Continuum limit of proton decay matrix elements in quenched lattice QCD *

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We present a lattice QCD calculation of the parameters α and β which are necessary in the theoretical estimation of the proton lifetime in grand unified theories (GUTs) using chiral lagrangian approach. The simulation is carried out using the Wilson quark action at three gauge coupling constants in the quenched approximation. We obtain $|\alpha(2\text{GeV})|$=0.0091(08)$(^{+10}_{-19})$GeV^3 and $|\beta(2\text{GeV})|$=0.0098(08)$(^{+10}_{-20})$GeV^3 in the continuum limit where the first error is statistical and the second one is due to scale setting.

1. Introduction

Proton decay is one of the most important predictions of grand unified theories (GUTs) and a lot of experimental effort has been devoted to detect it. Up to now, however, no such decay process have been observed, and the lower bound of the proton lifetime has been pushed up to $O(10^{32})$ years by the Super-Kamiokande experiment.

To constrain the parameter space of GUTs by the proton lifetime, we need a reliable estimation of the hadronic matrix elements $\langle PS|\mathcal{O}|N\rangle$, where PS and N represents pseudoscaler meson and nucleon respectively and $\mathcal{O}$ is a three-quark operator. In principle, lattice QCD allows a precise determination of the matrix elements from the first principles. In previous work, we made a GUT-model-independent calculation of the matrix elements for all dimension-six three-quark operators using the Wilson quark action and the plaquette gauge action in the quenched approximation[1].

There remain two major systematic errors in the previous calculation: scaling violation and quenching effects. As a first step to reduce the systematic errors, this work is devoted to study the scaling violation effects. We leave incorporation of the dynamical quark effects to future work.

2. Method

With the aid of chiral perturbation theory[2], the proton decay matrix elements $\langle PS|\mathcal{O}|N\rangle$ are related to the so-called α and β parameters, which are defined by

$$\langle 0|\epsilon_{ijk}(u^iCP_Rd^j)P_Lu^k|p(\vec{k}=\vec{0})\rangle = \alpha P_L u_p, \quad (1)$$
$$\langle 0|\epsilon_{ijk}(u^iCP_Ld^j)P_Lu^k|p(\vec{k}=\vec{0})\rangle = \beta P_L u_p, \quad (2)$$

where u_p denotes the proton spinor. Calculation of these parameters is much simpler than the direct calculation of the matrix elements $\langle PS|\mathcal{O}|N\rangle$: the former is obtained from two-point

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Table 1
Simulation parameters. The lattice spacing a is determined from m_ρ.

β	$L^3\times T$	a^{-1}[GeV]	#conf. (Ref. [3])
5.90	$32^3\times56$	1.934(16)	300 (800)
6.10	$40^3\times70$	2.540(22)	200 (600)
6.25	$48^3\times84$	3.071(34)	140 (420)

functions with zero spatial momentum projection, while the latter requires three-point functions with finite spatial momenta. It should be noted that our previous study showed that the matrix elements obtained from the three-point functions are roughly comparable with the tree-level predictions of the chiral lagrangian with the α and β parameters determined on the lattice.

The lattice operators relevant for the α and β parameters are renormalized as

$$\mathcal{O}^{\rm cont}_{R/L,L}(\mu) = Z(\alpha_s,\mu a)\mathcal{O}^{\rm latt}_{R/L,L}(a)$$

$$+\frac{\alpha_s}{4\pi}Z_{\rm mix}\mathcal{O}^{\rm latt}_{L/R,L}(a) \mp \frac{\alpha_s}{4\pi}Z'_{\rm mix}\mathcal{O}^{\rm latt}_{\gamma_\mu L}(a), \qquad (3)$$

where

$$\mathcal{O}_{R/L,L} = \epsilon_{ijk}(u^iCP_{R/L}d^j)P_Lu^k, \qquad (4)$$

and the additional mixing operator is

$$\mathcal{O}_{\gamma_\mu L} = \epsilon_{ijk}(u^iC\gamma_\mu\gamma_5 d^j)P_L\gamma_\mu u^k. \qquad (5)$$

The renormalization constants Z, $Z_{\rm mix}$, and $Z'_{\rm mix}$ are calculated perturbatively at one-loop level[1]. The continuum operators are defined in NDR (naive dimensional regularization) with the $\overline{\rm MS}$ subtraction scheme.

To obtain the matrix elements, we compose a ratio

$$R(t) = \frac{\sum_{\vec{x}}\langle\mathcal{O}_{R/L,L}(\vec{x},t)\bar{J}'_{p,s}(0)\rangle}{\sum_{\vec{x}}\langle J_{p,s}(\vec{x},t)\bar{J}'_{p,s}(0)\rangle}\sqrt{Z_p}, \qquad (6)$$

where $\sqrt{Z_p}$ is defined by $\langle 0|J_{p,s}(\vec{0},0)|p^{(s')}(\vec{0})\rangle = \sqrt{Z_p}u_s^{(s')}$. This factor is estimated from the proton correlator with local source and local sink. It is well known that a precise determination of $\sqrt{Z_p}$ is rather hard because of large statistical fluctuations of the local-local correlator. On the other hand, the ratio of two-point functions can be calculated using the smear-local proton correlator whose statistical fluctuation is fairly small compared to the local-local correlator.

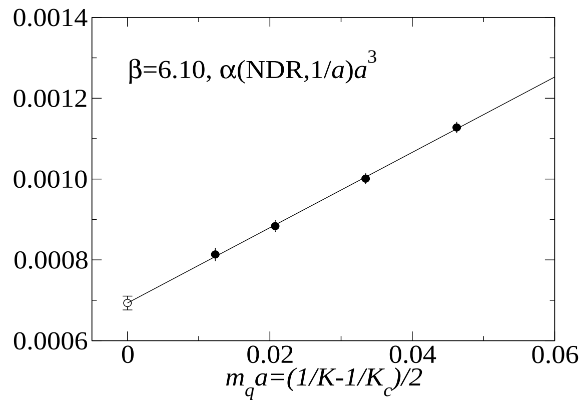

Figure 1. Typical light quark mass dependence of the α parameter.

This situation leads us to the following strategy: (i) For $\sqrt{Z_p}$ we resort to the result obtained from high statistical calculation of the quenched light hadron spectrum performed by the CP-PACS collaboration[3]. (ii) Employing the same parameters as the CP-PACS calculation, we make a new simulation to estimate the ratio of two-point functions including the mixing operators, which were not incorporated in the previous CP-PACS simulation. After combining the results of $\sqrt{Z_p}$ and the ratio of two-point functions we can achieve a few % level of statistical accuracy for the matrix elements at each β and hopping parameter.

3. Simulation details

Our calculation is carried out in quenched QCD with the Wilson quark and the plaquette gauge actions. The simulation parameters are given in Table1. We adopt three values of lattice spacings in the range $a \approx 0.1-0.064$ fm. The spatial lattice size is about 3 fm to avoid finite size effects. We employ four quark masses corresponding to $m_{PS}/m_V \approx 0.75-0.5$ for each β.

For smearing the source, the quark wave function in the pion is measured for each hopping parameter on gauge configurations fixed to the Coulomb gauge except for t=0 time slice where the wall source is placed[4]. To estimate $\sqrt{Z_p}$ we first fit the smeared-local proton correlator to a single exponential form $Z_p'\exp(-m_pt)$, and then the local-local proton correlator is fitted to $Z_p\exp(-m_pt)$ with m_p fixed to the value determined by the smeared-local correlator.

The matrix elements defined on the lattice are converted to those in the continuum at $\mu = 1/a$ and the scale is evolved up to μ=2GeV using the two-loop renormalization factor in the continuum[5].

4. Results

Figure 1 shows the light quark mass dependence of the α parameter at $\beta = 6.1$, which is well described by a linear function. Similar quark mass dependence is found for the β parameter. We check that linear and quadratic extrapolations yield consistent results within error bars in the chiral limit at all the lattice spacings.

In Fig. 2, we plot the parameters α and β in physical unit as a function of lattice spacing a, where we experiment with three choices m_N, m_ρ and f_π as physical input to determine the lattice spacing. In case of the nucleon mass m_N as physical input we find little scaling violation, which allows us to take the continuum limit of the parameters by linear extrapolation. We take the extrapolated value as the central one of the α and β parameters in the continuum limit. On the other hand, the parameters show rather large scaling violation if we set the lattice spacing by the rho meson mass m_ρ or the pion decay constant f_π. Although in these cases the values at the continuum limit possibly deviate from the result obtained with m_N as physical input, simple linear extrapolation is not reliable to estimate the deviation. Instead, we estimate this ambiguity from the result of quenched light hadron mass spectrum obtained by the CP-PACS collaboration[3], which shows that the values of m_N and f_π in quenched QCD deviate from the experiment by about 7% and 10% respectively in the continuum limit, once we set the lattice spacing by m_ρ.

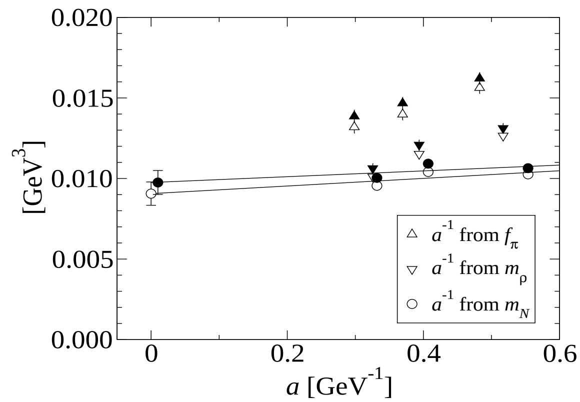

Figure 2. α (open) and β (filled) parameters as a function of lattice spacing.

Taking account of this ambiguity, we obtain

$$|\alpha(2\text{GeV})| = 0.0091(08)\left(^{+10}_{-19}\right)\text{GeV}^3, \quad (7)$$

$$|\beta(2\text{GeV})| = 0.0098(08)\left(^{+10}_{-20}\right)\text{GeV}^3, \quad (8)$$

with $\alpha/\beta < 0$ in the continuum limit, where the first error is statistical and the second one is due to scale setting. We note that the relative sign of α and β could be important, whereas the overall one is irrelevant.

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