Using Rough Sets to predict insolvency of Spanish non-life insurance companies

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Abstract

Insolvency of insurance companies has been a concern of several parties stemmed from the perceived need to protect the general public and to try to minimize the costs associated to this problem such as the effects on state insurance guaranty funds or the responsibilities for management and auditors. Most methods applied in the past to predict business failure in insurance companies are techniques of statistical nature and use financial ratios as explicative variables. These variables do not normally satisfy statistical assumptions so we propose an approach to predict insolvency of insurance companies based on Rough Set Theory. Some of the advantages of this approach are: first, it is a useful tool to analyse information systems representing knowledge gained by experience; second, elimination of the redundant variables is got, so we can focus on minimal subsets of variables to evaluate insolvency and the cost of the decision making process and time employed by the decision maker are reduced; third, a model consisted of a set of easily understandable decision rules is produced and it is not necessary the interpretation of an expert and, fourth, these rules based on the experience are well supported by a set of real examples so this allows the argumentation of the decisions we make.

This study completes previous researches for bankruptcy prediction based on Rough Set Theory developing a prediction model for Spanish non-life insurance companies and using general financial ratios as well as those that are specifically proposed for evaluating insolvency of insurance sector.

The results are very encouraging in comparison with discriminant analysis and show that Rough Set Theory can be a useful tool for parties interested in evaluating insolvency of an insurance firm.

C49 - Keywords: Business failure, insolvency, insurance companies, rough set, discriminant analysis

1.- INTRODUCTION

Insolvency, early detection of financial distress, or conditions leading to insolvency of insurance companies have been a concern of parties such as insurance regulators, investors, management, financial analysts, banks, auditors, policy holders and consumers. This concern has stemmed from the perceived need to protect the

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general public against the consequences of insurers insolvencies and to try to minimize the costs associated to this problem such as the effects on state insurance guaranty funds or the responsibilities for management and auditors.

In the past a large number of methods have been proposed to predict business failure; however, the special characteristics of the insurance sector have made most of them unfeasible. Up to date, just a few of them have been applied to this sector. Most approaches applied to insurance companies are statistical methods such as discriminant or logit analysis (Ambrose and Carroll (1994); Bar-Niv and Smith (1987); Mora (1994); Sanchis (2000, 2002)) and use financial ratios as explicative variables. This kind of variables do not usually satisfy statistical assumptions. So in order to avoid this inconveniencies of statistical methods, we propose an approach to predict insolvency of insurance companies based on Rough Set Theory (RS Theory). Some of the advantages of this approach are: first, it is a useful tool to analyse information systems representing knowledge gained by experience; second, through this analysis the elimination of the redundant variables is got, so we can focus on minimal subsets of variables to evaluate insolvency and, therefore the cost of the decision making process and time employed by the decision maker are reduced; third, the analysis process results in a model consisted of a set of easily understandable decision rules so usually it is not necessary the interpretation of an expert and finally, fourth, these rules are based on the experience and they are well supported by a set of real examples so this allows the argumentation of the decisions we make.

In short, in this paper, we propose an approach to predict insolvency of insurance companies based on RS Theory which classifies companies into solvent and insolvent and performs a selection among the financial ratios. This study completes previous researches for prediction of bankruptcy based on RS Theory (Dimitras *et al.,* 1999; Greco *et al.*, 1998; McKee, 2000; Slowinski and Zopounidis, 1995) developing a prediction model for insurance companies. A sample of Spanish non-life insurance firms is used and general financial ratios as well as those that are specifically proposed for evaluating insolvency of insurance sector are employed.

The results are very encouraging in comparison with discriminant analysis and show that RS Theory can be a useful tool for parties interested in evaluating insolvency of an insurance firm.

The rest of the paper is structured as follows: Section 2 introduces some concepts of the RS theory; in section 3 we describe the data and input variables; in section 4 the results of the RS approach are presented; in section 5 these results are compared with the ones obtained with discriminant analysis and finally, section 6 describes our conclusions.

We want to mention that rough set analysis has been performed using ROSE software provided by the Institute of Computing Science of Poznan University of Technology. Any personal computer with a link to internet can access to the web www-idss.cs.put.poznan.pl/rose. where ROSE software and its manual can be unloaded. More details about this software are given in Predki. *et al.,* (1998) and Predki and Wilk (1999).

2.- **MAIN CONCEPTS OF THE ROUGH SET THEORY**

RS Theory was firstly developed by Pawlak (1991) in the 1980s as a mathematical tool to deal with the uncertainty or vagueness inherent in a decision making process. Though nowadays this theory has been extended (Greco *et al*, 1998), we refer to classical approach.

This theory involves the calculus of classes, partitions or divisions, as we prefer. It is somewhat different to statistical probability, that deals with random events in nature or fuzzy set theory, that deals with objects that may not belong only to one category but may belong to more than one category by differing degrees.

On the other hand, RS Theory is very well fitted when the classes into which the objects have to be classified are imprecise but can be approximate with precise sets (Nurmi *et al.*, 1996). Therefore, these differences show one of the main advantages of this theory: an agent is not required to assign precise numerical values to express imprecision of his knowledge, such as probability distributions in statistics or grade of membership in fuzzy set theory (Pawlak, 1991).

This section presents some concepts of RS Theory following Pawlak´s reference and some remarks by Slowinski (1993) and Dimitras *et al.* (1999).

The philosophy of this approach is based on the assumption that with every object of the universe we are considering we can associate knowledge, data. Knowledge is regarded as ability to classify objects. Therefore knowledge consists of a family of various classification patterns of a domain of interest. Objects described by the same data or knowledge are indiscernible in view of such knowledge. The indiscernibility relation leads to mathematical basis for the RS Theory. Vague information causes indiscernibility of objects by means of data available and, as a result, this prevents their precise assignment to a set. Intuitively, a rough set is a set or a subset of objects that cannot be expressed exactly by employing available knowledge. If this information or knowledge consists of a set of objects described by another set of attributes, we consider a rough set as a collection of objects that, in general, cannot be precisely characterized in terms of the values of the set of attributes.

RS Theory represents knowledge about the objects as a data table, that is, an *information table*. Rows of which are labelled by objects (states, processes, firms, patients, candidates,…) and columns are labelled by attributes. Entries of the table are attribute values. Therefore, for each pair object-attribute, *x-q*, there is known a value called *descriptor*, *f(x, q)*. The *indiscernibility relation* would occur if for two objects, *x* and *y*, all their descriptors in the table have the same values, that is iff f(x, q) = f(y, q) .

Approximation of sets, accuracy and quality of approximation

In general, all properties of rough sets are not absolute, but are related to what we know about them. Indiscernible objects by means of attributes prevent their precise assignment to a class. Therefore, some categories (subsets of objects) can not be expressed exactly by employing available knowledge and, consequently, the idea of approximation of a set by other sets is reached. A rough set is a pair of a *lower and an upper approximation* of a set in terms of the classes of indiscernible objects. That is, it is a collection of objects that, in general, cannot be precisely characterized in terms of the values of the set of attributes, while a lower and an upper approximation of the collection can be. Therefore, each rough set has boundary-line cases, that is, objects which cannot be classified certainty as members of the set or of its complement and can be represented by a pair of crisp sets, called the lower and the upper approximation. The lower approximation consists of all objects which certainly belong to the set and can be certainly classified as elements of that set, employing the set of attributes in the table (the knowledge we are considering). The upper approximation contains objects which possibly belong to the set and can be possibly classified as elements of that set using the set of attributes in the table. The *boundary* or *doubtful region* is the difference between the lower and the upper approximation and is the set of elements which cannot be certainly classified to a set using the set of attributes. Therefore, the borderline region is the undecidable area of the universe, that is, none of the objects belonging to the boundary can be classified with certainty into a set or its complement as far as knowledge is concerned.

Inexactness of a set is due to the existence of the boundary. The greater the doubtful region of a set, the lower the accuracy of that set. The *accuracy of approximation* is defined as the quotient between the cardinality of the lower approximation and the cardinality of the upper one. This ratio expresses the percentage of possible correct decisions when classifying objects employing knowledge available. Therefore, using the lower and the upper approximation we can define those subsets- that cannot be expressed exactly using the available attributes- precisely.

Because we are interested in classifications, the *quality of classification* is defined as the quotient between the addition of the cardinalities of all the lower approximations of the classes in which the objects set is classified, and the cardinality of the objects set. It expresses the percentage of objects which can be correctly classified to classes employing the knowledge available.

Reduction and dependency of attributes

A fundamental problem in the rough set approach is discovering dependencies between attributes in an information table because it enables to reduce the set of attributes removing those that are not essential (unnecessary) to characterize knowledge.

This problem will be referred to as knowledge reduction and the main concepts related to this question are the *core* and the *reduct.* A reduct is the minimal subset of attributes which provides the same quality of classification as the set of all attributes. If the information table have more than one reduct, the intersection of all of them is called the core and is the collection of the most relevant attributes in the table.

Decision rules

An information table which contains condition and decision attributes is referred as a decision table. A decision table specifies what decisions (actions) should be undertaken when some conditions are satisfied. So a reduced information table may provide decision rules of the form "*if* conditions *then* decisions".

These rules can be *deterministic* when the rules describe the decisions to be made when some conditions are satisfied and *non-deterministic* when the decisions are not uniquely determined by the conditions so they can lead to several possible decisions if their conditions are satisfied. The number of objects that satisfy the condition part of the rule is called the *strength* of the rule and is a useful concept to assign objects to the strongest decision class when rules are non-deterministic.

The rules derived from a decision table do not usually need to be interpreted by an expert as they are easily understandable by the user or decision maker. The most important result in this approach is the generation of decision rules because they can be used to assign new objects to a decision class by matching the condition part of one of the decision rule to the description of the object. So rules can be used for decision support.

RS Theory can analyse several multiattribute decision problems. It is specially well suited to sorting problems. One of these problems is multiattribute sorting problem which consists of the assignment of each object, described by values of attributes, to a predefined class or category. Business failure is an example of this kind of problem as we try to assign firms (objects) described by a set of financial ratios (attributes) to a category (failed or "healthy" firm).

3.- SELECTION OF VARIABLES AND DATA

In this stage of our research, we have proceeded to the election of the data and variables that will be used to develop our model.

As for the *data*, we have used the sample of Spanish firms used by Sanchis (2000, 2002). This data sample consists of non-life insurance firm data five years prior to failure. The firms were in operation or went bankrupt between 1983 and 1994. In each period, 72 firms (36 failed and 36 non-failed) are selected. As a control measure, a failed firm is matched with a non failed one in terms of industry and size (premiums volume). More details about the design of the sample are given in Sanchis, A. (2000, 2002). In our analysis we have used data one year prior to failure to obtain the decision rules and we have tested the rules with data from years 2, 3, 4 and 5.

As for the *variables*, we have to mention that choosing the variables for bankruptcy prediction is a critical issue. These variables could influence the quality of the results generated. In this research, each firm is described by 21 financial ratios that have come from a detailed analysis of the variables, previous bankruptcy studies for insurance companies and our preferences and knowledge. We have to draw particular attention to the fact that special financial characteristics of insurance companies require general financial ratios as well as those that are specially proposed for evaluating insolvency of insurance sector.

The ratios have been calculated from the last financial statements (balance sheets and income statements) issued before the firms declared bankruptcy. Thus, the prediction of bankruptcy is to be made about one year in advance. The ratios are shown in **Table 1**.

Though we have calculated ratios 15 and 16, we have not introduced them in our model because most of the firms have not "other income" so there is no sense in using them for an economic analysis.

On the other hand, we have calculated the main statistical variables for the whole ratios (average, standard deviation, etc.) except for the autocorrelation matrix because this is not a statistical method and we can introduce the whole ratios regardless of this question.

4.- ROUGH SETS ANALYSIS AND EMPIRICAL RESULTS

The information table for year 1 which consisted of 72 firms described with 19 ratios was entered into an input file in ROSE.

The first analysis we have made was to recode the ratios (continuous variables) into qualitative terms (low, medium, high and very high) with corresponding numeric values such us 1, 2, 3 and 4. This recoding has been made dividing the original domain into subintervals. This recoding is not imposed by the RS theory but it is very useful in order to draw general conclusions from the ratios in terms of dependencies, reducts and decision rules (Dimitras *et al.,*1999).

We have decided to recode the information table using 4 subintervals (see **Table 2**) based on the quartiles for the actual ratio values (year 1) for the whole sample.

The ROSE software permits to recode automatically the data generating a file with the coded ratios. We have used the subintervals assigning the higher code to the better subinterval to derive a coded information table, so for decreasing attributes (for which the lower the ratio the better for the firm) we have given the codes in the inverse orders of the subintervals. Moreover, RS Theory allows us to make corrections of the scale in the case our experience or knowledge are not concordant with the increasing or decreasing sequence of subintervals. The assignment of codes to quartiles is presented in **Table 3.**

The coded table consisted of 72 firms described by 19 coded financial ratios using data from one year prior to bankruptcy and by a binary assignment to a decision class (healthy or not) represented by 1 or 0, respectively.

As we have previously mentioned, the analysis using the rough set approach has been performed using ROSE software. The results were: the approximation of the decision class and the quality of classification were equal to one and the core of attributes was empty. These results indicate that the firms are very well discriminated among them (so the boundary region is empty for the two decision class) and that none of the attributes are indispensable for the approximation of the two decision classes.

Next step of the rough set analysis was the generation of the reducts. We have obtained 241 reducts from the table which contain 5-8 attributes. These results mean that at least 11 attributes are redundant (and, therefore, they could be eliminated) because the reducts contain fewer attributes but ensuring the same value (1 in our sample) of the quality of approximation as the whole set of attributes. Consequently, this result shows the strong support of this approach in feature selection. The ratios that have the highest frequency of occurrence (more than 40%) in reducts are R1, R3, R4, R9, R17, R18 and R19. This indicates that these variables are highly discriminatory between solvent and insolvent firms in our sample.

We have selected the reduct consisted of R1, R3, R9, R14, R17 y R19 taking into account three questions:

a) The reduct should have a small number of attributes as possible

b) It should have the most significant attributes in our opinion for the evaluation of the companies

c) After having selected a few reducts containing the most significant attributes, the reduct chosen should not contain ratios with a very high value for the autocorrelation coefficient. Therefore we have calculated this coefficient in some cases.

Once we have chosen the reduct, the rest of attributes of the coded information table can be eliminated, so we have got an information system with only six columns instead of the initial one consisted of 19 columns. The reduced table will be used to obtain the decision rules. The strategy we have followed to obtain the decision rules consists in the generation of a minimal subset of rules covering all the objects from the decision table. This strategy is implemented in the ROSE software.

We have obtained 25 rules. All of them are deterministic because the quality of the classification is equal to 1 and this means that the doubtful region is empty, so all the firms are highly discriminated among them. The 25-rules algorithm is presented in **Table 4**.

The 25-rules decision model has been tested on data from 2, 3, 4 and 5 years before the actual ratio values (year 1 or year prior to bankruptcy) that were used to obtain the decision rules.

The classifications accuracies in percent of correctly classified firms by the set of 25 rules for the five years prior to the reference year (year 1) are:

	Year 1	**Year 2**	**Year 3**	**Year 4**	**Year 5**
Classifications Accuracy	100%	78.57%	66.67%	64,71%	70%

As we can see, the classification accuracy is decreasing going back from year 1 (year prior to bankrupt), except for year 5. One possible reason is that five years prior to bankrupt firms do not try to manipulate their financial statements in order to conceal the real financial situation, and two years prior to bankruptcy it begins to be impossible to conceal the financial problems.

We have to mention that ROSE software has implemented an strategy whether an object does not match any rule or if it can be classified by several rules pointing at different decision classes (see included references about the software)

5.- COMPARISON OF THE ROUGH SET APPROACH WITH DISCRIMINANT ANALYSIS

We have compared rough set model with Discriminant Analysis (DA). DA is a statistical technique used to classify objects into distinct groups on the basis of their observed characteristics. Basically, a linear discriminant function is developed which will compute a “score” for an object. This function is a weighted linear combination of the object’s observed values on discriminating characteristics. These weights represent, essentially, the relative importance and impact of the various characteristics. On the basis of its discriminant score, an object is then classified.

The discriminant analysis is subject to a number of restrictive assumptions: each group follows a multivariate normal distribution, the covariance matrices of each group are identical, and, the mean vectors, covariance matrices, prior probabilities and misclassifications costs are known. If this theoretical assumptions are violated, the results obtained may be erroneous.

Unfortunately in practice, violations of these statistical assumptions occur regularly and its applicability has been questioned by several researchers. However, although these assumptions are not satisfied in the case of financial ratios, DA has provided good empirical results in real problems dealing with this kind of variables. This explains why this technique is one of the most used in prediction problems and the reasons why it has been chosen.

To compare the two methods we have derived a discriminant function using the ratios of the selected reduct in its non-discretised form. The discriminant function's coefficients are:

Variable	**Coefficient**
Constant term	-0,62358
R1	4,68606
R3	-0,29266
R9	0,24881
R14	-0,03074
R17	-0,76597
R19	1,86133

The classifications accuracies[1] in percent of correctly classified firms by the discriminant function for the five years prior to the reference year (year 1) are:

	Year 1	Year 2	Year 3	Year 4	Year 5
Classifications Accuracy	66.17%	71,75%	74,51%	58,09%	65,62%

The classification accuracy of the rough set model has been compared with that of linear discriminant model. In general, the rough set model, except for year 3, has outperformed the comparative discriminant model.

6.- CONCLUSIONS

We have presented a new approach to insurance insolvency prediction using rough sets.

Through the exposition we have mentioned some advantages of this approach so we can conclude that this method is an effective tool for supporting managerial decision making in general, and for insurance sector, in particular.

In the light of the experiments carried out, this method is a competitive alternative to existing bankruptcy prediction models in insurance sector and have great potential capacities that undoubtedly make it attractive for application to the field of business classification.

Our empirical results show that rough set model offers better predictive accuracy than the discriminant one we have developed. Moreover, it does not require the pre-specification of a functional form, nor the adoption of restrictive assumptions about the characteristics of statistical distributions of the variables and errors of the model. In short, by their nature, rough set approach makes working with imprecise variables

[1] Prior probabilities and misclassifications costs are set for 0.5.

possible. The flexibility of the decision rules with changes of the models over the time allows us to adapt them gradually to the appearance of new cases representing changes in the situation. Consequently, for some real-world problems, the method we have presented is more attractive than the discriminant analysis showing that it is a very robust technique especially in the areas of forecasting and classification decision problems.

In practical terms, the decision rules generated can be used to preselect companies to examine more thoroughly, quickly and inexpensively, thereby, managing the financial user's time efficiently. They can also be used to check and monitor insurance firms as a "warning system" for insurance regulators, investors, management, financial analysts, banks, auditors, policy holders and consumers.

We know the model obtained has some problems and limitations but in spite of them, our objective is to show the suitability of this methodology as a support decision method for insurance sector.

In short, we believe that RS Methodology, without replacing analyst's opinion and in combination with other methods, whether statistical or otherwise, will play a bright role in the decision making process in insurance sector.

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Table 1: **List of Ratios**

RATIO	**DEFINITION**
R1	Working capital/ Total Assets
R2	Earnings before Taxes (EBT)/ (Capital+ Reserves)
R3	Investment Income/ Investments
R4	EBT*/ Total Liabilities EBT* = EBT+ Reserves for Depreciation+ Provisions + (Extraordinary Income-Extraordinary Charges)
R5	Earned Premiums/ (Capital+ Reserves)
R6	Earned Premiums Net of Reinsurance/ (Capital+ Reserves)
R7	Earned Premiums/ (Capital+ Reserves+ Technical Provisions)
R8	Earned Premiums Net of Reinsurance/ (Capital+ Reserves+ Technical Provisions)
R9	(Capital +Reserves)/ Total Liabilities
R10	Technical Provisions/ (Capital + Reserves)
R11	Claims Incurred/ (Capital+ Reserves)
R12	Claims Incurred Net of Reinsurance/ (Capital+ Reserves)
R13	Claims Incurred / (Capital+ Reserves + Technical Provisions)
R14	Claims Incurred Net of Reinsurance/ (Capital+ Reserves+ Technical provisions)
R15	Combined Ratio 1 = (Claims Incurred/ Earned Premiums)+ (Other Charges and Commissions/ Other Income)
R16	Combined Ratio 2 = (Claims Incurred Net of Reinsurance/ Earned Premiums Net of Reinsurance)+ (Other Charges and Commissions/ Other income)
R17	(Claims Incurred + Other Charges and Commissions)/ Earned Premiums
R18	(Claims Incurred Net of Reinsurance + Other Charges and Commissions)/ Earned Premiums Net of Reinsurance
R19	Technical Provisions of Assigned Reinsurance/ Technical Provisions
R20	Claims Incurred / Earned Premiums
R21	Claims Incurred Net of Reinsurance / Earned Premiums Net of Reinsurance

Table 2: List of Subintervals (quartiles)

Ratio	1^{st}	2^{nd}	3^{rd}	4^{th}
R1	$(-\infty, 0.115]$	$(0.115, 0.295]$	$(0.295, 0.475]$	$(0.475, +\infty)$
R2	$(-\infty, 0]$	$(0, 0.1]$	$(0.1, 0.07]$	$(0.07, +\infty)$
R3	$(-\infty, 0.03]$	$(0.03, 0.06]$	$(0.06, 0.11]$	$(0.11, +\infty)$
R4	$(-\infty, 0.03]$	$(0.03, 0.08]$	$(0.08, 0.26]$	$(0.26, +\infty)$
R5	$(-\infty, 0.565]$	$(0.565, 1.565]$	$(1.565, 3.29]$	$(3.29, +\infty)$
R6	$(-\infty, 0.525]$	$(0.525, 1.38]$	$(1.38, 2.715]$	$(2.715, +\infty)$
R7	$(-\infty, 0.455]$	$(0.455, 0.725]$	$(0.725, 1.22]$	$(1.22, +\infty)$
R8	$(-\infty, 0.46]$	$(0.46, 0.7]$	$(0.7, 1.18]$	$(1.18, +\infty)$
R9	$(-\infty, 0.14]$	$(0.14, 0.35]$	$(0.35, 0.68]$	$(0.68, +\infty)$
R10	$(-\infty, 0.04]$	$(0.04, 0.545]$	$(0.545, 2.97]$	$(2.97, +\infty)$
R11	$(-\infty, 0.27]$	$(0.27, 1.095]$	$(1.095, 2.43]$	$(2.43, +\infty)$
R12	$(-\infty, 0.27]$	$(0.27, 0.845]$	$(0.845, 1.815]$	$(1.815, +\infty)$
R13	$(-\infty, 0.27]$	$(0.27, 0.49]$	$(0.49, 0.82]$	$(0.82, +\infty)$
R14	$(-\infty, 0.225]$	$(0.225, 0.435]$	$(0.435, 0.765]$	$(0.765, +\infty)$
R17	$(-\infty, 0.98]$	$(0.98, 1.055]$	$(1.055, 1.27]$	$(1.27, +\infty)$
R18	$(-\infty, 1]$	$(1, 1.09]$	$(1.09, 1.29]$	$(1.29, +\infty)$
R19	$(-\infty, 0]$	$(0, 0.065]$	$(0.065, 0.19]$	$(0.19, +\infty)$
R20	$(-\infty, 0.515]$	$(0.515, 0.68]$	$(0.68, 0.785]$	$(0.785, +\infty)$
R21	$(-\infty,0.515]$	$(0.515, 0.655]$	$(0.655, 0.75]$	$(0.75, +\infty)$

Table 3: Assignment of codes to subintervals

	Code numbers			
Ratio	**1st**	**2nd**	**3rd**	**4th**
R1	1	2	3	4
R2	1	2	3	4
R3	1	2	3	4
R4	1	2	3	4
R5	1	3	4	2
R6	1	3	4	2
R7	1	3	4	2
R8	1	3	4	2
R9	1	3	4	2
R10	1	3	4	2
R11	1	4	3	2
R12	1	4	3	2
R13	1	4	3	2
R14	1	4	3	2
R17	1	4	3	2
R18	1	4	3	2
R19	1	3	3	2
R20	4	3	2	1
R21	4	3	2	1

Table 4: The 25 rules algorithm

Nº Rules	R1	R3	R9	R14	R17	R19	Decision	Strength	Firms
1	2	2					0	6	2,13,20,24,27,32
2				2	1		0	6	7,23,26,30,35,36
3		1	1				0	8	6,7,10,11,12,16,17,19
4	1		4				0	3	22,28,34
5		1	3				0	3	3,4,14
6					4	3	0	4	1,4,9,11
7			2			2	0	2	5,29
8		4		1			0	4	16,18,21,25
9	2				2	1	0	1	31
10	4		4		4		0	1	33
11	2			3			0	4	4,8,13,15
12		3		4			1	7	43,46,50,53,54,56,60
13			2		4		1	5	59,62,67,69,70
14	4			1			1	5	37,38,41,45,72
15	3	4	1				1	4	42,44,47,55
16	3				3		1	6	40,47,48,52,56,68
17	1		2			1	1	3	64,66,70
18	1		3			3	1	2	51,61
19			2	3			1	3	39,59,64
20		1		1		3	1	1	71
21	2	1		2			1	1	57
22	3		4		4		1	1	63
23	4		3				1	4	37,43,45,49
24				1	3		1	3	38,58,68
25	3			4			1	6	40,46,53,55,56,65