:

$$\begin{cases} (1)\ \sin\beta = \frac{\lambda_1 m_{12}^2}{|m_{22}^2 \lambda_1 - m_{11}^2 \lambda_{34}|}, & \cos\theta' = \text{sign}(m_{22}^2 \lambda_1 - m_{11}^2 \lambda_{34}), \\ (2)\ \cos\beta = \frac{\lambda_2 m_{12}^2}{|m_{11}^2 \lambda_2 - m_{22}^2 \lambda_{34}|}, & \cos\theta' = \text{sign}(m_{11}^2 \lambda_2 - m_{22}^2 \lambda_{34}), \\ (3)\ \cos2\beta = \frac{m_{11}^2(\lambda_{34} + \lambda_2) - m_{22}^2(\lambda_{34} + \lambda_1)}{m_{11}^2(-\lambda_{34} + \lambda_2) + m_{22}^2(-\lambda_{34} + \lambda_1)} + O(r_4). \end{cases} \tag{18}$$

Corresponding to each solution, (1) ~ (3) of Eq. (18), the vacuum expectation value v^2 and the minimum of the potential are obtained.

$$(v^2, V_{\text{min}}) = \begin{cases} (1) \left(-\frac{2m_{11}^2}{\lambda_1} + 2\lambda_1(m_{22}^2 - m_{11}^2)\left(\frac{m_{12}^2}{m_{22}^2 \lambda_1 - m_{11}^2 \lambda_{34}}\right)^2, \ -\frac{m_{11}^4}{2\lambda_1} + \frac{m_{12}^4 m_{11}^2}{m_{22}^2 \lambda_1 - m_{11}^2 \lambda_{34}} \right), \\ (2) \left(-\frac{2m_{22}^2}{\lambda_2} + 2\lambda_2(m_{11}^2 - m_{22}^2)\left(\frac{m_{12}^2}{m_{11}^2 \lambda_2 - m_{22}^2 \lambda_{34}}\right)^2, \ -\frac{m_{22}^4}{2\lambda_2} + \frac{m_{12}^4 m_{22}^2}{m_{11}^2 \lambda_2 - m_{22}^2 \lambda_{34}} \right), \\ (3) \left(2\frac{(\lambda_{34} - \lambda_2)m_{11}^2 + (\lambda_{34} - \lambda_1)m_{22}^2}{\lambda_1 \lambda_2 - \lambda_{34}^2} + O(r_4), \ -\frac{\lambda_2 m_{11}^4 - 2m_{11}^2 m_{22}^2 \lambda_{34} + \lambda_1 m_{22}^4}{2(\lambda_1 \lambda_2 - \lambda_{34}^2)} + O(r_4) \right). \end{cases} \tag{19}$$

The leading terms of the vacuum expectation values agree with those obtained in Z_2 symmetric model [18]. If $\sin2\beta = 0$, then r_4 must be vanishing and $\cos\theta' = 0$ from Eq. (13) and (14). The vacuum energies of the nonzero $\sin2\beta$ solutions are shown in Tables I. In Table II, the vacuum energies of the solutions with $\sin2\beta = 0$ are summarized.

Next, we derive the constraints on the parameters so that the solution corresponding to (1) in Table I becomes the

TABLE I.   Classification of the solutions with nonzero $\sin2\beta$ of the stationary conditions of Higgs potential. For (3), $O(r_4)$ correction is not shown.

(1) $\sin\beta = O(r_4)$	$-\frac{m_{11}^4}{2\lambda_1} - \frac{m_{12}^4}{\lambda_3 + \lambda_4 - \frac{m_{22}^2}{m_{11}^2}\lambda_1}$
(2) $\cos\beta = O(r_4)$	$-\frac{m_{22}^4}{2\lambda_2} - \frac{m_{12}^4}{\lambda_3 + \lambda_4 - \frac{m_{11}^2}{m_{22}^2}\lambda_2}$
(3) $\cos2\beta = O(1)$	$-\frac{\lambda_1 m_{11}^4 - 2m_{11}^2 m_{22}^2(\lambda_3 + \lambda_4) + \lambda_2 m_{22}^4}{2(\lambda_1\lambda_2 - (\lambda_3 + \lambda_4)^2)}$

global minimum of the potential. Since the other cases (2)–(5) do not have desired properties, we restrict the parameter space so that these solutions can not be a global minimum. Since v must have large positive vacuum expectation value, m_{11}^2 must be negative. In order that the vacuum energy of (1) is lower than that of (4),

$$m_{22}^2 \lambda_1 - m_{11}^2 \lambda_{34} > 0, \qquad (\cos\theta' = 1). \tag{20}$$

When Eq. (20) is satisfied and the solution (1) does exist, one can show that the vacuum energy of solution (3) is higher than that of (1). Furthermore, when $m_{22}^2 > 0$, the solutions corresponding to (2) and (5) are not realized. Then one can state the region of parameter space, which

TABLE II.   Classification of the solutions with $\sin2\beta = 0$.

	$\cos\theta' = 0$
(4) $\sin\beta = 0$	$-\frac{m_{11}^4}{2\lambda_1}$
(5) $\cos\beta = 0$	$-\frac{m_{22}^4}{2\lambda_2}$

is consistent with the case that the vacuum (1) becomes global minimum is

$$m_{11}^2 < 0, \qquad m_{22}^2 > 0, \qquad \lambda_{34} > \frac{m_{22}^2}{m_{11}^2} \lambda_1. \qquad (21)$$

Next, we consider the case with negative m_{22}^2. In this case, we impose the additional condition so that the vacuum energies corresponding to (2) and (5) are higher than that of (1):

$$\frac{m_{11}^4}{\lambda_1} > \frac{m_{22}^4}{\lambda_2}. \qquad (22)$$

Then, the condition for (1) is global minimum in this case is

$$m_{11}^2 < 0, \qquad m_{22}^2 < 0, \qquad \lambda_{34} > \frac{m_{22}^2}{m_{11}^2} \lambda_1,$$

$$\lambda_2 \frac{m_{11}^2}{m_{22}^2} > \lambda_1 \frac{m_{22}^2}{m_{11}^2}. \qquad (23)$$

In the following sections, we explore the regions for the parameters obtained in Eq. (21), (23), (8), and (9).

III. EFFECTIVE POTENTIAL IN ONE-LOOP AND RENORMALIZATION

In this section, we derive the effective potential within one-loop approximation. We introduce a real scalar fields with eight components as $\phi^i = (\phi_1^1, \phi_1^2, \phi_1^3, \phi_1^4, \phi_2^1, \phi_2^2, \phi_2^3, \phi_2^4)^T$, ($i = 1 \sim 8$). With the notation above, the one-loop effective action is given as

$$\Gamma_{\text{eff}}^{\text{1loop}} = i\frac{1}{2} \ln\det D^{-1}(\phi), \qquad D^{-1} = \Box + M_T^2, \qquad (24)$$

where M_T^2 is the mass squared matrix of the Higgs potential,

$$M_T^2 = M^2(\phi) + \begin{pmatrix} m_{11}^2 \times 1 & 0 \\ 0 & m_{22}^2 \times 1 \end{pmatrix} - m_{12}^2 \sigma_1,$$

$$M^2(\phi)_{ij} = \frac{\partial^2 V_{\text{tree}}^{(4)}}{\partial \phi_i \partial \phi_j}, \qquad (25)$$

and where 1(0) denotes 4×4 unit (zero) matrix. σ_1 is defined as

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \qquad (26)$$

In Eq. (26), 1(0) also denotes a four by four unit (zero) matrix. In modified minimal subtraction scheme, the finite part of the one-loop effective potential becomes

$$V_{\text{1loop}} = \frac{\mu^{4-d}}{2} \int \frac{d^d k}{(2\pi)^d i} \text{TrLn}(M_T^2 - k^2) + V_c,$$

$$= \frac{1}{64\pi^2} \text{Tr}\left(M_T^4\left(\text{Ln}\frac{M_T^2}{\mu^2} - \frac{3}{2}\right)\right). \qquad (27)$$

V_c denotes the counterterms and the derivation of V_c can be found in Appendix A.

IV. ONE-LOOP CORRECTIONS TO THE VACUUM EXPECTATION VALUES

In this section, we compute the one-loop corrections to the vacuum expectation values. Using the symmetry of the model, in general, one can choose $\varphi_I = (v, \beta, \alpha, \theta')$ as the vacuum expectation values of Higgs potential. Their values are obtained as the stationary points of the one-loop corrected effective potential $V = V_{\text{tree}} + V_{\text{1loop}}$,

$$\frac{\partial V}{\partial \varphi_I} = 0. \qquad (28)$$

By denoting the vacuum expectation values as sum of the tree level ones and the one-loop corrections to them, $\varphi_I = \varphi_I^{(0)} + \varphi_I^{(1)}$, one obtains the one-loop corrections,

$$\varphi_I^{(1)} = -(L^{-1})_{IJ} \frac{\partial V_{\text{1loop}}}{\partial \varphi_J}\bigg|_{\varphi=\varphi^{(0)}},$$

$$= -\frac{1}{32\pi^2}(L^{-1})_{IJ} \sum_{i=1}^{8} \left(O^T \frac{\partial M^2}{\partial \varphi_J}\bigg|_{\varphi=\varphi^{(0)}} O\right)_{ii}$$

$$\times M_{Di}^2\left(\ln\frac{M_{Di}^2}{\mu^2} - 1\right), \qquad (29)$$

where M_D^2 is a diagonal 8×8 tree level mass squared matrix of Higgs sector and L_{IJ} is 4×4 matrix given by the second derivatives of the tree level Higgs potential with respect to the order parameters,

$$L_{IJ} = \frac{\partial^2 V_{\text{tree}}}{\partial \varphi_I \partial \varphi_J}\bigg|_{\varphi=\varphi^{(0)}}. \qquad (30)$$

The diagonal Higgs mass matrix squared M_D^2 is related to 8×8 Higgs mass matrix squared M_T^2 in Eq. (25).

$$O^T M_{T0}^2 O = M_D^2$$

$$= \begin{pmatrix} M_{H^+}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{H^+}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_A^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_h^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_H^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad (31)$$

where M_{T0}^2 is obtained by substituting the vacuum expectation values to M_T^2. O is shown in Appendix D. Since M_D is the 8×8 diagonal matrix which elements correspond to the Higgs masses and zero mass of the would be Nambu-Goldstone bosons, one may write Eq. (29) in a simple form. The Higgs masses squared in Eq. (31) are given by

$$M_{H^+}^2 = \frac{1}{2}\left[\frac{1}{8}(\lambda_1 + \lambda_2 + 6\lambda_3 - 2\lambda_4 - \cos(4\beta)(\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4)))v^2 + (1 - \cos(2\beta))m_{11}^2 + (\cos(2\beta) + 1)m_{22}^2\right.$$
$$\left. + 2\sin(2\beta)m_{12}^2\right],$$

$$M_A^2 = M_{H^+}^2 + \frac{\lambda_4 v^2}{2}, \qquad \frac{M_h^2 + M_H^2}{2} = \frac{1}{4}((3\lambda_1\cos^2(\beta) + 3\sin^2(\beta)\lambda_2 + \lambda_3 + \lambda_4)v^2 + 2m_{11}^2 + 2m_{22}^2),$$

$$\frac{M_H^2 - M_h^2}{2} = \frac{1}{8}\left[\{6\cos(2\gamma)(\cos^2(\beta)\lambda_1 - \sin^2(\beta)\lambda_2) + (\cos(2(\beta + \gamma)) - 3\cos(2(\beta - \gamma)))(\lambda_3 + \lambda_4)\}v^2\right.$$
$$\left. + 4\cos(2\gamma)m_{11}^2 - 4\cos(2\gamma)m_{22}^2 + 8\sin(2\gamma)m_{12}^2\right], \tag{32}$$

where γ is an angle with which one can diagonalize the 2×2 mass matrix for CP-even neutral Higgs. $\tan2\gamma$ is given as

$$\tan2\gamma = \frac{-4m_{12}^2 + 2\sin2\beta(\lambda_3 + \lambda_4)v^2}{(3(-\lambda_1\cos^2\beta + \lambda_2\sin^2\beta) + \cos2\beta(\lambda_3 + \lambda_4))v^2 - 2(m_{11}^2 - m_{22}^2)}. \tag{33}$$

To compute Eq. (29), we still need to calculate $O^T \frac{\partial M^2}{\partial \varphi_I} O$ and L_{IJ}. They are shown in Appendix C. Using Eqs. (29) and (C1), one can find the quantum corrections for α and θ' vanish:

$$\alpha^{(1)} = 0, \qquad \theta'^{(1)} = 0. \tag{34}$$

For $v^{(1)}$ and $\beta^{(1)}$, one obtains,

$$v^{(1)} = -\frac{1}{32\pi^2}\frac{1}{\det L'}\left(L_{22}\sum_{j=1}^{5}\left[O^T\frac{\partial M^2}{\partial\varphi_1}O\right]_{jj}M_{Dj}^2\left(\ln\frac{M_{Dj}^2}{\mu^2} - 1\right) - L_{12}\sum_{j=1}^{5}\left[O^T\frac{\partial M^2}{\partial\varphi_2}O\right]_{jj}M_{Dj}^2\left(\ln\frac{M_{Dj}^2}{\mu^2} - 1\right)\right),$$

$$\beta^{(1)} = -\frac{1}{32\pi^2}\frac{1}{\det L'}\left(-L_{12}\sum_{j=1}^{5}\left[O^T\frac{\partial M^2}{\partial\varphi_1}O\right]_{jj}M_{Dj}^2\left(\ln\frac{M_{Dj}^2}{\mu^2} - 1\right) + L_{11}\sum_{j=1}^{5}\left[O^T\frac{\partial M^2}{\partial\varphi_2}O\right]_{jj}M_{Dj}^2\left(\ln\frac{M_{Dj}^2}{\mu^2} - 1\right)\right), \tag{35}$$

where L' is

$$L' = \begin{pmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{pmatrix}. \tag{36}$$

The elements of L' are shown in Eq. (C4). Equation (35) corresponds to the one-loop exact formulas and is a main result of the present paper. In the leading order of the expansion with respect to the symmetry breaking term m_{12}^2, the correction to v becomes

$$v^{(1)} = -\frac{v}{32\pi^2}\left\{3\lambda_1\left(\ln\frac{M_H^2}{\mu^2} - 1\right) + 2\lambda_3\frac{M_{H^+}^2}{M_H^2}\left(\ln\frac{M_{H^+}^2}{\mu^2} - 1\right) + (\lambda_3 + \lambda_4)\left(\frac{M_A^2}{M_H^2}\left(\ln\frac{M_A^2}{\mu^2} - 1\right) + \frac{M_h^2}{M_H^2}\left(\ln\frac{M_h^2}{\mu^2} - 1\right)\right)\right\}. \tag{37}$$

The Higgs masses in the formulas are the ones in the limit of $m_{12} \to 0$,

$$M_H^2 \simeq m_{11}^2 + \frac{3}{2}\lambda_1 v^2, \qquad M_A^2 \simeq M_h^2 \simeq m_{22}^2 + \frac{\lambda_3 + \lambda_4}{2}v^2, \qquad M_{H^+}^2 \simeq m_{22}^2 + \frac{\lambda_3}{2}v^2, \tag{38}$$

where v is related to m_{11}^2 as,

$$\frac{\lambda_1}{2}v^2 \simeq -m_{11}^2. \tag{39}$$

The approximate formulas for the physical Higgs masses in Eq. (38), which are valid to the limit $m_{12} \to 0$, agree with the ones given in [1] except the notational difference of M_H and M_h. The one-loop correction to β in the leading order expansion of m_{12}^2 is given as

$$\beta^{(1)} = -\frac{\beta}{32\pi^2}\Bigg\{2\Big(\lambda_2 - \lambda_4 - \frac{\lambda_3(\lambda_3 + \lambda_4)}{\lambda_1}\Big)\frac{M_{H^+}^2}{M_A^2}\Big(\ln\frac{M_{H^+}^2}{\mu^2} - 1\Big) + \Big(\lambda_2 - \frac{(\lambda_3 + \lambda_4)^2}{\lambda_1}\Big)\Big(\ln\frac{M_A^2}{\mu^2} - 1\Big)$$
$$+ \Big(3\lambda_2 + \Big(2\Gamma - \frac{\lambda_3 + \lambda_4}{\lambda_1}\Big)(\lambda_3 + \lambda_4)\Big)\frac{M_h^2}{M_A^2}\Big(\ln\frac{M_h^2}{\mu^2} - 1\Big) - 2(1 + \Gamma)(\lambda_3 + \lambda_4)\frac{M_H^2}{M_A^2}\Big(\ln\frac{M_H^2}{\mu^2} - 1\Big)\Bigg\}, \quad (40)$$

where

$$\Gamma = \lim_{m_{12}\to 0}\frac{\gamma}{\beta} = \frac{M_A^2 - M_H^2\frac{\lambda_3 + \lambda_4}{\lambda_1}}{M_H^2 - M_A^2}. \quad (41)$$

Equation (40) shows that the quantum correction is also proportional to the soft-breaking parameter m_{12}^2, which is expected. We also note that the correction depends on the Higgs mass spectrum and quartic couplings. The correlation to Higgs spectrum is studied in the next section.

V. NUMERICAL CALCULATION

In this section, we study the quantum correction to β and υ numerically. As shown in Eq. (37) and (40), the quantum corrections are written with four Higgs masses and the four quartic couplings. Since the neutral CP even and CP-odd Higgs of the second Higgs doublet are degenerate as $M_A = M_h$ in the limit $m_{12} \to 0$ (See Eq. (38)), the three Higgs masses (M_H, M_A, M_{H^+}) are independent. Moreover, for a given charged Higgs mass and neutral Higgs mass, λ_1 and λ_4 are given as

$$\lambda_1 = \frac{M_H^2}{\upsilon^2}, \qquad \lambda_4 = 2\frac{M_A^2 - M_{H^+}^2}{\upsilon^2}. \quad (42)$$

λ_2 and λ_3 are the remaining parameters to be fixed. The lower limit of λ_3 obtained from Eq. (8) and (9) is written as

$$\text{Max}\Big(-\frac{M_H}{\upsilon}\sqrt{\lambda_2}, -\frac{M_H}{\upsilon}\sqrt{\lambda_2} - 2\frac{M_A^2 - M_{H^+}^2}{\upsilon^2}\Big) < \lambda_3. \quad (43)$$

One can also write λ_3 with the charged Higgs mass formulas,

$$\lambda_3 = \frac{2}{\upsilon^2}(M_{H^+}^2 - m_{22}^2). \quad (44)$$

Depending on the sign of m_{22}^2, the upper bound and the lower bound of λ_3 can be obtained for a given charged Higgs mass. Combining it with Eq. (43), the constraints for positive m_{22}^2 case are,

$$\text{Max}\Big(-\frac{M_H}{\upsilon}\sqrt{\lambda_2}, -\frac{M_H}{\upsilon}\sqrt{\lambda_2} - 2\frac{M_A^2 - M_{H^+}^2}{\upsilon^2}\Big) < \lambda_3$$
$$< \frac{2M_{H^+}^2}{\upsilon^2}, \qquad (m_{22}^2 > 0). \quad (45)$$

When $m_{22}^2 \le 0$, in addition to the lower bound on λ_3, the constraint on λ_2 in Eq. (22) should be satisfied:

$$\frac{2M_{H^+}^2}{\upsilon^2} \le \lambda_3, \qquad \sqrt{\lambda_2} > \Big(\lambda_3 - 2\frac{M_{H^+}^2}{\upsilon^2}\Big)\frac{\upsilon}{M_H},$$
$$(m_{22}^2 < 0). \quad (46)$$

Now we study the quantum corrections numerically. We fix the standard model like Higgs mass as $M_H = 130$ (GeV). There are still four parameters to be fixed and they are λ_2, λ_3, M_A, and M_{H^+}. Focusing on the Higgs mass spectrum of the extra Higgs, we study the radiative corrections for the following scenarios for Higgs spectrum and the coupling constants.

A. Case for $M_A = M_{H^+}$; degenerate charged Higgs and pseudoscalar Higgs and a relation for vanishing quantum correction $\beta^{(1)}$

We first study the corrections for degenerate charged Higgs and pseudoscalar Higgs. In this case, for a given degenerate mass, one can identify the values of coupling constants λ_2 and λ_3, for which $\beta^{(1)}$ vanishes. With $M_A = M_{H^+}$, the relation for coupling constants which satisfies $\beta^{(1)} = 0$ is

$$\lambda_2 = \frac{\lambda_3^2}{3\lambda_1}\Bigg\{2 + \frac{M_H^2}{M_H^2 - M_{H^+}^2}\Big(1 - \frac{M_H^2}{M_{H^+}^2}\frac{\log\frac{M_H^2}{\mu^2} - 1}{\log\frac{M_{H^+}^2}{\mu^2} - 1}\Big)\Bigg\}$$
$$- \frac{\lambda_3}{3}\Big(\frac{M_{H^+}^2}{M_H^2 - M_{H^+}^2} - \frac{M_H^2}{M_H^2 - M_{H^+}^2}\frac{M_H^2}{M_{H^+}^2}\frac{\log\frac{M_H^2}{\mu^2} - 1}{\log\frac{M_{H^+}^2}{\mu^2} - 1}\Big). \quad (47)$$

The set of coupling constants (λ_3, λ_2), which satisfy the relation Eq. (47), are shown in Table III. We note that when λ_2 is as large as 10, λ_3 is at most about 3. If λ_2 is 1, λ_3 is lies in the range $0.55 \sim 0.7$.

TABLE III. The coupling constants (λ_3, λ_2) which satisfy the relation, Eq. (47) for the three degenerate masses $M_{H^+} = M_A = 100$, 200 and 500 (GeV).

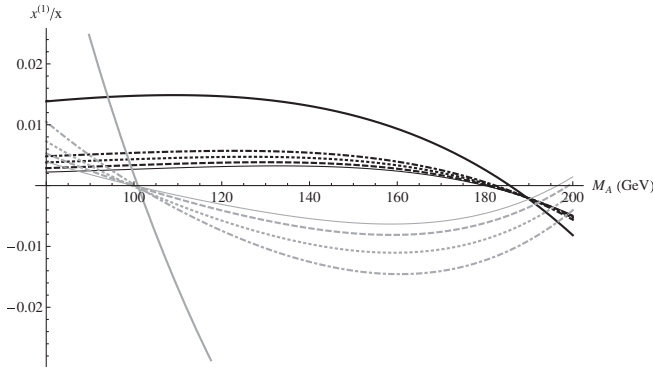| λ_2	λ_3 ($M_{H^+} = 100$)	λ_3 ($M_{H^+} = 200$)	λ_3 ($M_{H^+} = 500$)
0.14	0.19	0.16	0.18
0.28	0.28	0.28	0.28
0.56	0.41	0.47	0.42
1.0	0.55	0.69	0.59
10	1.8	2.8	2.0

FIG. 1.   The quantum correction $\frac{\beta^{(1)}}{\beta}$ (gray lines) and $\frac{v^{(1)}}{v}$ (black lines) due to the nondegeneracy of charged Higgs and pseudo-scalar Higgs masses. The pseudoscalar Higgs mass M_A (GeV) dependence of the quantum corrections $\frac{x^{(1)}}{x}$ ($x = \beta, v$) is shown, while the charged Higgs mass is fixed as $M_{H^+} = 100$ (GeV). The set of parameters (λ_3, λ_2) are chosen so that the correction $\beta^{(1)}$ vanishes for the degenerate case; $M_{H^+} = M_A = 100$ (GeV). The values (λ_3, λ_2) are taken from Table III and they are $(0.19, 0.14)$ (solid line), $(0.28, 0.28)$ (dashed line), $(0.41, 0.56)$ (dotted line), $(0.55, 1)$ (dotdashed line), and $(1.8, 10)$ (thick solid line).
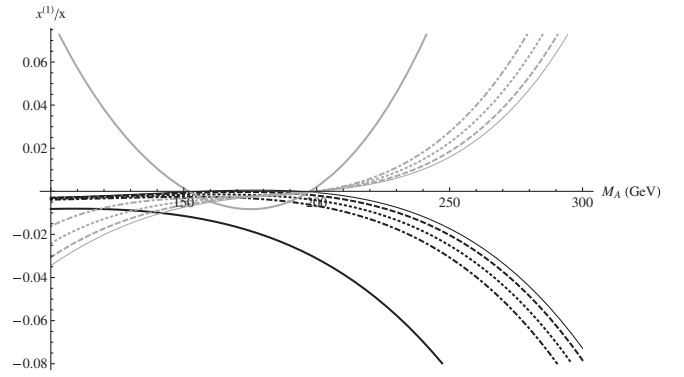

FIG. 2.   The quantum correction $\frac{\beta^{(1)}}{\beta}$ (gray lines) and $\frac{v^{(1)}}{v}$ (black lines) due to the nondegeneracy of charged Higgs and pseudo-scalar Higgs masses. The pseudoscalar Higgs mass M_A (GeV) dependence of the quantum corrections $\frac{x^{(1)}}{x}$ ($x = \beta, v$) is shown while charged Higgs mass is fixed as $M_{H^+} = 200$ (GeV). The set of parameters (λ_3, λ_2) are chosen so that the correction $\beta^{(1)}$ vanishes for the degenerate case; $M_{H^+} = M_A = 200$ (GeV). The values (λ_3, λ_2) are taken from Table III and they are $(0.16, 0.14)$ (solid line), $(0.28, 0.28)$ (dashed line), $(0.47, 0.56)$ (dotted line), $(0.69, 1)$ (dotdashed line), and $(2.8, 10)$ (thick solid line).

B. Non-Degenerate case $M_A \neq M_{H^+}$ with the coupling constants satisfying Eq. (47)

Next we lift the degeneracy by shifting the pseudoscalar Higgs mass from the charged Higgs mass and study the effect on $\beta^{(1)}$ and $v^{(1)}$. The nondegeneracy of the charged Higgs mass and the pseudoscalar Higgs mass is constrained by ρ parameter. We change the pseudoscalar Higgs mass within the range $|M_A - M_{H^+}| < 100$ (GeV) allowed from the electro-weak precision studies. The coupling constants (λ_3, λ_2) are chosen from the sets of their values satisfying the relation Eq. (47). In Fig. 1, we show $\frac{\beta^{(1)}}{\beta}$ as a function of M_A with charged Higgs mass $M_{H^+} = 100$ (GeV). When $M_A = 100$ (GeV), the correction vanishes exactly. As we increase M_A from 100 (GeV) (the mass of charged Higgs), the correction becomes nonzero and is negative. The corrections are at most about 1.3% when $\lambda_2 \sim 1$. By increasing M_A further, we meet the point around at $M_A \simeq 200$ (GeV) corresponding to that the correction vanishes again. In Fig. 2, we study the correction $\beta^{(1)}$ with larger charged Higgs mass case, $M_{H^+} = 200$ (GeV). In contrast to the case for $M_{H^+} = 100$ (GeV), by increasing M_A from 200 (GeV) where the correction vanishes, it increases and becomes positive. We also note that the correction tends to be larger than the lighter charged Higgs mass case. When $\lambda_2 \sim 1$, increasing the pseudoscalar Higgs mass from 200 (GeV) to 300 (GeV), the correction is about 10%. As the pseudoscalar Higgs mass decreases from 200 (GeV) to 100 (GeV), the correction becomes negative for $0 < \lambda_2 \leq 1$. With the larger value $\lambda_2 = 10$, we meet the point around at

$M_A \simeq 150$ (GeV) where the correction vanishes again. In Fig. 3, we study the further larger charged Higgs mass case, i.e., $M_{H^+} = 500$ (GeV). With $M_A \simeq 600$ (GeV), the correction is positive and about 100%. The correction stays small for $0 < \lambda_2 \leq 1$ when decreasing M_A from 500 (GeV) to 400 (GeV).
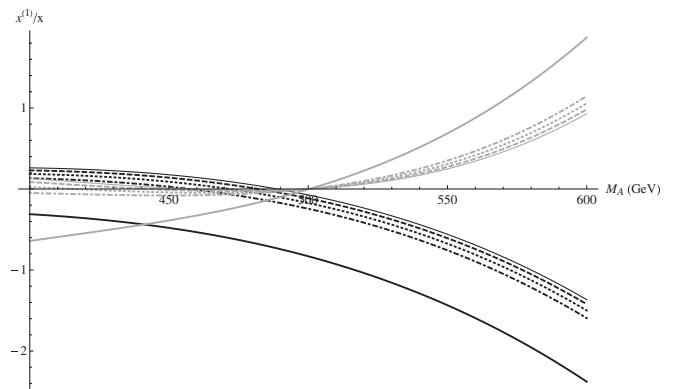

FIG. 3.   The quantum correction $\frac{\beta^{(1)}}{\beta}$ (gray lines) and $\frac{v^{(1)}}{v}$ (black lines) due to the nondegeneracy of charged Higgs and pseudo-scalar Higgs masses. The pseudoscalar Higgs mass M_A (GeV) dependence of the quantum corrections $\frac{x^{(1)}}{x}$ ($x = \beta, v$) is shown while charged Higgs mass is fixed as $M_{H^+} = 500$ (GeV). The set of parameters (λ_3, λ_2) are chosen so that the correction $\beta^{(1)}$ vanishes for the degenerate case; $M_{H^+} = M_A = 500$ (GeV). The values (λ_3, λ_2) are taken from Table III and they are $(0.18, 0.14)$ (solid line), $(0.28, 0.28)$ (dashed line), $(0.42, 0.56)$ (dotted line), $(0.59, 1)$ (dotdashed line), and $(2, 10)$ (thick solid line).
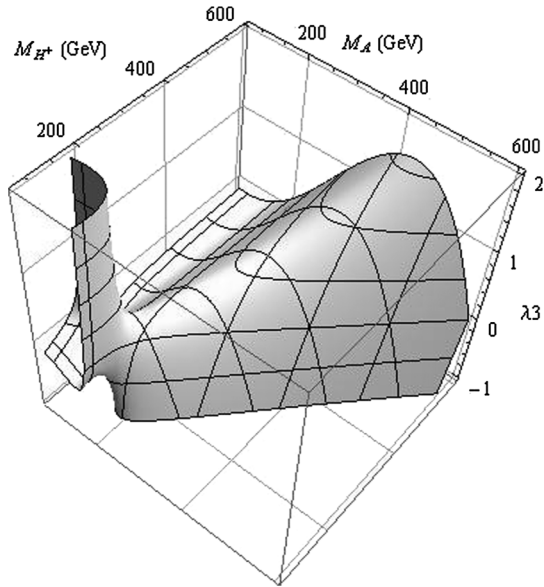
FIG. 4.   The two dimensional surface for $v^{(1)} = 0$.

C. The correction $\frac{v^{(1)}}{v}$

In Figs. 1–3, we also show the correction $\frac{v^{(1)}}{v}$ as functions of M_A. $v^{(1)}$ is independent of λ_2 and does not necessarily vanish at the same points where $\beta^{(1)}$ vanishes. With $\lambda_3 \gtrsim 2$ and $M_{H^+} \gtrsim 200$ (GeV), when the pseudoscalar Higgs mass is much larger than that of charged Higgs mass; we find a very large correction to v. In Fig. 4, we show that the two dimensional surface, which corresponds to $v^{(1)} = 0$. We find that the interior of the surface corresponds to the
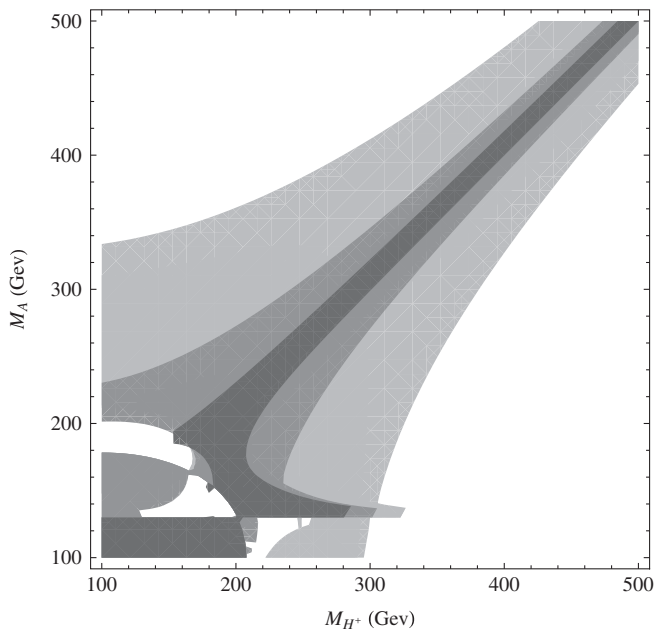

FIG. 5.   The regions of (M_{H^+}, M_A), which correspond to $(|\frac{v^{(1)}}{v}|, |\frac{\beta^{(1)}}{\beta}|) = (0, 0)$ (dark gray), (0.01, 0.01) (gray), and (0.1, 0.1) (light gray).

region of the positive correction $v^{(1)} > 0$, while the exterior region of the surface corresponds to the negative correction $v^{(1)} < 0$.

In Fig. 5, we have shown the regions of (M_{H^+}, M_A) which correspond to that the corrections of $|v^{(1)}|$ and $|\beta^{(1)}|$ have the definite values (0, 0.01, 0.1). The dark gray shaded area corresponds to the region where both $v^{(1)}$ and $\beta^{(1)}$ can vanish with taking account of the conditions in Eqs. (7)–(9). We note that for $M_{H^+}, M_A > 200$ (GeV), the quantum corrections vanish around the region where the charged Higgs degenerates with the pseudoscalar Higgs. When the corrections become larger, the larger mass splitting of the pseudoscalar Higgs and charged Higgs is allowed. However, as the average mass of the charged Higgs and pseudoscalar Higgs increases, the allowed mass splitting becomes smaller.

VI. DISCUSSION AND CONCLUSION

In this paper, the Dirac neutrino mass model of Davidson and Logan is studied. In the model, one of the vacuum expectation values of two Higgs doublets is very small and it becomes the origin of the mass of neutrinos. The ratio of the small vacuum expectation value v_2 and that of the standard-like Higgs v_1 is $\tan\beta = \frac{v_2}{v_1}$. Therefore, $\tan\beta$ is very small and typically it is O(10^{-9}). The smallness of $\tan\beta$ is guaranteed by the smallness of the soft breaking term of U(1)$'$.

We have treated the soft-breaking term as perturbation and calculated, in particular, the vacuum expectation of Higgs in the leading order of the perturbation precisely. As summarized in Table I, only by including the soft breaking terms, one can argue which of the local minima minimizes the potential and becomes the global minimum. We have studied the global minimum of the tree-level Higgs potential, including the effect of the soft breaking term as perturbation.

Beyond the tree level, we study the quantum correction to the vacuum expectation values and $\tan\beta$ in a quantitative way. In one-loop level, we confirmed that tree-level vacuum is stable, i.e., the order parameters which vanish at tree level do not have the vacuum expectation value as quantum correction. In one-loop level, we derived the exact formulas for the quantum correction to β in the leading order of expansion of the soft breaking parameter m_{12}^2. We have confirmed not only that the loop correction to $\tan\beta$ is proportional to the soft breaking term, but also found that the correction depends on the Higgs mass spectrum and some combination of the quartic coupling constants of the Higgs potential. Technically, we carried out the calculation of the one-loop effective potential by employing O(4) real representation for SU(2) Higgs doublets.

Dependence of the corrections on the Higgs spectrum is studied numerically. We first derive a relation of the

coupling constants, which corresponds to the condition that the correction to β vanishes for degenerate extra Higgs masses. Next, we study the effect of nondegeneracy of the charged Higgs and pseudoscalar Higgs on the correction. If the charged Higgs mass is as light as 100 (GeV) $\sim$ 200 (GeV), allowing the mass difference of charged Higgs and pseudoscalar Higgs is about 100 (GeV), the quantum corrections to both β and v are within a few % for $(\lambda_3, \lambda_2) \sim (0.5, 1)$. If the charged Higgs is heavy $M_{H^+} = 500$ (GeV), a slight increase of the pseudoscalar Higgs mass from the degenerate point leads to very large corrections to β and v.

One can argue the size of the quantum corrections to the neutrino mass of the model, because the ratio of the tree level neutrino mass and one-loop correction can be written as

$$\frac{m_\nu^{(1)}}{m_\nu} = \frac{v^{(1)}}{v} + \frac{\beta^{(1)}}{\beta}, \tag{48}$$

where we take account of the corrections only due to Higgs vacuum expectation values. The formulas in Eq. (48) imply that radiative correction to neutrino mass is related to the Higgs mass spectrum. Therefore, once Higgs mass spectrum is measured in LHC, one can compute the radiative correction to the mass of neutrinos using the formulas of Eq. (48).

Note added.—After submitting the paper, we became aware that the stability of the model studied in this paper was also discussed in [20]. Compared to their analysis, we derived the one-loop effective potential taking into account all the interactions of Higgs sector while they consider a part of the interactions and study the stability in a qualitative way. Using the effective potential, we carried out the quantitative analysis of the quantum corrections.

APPENDIX A: DERIVATION OF ONE-LOOP EFFECTIVE POTENTIAL

In this appendix, we give the details of the derivation of the one-loop effective potential and the counterterm in Eq. (27). One can split $M^2(\phi)_{ij}$ in Eq. (25) into the diagonal part and the off-diagonal part as $\delta M^2(\phi)_{ij} = M^2(\phi)_{ij} - M^2(\phi)_{ii}\delta_{ij}$. The divergent part of V_{1loop} can be easily computed by expanding it up to the second order of δM^2,

$$V_{1loop} = V^{(1)} + V_c,$$

$$V^{(1)} = \frac{\mu^{4-d}}{2} \int \frac{d^dk}{(2\pi)^d i} \mathrm{TrLn}\{(D_{ii}^{0-1} + M_{ii}^2(\phi))\delta_{ij} + \delta M_{ij}^2 - \sigma_1 m_{12}^2\}$$

$$= \sum_{i=1}^{8} \frac{\mu^{4-d}}{2} \int \frac{d^dk}{(2\pi)^d i} \ln\{D_{ii}^{0-1} + M_{ii}^2(\phi)\} - \sum_{i,j=1}^{8} \frac{\mu^{4-d}}{4} \int \frac{d^dk}{(2\pi)^d i} D_{ii}(\delta M^2 - \sigma_1 m_{12}^2)_{ij} D_{jj}(\delta M^2 - \sigma_1 m_{12}^2)_{ji} + \ldots,$$

$$\tag{A1}$$

where

$$D_{ii}^{-1} = D_{ii}^{0-1} + M_{ii}^2(\phi),$$
$$= \begin{cases} M_{ii}^2 + m_{11}^2 - k^2 & (1 \le i \le 4), \\ M_{ii}^2 + m_{22}^2 - k^2 & (5 \le i \le 8). \end{cases} \tag{A2}$$

The diagonal parts of the propagators are given as,

$$D_{ii} = \begin{cases} \frac{1}{M_{ii}^2 + m_{11}^2 - k^2} & (1 \le i \le 4), \\ \frac{1}{M_{ii}^2 + m_{22}^2 - k^2} & (5 \le i \le 8). \end{cases} \tag{A3}$$

In the modified minimal subtraction scheme, Feynman integration is carried out with help of the well known formulas of dimensional regularization

$$\mu^{4-d} \frac{1}{2} \int \frac{d^dk}{(2\pi)^d i} \log(m^2 - k^2)$$
$$= -\frac{1}{64\pi^2 \bar\epsilon} m^4 + \frac{m^4}{64\pi^2}\left(\log\frac{m^2}{\mu^2} - \frac{3}{2}\right), \tag{A4}$$

and

$$\mu^{4-d} \int \frac{d^dk}{(2\pi)^d i} \frac{1}{(m_i^2 - k^2)(m_j^2 - k^2)}\bigg|_{div} = \frac{1}{16\pi^2}\frac{1}{\bar\epsilon}, \tag{A5}$$

with $\frac{1}{\bar\epsilon} = \frac{1}{\epsilon} - \log 4\pi$ and $\epsilon = 2 - \frac{d}{2}$. The divergent part of $V^{(1)}$ is

$$V_{\text{div}}^{(1)} = -\frac{1}{64\pi^2\bar{\epsilon}}\left\{\sum_{i=1}^{4}(M_{ii}^2 + m_{11}^2)^2 + \sum_{i=5}^{8}(M_{ii}^2 + m_{22}^2)^2\right\} - \frac{1}{64\pi^2\bar{\epsilon}}\sum_{i\neq j=1}^{8}(\delta M^2 - m_{12}^2\sigma_1)_{ij}(\delta M^2 - m_{12}^2\sigma_1)_{ji},$$

$$= -\frac{1}{32\pi^2\bar{\epsilon}}\left(m_{11}^2\sum_{i=1}^{4}M_{ii}^2(\phi) + m_{22}^2\sum_{i=5}^{8}M_{ii}^2(\phi) + 2(m_{11}^4 + m_{22}^4)\right) - \frac{1}{64\pi^2\bar{\epsilon}}\text{Tr}[(M^2(\phi) - m_{12}^2\sigma_1)(M^2(\phi) - m_{12}^2\sigma_1)],$$

$$= -\frac{1}{64\pi^2\bar{\epsilon}}\text{Tr}[M_T^4]. \tag{A6}$$

The trace of Eq. (A6) is calculated in Eq. (B6) and (B11) of Appendix B, and the result is,

$$V_{\text{div}}^{(1)} = -\frac{1}{32\pi^2\bar{\epsilon}}[m_{11}^2\{6\lambda_1(\Phi_1^\dagger\Phi_1) + 2(2\lambda_3 + \lambda_4)(\Phi_2^\dagger\Phi_2)\} + m_{22}^2\{2(2\lambda_3 + \lambda_4)(\Phi_1^\dagger\Phi_1) + 6\lambda_2(\Phi_2^\dagger\Phi_2)\}]$$

$$+ \frac{2m_{12}^2}{64\pi^2\bar{\epsilon}}[(2\lambda_3 + 4\lambda_4)(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1)] - \frac{8m_{12}^4 + 4(m_{11}^4 + m_{22}^4)}{64\pi^2\bar{\epsilon}}$$

$$- \frac{1}{64\pi^2\bar{\epsilon}}\Big[(12\lambda_1^2 + 4\lambda_3\lambda_4 + 4\lambda_3^2 + 2\lambda_4^2)(\Phi_1^\dagger\Phi_1)^2 + (12\lambda_2^2 + 4\lambda_3\lambda_4 + 4\lambda_3^2 + 2\lambda_4^2)(\Phi_2^\dagger\Phi_2)^2$$

$$+ (12\lambda_1\lambda_3 + 4\lambda_1\lambda_4 + 8\lambda_3^2 + 4\lambda_4^2 + 12\lambda_2\lambda_3 + 4\lambda_2\lambda_4)(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$$

$$+ (4\lambda_1\lambda_4 + 16\lambda_3\lambda_4 + 8\lambda_4^2 + 4\lambda_2\lambda_4)|\Phi_1^\dagger\Phi_2|^2\Big]. \tag{A7}$$

Now the counterterms for the one-loop effective potential are simply given by changing the sign of the divergent part of Eq. (A7),

$$V_c = -V_{\text{div}}^{(1)} = \frac{1}{64\pi^2\bar{\epsilon}}\text{Tr}[M_T^4]. \tag{A8}$$

Using Eq. (A8) and (A4), one can derive the finite part of the one-loop effective potential given in Eq. (27).

APPENDIX B: DERIVATION OF EQ. (A7)

In this section, we present the derivation of Eq. (A7). We start with the quartic interaction terms of the Higgs potential,

$$V^{(4)} = \frac{\lambda_1}{8}\left(\sum_{i=1}^{4}\phi_i^2\right)^2 + \frac{\lambda_2}{8}\left(\sum_{i=5}^{8}\phi_i^2\right)^2 + \frac{\lambda_3}{4}\left(\sum_{i=1}^{4}\phi_i^2\right)\left(\sum_{j=5}^{8}\phi_j^2\right)$$

$$+ \frac{\lambda_4}{4}((\phi_1\phi_5 + \phi_2\phi_6 + \phi_3\phi_7 + \phi_4\phi_8)^2 + (\phi_1\phi_6 + \phi_3\phi_8 - \phi_2\phi_5 - \phi_4\phi_7)^2). \tag{B1}$$

By taking the derivatives of $V^{(4)}$, one can obtain the mass squared matrix $M^2(\phi)$. One first computes the first derivative of $V^{(4)}$ with respect to ϕ_i,

$$\frac{\partial V^{(4)}}{\partial\phi_i} = \begin{cases} \frac{\lambda_1}{8}2\left(\sum_{j=1}^{4}\phi_j^2\right)2\phi_i + \frac{\lambda_3}{2}\phi_i\sum_{j=5}^{8}\phi_j^2 + \frac{\lambda_4}{2}\{(\phi_1\phi_5 + \phi_2\phi_6 + \phi_3\phi_7 + \phi_4\phi_8)\phi_{i+4} \\ \quad + (\phi_1\phi_6 + \phi_3\phi_8 - \phi_2\phi_5 - \phi_4\phi_7)(\delta_{1i}\phi_6 - \delta_{2i}\phi_5 + \delta_{3i}\phi_8 - \delta_{4i}\phi_7)\}, & (1 \leq i \leq 4) \\ \frac{\lambda_2}{8}2\left(\sum_{j=5}^{8}\phi_j^2\right)2\phi_i + \frac{\lambda_3}{2}\phi_i\sum_{j=1}^{4}\phi_j^2 + \frac{\lambda_4}{2}\{(\phi_1\phi_5 + \phi_2\phi_6 + \phi_3\phi_7 + \phi_4\phi_8)\phi_{i-4} \\ \quad + (\phi_1\phi_6 + \phi_3\phi_8 - \phi_2\phi_5 - \phi_4\phi_7)(-\delta_{5i}\phi_2 + \delta_{6i}\phi_1 - \delta_{7i}\phi_4 + \delta_{8i}\phi_3)\}. & (5 \leq i \leq 8). \end{cases} \tag{B2}$$

The second derivatives are given as

$$\frac{\partial^2 V^{(4)}}{\partial \phi_i \partial \phi_j} = \begin{cases} \frac{\lambda_1}{2}\left(\delta_{ij}\sum_{k=1}^{4}\phi_k^2 + 2\phi_j\phi_i\right) + \frac{\lambda_3}{2}\delta_{ij}\left(\sum_{k=5}^{8}\phi_k^2\right) + \frac{\lambda_4}{2}\{\phi_{j+4}\phi_{i+4} \\ \quad + (\delta_{1j}\phi_6 - \delta_{2j}\phi_5 + \delta_{3j}\phi_8 - \delta_{4j}\phi_7)(\delta_{1i}\phi_6 - \delta_{2i}\phi_5 + \delta_{3i}\phi_8 - \delta_{4i}\phi_7)\}, \qquad (1 \le i, j \le 4), \\[2mm] \lambda_3\phi_i\phi_j + \frac{\lambda_4}{2}\Big\{\phi_{i+4}\phi_{j-4} + \sum_{k=1}^{4}\delta_{i+4,j}\phi_k\phi_{k+4} + (-\delta_{5j}\phi_2 + \delta_{6j}\phi_1 - \delta_{7j}\phi_4 + \delta_{8j}\phi_3) \\ \quad \times (\delta_{1i}\phi_6 - \delta_{2i}\phi_5 + \delta_{3i}\phi_8 - \delta_{4i}\phi_7) + (\phi_1\phi_6 + \phi_3\phi_8 - \phi_2\phi_5 - \phi_4\phi_7) \\ \quad \times (\delta_{1i}\delta_{6j} + \delta_{3i}\delta_{8j} - \delta_{2i}\delta_{5j} - \delta_{4i}\delta_{7j})\Big\}, \qquad (1 \le i \le 4, 5 \le j \le 8), \\[2mm] \lambda_3\phi_i\phi_j + \frac{\lambda_4}{2}\Big\{\phi_{i-4}\phi_{j+4} + \sum_{k=1}^{4}\delta_{i-4,j}\phi_k\phi_{k+4} + (\delta_{1j}\phi_6 - \delta_{2j}\phi_5 + \delta_{3j}\phi_8 - \delta_{4j}\phi_7) \\ \quad \times (-\delta_{5i}\phi_2 + \delta_{6i}\phi_1 - \delta_{7i}\phi_4 + \delta_{8i}\phi_3) + (\phi_1\phi_6 + \phi_3\phi_8 - \phi_2\phi_5 - \phi_4\phi_7) \\ \quad \times (\delta_{1i}\delta_{6j} + \delta_{3i}\delta_{8j} - \delta_{2i}\delta_{5j} - \delta_{4i}\delta_{7j})\Big\}, \qquad (5 \le i \le 8, 1 \le j \le 4), \\[2mm] \frac{\lambda_2}{2}\left(\delta_{ij}\sum_{k=5}^{8}\phi_k^2 + 2\phi_j\phi_i\right) + \frac{\lambda_3}{2}\delta_{ij}\left(\sum_{k=1}^{4}\phi_k^2\right) \\ \quad + \frac{\lambda_4}{2}\{\phi_{j-4}\phi_{i-4} + (-\delta_{5j}\phi_2 + \delta_{6j}\phi_1 - \delta_{7j}\phi_4 + \delta_{8j}\phi_3) \\ \quad \times (-\delta_{5i}\phi_2 + \delta_{6i}\phi_1 - \delta_{7i}\phi_4 + \delta_{8i}\phi_3)\}, \qquad (5 \le i, j \le 8). \end{cases}$$

$$(B3)$$

With Eq. (B3), the diagonal sums of M^2 are given as

$$\sum_{i=1}^{4} M_{ii}^2 = 3\lambda_1\sum_{i=1}^{4}\phi_i^2 + 2\lambda_3\sum_{i=5}^{8}\phi_i^2 + \lambda_4\sum_{i=5}^{8}\phi_i^2 = 6\lambda_1\Phi_1^\dagger\Phi_1 + (4\lambda_3 + 2\lambda_4)\Phi_2^\dagger\Phi_2, \qquad (1 \le i \le 4),$$

$$\sum_{i=5}^{8} M_{ii}^2 = 3\lambda_2\sum_{i=5}^{8}\phi_i^2 + 2\lambda_3\sum_{i=1}^{4}\phi_i^2 + \lambda_4\sum_{i=1}^{4}\phi_i^2 = 6\lambda_2\Phi_2^\dagger\Phi_2 + (4\lambda_3 + 2\lambda_4)\Phi_1^\dagger\Phi_1, \qquad (5 \le i \le 8). \qquad (B4)$$

The counterterm in Eq. (A8) includes the following contribution:

$$\text{Tr}\left[(M^2(\phi) - m_{12}^2\sigma_1)(M^2(\phi) - m_{12}^2\sigma_1)\right] = \text{Tr}[M^2(\phi)M^2(\phi) - 2m_{12}^2\sigma_1 M^2] + 8m_{12}^4. \qquad (B5)$$

The second term of Eq. (B5) is proportional to

$$\text{Tr}[m_{12}^2\sigma_1 M^2] = (2\lambda_3 + 4\lambda_4)(\phi_1\phi_5 + \phi_2\phi_6 + \phi_3\phi_7 + \phi_4\phi_8)m_{12}^2 = (2\lambda_3 + 4\lambda_4)(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1)m_{12}^2. \qquad (B6)$$

The first term of Eq. (B5) can be decomposed as

$$\text{Tr}[M^2(\phi)M^2(\phi)] = \sum_{i,j=1}^{4} M^2(\phi)_{ij}M^2(\phi)_{ji} + 2\sum_{i=1}^{4}\sum_{j=5}^{8} M^2(\phi)_{ij}M^2(\phi)_{ji} + \sum_{i,j=5}^{8} M^2(\phi)_{ij}M^2(\phi)_{ji}. \qquad (B7)$$

Each term of Eq. (B7) is given as

$$\sum_{i,j=1}^{4} M^2(\phi)_{ij}M^2(\phi)_{ji} = 3\lambda_1^2\left(\sum_{i=1}^{4}\phi_i^2\right)^2 + 3\lambda_1\lambda_3\sum_{i=1}^{4}\phi_i^2\sum_{j=5}^{8}\phi_j^2 + \lambda_1\lambda_4\left\{\sum_{i=5}^{8}\phi_i^2\sum_{j=1}^{4}\phi_j^2 + (\phi_1\phi_5 + \phi_2\phi_6 + \phi_3\phi_7 + \phi_4\phi_8)^2\right.$$

$$\left. + (\phi_1\phi_6 + \phi_3\phi_8 - \phi_2\phi_5 - \phi_4\phi_7)^2\right\} + \lambda_3\lambda_4\left(\sum_{i=5}^{8}\phi_i^2\right)^2 + \lambda_3^2\left(\sum_{i=5}^{8}\phi_i^2\right)^2 + \frac{\lambda_4^2}{2}\left(\sum_{i=5}^{8}\phi_i^2\right)^2$$

$$= 12\lambda_1^2(\Phi_1^\dagger\Phi_1)^2 + (12\lambda_1\lambda_3 + 4\lambda_1\lambda_4)(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + 4\lambda_1\lambda_4|\Phi_1^\dagger\Phi_2|^2$$

$$+ (4\lambda_3\lambda_4 + 4\lambda_3^2 + 2\lambda_4^2)(\Phi_2^\dagger\Phi_2)^2, \qquad (B8)$$

$$\sum_{i=1}^{4}\sum_{j=5}^{8} M^2(\phi)_{ij}M^2(\phi)_{ji} = \lambda_3^2 \sum_{i=5}^{8}\phi_i^2 \sum_{j=1}^{4}\phi_j^2 + 2\lambda_3\lambda_4\Big\{\sum_{i=1}^{4}\phi_i\phi_{i+4}\sum_{j=1}^{4}\phi_j\phi_{j+4} + (\phi_1\phi_6 - \phi_2\phi_5 + \phi_3\phi_8 - \phi_4\phi_7)^2\Big\}$$

$$+ \frac{\lambda_4^2}{2}\Big\{\sum_{i=1}^{4}\phi_i^2 \sum_{j=5}^{8}\phi_j^2 + 2\Big(\sum_{i=1}^{4}\phi_i\phi_{i+4}\Big)^2 + 2(\phi_1\phi_6 - \phi_2\phi_5 + \phi_3\phi_8 - \phi_4\phi_7)^2\Big\}$$

$$= (4\lambda_3^2 + 2\lambda_4^2)(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + (8\lambda_3\lambda_4 + 4\lambda_4^2)|\Phi_1^\dagger\Phi_2|^2, \tag{B9}$$

$$\sum_{i,j=5}^{8} M^2(\phi)_{ij}M^2(\phi)_{ji} = 3\lambda_2^2\Big(\sum_{i=5}^{8}\phi_i^2\Big)^2 + 3\lambda_2\lambda_3\sum_{i=5}^{8}\phi_i^2\sum_{j=1}^{4}\phi_j^2 + \lambda_2\lambda_4\Big\{\sum_{i=1}^{4}\phi_i^2\sum_{j=5}^{8}\phi_j^2 + (\phi_1\phi_5 + \phi_2\phi_6 + \phi_3\phi_7 + \phi_4\phi_8)^2$$

$$+ (\phi_1\phi_6 + \phi_3\phi_8 - \phi_2\phi_5 - \phi_4\phi_7)^2\Big\} + \lambda_3\lambda_4\Big(\sum_{i=1}^{4}\phi_i^2\Big)^2 + \lambda_3^2\Big(\sum_{i=1}^{4}\phi_i^2\Big)^2 + \frac{\lambda_4^2}{2}\Big(\sum_{i=1}^{4}\phi_i^2\Big)^2$$

$$= 12\lambda_2^2(\Phi_2^\dagger\Phi_2)^2 + (12\lambda_2\lambda_3 + 4\lambda_2\lambda_4)(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + 4\lambda_2\lambda_4|\Phi_1^\dagger\Phi_2|^2$$

$$+ (4\lambda_3\lambda_4 + 4\lambda_3^2 + 2\lambda_4^2)(\Phi_1^\dagger\Phi_1)^2. \tag{B10}$$

From Eqs. (B8)–(B10), one obtains,

$$\mathrm{Tr}[M^2(\phi)M^2(\phi)] = (12\lambda_1^2 + 4\lambda_3\lambda_4 + 4\lambda_3^2 + 2\lambda_4^2)(\Phi_1^\dagger\Phi_1)^2 + (12\lambda_2^2 + 4\lambda_3\lambda_4 + 4\lambda_3^2 + 2\lambda_4^2)(\Phi_2^\dagger\Phi_2)^2$$

$$+ (12\lambda_1\lambda_3 + 4\lambda_1\lambda_4 + 8\lambda_3^2 + 4\lambda_4^2 + 12\lambda_2\lambda_3 + 4\lambda_2\lambda_4)(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$$

$$+ (4\lambda_1\lambda_4 + 16\lambda_3\lambda_4 + 8\lambda_4^2 + 4\lambda_2\lambda_4)|\Phi_1^\dagger\Phi_2|^2. \tag{B11}$$

Using Eqs. (B4)–(B6) and (B11), one can derive Eq. (A7).

APPENDIX C: $[O^T \frac{\partial M^2}{\partial \varphi_I} O]_{jj}$ AND L_{IJ}

In this appendix, we show $[O^T \frac{\partial M^2}{\partial \varphi_I} O]_{jj}$ and L_{IJ}, which are needed to calculate one-loop corrections to the order parameters $\varphi_I^{(1)}$ in Eq. (29). $[O^T \frac{\partial M^2}{\partial \varphi_I} O]_{jj}$ ($I = 1, 2, 3, 4$) are given as

$$\Big[O^T \frac{\partial M^2}{\partial \alpha} O\Big]_{jj} = 0, \qquad \Big[O^T \frac{\partial M^2}{\partial \theta'} O\Big]_{jj} = 0. \tag{C1}$$

$$\Big[O^T \frac{\partial M^2}{\partial v} O\Big]_{jj} = 2v\Big[O^T \frac{\partial M^2}{\partial v^2} O\Big]_{jj}$$

$$= \frac{v}{4}\begin{pmatrix} \frac{1}{2}(\lambda_1 + \lambda_2 + 6\lambda_3 - 2\lambda_4 - \cos(4\beta)(\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4))) \\ \frac{1}{2}(\lambda_1 + \lambda_2 + 6\lambda_3 - 2\lambda_4 - \cos(4\beta)(\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4))) \\ \frac{1}{2}(\lambda_1 + \lambda_2 + 6\lambda_3 + 6\lambda_4 - \cos(4\beta)(\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4))) \\ 12\{\lambda_2\cos^2\gamma\sin^2\beta + \cos^2\beta\sin^2\gamma\lambda_1\} + (3\cos2(\beta - \gamma) - \cos2(\beta + \gamma) + 2)(\lambda_3 + \lambda_4) \\ 12\{\lambda_1\cos^2\beta\cos^2\gamma + \sin^2\beta\sin^2\gamma\lambda_2\} + (-3\cos2(\beta - \gamma) + \cos2(\beta + \gamma) + 2)(\lambda_3 + \lambda_4) \end{pmatrix}, \tag{C2}$$

and

$$\Big[O^T \frac{\partial M^2}{\partial \beta} O\Big]_{jj} = v^2\frac{\sin2\beta}{2}\begin{pmatrix} \lambda_2\cos^2(\beta) - \sin^2(\beta)\lambda_1 - \cos(2\beta)(\lambda_3 + \lambda_4) \\ \lambda_2\cos^2(\beta) - \sin^2(\beta)\lambda_1 - \cos(2\beta)(\lambda_3 + \lambda_4) \\ \lambda_2\cos^2(\beta) - \sin^2(\beta)\lambda_1 - \cos(2\beta)(\lambda_3 + \lambda_4) \\ 3\lambda_2\cos^2(\gamma) - 3\sin^2\gamma\lambda_1 + \frac{1}{2\sin2\beta}(\sin(2(\beta + \gamma)) - 3\sin(2(\beta - \gamma)))(\lambda_3 + \lambda_4) \\ -3\lambda_1\cos^2(\gamma) + 3\sin^2(\gamma)\lambda_2 - \frac{1}{2\sin2\beta}(\sin(2(\beta + \gamma)) - 3\sin(2(\beta - \gamma)))(\lambda_3 + \lambda_4) \end{pmatrix}. \tag{C3}$$

Next, we show L_{IJ} in Eq. (30). Note that L_{IJ} is symmetric $L_{IJ} = L_{JI}$ and its nonzero elements are:

$$L_{11} = \cos^2\beta m_{11}^2 + \sin^2\beta m_{22}^2 - 2\cos(\beta)\sin(\beta)m_{12}^2 + \frac{1}{2}[3v^2\{\lambda_1\cos^4(\beta) + \sin^2(\beta)(2(\lambda_3 + \lambda_4)\cos^2(\beta) + \sin^2(\beta)\lambda_2)\}],$$

$$L_{22} = v^2\left\{-\frac{\cos4\beta}{4}(\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4))v^2 + \frac{\cos2\beta}{4}(\lambda_2 - \lambda_1)v^2 + 2m_{12}^2\sin2\beta - \cos2\beta(m_{11}^2 - m_{22}^2)\right\},$$

$$L_{12} = L_{21} = v\left\{-\frac{\sin4\beta}{4}(\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4))v^2 + \frac{1}{2}\sin2\beta(\lambda_2 - \lambda_1)v^2 - 2m_{12}^2\cos2\beta - \sin2\beta(m_{11}^2 - m_{22}^2)\right\}$$

$$L_{33} = -\frac{1}{8}v^2\sin(2\beta)(v^2\sin(2\beta)\lambda_4 - 4m_{12}^2),$$

$$L_{44} = v^2\cos(\beta)\sin(\beta)m_{12}^2. \tag{C4}$$

APPENDIX D: ORTHOGONAL MATRIX O IN EQ. (31)

Here we show the orthogonal matrix O in Eq. (31).

$$O = \begin{pmatrix} 0 & -\sin\beta & 0 & 0 & 0 & 0 & \cos\beta & 0 \\ -\sin\beta & 0 & 0 & 0 & 0 & \cos\beta & 0 & 0 \\ 0 & 0 & 0 & \sin\gamma & \cos\gamma & 0 & 0 & 0 \\ 0 & 0 & -\sin\beta & 0 & 0 & 0 & 0 & \cos\beta \\ 0 & \cos\beta & 0 & 0 & 0 & 0 & \sin\beta & 0 \\ \cos\beta & 0 & 0 & 0 & 0 & \sin\beta & 0 & 0 \\ 0 & 0 & 0 & \cos\gamma & -\sin\gamma & 0 & 0 & 0 \\ 0 & 0 & \cos\beta & 0 & 0 & 0 & 0 & \sin\beta \end{pmatrix}. \tag{D1}$$

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