B^0-$\bar{B}^0$ mixing with quenched lattice NRQCD[*]

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We present our recent results for the B-parameters, which parameterize the $\Delta B=2$ transition amplitudes. Calculations are made in quenched QCD at $\beta=5.7$, 5.9, and 6.1, using NRQCD for heavy quark and the $O(a)$-improved action for light quark. The operators are perturbatively renormalized including corrections of $O(\alpha_s/am_Q)$. We examine scaling behavior of the B-parameters in detail, and discuss the systematic uncertainties using scatter of results with different analysis procedures adopted. As a result, we find $B_{B_d}(m_b) = 0.84(2)(8)$, $B_{B_s}/B_{B_d} = 1.017(10)\binom{+4}{-0}$ and $B_{S_s}(m_b) = 0.87(1)(9)\binom{+1}{-0}$ in the quenched approximation.

1. Introduction

The precise determination of $\Delta B=2$ transition amplitudes would allow us to explore new physics beyond the Standard Model. A number of works to compute the amplitudes have been already done and are still in progress using the lattice QCD [1]. The $\Delta B=2$ transition amplitudes are conventionally expressed in terms of the corresponding B-parameters, which parameterize the deviation from the vacuum saturation approximation of the amplitudes. Here we present our recent results on the two phenomenologically important B-parameters, B_{B_q} and B_{S_q}, which are defined in the $\Delta B=2$ transitions with $\mathcal{O}_L = \bar{b}\gamma_\mu P_L q \, \bar{b}\gamma_\mu P_L q$ and $\mathcal{O}_S = \bar{b}P_L q \, \bar{b}P_L q$, respectively, where $q=s$ or d and $P_L = 1 - \gamma_5$.

One of the features in this work is the use of lattice NRQCD [2] for heavy quark, which enables us to obtain a better control of a systematic error than other approaches since it does not need any extrapolation in heavy quark mass. The price one has to pay, however, is that in using effective theory one can not take the continuum limit

and so that the power-law divergence of the form $\alpha_s^n/(am_b)^l$ and the discretization error of $a^2\Lambda_{\mathrm{QCD}}^2$ appear simultaneously. This means that we have to choose lattice spacing such that both sources of systematic errors are under control. To see this we perform quenched QCD simulations at $\beta=5.7$, 5.9, and 6.1 and investigate stability of the results against the lattice spacing.

The simulation parameters are summarized in Table 1. We use the standard plaquette action for gauge field and the $O(a)$-improved action for light quark. In order to see the chiral behavior, we take four different masses for light quarks in the range $[m_s/2, m_s]$. We also take five different masses for heavy quarks in the range $[2m_b/3, 4m_b]$ to investigate the convergence of $1/m_Q$ expansion. The lattice spacing for each lattice, shown in Table 1, is determined by the string tension.

2. Method

The operators $\mathcal{O}_X$ in the continuum are written in terms of those on the lattice $\mathcal{O}_X^{\mathrm{lat}}$ as

$$\mathcal{O}_X = \mathcal{O}_X^{\mathrm{lat}} + \sum_Y z_{X,Y} \mathcal{O}_Y^{\mathrm{lat}}. \tag{1}$$

β	6.1	5.9	5.7
size	$24^3 \times 64$	$16^3 \times 48$	$12^3 \times 32$
# of conf.	518	419	420
$1/a_\sigma$ [GeV]	2.29	1.64	1.08

Table 1
Lattice parameters in this work.

The one-loop matching coefficients $z_{X,Y}$ ($X=L$, S) as well as the precise definitions of operators and other details can be found in Ref. [3], where the perturbative calculations are performed including $O(\alpha_s/am_Q)$ corrections.

The b quark field defining each operator such as $\mathcal{O}_L$ and $\mathcal{O}_S$ is related to the non-relativistic field $h=(Q \ \chi^\dagger)^t$ through the inverse of Foldy-Wouthuysen-Tani transformation as

$$b = \left[1 - \frac{\vec{\gamma} \cdot \vec{D}}{2m_Q} \right] h. \tag{2}$$

It should be noted that because of a lack of the matching coefficients at the higher order of $O(\alpha_s\Lambda_{QCD}/m_Q)$ and $O(\alpha_s a\Lambda_{QCD})$, replacing $\mathcal{O}_Y^{\text{lat}}$ in eq. (1) by an alternative set of operators $\mathcal{O'}_Y^{\text{lat}} = \bar{h}\Gamma_Y q \ \bar{h}\Gamma_Y q$ is equivalent to eq. (1) itself up to the higher order corrections.

In the following, we explain our method by taking B_B as an example. First we define the "lattice B-parameter" by

$$B_Y^{\text{lat}} = \frac{\langle \bar{B}^0|\mathcal{O}_Y^{\text{lat}}|B^0\rangle}{\frac{8}{3}\langle \bar{B}^0|A_0^{\text{lat}}|0\rangle\langle 0|A_0^{\text{lat}}|B^0\rangle}, \tag{3}$$

where $A_0^{\text{lat}}=\bar{b}\gamma_0\gamma_5 q$. B'^{lat}_Y is also defined in a similar manner but with $\mathcal{O'}_Y^{\text{lat}}$ and $A'^{\text{lat}}_0=\bar{h}\gamma_0\gamma_5 q$. The B_B is, then, calculated in the following four forms,

1: $B_B = \left(B_L^{\text{lat}} + \sum_Y z_{L,Y}B_Y^{\text{lat}} \right)/(1+z_A)^2$

2: $B_B = B_L^{\text{lat}} + \sum_Y z_{L,Y-A^2}B_Y^{\text{lat}}$

3: $B_B = \left(B_L^{\text{lat}} + \sum_Y z_{L,Y}B'^{\text{lat}}_Y \right)/(1+z_A)^2$

4: $B_B = B_L^{\text{lat}} + \sum_Y z_{L,Y-A^2}B'^{\text{lat}}_Y$,

where z_A is the one-loop matching coefficient of axial vector current and $z_{L,Y-A^2} = z_{L,Y}$ except for $z_{L,L-A^2} = z_{L,L} - 2z_A$. The difference between the method 1 and 2 (or between the method 3 and 4) is of $O(\alpha_s^2)$, since in the former expression the factor $1/(1+z_A)^2$, which arises as the renormalization factor of A_0^{lat} in the denominator, is kept untouched and in the latter it is expanded to leave only the linear term in α_s. On the other hand, the difference between the method 1 and 3 (or between the method 2 and 4) is of $O(\alpha_s p/m_Q)$ or $O(\alpha_s^2/am_Q)$, since the former uses $\mathcal{O}_Y^{\text{lat}}$ and the latter $\mathcal{O'}_Y^{\text{lat}}$. Note that all these definitions of B_B have equal accuracy up to higher order contributions as just discussed. A typical size of these uncertainties may be estimated from scatter of the results. For the coupling constant, which is included in $z_{X,Y}$, $\alpha_V(q^*)$ with $q^* = 2/a$ is used throughout this work.

In the case of B_S, the constant $8/3$ and axial vector current A_0^{lat} in the denominator of eq. (3), z_A and $z_{L,Y-A^2}$ are replaced by $5/3$ and pseudo-scalar current $P^{\text{lat}} = \bar{b}\gamma_5 q$, z_P and $z_{S,Y-P^2}$, respectively.

3. Results

In the following, we show the results of $B_{B_d}(m_b)$, B_{B_s}/B_{B_d} and $B_{S_s}(m_b)$ obtained with the four methods. Figure 1 shows the $1/M_P$ dependence of $B_{B_d}(m_b)$ obtained at $\beta=5.9$, where M_P is the pseudo-scalar heavy-light meson mass. The renormalization scale of the continuum operator is set to $\mu=m_b=4.8$ GeV. First we find that four plots in Fig. 1 show various $1/M_P$ dependences and there is no clear tendency. This means that B_B depends on $1/M_P$ only weakly, at least, in the region of $m_Q \geq 2m_b/3$. Next one can see that the difference between the method 1 and 2 (or 3 and 4) becomes larger as one goes to the static limit and dominate the scatter of the results near the static limit. This might be interpreted as a typical size of the $O(\alpha_s^2)$ uncertainty as discussed before. We also find a deviation at the lightest M_P, which is dominated by the difference between the method 1 and 3 (or method 2 and 4). At this lightest M_P, the bare heavy quark mass in lattice unit is not much larger than one ($am_Q=1.3$), so we might infer that this difference indicates a typical size of the $O(\alpha_s^2/am_Q)$ error rather than the $O(\alpha_s p/m_Q)$. Interpolating
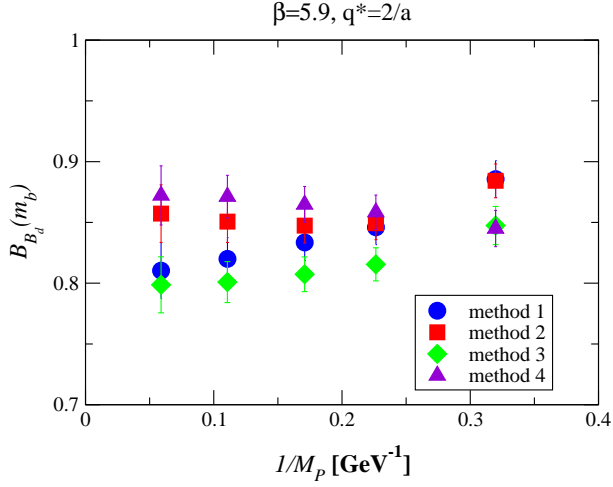
Figure 1. $1/M_P$ dependence of $B_{B_d}(m_b)$ at $\beta = 5.9$.

Figure 2. $B_{B_d}(m_b)$ as a function of lattice spacing.

all the data to the physical B meson mass, typical sizes of the uncertainties are estimated as

$$
\begin{aligned}
O(\alpha_s^2) &\sim 5\% \\
O(\alpha_s^2/am_Q) + O(\alpha_s p/m_Q) &\sim 4\%
\end{aligned} \tag{4}
$$

Repeating the same analysis at three lattice spacings, we obtain the lattice spacing dependence of B_{B_d}, which is plotted in Fig. 2. From this figure we can see that the three data are consistent with each other within the magnitude of scatter of the results. This suggests that the scaling violation for B_{B_d} is not too large. We take our best estimate from the central value of the results at β=5.9. The remaining uncertainty is estimated by the naive order counting, which gives

$O(\alpha_s^2) \sim 7\%$, $O(\alpha_s p/m_b) \sim 2\%$,
$O(\alpha_s^2/am_b) \sim 4\%$, $O(a^2 p^2) \sim 3\%$, $O(\alpha_s ap) \sim 5\%$.

Adding them in quadrature we obtain 10% as our estimate for the systematic error, which is more conservative than eq. (4), that is, just taking scatter of the results as systematic uncertainty. Our preliminary result in the quenched approximation is $B_{B_d}(m_b)$=0.84(2)(8), where the errors are the statistical and systematic ones, respectively.

Repeating the similar analysis, we obtain the lattice spacing dependences of B_{B_s}/B_{B_d} shown in Fig. 3. For this quantity, we observe neither method dependence nor large scaling violation.
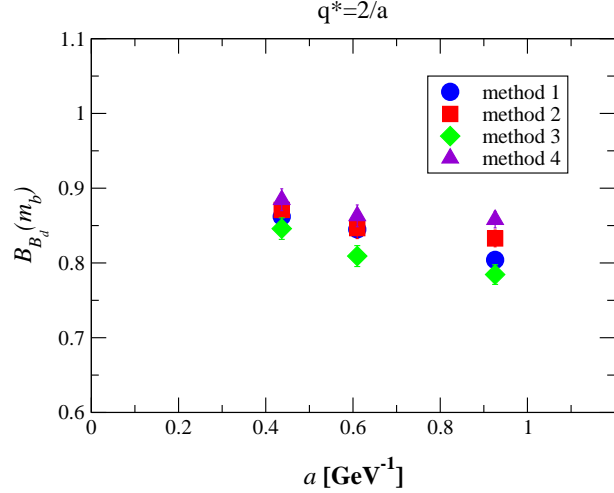
We take our best estimate from β=5.9 lattice as before, and obtain $B_{B_s}/B_{B_d} = 1.017(10)\binom{+4}{-0}$ by only considering the uncertainty in κ_s.

Figure 4 shows the lattice spacing dependence of B_{S_s}. The data plotted in this figure does not show a large scaling violation within the systematic uncertainty indicated by the scatter of the results. Following the same analysis as in the case of B_{B_d}, we obtain B_{S_s}=0.87(1)(9)$\binom{+1}{-0}$ in the quenched approximation, where the errors are statistical, systematic and coming from the uncertainty of κ_s, respectively.

4. Applications

Let us now consider a few applications of our results. The first one is the mass difference in the neutral B_q meson systems, which is given by the CKM matrix elements ($|V_{tb}^* V_{tq}|$) and $f_{B_q}^2 B_{B_q}$ up to a known factor as

$$
\Delta M_{B_q} \quad \propto \quad |V_{tb}^* V_{tq}|^2 f_{B_q}^2 B_{B_q}. \tag{5}
$$

Assuming the additional 10% uncertainty due to the quenched approximation to above result for B_{B_d} and the recent world average of the decay constant from n_f=2 simulations f_{B_d}=210(30) MeV, we obtain $|V_{tb}^* V_{td}|$=0.0074(12). This result can be used to estimate the frequency of the B_s^0-$\bar{B}_s^0$ mixing. If we take $|V_{ts}|$=$|V_{cb}|$=0.0404(18) and
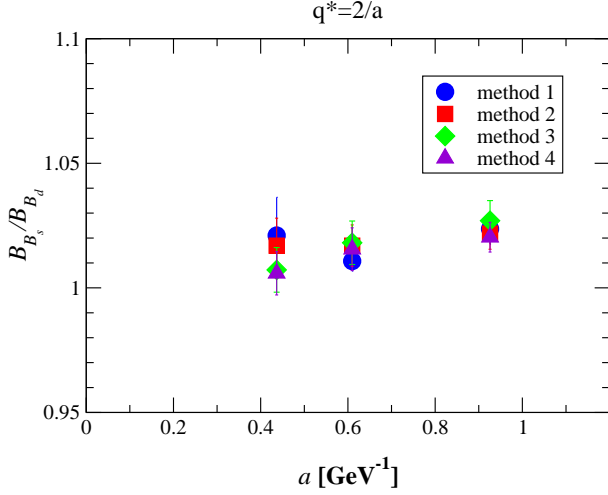
Figure 3. B_{B_s}/B_{B_d} as a function of lattice spacing.

Figure 4. $B_{S_s}(m_b)$ as a function of lattice spacing.

$f_{B_s}/f_{B_d}=1.16(4)$ and assume the uncertainty due to the quenched approximation for B_{B_s}/B_{B_d} to be 10% again, we obtain $\Delta M_{B_s}= 20.0(4.7)$ ps^{-1}. The current experimental bound is $\Delta M_{B_s} > 14.9$ ps^{-1} [4].

The second application is the width difference of B_s mesons. The width difference is theoretically calculated using the $1/m_b$ expansion. Beneke *et al.* have completed the formula with the next-to-leading order correction in Ref. [5]. Following their analysis but with $f_{B_s}=245(30)$MeV [1], the formula is given by

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{245(30)\text{MeV}}\right)^2 \Big[\ 0.008B_{B_s}(m_b) + 0.227(17)B_{S_s}(m_b) - 0.086(17)\ \Big]. \quad (6)$$

Using the results for B-parameters but with the additional 10% error due to the quenched approximation, we obtain $(\Delta\Gamma/\Gamma)_s = 0.119(29)(32)(17)$, where errors come from the uncertainties of f_{B_s}, B_{S_s} and the last term in eq. (6), which shows $O(1/m_b)$ contribution, respectively. The current experimental bound is $(\Delta\Gamma/\Gamma)_s = 0.16(^{+16}_{-13})$ [4]. It would be quite interesting to compare our result with the coming Tevatron Run II data, which are expected to improve the current experimental results significantly soon.
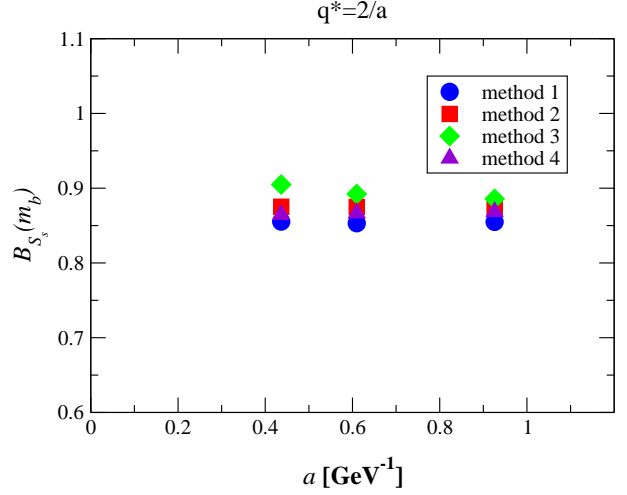
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