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Non-perturbative renormalization of vector and axial vector currents in quenched QCD for a renormalization group improved gauge action *

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Renormalization constants of vector (Z_V) and axial-vector (Z_A) currents are determined non-perturbatively in quenched QCD for an RG-improved gauge action and a tadpole-improved clover quark action using the Schrödinger functional method. Meson decay constants f_ρ and f_π show much better scaling when Z_V and Z_A estimated for infinite physical volume are used instead of Z-factors from tadpole-improved one-loop perturbation theory.

1. Introduction

In a recent comprehensive study by the CP-PACS Collaboration of N_f=2 full QCD [1], meson decay constants were found to exhibit a very large scaling violation over the range of lattice spacing $a^{-1} \approx 1 - 2$GeV. This was disappointing since an RG-improved gluon action and Sheikoleslami-Wohlert quark action with tadpole-improved $c_{\rm sw}$ were used. In this calculation, however, one-loop perturbative Z-factors, albeit tadpole-improved, were used for currents. A natural question was whether scaling becomes improved if non-perturbative Z-factors are employed instead.

At Lattice2001, we reported an initial study of this problem using the Schrödinger functional (SF) method [2] within quenched QCD. We found the problem of anomalously large values appearing in the ensemble of hadron correlators toward strong coupling where CP-PACS data of decay constants had been taken. In this report, we have analyzed this problem in some detail. Here we present our final results on the Z-factors including these analysis.

*Talk presented by K. Ide.

2. Method

We follow the method developed by the ALPHA collaboration [3], and work with a lattice geometry of $L^3 \times T$ with $T = 2L$ for Z_V with a vector operator at $t = L$, and for Z_A with two axial vector operators at $t = 3T/8$ and $t = 5T/8$.

Tree-level values are used for the coefficients of boundary counter terms of the action. For improving the axial current, we adopt the one-loop perturbative value for the coefficient c_A.

Values of Z_V and Z_A are determined for $\beta = 2.2 - 8.0$ which approximately covers the range of the CP-PACS quenched calculation [1], $\beta = 2.187 - 2.575$. We have analyzed 200–20000 configurations depending on β value and lattice size.

3. Exceptional Configurations

It is straight-forward to calculate Z-factors for $\beta \geq 2.6$. For lower β values on large lattices such as $8^3 \times 16$, however, anomalously large values appear in the ensemble of hadron correlators. This makes it difficult to determine quark mass precisely, and since this means uncertainties in κ_c, also that of Z-factors.

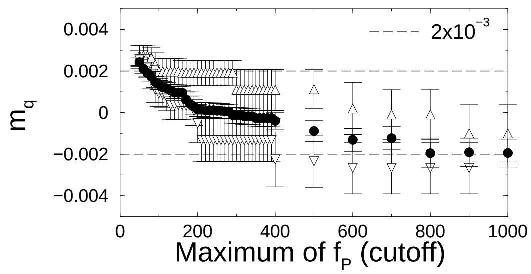

Figure 1. Cutoff dependence of m_q at $\beta = 2.4$ on an $8^3 \times 16$ lattice for three κ's around κ_c.

We suspect that these "exceptional" configurations are an artifact of quenched approximation; having very small or negative eigenvalues of the Wilson-Dirac operator, they would be suppressed in full QCD. Since one cannot distinguish "exceptional" configurations from "normal" ones on some rigorous basis, we restrict the configurations used for averaging to those having the value of a relevant hadron correlator below some cutoff. We then examine if uncertainties under variation of the cutoff are contained within some acceptable magnitude.

In Fig. 1 we illustrate this test for m_q for which a cutoff is set for f_P (see Ref. [3] for definition). We estimate κ_c from m_q with the cutoff value of 300, because m_q is rather stable there. The uncertainty in m_q at the κ_c is $\approx \pm 2 \times 10^{-3}$, once the cutoff of f_P is taken in the range $200 - 1000$.

In Fig. 2 we show how much the Z-factors depend on m_q. Z_V is insensitive to m_q, and Z_A is consistent within 10% or so, albeit apparently exhibiting a more pronounced dependence.

We analyze the uncertainties in the statistical averaging of Z-factors themselves by applying a cutoff in f_1, as carried out in Ref. [2]. The conclusion is similar; Z_V is very stable against variation of the cutoff, and Z_A shows a more conspicuous variation of 5% or so.

Uncertainties of Z_A on an 8^3 lattice of order 15% in total lead to uncertainty of Z_A normalized at infinite volume of order 30%. The uncertainty, however, has little effect in a Padé fit of Z_A and hence final results; Z_A varies less than 3% at the largest coupling $\beta = 2.187$, even if we artificially shift Z_A at $\beta = 2.4$ by 30%.

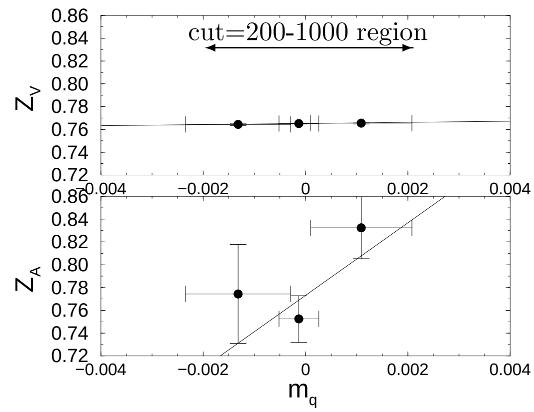

Figure 2. m_q dependence of Z_V and Z_A at $\beta = 2.4$ on an $8^3 \times 16$ lattice.

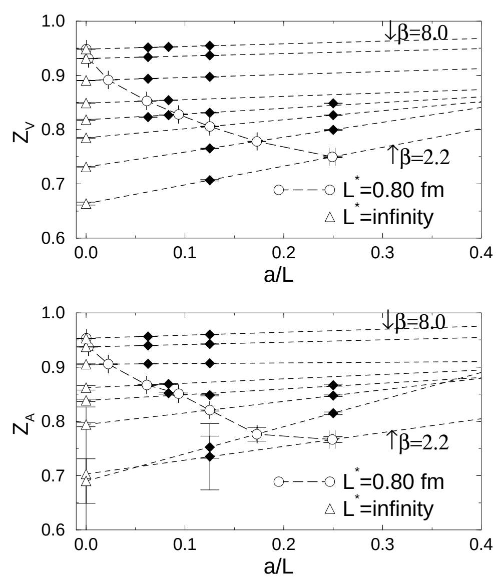

Figure 3. Size dependence of Z_V and Z_A.

4. Results for Z-factors

We determine the Z-factors for infinite volume ($L^* = \infty$) and also for a fixed finite physical volume ($L^* = 0.8$ fm corresponding to 8^3 lattice at $\beta = 2.6$) for comparison. The lattice scale is set through the string tension

As shown in Fig. 3, size dependence of Z-factors becomes sizable toward strong couplings. Since our quark action employs a tadpole-improved value of $c_{\rm sw}$, we expect $O(a)$ errors in the Z-factors. Therefore we extrapolate or inter-

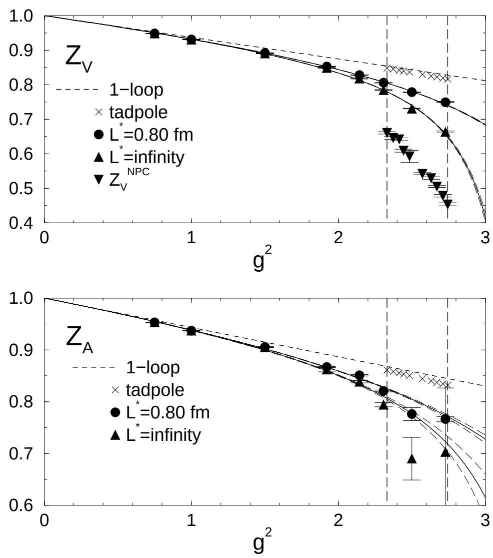

Figure 4. Results for Z_V and Z_A normalized at $L^* = 0.8$ fm and $L^* = \infty$.

polate results linearly in a/L to obtain estimates at $L^* = 0.8$ fm and at $L^* = \infty$.

In Fig. 4 we show results of Z-factors as a function of bare coupling g^2, together with Padé fits (solid curves in the figure) to them. Non-perturbative estimates give values smaller than the one-loop perturbative ones (dashed lines) by about 20 % (15%) for Z_V (Z_A) at the largest coupling of the CP-PACS simulation, $\beta = 2.187$.

Z_V^{NPC} determined from the ratio of the conserved vector current to the local one differs significantly from Z_V from the SF method, because the local current is not $O(a)$-improved.

5. Scaling Property of Decay Constants

We compare in Fig. 5 f_π and f_ρ determined with non-perturbative Z-factors normalized at $L^* = \infty$ (filled circles) with those using perturbative Z-factors (open up triangles). For comparison, open squares are the results from the standard plaquette and Wilson action[4].

We observe a very encouraging result that with the non-perturbative Z-factors scaling violations are sizably reduced. Furthermore the continuum extrapolation yields values consistent with those from the standard action.

In the same figure, we overlay f_π and f_ρ determined with Z-factors normalized at finite $L^* = 0.8$ fm (open circles). Scaling is best improved when Z-factors are normalized at $L^* = \infty$. This property is likely related to the fact that $O(a/L)$ errors in Z-factors are removed in the limiting procedure $L^* \to \infty$.

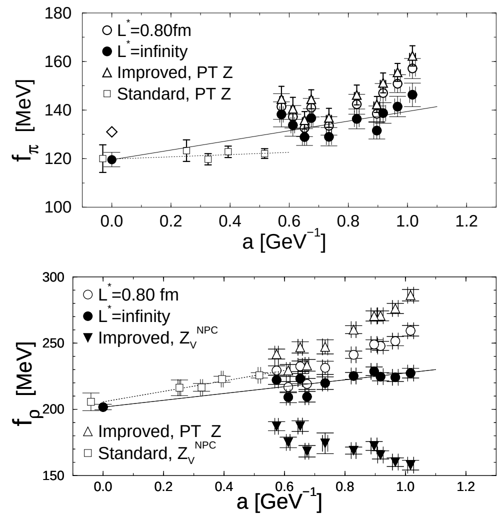

Figure 5. f_π and f_ρ vs. a for our improved action with non-perturbative and perturbative (PT) Z-factors together with results for the standard action [4].

We also find that f_ρ determined from the conserved vector current (filled down triangle in Fig. 5) exhibits a large scaling violation.

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