Comment on "Viable Singularity-Free $f(R)$ Gravity without a Cosmological Constant"

A modified $f(R)$ gravity model has been recently proposed in [1], whose cosmological behavior is clearly distinguishable from ΛCDM. Contrary to previous opinions which consider that self-consistent $f(R)$ gravity models distinct from ΛCDM are almost ruled out, the authors claim that the proposed model is cosmologically viable. Here we show that although the model satisfies some consistency conditions, precisely because of its departure from ΛCDM behavior, it does not satisfy local gravity constraints and, in addition, the predicted matter power spectrum conflicts with SDSS data.

Out of the four viability conditions imposed on $f(R)$ theories [2], the proposed model satisfies three of them. The fourth condition, namely, $|f_R - 1| \ll 1$ at recent epochs, is imposed by local gravity tests. Although it is still not clear what is the actual limit on this parameter, certain estimates [3] give $|f_R - 1| < 10^{-6}$ today, [4]. This condition also ensures that the cosmological evolution at late times resembles that of ΛCDM. However, in the proposed model, $|f_R - 1| \sim 0.2$ today for $\alpha = 2$ and $q_0 \sim -0.25$. In principle, if we are only interested in large scales, we could ignore local gravity inconsistencies, but still the deviations from ΛCDM can have drastic cosmological consequences on the evolution of density perturbations, as discussed by several authors [5–7].

Thus, the linear evolution of matter density perturbations for sub-Hubble ($k\eta \gg 1$) modes in ΛCDM is given by the well-known expression

$$\delta'' + \mathcal{H}\delta' - 4\pi G\rho_0 a^2 \delta = 0, \tag{1}$$

where $\delta = \delta\rho/\rho_0$, $\mathcal{H} = a'/a$, and prime denotes derivative with respect to conformal time η. Notice that the evolution of the Fourier modes does not depend on k. This means that once the density contrast starts growing after matter-radiation equality, the mode evolution only changes the overall normalization of the matter power spectrum $P(k)$, but not its shape.

However, in $f(R)$ theories, the corresponding equation reads [7]

$$\delta'' + \mathcal{H}\delta'$$
$$+ \frac{f_R^5 \mathcal{H}^2(-1+\kappa_1)(2\kappa_1-\kappa_2) - \frac{16}{a^8}f_{RR}^4(\kappa_2-2)k^8 8\pi G\rho_0 a^2}{f_R^5(-1+\kappa_1) + \frac{24}{a^8}f_{RR}^4 f_R(\kappa_2-2)k^8}\delta$$
$$= 0, \tag{2}$$

where $\kappa_1 = \mathcal{H}'/\mathcal{H}^2$ and $\kappa_2 = \mathcal{H}''/\mathcal{H}^3$. Notice the k^8 dependence in the last term which appears due to the fact that $f_{RR} \neq 0$. This means that the matter power spectrum is further processed after equality and the transfer function is modified with respect to that of ΛCDM. This drastically
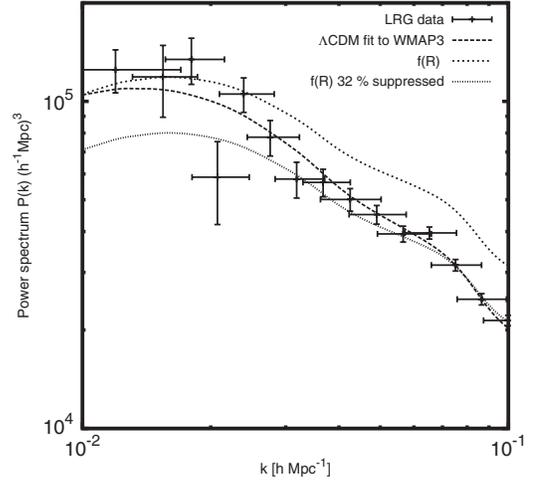

FIG. 1. Linear matter power spectra for ΛCDM and $f(R)$ in [1] with $\alpha = 2$. Data from SDSS [8].

changes the shape of $P(k)$, as shown in Fig. 1, where normalization to WMAP3 has been imposed. In the figure, SDSS data from luminous red galaxies [8] and the ΛCDM power spectrum from the linear perturbation theory with WMAP3 cosmological data are also shown. Notice that ΛCDM gives an excellent fit to data with $\chi^2 = 11.2$, whereas for the $f(R)$ theory $\chi^2 = 178.9$, i.e., 13σ out. Even if we drastically reduced the overall normalization in a 20%, the discrepancy would still remain at the 7σ level. Actually, the best fit would require a 32% normalization reduction and still would be 4.8σ away (see Fig. 1).

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