Atomicity and Non-anonymity in Population-like Games for the Energy Efficiency of Hybrid-power HetNets

L. A. Fletscher, *Member, IEEE,* J. Barreiro-Gomez, *Member, IEEE,* C. Ocampo-Martinez, *Senior Member, IEEE,* C. Valencia Peroni, and J. M. Maestre, *Senior Member, IEEE* .

***Abstract*—In this paper, the user–base station (BS) association problem is addressed to reduce grid consumption in heterogeneous cellular networks (HetNets) powered by hybrid energy sources (grid and renewable energy). The paper proposes a novel distributed control scheme inspired by population games and designed considering both *atomicity* and *non-anonymity* – i.e., describing the individual decisions of each agent. The controller performance is considered from an energy–efficiency perspective, which requires the guarantee of appropriate quality-of-service (QoS) levels according to renewable energy availability. The efficiency of the proposed scheme is compared with other heuristic and optimal alternatives in two simulation scenarios. Simulation results show that the proposed approach inspired by population games reduces grid consumption by 12% when compared to the traditional best-signal-level association policy.**

***Index Terms*—Energy efficiency, distributed control, HetNets, population games, atomicity, non-anonymity**

I. Introduction

THE energy efficiency of next-generation telecommunication networks is a field of special interest today [1]–[4], particularly with the exponential growth of users expected in 5G mobile systems [5]. Additionally, previous studies have shown that about 0.5% of the global energy supply is consumed by cellular networks [6], [7]. This phenomenon has motivated different projects focused on the study of ways to reduce grid consumption in cellular networks, e.g., ICT-EARTH [8], Trend [9], and 5GrEEn [10]. One of the conclusions of these projects is that most of the grid consumption in cellular networks is caused by base stations (BSs) and also depends on the traffic load [11]. For this reason, the study of optimal mechanisms that balance the load of users over the available BSs is a key issue in the field of energy efficiency in cellular networks.

Among different proposed alternatives to improve the energy efficiency in cellular networks [12], the utilization of renewable energies as the power source for BSs has become increasingly relevant in recent years. Some studies have shown a reduction of network costs – capital expenditure (CAPEX) and – operating expense (OPEX) – and environmental impact using renewable energy sources in HetNets [13]–[16]. In the same way, the possibility of deploying infrastructure in off-grid and connection-limited scenarios (e.g., in developing countries) allows us to think of renewable energies as a complementary element in next-generation cellular networks (NGCNs). However, from the control viewpoint, the integration of renewable energies into NGCNs presents various challenges related to network architecture and the stochastic behaviour of renewable sources [17]. In particular, short response times, network stability and service availability must be guaranteed, especially considering the variability of renewable energy sources and the increased number of agents (users, BSs, network operators).

These challenges are deeply connected with the design of the user–BS association algorithm, for it determines how the network uses its resources to serve the users. The user–BS association problem has been treated in different ways in the literature [18], [19], as can be seen in [20], where Andrews *et al.* present a survey of approaches for load balancing in HetNets. Despite these efforts, there is still a need for exploring new load-balancing mechanisms, as the problem of associating users to base stations is nondeterministic polynomial-time hard (NP-hard) and may not be tractable even for small-sized HetNets.

A. Contribution

The main contribution of this paper is a novel, distributed user–BS association scheme inspired by population games [21], [22] to reduce grid consumption in HetNets powered by hybrid sources without storage systems. In particular, characteristics of atomicity and non-anonymity are considered to take into account that even one user's decision affects

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L. Fletscher is with Departamento de Ingeniería Electrónica y Telecomunicaciones, Facultad de Ingeniería, Universidad de Antioquia UdeA, Medellín, Colombia (e-mail: luis.fletscher@udea.edu.co).

J. Barreiro-Gomez is with Learning & Game Theory Laboratory, Division of Engineering, New York University Abu Dhabi (NYUAD), Saadiyat Campus PO Box 129188, United Arab Emirates (e-mail: jbarreiro@nyu.edu)

C. Ocampo-Martinez is with the Automatic Control Department, Universitat Politècnica de Catalunya, Institut de Robòtica i Informàtica Industrial (CSIC - UPC), Barcelona, Spain (e-mail: cocampo@iri.upc.edu).

J. Maestre is with System Engineering and Automation Department, School of Engineering, Universidad de Sevilla, Seville, Spain (e-mail: pepemaestre@us.es).

C. Valencia Peroni is with Process and Energy Department, Universidad Nacional de Colombia, Medellín, Colombia (e-mail: cavalenciapa@unal.edu.co).

the global performance of the system. In general, the overall behaviour of a large number of agents in a strategic interaction can be represented by a simplified aggregated model – e.g., considering proportions of agents. In contrast, when the number of agents is not that large, or when they are not homogeneous, it is more appropriate to represent the behaviour of each agent individually [23]. Also, when the scenario of the game changes dynamically over time, then it is necessary to implement dynamic-game approaches such as the one discussed in this paper – i.e., a game-theoretical approach performing as a learning algorithm seeking Nash equilibria. It is important to note that atomicity and non-anonymity are novel features of the proposed population-like-games approach.

The implemented revision protocol maintains the optimization problem constraints and attains grid consumption reduction and energy efficiency in a tractable way. Likewise, the system utility is maximized while the users' decisions are taken using partial information of the network state. The simplicity of the decision process and the computational time to reach a steady state in the proposed mechanism allows for its implementation in large-scale scenarios.

B. Assessment

To evaluate the proposed mechanisms, a two-tier HetNet with small cell-base stations (SCBSs) powered by renewable energy only is utilized, requiring more demanding control strategies to guarantee QoS levels. Wind is the only green-energy source considered, and three different wind scenarios are used to evaluate the proposed mechanism, with one of them corresponding to real data of Medellín, Colombia. It is important to note that wind is a highly fluctuating disturbance, which has a significant effect on user–BS association dynamics, thus increasing the control requirements. This feature is different to previous works in which more stable sources such as photovoltaic power generation were assumed [24]–[26].

Different user–BS association mechanisms are also used for comparison. Currently, in cellular networks, the default association scheme is based on the maximum signal-to-interference-plus-noise ratio (max-SINR), which maximizes the probability of coverage – i.e., $p(\mathrm{SINR} > \varphi)$, where φ is a target SINR. Consequently, max-SINR is the base mechanism used to compare the efficiency of the proposed population-like-games approach. Furthermore, two other user–BS association mechanisms are assessed in this work: a greedy algorithm used to select the best BS for a user based on the energy source and the signal level provided [27], and a discrete branch-and-bound optimization that assigns users to BSs.

The outline of the rest of the article is as follows. In Section II, some related works are presented. In Section III, the problem statement is described. Section IV presents the atomicity and non-anonymity approaches in population-like games. Section V describes the assessed user–BS association mechanism. Section VI presents the simulation scenario. In Section VII, the performance of the proposed schemes is evaluated, including the analysis of results. Finally, in Section VIII, the conclusions are provided. A summary of the notation used in this work can be found in Table I.

TABLE I
NOTATION

Parameter	Description
$\mathcal{B}$	Set of base stations
b	Number of base stations
ℓ	Base station's index
$\mathcal{U}$	Set of users
u	Number of users
i	User's index
p	Possible location
k	Discrete time step
N	Simulation horizon
$\mathcal{B}_k$	Active base stations at time k
$\mathcal{B}_{i,k}$	Available BSs providing service to $i \in \mathcal{U}$ at k
$C_{\ell,k}$	Energy consumption of BS ℓ at k
$r_{i,\ell}^{p}$	Transmission rate of $i \in \mathcal{U}$, connected with a BS ℓ at k
$\psi_{i,\ell}^{p}$	SINR perceived by $i \in \mathcal{U}$ in p from BS ℓ
φ	Threshold: minimum SINR required to have service
$y_{i,\ell,k}$	User-BS association indicator for user i with BS ℓ at time k
$z_{A,k}$	Number of active users at k
$z_{\ell}^{\max}$	Users that can be served by a SCBS ℓ simultaneously
$f_{i,\ell}$	Fitness function perceived for $i \in \mathcal{U}$ from BS ℓ at k
$\varrho_{i,k}^{h,\ell}$	Switching rule from strategy ℓ to strategy h for $i \in \mathcal{U}$

II. Related Works

In NGCNs, many efforts have been dedicated to developing BS-topology management approaches, including load balancing and traffic redistribution. From the energy efficiency perspective, Zhou *et al.* proposed a heuristic algorithm for target-cell selection combined with a power-control algorithm for coverage optimization to guide users towards BSs with a renewable energy supply in the handover process [28]. Likewise, Han and Ansari proposed optimizing the utilization of green–energy in cellular networks by cell-size optimization [29]. To this end, they decomposed the problem into two parts: a multi-stage energy-allocation problem, and a multi-BS energy-balancing problem. Liu *et al.* proposed, in [30], an off-line algorithm to optimize the green-energy allocation across different time instants to minimize the on-grid energy consumption of a BS. Silva *et al.* used the classic optimal-transportation approach to study the mobile association problem in cellular networks [31]. These approaches to reducing consumption in cellular networks differ from our proposal in the utilization of energy-storage elements as part of the system and the centralized nature of the proposed algorithms.

Likewise, the high number of interacting users and the demand for short response times and lower overhead information exchange are challenging issues for distributed control strategies [32]–[34]. From this perspective, in [25], Han and Ansari presented a virtually distributed algorithm named vGALA to reach a trade-off between network utilities and green-energy utilization in software-defined radio access networks powered by hybrid energy sources. Also, in [35], Ye *et al.* proposed a low-complexity distributed algorithm to solve the association problem jointly with resource allocation in an on-grid HetNet. They assume that users can be associated with more than one BS at the same time as a relaxation of the NP-hard problem. Unlike these proposals, in this paper the user–BS association problem is not relaxed, maintaining the constraint of one user being attended by only one BS at any time instant, and responding to the variations of renewable sources using

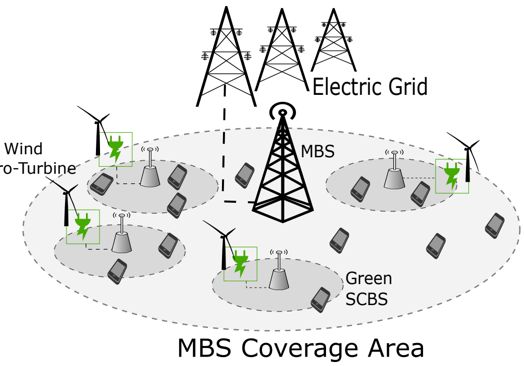


Fig. 1. Scenario: A HetNet powered by hybrid energy sources.

the grid as a backup system.

Note that game theory (GT) has also been used to solve the user–BS association problem. In [36], the authors presented a scheme based on a game of two players moving between a macro-base station and a small cell, with both BSs connected to the grid. To make the association decision, players used a distributed algorithm, trying to maximize their utilities independently. In [37], the user–BS association problem in HetNets was modelled as a non-cooperative game and solved with a distributed algorithm inspired by machine learning techniques. In [38], Khan and Tembine studied the network selection problem using coalitional games with an evolutionary perspective. They stated the need for a user-centric paradigm in fully distributed environments with the multi-objective characteristics of next-generation network systems. In [39], a coalitional planning scheme for HetNets is proposed.

III. Problem Statement

The major sources of energy consumption in a cellular network are base stations (BSs), whose consumption depends on the number of active users in a given time instant [11]. Hence, a suitable user–BS association mechanism is key to reducing on-grid consumption. However, lower grid consumption involves lower average transmission rates [40], thus rendering it necessary to propose schemes that reduce on-grid consumption while maintaining appropriate transmission rate values. Another complicating issue is the need for short decision times, even when there are many nodes. Hence, control strategies capable of responding adequately to these requirements must be developed.

A. Network Scenario

HetNets have been designed to respond to NGCN requirements and are used in this paper. Consider a two-tier downlink HetNet such as that in Figure 1, which is composed of one macro-base station (MBS) and multiple SCBSs. The MBS is always on and is powered by on-grid energy, while the SCBSs are powered exclusively by renewable energy without a battery system. The MBS provides basic coverage, while the SCBSs are deployed to enhance network capacity and receive traffic load from the MBS.

Let us define a geographical area $\mathcal{A} \subset \mathbb{R}^2$ where base stations and users are located. The set of $n_b \in \mathbb{Z}_{>0}$ base stations is denoted by $\mathcal{B} = \{1, \dots, n_b\}$, and a set of $u \in \mathbb{Z}_{>0}$ users is denoted by $\mathcal{U} = \{1, \dots, u\}$. Let $p \in \mathcal{A}$ denote a possible location, and let $b = 1 \in \mathcal{B}$ represent the MBS. Let $k \in \mathbb{Z}_{\geq 0}$ denote the discrete time with a sampling time given by $\tau \in \mathbb{R}_{>0}$ seconds, and let $N \in \mathbb{Z}_{>0}$ be a simulation horizon. Each SCBS updates its cell size every τ seconds by changing the transmission power according to the amount of renewable energy available at its location. In each time instant k, a set of $\mathcal{B}_{i,k} \subset \mathcal{B}$ base stations is available to provide service to user $i \in \mathcal{U}$.

For simplicity, the inter-BS interference will be modelled as a static value that includes the influence of other BSs present in the network, as in [25], [41]. This value varies depending on the activities in the interfering BSs, which can be coordinated via time-domain, frequency-domain, and power-control techniques [42]. Additionally, this simplification allows us to model the network as one MBS and multiple SCBSs without lacking generality.

B. Energy Model

The energy-consumption model used in this paper was proposed by Project EARTH and has been widely used in works related to energy efficiency in cellular networks [24], [25], [43]. According to Project EARTH, the energy consumption of a BS consists of two parts: the static power consumption and the dynamic power consumption [11]. The energy consumption can be expressed as

$$C_{\ell,k} = \Delta_\ell \delta_{\ell,k} T_{\ell,k} + E_\ell^S,\ \forall \ell \in \mathcal{B}, \tag{1}$$

where Δ_ℓ is the slope of load-dependent energy consumption of BS ℓ, $T_{\ell,k}$ is the transmission power of BS ℓ at the k^{th} time instant, $\delta_{\ell,k}$ is the traffic load of BS ℓ at the k^{th} time instant and E_ℓ^S is the static energy consumption of BS ℓ in each time instant. Static power consumption is related to the energy required for the normal operation of a BS, and dynamic power consumption is the additional energy demand caused by the traffic load, which is approximated by a linear function of the load.

Here, the total energy consumption of the network scenario in a given time instant is the sum of the grid consumption (due to the MBS) and the green consumption (due to the SCBSs). Hence, the reduction of consumption in BS $\ell = 1$ (MBS) is the key to increasing energy efficiency.

Regarding renewable energy, wind is considered as the source of renewable power in this work. In particular, real data is used to define a Weibull probability distribution that represents the expected wind speed at a specific location and time interval. In this case, the Weibull parameters are fitted with real data from Medellín (Colombia) [44]. This fact allows a calculation of the amount of energy that can be produced by a micro-turbine in a time period.

C. Traffic Model

According to the features of renewable energy sources and the traffic behaviour, it is possible to assume that the network operates with two timescales: a long timescale and a short timescale [45], [46]. In the long timescale, traffic changes with time (temporal variability of traffic), and decisions about network planning are taken. In the short timescale, cell-selection decisions are taken on the assumption that the operational states of the base stations are almost constant within a time instant. For this reason, the conditions of the system are assumed to be constant in each time instant to solve the game and make the corresponding decisions. In the next time instant, the state of the system is measured again and a new game is solved following the same principle. Hence, since the user–BS association problem deals with short-timescale decisions, the temporal variability of traffic over the cellular network can be ignored. Nevertheless, the spatial variability of traffic requests is considered and modelled as an inhomogeneous Poisson point process, as in [41]. The traffic size, the arrival rate per area $\lambda^p \in \mathbb{R}_{\geq 0}$, for all $p \in \mathcal{A}$, and the average traffic size $\mu^p \in \mathbb{R}_{\geq 0}$, as well as for all $p \in \mathcal{A}$, are independently distributed.

A mobile user $i \in \mathcal{U}$ at location $p \in \mathcal{A}$ associated with a BS $\ell \in \mathcal{B}_{i,k}$, has a transmission rate denoted by $r^p_{i,\ell} \in \mathbb{R}_{\geq 0}$, for all $p \in \mathcal{A}$, which can be generally expressed according to the Shannon–Hartley theorem [41] as

$$r^p_{i,\ell,k} = W_\ell \cdot \log_2(1 + \psi^p_{i,\ell,k}), \quad \forall i \in \mathcal{U}, \ell \in \mathcal{B}_{i,k}, p \in \mathcal{A}, \tag{2}$$

where $W_\ell \in \mathbb{R}_{\geq 0}$, for all $\ell \in \mathcal{B}$, is the operating bandwidth. The signal received by user $i \in \mathcal{U}$ at location $p \in \mathcal{A}$ from $\ell \in \mathcal{B}_{i,k}$ is given by the signal-to-interference-plus-noise ratio (SINR) denoted by $\psi^p_{i,\ell,k} \in \mathbb{R}_{\geq 0}$, for all $p \in \mathcal{A}$, and computed as

$$\psi^p_{i,\ell} = \frac{T_\ell g^p_\ell}{\sigma^2 + \sum_{j \in \mathcal{B} \setminus \{\ell\}} T_j g^p_j}, \ \forall j, \ell \in \mathcal{B}, \tag{3}$$

where $T_\ell \in \mathbb{R}$ denotes the transmission power, for all $\ell \in \mathcal{B}$, and g^p_ℓ is the channel gain between the ℓ^{th} BS and the user at location p. Additionally, the parameter $\sigma^2 \in \mathbb{R}$ denotes the noise power level. Note that the channel gain here reflects only the slow fading, including the path loss and the shadowing; fast fading is not considered. In (3), the denominator representing the interfering BS's transmission towards a user at location p. $\psi^p_{i,\ell}$ must be higher than a threshold denoted by $\varphi \in \mathbb{R}$ so that user $i \in \mathcal{U}$ has a useful signal. For simplicity, the location indicator p is omitted when referring to user $i \in \mathcal{U}$.

It is also assumed that the network's frequency scheduling is such that each SCBS can only serve a fixed number $z^{\max}_\ell$ of users simultaneously for all $\ell \in \mathcal{B} \setminus \{1\}$. Nevertheless, the MBS limit for the number of served users is defined by all active users at a time instant $z_{A,k}$. This assumption guarantees service availability for all time instants without renewable energy.

Hence, note that the bandwidth assigned to user $i \in \mathcal{U}$ is affected by the number of users connected to the $\ell \in \mathcal{B}_{i,k}$, as the channels available must be shared between the active users.

Assuming that mobile users are uniformly distributed in the coverage area of all BSs, the traffic load of the ℓ^{th} in the k^{th} time instant can be expressed as

$$\delta_{\ell,k} = \frac{\sum_{i \in \mathcal{U}} y_{i,\ell,k}}{U_{\ell,k}}, \ \forall \ell \in \mathcal{B}, \tag{4}$$

with

$$U_{\ell,k} = \begin{cases} z_{A,k}, & \text{if } \ell = 1, \\ z^{\max}_\ell, & \text{otherwise,} \end{cases}$$

where $y_{i,\ell}$ is the user association indicator – i.e., if user $i \in \mathcal{U}$ is associated to the $\ell \in \mathcal{B}$, then $y_{i,\ell} = 1$, and $y_{i,\ell} = 0$ otherwise. Moreover, note that $0 \leq \delta_\ell \leq 1, \forall \ell \in \mathcal{B}$.

The average transmission rate per user in the k^{th} time instant depends on ψ and the number of users connected to the serving BS [47], which allows us to express (2) as

$$\tilde{r}^p_{i,\ell,k} = \frac{W_\ell}{\sum_{i \in \mathcal{U}} y_{i,\ell,k}} \log_2(1 + \psi^p_{i,\ell,k}), \ \forall i \in \mathcal{U}, \ell \in \mathcal{B}_{i,k}. \tag{5}$$

In addition, it is assumed that, at each time instant, a user can be associated with the ℓ^{th} BS if the received signal level $\psi^p_{i,\ell,k}$ is greater than a threshold φ that indicates the minimum signal level required by a user to have service.

D. Quality-of-service Objective

As stated before, an important element in the on-grid consumption reduction problem is to maintain appropriate QoS levels according to the availability of renewable energy. In this paper, the lower QoS band is defined as a 5% reduction of the average transmission rate. This percentage is equivalent to the degradation caused by changing the QoS classes in LTE Networks [48].

E. On-grid Energy-consumption Optimization Problem

As previously mentioned, on-grid consumption reduction and adequate transmission rates are design requirements in NGCNs. From (1), notice that the BS energy consumption depends on the traffic load – i.e., the number of active users connected to a BS. For this reason, the grid-energy-consumption reduction objective will be formulated as the minimization of the number of users connected to the MBS. Hence, the optimization problem has two objectives: *(i)* to reduce the overall system grid consumption and, *(ii)* to maximize the average transmission rate per user. According to this, it is possible to formulate the following optimization problem:

$$\min_{y_{i,1,k},\ldots,y_{n,1,k}} J(y_{i,1,k}, \ldots, y_{n,1,k}) = \\ \sum_{k=1}^{N} \left\{ \gamma_1 \sum_{i \in \mathcal{U}} y_{i,1,k} - \gamma_2 \frac{W_\ell}{\sum_{i \in \mathcal{U}} y_{i,\ell,k}} \log_2(1 + \psi^p_{i,\ell,k}) \right\}, \tag{6}$$

s.t.

$$\sum_{i \in \mathcal{U}} y_{i,\ell,k} \leq z^{\max}_\ell, \ \forall \ell \in \mathcal{B} \setminus \{1\}, k \in [0, N] \cap \mathbb{Z}_{\geq 0}, \tag{7a}$$

$$y_{i,\ell,k} . \psi^p_{i,\ell,k} \geq \varphi, \qquad \forall i \in \mathcal{U}, \ell \in \mathcal{B}_{i,k}, k \in [0, N] \cap \mathbb{Z}_{\geq 0}, \tag{7b}$$

$$\sum_{\ell \in \mathcal{B}} y_{i,\ell,k} \leq 1, \qquad \forall i \in \mathcal{U}, k \in [0, N] \cap \mathbb{Z}_{\geq 0}, \tag{7c}$$

$$y_{i,\ell,k} \in \{0, 1\}, \ \forall i \in \mathcal{U}, \ell \in \mathcal{B}, k \in [0, N] \cap \mathbb{Z}_{\geq 0}, \tag{7d}$$

where (6) is the objective function, which focuses on minimizing consumption from the grid and maximizing the user's transmission rate with an optimal assignment of active users to available BSs in each time instant. Moreover $\gamma_1, \gamma_2 \in \mathbb{R}$ are weights assigned to each objective. Inequalities (7a - 7d) are the problem constraints: (7a) establishes that a small cell $\ell \in \mathcal{B}\backslash\{1\}$ can serve a maximum of $z_\ell^{\max}$ users simultaneously; (7b) is the user's received signal level constraint; (7c) requires that a user is served by only one BS in a time instant; and (7d) establishes that $y_{i,\ell}$ is a binary variable.

The optimization problem can be solved according to the specific network characteristics at each time instant k, including the variability of the renewable energy sources. Nevertheless, the optimization problem involves mixed-integer variables and requires full information about the whole status of the base stations and user to be solved. In the next section, a population-like-games approach is proposed in order to find a feasible solution for the problem associated with the minimization of energy consumption and maximization of the transmission rate in a distributed fashion.

IV. POPULATION-LIKE GAME

This section addresses the design of a population-like game by means of the appropriate incentives. To this end, the same reasoning used to state the optimization problem in (6) and (7) is followed, and the relationships between the game-theoretical approach and the optimization problem are highlighted when necessary. Two of the main characteristics of population dynamics, which can be seen as restrictive features for applying this game-theoretical approach in some specific engineering applications, are the anonymity and non-atomicity [23].

Definition 1. *(Anonymity [23]) The anonymity describes the situation in which the index of decision makers does not affect the utility function. This concept can also be associated with the homogeneousness of decision makers selecting strategies – i.e., decision makers are assumed to be indistinguishable within the same strategy.* $\diamondsuit$

Definition 2. *(Atomicity [23]) The atomicity describes the situation in which a single decision maker affects the global utility – i.e., decisions made by an individual player impact the overall performance.* $\diamondsuit$

This paper presents an alternative population-like-games approach that allows us to deal with atomicity and non-anonymity. In fact, it is assumed that each decision maker within the population is different, and consequently, each decision maker has a different utility. Therefore, each individual decision maker affects the global utility. In addition, all the decision makers selecting the same strategy are considered to be different even though they belong to the same strategy. Also, given that the population-like-games approach looks at the optimization problem (6) from a different perspective, the equivalence between elements in the population dynamics approach with elements in the optimization problem is shown in Table II.

A. Atomicity and Non-anonymity in Population-like Games

Let $\mathcal{U}$ be the set of rational decision makers in a population located throughout a bi-dimensional geographical area denoted by $\mathcal{A} \subset \mathbb{R}^2$. These agents are rational in the sense that they make decisions to improve their individual benefits. Moreover, let $\mathcal{B} = \{1, \dots, n_b\}$ denote the set of choices that the set of decision makers have. More precisely, let $\mathcal{B}_{i,k}^p \subset \mathcal{B}$ denote the possible choices that the i^{th} decision maker has at time instant k depending on its geographical position $p \in \mathcal{A}$, where $\mathcal{B}_{i,k}^p \neq \emptyset$, for all $i \in \mathcal{U}$, $k \in \mathbb{Z}_{>0}$, $p \in \mathcal{A}$. In other words, the sets $\mathcal{B}_i$, for all $i \in \mathcal{U}$ define possible interaction sets. For simplicity, the superscript p is omitted, indicating that the set of available strategies for each decision maker depends on its position – i.e., $\mathcal{B}_{i,k} = \mathcal{B}_{i,k}^p$. In addition, BSs in the set $\mathcal{B}_{i,k}^p$ are the ones satisfying the transmission threshold in (7b).

The set of decision makers selecting the strategy $\ell \in \mathcal{B}$ at time instant k is given by $\mathcal{U}_k^\ell \subseteq \mathcal{U}$. Note that the cardinality $|\mathcal{U}_k^\ell| = \sum_{i\in\mathcal{U}} y_{i,\ell,k}$ for all k and $|\mathcal{U}| = \sum_{\ell\in\mathcal{B}}\sum_{i\in\mathcal{U}} y_{i,\ell,k}$. Moreover, consider a strategic profile given by a distribution of decision makers $\mathcal{U}$ throughout the set of choices $\mathcal{B}$ – i.e., $\left(\mathcal{U}_k^1, \dots, \mathcal{U}_k^b\right)$, which represents the population state, where $\bigcap_{\ell\in\mathcal{B}} \mathcal{U}_k^\ell = \emptyset$, and $\bigcup_{\ell\in\mathcal{B}} \mathcal{U}_k^\ell = \mathcal{U}$. Also, let $g_i = \{\ell \in \mathcal{B} : i \in \mathcal{U}^\ell\}$ return the strategy that a decision maker $i \in \mathcal{U}$ choose. In addition, let the amount of decision makers be constrained at each possible choice, i.e., $|\mathcal{U}_k^\ell| \leq z_\ell^{\max}$, being $z_\ell^{\max} \in \mathbb{Z}_{>0}$, for all $\ell \in \mathcal{B}$.

Assumption 1. *The initial distribution of decision makers $\left(\mathcal{U}_0^1, \dots, \mathcal{U}_0^b\right)$ in the population is feasible – i.e., $|\mathcal{U}_0^\ell| \leq z_\ell^{\max}$, for all $\ell \in \mathcal{B}$. This implies that $n = |\mathcal{U}| \leq \sum_{\ell\in\mathcal{B}} z_\ell^{\max}$. Moreover, $\bigcap_{\ell\in\mathcal{B}} \mathcal{U}_0^\ell = \emptyset$, and $\bigcup_{\ell\in\mathcal{B}} \mathcal{U}_0^\ell = \mathcal{U}$.* $\diamondsuit$

Let $f_{i,\ell,k} \in \mathbb{R}$ be the fitness function for decision maker $i \in \mathcal{U}$ selecting strategy $\ell \in \mathcal{B}$ at time instant $k \in \mathbb{Z}_{>0}$. If two decision makers $i, u \in \mathcal{U}$ select the same strategy $\ell \in \mathcal{B}$, then $f_{i,\ell,k} \neq f_{u,\ell,k}$, since the population considers non-anonymity. Furthermore, since decision maker $i \in \mathcal{U}$ cannot select the strategies $\mathcal{B}\backslash\mathcal{B}_{i,k}$, then for simplicity it is considered that the decision maker has no incentives to move to such a strategy – i.e., $f_{i,\ell,k} = 0$, for all $\mathcal{B}\backslash\mathcal{B}_{i,k}$. The objective within the population is to achieve a local ε-equilibrium [49], as presented in Definition 3, which also provides notions about the local Nash equilibrium [50], [51].

Definition 3. *(Local ε-equilibrium). Let $\varepsilon \in \mathbb{R}_{\geq 0}$. A population distribution $\left(\mathcal{U}^{1*}, \dots, \mathcal{U}^{b*}\right)$ is a local ε-equilibrium with respect to the interaction sets $\mathcal{B}_i$ if all decision makers $i \in \mathcal{U}^{\ell*}$, for all $\ell \in \mathcal{B}$, satisfy the following condition:*

$$f_{i,\ell} \geq f_{i,\ell'} - \varepsilon, \quad \forall\, \ell, \ell' \in \left\{h \in \mathcal{B}_i : |\mathcal{U}^h| < z_h^{\max}\right\}. \tag{8}$$

On the other hand, if condition (8) holds with $\varepsilon = 0$, then $\left(\mathcal{U}^{1*}, \dots, \mathcal{U}^{b*}\right)$ is a local Nash equilibrium with respect to the interaction sets $\mathcal{B}_i$. $\diamondsuit$

The population evolves according to switching rules, which determine the timing and the result of decision makers' choices. Let $\varrho_{i,k}^{h,\ell} \in \mathbb{R}_{\geq 0}$ represent the switching rule for the i^{th} decision maker. Therefore, if $\varrho_{i,k}^{h,\ell} > 0$, then decision maker $i \in \mathcal{U}$ has incentives to move from the h^{th} strategy to the ℓ^{th} at time instant k. The evolution of the population is made by assigning a revision opportunity as described in [52]: a decision maker is chosen randomly from the population, and

TABLE II
EQUIVALENCE BETWEEN POPULATION-LIKE DYNAMICS AND THE OPTIMIZATION PROBLEM

Variable	Population-like dynamics	Optimization problem
$\mathcal{B}$	Set of possible strategies	Set of base stations
b	Number of strategies	Number of base stations
ℓ	Strategy's index	Base station's index
$\mathcal{U}$	Set of rational decision makers	Set of users
u	Number of decision makers	Number of users
i	Decision maker's index	User's index
$\mathcal{B}_{i,k}^{p}$	Set of possible strategies for decision maker i at time instant k depending on its geographical position p	Set of base stations available to provide service to user i at location p in a time instant k
$\mathcal{U}_k^\ell$	Set of decision makers selecting the strategy ℓ at time instant k	Number of users connected to BS ℓ at time instant k
$y_{i,\ell}$	Agent-strategy choice indicator for decision maker i with strategy ℓ	User-BS association indicator for user i with BS ℓ
$z_\ell^{\max}$	Maximum number of possible decision makers selecting strategy ℓ simultaneously	Number of users that can be served by a SCBS ℓ simultaneously

it receives an opportunity to decide whether or not it should *move* to another strategy by comparing its utility with those it would obtain by selecting the strategy with a higher fitness function from the set $\mathcal{B}_{i,k}$.

Assumption 2. *Suppose that decision maker $i \in \mathcal{U}$ receives a revision opportunity. Then, before its next revision opportunity, all decision makers $u \in \mathcal{U}\backslash\{i\}$ receive a revision opportunity.* ♢

With $i \in \mathcal{U}$ being the decision maker with a revision opportunity, the procedure is as follows:

$$\mathcal{U}_{k+1}^{g_i} = \mathcal{U}_k^{g_i} \backslash \left\{ i \,\mathrm{sgn}\left(\varrho_{i,k}^{g_i,\ell}\right)\right\}, \text{ for any } i \in \mathcal{U}, \tag{9a}$$

$$\mathcal{U}_{k+1}^{\ell} = \mathcal{U}_k^{\ell} \cup \left\{ i \,\mathrm{sgn}\left(\varrho_{i,k}^{g_i,\ell}\right)\right\} \backslash \{0\}, \text{ for any } \ell \in \mathcal{B}_{i,k}. \tag{9b}$$

Notice that the equilibrium in (9) is achieved when $\varrho_i^{g_i,\ell} = 0$, for all $i \in \mathcal{U}$, $\ell \in \mathcal{B}_i$ – i.e., when decision maker $i \in \mathcal{U}$ has no incentives to move to any strategy $\ell \in \mathcal{B}_{i,k}$. In such a case, $\mathcal{U}_{k+1}^{\ell*} = \mathcal{U}_k^{\ell*}$, for all $\ell \in \mathcal{B}$.

Remark 1. *Notice that in (9) each decision maker $i \in \mathcal{U}$ does not require full information from the population, but only local information from $\mathcal{B}_i$.* ♢

Now, it is necessary to define an appropriate switching rule for the population. Consider the following switching rule:

$$\varrho_{i,k}^{h,\ell} = \left(z_\ell^{\max} - |\mathcal{U}_k^\ell|\right) \max\left(0, f_{i,\ell,k} - f_{ih,k} - \varepsilon\right), \forall h, \ell \in \mathcal{B}_i, \tag{10}$$

where $\varepsilon \in \mathbb{R}_{\geq 0}$. Notice that the switching rule in (10) indicates that the i^{th} decision maker has incentives to *move* from the h^{th} to the ℓ^{th} strategy only if it represents an improvement over its utilities greater than ε and there is available capacity at the ℓ^{th} strategy. Associating the designed revision protocol with the optimization problem in (6) and (7), it can be seen that the switching rate guarantees the satisfaction of constraint (7a) through the term $(z_\ell^{\max} - |\mathcal{U}_k^\ell|)$. Hence, Proposition 1 shows that an equilibrium in dynamics (9) with the aforementioned switching rule implies a local ε-equilibrium with respect to the allowed interactions within the population.

Proposition 1. *(ε-equilibrium point) The equilibrium point of the dynamics in (9) with the switching rule in (10) implies a local ε-equilibrium with respect to the interaction sets $\mathcal{B}_i$, for all $i \in \mathcal{U}$.*

Proof. It immediately follows from the fact that the equilibrium in (9) implies that $\varrho_i^{g_i,\ell} = 0$, with $i \in \mathcal{U}^{g_i*}$, for all $i \in \mathcal{U}$, $\ell \in \mathcal{B}_i$. Therefore, $f_{i,\ell,k} \leq f_{ig_i,k} + \varepsilon$, with $i \in \mathcal{U}^{g_i*}$, for all $i \in \mathcal{U}$, $\ell \in \mathcal{B}_i$ such that $z_\ell^{\max} < |\mathcal{U}_k^\ell|$, which is the required conclusion according to Definition 3. □

In addition to obtaining a local equilibrium (Definition 3), Proposition 2 shows the satisfaction of the stated constraints involving the initial condition in Assumption 1, for all the time instants $k \in \mathbb{Z}_{>0}$.

Proposition 2. *(Satisfaction of constraints) If Assumption 1 holds, then $|\mathcal{U}_k^\ell| \leq z_\ell^{\max}$, $\bigcap_{\ell\in\mathcal{B}} \mathcal{U}_k^\ell = \emptyset$, and $\bigcup_{\ell\in\mathcal{B}} \mathcal{U}_k^\ell = \mathcal{U}$ under the dynamics in (9) for all time instants $k \in \mathbb{Z}_{>0}$.*

Proof. Regarding the first constraint, it is assumed that $|\mathcal{U}_0^\ell| \leq z_\ell^{\max}$, for all $\ell \in \mathcal{B}$. Moreover, notice that the cardinality $|\mathcal{U}_k^\ell|$ can only grow one by one, for all $\ell \in \mathcal{B}$, due to the fact that $z_\ell^{\max} \in \mathbb{Z}_{>0}$ and that $\varrho_{i,k}^{g_i,\ell} = 0$ in (9) if constraint $|\mathcal{U}_k^\ell| \leq z_\ell^{\max}$ is active. Then $|\mathcal{U}_k^\ell| \leq z_\ell^{\max}$, for all $\ell \in \mathcal{B}$ and all $k \in \mathbb{Z}_{>0}$. Regarding the second constraint, notice that $0 \notin \mathcal{U}_k^\ell$, for all $\ell \in \mathcal{B}$. It follows that if $\bigcap_{\ell\in\mathcal{B}} \mathcal{U}_k^\ell = \emptyset$, then $i \in \mathcal{U}$, $\mathcal{U}_k^{g_i} \cap \mathcal{U}_k^\ell = \emptyset$, and $\{\mathcal{U}_k^{g_i} \backslash \mathcal{T}\} \cap \{\mathcal{U}_k^\ell \cup \mathcal{T}\} = \emptyset$, for any set $\mathcal{T}$. Regarding the third constraint: $\mathcal{U}_{k+1}^{g_i} \cup \mathcal{U}_{k+1}^\ell = \left\{\mathcal{U}_k^{g_i} \backslash \left\{ i \,\mathrm{sgn}\left(\varrho_{i,k}^{g_i,\ell}\right)\right\}\right\} \cup \left\{\mathcal{U}_k^\ell \cup \left\{ i \,\mathrm{sgn}\left(\varrho_{i,k}^{g_i,\ell}\right)\right\} \backslash \{0\}\right\}$, or equivalently, $\mathcal{U}_{k+1}^{g_i} \cup \mathcal{U}_{k+1}^\ell = \mathcal{U}_k^{g_i} \cup \mathcal{U}_k^\ell$, completing the proof. □

Finally, note that the result presented in Proposition 2 guarantees that each user is served by only one base station, which corresponds to the constraint presented in (7c).

B. Population-like-game Statement

The optimization problem (6) is a multi-objective mixed integer problem (MIP), a well-known NP-hard problem [53]. However, the distributed control strategy based on population games proposed in this paper is a suitable alternative for reducing the computational burden. Reducing computational burden is possible since each user solves a limited maximization problem based only on the comparison of its current fitness function offered by the subset of BSs ($\mathcal{B}_{i,k}$).

Another key element in the proposed game-theory-based-mechanism is the possibility of designing a fitness function according to the cost function in the optimization problem. In this case, the fitness function maintains the weights γ_1 and γ_2 defined in (6) for each objective and includes an incentive to choose a BS powered by renewable energy. In other words, the incentive allows prioritization of SCBS selection even if the received signal level of an MBS is better. The fitness function is expressed as

$$f_{i,\ell,k} = \gamma_1 P_{i,\ell} + \gamma_2 \tilde{r}^p_{i,\ell,k},\ \forall \ell \in \mathcal{B}_{i,k}, \tag{11}$$

where the condition $\gamma_1+\gamma_2 = 1$ must hold, $P_{i,\ell}$ is the incentive received for user $i \in \mathcal{U}$ to choose a cell $\ell \in \mathcal{B}_i$ according to the energy source, and $\tilde{r}^p_{i,\ell,k}$ is the normalized transmission rate that can receive user $i \in \mathcal{U}$ from $\ell \in \mathcal{B}_i$ at time instant $k \in [0, N] \cap \mathbb{Z}_{\geq 0}$. As previously mentioned, on-grid energy has a higher economic and environmental impact compared to green-energy, and thus it is suitable to consider a network-operator policy focused on encouraging users to use cells powered by renewable energies. For this reason, this paper proposes a green incentive G for users such that

$$P_{i\ell} = \begin{cases} G, & \text{if } \ell = 1, \\ 2G, & \text{if } \ell \in \mathcal{B}\backslash\{1\}. \end{cases} \tag{12}$$

The definition of (12) is arbitrary and other options could be considered as well. Note that an advantage of the green incentive is the possibility for the network operator to modify the relation between the energy sources according to the desired priorities. Given the energy-efficiency and grid-consumption-reduction approaches considered in this paper, it was defined as having a double priority in BSs powered by renewable energies.

It is important to note that the green incentive does not have a physical meaning itself, as it is a tuning parameter that allow operators to design policies focusing on prioritizing the green-energy use. It is possible to interpret G as the maximum extent of the trade-off between a user's normalized transmission rate and the use of green energy when equal weights are applied to the two objectives in (11) – i.e., when $\gamma_1 = \gamma_2$. In this way, the value assigned to G is directly related to the importance that the network operator assigns to green energies.

In the stated case, different values for G were defined arbitrarily with the aim of analysing their impact on energy consumption. However, in real applications, the G weight must be defined by the operator according to its management policies. Finally, it must be mentioned that the green-incentive definition is related to future work focused on analysing the economic impact of integrating renewable energies in HetNets.

C. Relationship between Optimization and the Game

As has been stated, the original optimization problem (6) with constraints (7) presents serious difficulties to be solved in real time. For this reason, the present game-theoretical approach is proposed. In this subsection, we show how the original optimization and the game are connected.

First, it must be noted that the solution provided by the proposed Algorithm 1 is feasible for the original optimization problem. In particular, constraint (7a) is satisfied by the term $(z^{\max}_\ell - |\mathcal{U}^\ell_k|)$ in the switching rule; constraint (7b) is fulfilled because the base stations in set $\mathcal{B}^p_{i,k}$ are the only ones satisfying the requested transmission threshold; constraint (7c) is satisfied by Proposition 2; and, finally, (7d) is a constraint to specify the binary nature of certain optimization variables in the original problem, which are directly satisfied by Algorithm 1.

Once it has been established that the solution of the game theoretical approach is a feasible solution of the original optimization problem, it is necessary to analyse the optimality of the solution. As can be seen, the function optimized in (6) is centralized, whereas the population-like-game approach uses a switching rule based on local information. In addition, the optimization problem uses a time horizon of length N so it can exploit the centralized decision-making capabilities in a proactive manner. From this viewpoint, it becomes clear that the game-theoretical approach provides a suboptimal solution of problem (6). Nevertheless, the fitness function and the corresponding switching rule overcome some of the aforementioned limitations by promoting similar goals to those of (6). In particular, notice that the utility function in (11) takes into account the same two objectives corresponding to the cost function in (6). On one hand, the first term in (6) aims to minimize the number of agents served by the MBS, while the first incentive in (11) assigns more priority to avoiding connections to the MBS. On the other hand, the second term in (6) intends to improve the transmission rate for the users, which is the same intention of the second term in the utility function in (11). Thus, the relationship between the objectives in the optimization problem and the incentives in the game approach are related to each other.

All things considered, the proposed population-like game approach can be considered a heuristical method that provides a suboptimal solution with feasibility guarantees for the original problem (6) with constraints (7). Nevertheless, and as will be seen in the simulation section, the degree of suboptimality of the proposed approach is small, and much can be gained in terms of computation speed by using Algorithm 1.

V. Assessed Methods

In this section, three other techniques are presented and compared with the proposed game-theory-based scheme. The first one is the standard best-signal-level mechanism. The second is based on traditional discrete optimization techniques. Finally, a greedy algorithm is used.

A. Game-theory Scheme

The energy–efficiency problem is studied using a distributed population-like dynamic approach with atomicity and non-anonymity characteristics. Hence, in each time instant $k \in [0, N]$ a user $i \in \mathcal{U}$ with a revision opportunity evaluates the fitness function $f_{i,\ell}$ among available choices $\mathcal{B}^p_{i,k} \subset \mathcal{B}$ and selects the destination BS according to the switching rule $\varrho^{h,\ell}_{i,k}$. The distributed user–BS association mechanism is implemented according to Algorithm 1 and Algorithm 2.

Algorithm 1 Distributed population-like dynamic for user–BS association

```
/*Initial association of decision makers to strategy 1 (MBS)*/
for i = 1 to u do
    y_{i,1} = 1
    for h = 2 to b do
        y_{i,h} = 0
    end for
end for
/*Evaluation of switching rule from strategy h to strategy ℓ for user i according to the best fitness function in his neighbourhood (B_{i,k})*/
k = 0
while 1 do
    k = k + 1
    Obtain U_k from the current total users in the network
    while U_k ≠ ∅ do
        i=rand (U_k), U_k = U_k \ {i}
        (ℓ, f_{i,ℓ})=bestneighbour(i)
        Compute ϱ^{g_i,ℓ}_{i,k} according to (10)
        if ϱ^{g_i,ℓ}_{i,k} > 0 then
            y_{i,g_i} = 0, y_{i,ℓ} = 1
        end if
    end while
end while
```

Algorithm 2 Function: *bestneighbour*

```
/*Calculation of the highest utility function in the neighbourhood of user i (B_{i,k})*/
Require: i
fitness = 0, bestneighbour = 0
for ℓ = 1 to b do
    if ψ_{i,ℓ} ≥ φ and bestneighbour < f_{i,ℓ} then
        bestneighbour = ℓ
        fitness = f_{i,ℓ}
    end if
end for
return bestneighbour, fitness
```

B. Best-signal-level Policy (SLP)

In traditional cellular networks, mobile users connect to the BS that offers the best SINR, which depends on BS power transmission, path loss, and interference from other BSs. However, this mechanism is not entirely adequate for HetNets since SCBSs with available resources can be ignored by users when receiving a stronger signal from an MBS [47]. This procedure will be referred to as the *traditional scheme*, and it will be the baseline for evaluating the performance of the proposed game-theory-based mechanism.

C. Direct Optimization (DO)

The optimal connection policy is attained through solving (6) by means of a mixed-integer linear optimization problem (MILP). To find a solution, a constraint and a discrete optimizer based on a branch-and-bound method is used [54].

D. Greedy Algorithm (GA)

The third user–BS association method considered is a greedy algorithm based on the best signal level received by a user and the energy source of each BS [27]. This algorithm allows ranking of BSs according to the best function objective value perceived by each user. The greedy mechanism works as follows: in a first round, a user assesses all potential BSs and ranks them according to the objective function value obtained. This process is repeated for all users. Once all BSs are assessed by all users, the association process is made according to the best values until reaching the BS capacity. The process continues until assigning all users to a BS.

VI. Case Study

The scenario described in Section III was implemented to evaluate the proposed mechanism. The case study considered is composed of one MBS and 36 overlapping SCBSs. The MBS is powered by on-grid energy, which is always on, ensuring constant coverage over the area. Only large-scale path loss between users and BSs is considered in the simulation – i.e., the signal level received decreases with distance. Another type of path loss such as small-scale fading was not considered, since its duration is shorter than the duration of the user–BS association process. The file transfer requests follow an homogeneous Poisson point process where $\lambda^p = \lambda$ for the sake of simplicity.

This case study presents different simplifications but is complex enough to show the potential of the method proposed in the paper. Despite some simplifications made in the case study, it maintains generality and is representative of a real scenario where the complexity of the association process is caused in part by the number of BSs and active users.

From a telecommunications viewpoint, technical parameters of the simulation are defined according to a Long-Term-Evolution system in a coverage area of 3.5 km^2 [55]. The distance between BSs is 500 m and users are uniformly distributed across the coverage area. Table III summarizes the parameters used in the simulation.

A. Simulation Scenarios

To evaluate the performance of the proposed game-theory-based mechanism, three scenarios were used:

1 Static scenario. In this scenario, users are not moving and a constant wind it is assumed to have all BSs active during the simulation horizon.

2 Dynamic scenario with controlled wind. This scenario uses a controlled wind profile to enable different groups

TABLE III
SIMULATION PARAMETERS

Parameter	Value	Units
Coverage Area	3.5	km^2
System	LTE	-
BW LTE	20	MHz
RB per BS	100	-
N. Macro-base-stations	1	-
N. MBS Sectors	1	-
N. SCBSs	36	-
Inter-site distance	500	m
Tx power MBS	43	dBm
Tx power SCBS	22	dBm
Static Power Cons. MBS	130	W
Static Power Cons. SCBS	6.8	W
Consumption Slope MBS	4.7	-
Consumption Slope SCBS	4.0	-
Path Loss between MBS and User	Cost 231 model	-
Antenna Gain	15	dBi
Max. Users Simultaneously for an SCBS	100	-
Receiver Sensitivity	-107.5	dBm
Size of Request File	500	KB
Time-instant Length	1	s
Mobility Model	Random walk point	-
Mobility Speed	4	km/h
γ_1	0.6	-
γ_2	0.4	-
G	0.5	-

of SCBSs during specific time periods. The number of active BSs changes according to a pre-defined sequence, and users are moving.

3 Fully dynamic scenario. In this scenario, the number of active BSs is defined by the stochastic wind behaviour. Also, users are moving and data transmission requests are variable. This configuration uses a variable wind profile fitted from real data. In particular, the simulation considers the behaviour of the wind in Medellín, based on 3 years of data provided by weather stations of SIATA [44]. Using @Risk7, it was possible to define three sectors with different wind behaviours in the geographic area. Sector 1 presents a mean wind speed of 1.787 m/s, Sector 2 has a mean wind speed of 1.880 m/s, and Sector 3 has a mean wind speed of 2.238 m/s.

Scenario 1 is used to evaluate the stability of the game. In Scenarios 2 and 3, the game-theory-based scheme is compared to the traditional best-signal-level mechanism and evaluated by using key performance indicators (KPI) related to grid consumption, average transmission rate per user, and average utility per user.

It is important to note that the spatial variability of traffic allows variations in the number of active users along the simulation time – in other words, the average arrival rate per area λ^p is the same along the simulation time, but the number of active users changes in each time instant.

B. Renewable Power Potential

As mentioned previously, the renewable source selected to power the SCBSs is wind. According to the average wind speed, a micro-turbine was selected for the SCBSs with a start-up wind speed of 2 m/s and power potential of 26 watts.

The simulation is configured with different average wind speeds in the sectors under the coverage area of the MBS. Wind dynamics vary every minute, and therefore there are three possible green-energy scenarios: (1) no SCBS has sufficient green energy, (2) the SCBSs of only one sector have green energy to operate, and (3) the SCBSs of two or more sectors have green energy (this case could even include all SCBSs having energy in the same period).

VII. RESULTS AND DISCUSSION

Using MATLAB®, it was possible to evaluate the proposed schemes and their impact on grid-power consumption and users' transmission rates. As stated previously, the dynamics of the user–BS association problem are made faster than the temporal variability of traffic on the cellular network, which is considered as constant. Nevertheless, the wind speed changes every minute, and there is spatial variability in the number of users due to changes in the arrival of active users and cell selection. For this reason, a simulation horizon of nine minutes (540 time instants) is sufficient to assess the behaviour of the proposed approach.

A. Analysis of Scenarios

Figure 2 presents the behaviour of the proposed mechanism in the static scenario with 1000 users. This scenario is configured with all BSs active during the simulation horizon – Figure 4(a) – and has a limit of 100 active users connected simultaneously at each BS. The initial condition ($k = 0$) to start the process is that users are associated to the BS with the bets signal level.

In Figure 2(a), it is possible to observe that users perceive greater utility from SCBSs, represented by grey lines, compared to the MBS, represented by the black line. This result is in accordance with the proposed utility function, whereby the decision process contains an incentive to use green energy. Another element to note is the stability of average utility per BS after 300 time instants – the time in which the equilibrium between the energy source of a BS, the number of users connected, and the transmission rate that BS can offer reach the steady state in each cell and users have no incentive to deviate from their decision.

In Figure 2(b), it is possible to observe the tendency of the number of users connected to each BS. The solid black line represents the MBS, the pointed line is the number of users not served, and the grey lines are users connected to small cells. The first element to note is the accomplishment of the base station capacity constraint, where all values are less than 100. Also, it is possible to observe an initial concentration of users of the MBS according to the initial condition of selecting the BS with the best signal level. Once the process begins, the distribution of users is modified to reach an equilibrium, as can be seen after 300 time instants. In the same way, the number of users that are not served is high in the initial time instants, which is caused by the limit on the BSs' capacity, but the game balances the distribution of users until a null value.

Regarding the performance of the proposed mechanism, in Figure 4(d) it is possible to observe a gradual reduction of grid consumption along the simulation time. This behaviour is in line with the first element of the objective function presented in (6), and it is explained because, over time, users find more utility in an SCBS compared to the MBS, changing their associated decision to a cell powered by renewable energy that offers a similar transmission rate to the current BS. In the same way, Figure 4(g) shows a gradual maximization of average user rate until a stable point.

The average utility per user behaviour can be observed in Figure 4(j), with a continuous increase in the first 100 time instants, followed by a stabilization period until 300 instants. Once a steady state is reached, utility remains constant because the users have no incentive to deviate from their BS selection. This behaviour is explained because in the first time instants users easily find base stations with better utility and deviate their initial cell selection, but over time a smaller number of users have incentives to change their current decision until reaching the point where no user has the incentive to select a base station other than the one to which they are already connected.

The results obtained in the static scenario allow us to observe the utility function's maximization and the steady state of the game, as well as key elements in game theory, making it possible to extend its application to scenarios with dynamic features. Additionally, the use of a constant wind profile allows us to observe the efficiency of the proposed method with stable energy conditions. According to this, it is possible to affirm that the proposed approach is easily applicable in another hybrid energy configurations such as those equipped with a battery-based system. The dynamic scenarios are configured with 1000 users, different wind profiles, and and MBS without a limit to serve users simultaneously. In the scenario with the controlled wind, the number of active base stations changes in defined time intervals, as can be seen in Figure 4(b). The fully dynamic scenario uses a wind profile fitted from real data of Medellín city to reflect the stochastic behaviour of the wind – as in Figure 4(c).

To analyse the impact and behaviour of the proposed approach in an environment with changing characteristics of renewable sources, Figure 3 shows the user–BS association process in the scenario with the controlled wind. Initially, users are connected to the MBS because it is the only one active. In time instant 61 – Figure 3(a) – a sector of small cells is activated and some users change their BS according to the revision protocol. It is important to note that the revision opportunity is a probabilistic process, hence not all users make a decision at the same time. In Figure 3(b), it is possible to observe the game evolution at the end of this wind behaviour ($k = 120$), in which the largest number of users in the area with active SCBSs are distributed over green cells. It is important to note that some users continue to connect to the MBS despite being in a location with coverage from green cells. This result is the consequence of finding a better utility in the MBS due to the cells overloading, or because these users are located on the edges of small cells where the transmission rate is better from the MBS. In this case, the second element of the objective function presented in (6) has a dominant role in the utility function.

Figure 3(c) presents the game evolution at $k = 360$. Here, it is observed that the largest number of users with the possibility of accessing a green cell is associated with one of these. Finally, Figure 3(d) shows the last time instant in the period with all SCBSs active. It is possible to observe a uniform distribution of users over green cells and the accomplishment of the objective of discharging traffic load from the MBS to cells powered by renewable energies. Regarding the users connected to the MBS, besides the evaluation of the utility function mentioned previously, it is important to remember that the process of user generation is dynamic and, for this reason, at each time interval there will be users who have not started the game. These new active users are connected to the base station with the best signal level, in this case the MBS.

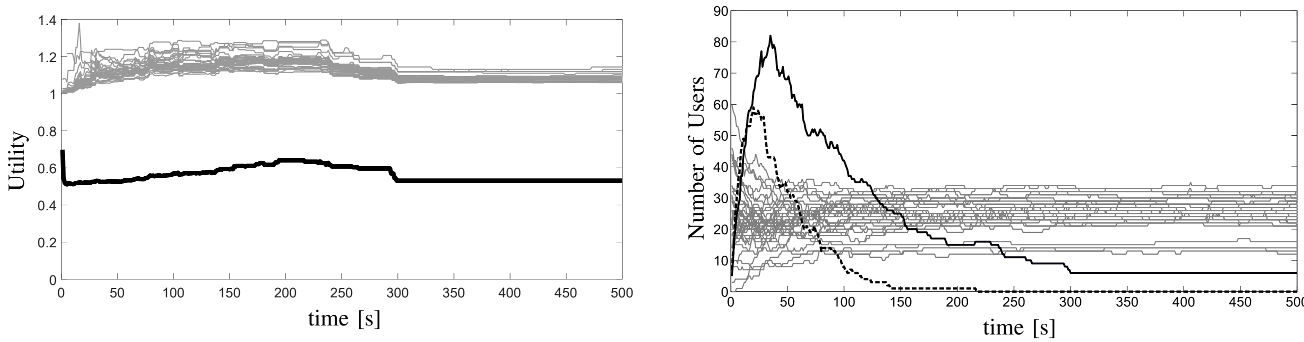
Regarding grid consumption, in the scenario with the controlled wind it is possible to observe the proper response of the proposed mechanism in the presence of renewable energy, minimizing grid consumption to levels near to static consumption (130 watts) when all SCBSs are active – Figure 4(e). This result is in accordance with the behaviour observed in the user–BS association process presented in the previous sections. Here, it is possible to observe that reducing the number of users connected to the MBS has a positive impact on the energy efficiency of the network. The reduction of grid utilization is maintained in the fully dynamic scenario, as can be seen in Figure 4(f). It is possible to observe during all simulation horizons a lower grid consumption with the game-theory-based scheme compared to the traditional mechanism. The consumption difference between both schemes is more noteworthy in the presence of green cells.

Regarding the average transmission rate, Figures 4(h-i) show that it is lower with the game-theory-based mechanism compared to the traditional scheme. This is caused by the relation between $r^p_{i,\ell}$ and ψ^p_ℓ (3). Therefore, if the best signal level is not the main criterion for selecting a base station, a reduction in the average rate can be achieved. However, despite the reduction in the average transmission rate, Figures 4(h-i) allow us to observe that degradation is not sufficient to consider it a critical problem. It is important to remember that the transmission power of an SCBS is lower than the MBS, and this has an important impact on the perceived rate by users. This situation can be countered with accurate scheduling methods to assign more bandwidth to users connected to small cells, but this problem is beyond the scope of this paper.

As was expected, it is possible to observe that the average utility received by users has a high sensitivity to wind variations, especially in the fully-dynamic scenario – Figure 4(l) – compared to the wind-controlled scenario – Figure 4(k)– where changes in wind speed are less drastic and values between both schemes are closer. Here, it is important to note the suitable utilization of renewable energies by the designed mechanism compared to the traditional scheme. In this way, it is very clear how the system utility increases when the number of active green base stations grows.

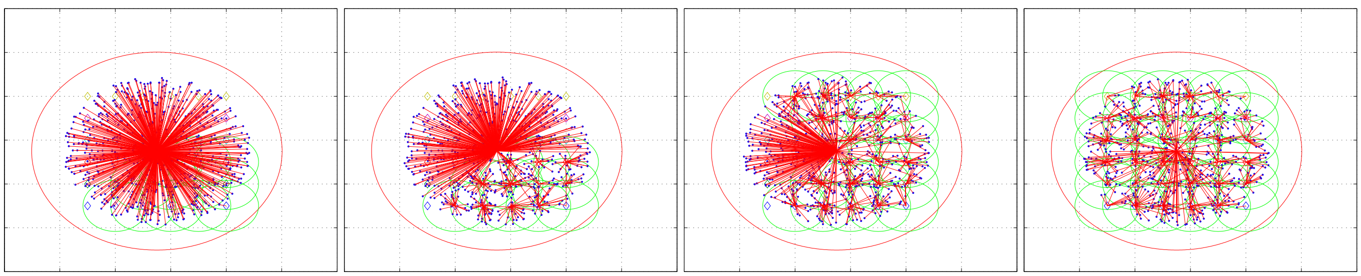
B. Impact of Parameters on Network Performance

This section presents the sensitivity of green consumption and the average user rate with respect to γ_1, γ_2, and G.



(a) (b)

Fig. 2. (a) Average utility by BS. Grey lines are the SCBS utility and the black line is the MBS. (b) Number of users connected to a BS. Grey lines are SCBSs, the solid black line is the MBS, and the dashed black line is the number of users without service.



(a) (b) (c) (d)

Fig. 3. User–BS association process with the proposed game-theory-based scheme. State of the network at different time instants (a) $k = 61$, (b) $k = 120$, (c) $k = 360$, and (d) $k = 480$.

The analysis was performed using the dynamic scenario with controlled wind.

First, it is important to note that, regardless of the value of G, the game-theory-based mechanism reduces on-grid consumption when compared to the traditional scheme, with the biggest reduction attained for $G = 4$.

Table IV presents the simulated network performance for each value of G according to γ_1 and γ_2 variations. It is possible to observe that the highest reduction in grid consumption is obtained when G=4 and $\gamma_1 = \gamma_2$. Likewise, this combination presents a lower reduction in the average transmission rate compared to the traditional scheme. Another remarkable combination is G=2, γ_1=0.9, and γ_2=0.1, which reduces grid consumption by up to 8.14% and keeps the transmission rate over the defined lower bound.

TABLE IV
IMPACT OF γ_1, γ_2, AND G VARIATIONS.

G	γ_1	γ_2	% of Grid Consumption Reduction	% of Transmission Rate Reduction
1	0	1	4.98	7.28
	0.1	**0.9**	**5.81**	**4.01**
	0.3	0.7	5.17	5.61
	0.5	0.5	5.24	5.24
	0.7	0.3	1.05	12.61
	0.9	0.1	3.32	12.48
	1	0	1.47	12.56
2	0	1	5.56	4.19
	0.1	0.9	6.10	9.42
	0.3	0.7	6.32	5.41
	0.5	0.5	6.94	7.41
	0.7	0.3	6.61	9.21
	0.9	**0.1**	**8.14**	**4.47**
	1	0	7.44	8.71
4	0	1	6.46	4.13
	0.1	0.9	7.23	7.08
	0.3	0.7	7.92	5.10
	0.5	**0.5**	**8.41**	**4.85**
	0.7	0.3	8.14	6.24
	0.9	0.1	7.75	9.19
	1	0	7.97	8.08

C. Computational Time Performance

As mentioned previously, the solution of the optimization problem formulated in (6) leads to a combinatorial explosion. Hence, there is a need to develop approximate methods such as those presented in Section IV. A way to evaluate the computational efficiency of the proposed mechanisms is to analyse the computational time spent to find a solution. For this reason, the scenario with controlled wind was used to evaluate, under equal conditions, the computational efficiency from all mechanisms.

Regarding the computational time required for the simulations, Table V shows a comparison of the results when implementing all the considered mechanisms with different numbers of users in the network. It can be observed that the optimizer increases its computational time significantly when the number of users grows. Another interesting result is the processing time of the proposed game-theory-based mechanism, which is lower than the discrete optimizer and remains practically constant despite the growth of users. Thus, it represents a suitable option for improving consumption and maintaining QoS levels in scenarios with a large number of users.

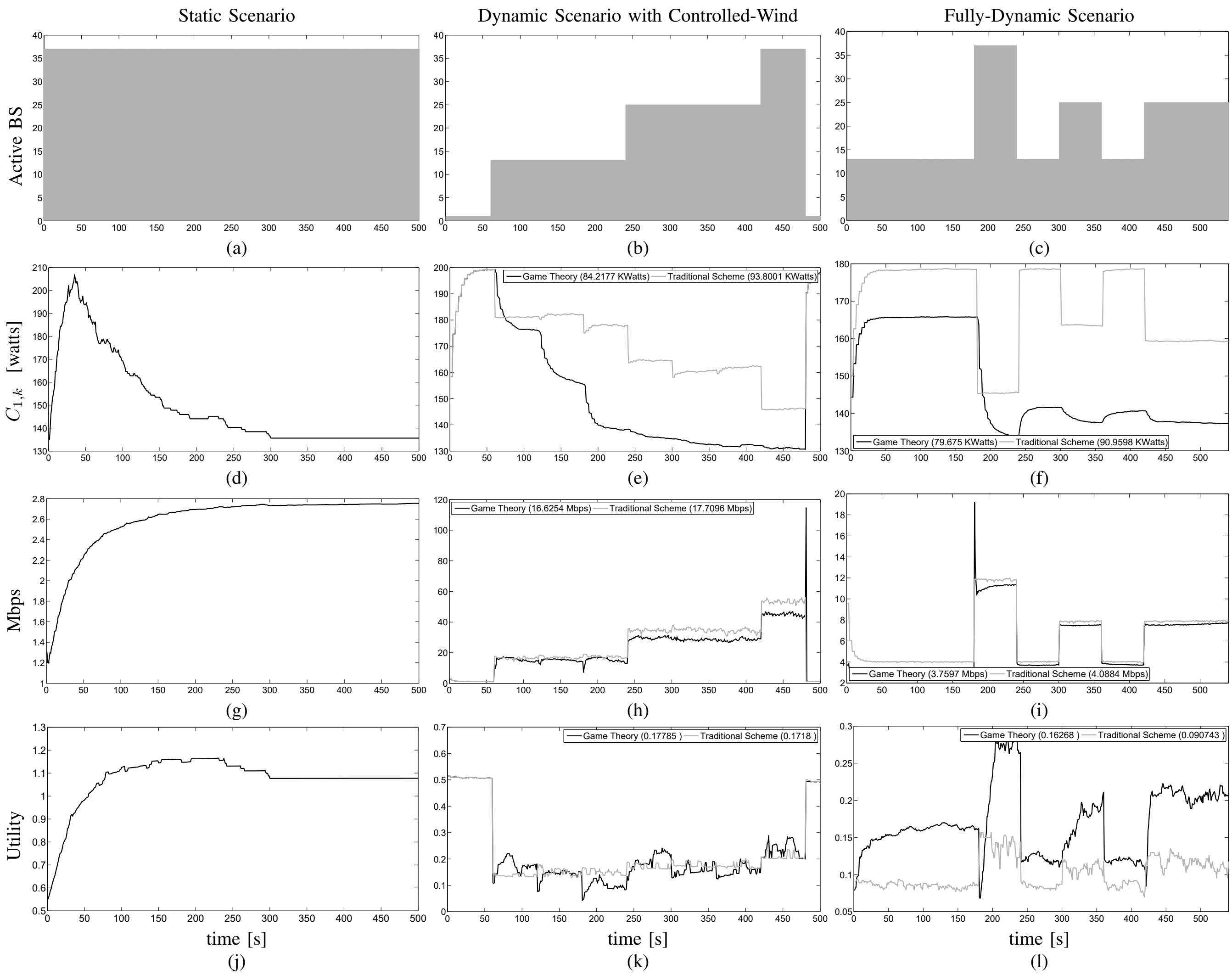


Fig. 4. Behaviour comparison of proposed scheme in different scenarios.

TABLE V
COMPUTATION TIME FOR THE SIMULATIONS [S]

Association Scheme	500 Users	750 Users	1000 Users
SLP	141.94 ±10	211.53 ±10	236.07 ±12
GA	321.43 ±11	436.89 ±12	445.09 ±12
DO	1740.46 ±14	4474.6 ±22	7434.1 ±29
G.T Scheme	202,95 ±12	293.17 ±14	374.74 ±16

D. Comparison with Other User–BS Association Mechanisms

To compare the performance of the proposed game-theory-based mechanism with the schemes presented in Section IV, six KPIs are proposed: grid consumption (kWatts-h), percentage of consumption reduction in comparison to the traditional scheme, average transmission rate per user (Mbps), percentage of variation in the transmission rate, transmitted bits per grid consumption (kbits/Watts), and variation in the kbits/Watts ratio. The fully-dynamic scenario with 1.000 users was used for comparison purposes. Table VI shows the KPI results for each scheme.

As can be seen in Table VI, the highest percentage of grid consumption reduction compared with the traditional scheme is achieved with the discrete optimizer (17.54%), followed by the game-theory-based mechanism (17.13%), and the greedy mechanism (12.02%). This result is quite relevant since the discrete optimizer delivers the optimal consumption of the system and the proposed game-theory-based mechanism achieves similar values with better computational time.

Regarding average transmission rates, all mechanisms have reductions close to 5% when compared to the traditional scheme. The user-rate results are caused by the relation between the signal level and the user rate presented in (2), with this being an expected result.

Considering the grid consumption and the average transmission rate, it is possible to introduce another energy-efficiency-related KPI as the ratio between the transmitted bits and grid consumption. This KPI represents the amount of grid energy required to transmit a kbit of information. Table VI shows that the proposed game-theory-based mechanism has a better performance, with it being possible to improve the kbit/Watts ratio by up to 14.7% compared to the traditional best-signal-level mechanism. The best relationship is achieved with the

TABLE VI
SCHEMES COMPARISON IN FULLY-DYNAMIC SCENARIO

Association scheme	Grid consumption (kWatts-h)	% reduction	Average transmission rate (Mbps)	% reduction	Transmitted bits per grid consumed (kbits/Watts)	% increase
SLP	391.2	-	2.43	-	22.89	-
GA	345.6	12.02	2.32	4.5	24.75	8.07
DO	322.56	17.54	2.32	4.5	26.51	15.78
GT scheme	324.16	17.13	2.31	4.9	26.27	14.72

discrete optimizer, but it is not a viable option due to its high demand of computation time. It is important to emphasize the suitable response of the transmitted bits per grid consumption KPI since this is an objective measure of the energy efficiency of the system to compare the mechanisms.

VIII. CONCLUSIONS

The goal of this paper was to study a distributed game-theory-based mechanism to control the user–BS association process in a HetNet powered by renewable energy, reducing grid consumption and improving energy efficiency. The proposed mechanism is based on a population-like game with characteristics of atomicity and non-anonymity, elements not considered previously in proposals based on this methodology. Three scenarios with different wind behaviour were considered to test the performance of the proposed mechanism and to compare it with the traditional best signal level policy.

The distributed population-like dynamics mechanism has been shown to be a suitable option for balancing traffic in dense HetNets and reducing grid consumption through traffic discharge from an MBS to green SCBSs. Another important characteristic observed is the possibility to reduce the users' search space to a subset of strategies, which facilitates solving the integer-association problem, being a proper option for controlling systems with a large amount of users, as expected in next-generation cellular networks. In this sense, based on the results, it is possible to conclude that the proposed game-theory-based approach improves the energy efficiency of HetNets powered by hybrid energy sources in real scenarios with similar characteristics to those presented in this paper.

Also, it is important to emphasize that the proposed distributed game-theory-based mechanism can be used to attain other goals related to the performance of the network through the modification of the utility function.

Regarding atomicity and non-anonymity, it was possible to demonstrate under a population-like dynamic approach that one agent's decision can influence the global utility of the system, as can be observed in the reduction of grid consumption when users are transferred from a macro-base to small cells.

The case study has simplifications but is representative of a real scenario with an appropriate level of complexity due to the absence of batteries and the number of users and BSs considered in the simulations. Even when the results obtained are approximate due to the simplicity of the models used, they are an indication of the potential of the proposed game-theory-based mechanism.

The next stage in the study of alternatives for improving the energy consumption of these types of network should include the scheduling of resource-block assignation from SCBSs to users to improve the transmission rate. In the same way, it is important to note that, to reach minimum grid consumption levels, it is necessary to guarantee a continuous provisioning of green energy, which can be achieved by means of storage systems or by using more stable renewable sources.

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